Baryon χPT for Two-Boson Exchange Graph

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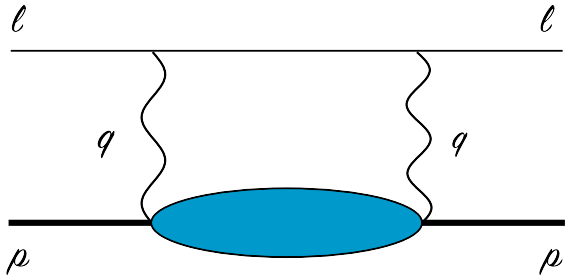
Motivation



- Consider the $\gamma\gamma$ box at low energies (important corrections to *ep* scattering, μ atoms)
- χPT — the low-energy EFT of QCD — is a suitable tool in this regime
- Remove the lepton line $\Longrightarrow$ proton Compton scattering (CS) — wealth of exp. data
- Calculate CS in χPT, confront data, make predictions for $\gamma\gamma$ box

- Extend to γZ and γW at low energies?

Outline

1. χPT framework
2. Results for nucleon Compton scattering and μH
   - RCS
   - VVCS
   - VCS
   - Lamb shift and HFS
3. Some thoughts on extension to γZ and γW

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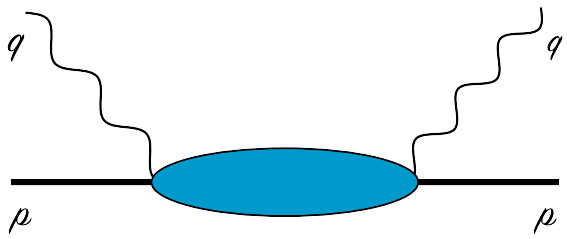
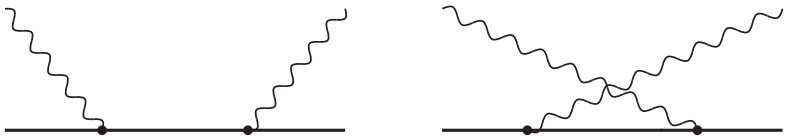
χPT framework



- inlcude nucleons, photons, pions Weinberg, Gasser and Leutwyler, ...
  - count powers of small momenta $p \sim m_\pi$
  - numerically $e \sim m_\pi / M_N$ — count as p
- also include the Delta isobar Hemmert, Holstein, ...
  - $\Delta = M_\Delta - M_N$ is a new energy scale $\Longrightarrow$ δ-counting: Pascalutsa, Phillips (2002) numerically $\delta = \Delta / M_N \sim \sqrt{m_\pi / M_N}$, count $\Delta \sim p^{1/2}$ if $p \sim m_\pi$
  - complications due to the spin-3/2 field (consistent couplings etc.)
- two energy regimes:
  - $\omega \sim m_\pi$:
$$n = 4L - 2N_\pi - N_N - \frac{1}{2} N_\Delta + \sum kV_k$$
  - $\omega \sim \Delta$:
$$n = 4L - 2N_\pi - N_N - N_\Delta - 2N_{1\Delta R} + \sum kV_k$$

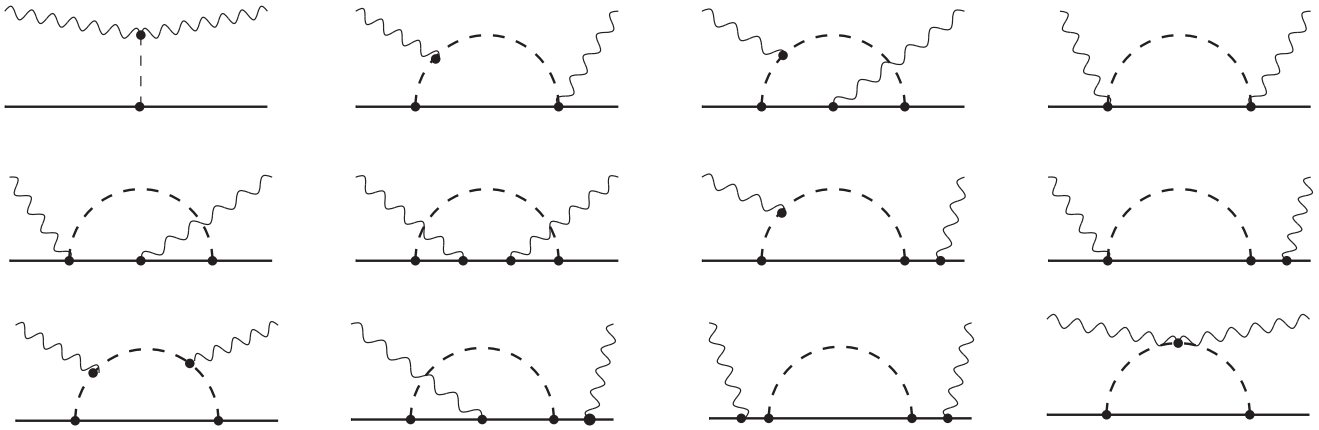
NLO: Born and πN Loops

- Born graphs



  - responsible for low energy (Thomson) limit; point-like nucleon
  - $\mathcal{O}(p^2)$ and $\mathcal{O}(p^3)$ (a.m.m. coupling)

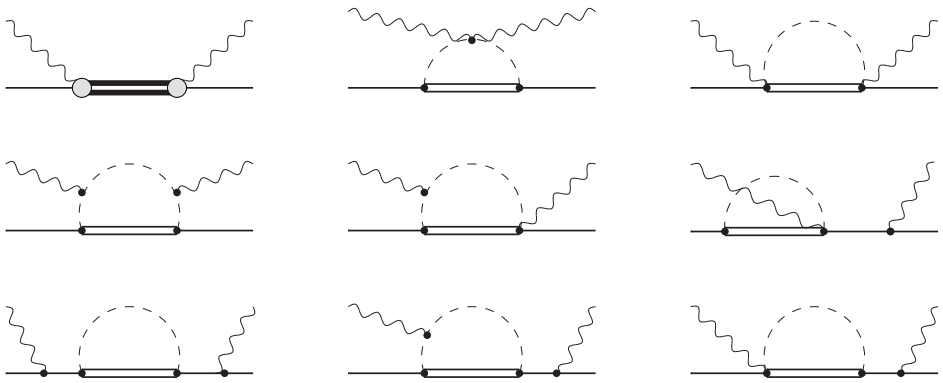
- π^0 anomaly and πN loops



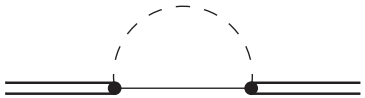
  - leading-order contribution to *polarisabilities*: $\mathcal{O}(p^3)$

NNLO: Delta pole and $\pi\Delta$ loops

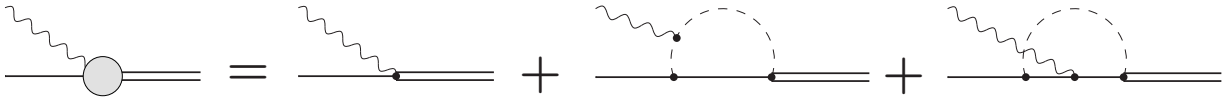
- Delta pole and $\pi\Delta$ loops



  - different counting in different energy regimes
    - Delta pole: $\mathcal{O}(p^4/\Delta) = \mathcal{O}(p^{7/2})$ at $\omega \sim m_\pi$; $\mathcal{O}(p)$ at $\omega \sim \Delta$
    - $\pi\Delta$ loops: $\mathcal{O}(p^4/\Delta) = \mathcal{O}(p^{7/2})$ at $\omega \sim m_\pi$; $\mathcal{O}(p^3)$ at $\omega \sim \Delta$
  - at $\omega \sim \Delta$ one needs to dress the $1\Delta R$ propagator

$i\Sigma =$ 

  - at $\omega \sim \Delta$ corrections to $\gamma N\Delta$ vertex are $\mathcal{O}(p^2)$



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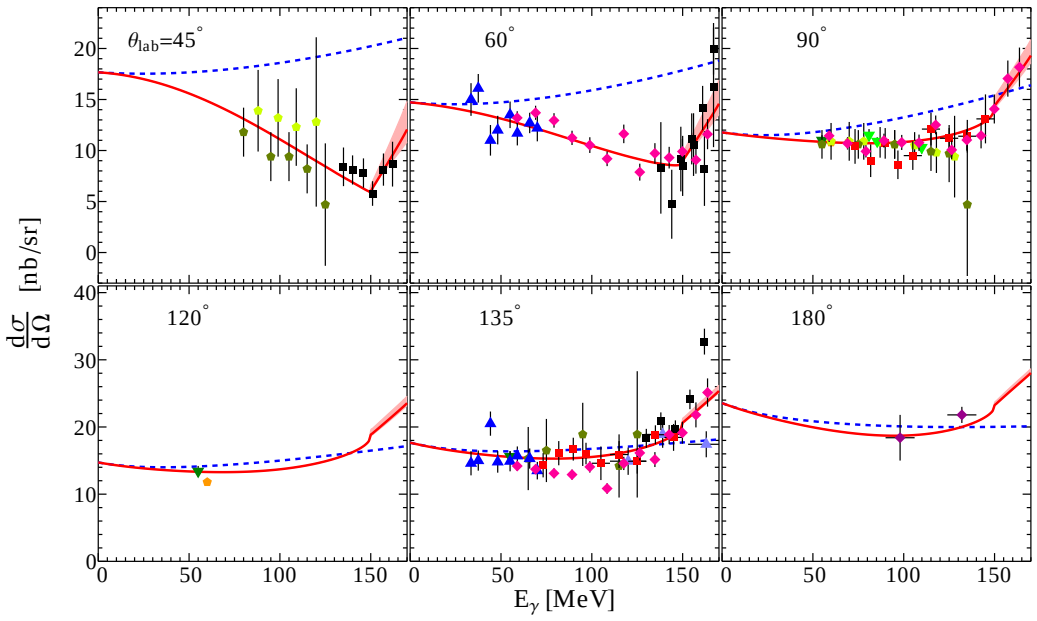
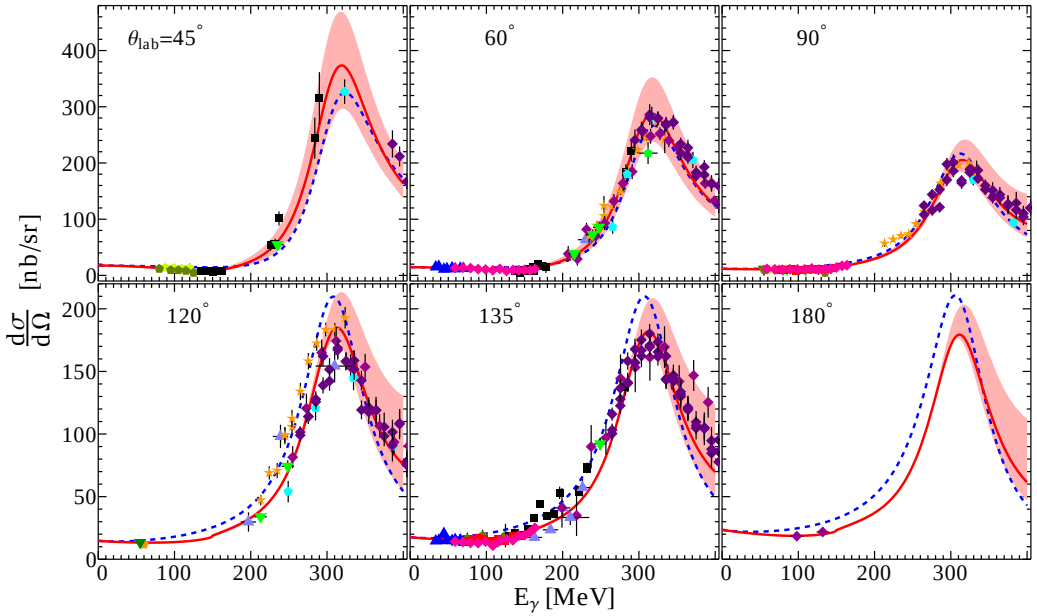
Results: RCS observables

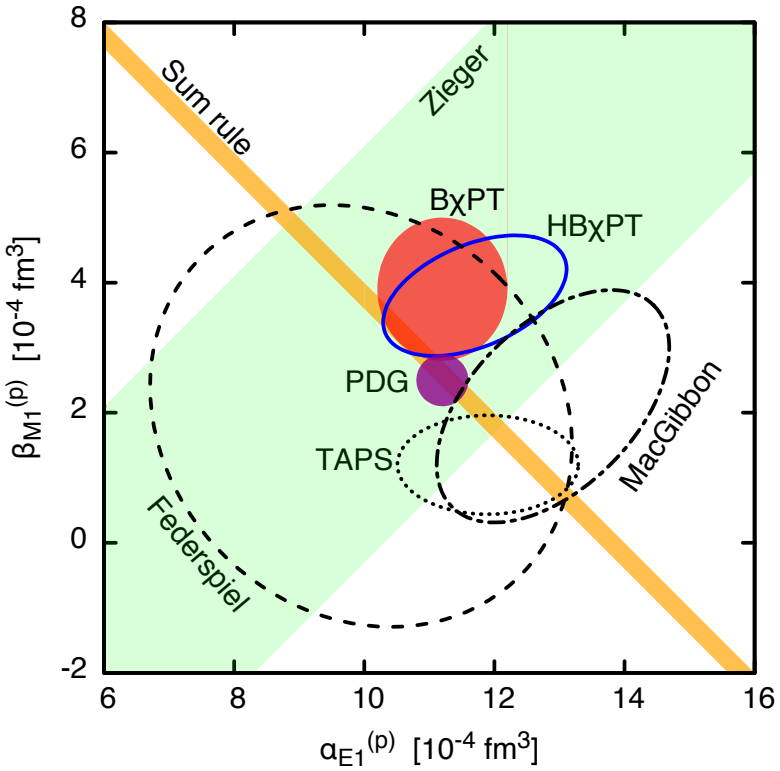
- covariant baryon calculation
  VL, McGovern, Pascalutsa (2015)
  VL, Pascalutsa (2009)
- NNLO ($\mathcal{O}(p^{7/2})$) at $\omega \sim m_\pi$
- NLO ($\mathcal{O}(p^2)$) at $\omega \sim \Delta$
- prediction of ChPT

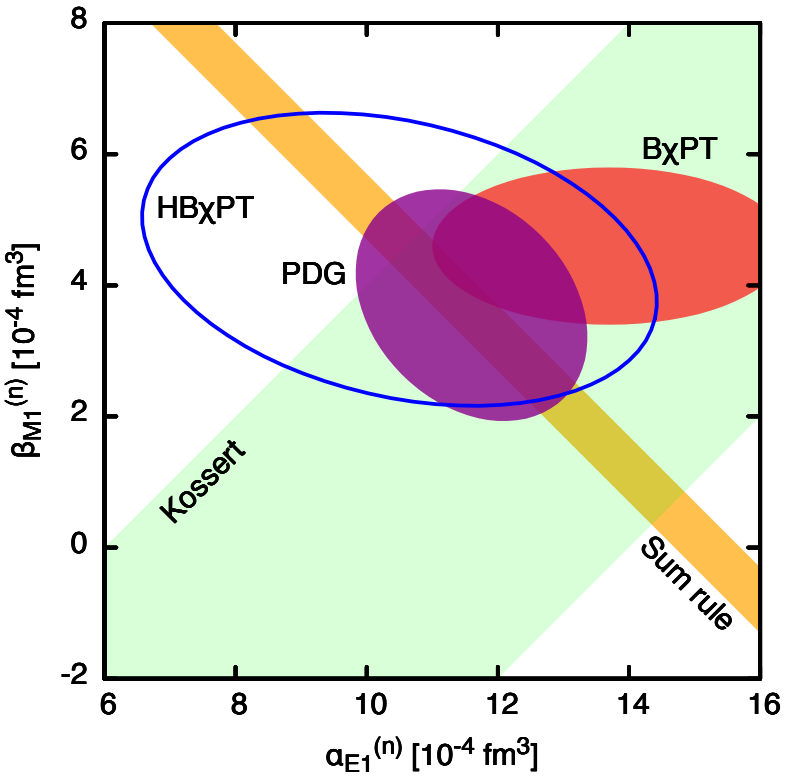


- polarisabilities are seen starting at $\sim$ 50 MeV
- pion loops are important at low energies and around pion production threshold
- Delta pole dominates in the resonance region



Scalar polarisabilities: status





- for the proton, χPT calculations somewhat differ from other extractions (in particular, TAPS value)
  - there are hints that the issue might be due to exp. data

    Krupina, VL, Pascalutsa, *in preparation*
- neutron polarisabilities are less well constrained — there is no free neutron target

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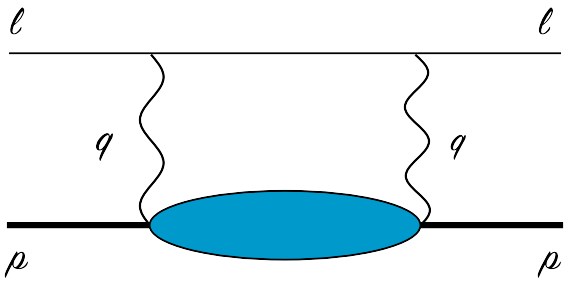
VVCS



- forward VVCS amplitude

$$T(\nu, Q^2) = f_L(\nu, Q^2) + (\vec{\epsilon}'^* \cdot \vec{\epsilon})\, f_T(\nu, Q^2) + i\vec{\sigma} \cdot (\vec{\epsilon}'^* \times \vec{\epsilon})\, g_{TT}(\nu, Q^2) - i\vec{\sigma} \cdot [(\vec{\epsilon}'^* - \vec{\epsilon}) \times \hat{q}]\, g_{LT}(\nu, Q^2)$$

- low-energy expansion of the amplitude is

$$\begin{aligned} f_T(\nu, Q^2) &= f_T^{\mathrm{B}}(\nu, Q^2) + 4\pi \left[Q^2 \beta_{M1} + (\alpha_{E1} + \beta_{M1})\nu^2 \right] + \ldots \\ f_L(\nu, Q^2) &= f_L^{\mathrm{B}}(\nu, Q^2) + 4\pi(\alpha_{E1} + \alpha_L \nu^2) Q^2 + \ldots \\ g_{TT}(\nu, Q^2) &= g_{TT}^{\mathrm{B}}(\nu, Q^2) + 4\pi \gamma_0 \nu^3 + \ldots \\ g_{LT}(\nu, Q^2) &= g_{LT}^{\mathrm{B}}(\nu, Q^2) + 4\pi \delta_{LT} \nu^2 Q + \ldots \end{aligned}$$

- ν-dependent terms can be treated as functions of Q^2 and related to moments of nucleon structure functions

VVCS: results

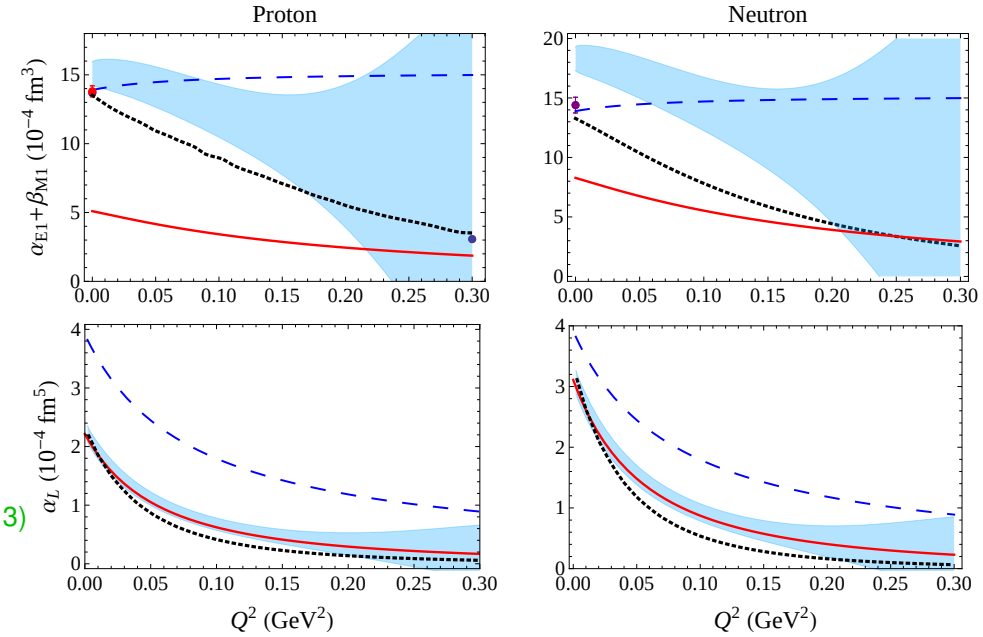
NLO/LO $[\mathcal{O}(p^4/\Delta)/\mathcal{O}(p^3)]$
VL, Alarcón, Pascalutsa (2014)

HB χPT $\mathcal{O}(p^4)$
Kao, Spitzenberg, Vanderhaeghen (2003)

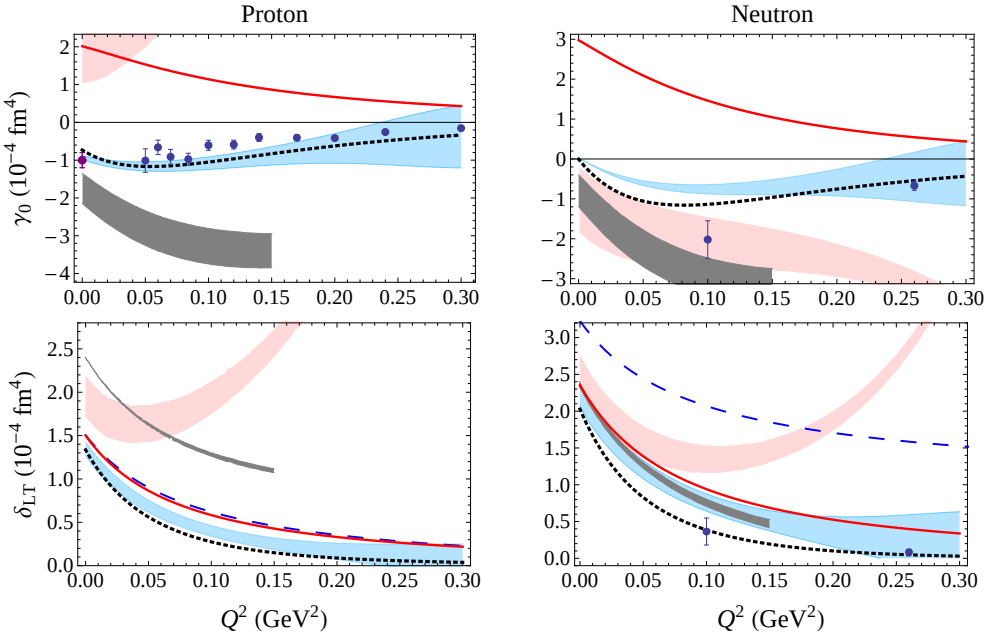
IR $\mathcal{O}(p^4)$
Bernard, Hemmert, Meissner (2003)

covariant χPT $\mathcal{O}(\epsilon^3)$
Bernard, Epelbaum, Krebs, Meissner (2013)

MAID



- HB and IR do not provide adequate description
- covariant χEFT works much better, especially in γ_0 (HB is off the scale there)
- δ_{LT} puzzle: difference between the two covariant calculations (the one of Bernard et al. contains $\pi\Delta$ loops subleading in our counting)
- data on δ_{LT} from JLab expected



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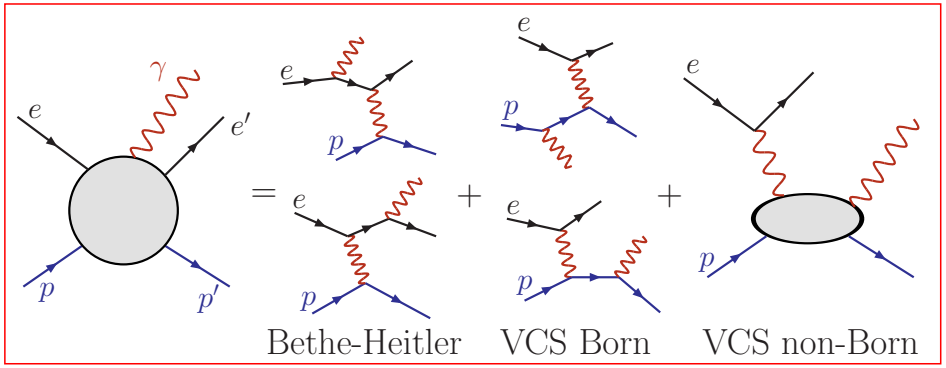
VCS: response functions



- low-energy expansion (small ω'):
  Guichon, Liu, Thomas (1995)

$$d^5\sigma^{\text{VCS}} = d^5\sigma^{\text{BH+Born}} + \omega' \Phi \, \Psi_0(Q^2, \epsilon, \theta, \phi) + \mathcal{O}(\omega'^2);$$

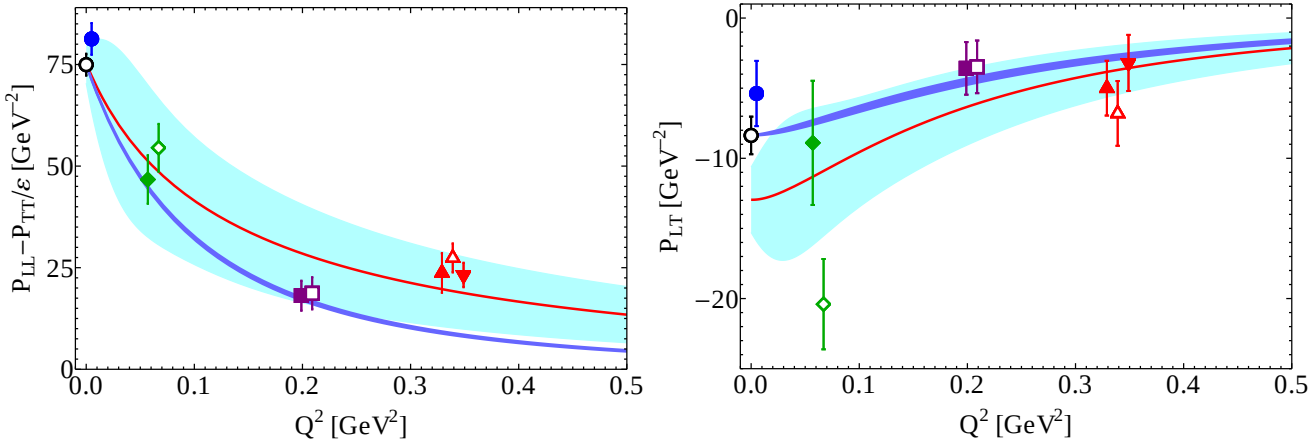
$$\Psi_0(Q^2, \epsilon, \theta, \phi) = V_1 \left[P_{LL}(Q^2) - \frac{P_{TT}}{\epsilon} \right] + V_2 \sqrt{\epsilon(1+\epsilon)} P_{LT}(Q^2)$$

- at $Q^2 = 0$:

$$P_{LL}(0) = \frac{6M}{\alpha_{\text{em}}} \alpha_{E1}$$

$$P_{LT}(0) = -\frac{3M}{4\alpha_{\text{em}}} \beta_{M1}$$

- response functions of VCS in covariant χET
  VL, Pascalutsa, Vanderhaeghen (2016)



- more data from MAMI expected soon
- low-Q^2 expts. at MESA?

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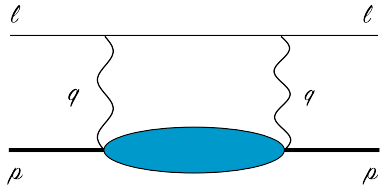
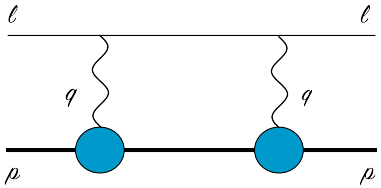
Two-photon corrections

TPE — all

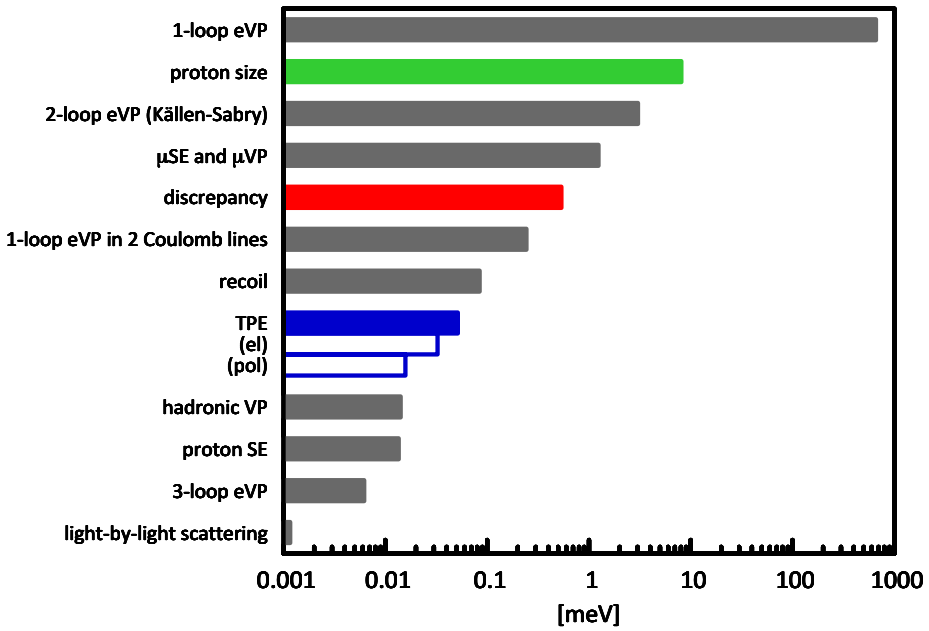


TPE — elastic



polarisability = all − elastic

elastic is calculated from formfactors (empirical data)



review by Hagelstein, Miskimen, Pascalutsa (2016)

- muonic hydrogen Lamb shift in theory [energies in meV, R_p in fm]:

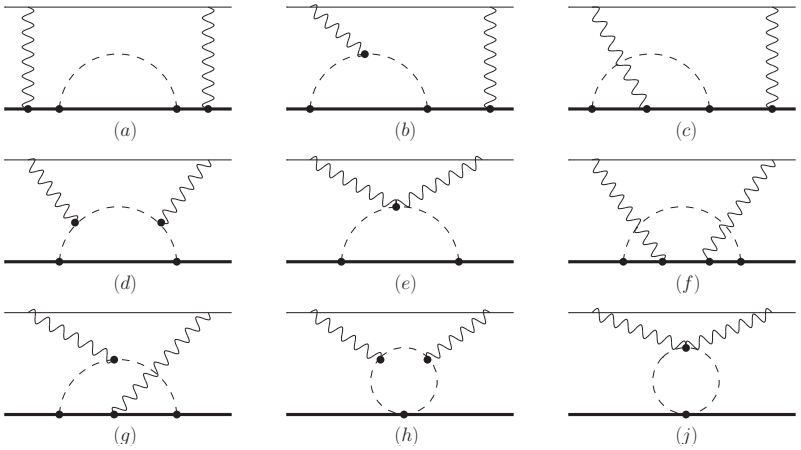
$$\Delta E_{\rm LS} = 206.0336(15) - 5.2275(10) R_p^2 + E^{\rm TPE}$$ Antognini et al (2013)

$$E^{\rm TPE} = 0.0332(20); \quad E^{\rm (pol)} = 0.0085(11)$$ Birse, McGovern (2012)

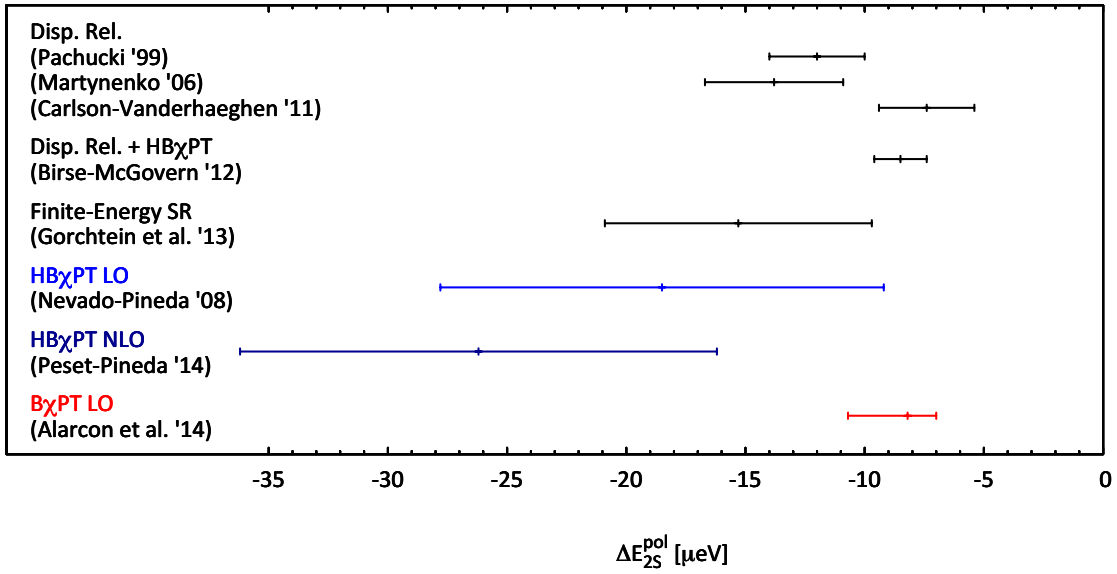
- $E^{\rm TPE}$ is an important source of th. uncertainty
- polarisability corrections depend on the VVCS amplitude

Lamb shift

- πN loops give the leading contribution
- Delta pole strongly suppressed
- $\pi\Delta$ loops not included

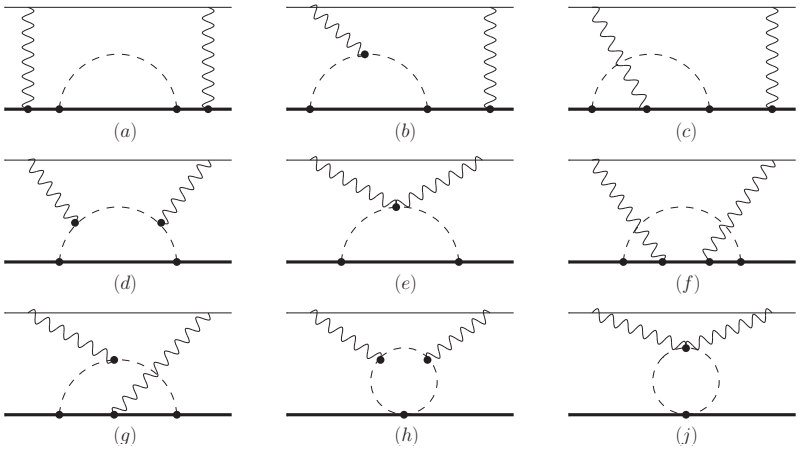


- the $\mathcal{O}(p^3)$ result is
  $$\Delta E_{2S}^{(\mathrm{pol})} = -8.2\left(^{+1.2}_{-2.5}\right)\ \mu\mathrm{eV}$$
  Alarcon, VL, Pascalutsa (2014)
- consistent with other calculations



HFS (preliminary)

- πN loops give the leading contribution
- Delta pole important
- $\pi\Delta$ loops not included

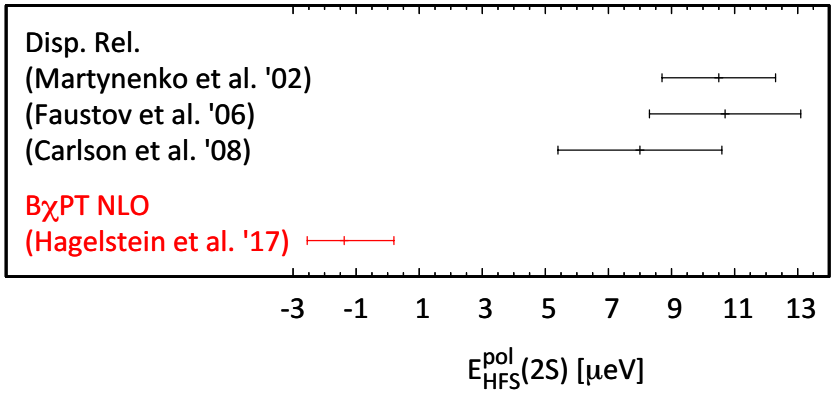


- the $\mathcal{O}(p^3)$ result is

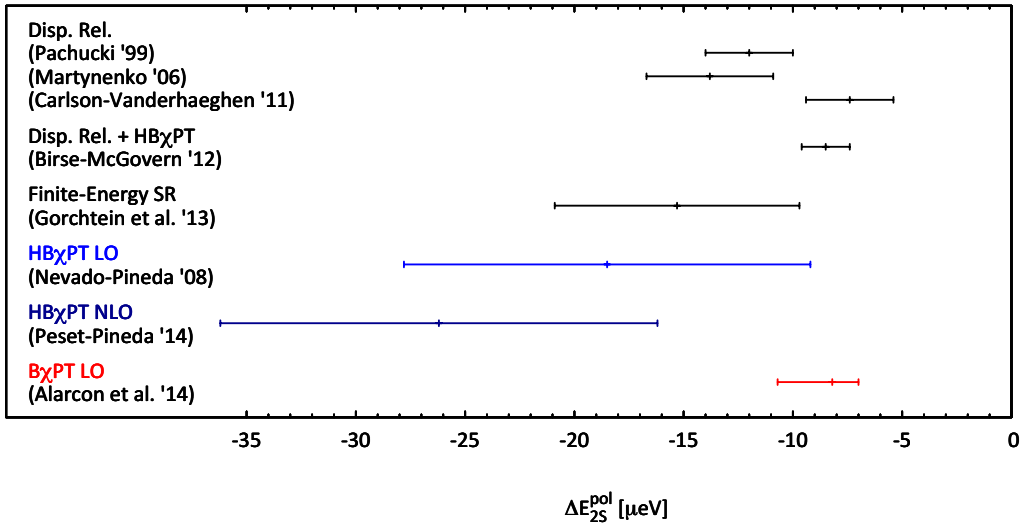
$$E^{(\mathrm{pol})}_{2S,HFS} = -1.4\left(^{+1.6}_{-1.2}\right)\ \mu\mathrm{eV}$$

Hagelstein, VL, Pascalutsa *in preparation*

- large cancellations
- does not agree with dispersive calculations

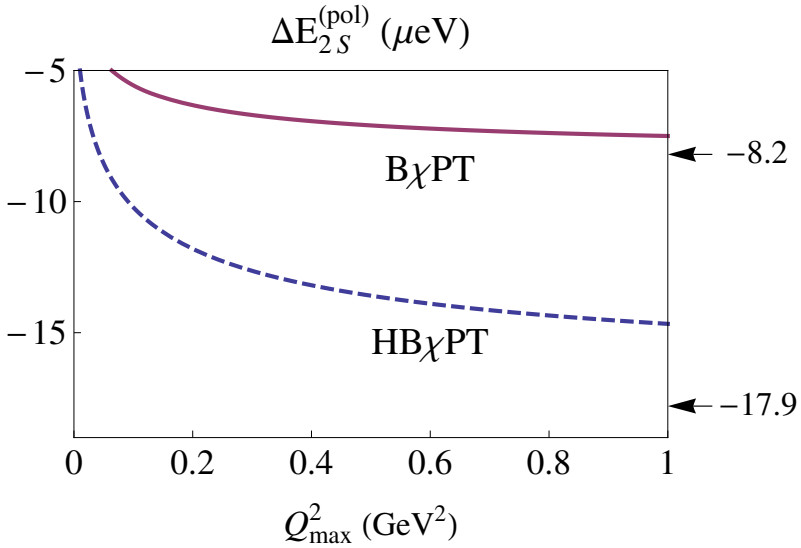


Lamb shift: HB vs. covariant



$$\Delta E_{nS}^{(\mathrm{pol})} = \frac{\alpha_{em}}{\pi}\,\phi_n^2 \int_0^{Q_{\max}} \frac{dQ}{Q^2}\, w(\tau_\ell)\, \left[T_1^{(\mathrm{NB})}(0,Q^2) - T_2^{(\mathrm{NB})}(0,Q^2)\right]$$

- one can expect larger error in HB since the integral converges more slowly there
- neither HB nor covariant (nor other results), however, can explain the missing $\sim 300\ \mu$eV



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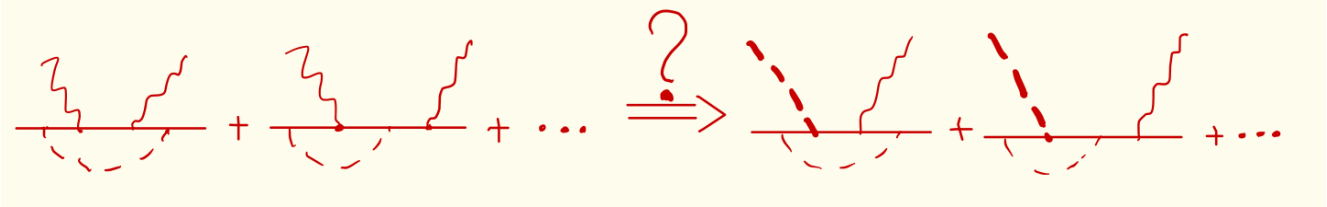
Some thoughts (and summary)

- covariant baryon χEFT works well in nucleon CS (RCS/VCS/VVCS) and gives a $\mathcal{O}(p^3)$ prediction for μH Lamb shift and hyperfine splitting
- by connecting $\gamma\gamma$ box to nucleon CS, we cross-check our χPT calculation and better understand these processes
- how can χPT be applied also to γZ and γW boxes?



- naïvely: can work at low energies (P2@MESA, β-decays)
- could follow a route similar to $\gamma\gamma$ box:
  - calculate the analog of CS ("vector boson production")
  - calculate the boxes
- could provide a systematic evaluation of γZ and γW boxes
- feedback and suggestions welcome!

Backup

χPT for nucleon CS and μH

- real CS in χPT
  - prediction at $\mathcal{O}(p^4/\Delta)$
  - compare with data
  - predict nucleon polarizabilities
- virtual and doubly-virtual CS in χPT
  - prediction at $\mathcal{O}(p^4/\Delta)$
  - nucleon generalized polarizabilities (different in VCS and VVCS!)
- we can calculate μH (Lamb shift and HFS) in χPT
  - prediction at $\mathcal{O}(p^3)$

$\Longrightarrow$ RCS, VCS, VVCS and Lamb shift calculated in a single χPT framework

- complementary information about properties of the nucleon (verify sum rules in χPT etc.)

- our choice is to use the covariant formulation of χPT

Lamb shift and Compton scattering

- Compton scattering amplitude (forward VVCS)

$$T^{\mu\nu}(q,p) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu, Q^2) + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}\, q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}\, q^\nu\right) T_2(\nu, Q^2)$$
$$- \frac{1}{M}\gamma^{\mu\nu\alpha} q_\alpha\, S_1(\nu, Q^2) - \frac{1}{M^2}\left(\gamma^{\mu\nu} q^2 + q^\mu \gamma^{\nu\alpha} q_\alpha - q^\nu \gamma^{\mu\alpha} q_\alpha\right) S_2(\nu, Q^2)$$

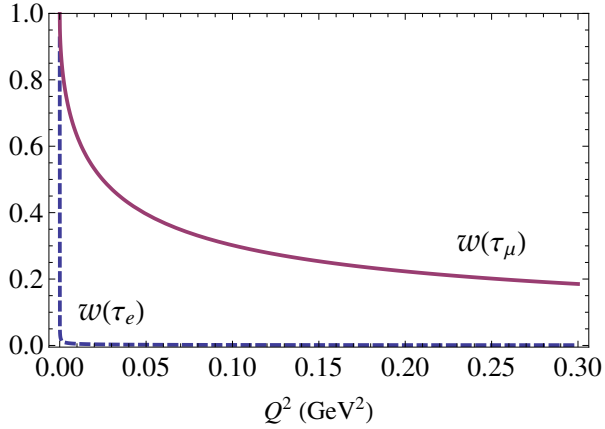
- spin-dependent terms contribute to hyperfine splitting
- n^{th} S-level shift is given by

$$\Delta E_{nS}^{(\text{pol})} = \frac{\alpha_{em}}{\pi}\, \phi_n^2 \int_0^\infty \frac{dQ}{Q^2}\, w(\tau_\ell)\, \left[T_1^{(\text{NB})}(0, Q^2) - T_2^{(\text{NB})}(0, Q^2)\right]$$

$w(\tau_\ell)$: the lepton weighting function

$$w(\tau_\ell) = \sqrt{1+\tau_\ell} - \sqrt{\tau_\ell}, \qquad \tau_\ell = \frac{Q^2}{4m_\ell^2}$$

weighted at low virtualities



Compton amplitudes

- $T_1(\nu, Q^2)$ and $T_2(\nu, Q^2)$ can be related, via dispersive integrals, with nucleon structure functions ($\tilde{\nu} = 2M\nu/Q^2$):

$$T_1(\nu, Q^2) = \frac{8\pi\alpha}{M} \int_0^1 \frac{dx}{x} \frac{f_1(x, Q^2)}{1 - x^2\tilde{\nu}^2 - i0}, \quad T_2(\nu, Q^2) = \frac{16\pi\alpha M}{Q^2} \int_0^1 dx \frac{f_2(x, Q^2)}{1 - x^2\tilde{\nu}^2 - i0}$$

- the integral for T_1 needs a subtraction: unknown function $T_1(0, Q^2)$
- high-Q^2 behaviour of $T_1(0, Q^2)$ needs to be modelled
  - formfactors Pachucki (1999), Martynenko (2006)
  - χPT-inspired formfactors Carlson, Vanderhaeghen (2011), Birse, McGovern (2012)
  - empirical fits Tomalak, Vanderhaeghen (2016)
- something we know about $T_1(0, Q^2)$: low-energy theorem

$$T_1^{\mathrm{NB}}(0, Q^2) = 4\pi\beta_{M1} Q^2 + \cdots, \quad T_2^{\mathrm{NB}}(0, Q^2) = 4\pi(\alpha_{E1} + \beta_{M1}) Q^2 + \cdots$$

$\Longrightarrow$ nucleon polarisabilities

Polarisabilities

- point particle (or low energies) $\Longrightarrow$ charge, mass, a.m.m.
- higher energies: response of the nucleon to external e.m. field

$\Longrightarrow$ static polarisabilities: low-energy constants of effective γN interaction

$$\mathcal{H}_{\rm eff}^{(2)} = -\frac{1}{2}\,4\pi(\alpha_{E1}\vec{E}^2 + \beta_{M1}\vec{H}^2),$$

$$\mathcal{H}_{\rm eff}^{(3)} = -\frac{1}{2}\,4\pi\Big(\gamma_{E1E1}\vec{\sigma}\cdot\vec{E}\times\dot{\vec{E}} + \gamma_{M1M1}\vec{\sigma}\cdot\vec{H}\times\dot{\vec{H}} - 2\gamma_{M1E2}E_{ij}\sigma_i H_j + 2\gamma_{E1M2}H_{ij}\sigma_i E_j\Big),$$

$$\mathcal{H}_{\rm eff}^{(4)} = -\frac{1}{2}\,4\pi(\alpha_{E1\nu}\dot{\vec{E}}^2 + \beta_{M1\nu}\dot{\vec{H}}^2) - \frac{1}{12}\,4\pi(\alpha_{E2}E_{ij}^2 + \beta_{M2}H_{ij}^2),\ldots$$

$$A_{ij} = \frac{1}{2}(\nabla_i A_j + \nabla_j A_i), \quad A = \vec{E}, \vec{H}$$

- this EFT breaks down around the pion production threshold
- we can calculate the polarisabilities from our more high-energy theory — χPT
- ... or find from fits to data (with some help of χPT)

One more step: fit at $\mathcal{O}(p^4)$ (partial)

- add a dipole polarisabilities contact term; fit $\delta\alpha_{E1}$ and $\delta\beta_{M1}$ to data

$$\mathcal{L}^{(4)}_{\pi N} = \pi e^2 \overline{N}\big(\delta\beta_{M1} F^{\mu\rho} F_{\mu\rho} + \tfrac{2}{M^2}(\delta\alpha_{E1} + \delta\beta_{M1})\partial_\mu F^{\mu\rho} F^{\nu}{}_{\rho}\partial_\nu\big) N$$

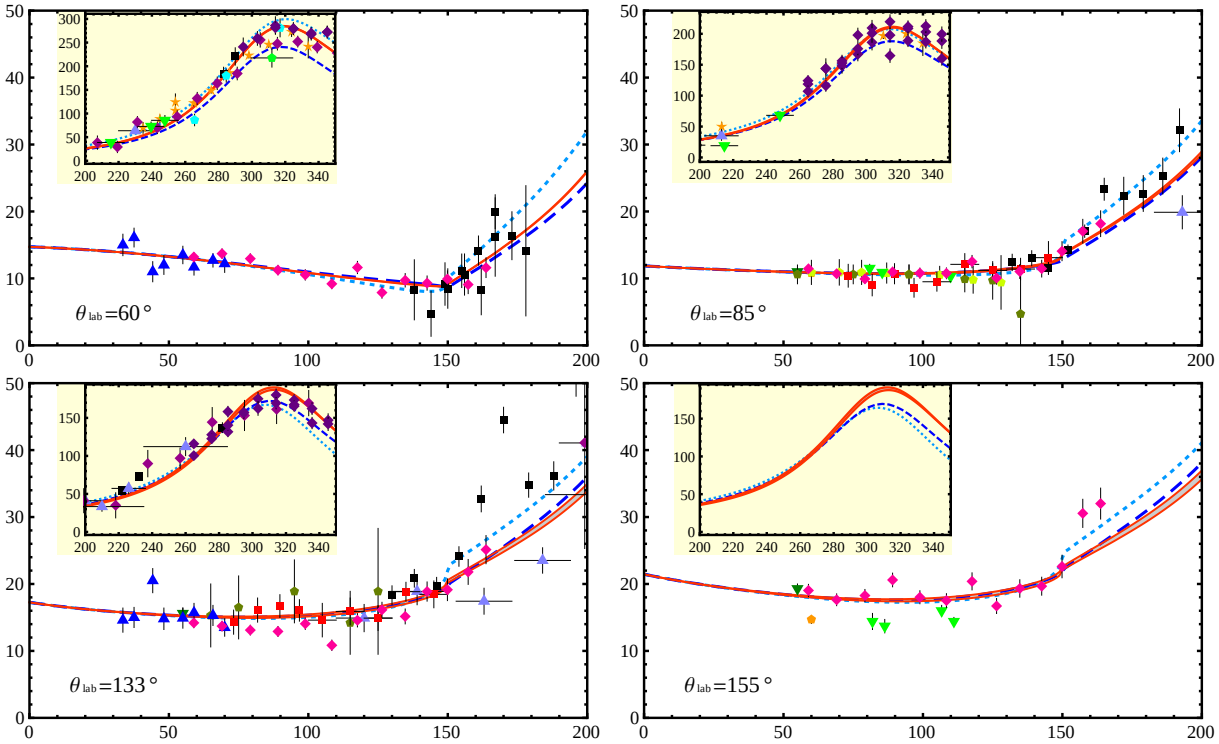
- results:

  VL, McGovern (2014)

$$\alpha_{E1} = 10.6 \pm 0.5$$

$$\beta_{M1} = 3.2 \pm 0.5$$

Baldin constrained
$\chi^2/\text{d.o.f.} = 112.5/136$



Sum rules

- sum rules connecting VCS, RCS, and VVCS:
  Pascalutsa, Vanderhaeghen (2015)

$$\delta_{LT} = -\gamma_{E1E1} + 3M\alpha_{\rm em}\left[P'^{\,(M1M1)1}(0) - P'^{\,(L1L1)1}(0)\right],$$

$$I_1'(0) = \frac{\kappa_N^2}{12}\langle r_2^2\rangle + \frac{M^2}{2}\left\{\frac{1}{\alpha_{\rm em}}\gamma_{E1M2} - 3M\left[P'^{\,(M1M1)1}(0) + P'^{\,(L1L1)1}(0)\right]\right\}$$

- verified in covariant and HB χPT VL, Pascalutsa, Vanderhaeghen, Kao (2017)
- connect experimentally accessible quantities
- allow to obtain complementary information on, e.g., static spin polarisabilities
- higher-order scalar sum rules VL, Pascalutsa, Vanderhaeghen, Hagelstein *in preparation*

Sum rules and δ_{LT} puzzle

$$\delta_{LT} = -\gamma_{E1E1} + 3M\alpha_{\rm em}\left[P'^{(M1M1)1}(0) - P'^{(L1L1)1}(0)\right]$$

- δ_{LT} puzzle shows here
- the result of Bernard et al. for δ_{LT} seems to be in contradiction with MAID
- new JLab data for the proton δ_{LT} are expected to shed light on this puzzle