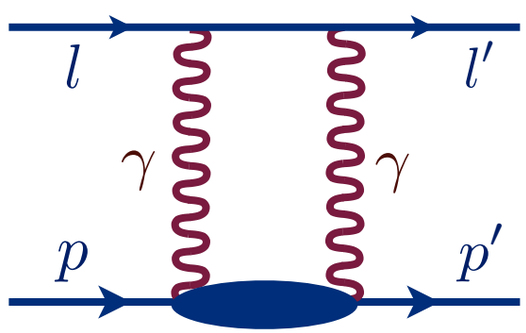
Amherst Center for Fundamental Interactions
Physics at the interface: Energy, Intensity, and Cosmic frontiers
University of Massachusetts Amherst

28 September, 2017

Two-photon exchange calculations versus data



Oleksandr Tomalak
Johannes Gutenberg University,
Mainz, Germany

Scattering experiments and 2γ

- 2γ is not among standard radiative corrections

$$\sigma^{\rm exp} \equiv \sigma_{1\gamma}(1 + \delta_{\rm rad} + \delta_{\rm soft} + \delta_{2\gamma})$$

- charge radius insensitive to 2γ model

- magnetic radius depends on 2γ model

J. C. Bernauer et al. (2014)

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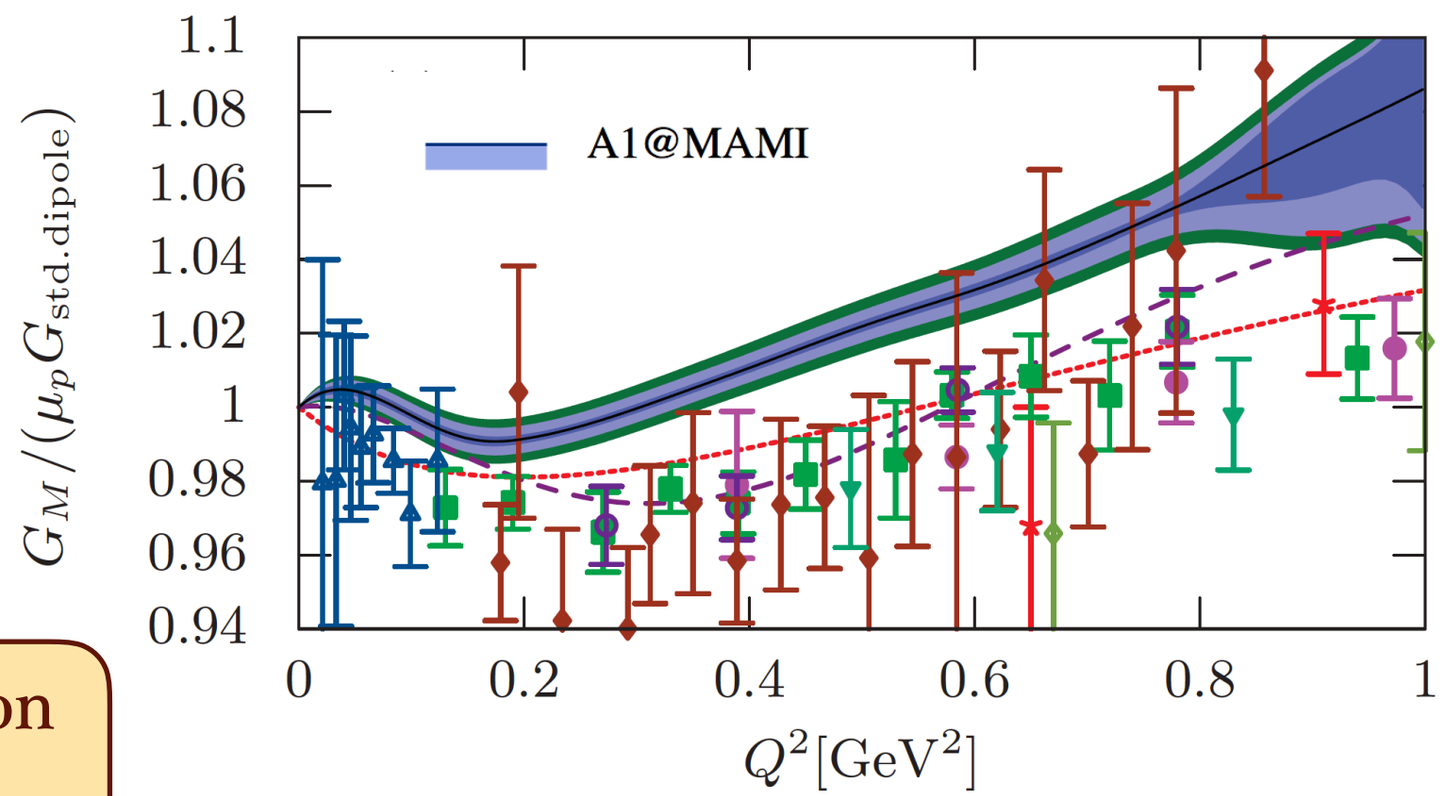
magnetic form factor



- 2 % systematic deviation MAMI vs. world data

μH hyperfine splitting and 2γ

1S HFS in μH
PSI, J-PARC, RIKEN-RAL
1 ppm accuracy

R. Pohl et al. (2016)

- leading theoretical uncertainty: 213 ppm from 2γ, 109 ppm from 2γ

C. Carlson, V. Nazaryan, K. Griffioen (2011) Cl. Peset and A. Pineda (2017)

- HFS in terms of forward lepton-proton scattering amplitudes

O. Tomalak (2017)

- traditional decomposition:

$$\Delta_{\rm HFS} = \Delta_{\rm Z} + \Delta^{\rm R} + \Delta^{\rm pol}$$

Zemach term
$G_E,\ G_M$

recoil correction
$G_E,\ G_M$

polarizability
$F_2,\ g_1,\ g_2$

- A1@MAMI fit allows to quantify 2γ uncertainty

J. C. Bernauer et al. (2014)

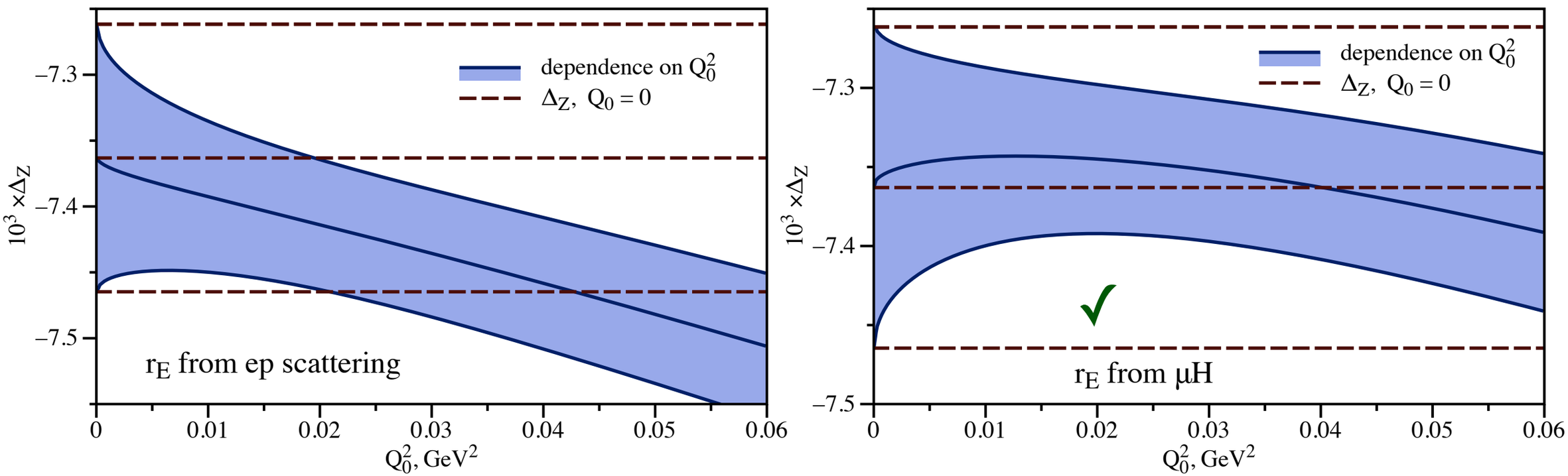
- proton radii, form factors and spin structure are important

Zemach contribution

- Zemach correction expanding form factors

$$\Delta_Z = \frac{8\alpha m_r}{\pi}\int_{Q_0}^{\infty}\frac{dQ}{Q^2}\left(\frac{G_M(Q^2)\,G_E(Q^2)}{\mu_P}-1\right)+\frac{4\alpha m_r Q_0}{3\pi}\left(-r_E^2-r_M^2+\frac{r_E^2 r_M^2}{18}Q_0^2\right)$$

- dependence on splitting: consistency check



O. Tomalak (2017)

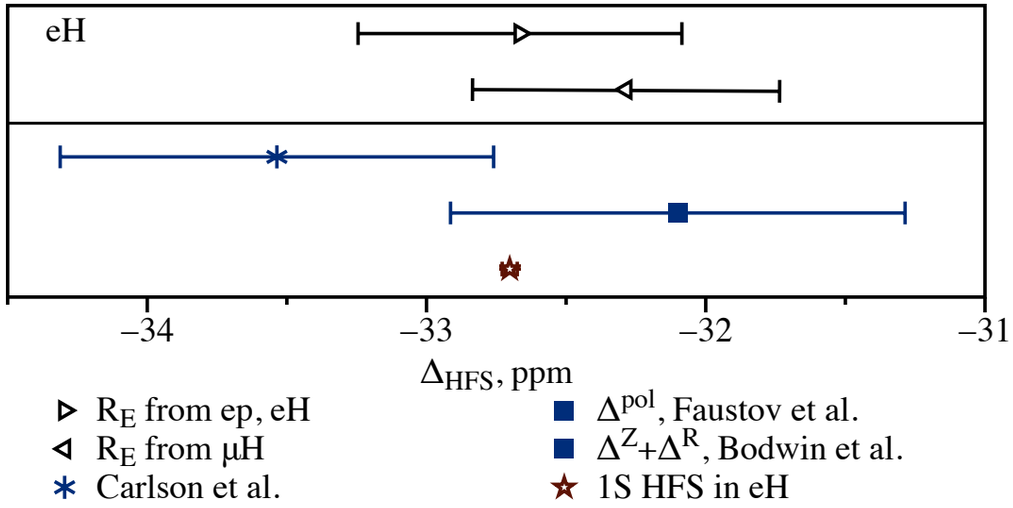
- 95 ppm change for μH and ep radii with $Q_0 = 0.2$ GeV

- 3 times more precise: 140 ppm → 49 ppm
- magnetic radius is important

Hyperfine splitting and 2γ

- compare with precise 1S HFS from eH

achieved accuracy in 70th: 10^{-12}



- dispersive evaluation and phenomenological extractions agree

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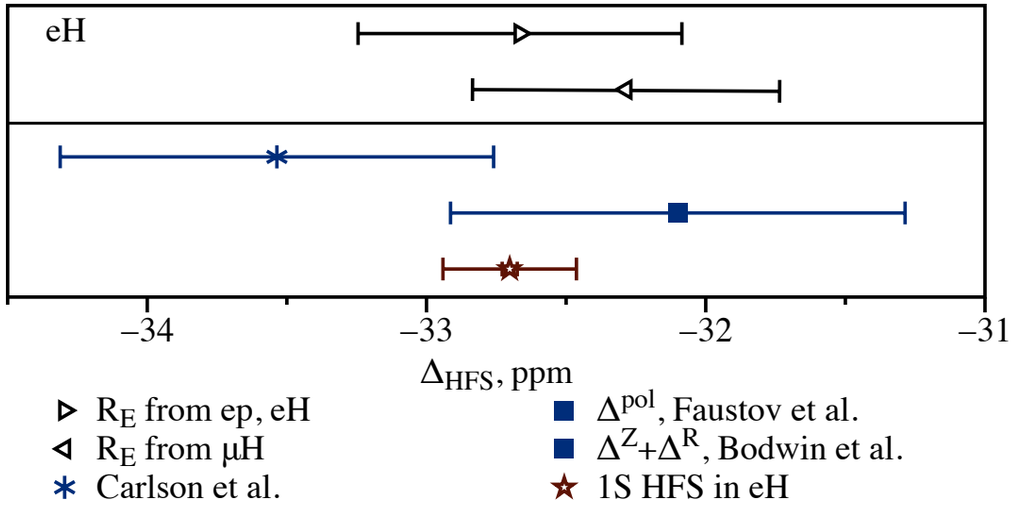

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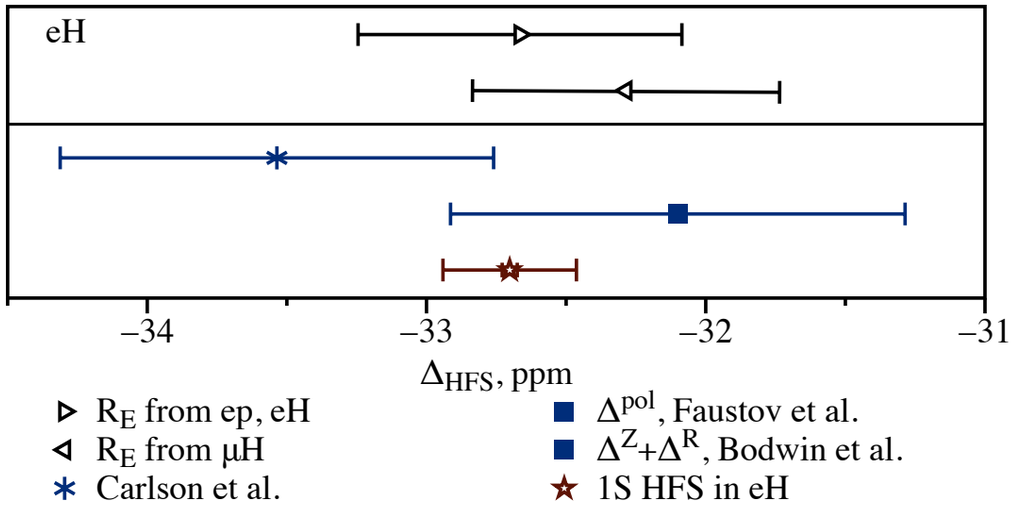


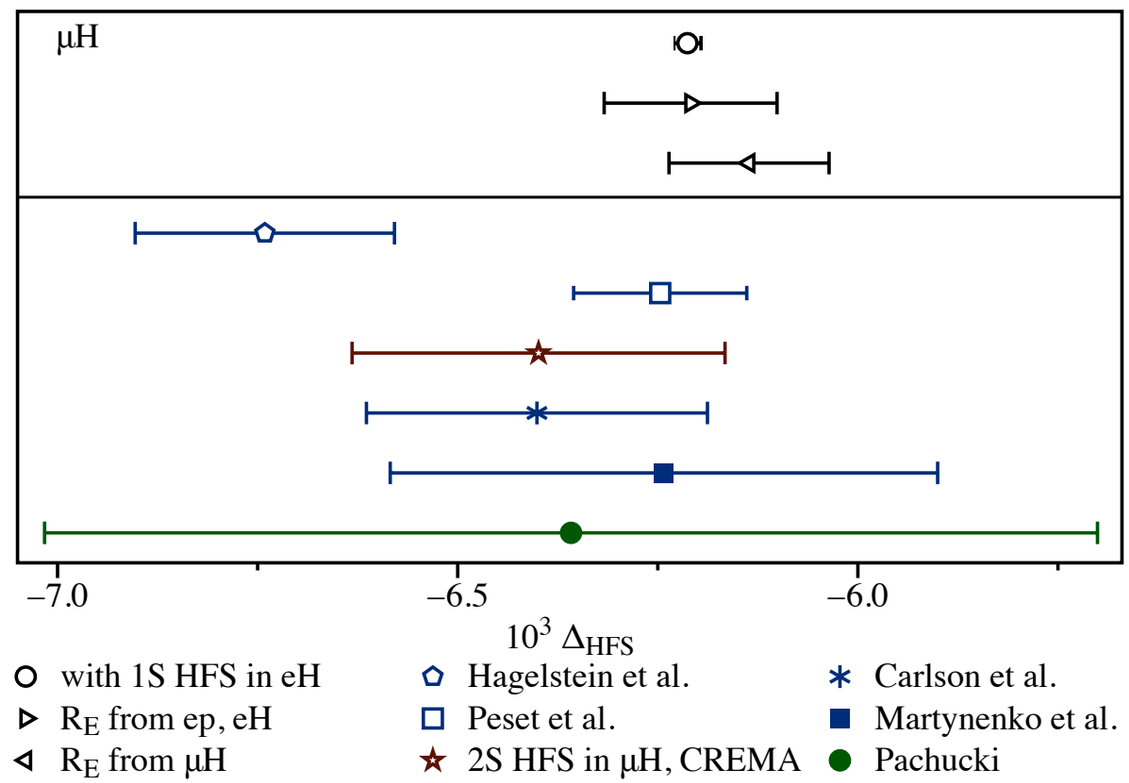
- exploit eH HFS measurements

  scaled by a reduced mass m_r

$$\Delta(\mu\mathrm{H}) = \frac{m_r(m_\mu)}{m_r(m_e)}\Delta\,(\mathrm{eH}) +$$

$$\Delta_{\mathrm{HFS}}\,(m_\mu) - \frac{m_r(m_\mu)}{m_r(m_e)}\Delta_{\mathrm{HFS}}\,(m_e)$$

- uncertainty: 100 ppm → 16 ppm !!!



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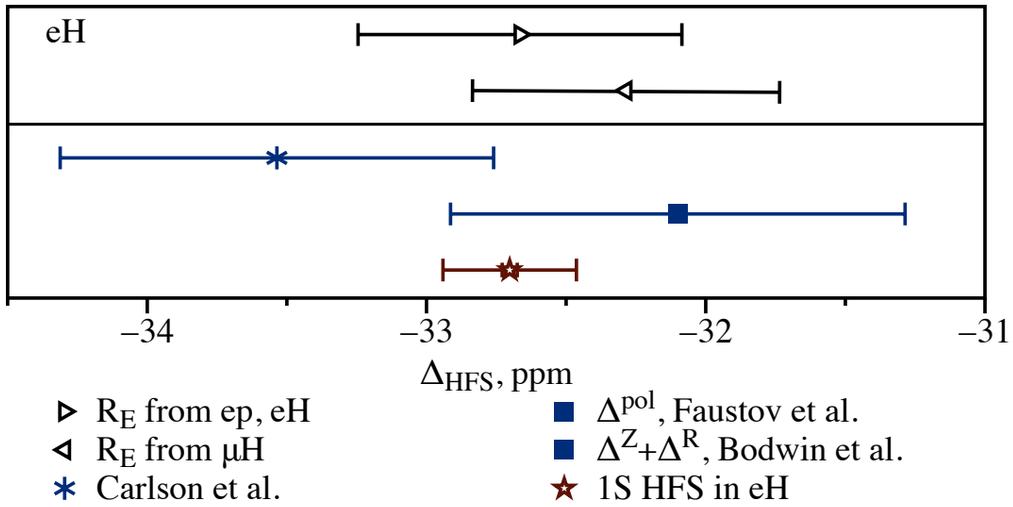

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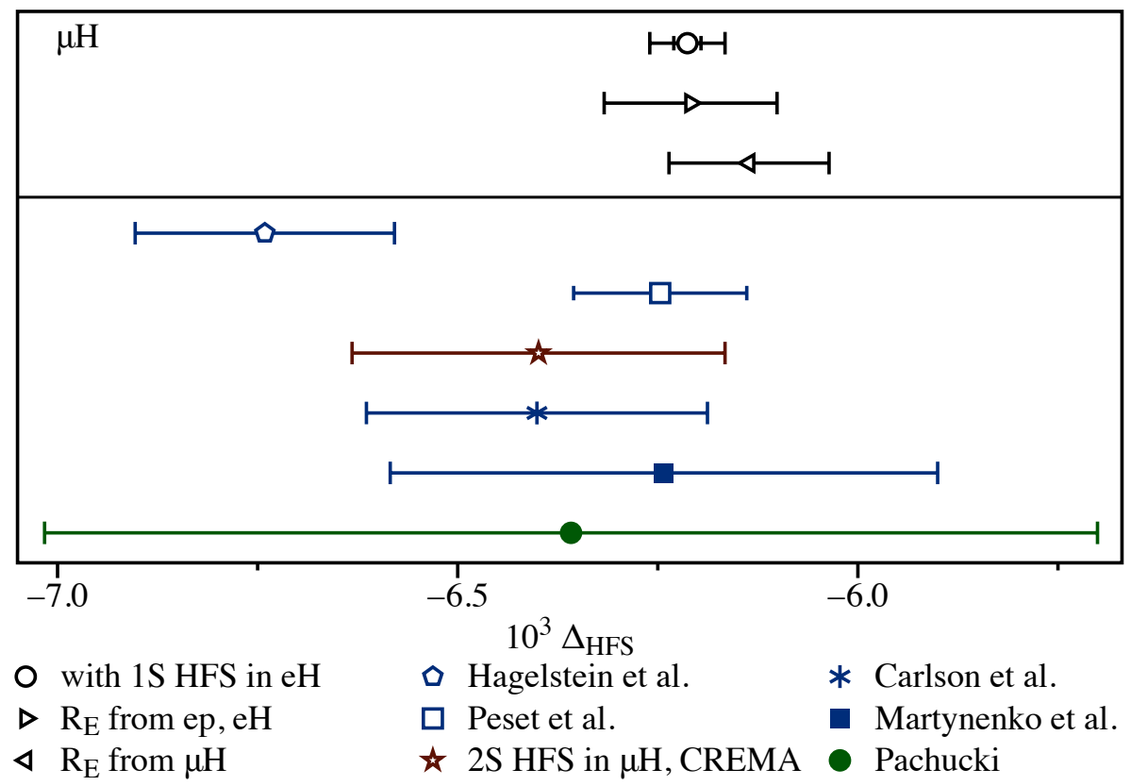


- dispersive evaluation and phenomenological extractions agree

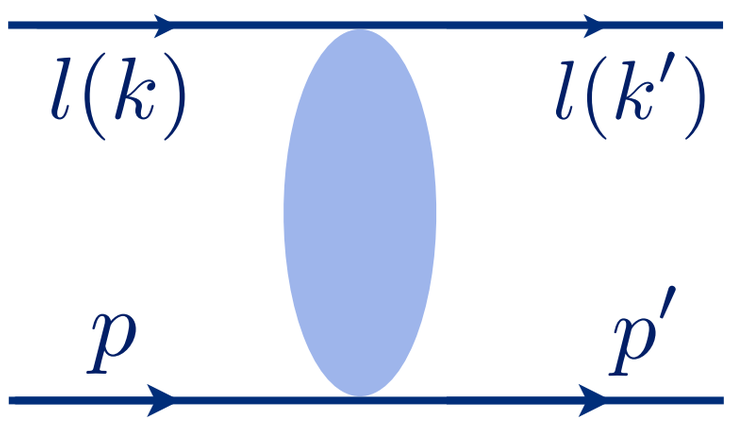
Elastic lepton-proton scattering

momentum transfer

$$Q^2 = -(k - k')^2$$

crossing-symmetric variable

$$\nu = \frac{(k, p + p')}{2}$$



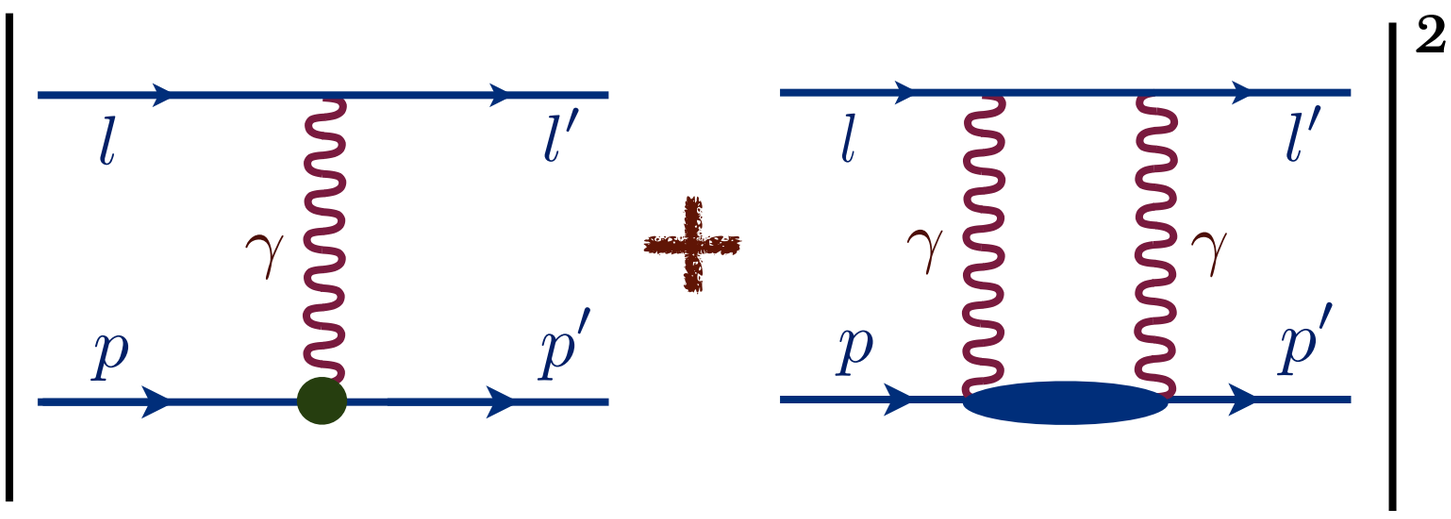
photon polarization parameter

$$\varepsilon$$

forward scattering

$$\varepsilon \to 1$$

- leading 2ɣ contribution: interference term

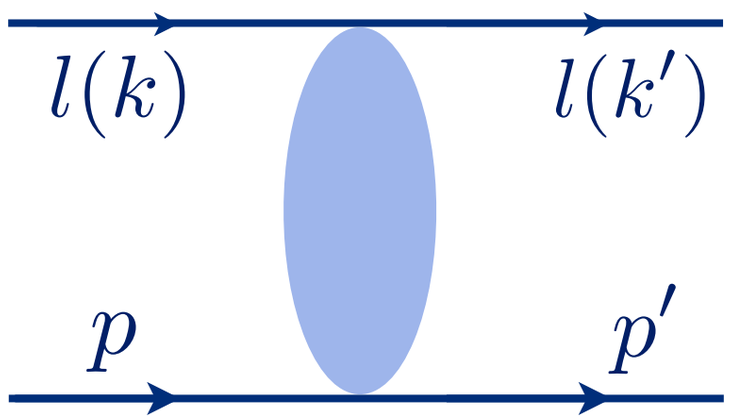


$$\delta_{2\gamma} = \frac{2 \sum_{\text{spin}} T^{1\gamma} \Re T^{2\gamma}}{\sum_{\text{spin}} |T^{1\gamma}|^2}$$

- 2ɣ correction to cross section is given by amplitudes real parts

Elastic lepton-proton scattering

$$K = \frac{k + k'}{2}$$



$$P = \frac{p + p'}{2}$$

- electron-proton scattering: 3 structure amplitudes

$$T^{\text{non-flip}} = \frac{e^2}{Q^2} \bar{l}\gamma_\mu l \cdot \bar{N} \left(\mathcal{G}_M(\nu, Q^2)\gamma^\mu - \mathcal{F}_2(\nu, Q^2)\frac{P^\mu}{M} + \mathcal{F}_3(\nu, Q^2)\frac{\hat{K}P^\mu}{M^2} \right) N$$

P.A.M. Guichon and M. Vanderhaeghen (2003)

- muon-proton scattering: add helicity-flip amplitudes

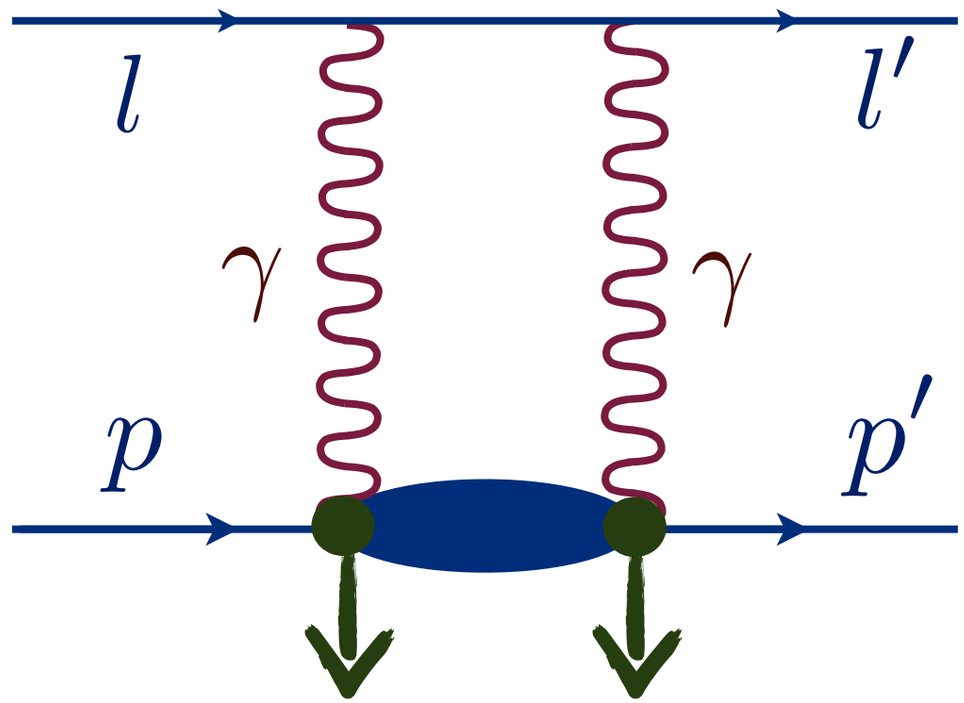
$$T^{\text{flip}} = \frac{e^2}{Q^2}\frac{m}{M}\bar{l}l \cdot \bar{N} \left(\mathcal{F}_4(\nu, Q^2) + \mathcal{F}_5(\nu, Q^2)\frac{\hat{K}}{M} \right) N + \frac{e^2}{Q^2}\frac{m}{M}\mathcal{F}_6(\nu, Q^2)\bar{l}\gamma_5 l \cdot \bar{N}\gamma_5 N$$

M. Gorchtein, P.A.M. Guichon and M. Vanderhaeghen (2004)

- 2ɣ correction to cross section is given by amplitudes real parts

non-forward scattering

at low momentum transfer



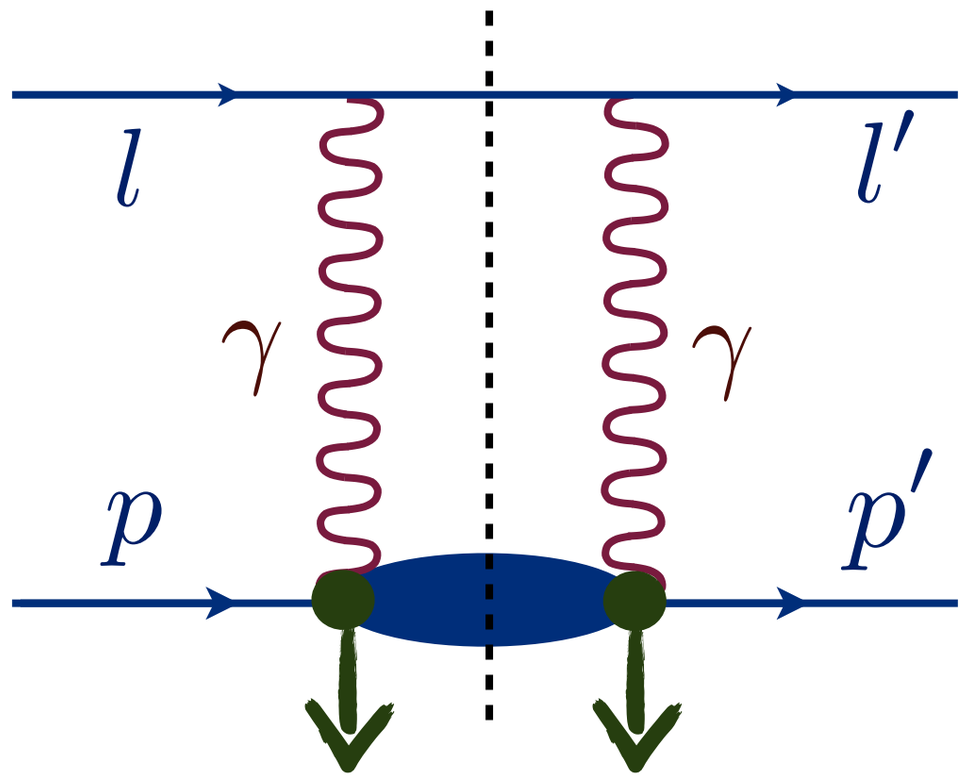
photoproduction vertex or Compton tensor

box diagram

assumption about the vertex

non-forward scattering

at low momentum transfer



photoproduction vertex or Compton tensor

box diagram

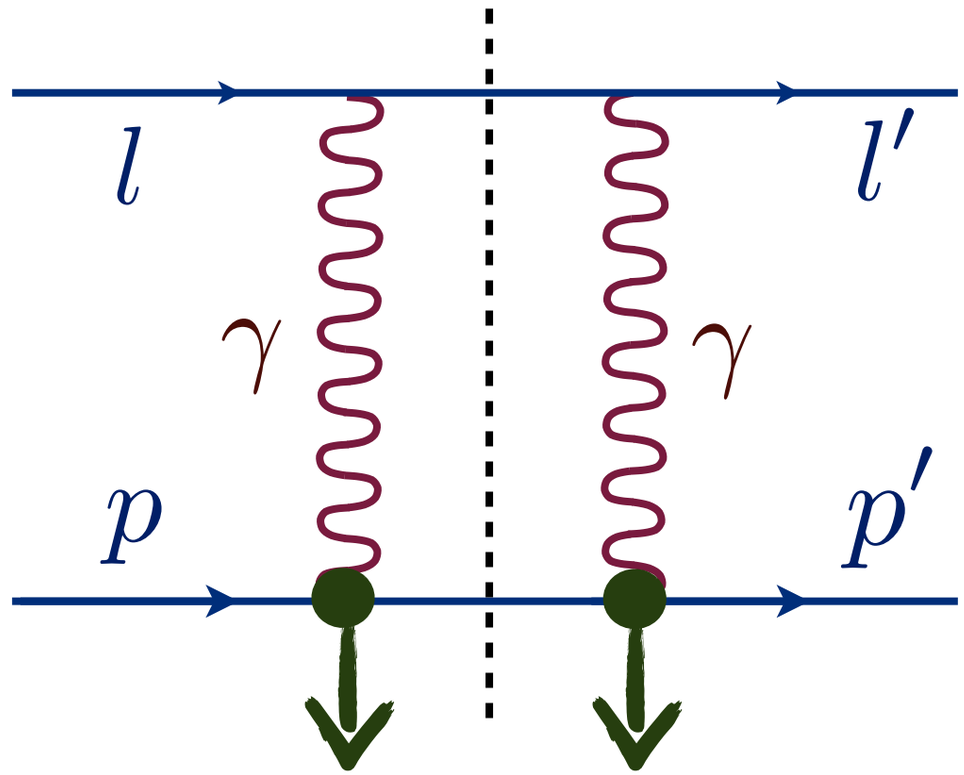
dispersion relations

assumption about the vertex

based on on-shell information

non-forward scattering

proton state



Dirac and Pauli form factors

box diagram

dispersion relations

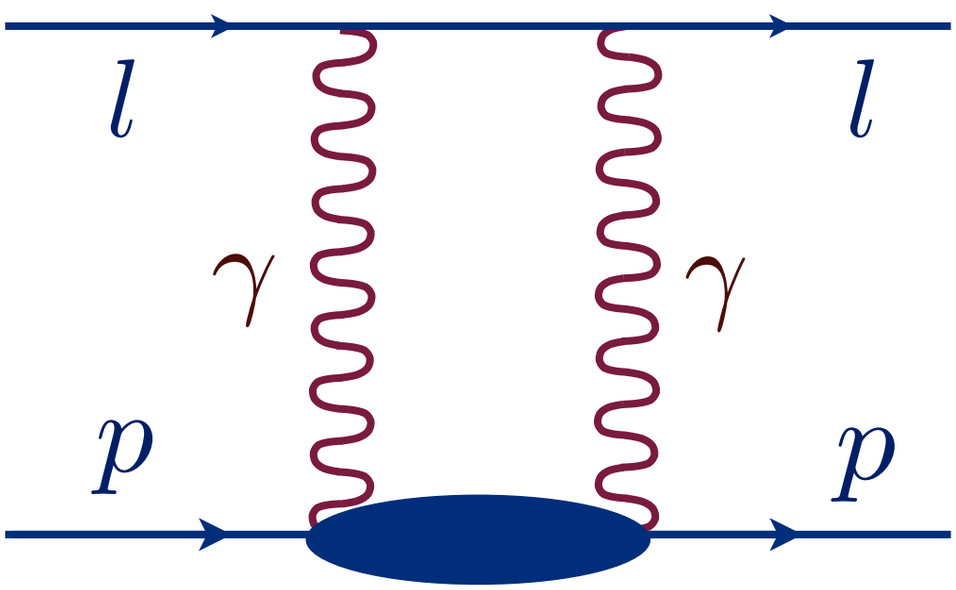
assumption about the vertex

Blunden, Melnitchouk and Tjon (2003)

based on on-shell information

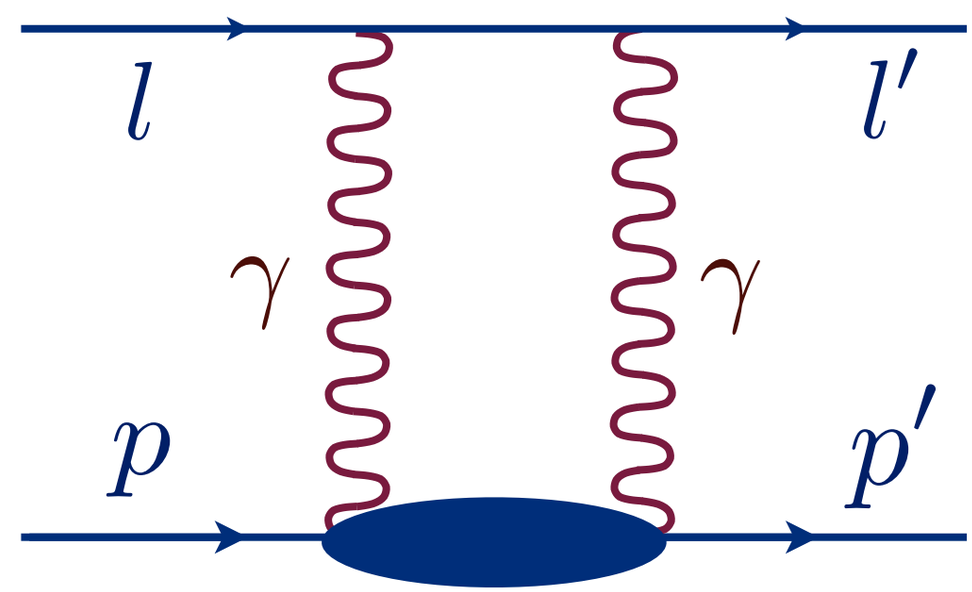
Borisyuk and Kobushkin (2008), O. T. and M. Vanderhaeghen (2014)

forward scattering

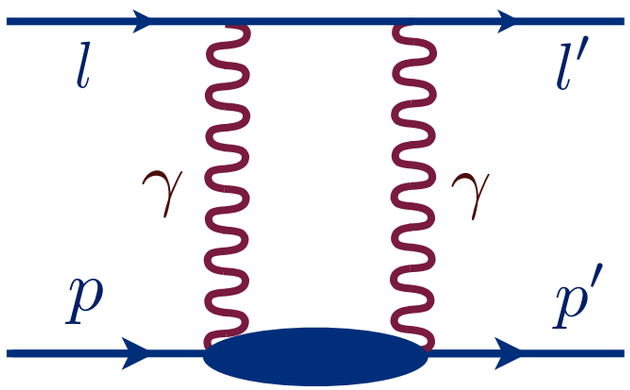


near-forward scattering

account for all inelastic 2γ



Low-Q^2 inelastic 2γ correction (e^-p)



- 2γ blob: near-forward virtual Compton scattering

Feshbach inelastic elastic

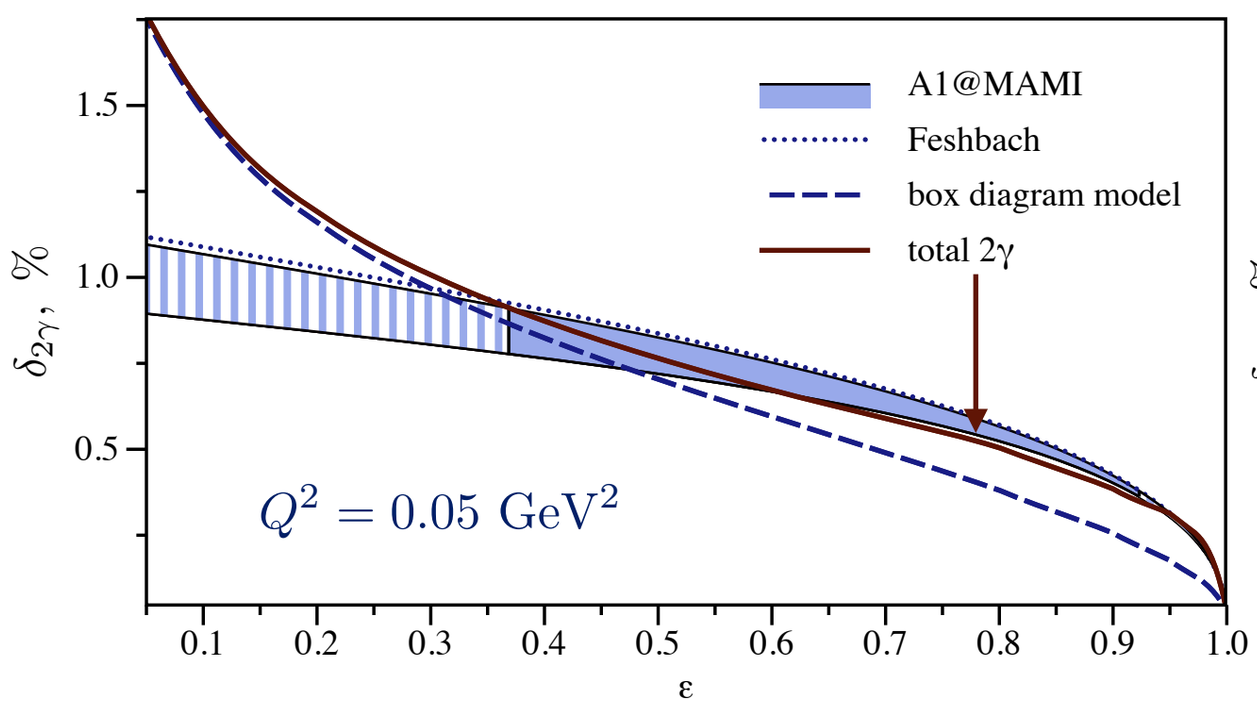
$$\delta_{2\gamma} \sim a\,\sqrt{Q^2} + b\,Q^2 \ln Q^2 + c\,Q^2 \ln^2 Q^2$$

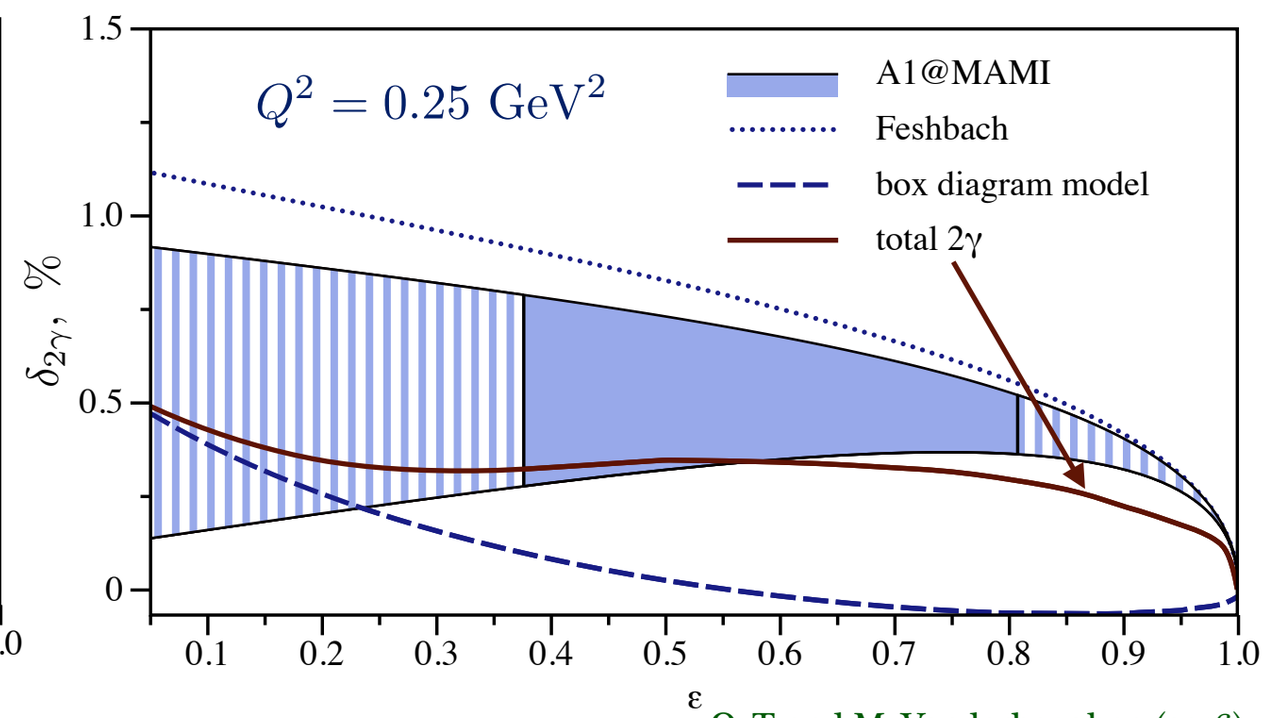
R. W. Brown (1970), M. Gorchtein (2013), O. T. and M. Vanderhaeghen (2014)

unpolarized proton structure

M. E. Christy, P. E. Bosted (2010)

$$\delta_{2\gamma} = \int \mathrm{d}\nu_\gamma \mathrm{d}Q^2 (w_1(\nu_\gamma, Q^2) \cdot F_1(\nu_\gamma, Q^2) + w_2(\nu_\gamma, Q^2) \cdot F_2(\nu_\gamma, Q^2))$$





O. T. and M. Vanderhaeghen (2016)

- 2γ at large ε agrees with empirical fit

r_E extraction ✓

Scattering experiments and 2γ

- charge radius extractions:

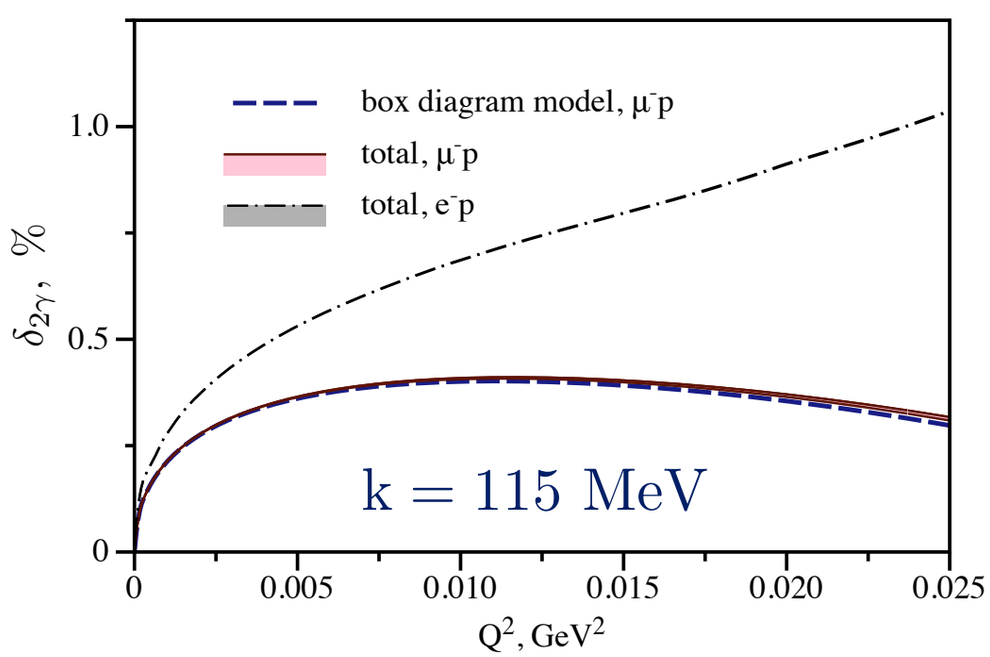
eH, eD spectroscopy	ep scattering
μH, μD spectroscopy	μp scattering ????

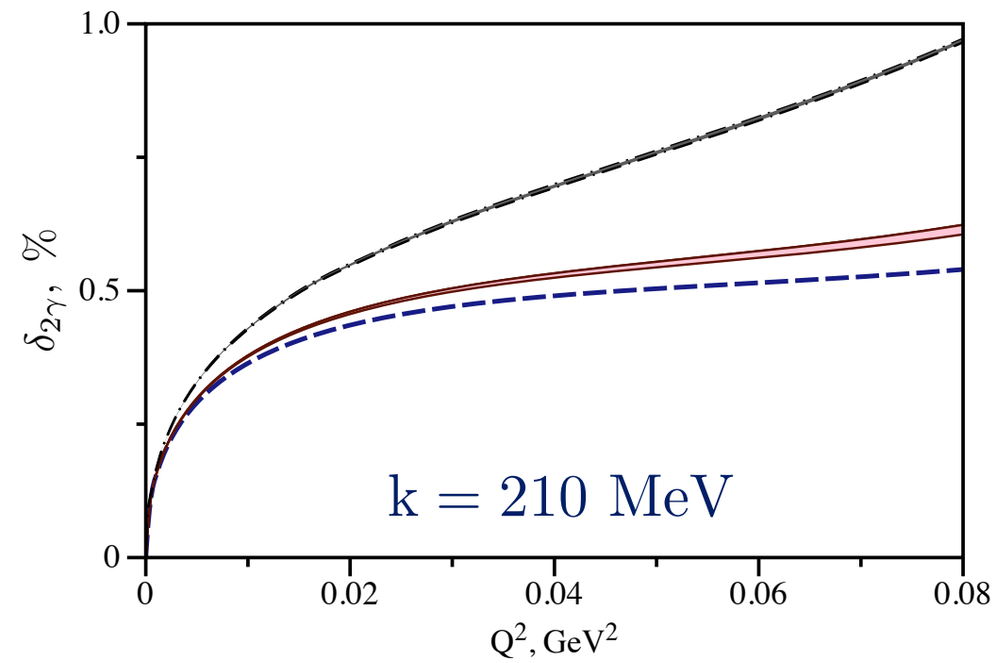
- μp elastic scattering is planned by MUSE@PSI(2018-19)
  measure with both electron/muon charges

- 2γ correction in MUSE ?

MUSE@PSI (2018-19) estimates (μ^-p)

- proton box diagram model + inelastic 2ɣ





O. T. and M. Vanderhaeghen (2014, 2016)

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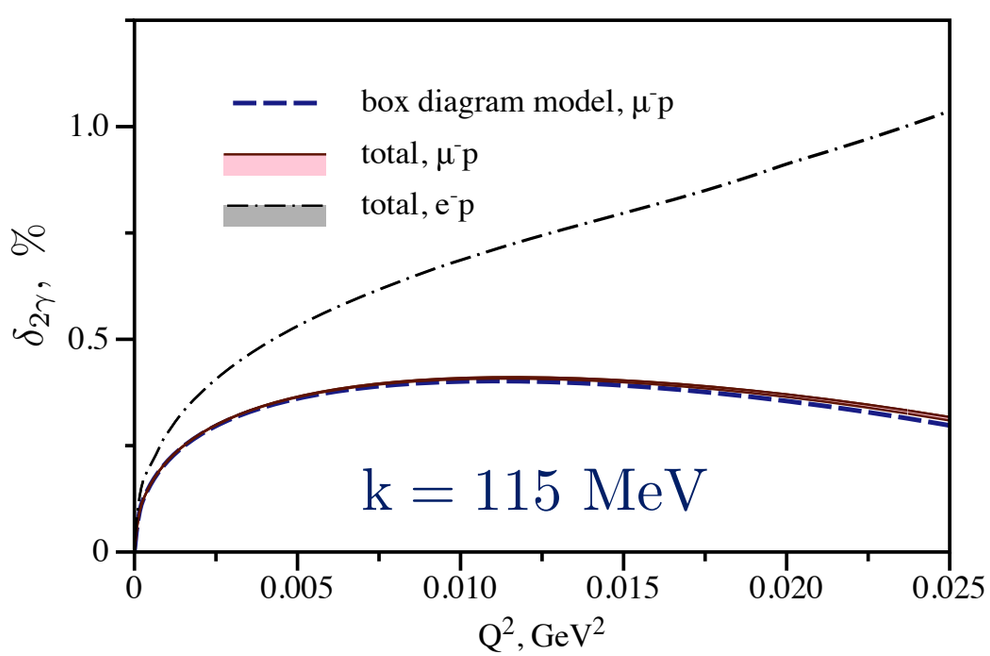
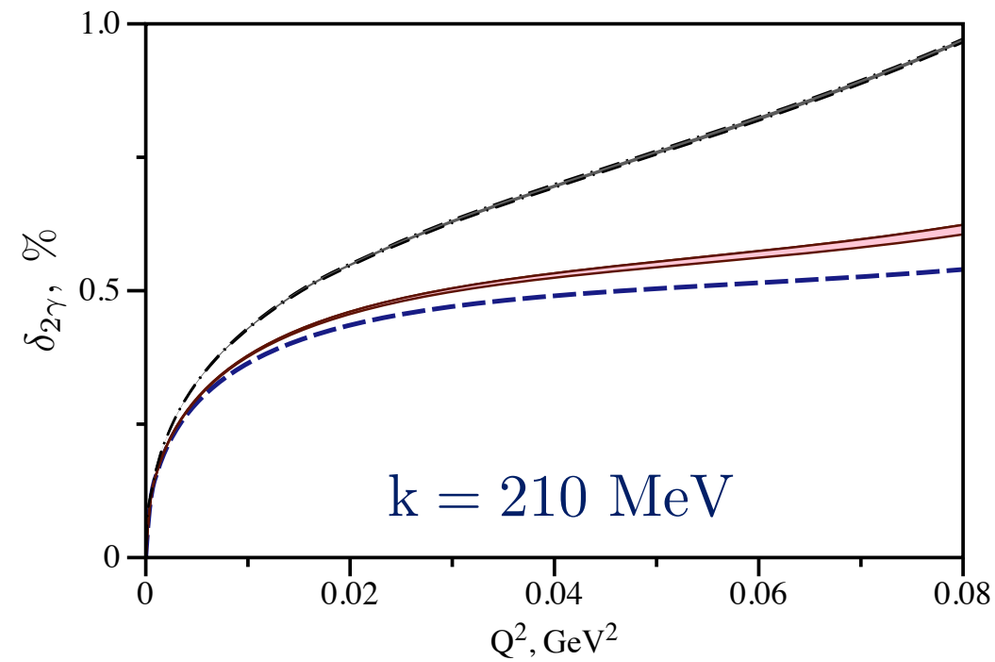




O. T. and M. Vanderhaeghen (2014, 2016)

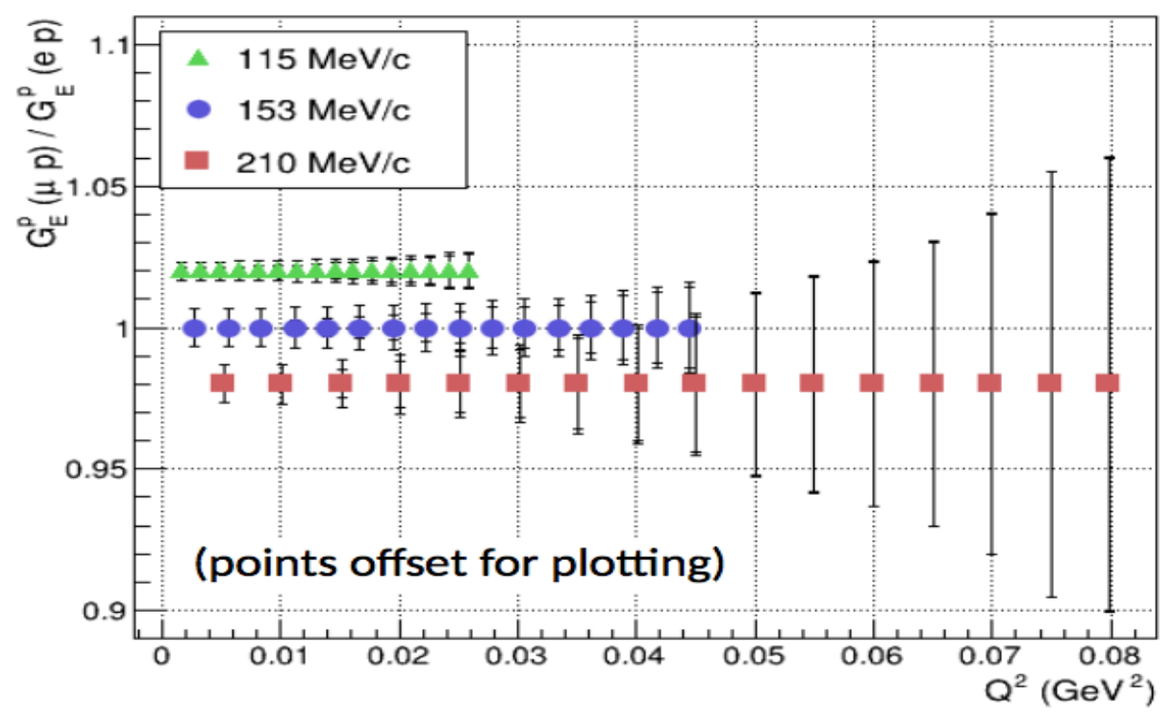
- expected muon over electron ratio

small inelastic 2ɣ

↓

small 2ɣ uncertainty

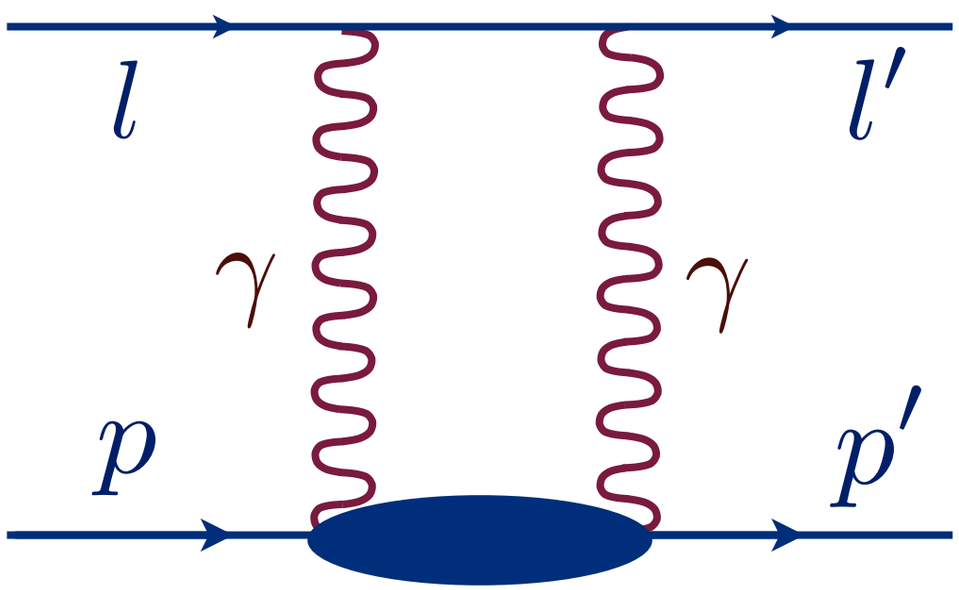
- MUSE can test r_E in one charge channel



K. Mesick talk (PAVI 2014), MUSE TDR (2016)

near-forward scattering

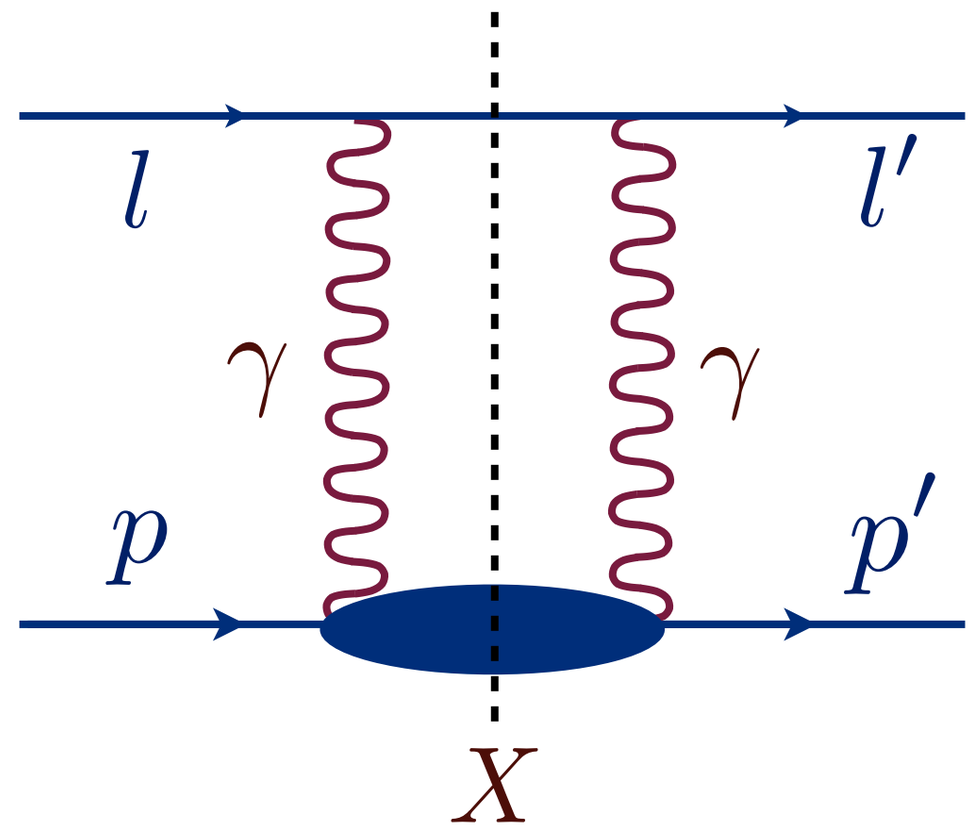
(large ε)



p + all inelastic

dispersion relations

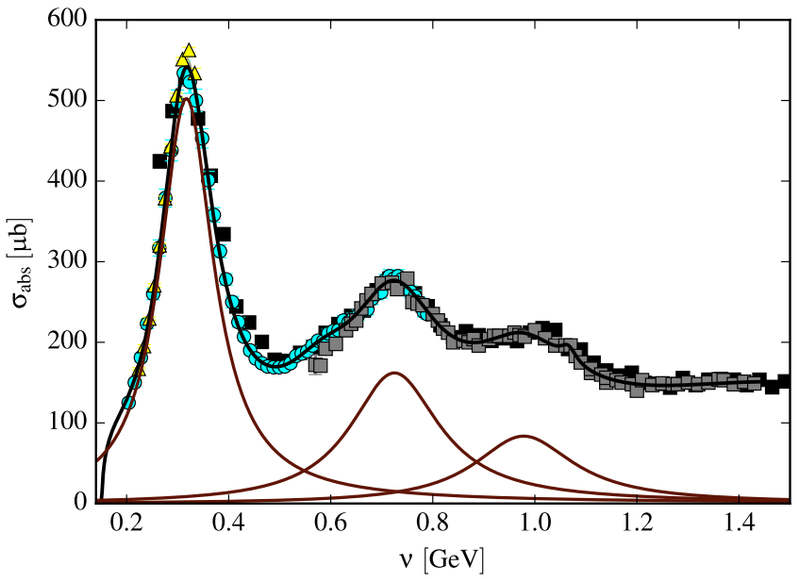
(arbitrary ε)



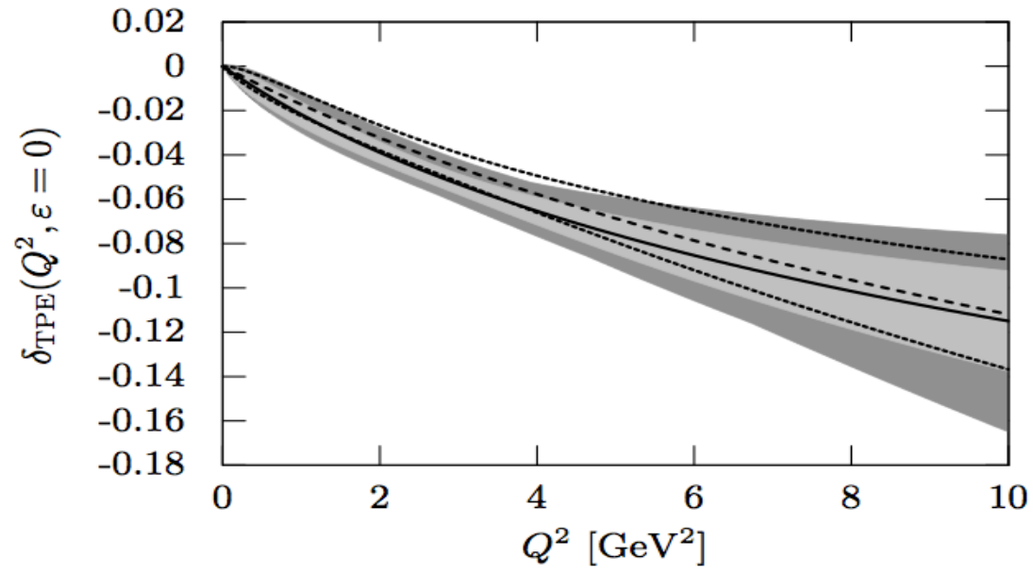
$X = p + \pi N$

Fixed-Q^2 dispersion relation framework

on-shell 1γ amplitudes



2γ prediction



experimental data

cross section correction

unitarity

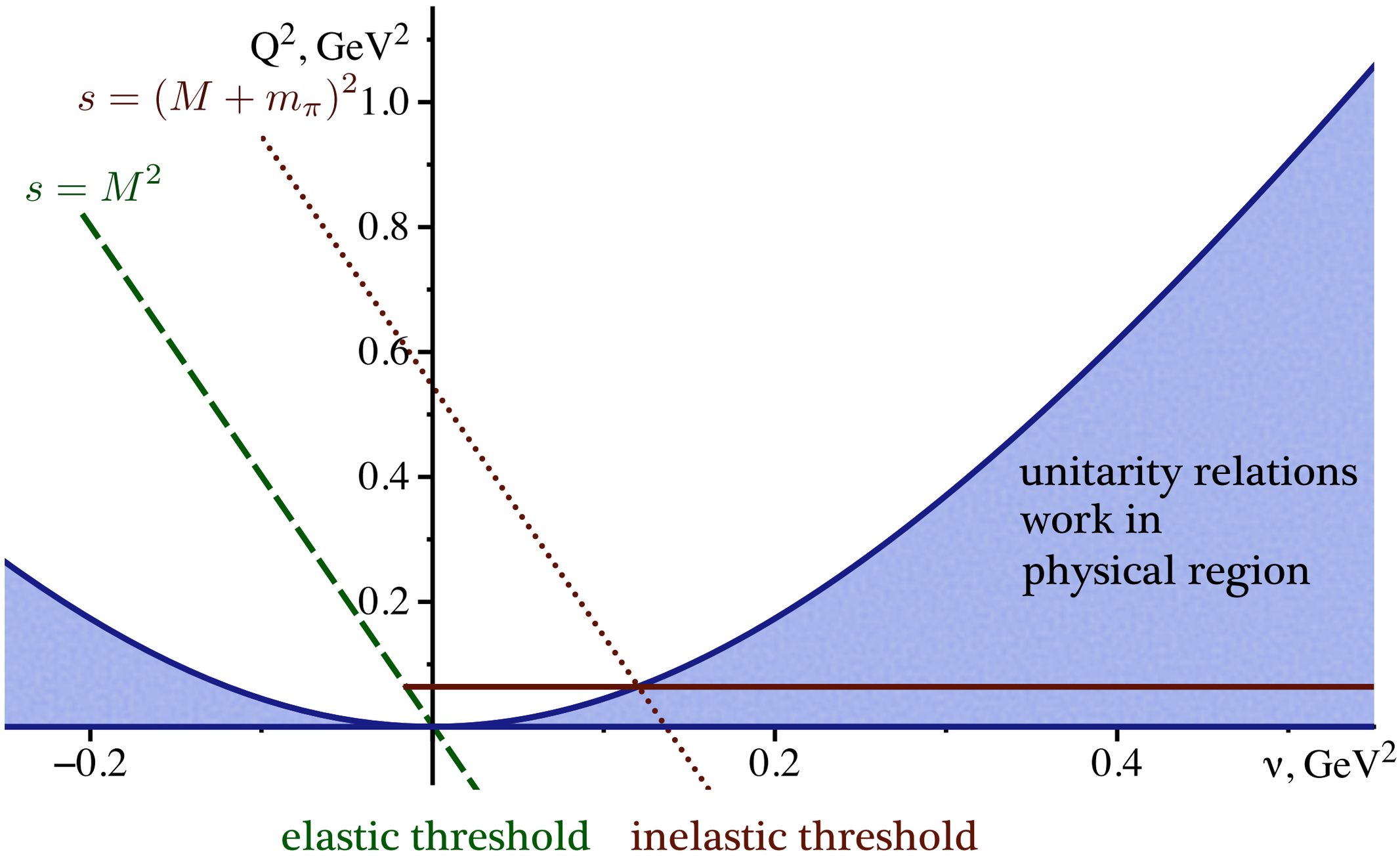
$$\Re\mathcal{F}(\nu) = \frac{2\nu}{\pi}\mathcal{P}\int_{\nu_{min}}^{\infty}\frac{\Im\mathcal{F}(\nu' + i0)}{\nu'^2 - \nu^2}\mathrm{d}\nu'$$

2γ imaginary parts

disp. rel.

2γ real parts

Mandelstam plot (ep)



- proton intermediate state is outside physical region for $Q^2 > 0$
- πN intermediate state is outside physical region for $Q^2 > 0.064$ GeV^2

O. T. and M. Vanderhaeghen (2014)

Analytical continuation. Elastic state

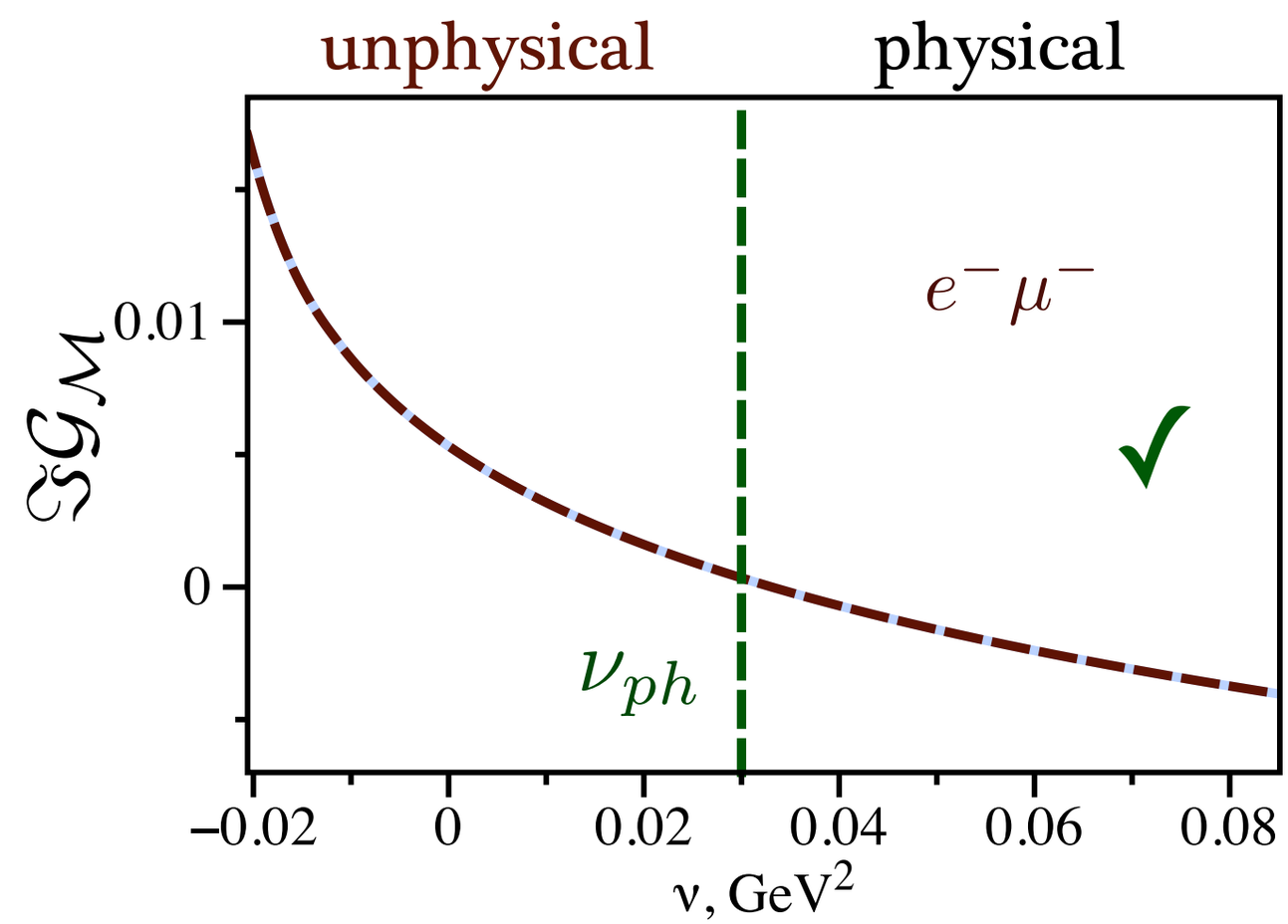
- contour deformation method:

O. T. and M. Vanderhaeghen (2014), Blunden and Melnitchouk (2017)

$\int d\Omega$ → angular integration to integration on curve in complex plane → deform integration contour keeping poles inside going to unph. region

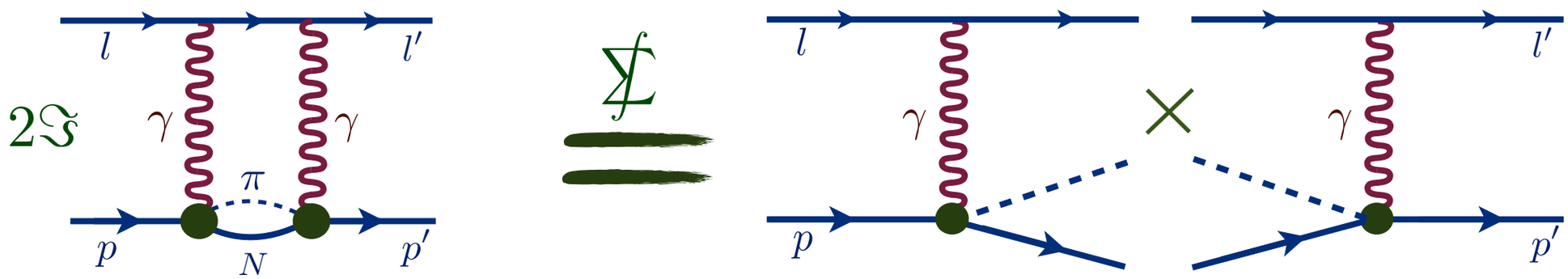
- analytical continuation reproduces results in unphysical region

$Q^2 = 0.1\ \mathrm{GeV}^2$



- central value: form factor fit of A1@MAMI (2014)
- uncertainty: difference to 2γ with dipole form factors

Analytical continuation. πN states

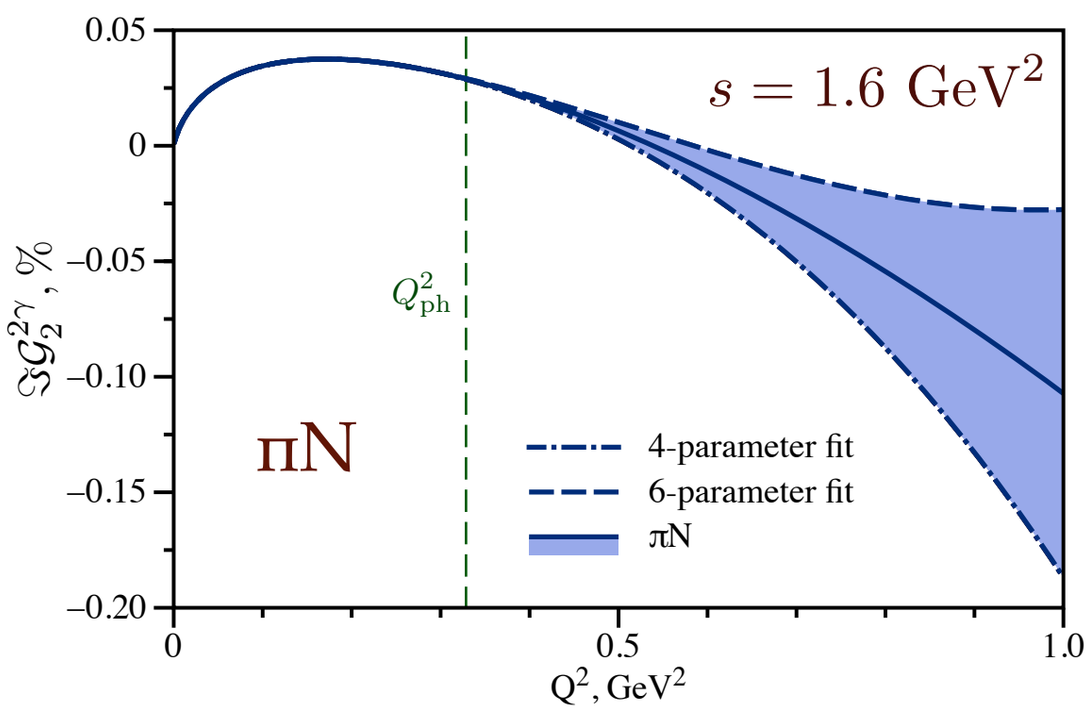


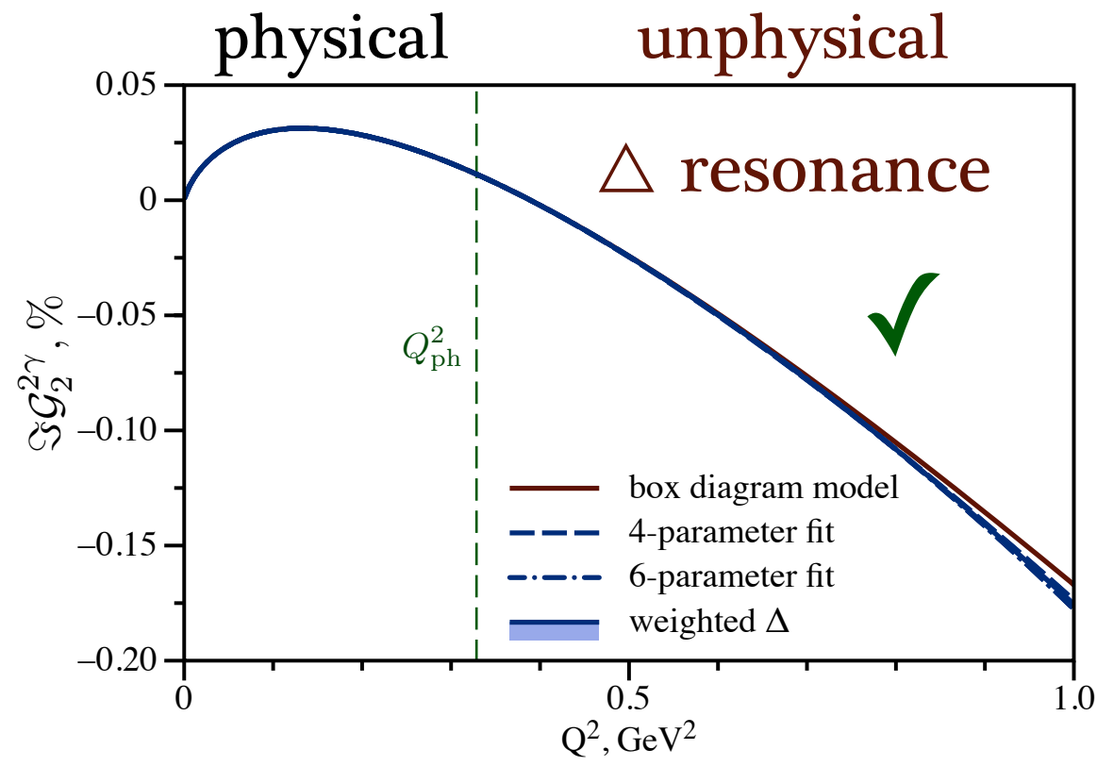
- pion electroproduction amplitudes: MAID2007

D. Drechsel. S. Kamalov and L. Tiator (2007)

- analytical continuation: fit of low-Q^2 expansion in physical region

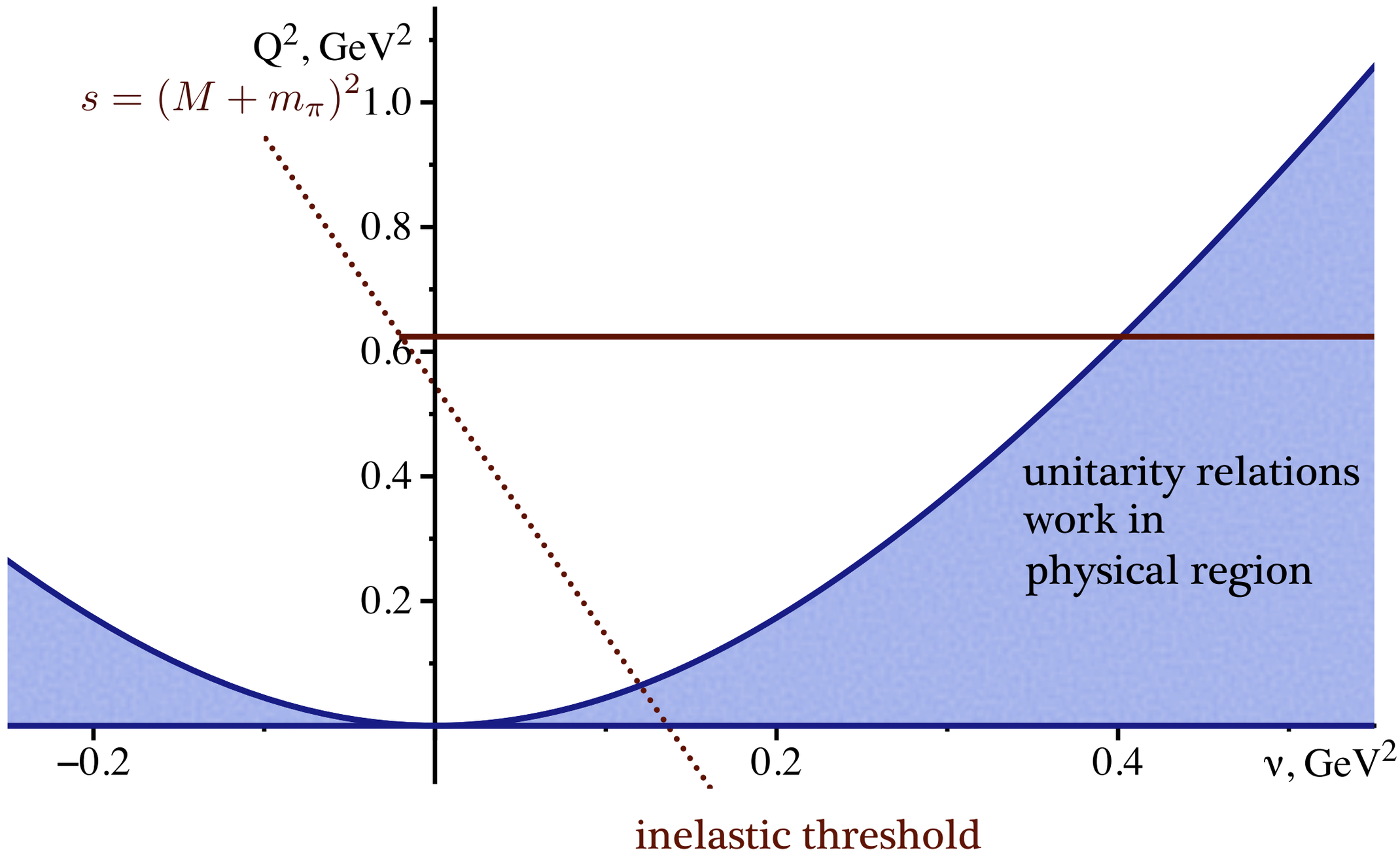
$$\mathcal{G}_{1,2}(s,\ Q^2),\ Q^2\mathcal{F}_3(s,\ Q^2) \sim a_1 Q^2 \ln Q^2 + a_2 Q^2 + a_3 Q^4 \ln Q^2 + \ldots$$





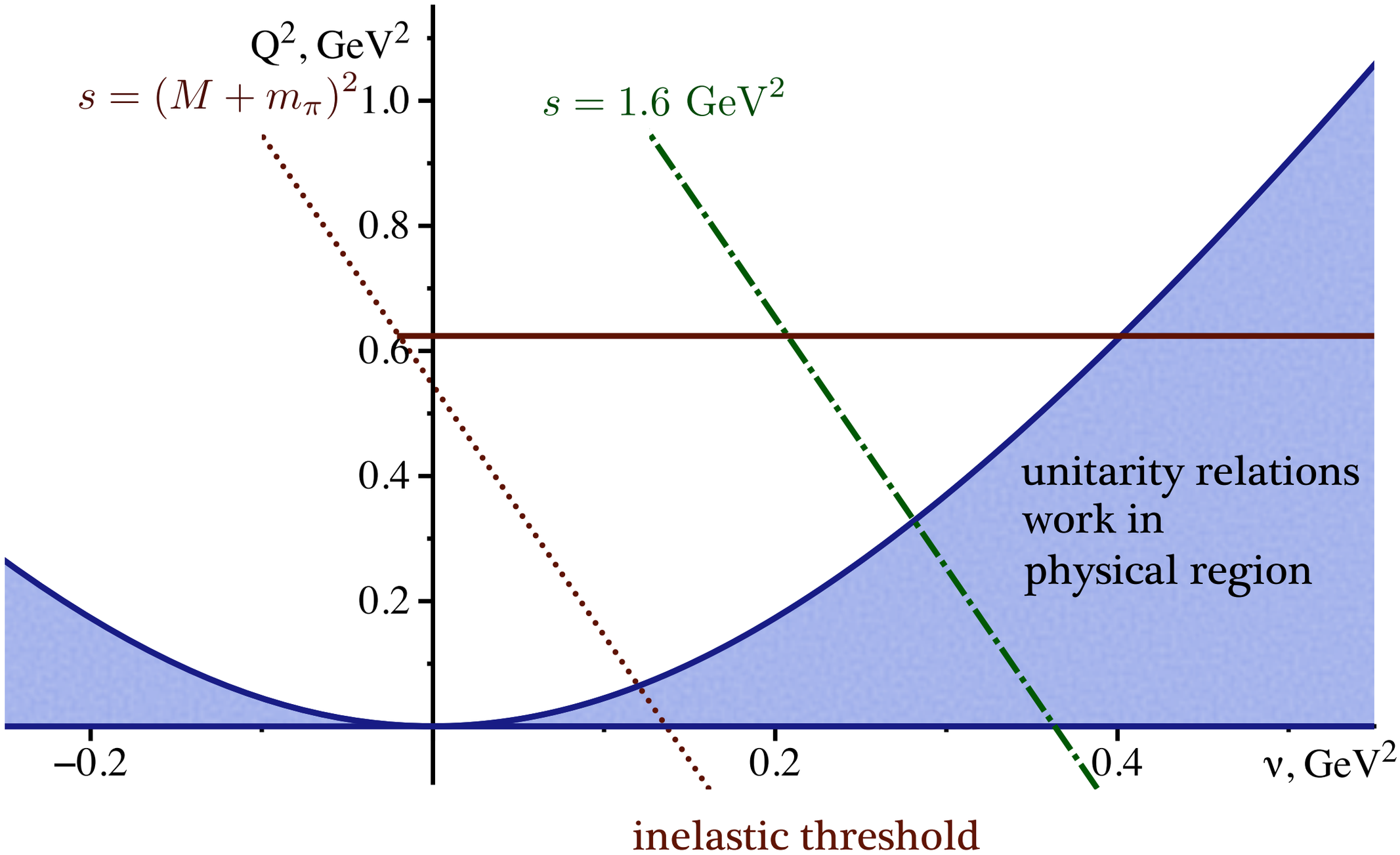
- uncertainty: extrapolation + large invariant masses

Mandelstam plot (ep)



- πN intermediate state is outside physical region for $Q^2 > 0.064$ GeV^2

Mandelstam plot (ep)



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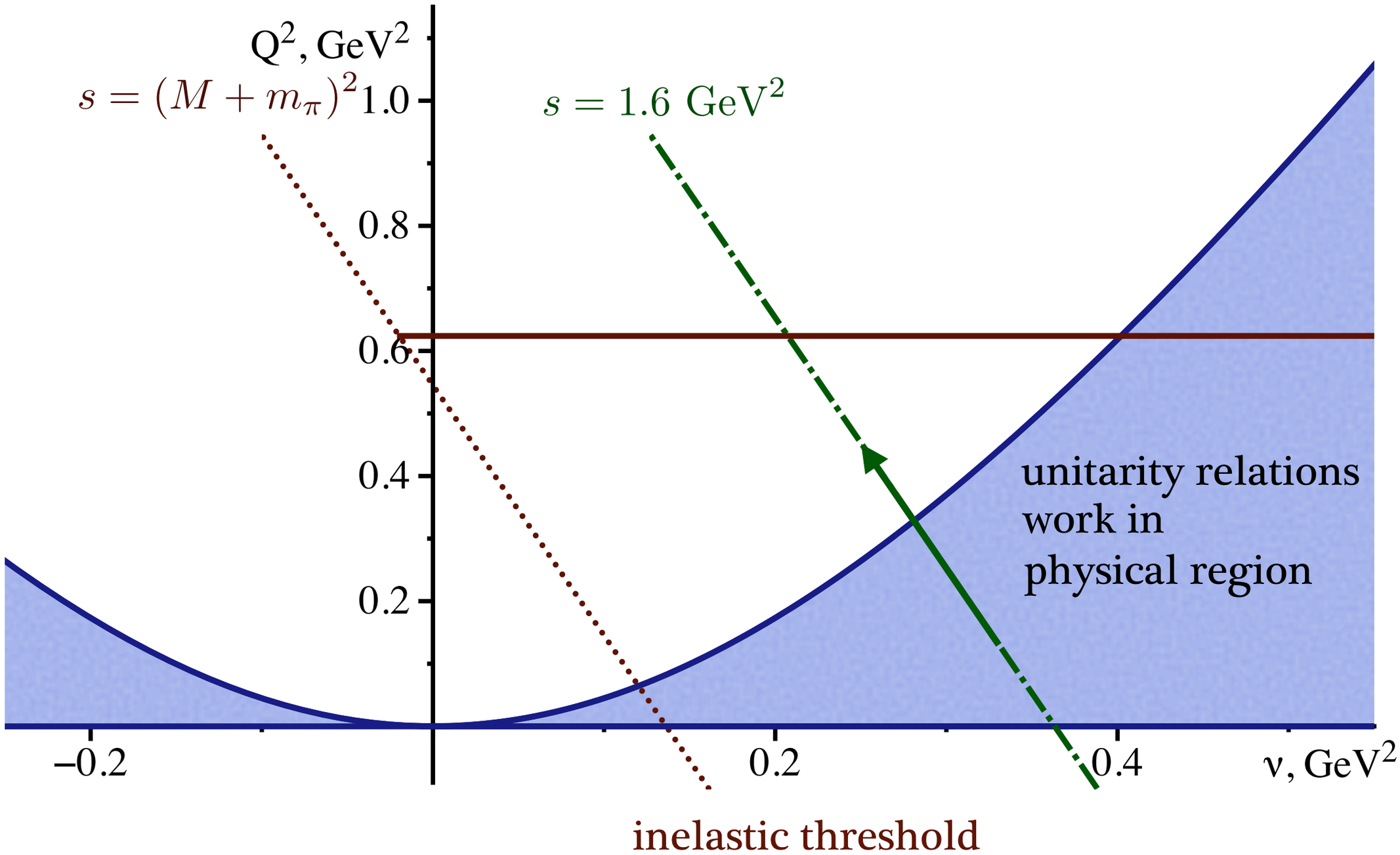

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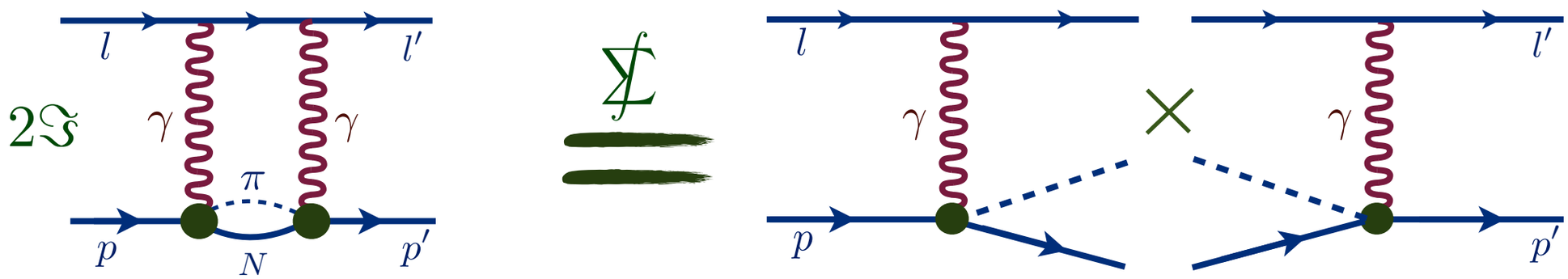
Analytical continuation. πN states



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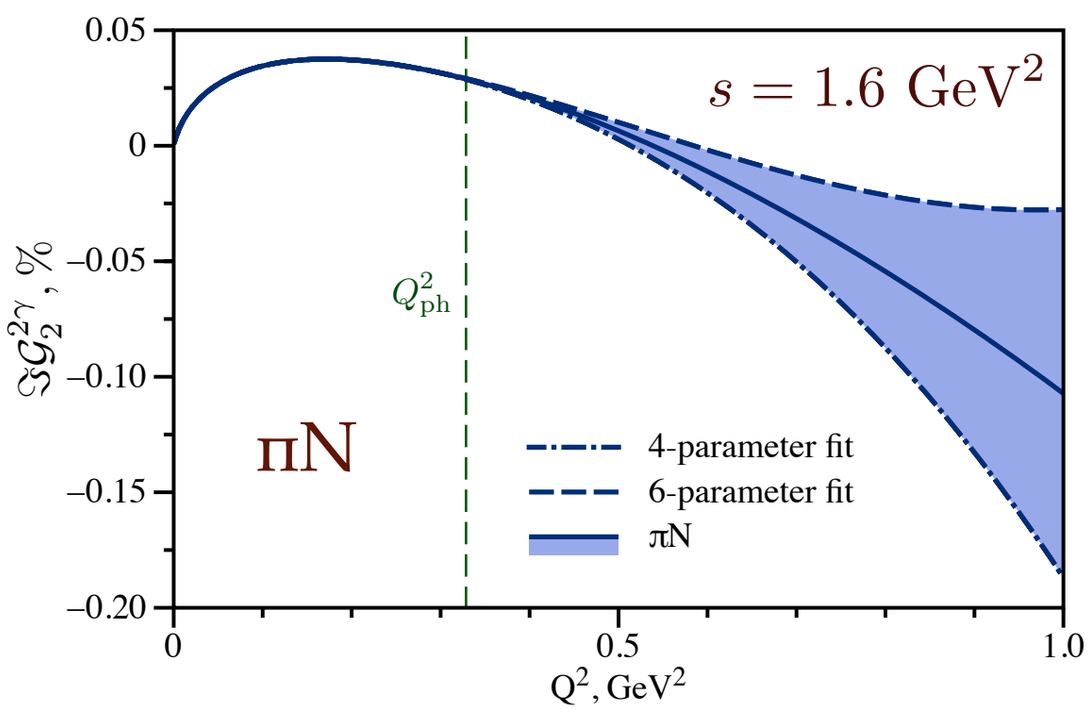
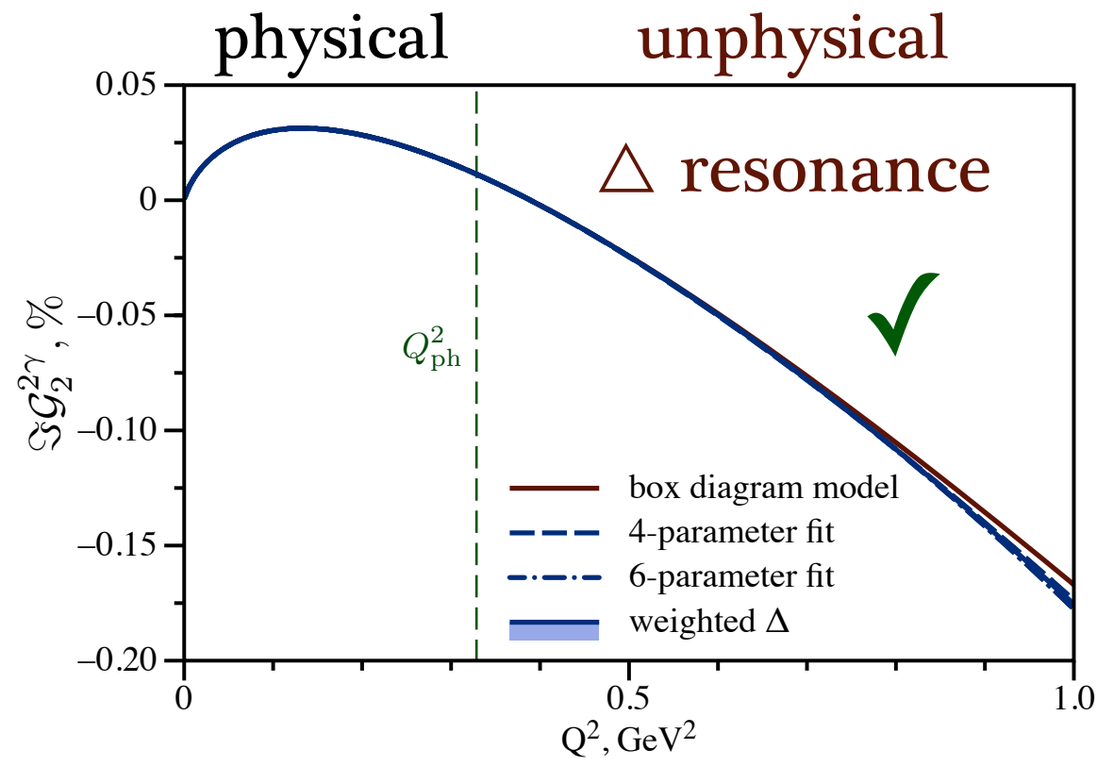
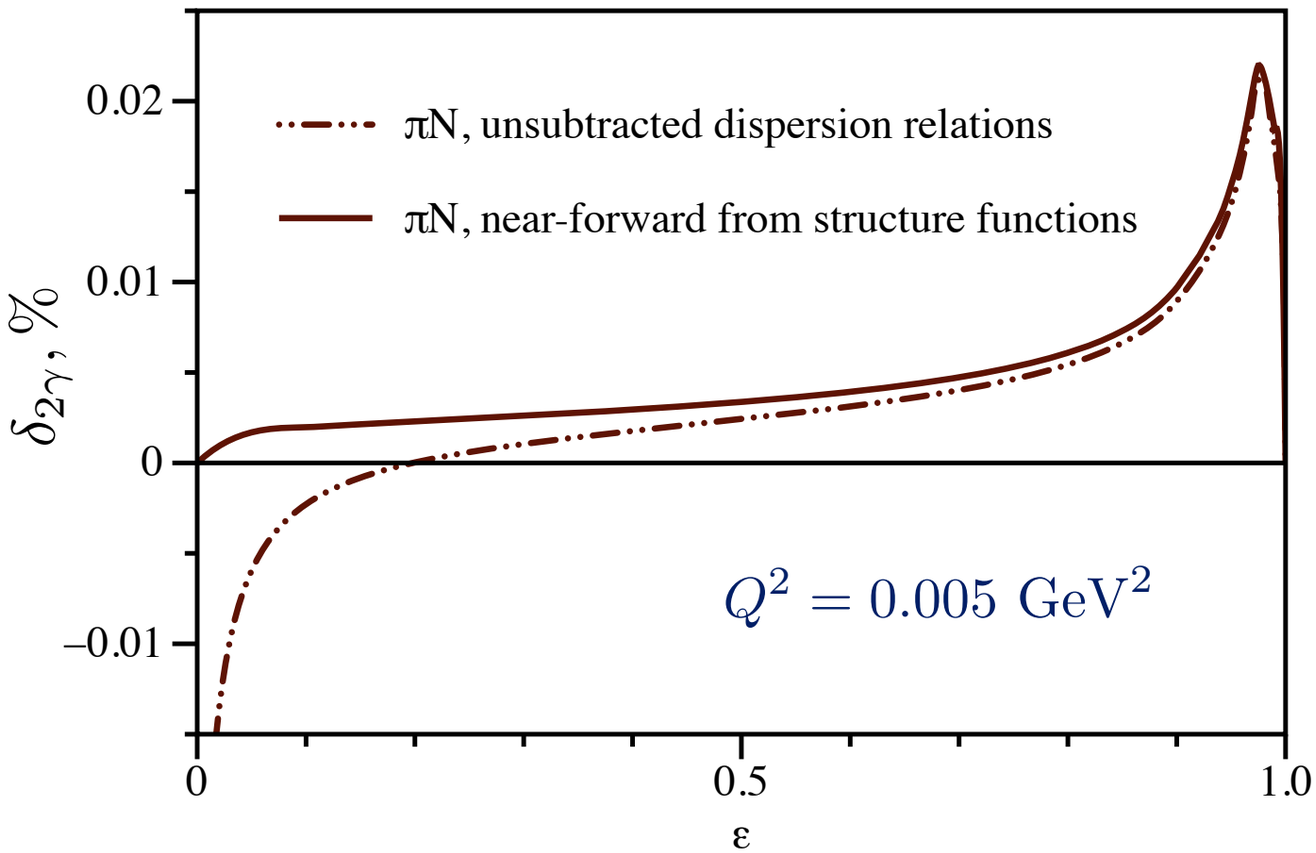
$$\mathcal{G}_{1,2}(s,\ Q^2),\ Q^2\mathcal{F}_3(s,\ Q^2) \sim a_1 Q^2 \ln Q^2 + a_2 Q^2 + a_3 Q^4 \ln Q^2 + \ldots$$





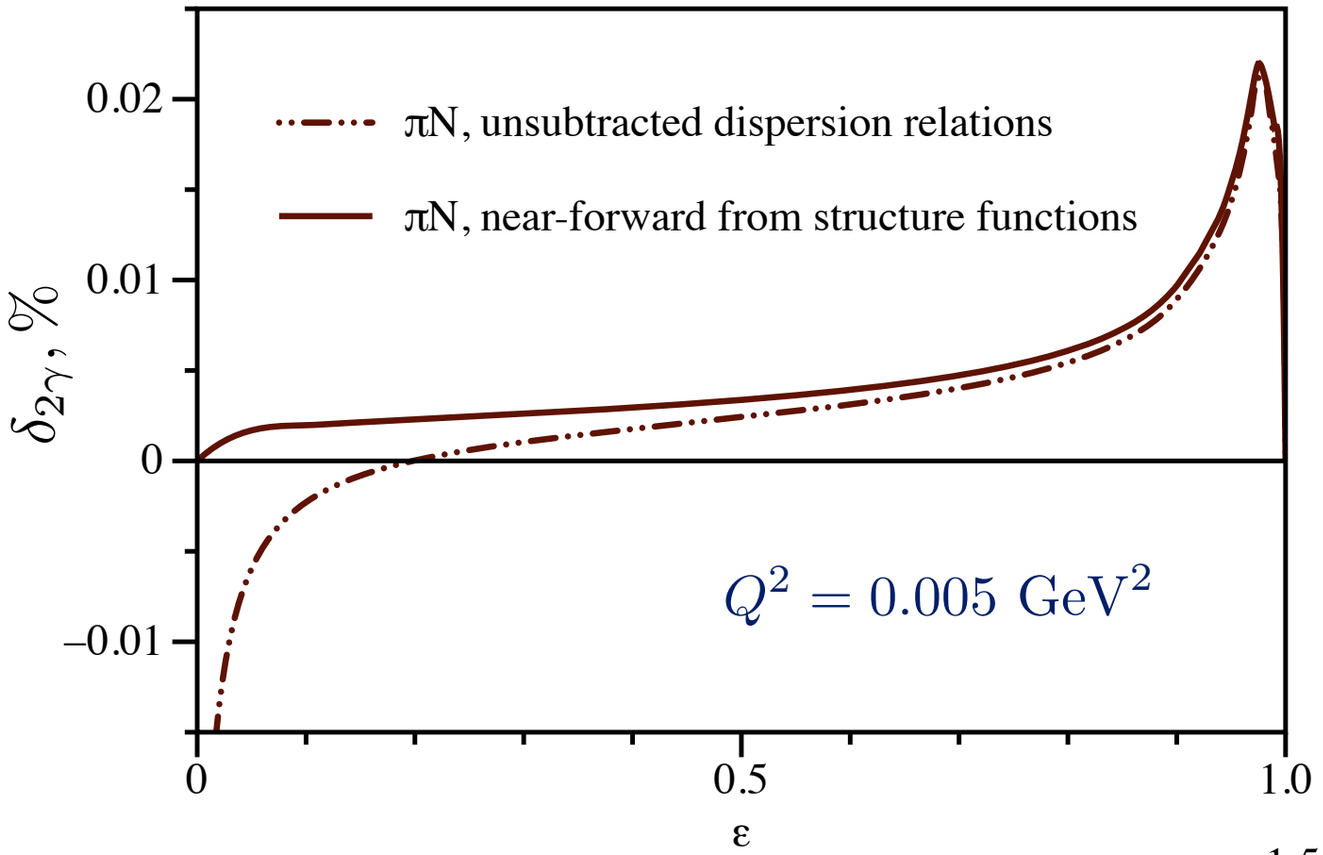
- uncertainty: extrapolation + large invariant masses

πN in dispersive framework (e-p)

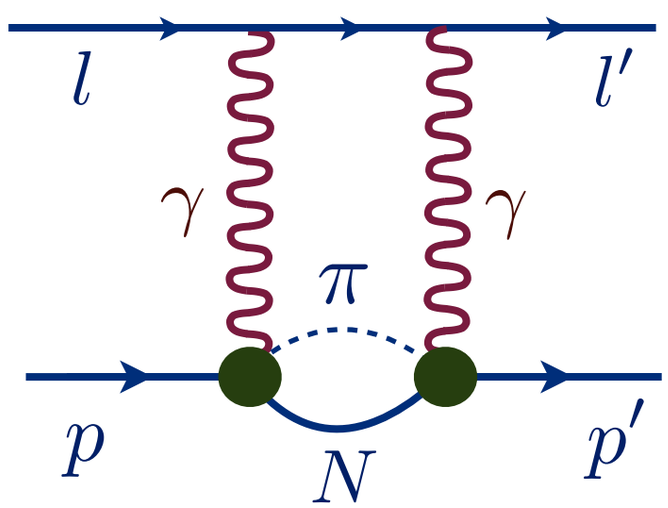


- dispersion relations agree with near-forward at large ε

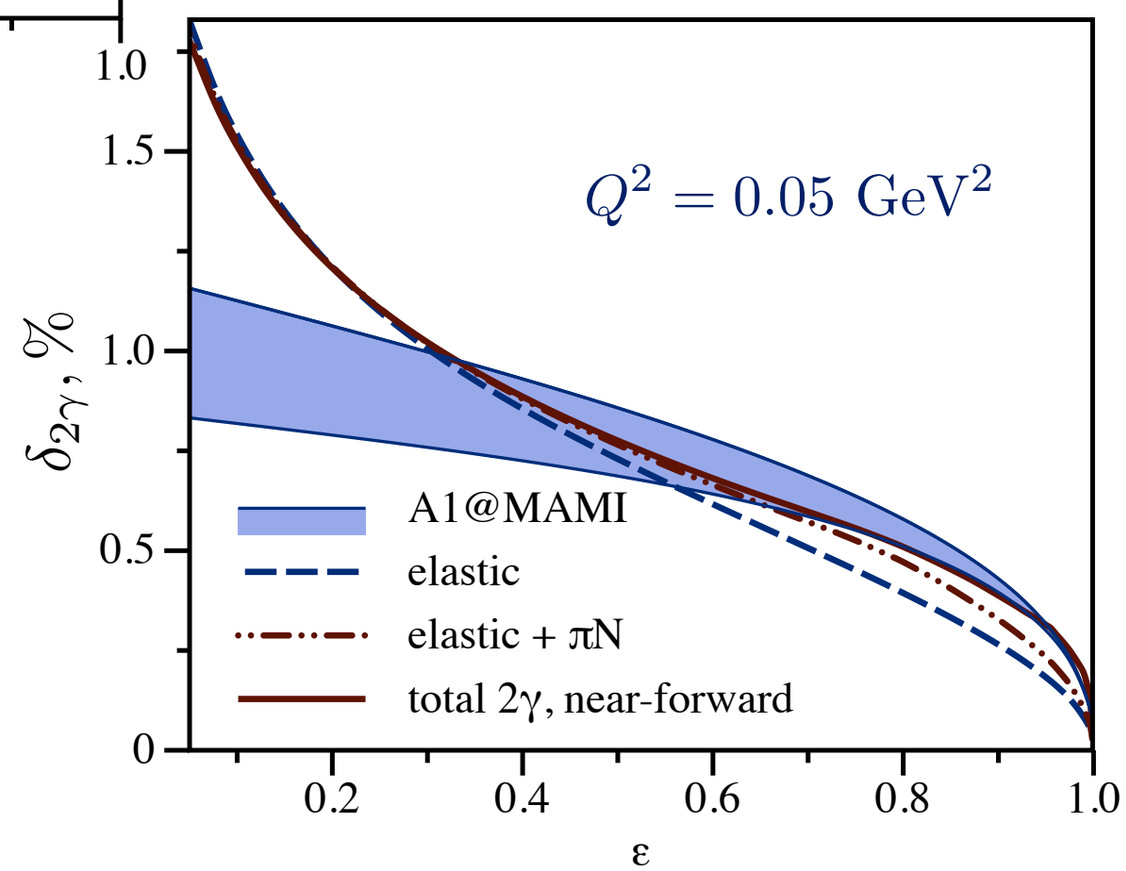
πN in dispersive framework (e-p)



- dispersion relations agree with near-forward at large ε

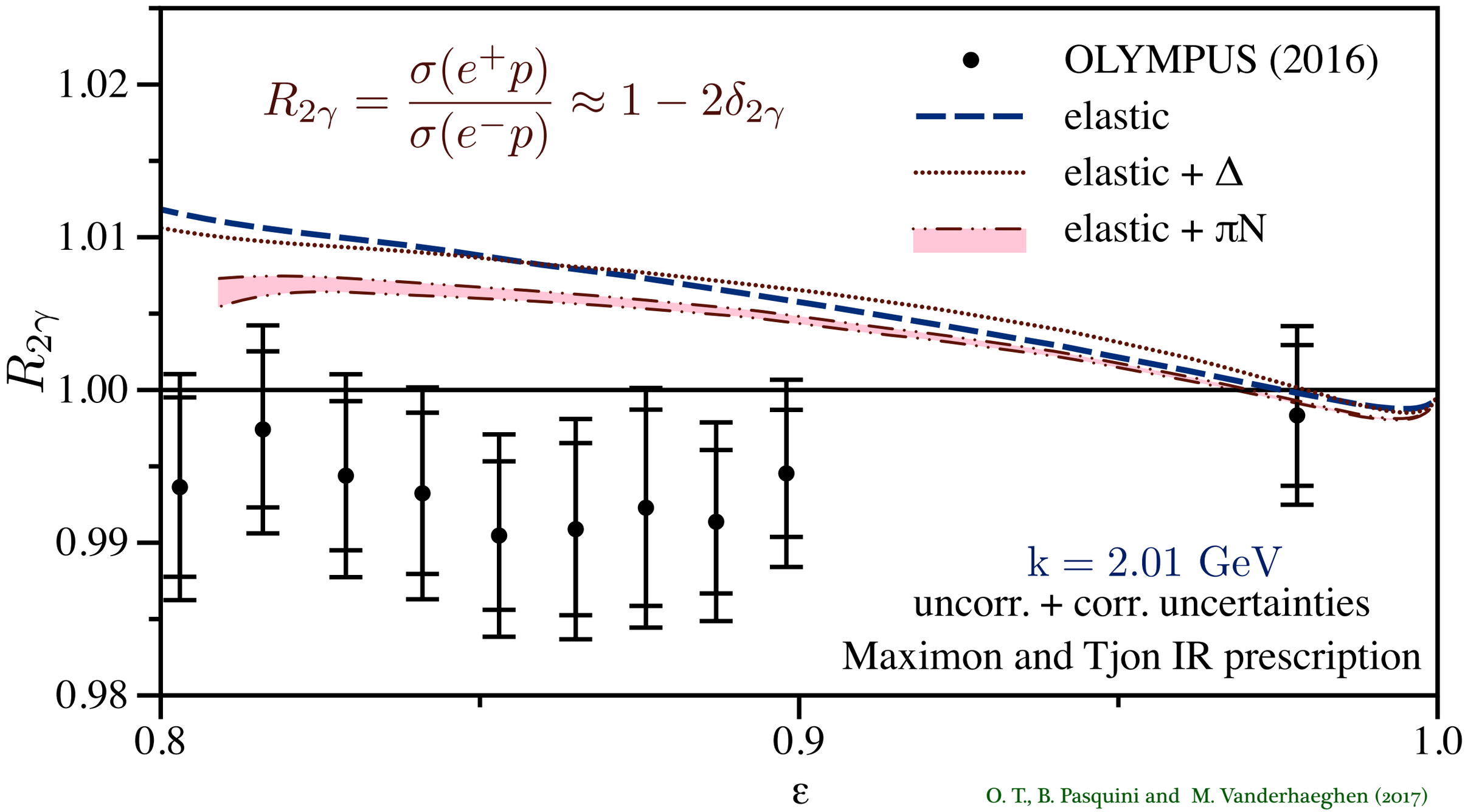


- πN is dominant inelastic 2γ



O. T., B. Pasquini and M. Vanderhaeghen (2017)

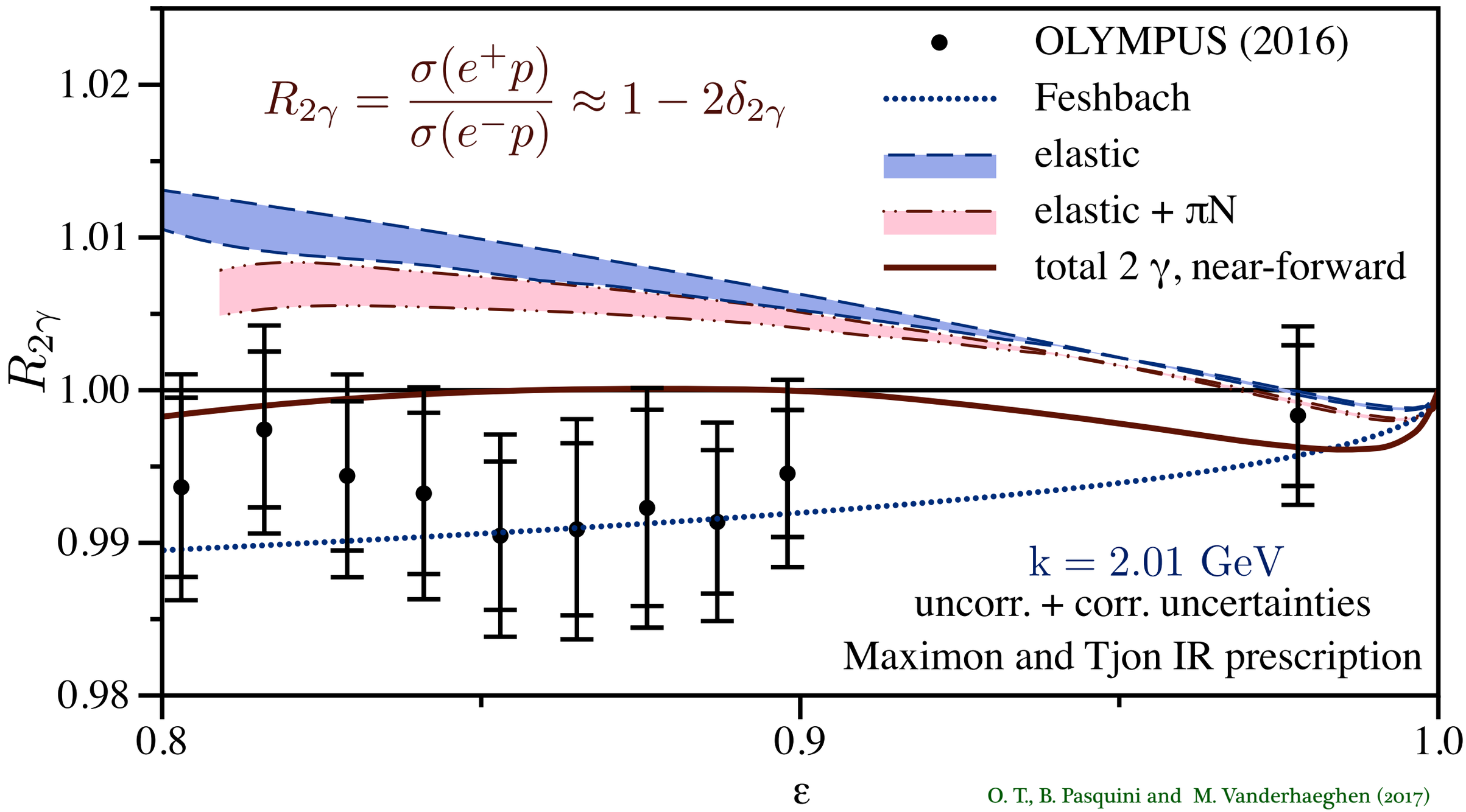
Comparison with data



O. T., B. Pasquini and M. Vanderhaeghen (2017)

- weighted △ is similar to narrow one of Blunden et al. (2017)
- πN contribution is closer to data than △ only

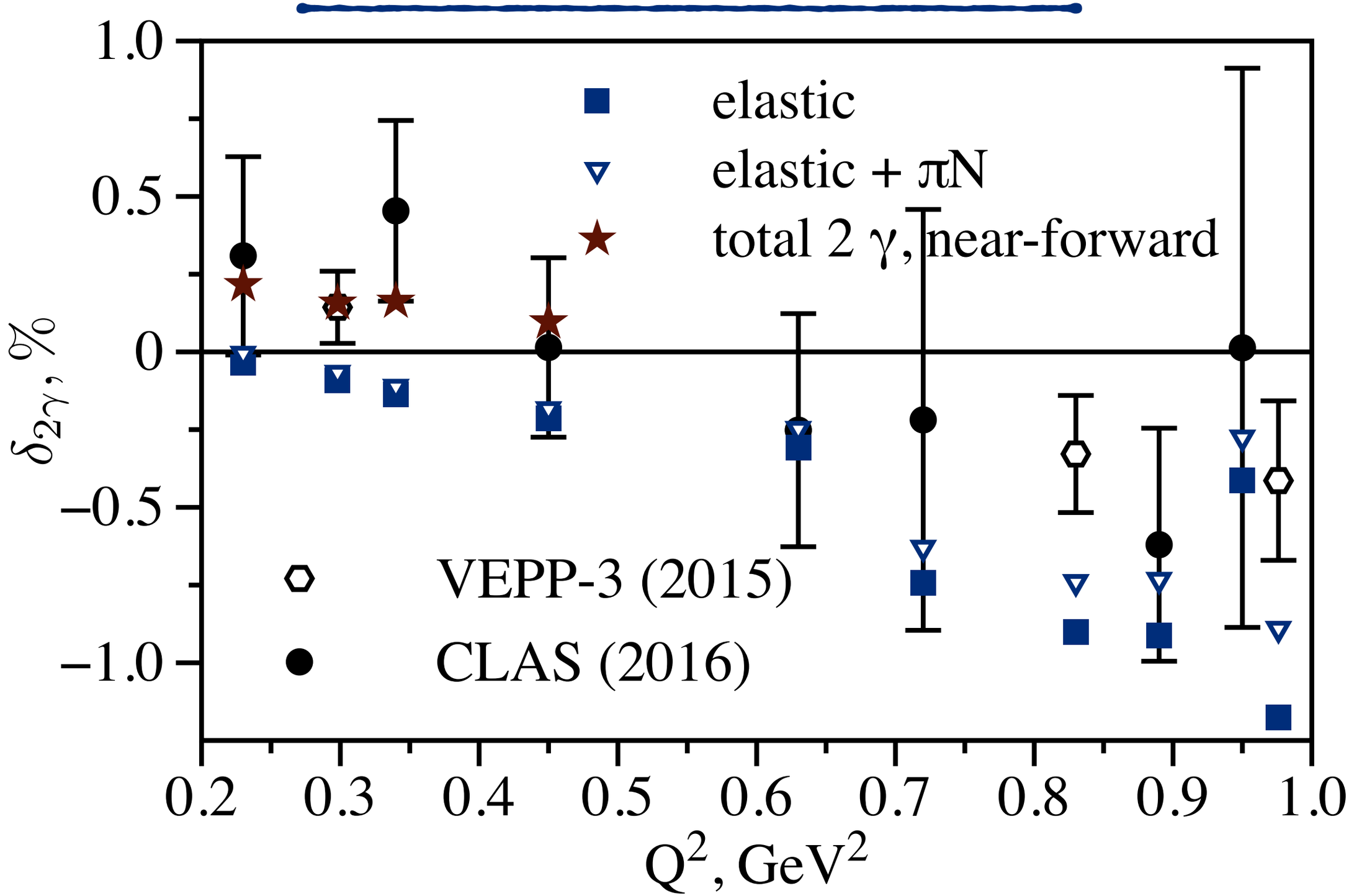
Comparison with data



O. T., B. Pasquini and M. Vanderhaeghen (2017)

- near-forward 2γ agree with data
- multi-particle 2γ, e.g. ππN, is important

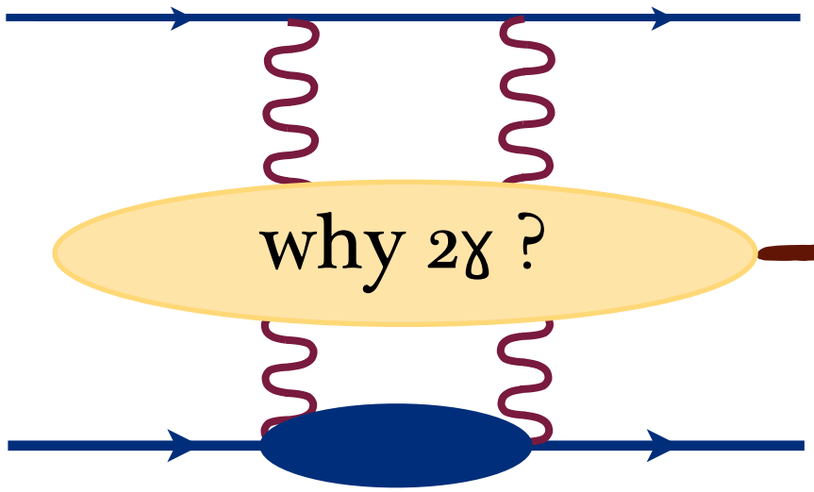
Comparison with data



O. T., B. Pasquini and M. Vanderhaeghen (2017)

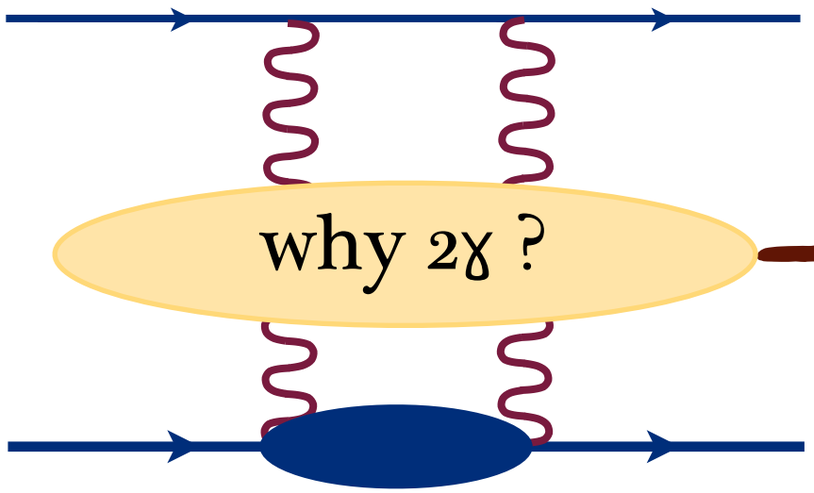
- dispersion relations agree with CLAS data

Conclusions



largest theoretical uncertainty
in low-energy proton structure

Conclusions



why 2γ ?

largest theoretical uncertainty in low-energy proton structure

how to study ?

box diagram

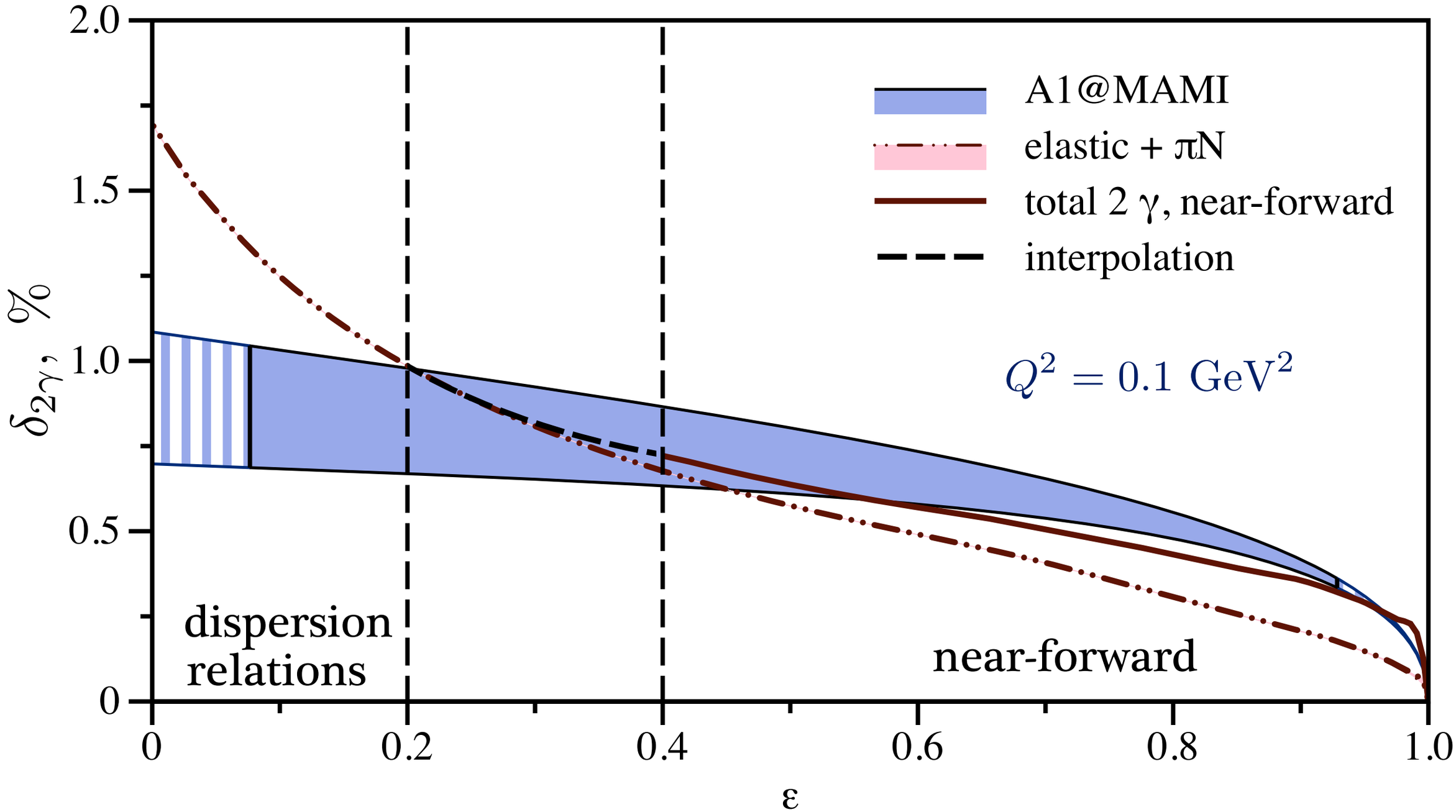
dispersion relations

small scatt. angles
ep, μp (all states)

all scatt. angles
ep (p + πN states)

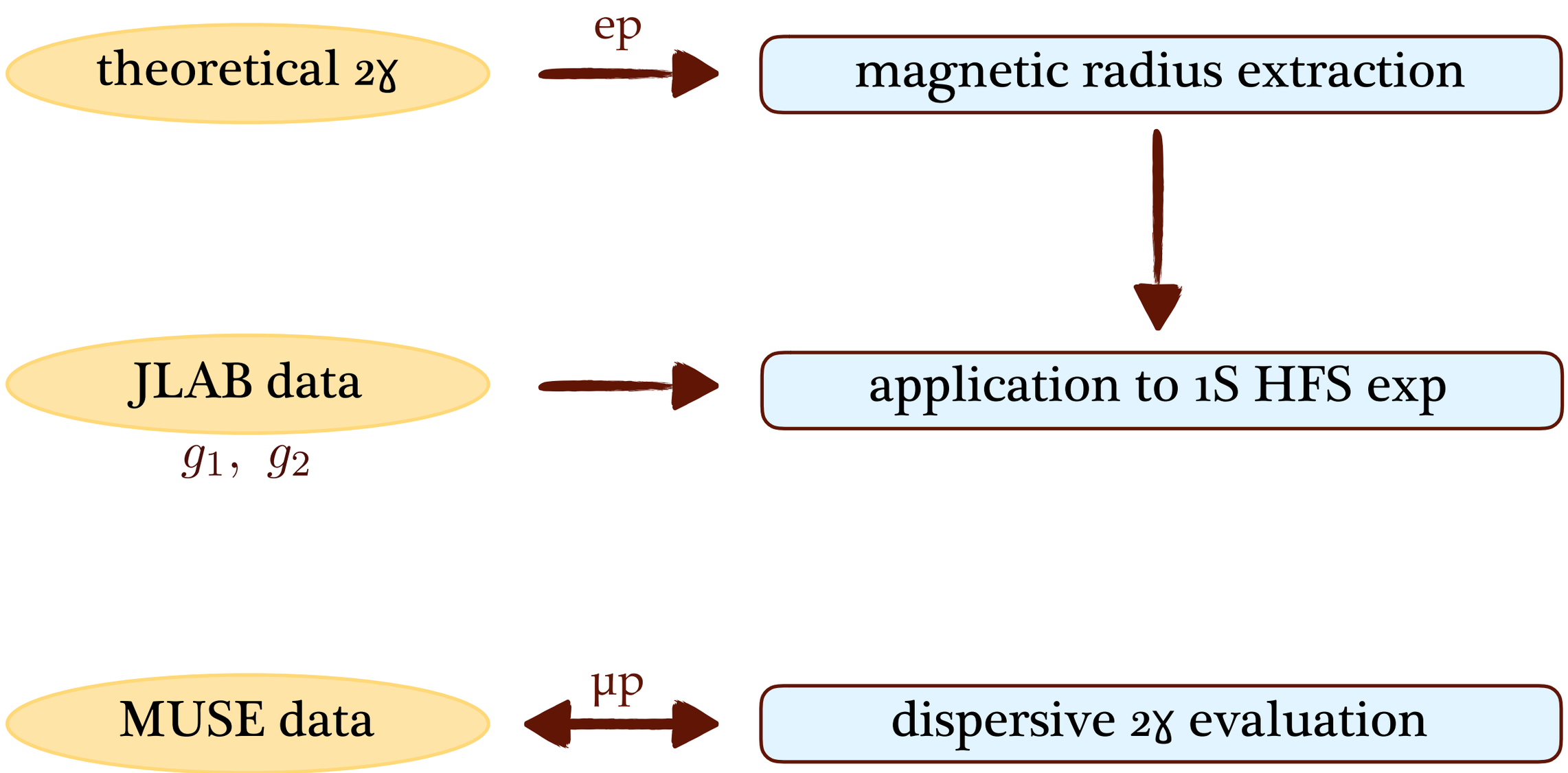
- multi-particle 2γ, e.g. ππN, within dispersion relations is important

Our best 2γ knowledge



- small Q^2: near-forward at large ε, all inelastic states
- $Q^2 \lesssim 1$ GeV^2: elastic+πN within dispersion relations
- intermediate range: interpolation

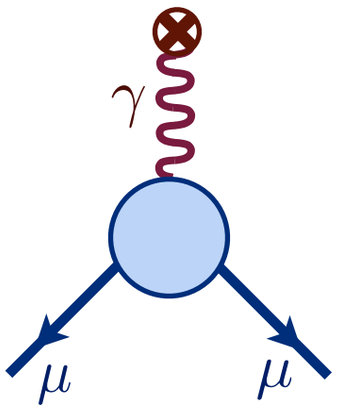
Outlook


theoretical 2γ
ep
magnetic radius extraction
JLAB data
g_1, g_2
application to 1S HFS exp
MUSE data
μp
dispersive 2γ evaluation

Thanks for your attention !!!

Muon discrepancies: new physics?

anomalous magnetic moment



3.6 σ
theory vs exp.

μH Lamb shift

proton size discrepancy

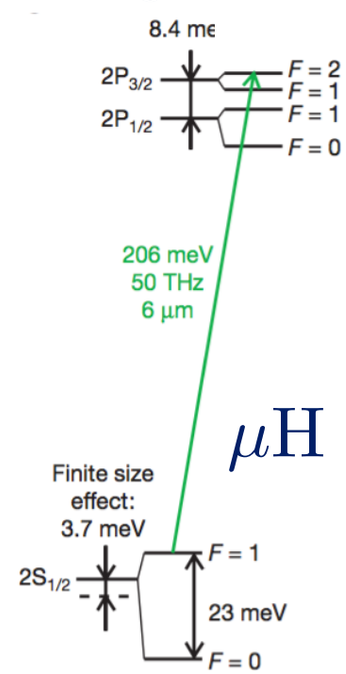
up to 7σ
electron vs muon

hadronic uncertainty is dominant in theory

Our goals

decrease hadronic uncertainties

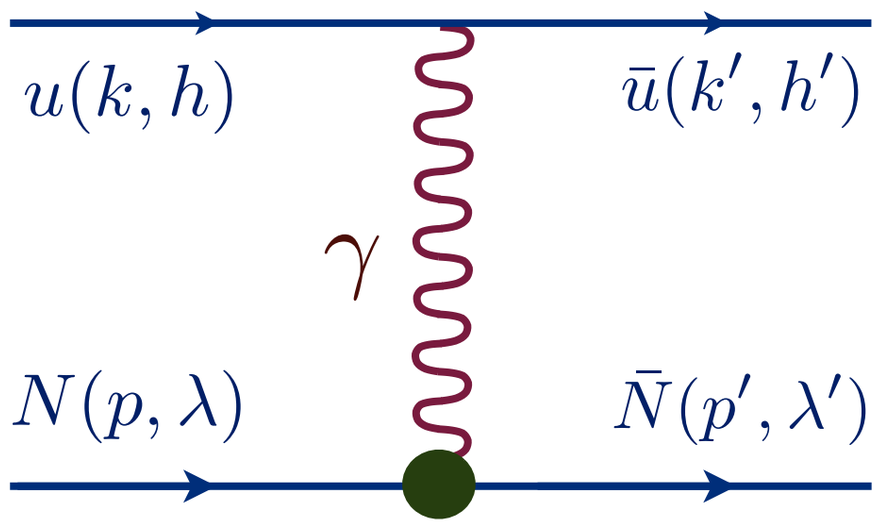
study the low energy proton structure



Pohl et al., Nature (2010)

39

Tool to explore the proton structure



photon-proton vertex

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_P(Q^2)$$

Dirac and Pauli form factors

lepton energy ω

momentum transfer $Q^2 = -(k-k')^2$

l-p amplitude

$$T = \frac{e^2}{Q^2}\left(\bar{u}\left(k',h'\right)\gamma_\mu u\left(k,h\right)\right)\cdot\left(\bar{N}\left(p',\lambda'\right)\Gamma^\mu(Q^2)N\left(p,\lambda\right)\right)$$

Form factors measurement

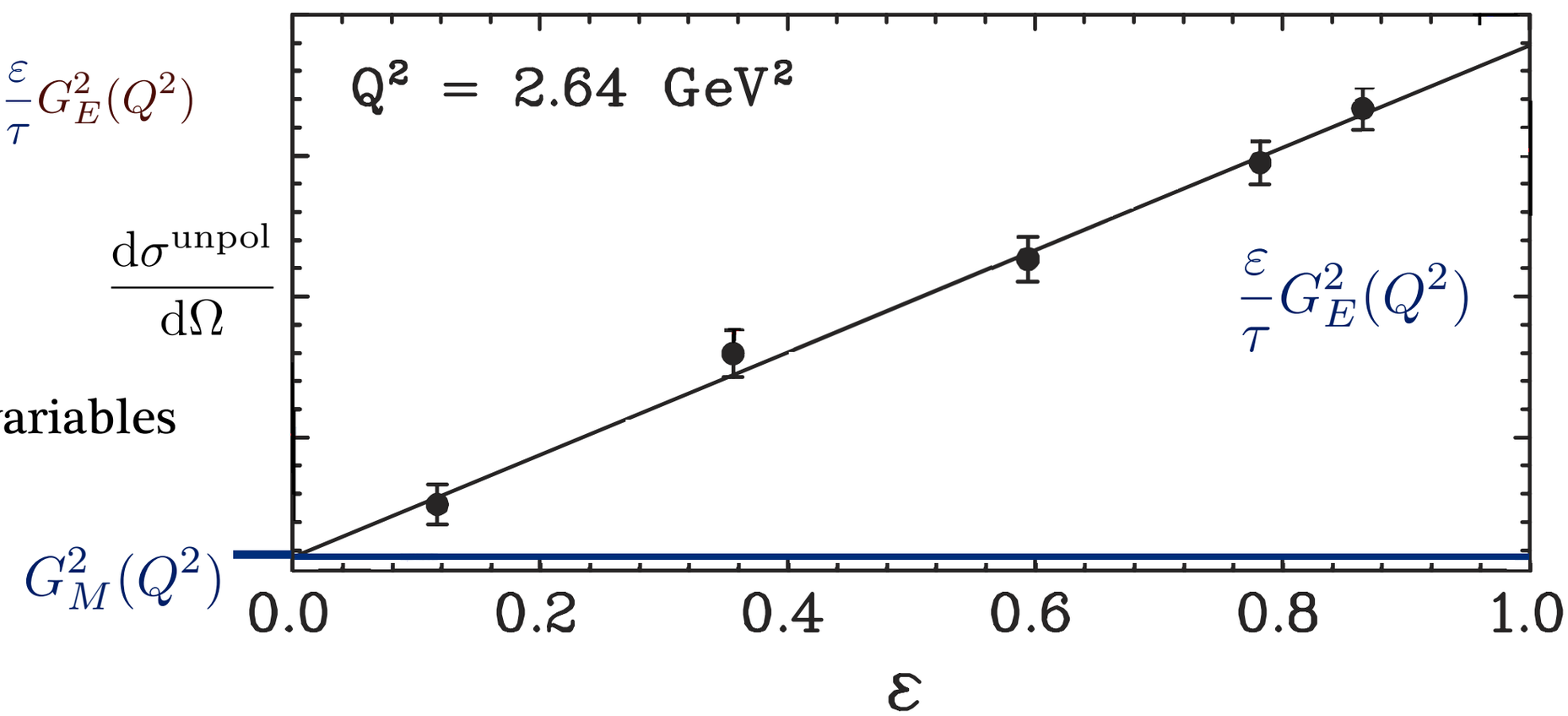
- Sachs electric and magnetic form factors:

$$G_E = F_D - \tau F_P \qquad G_M = F_D + F_P$$

- Rosenbluth separation:

$$\frac{d\sigma^{\text{unpol}}}{d\Omega} \sim G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

τ ε kinematical variables



Qattan et al. (2005)

- Rosenbluth slope is sensitive to corrections beyond 1γ

Form factors measurement

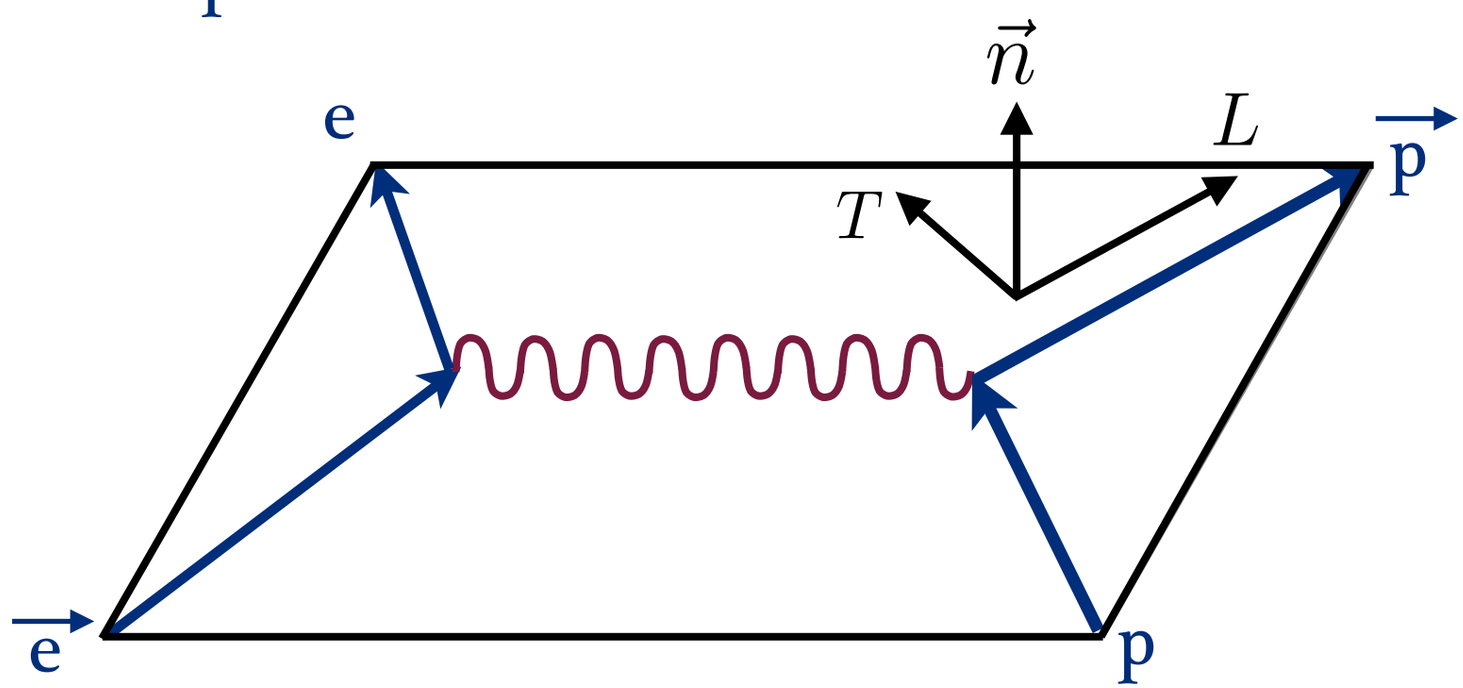
- Sachs electric and magnetic form factors:

$$G_E = F_D - \tau F_P \qquad G_M = F_D + F_P$$

- polarization transfer method:

$\vec{e} + p \rightarrow e + \vec{p}$

realized in 2000 at JLab



$$P_T \sim G_E(Q^2) G_M(Q^2)$$

$$P_L \sim G_M^2(Q^2)$$

$$\frac{P_T}{P_L} \sim \frac{G_E(Q^2)}{G_M(Q^2)}$$

Proton form factors puzzle

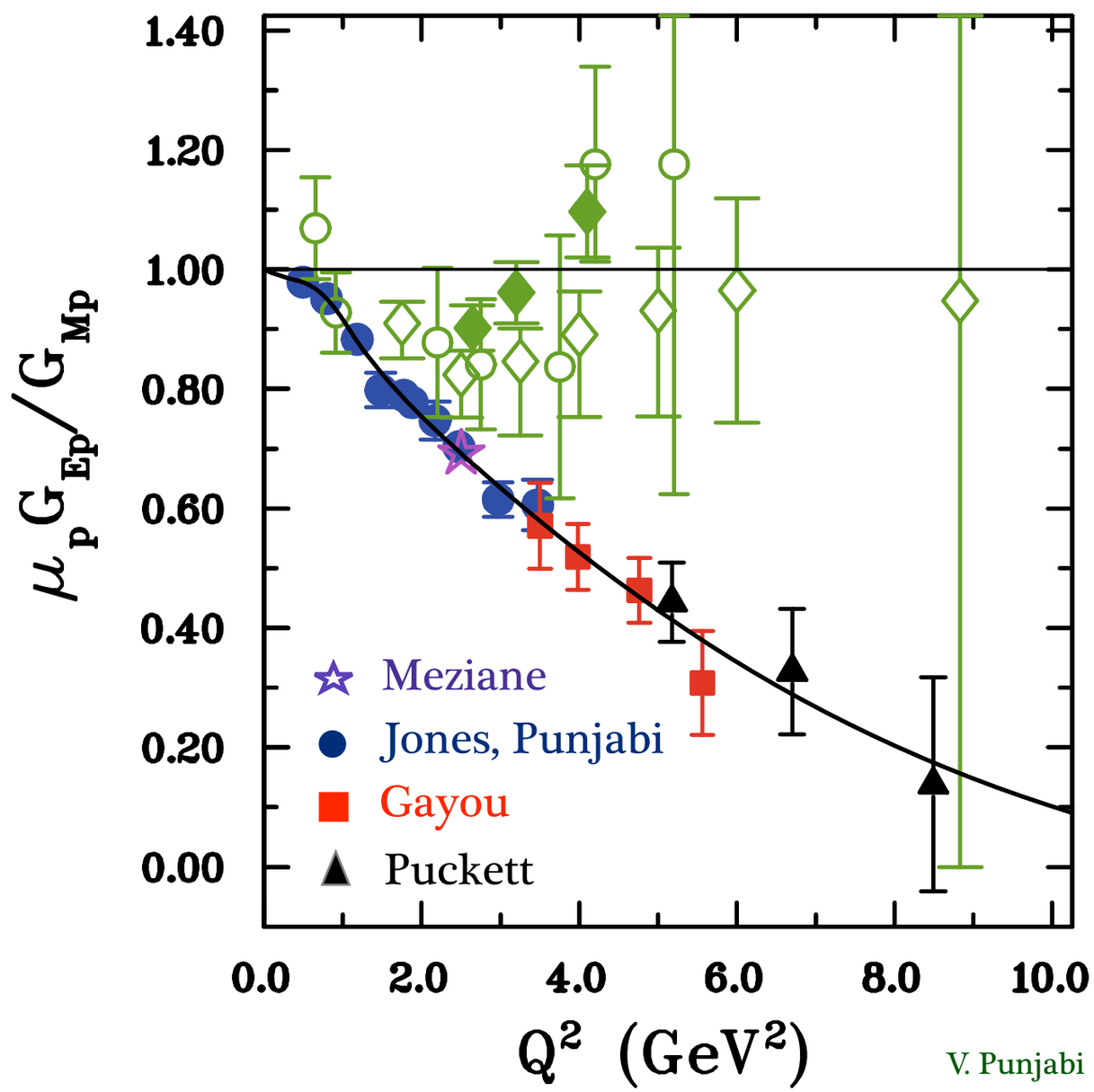
Polarization transfer

JLab (Hall A, C)

vs.

Rosenbluth separation

SLAC, JLab (Hall A, C)



V. Punjabi et al. (2015)

Proton form factors puzzle

Polarization transfer

JLab (Hall A, C)

vs.

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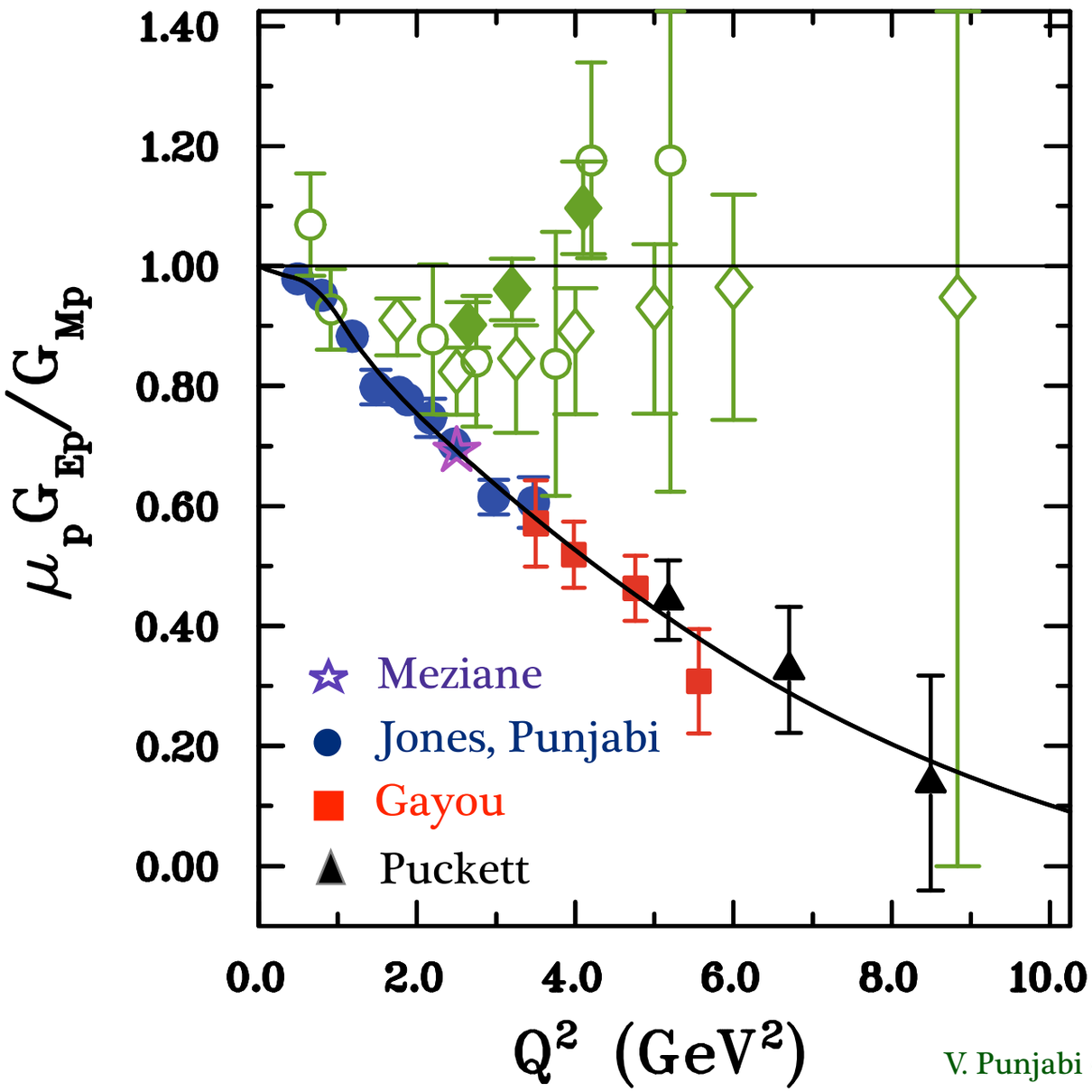
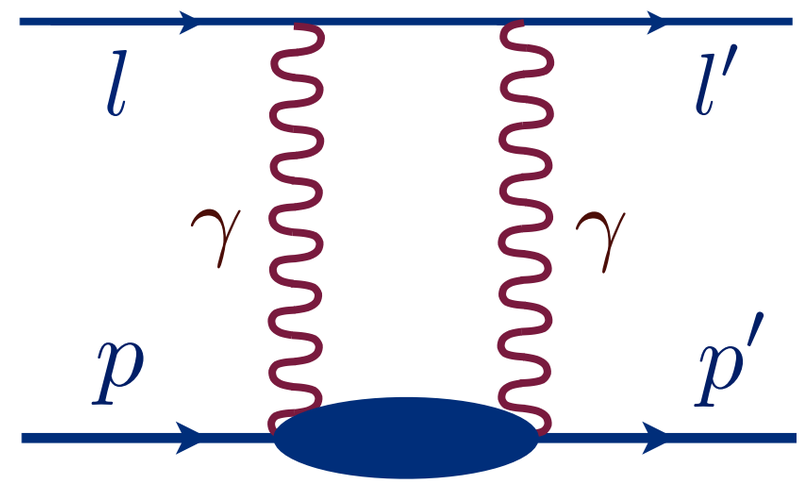
SLAC, JLab (Hall A, C)



V. Punjabi et al. (2015)



a possible explanation
two-photon exchange

↓

2γ measurements
e^+p/e^-p cross section ratio

$$R_{2\gamma} = \frac{\sigma(e^+p)}{\sigma(e^-p)} \approx 1 - 2\delta_{2\gamma}$$

- discrepancy motivates model-independent study of 2γ

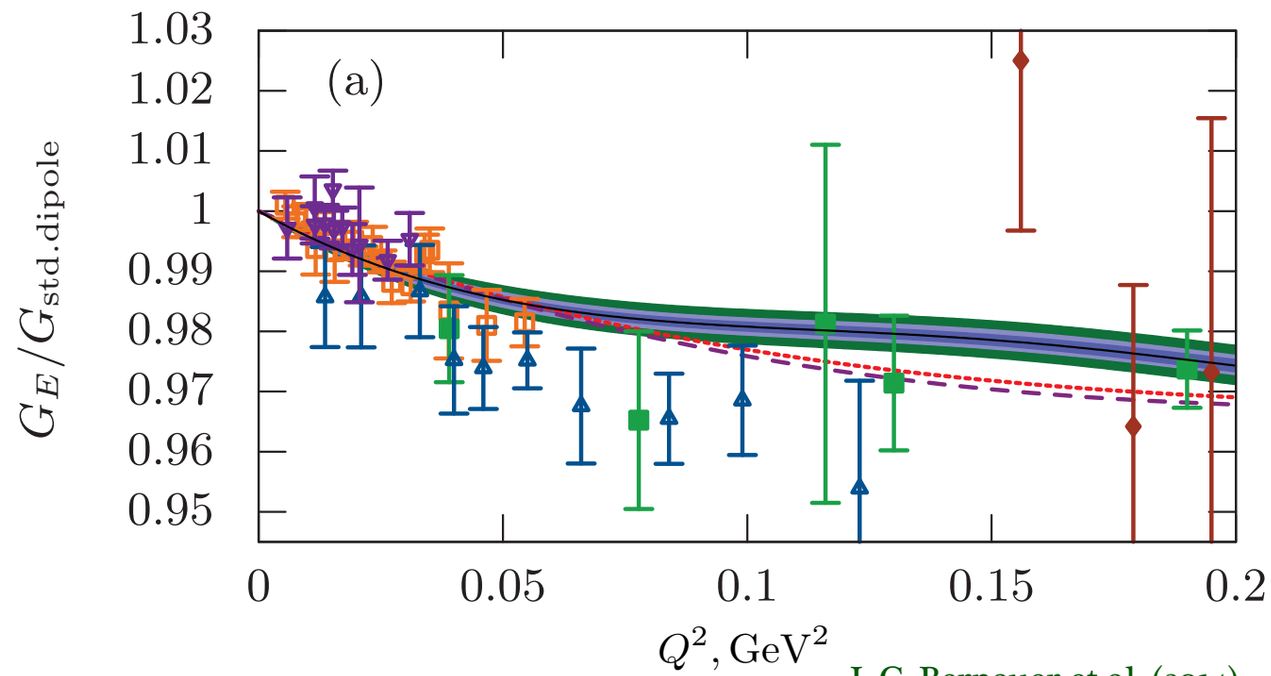
Proton charge radius

electric charge radius

$$< r_E^2 > \equiv -6 \left. \frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2} \right|_{Q^2=0}$$

ep elastic scattering

$$r_E = 0.879 \pm 0.008 \text{ fm}$$



J. C. Bernauer et al. (2014)

Proton charge radius

electric charge radius

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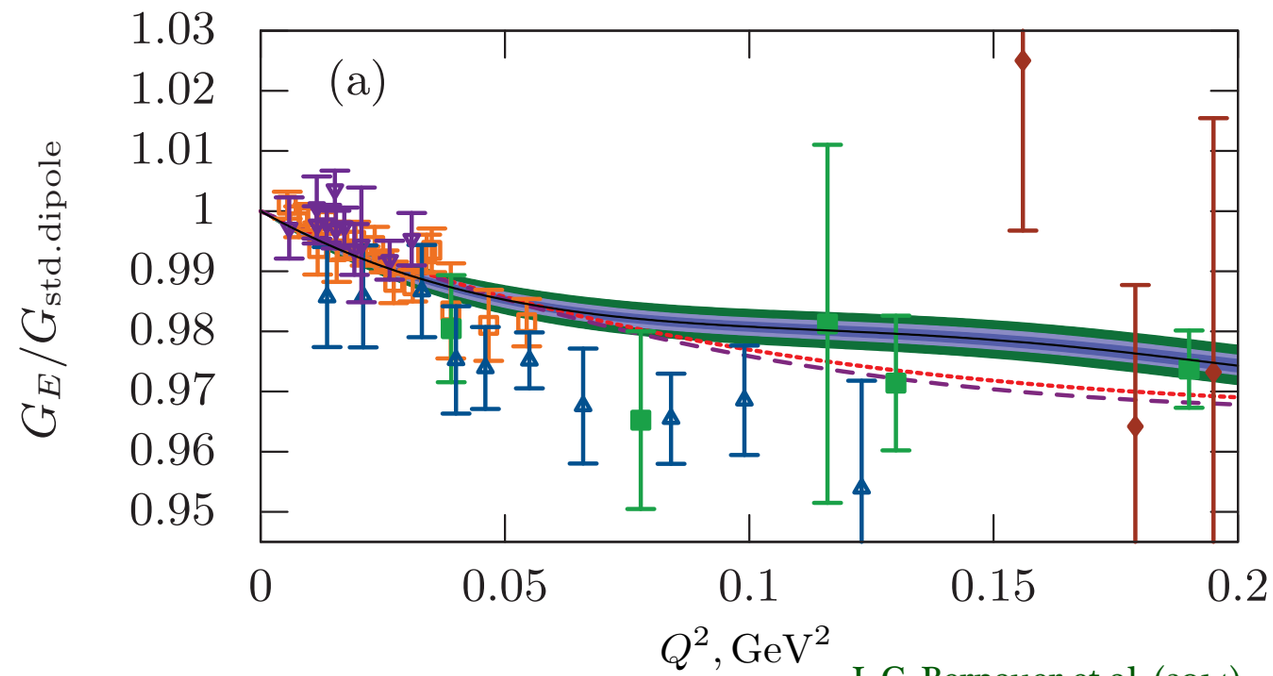


J. C. Bernauer et al. (2014)

ep elastic scattering

$$r_E = 0.879 \pm 0.008 \text{ fm}$$

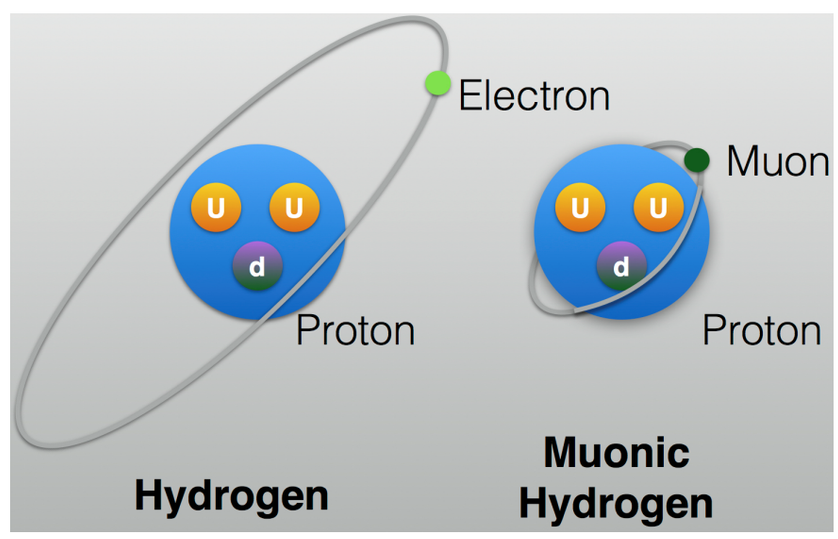
atomic spectroscopy

$$\Delta E_{\mathrm{nS}} \sim m_r^3 < r_E^2 >$$

H, D spectroscopy

$$r_E = 0.8758 \pm 0.0077 \text{ fm}$$

CODATA 2010



μH Lamb shift

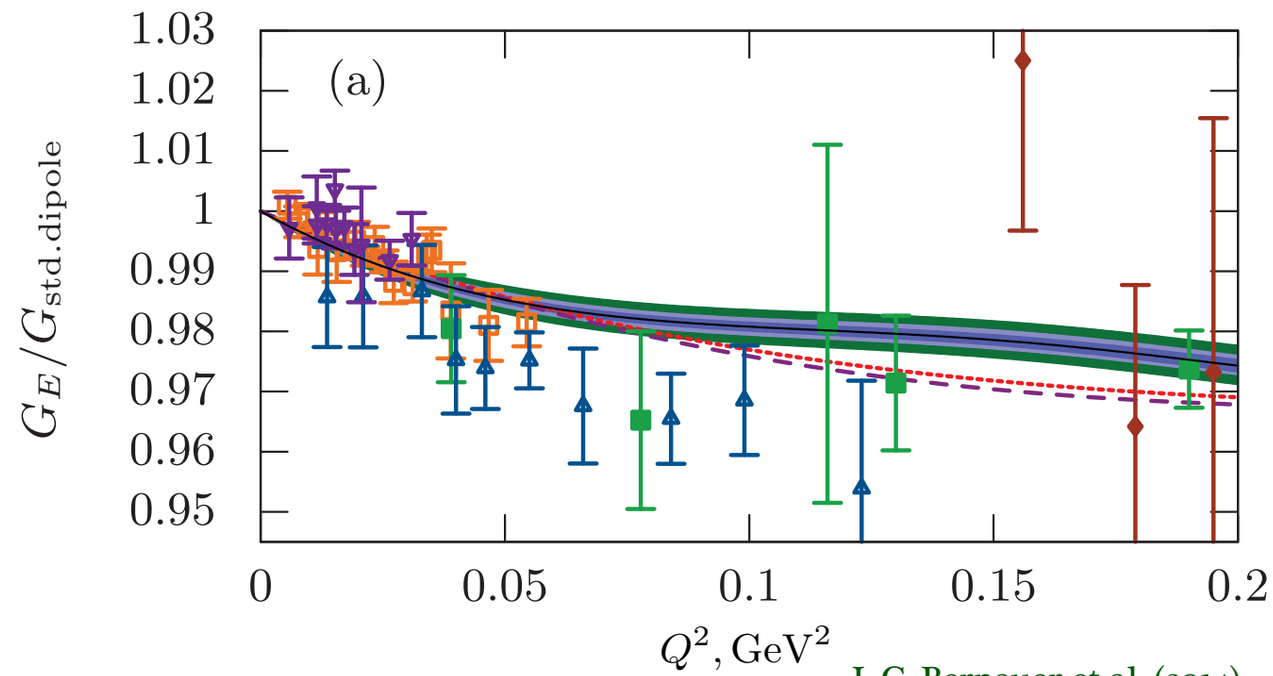
$$r_E = 0.8409 \pm 0.0004 \text{ fm}$$

CREMA (2010, 2013)

Proton radius puzzle

electric charge radius

$$< r_E^2 > \equiv -6 \left. \frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2} \right|_{Q^2=0}$$



J. C. Bernauer et al. (2014)

ep elastic scattering

$$r_E = 0.879 \pm 0.008 \text{ fm}$$

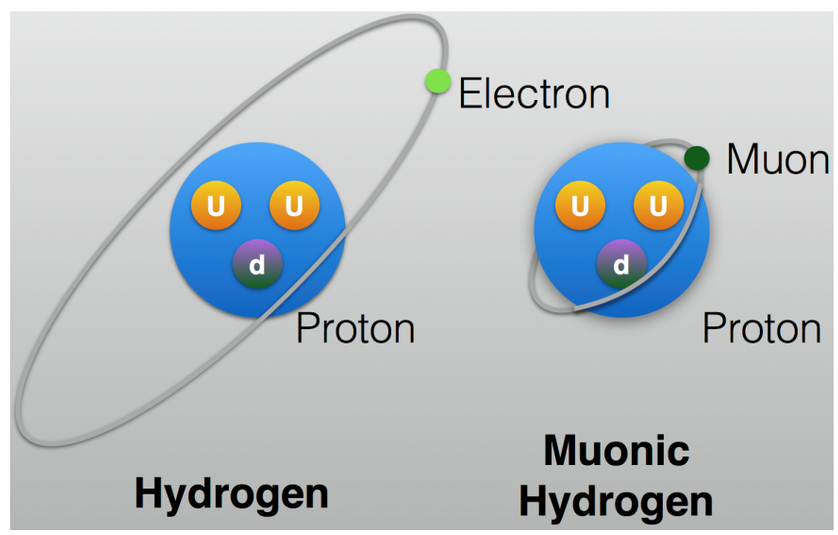
atomic spectroscopy

$$\Delta E_{\mathrm{nS}} \sim m_r^3 < r_E^2 >$$

H, D spectroscopy

$$r_E = 0.8758 \pm 0.0077 \text{ fm}$$

CODATA 2010



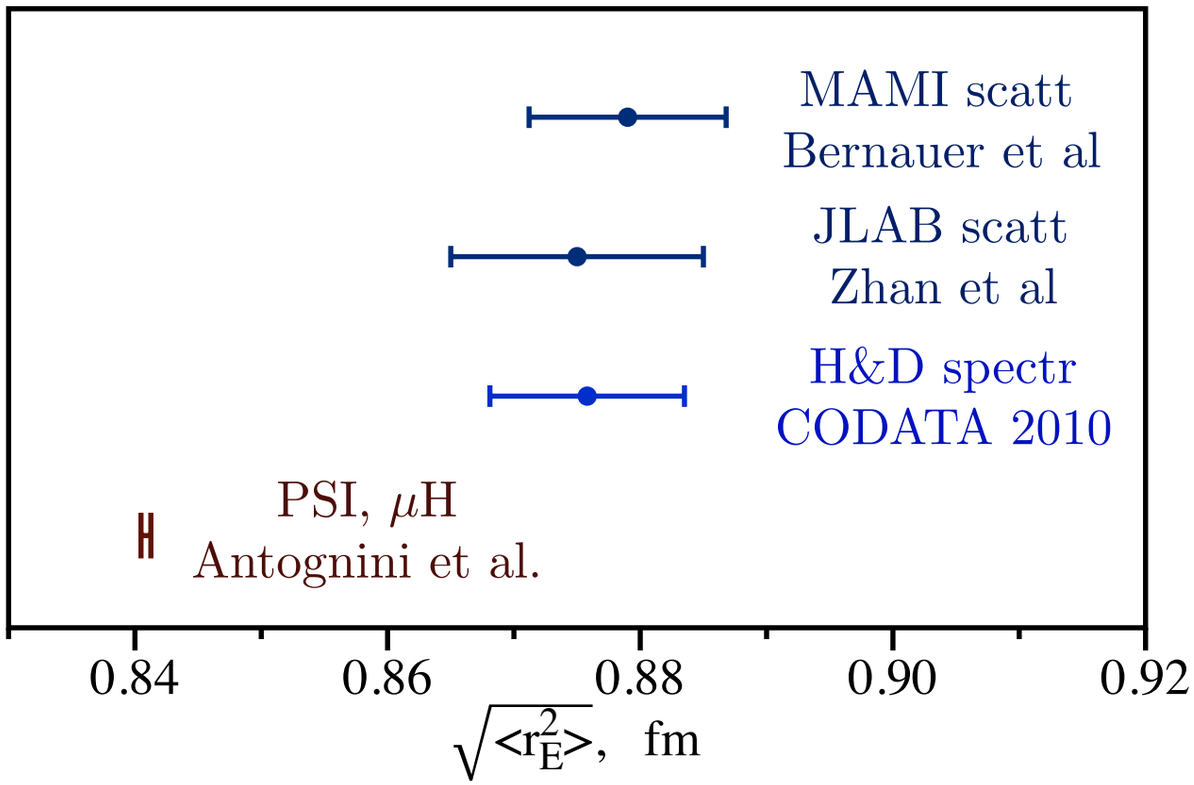
μH Lamb shift

$$r_E = 0.8409 \pm 0.0004 \text{ fm}$$

CREMA (2010, 2013)

4 % difference

μH Lamb shift and 2γ

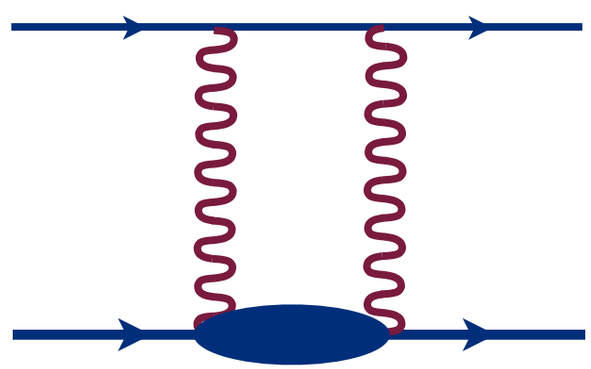


2P-2S transition in μH

charge radius discrepancy
310 μeV

μH experimental uncertainty
2.5 μeV

2γ hadronic correction

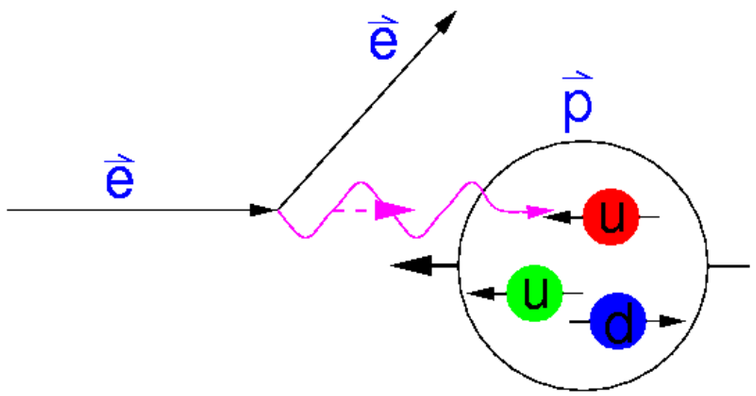


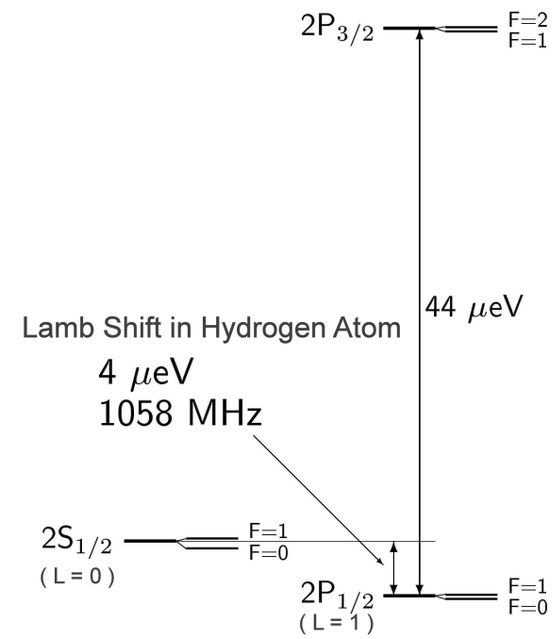
$$\Delta E^{2\gamma}_{2\mathrm{P}-2\mathrm{S}} = 33 \pm 2\ \mu\mathrm{eV}$$

C. Carlson, M. Vanderhaeghen (2011) + M. Birse, J. McGovern (2012)

important to reduce ambiguities of 2γ

Dispersion relation framework





$f(z)$

analyticity

experimental cross sections

energy levels correction

optical theorem

$$\Re\mathcal{F}(\nu) = \frac{2\nu}{\pi}\mathcal{P}\int_{\nu_{min}}^{\infty}\frac{\Im\mathcal{F}(\nu'+i0)}{\nu'^2-\nu^2}d\nu'$$

amplitudes: imaginary parts $\xrightarrow{\text{DR}}$ amplitudes: real parts