Self-consistent radiative corrections to bubble nucleation rates

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Based upon work in collaboration with **Björn Garbrecht**:
PRD**91** (2015) 105021 [1501.07466]; PRD**92** (2015) 125022 [1509.07847];
NPB**906** (2016) 105–132 [1509.08480]; 1703.05417 (summary);
and work in progress with **Wen-Yuan Ai** and **Björn Garbrecht**

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Outline

- Introduction and motivation
- Coleman's bounce and the semi-classical tunneling rate
- Quantum corrections and the effective action
- **Green's function method**: accounting for **gradient effects**
  - **Tree-level SSB:** $V(\phi) = \lambda\phi^4/24 - \mu^2\phi^2/2$, $\mu^2 > 0$

    [Garbrecht & Millington, PRD91 (2015) 105021]
  - **Radiative SSB:** the Coleman-Weinberg mechanism

    [Garbrecht & Millington, PRD92 (2015) 125022; see also NPB906 (2016) 105–132]
  - **Extensions**: fermions/beyond thin wall

    [in progress with Wen-Yuan Ai & Björn Garbrecht]
- How important are **gradient effects**?
- Conclusions and future/ongoing directions

First-order phase transitions in fundamental physics

Many examples across **high-energy** and **astro-particle physics**, and **cosmology**:

- **symmetry restoration** at finite temperature and early Universe **phase transitions**
  [Kirzhnits & Linde, PLB42 (1972) 471; Dolan & Jackiw, PRD9 (1974) 3320; Weinberg, PRD9 (1974) 3357]
  - generation of the **Baryon asymmetry of the Universe**
    [Everyone in this room! See, e.g., Morrissey & Ramsey-Musolf, New J. Phys. 14 (2012) 125003]
  - first-order phase transitions may produce **relic gravitational waves**
    [Well done, LIGO! Witten, PRD30 (1984) 272; Kosowsky, Turner & Watkins, PRD45 (1992) 4514; Caprini, Durrer, Konstandin & Servant, PRD79 (2009) 083519]
- the perturbatively-calculated **SM** effective potential develops an **instability** at $\sim 10^{11}$ GeV, given a ~ 125 GeV Higgs and a ~ 174 GeV top quark.
  [Cabibbo, Maiani, Parisi & Petronzio, NPB158 (1979) 295; Sher, Phys. Rep. 179 (1989) 273; PLB317 (1993) 159; Isidori, Ridolfi & Strumia, NPB609 (2001) 387; Elias-Miró, Espinosa, Giudice, Isodori, Riotto & Strumia, PLB709 (2012) 222; Degrassi, Di Vita, Elias-Miró, Espinosa, Giudice, Isidori & Strumia, JHEP1208 (2012) 098; Alekhin, Djouadi & Moch, PLB716 (2012) 214; Bednyakov, Kniehl, Pikelner & Veretin, PRL115 (2015) 201802; Di Luzio, Isidori & Ridolfi, PLB753 (2016) 150–160; . . .]
- dynamics of both **topological** and **non-topological defects**, and non-perturbative phenomena in non-linear field theories, e.g., domain walls, Q balls, oscillons, etc.

Pete's tunneling-rate checklist

- **phenomenology:** impact of non-renormalizable operators/sensitivity to UV completion/new (or other) physics?
- **experiment:** measurement (or limit setting) on model parameters?
- **environment:** impact of "seeds;" is it sufficient to consider the decay of an initially homogeneous state?

  [Grinstein & Murphy, JHEP 1512 (2015) 063; Gregory, Moss and Withers JHEP 1403 (2014) 081; Burda, Gregory and Moss PRL115 (2015) 071303; JHEP 1508 (2015) 114; JHEP 1606 (2016) 025]
- **theory:**
  - gauge dependence?

    [Tamarit and Plascencia, JHEP1610 (2016) 099]
  - interpretation of the non-convexity of the effective potential?

    [Weinberg & Wu, PRD36 (1987) 2474; Alexandre & Farakos, JPA41 (2008) 015401; Branchina, Faivre & Pangon, JPG36 (2009) 015006; Einhorn & Jones, JHEP0704 (2007) 051]
  - implementation of RG improvement?

    [Gies & Sondenheimer, EPJC75 (2015) 68]
  - incorporation of the inhomogeneity of the solitonic background (this talk); how important are gradients?

    [Garbrecht & Millington, PRD91 (2015) 105021, cf. Goldstone & Jackiw, PRD11 (1975) 1486; Garbrecht & Millington, PRD92 (2015) 125022; for a summary, see arXiv: 1703.05417]

Semi-classical tunneling rate

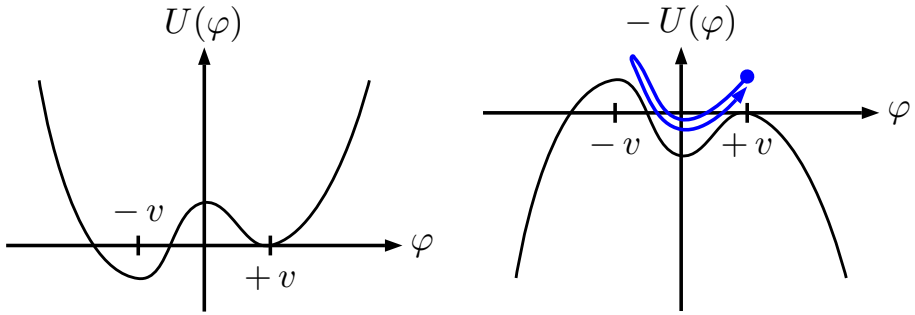
Archetype: Euclidean Φ^4 theory with tachyonic mass ($\mu^2 > 0$):

$$\mathcal{L} \;=\; \frac{1}{2!}\left(\partial_\mu \Phi\right)^2 \;-\; \frac{1}{2!}\,\mu^2\Phi^2 \;+\; \frac{1}{3!}\,g\Phi^3 \;+\; \frac{1}{4!}\,\lambda\Phi^4 \;+\; U_0$$

[for self-consistent numerical studies, see Bergner & Bettencourt, PRD69 (2004) 045002; PRD69 (2004) 045012; Baacke & Kevlishvili, PRD71 (2005) 025008; PRD75 (2007) 045001]

Non-degenerate minima:

$$\varphi \;\equiv\; \langle\Phi\rangle \;=\; v_\pm \;\approx\; \pm\, v \;-\; \frac{3g}{2\lambda}\,, \qquad v^2 \;=\; \frac{6\mu^2}{\lambda}$$



The **Coleman bounce**:

$$\varphi\big|_{x_4\,\to\,\pm\infty} \;=\; +\,v\,, \qquad \dot\varphi\big|_{x_4\,=\,0} \;=\; 0\,, \qquad \varphi\big|_{|\mathbf{x}|\,\to\,\infty} \;=\; +\,v$$

[Coleman, PRD15 (1977) 2929; Callan, Coleman, PRD16 (1977) 1762; Coleman Subnucl. Ser. 15 (1979) 805; Konoplich, Theor. Math. Phys. 73 (1987) 1286]

Semi-classical tunneling rate

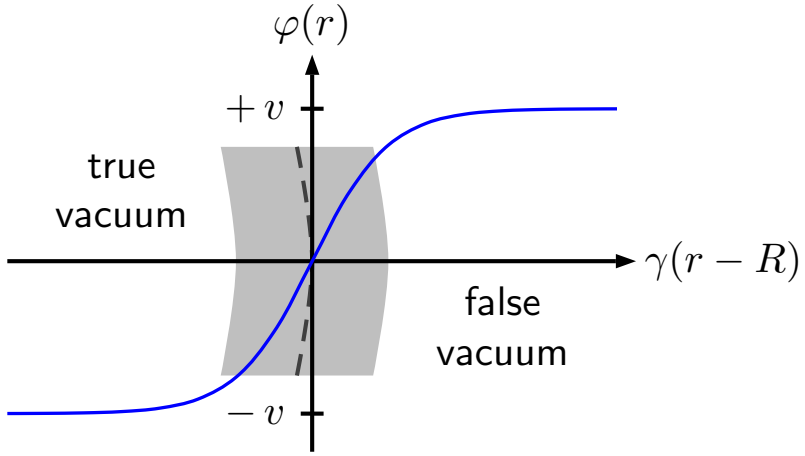
In hyperspherical coordinates, the boundary conditions are

$$\varphi\big|_{r\to\infty} \;=\; +v\,, \qquad \mathrm{d}\varphi/\mathrm{d}r\big|_{r=0} \;=\; 0\,,$$

with the bounce corresponding to the **kink**

[Dashen, Hasslacher & Neveu, PRD10 (1974) 4114; *ibid.* 4130; *ibid.* 4138]

$$\varphi(r) \;=\; v\tanh\gamma(r-R)\,, \qquad \gamma \;=\; \mu/\sqrt{2}\,.$$



The bounce looks like a **bubble** of radius $R = 12\lambda/g/v$, where the latter is found by minimizing the energy difference between the **latent heat** of the true vacuum and the **surface tension** of the bubble.

Semi-classical tunneling rate

The **tunneling rate** Γ is calculated from the path integral

$$Z[0] \;=\; \int [\mathrm{d}\Phi] e^{-S[\Phi]/\hbar}\,, \qquad \Gamma/V \;=\; 2\big|\mathrm{Im}\, Z[0]\big|/V/T\,.$$

[see Callan & Coleman, PRD16 (1977) 1762]

Expanding around the kink $\Phi = \varphi^{(0)} + \hbar^{1/2}\hat{\Phi}$, the spectrum of the operator

$$G^{-1}(\varphi^{(0)}; x, y) \;\equiv\; \left.\frac{\delta^2 S[\Phi]}{\delta\Phi(x)\delta\Phi(y)}\right|_{\Phi\,=\,\varphi^{(0)}} \;=\; \delta^{(4)}(x-y)\big(-\,\Delta^{(4)} \,+\, U''(\varphi^{(0)})\big)$$

contains **four zero eigenvalues** (translational invariance of the bounce action) and **one negative eigenvalue** (dilatations of the bounce).

Writing $B^{(0)} \equiv S[\varphi]$,

$$Z[0] \;=\; -\,\frac{i}{2}\, e^{-B^{(0)}/\hbar} \left| \frac{\lambda_0 \det{}^{(5)}\, G^{-1}(\varphi^{(0)})}{(VT)^2 \big(\frac{B^{(0)}}{2\pi\hbar}\big)^4 (4\gamma^2)^5 \det{}^{(5)}\, G^{-1}(v)} \right|^{-1/2}\,.$$

Non-perturbative treatment of quantum effects: the effective action

If the instability arises from **radiative effects** (including **thermal effects**), the **quantum (statistical) path** is **non-perturbatively** far away from the **classical (zero-temperature) path**.

Specifically, the tree-level fluctuation operator will have a positive-definite spectrum, whereas the one-loop fluctuation operator will **not**.

The **2PI effective action** is defined by the **Legendre transform**

$$\Gamma[\phi,\Delta] \;=\; \max_{J,K}\left[-\,\hbar\,\ln\,Z[J,K] \;+\; J_x\phi_x \;+\; \tfrac{1}{2}\,K_{xy}\left(\phi_x\phi_y \;+\; \hbar\,\Delta_{xy}\right)\right] \;.$$

[Cornwall, Jackiw & Tomboulis, PRD10 (1974) 2428]

Method of external sources: Turn the evaluation of the effective action on its head, such that the **physical limit** corresponds to **non-vanishing** sources.

[Garbrecht & Millington, NPB906 (2016) 105–132; see also PRD91 (2015) 105021]

By constraining these sources subject to the **consistency relation**

$$\left.\frac{\delta S[\phi]}{\delta\phi_x}\right|_{\phi=\varphi} \;-\; J_x[\phi,\Delta] \;-\; K_{xy}[\phi,\Delta]\varphi_y \;=\; \left.\frac{\delta\Gamma[\phi,\Delta]}{\delta\phi_x}\right|_{\phi=\varphi} \;=\; 0\;,$$

we can **force** the system along the **quantum (statistical) path**.

Quantum-corrected bounce

For the **tree-level instability**, we may find the leading corrections to the **bounce** and **tunneling rate** by making use of the **1PI** effective action.

[Jackiw, PRD9 (1974) 1686]

The **tunneling rate** per unit volume is related to the 1PI effective action via

$$\Gamma/V \;=\; 2|\mathrm{Im}\, e^{-\Gamma[\varphi^{(1)}]/\hbar}|/V/T\;.$$

The **quantum-corrected bounce** $\varphi^{(1)}(x) \;\equiv\; \varphi^{(0)} \;+\; \hbar\,\delta\varphi$ satisfies

$$-\,\partial^2\varphi^{(1)}(x) \;+\; U'(\varphi^{(1)};x) \;+\; \hbar\,\Pi(\varphi^{(0)};x)\,\varphi^{(0)}(x) \;=\; 0\;,$$

including the **tadpole correction**

$$\Pi(\varphi^{(0)};x) \;=\; \frac{\lambda}{2}\,G(\varphi^{(0)};x,x)\;.$$

If we employ the **method of external sources**, the **self-consistent** choice of $J_x[\phi]$ for this method of evaluation is

$$J_x[\phi] \;=\; -\,\hbar\,\Pi(\varphi^{(0)};x)\varphi^{(0)}(x)\;.$$

[see Garbrecht & Millington, PRD91 (2015) 105021; NPB906 (2016) 105–132]

Approximations

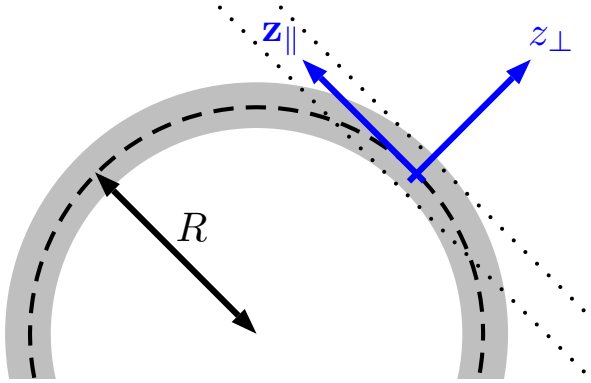
The **radial part** of the 1PI **Klein-Gordon** equation for the Φ Green's function is

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{3}{r}\frac{\mathrm{d}}{\mathrm{d}r} + \frac{j(j+2)}{r^2} - \mu^2 + \frac{\lambda}{2}\varphi^2(r) \right] G(r,r') = \frac{\delta(r-r')}{r^3} .$$

Making the following approximations, we can solve for the Φ Green's function **analytically**:

1. **Thin-wall** approximation $\mu R \gg 1$: drop the damping term.
2. **Planar-wall** approximation: replace the sum over discrete angular momenta by an integral over linear momenta, i.e.

$$\frac{j(j+2)\hbar}{\mu^2 R^2} \longrightarrow \frac{k^2}{\mu^2} .$$



Green's function

Defining

$$u^{(\prime)} \equiv \varphi^{(0)}(r^{(\prime)})/v \ ,$$
$$m \equiv \left(1 + k^2/4/\gamma^2\right)^{1/2} \ ,$$

the result for the **Green's function** is

[Garbrecht & Millington, PRD91 (2015) 105021]

$$G(u,u',m) \ = \ \frac{1}{2\gamma m}\left[\vartheta(u-u')\left(\frac{1-u}{1+u}\right)^{\frac{m}{2}}\left(\frac{1+u'}{1-u'}\right)^{\frac{m}{2}}\right.$$
$$\left.\times\left(1-3\,\frac{(1-u)(1+m+u)}{(1+m)(2+m)}\right)\left(1-3\,\frac{(1-u')(1-m+u')}{(1-m)(2-m)}\right)+(u\leftrightarrow u')\right] \ .$$

We can then find the (manifestly-real) **renormalized tadpole self-energy**:

$$\Pi^R(u) \ = \ \frac{3\lambda\gamma^2}{16\pi^2}\left[6+(1-u^2)\left(5-\frac{\pi}{\sqrt{3}}\,u^2\right)\right] \ .$$

The variation in the background field $u \in [-1,+1]$ gives order-1 corrections to the tadpole self-energy, i.e. gradient effects contribute at **LO** in the equation of motion.

Tunneling rate

Expanding the 1PI effective action $\Gamma[\varphi^{(1)}]$ about $\varphi^{(0)}$, the **tunneling rate per unit volume** is

$$\Gamma/V \;=\; \left(\frac{B}{2\pi\hbar}\right)^2 (2\gamma)^5 |\lambda_0|^{-\frac{1}{2}} \exp\left[-\frac{1}{\hbar}\left(B^{(0)} + \hbar B^{(1)} + \hbar^2 B^{(2)} + \hbar^2 B^{(2)\prime}\right)\right].$$

- one-loop corrections captured by the **fluctuation determinant**:

$$B^{(1)} \;=\; \frac{1}{2}\,\mathrm{tr}^{(5)}\Big(\ln G^{-1}(\varphi^{(0)}) \;-\; \ln G^{-1}(v)\Big)$$

- two-loop (**1PR**) corrections (i) $B^{(2)}$ from the action of the corrected bounce and (ii) $B^{(2)\prime}$ from expanding the fluctuation determinant:

$$B^{(2)} \;=\; -\frac{1}{2}\int \mathrm{d}^4x\; \varphi^{(0)}(x)\Pi(\varphi^{(0)};x)\delta\varphi(x) \;=\; -\frac{1}{2}\,B^{(2)\prime}$$

We have ignored $\mathcal{O}(\hbar^2)$ **2PI** corrections, and so we need to ensure that our perturbative truncation is meaningful . . .

Spectators

To this end and to enhance the radiative effects (while remaining in a perturbative regime), we consider an ***N*-field model**:

[see 't Hooft, NPB72 (1974) 461]

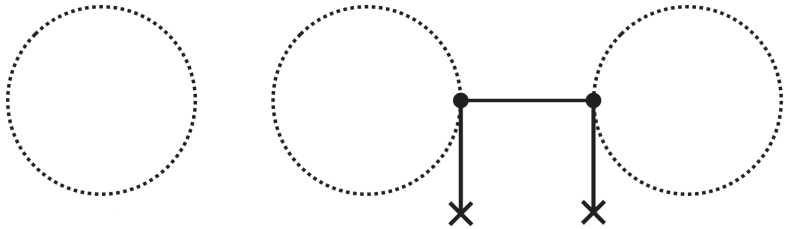
$$\mathcal{L} \supset \sum_{i=1}^{N} \left[\frac{1}{2} \left(\partial_\mu X_i\right)^2 + \frac{1}{2} m_X^2 X_i^2 + \frac{\lambda}{4} \Phi^2 X_i^2 \right] .$$

For $m_X^2 \gg \gamma^2$, the X **renormalized tadpole correction** is

[Garbrecht & Millington, PRD91 (2015) 105021]

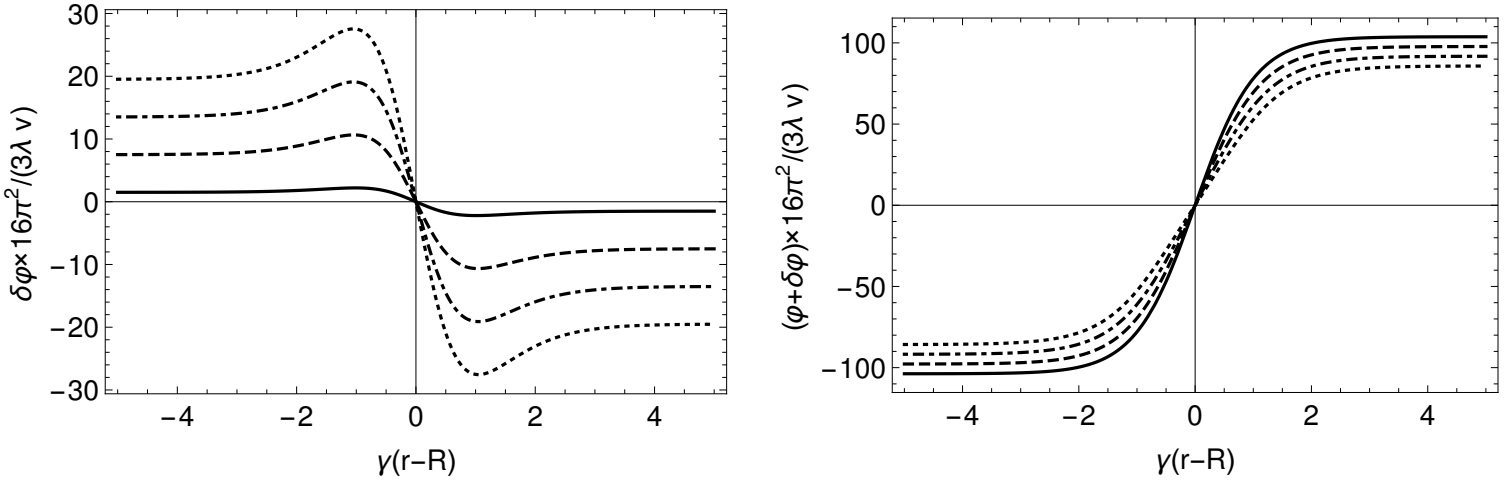
$$\Sigma^R(u) = \frac{\lambda\gamma^2}{8\pi^2} \frac{\gamma^2}{m_X^2} \left[72 + \left(1 - u^2\right)\left(40 - 3u^2\right)\right] .$$

Dominant $\hbar$ and $\hbar^2$ corrections in $1/N$ expansion:



Quantum-corrected bounce

$$\delta\varphi(u) \;=\; -\frac{v}{\gamma}\int_{-1}^{1}\mathrm{d}u'\,\frac{u'\,G(u,u',m)\big|_{k=0}}{1-u'^2}\left(\Pi^R(u')+N\Sigma^R(u')\right)$$



$N\gamma^2/m_X^2$: 0 (solid), 0.5 (dashed), 1 (dash-dotted) and 1.5 (dotted)

[Qualitative agreement with Bergner & Bettencourt, PRD69 (2004) 045002]

$\longrightarrow$ reduction in bounce action $\longrightarrow$ increase in tunneling rate.

Why go to all this trouble?

When determining the quantum corrections to the tunneling configuration, it is tempting to:

1. Calculate the **Coleman-Weinberg (or thermal) effective potential** assuming a homogeneous, constant field configuration.
2. Promote this homogeneous, constant field to a spacetime-dependent in order to obtain the quantum equation of motion for the bounce.

This procedure does not fully capture the back-reaction of the gradients of the tunneling configuration on the quantum corrections.

[e.g. of calculations in the homogeneous background, see e.g. Frampton, PRL37 (1976) 1378; PRD15 (1977) 2922; Camargo-Molina, O'Leary, Porod & Staub, EPJC73 (2013) 2588]

How significant can the impact of these gradients be, both on the bounce and the tunneling rate?

Gradients can be important classically

Consider the following Euclidean theory with **abyssal potential**:

$$\mathcal{L} \;=\; \frac{1}{2!}\left(\partial_\mu \Phi\right)^2 \;-\; \frac{\lambda}{4!}\,\Phi^4\,, \qquad \lambda \;>\; 0\,.$$

In hyperradial coordinates, the equation of motion is

$$-\,\frac{\mathrm{d}^2}{\mathrm{d}r^2}\,\varphi \;-\; \frac{3}{r}\,\frac{\mathrm{d}}{\mathrm{d}r}\,\varphi \;-\; \frac{\lambda}{3!}\,\varphi^3 \;=\; 0\,.$$

The damping term provides an effective barrier (a gradient barrier), and the bounce corresponds to the **Fubini-Lipatov instanton**

[Fubini, Nuovo Cim A 34 (1976) 521; Lipatov, Sov. Phys. JETP 45 (1977) 216]

$$\varphi(r) \;=\; \frac{\varphi(0)}{1 + r^2/R^2}\,, \qquad \varphi(0) = \sqrt{\frac{48}{\lambda}}\,\frac{1}{R}\,.$$

Can the impact of gradient effects on the quantum/thermal corrections have a significant impact on the tunneling barrier and therefore the tunneling rate?

[Garbrecht and Millington, in progress]

N-field Coleman-Weinberg model

[Coleman & Weinberg, PRD7 (1973) 1888;
Garbrecht & Millington, PRD92 (2015) 125022]

Start with a **classically scale-invariant model** (for $g = 0$):

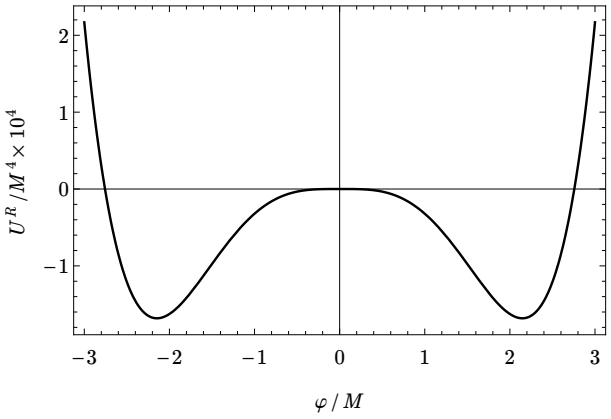
$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \Phi\right)^2 + \frac{1}{2}\sum_{i=1}^{N}\left(\partial_\mu X_i\right)^2 + \frac{1}{4}\lambda\,\Phi^2\sum_{i=1}^{N} X_i^2 + \frac{1}{4}\kappa\sum_{i,j=1}^{N} X_i^2 X_j^2 + \underbrace{\frac{1}{6}\,g\,\Phi^3}_{\mathbb{Z}_2-\text{breaking}} + \overbrace{U_0}^{\Rightarrow\ U^R\ =\ 0\ \text{in the false vacuum}}$$

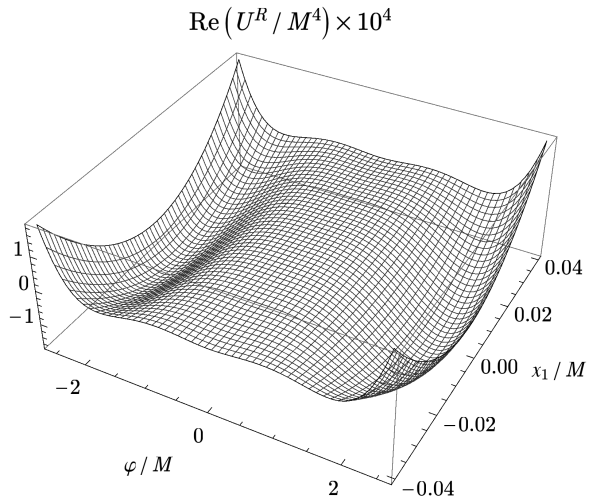
Renormalized one-loop effective potential ($\rho \equiv 6\kappa/\lambda$):

$$U^R_{\text{eff}}(\varphi) = \frac{\lambda^2}{16\pi^2}\,\varphi^4\left[N\left(\ln\frac{3\varphi^2}{\rho M^2} - \frac{3}{2}\right) + F(\rho) + \frac{g}{6}\,\varphi^3 + U_0\right] + \mathcal{O}(g^2)$$

The field φ obtains a vacuum expectation value for $\chi_1 = 0$:

$$v \approx \pm\sqrt{\frac{\rho M^2}{3}}\,\exp\left(\frac{1}{2} + \frac{F(\rho)}{2N}\right), \qquad F \approx 2 \text{ for } \rho = 3\,.$$





$1/N$ power counting

$1/N$ power counting tells us that we can consistently:

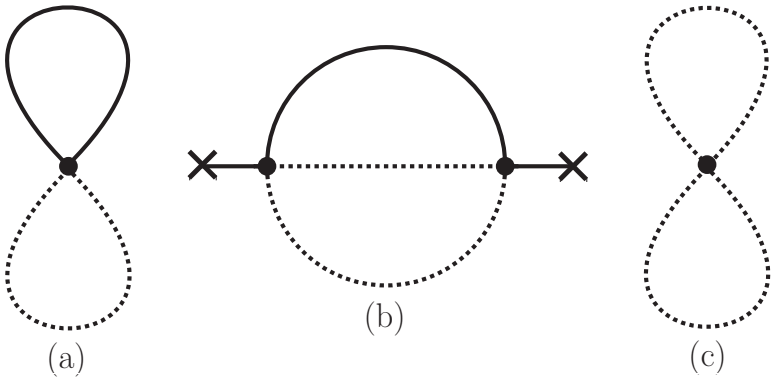
- treat the equation of motion for φ at the **1PI** level [only need diagram (a)]:

$$-\partial^2\varphi + \Pi(\varphi;x)\,\varphi(x) = 0\,, \qquad \Pi(\varphi;x) = \frac{\lambda N}{2}\,S(\varphi;x,x)\,.$$

- treat the equation of motion for the X propagator at **tree-level**:

$$\left(-\partial^2 + \frac{\lambda}{2}\varphi^2\right) S(\varphi;x,y) = \delta^{(4)}(x-y)\,.$$

- **neglect** the Φ propagator altogether.



Iterative procedure

Introducing a small $\mathbb{Z}_2$ breaking (g small), we use the **thin-** and **planar-wall approximations**, as before, and employ an iterative procedure:

[Garbrecht & Millington, PRD92 (2015) 125022]

1. Calculate a **first approximation** to the **bounce** by promoting the homogeneous field configuration in the CW effective potential to a spacetime-dependent one:

$$-\partial^2\varphi + U_{\text{eff}}^{R\,\prime}(\varphi) = 0\,.$$

2. Solve for the ***X* Green's function**.

3. Calculate the **tadpole correction**, renormalizing in the homogeneous false vacuum.

4. Insert the tadpole correction into the quantum equation of motion and solve for the **bounce**.

5. **Iterate** over steps 2 to 5 until solution has converged sufficiently.

This essentially undoes a **gradient expansion**, putting back the full dependence on the inhomogeneity of the solitonic configuration.

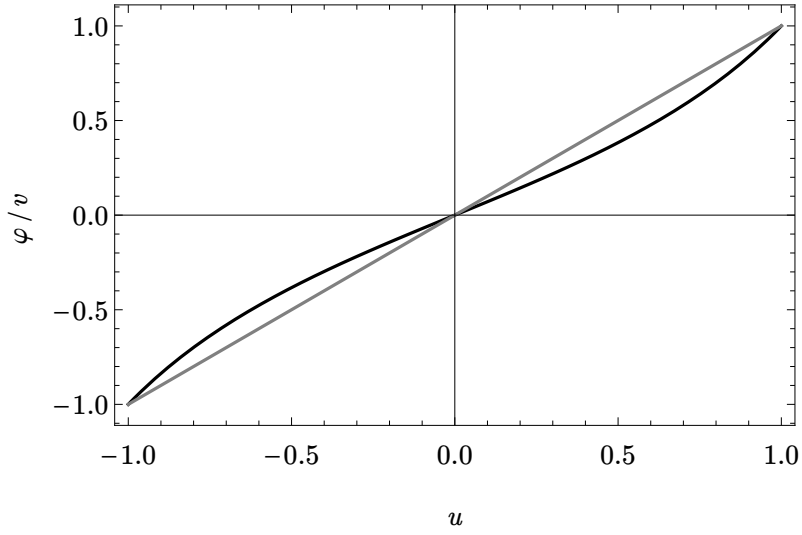
Numerical results

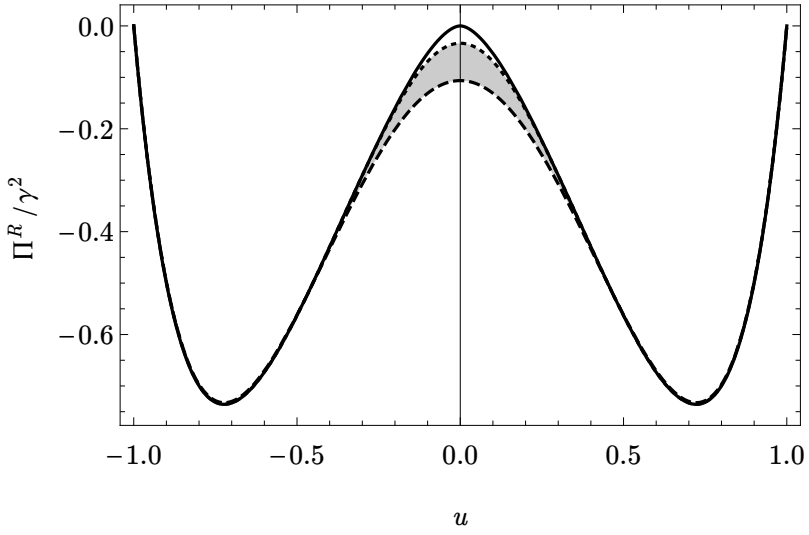
Numerical procedure cross-checked with 3 independent codes: 2 using `Mathematica`'s differential solvers and 1 using Chebyshev pseudo-spectral collocation methods.

[Boyd, *Chebyshev and Fourier Spectral Methods*, 2nd Ed., Dover Publications, New York (2001)]

Numerical sample: $M = 1$ with $0.04 \leq \lambda^2 N \leq 0.4$ (consistency of perturbative 1/N regime checked numerically).

Self-consistent bounce:

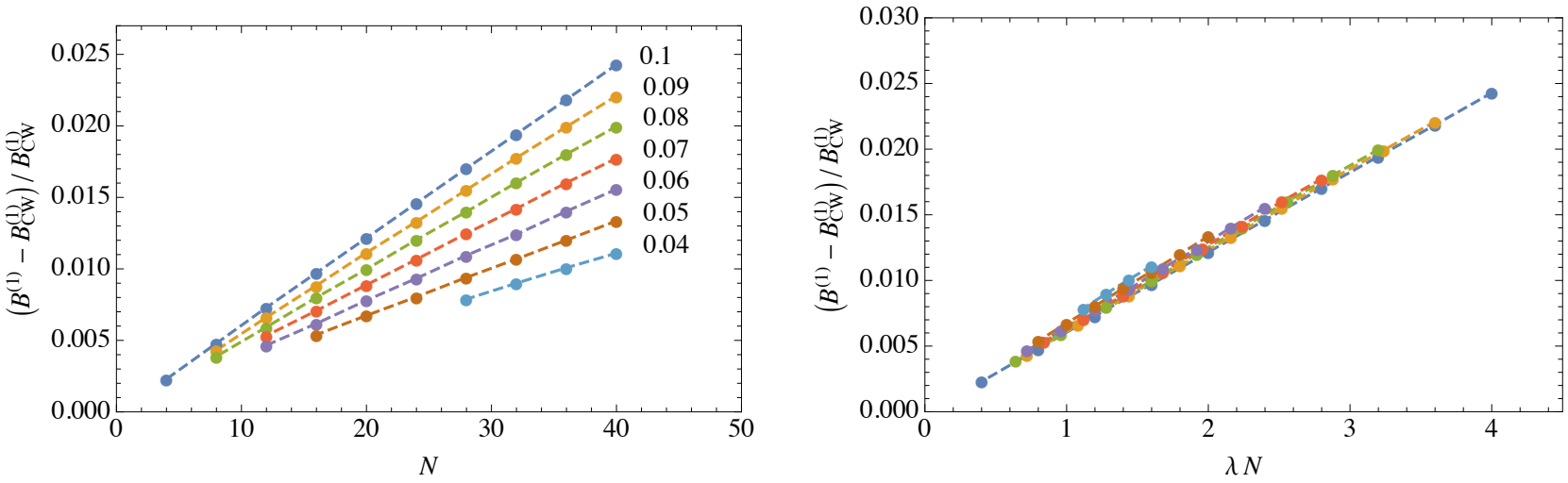




Numerical results

Dominant dependence on the gradients observed in the **one-loop fluctuation determinant** $B^{(1)}$.

[Garbrecht & Millington, PRD92 (2015) 125022; see also arXiv: 1703.05417]



Difference between the Coleman-Weinberg effective potential and self-consistent results shows scaling $\sim \lambda N$ relative to the one-loop CW corrections.

Thus, **gradient effects** can compete with **two-loop effects**, i.e. at **NLO** in the tunneling rate, confirming analytic arguments of E. Weinberg, PRD47 (1993) 4614.

Many methods for calculating the fluctuation determinant on the market: the **heat kernel method**, the **Gel'fand-Yaglom theorem** or . . .

Fluctuation determinant: direct integration

Instead of dealing with numerical Laplace transforms (as in the heat kernel method), we use the **direct integration** method due to **Baacke and Junker**.

[Baacke & Junker, MPLA8 (1993) 2869; PRD49 (1994) 2055; 50 4227; Baacke & Daiber, PRD51 (1995) 795; Baacke, PRD78 (2008) 065039]

Use a **partial-wave decomposition** of the eigenfunctions of the **fluctuation operator** with eigenvalues λ_{nj}:

$$f_{nj\{\ell\}}(x) \;=\; \phi_{nj}(r) \;\underbrace{Y_{j\{\ell\}}(\mathbf{e}_r)}_{\text{hyperspherical harmonics}}$$

The one-loop corrections can be written:

$$B^{(1)} \;=\; \frac{N}{2} \sum_{n,j,\{\ell\}} \ln \frac{\lambda_{nj}}{\lambda_{nj}^{(v)}} \;=\; \frac{N}{2} \sum_{n,j} (j+1)^2 \ln \frac{\lambda_{nj}}{\lambda_{nj}^{(v)}}$$

Shift the mass by an amount $s \in \mathbb{R}$ and decompose the Green's function as

$$S_s(\varphi; x, x) \;=\; \frac{1}{2\pi^2} \sum_{n,j} (j+1)^2 \frac{\phi_{nj}^*(r)\phi_{nj}(r)}{\lambda_{nj} + s} \;.$$

We can then show that

$$B^{(1)} \;=\; -\frac{N}{2} \int_0^{\Lambda^2} \mathrm{d}s \int \mathrm{d}^4x \left(S_s(\varphi; x, x) \;-\; S_s(v; x, x) \right) .$$

Extensions

Fermions?

Consider a toy Higgs-Yukawa theory with N fermions:

$$\mathcal{L} \supset \sum_{i=1}^{N} \bar{\Psi}_i \gamma_\mu \partial_\mu \Psi_i + \kappa \sum_{i=1}^{N} \bar{\Psi}_i \Phi \Psi_i \ .$$

We must carefully handle the **four-dimensional angular-momentum structure**:

[Ai, Garbrecht & Millington, in preparation]

$$D(x, x') = \sum_{\lambda} \left[a_\lambda(r, r') + b_\lambda(r, r') \gamma \cdot x \right] \tilde{D}_\lambda(\mathbf{e}_r, \mathbf{e}'_r) \ ,$$

$$\gamma \cdot x \left[\frac{r \cdot \partial - \mathcal{J}}{r^2} a_\lambda + m(r) b_\lambda \right] + m(r) a_\lambda + \left[\frac{\partial}{\partial r} + \frac{\mathcal{J} + 3}{r} \right] r b_\lambda = \frac{\delta(r - r')}{r^3}$$

The smaller the coupling of the scalar or fermion spectators, the larger the relative impact of the gradients (but the smaller the overall corrections).

Beyond thin wall?

Consider the scale-invariant, classical abyssal potential, highlighted earlier.

The quantum corrections play a pivotal role at **LO** in breaking the scale invariance. However, we must carefully handle the **zero and negative eigenmodes**, and perturbative truncations need to be treated delicately.

[Garbrecht & Millington, in preparation]

So how important are gradients?

It depends on . . .

. . . the parametric dependence of the vev on the couplings:

In the Coleman-Weinberg SSB example, the gradients had an impact (on the loop corrections) only at **NLO**. In the archetypal tree-level SSB example, there were corrections at **LO**.

$\Leftarrow$ In the tree-level case, the vev is enhanced by a factor of $1/\sqrt{\lambda}$ relative to the mass. In the Coleman-Weinberg example, the couplings were such that the vev was comparable to the mass.

. . . the symmetry of the critical bubble about the bubble wall:

In the **thin-wall regime**, the gradient corrections are suppressed due to the symmetry about the centre of the bubble wall: the field is going through zero just where the gradients are maximal. This is not expected to be the case in the **thick-wall regime**.

. . . the relevance of quantum effects to the negative-semi-definite eigenmodes:

Watch this space for full details of the abyssal example highlighted above . . .

Conclusions

- Described how a **Green's function method** can be used to calculate **self-consistent quantum corrections to tunnneling configurations**, whilst accounting fully for the background inhomogeneity of the tunneling soliton.
- Described the relative **importance of gradient effects** in relation to both the relative size of the mass and vev of the field, and the thin- vs. thick-wall regimes.
- Highlighted the impact quantum corrections can have on the **negative semi-definite eigenmodes**.
- What about first-order thermal phase transitions? Can we embed all of the above methodology into **non-equilibrium field theory** and, in so doing, account fully for gradients in the one-loop, finite-temperature corrections?
- Much more to come soon . . .

Thank you for your attention.

Back-up slides
Explicit results (tree-level example)

$$B = \frac{8\pi^2 R^3 \gamma^3}{\lambda}$$

$$B^{(1)} = -B\left(\frac{3\lambda}{16\pi^2}\right)\left[\frac{\pi}{3\sqrt{3}} + 21 + \frac{2542}{15}\frac{\gamma^2}{m_\chi^2}N\right]$$

$$\begin{aligned} B^{(2)} + B^{(2)\prime} &= \frac{1}{2}\int \mathrm{d}^4x\, \varphi(u)\left(\Pi^R(u) + N\Sigma^R(u)\right)\delta\varphi(u) \\ &= -\frac{B}{3}\left(\frac{3\lambda}{16\pi^2}\right)^2\left[\frac{291}{8} - \frac{37}{4}\frac{\pi}{\sqrt{3}} + \frac{5}{56}\frac{\pi^2}{3}\right. \\ &\qquad \left. + \left(\frac{667}{2} - \frac{2897}{42}\frac{\pi}{\sqrt{3}}\right)\frac{\gamma^2}{m_\chi^2}N + \frac{5829}{14}\frac{\gamma^4}{m_\chi^4}N^2\right] \end{aligned}$$

Back-up slides

Fluctuation determinant: heat-kernel method (used for tree-level example)

[Diakonov, Petrov & Yung, PLB130 (1983) 385; Sov. J. Nucl. Phys. 39 (1984) 150 [Yad. Fiz. 39 (1984) 240]; Konoplich, Theor. Math. Phys. 73 (1987) 1286 [Teor. Mat. Fiz. 73 (1987) 379; Vassilevich, Phys. Rept. 388 (2003) 279; Carson & McLerran, PRD41 (1990) 647; Carson, Li, McLerran &Wang, PRD42 (1990) 2127; Carson, PRD42 (1990) 2853]

Fluctuation determinant over the positive-definite modes:

$$\mathrm{tr}^{(5)} \ln G^{-1}(\varphi; x) \;=\; -\int \mathrm{d}^4x \int_0^\infty \frac{\mathrm{d}\tau}{\tau}\; K(\varphi; x, x|\tau)\;.$$

The **heat kernel** is the solution to the **heat-flow equation**

$$\partial_\tau K(\varphi; x, x'|\tau) \;=\; G^{-1}(\varphi; x) K(\varphi; x, x'|\tau)\;,$$

with $K(\varphi; x, x'|0) = \delta^{(4)}(x - x')$.

It's **Laplace transform**

$$\mathcal{K}(\varphi; x, x'|s) \;=\; \int_0^\infty \mathrm{d}\tau\; e^{-s\tau}\; K(\varphi; x, x'|\tau)$$

is just the **Green's function** with $k^2 \to k^2 + s$.

[cf. Gel'fand-Yaglom thoerem, JMP1 (1960) 48; Baacke & Kiselev PRD48 (1993); Dunne & Kirsten, JPA39 (2006) 11915; Dunne, JPA41 (2008) 304006]

Back-up slides
Additional numerical results