A Few Thoughts on Box Corrections in Nuclei

J. Engel

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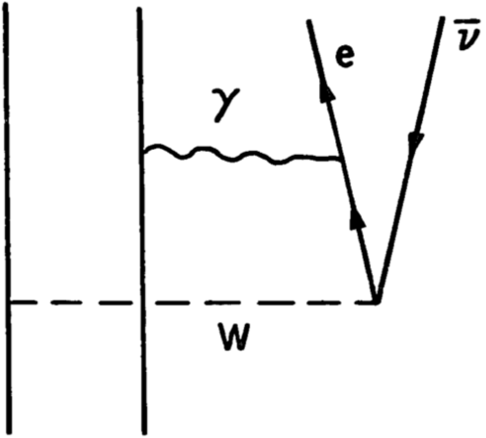
Similarity of Two-Body β Correction to 0νββ Graph



Radiative correction to β decay

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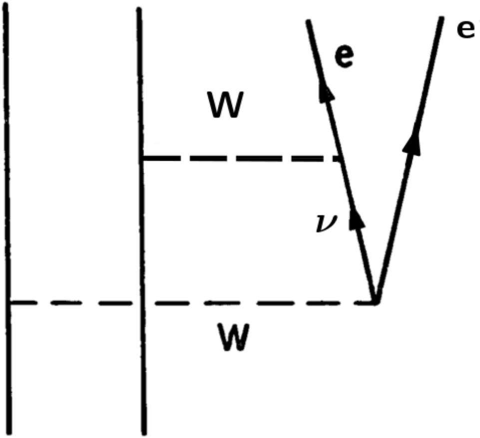

Neutrinoless ββ decay

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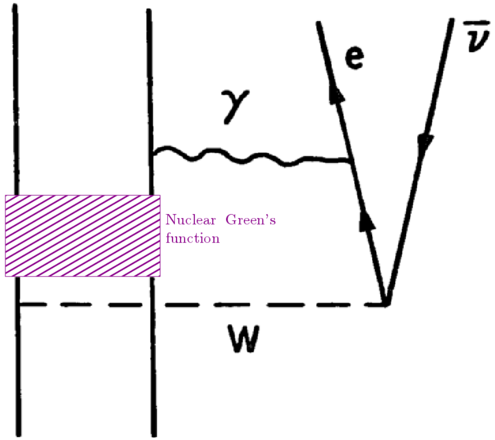
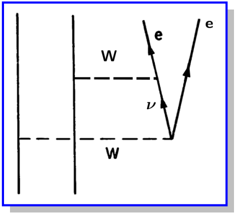


Omitting the nuclear Green's function is the "closure approximation."

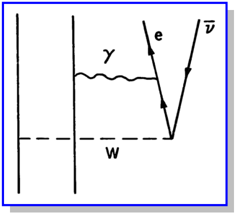
Closure Approximation Always Used

In $0\nu\beta\beta$ decay, closure thought to be OK because characteristic nuclear excitation energy is much less than characteristic neutrino momentum.

But is it really? Models try to integrate out short-range nuclear physics. Hasn't yet been done consistently.



For radiative corrections to β decay, the photon propagator adds another $1/q^2$, emphasizes low virtual-electron momenta, could make low-energy nuclear states more important.



Other Approximations in β Box

- Forbidden corrections thought to be of order 30% in $0\nu\beta\beta$ decay because of high virtual-momentum transfer, neglected in β box.
- Many-body currents under investigation in $0\nu\beta\beta$ decay, neglected in β box.
- $\vdots$

Nuclear-Structure Methods for This and Related Problems

The Field in One Slide

- **Density Functional Theory & Related Techniques:** Mean-field-like theory plus relatively simple corrections in very large single-particle space with phenomenological interaction.
- **Shell Model:** Partly phenomenological interaction in a small single-particle space – a few orbitals near nuclear Fermi surfac but with arbitrarily complex correlations.
- **Ab Initio Calculations:** Start from a well justified two-nucleon three-nucleon Hamiltonian, then solve full many-body Schröding equation to good accuracy in space large enough to include all important correlations. At present, works pretty well for energies in systems up to $A \approx 50$.

New!
(sort of)

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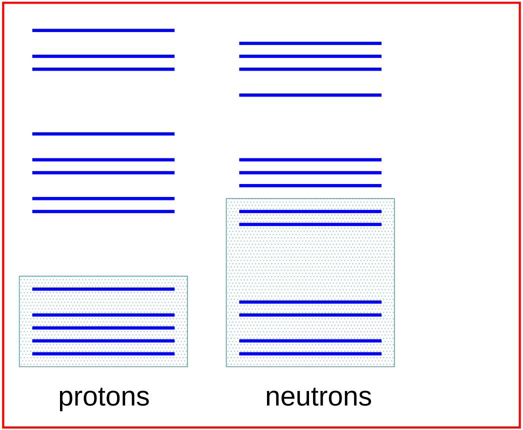
Has potential to combine and ground virtues of shell model and density functional theory.

Traditional Shell Model

Starting point: set of single-particle orbitals in an average potential.



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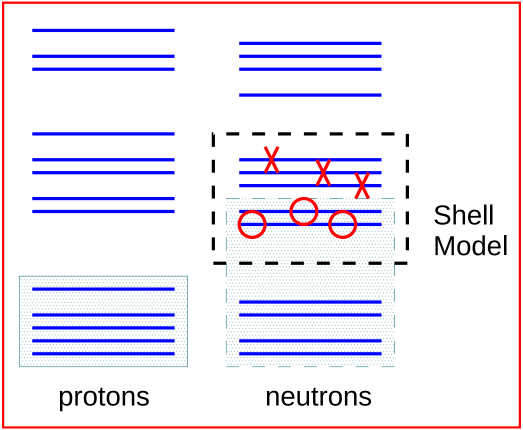
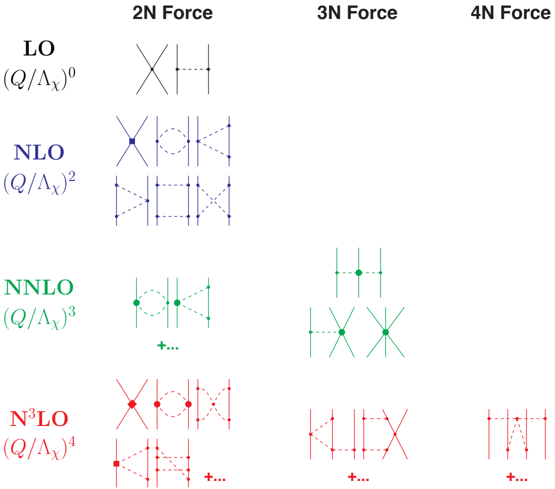


Shell model in heavy nuclei neglects all but a few orbitals around the Fermi surface, uses phenomenological Hamiltonian.

Ab Initio Nuclear Structure in Heavy Nuclei

Typically starts with chiral effective field theory; degrees of freedom are nucleons and pions below the chiral-symmetry breaking scale.



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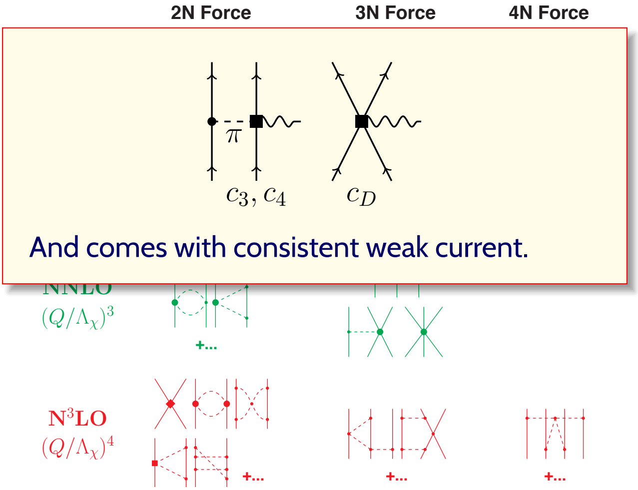
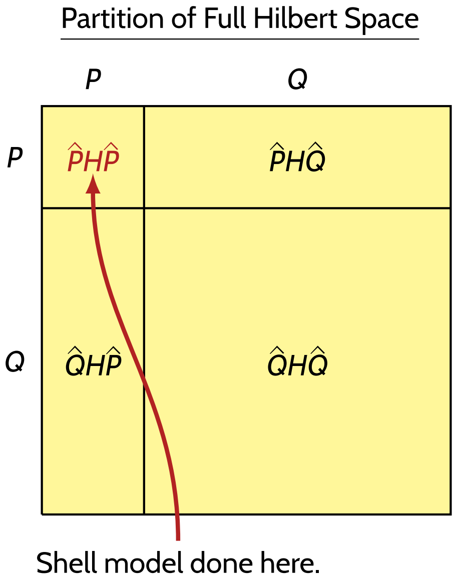


Ab Initio Shell Model

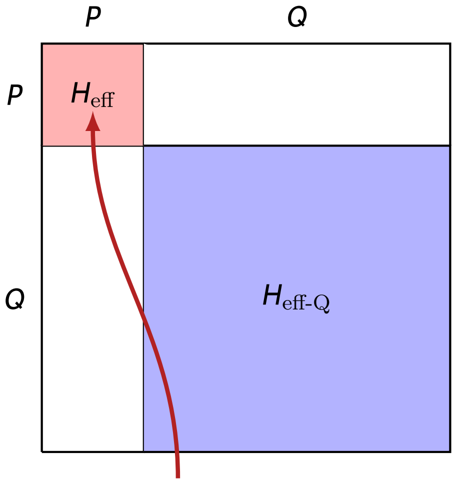


P = valence space
Q = the rest

Task: Find unitary transformation to make H block-diagonal in P and Q, with $H_{\rm eff}$ in P reproducing most important eigenvalues.

Ab Initio Shell Model

<u>Partition of Full Hilbert Space</u>



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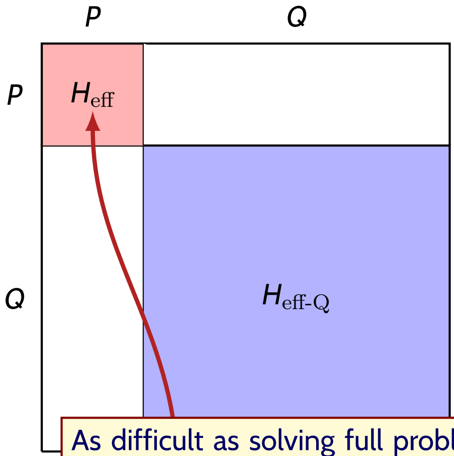

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As difficult as solving full problem. But idea is that N-body effective operators should not be important for $N > 2$ or 3.

Ab Initio Shell Model in Heavier Nuclei

Method 1: Coupled-Cluster Theory

Ground state in closed-shell nucleus:

$$|\Psi_0\rangle = e^T |\varphi_0\rangle \qquad T = \sum_{i,m} t_i^m a_m^\dagger a_i + \sum_{ij,mn} \frac{1}{4} t_{ij}^{mn} a_m^\dagger a_n^\dagger a_i a_j + \ldots$$

Slater determinant

$m,n>F \quad i,j<F$

States in closed-shell + a few constructed in similar way.

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Construction of Unitary Transformation to Shell Model:

1. Complete calculation of low-lying states in nuclei with 1, 2, and 3 nucleons outside closed shell (where calculations are feasible).
2. Lee-Suzuki mapping of lowest eigenstates onto shell-model space, determine effective Hamiltonian and transition operator.

   Lee-Suzuki maps lowest eigenvectors to orthogonal vectors in shell model space in way that minimizes difference between mapped and original vectors.
3. Use these operators in shell-model for nucleus in question.

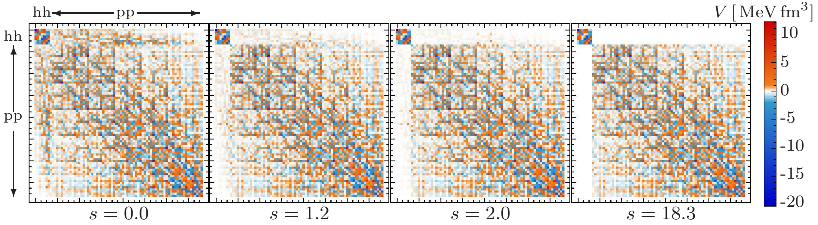
Ab Initio Shell Model in Heavier Nuclei

Method 2: In-Medium Similarity Renormalization Group

Flow equation for effective Hamiltonian.
Shell-model space asymptotically decoupled.

$$\frac{d}{ds}H(s) = [\eta(s), H(s)]\,, \qquad \eta(s) = [H_d(s), H_{od}(s)]\,, \quad H(\infty) = H_{\text{eff}}$$

d = diagonal $\qquad$ od = off diagonal



Hergert et al.

Development about as far along as coupled clusters.

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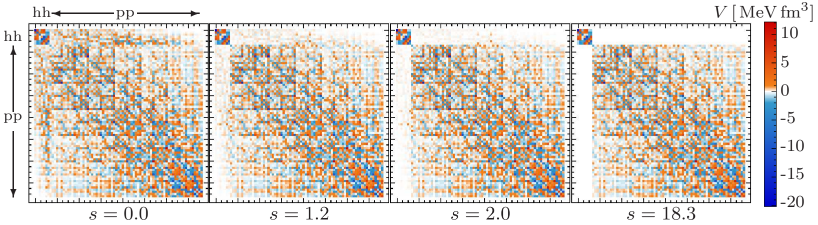

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Thanks for listening

…and thanks to Michael for the invitation.