Leptogenesis – Theory Developments

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Outline

- Explain basic concepts used in non-equilibrium field theory approaches to Leptogenesis.
- Omit further technical details and give an overview what the added value is
  - from the theory perspective,
  - for phenomenological models.

1. Standard Calculations of the Lepton Asymmetry
2. Non-Equilibrium Field Theory Approach
3. Application to Leptogenesis
   3.1. Unitarity without “by Hand” Subtraction of Real Intermediate States
   3.2. Thermal Production Rates for Right-Handed Neutrinos (RHNs)
   3.3. Coherence of Active Lepton Flavours (“Flavoured Leptogenesis”)
   3.4. Resonant Leptogenesis
   3.5. Leptogenesis from Oscillations of RHNs
   3.6. Leptogenesis from Mixing/Oscillations of Charged States
1. Standard Calculations of the Asymmetry
Standard Approach – Boltzmann Equations

- Calculate reaction rates in terms of vacuum S-Matrix elements (quantum).

\[ S = 1 + iT \]

\[ \langle \mathbf{k}_{B_1} \mathbf{k}_{B_2} \ldots | iT | \mathbf{P}_{A_1} \mathbf{P}_{A_2} \ldots \rangle = (2\pi)^4 \delta^n (k_{B_1} + k_{B_2} + \ldots - p_{A_1} - p_{A_2} - \ldots) \ i \mathcal{M}_{A_1 A_2 \ldots \rightarrow B_1 B_2 \ldots} \]

\[ \text{invariant matrix element} \]

- Plug these into Boltzmann equations (classical statistics).

\[ \nabla_k f_X^\mu = \partial_x n_x - \nabla \cdot \mathbf{f}_X + 3Hq_x = \mathcal{E}_x \]

\[ \mathcal{E}_x = \frac{1}{2E_x} \int \frac{d^3p_i}{(2\pi)^3 2E_i} \delta^n (p_{X+P_{A_1}+\ldots - P_{B_1} - \ldots}) \left\{ \left( 1 - f_x \right) (1 + f_{A_1}) \ldots (1 + f_{B_1}) \ldots | \mathcal{M}_{B_1 B_2 \ldots \rightarrow X A_1 A_2 \ldots} |^2 \right. \]

\[ - \left. f_x f_{A_1} \ldots (1 + f_{B_1}) \ldots | \mathcal{M}_{X A_1 A_2 \ldots \rightarrow B_1 B_2 \ldots} |^2 \right\} \]

- Sometimes reaction rates are also computed in equilibrium thermal field theory, if vacuum rates are insufficient.

- Standard approach is a “hybrid” of quantum/classical elements.
Decay asymmetries:

[Fukugita, Yanagida (1986), Covi, Roulet, Vissani (1996)]:

\[ \epsilon_{\nu_i \rightarrow \nu_j} = \frac{\Gamma_{\nu_i \rightarrow l_a n} - \Gamma_{\bar{\nu}_i \rightarrow \bar{l}_a n}}{\Gamma_{\nu_i \rightarrow l_a n} + \Gamma_{\bar{\nu}_i \rightarrow \bar{l}_a n}} \]

\[ \epsilon^{\text{w.r.t.}}_{\nu_i \rightarrow l_a} = \frac{1}{9i} \sum_{j>i} \frac{M_j}{M_i^2 - M_i^2} \text{Im} \left[ \left( Y^* Y^* Y^* \right)_{ij} M_j Y_{ja} + Y^* Y Y^* M_j Y_{ja} \right] \]

\[ \epsilon^{\text{vortex}}_{\nu_i \rightarrow l_a} = \frac{1}{9i} \sum_{j>i} \sqrt{M_j} \left[ 1 - \left( 1 + \frac{M_i}{M_j} \right) \log \left( 1 + \frac{M_i}{M_j} \right) \right] \text{Im} \left[ \left( Y^* Y^* Y^* \right)_{ij} M_j Y_{ja} \right] \]

Note the resonant enhancement of the wave-function contribution as \( M_1 \rightarrow M_2 \).
Standard Approach – Unitarity

- Interference of tree and loop amplitudes $\rightarrow$ CP violation.

\[
\left| \begin{array}{c}
\phi \quad l \\
N_a \end{array} \right| + \left| \begin{array}{c}
\phi \\
N_a \end{array} \right| + \left| \begin{array}{c}
\phi \\
N_a \end{array} \right|^{2}
\] 

- CP violating contributions from discontinuities $\rightarrow$ loop momentum where cut particles are on shell (Cutkosky rules).

- Is an extra process or is it already accounted for by

\[
\text{[diagram]}
\]

(cut particles for CP violation)

- Inlcuding (*) only $\rightarrow$ CP violation generated in equilibrium.

- Conflict with CPT theorem that excludes presence of asymmetries in equilibrium.
(Inverse) Decays & CP Asymmetry

Consider squared matrix elements, ε the parameter that quantifies the CP asymmetry.

\[ |U_{\nu\rightarrow \phi}|^2 \sim 1 + \varepsilon \]
\[ |U_{\bar{\nu}\phi^* \rightarrow \nu}|^2 \sim 1 + \varepsilon \]
\[ |U_{\nu \rightarrow \bar{\phi}}|^2 \sim 1 - \varepsilon \]
\[ |U_{\bar{\nu} \phi^* \rightarrow \bar{\nu}}|^2 \sim 1 - \varepsilon \]

Naive multiplication* suggests that an asymmetry is generated already in equilibrium:

\[ |\bar{\nu} \phi^* \rightarrow \nu \phi | \sim 1 + 2 \varepsilon \]

*Do not try this at home: the unstable N are not asymptotic states of a unitary S matrix → conflict with CPT theorem.

Usual Fix: RIS Subtraction

Generation of CP-asymmetry in equilibrium is at odds with CPT theorem.

Usual fix: subtract Real Intermediate States (RIS) from [Kolb & Wolfram (1980)].
Standard Approach: Applicability & Limitations

**Applicability:**
- Vacuum $S$-Matrix elements appropriate for finite-temperature environment?
- Should be OK if the time an individual reaction takes is below the reaction rate.
  
  - virtuality $\sqrt{|q^2|} \gg$ reaction rate
  - time for individual reaction $\ll$ time between two reactions

**Limitations:**
- Valid approximation e.g. for freeze out of WIMPs at leading order (LO).
  
  - t-channel divergences, LO effect
  - CP-violating cuts: on-shell intermediate states, LO effect
  - Higher order corrections, e.g. soft & collinear emissions, NLO effect

- Reaction time large (virtuality small), or it is unclear how to define it.
2. Non-Equilibrium Field Theory Approach
Relativistic Scattering Theory: S-Matrix

- The LSZ reduction formula relates S-Matrix elements to time-ordered Green functions. These quantities can be represented and calculated in terms of Feynman diagrams.

- Path integral representation of time-ordered Green-functions:

\[
Z[J] = N^{-1} \langle \text{vac}_{\text{out}} | \text{vac}_{\text{in}} \rangle_J = \int D\phi e^{i \int dx (L + J(x)\phi(x))} \\
\langle T[\phi(x)\phi(y)] \rangle = -\left. \frac{\delta^2}{\delta J(x) \delta J(y)} \log Z[J] \right|_{J=0}
\]

- We call the partition function \(Z\) here the in-out generating functional.

- Alternatively, use explicit time-evolution and derive Feynman rules from Dyson series and Wick’s theorem:

\[
\langle T(\phi(x)\phi(y)) \rangle = \lim_{T \to \infty(t-i\epsilon)} \frac{\langle T(\Sigma(x)\Sigma(y)U(T,T')) \rangle}{\langle U(T,T') \rangle}
\]
The Closed Time Path (CTP) Approach

- **In-in generating functional:**
  \[ Z[J_+, J_-] = \int \mathcal{D}\phi \mathcal{D}\phi^- \mathcal{D}\phi^+ \mathcal{D}\phi^- \langle \phi^- | \psi, \tau \rangle_y \langle \psi, \tau | \phi^+ \rangle_y \langle \phi^- | \phi^+ \rangle \]
  \[ = \int \mathcal{D}\phi^+ \mathcal{D}\phi^- e^{i \int dt \left\{ L[\phi^+] + J_+ \phi^+ - L[\phi^-] - J_- \phi^- \right\}} \langle \phi^- | \phi^+ \rangle \]

- **The Closed Time Path:**

- **Path-ordered Green functions:**
  \[ i \Delta_{\phi}^{ab}(u, v) = -\frac{\delta^2}{\delta J_a(u) \delta J_b(v)} \left. \log Z[J_+, J_-] \right|_{J_+ = 0} = i \langle \mathcal{C} [\phi^a(u) \phi^b(v)] \rangle \]

  - **Cf. canonical time evolution:**
  \[ \langle \phi(x, x_0) \phi(y) \rangle = \lim_{T \to \infty, t \to i\tau} \frac{\langle U(T, x_0) \phi(x) U(x, y) \phi(y) U(y, -T) \rangle}{\langle U(T, t) \rangle} \]

  - **Thermal density matrix can be incorporated by attaching path into imaginary direction to the point \( \tau_0 \).**

- Cf. canonical time evolution:
  
  - Schwinger (1961);
  - Keldysh (1965);
Path-Ordered Green Functions @ Tree Level

- Four propagators (two of which are linearly independent):
  \[
  i \Delta^L_\phi (u,v) = i \Delta^T_\phi (u,v) = \langle \phi (v) \phi (u) \rangle \\
  i \Delta^R_\phi (u,v) = i \Delta^+_\phi (u,v) = \langle \phi (u) \phi (v) \rangle \\
  i \Delta^T_\phi (u,v) = i \Delta^{++}_\phi (u,v) = \langle T [\phi (u) \phi (v)] \rangle \\
  i \Delta^T_\phi (u,v) = i \Delta^{--}_\phi (u,v) = \langle \overline{T} [\phi (u) \phi (v)] \rangle
  \]

- Perturbation theory can be formulated in terms of tree-level expressions:
  \[
  i \Delta^L_\phi (p) = 2 \pi \delta(p^2 - m^2) \left[ \Theta (p^0) \Theta (-p^0) + \Theta (-p^0) (1 + \overline{\Theta (-p^0)}) \right] \\
  i \Delta^R_\phi (p) = 2 \pi \delta(p^2 - m^2) \left[ \Theta (p^0) (1 + \Theta (p^0)) + \Theta (-p^0) \overline{\Theta (-p^0)} \right] \\
  i \Delta^T_\phi (p) = i \frac{m^2 - p^2 + i \epsilon}{m^2 - p^2 + i \epsilon} + 2 \pi \delta(p^2 - m^2) \left[ \Theta (p^0) \Theta (p^0) + \Theta (-p^0) \overline{\Theta (-p^0)} \right] \\
  i \Delta^{--}_\phi (p) = i \frac{m^2 - p^2 - i \epsilon}{m^2 - p^2 - i \epsilon} + 2 \pi \delta(p^2 - m^2) \left[ \Theta (p^0) \Theta (-p^0) + \Theta (-p^0) \overline{\Theta (-p^0)} \right]
  \]

- Similarly for spin-1/2 fermions, gauge bosons, ...
Feynman Rules

- Vertices either + or -
- Connect vertices \( a = t \) and \( b = t \) with \( i \Delta^{ab} \)
- Factor -1 for each - vertex

Schwinger-Dyson Equations

\[
i \Delta^{ab} = i \Delta^{(0)ab} + c d i \Delta^{(0)ac} \circ \Pi \circ d \circ \Delta^{db}
\]

- These describe in principle the real time evolution of the quantum system.
- Time evolution expressed in terms of Green functions. No reference to \( S \)-matrix elements for scattering processes in a vacuum background.
- Rather than choosing the detour via transition probabilities, directly calculate the time evolution of the quantum state in this approach.
Kinetic Equations from the CTP Approach

- The \( <,> \) parts of the Schwinger-Dyson equations are the celebrated Kadanoff-Baym equation:

\[
(-\partial^2 - m^2) \Delta^{<,>} - \Pi^{<,>}_H \Delta^{<,>} - \Pi^{<,>}_\theta \Delta^{\theta} = \frac{i}{2} (\Pi^{<,>}_H \Delta^{<,>} - \Pi^{<,>}_\theta \Delta^{\theta})
\]

- Remaining linear combination gives pole-mass equation:

\[
(-\partial^2 - m^2) i \Delta^{R,A} - \Pi^{R,A}_H \Delta^{R,A} = i \Sigma^A
\]

- Wigner transformation:

\[
A(k,x) = \int d^q x' e^{ikx'} A(x' + \frac{i}{2}, x' - \frac{i}{2})
\]

- Suitable for describing particle number & charge densities.
3. Application to Leptogenesis
3.1 Unitarity in CTP Approach to Leptogenesis

- The inclusiveness of the CTP approach guarantees that no over-/undercounting occurs. → No subtraction of RIS necessary.

- As byproduct, obtain corrections due to quantum statistical distributions (not important though for strong washout):

\[
\mathcal{Z}^L(k) = -\frac{1}{2} \int \frac{d^3p}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \delta^4(k-p-q) \sin^4(p^0) \left[ |Y_1^L|^2 |Y_2^L|^2 (P_R^+ Y_1^L Y_2^L P_R) \right] \left[ 1 - e(k^2) + 4e(q^2) \right]
\]

\[
= \left( \frac{1}{2} \right) + \left( \frac{1}{2} \right)
\]

\[
S = 3 \left[ \frac{|Y_1^L|}{M_2} \right] \left( -\frac{M_4}{M_2} \right) \int \frac{d^3k}{(2\pi)^3} \frac{1}{2 \sqrt{k^2 + M_1^2}} \left[ \Delta^0(k^2) - \Delta^1(k^2) \right] \sum_{\mu \nu} \left( \mathcal{L}_{\mu \nu}^0(k) \mathcal{L}_{\mu \nu}^0(k) \right) M_2 >> M_1
\]

Garny, Hohenegger, Kartavtsev, Lindner (2009,2010);
Beneke, BG, Herranen, Schwaller (2010);
Anisimov, Buchmüller, Drewes, Mendizabal (2010).
Can we connect phases that are observable at low energies (EDMs) with Leptogenesis? → Try “Soft Leptogenesis”.

Above diagrams lead to a lepton asymmetry, that is however cancelled when supplementing the stable external states and subtracting RIS.

Is there a loophole when accounting for the different equilibrium statistics of fermionic/bosonic external states?  

Not when computing the inclusive thermal production rate for leptons in CTP approach.

Loophole: Can generate lepton/slepton asymmetry above $10^7$ GeV, when chirality-flipping gaugino exchange is out-of equilibrium.

Unitarity to be considered for asymmetric DM freeze-in.

Comprehensive beastology of all possible CP-cuts @ finite $T$ in MSSM.
3.2 Thermal Production of Right-Handed Neutrinos

(Nearly) Massless (i.e. Relativistic) Right-Handed Neutrinos

- Of interest for Leptogenesis from oscillations (see below); lepton asymmetry can also influence subsequent DM abundance of RHNs.

- Tree-level production process:

- Thermal masses suggest different process:

\[ m_{\phi}^2 = \frac{A}{16} (3g_1^2 + g_2^2 + 4g_E^2 + 2\lambda)^2, \quad m_{\nu}^2 = \frac{1}{16} (3g_1^2 + g_2^2 + 2\lambda)^2, \quad m_{\nu} \ll 0 \Rightarrow m_{\phi} > m_{\nu} + m_{N} \]

- Yet, the phase space is suppressed by the square of gauge and Yukawa couplings. At this order, there are also contributions from scattering amplitudes:

- In fact, the radiative processes are the LO contribution to the production of relativistic RHNs.
Massless Right-Handed Neutrinos: $t$-Channel Divergence

- A $t$-channel divergence arises when integrating over thermal phase-space:

$$\mathcal{M} \sim \frac{1}{(p_{\nu} \cdot p_{\phi})^2}$$

- The following integral is divergent:

$$\int \frac{d^4 p_\ell \, d^4 p_{\nu} \, d^4 p_{\phi} \, d^4 p_w}{(2\pi)^4 \, 2E_{\ell} \, 2E_{\nu} \, 2E_{\phi} \, 2E_w} \, (2\pi)^4 \delta^4(p_{\ell} + p_{\nu} - p_{\phi} - p_w) \, \sum_{\text{pol.}} |\mathcal{M}|^2 \left(1 - \omega(\nu)\right) \left(1 - \omega(\phi)\right) \delta(p_{\phi}) \delta(w) \left|\psi(\phi)\right|^2$$

- Physically, expect screening of exchange of soft leptons by gauge interactions.
- Readily implemented in 2PI CTP approach. RHN production rate is given by:

Double lines indicate resummed propagators. To LO, the two-loop diagram can be expressed in terms of tree-level propagators. Kinematically accessible cut contributions.
**Screening of t-Channel Divergences**

- For the resummed propagators (spectral functions), take:

\[
\Sigma^u(m) = \frac{2(K - \mathcal{H}^u) \cdot \Sigma^u (k - \mathcal{H}^u) - K^u (k - \mathcal{H}^u)^2 + \mathcal{I}^u^3}{[(K - \mathcal{H}^u)^2 - \mathcal{I}^u^2]^2 + 4[(\Sigma^u_1 (k - \mathcal{H}^u)]^2}
\]

\[
\mathcal{H} = \gamma^u \Sigma^u
\]

\[
\Delta \Phi = \frac{\Pi^u}{(k^2 - \mathcal{H}^u)^2 + \Pi^u^2}
\]

Note reduction to delta-functions when the widths goes to zero.

- **Result** [Bödeker & Besak (2012); BG, Glowna, Schwaller (2013)]:

\[
\frac{\Gamma^u}{g^2 V} = (3.1 \times 10^{-3} - 3.7 \times 10^{-4} \log(\frac{3}{2} g_2^2 + \frac{4}{7} g_1^2)) \times \left(\frac{3}{2} g_2^2 + \frac{4}{7} g_1^2\right) T^4 + 5.2 \times 10^{-4} \mu_e^2 T^4
\]

- Due to the \( t \)-channel divergence, LO production of massless particles is already an involved calculation.

- Rate of lepton flavour violation for flavoured Leptogenesis:

  [BG, Glowna, Schwaller (2013)]

\[
\frac{\Gamma^{\delta l}}{h_0 \mu_e^2} = (5.4 \times 10^{-3} - 8.3 \times 10^{-4} \log(\frac{3}{2} g_2^2 + \frac{4}{7} g_1^2)) \times \left(\frac{3}{2} g_2^2 + \frac{4}{7} g_1^2\right) T
\]

\[
+ (4.7 \times 10^{-3} - 1.7 \times 10^{-3} \log g_1) g_1 T + 1.5 \times 10^{-3} \mu_e^2 T
\]
Massive (i.e. Non-Relativistic) Right-Handed Neutrinos: Soft and Collinear Divergences

- When the RHN mass is of the same order or above the temperature, the $1 \leftrightarrow 2$ contributions to RHN production become dominant/relevant.

- Due to the RHN mass, new cuts are possible, that may be interpreted as virtual corrections. The loops involved suffer from IR divergences:

- Soft and collinear divergences occur, when the intermediate propagator in the (inverse) decay diagrams is on-shell:

- At zero temperature, it is well known that the IR, soft & collinear divergences cancel when added (inclusive rates/cross sections $\to$ KLN theorem).
Cancellation of Soft and Collinear Divergences

Cancellation can be demonstrated @ NLO and arbitrary temperature by rearranging integrands to matching finite pieces [BG, Glowna, Herranen (2013); Laine (2013); Laine & Ghisiou (2014)].

Proof relies on detailed balance relations for collinear reactions:

\[
\frac{d\phi(E)(1+d\phi(aE))(1+d\lambda((1-a)E))}{1+d\lambda((1-a)E)} = (1+d\phi(E)) \frac{d\phi(aE) d\lambda((1-a)E)}{aE} - \frac{E^{(1-a)E}}{aE}
\]

\[
\frac{d\lambda(E)(1-d\lambda(aE))(1+d\lambda((1-a)E))}{1+d\lambda((1-a)E)} = (1-d\lambda(E)) \frac{d\lambda(aE) d\lambda((1-a)E)}{aE} + \frac{E^{(1-a)E}}{aE}
\]

Two scales: RHN mass \( M \) and temperature \( T \) → general results need to be obtained numerically and are somewhat intransparent.

When \( M \gg T \), result can be presented in a simple form that can even be expanded analytically in powers of \( T/M \). → Use approximations:

- RHNs are at rest and
- replace quantum-statistical distributions by Maxwell distributions.

[Lodone, Salvio, Strumia (2011); Laine, Schröder (2011); Biondini, Brambilla, Vairo (2013)].

\[
\mu^2 = \frac{\gamma^2 M}{2 \pi^2} \left[ 1 - \frac{\lambda g}{2^{\pi^2}} (3 g_s^2 + g_1^2) - \frac{\lambda}{2^{\pi^2}} \lambda - \lambda \frac{T^2}{\mu^2} \right] \text{ Mind also corrections to } \mu(T) \text{ ("susceptibility"), Bödeker & Laine (2014).}
\]


\( \mathcal{O}(T^2) \) correction to \( CP \)-violating rate still missing in the non-relativistic expansion to NLO.
3.3 Flavoured Leptogenesis

- RHN Yukawa couplings including flavour: \( Y_{ia} N_i \phi_a \)
- When insensitive to SM flavour \( a \), can transform SM lepton basis and bring \( Y \) to triangular form:

\[
\begin{pmatrix}
N_1 & N_2 & N_3 \\
x & 0 & 0 \\
x & x & 0 \\
x & x & x
\end{pmatrix}
\begin{pmatrix}
L_1 \\
L_2 \\
L_3
\end{pmatrix}
\]

only \( L_1 \) couples to \( N_1 \) \( \rightarrow \) single flavour leptogenesis, but \( L_7 = a_e L_e + a_\mu L_\mu + a_\tau L_\tau \) in general

- When \( T \) falls below \( 10^{12} \) GeV (\( 10^9 \) GeV, \( 10^4 \) GeV), \( h_{\tau\tau} (h_{\mu\mu}, h_{ee}) \) come into equilibrium (reactions faster than expansion rate \( H \)).

[Abada, Davidson, Josse-Michaux, Losada, Riotto (2006); Nardi, Nir, Roulet, Racker (2006)]

- Between \( 10^9 \) GeV and \( 10^{12} \) GeV, only the off-diagonal components involving \( \tau \) evaporate and we may effectively distinguish two flavours.
- Goal: Describe intermediate regime between full flavour coherence and decoherence.
Flavoured Leptogenesis

- Kinetic equations for (anti-) leptons:

\[
\frac{dN_{\ell\ell}}{dt} = \pm S_{ab} \mp i A_{\ell\ell} \delta N_{\ell\ell} - \left[ W_{ab}, \delta N_{\ell\ell} \right]_{ab} - \frac{1}{2} \left( \delta N_{\ell\ell}^f + \delta N_{\ell\ell}^\ell \right) - \Gamma_{ab}
\]

- CP-violating source
- oscillations induced by thermal masses
- washout
- flavour-blind pair creation & annihilation
- flavour-sensitive interactions

\[
\Gamma_{ab} \propto \frac{1}{h_\nu^2} \left( \delta N_{\ell\ell}^f - \delta N_{\ell\ell}^\ell \right)
\]

- Can interpolate between fully flavoured and unflavoured regimes:
  [Beneke, BG, Fidler, Herranen, Schwaller (2010)]

- Results (in particular freeze-out of oscillations) confirmed using density-matrix techniques.
  [Blanchet, Di Bari, Jones, Marzola (2011)]
3.4 Resonant Leptogenesis

- We can derive equations for the distribution functions that resemble equations for density-matrices:

\[ 2i\, k^0 \partial_k \left[ \delta_{\nu}(k) + \delta_{\bar{\nu}}(k) \right] - [\mathbf{M}^2, \delta_{\nu}(k)] = -i \{ \Pi_U(k), \delta_{\nu}(k) \} \]

- Mass commutators induce flavour oscillations:

\[ \mathbf{M}^2 = \begin{pmatrix} M_1^2 & 0 \\ 0 & M_2^2 \end{pmatrix} \Rightarrow [\mathbf{M}^2, \delta_{\nu}(k)] = (M_1^2 - M_2^2) \begin{pmatrix} 0 \\ \delta_{\nu_1}(k) \end{pmatrix} \begin{pmatrix} \delta_{\nu_2}(k) \\ 0 \end{pmatrix} \]

- Unless we are close to the resonance, we can average over the oscillations → recover standard decay asymmetry:

\[ M_1^2 - M_2^2 \gg \Pi U \Rightarrow \delta_{\nu_1} \propto \frac{\Pi U(k)}{M_1^2 - M_2^2} (\delta_{\nu_1} + \delta_{\nu_2}) \]

- Time derivative acts as regulator in mass-degenerate limit.

- Method applies to resonant as well as hierarchical limits. [BG, Herranen (2011); Garny, Hohenegger, Kartavtsev (2011)]

- Recently challenged: “ε-type” (as in the usual decay asymmetry) and “regenerative” (as here) asymmetries are different contributions → factor two enhancement. [Dev, Millington, Pilaftsis, Teresi (2014)]
Strong Washout Limit of Resonant Leptogenesis

- Kinetic equation for non-relativistic (for simplicity) RHNs:

\[
\frac{M}{d\gamma} \delta n_0 + \frac{i\gamma^2}{2M^2H} \left[ \mathcal{M}^2, \delta n_0 \right] + \frac{\gamma^2}{2M} \left\{ \Gamma, \delta n_0 \right\} + \frac{\mathcal{M}}{d\gamma} n_{eq} = 0
\]

where \( \mathcal{M}_1^2 - \mathcal{M}_2^2 = \frac{\Gamma}{\mathcal{M}} \), \( \Gamma = \frac{\text{Re} \left[ Y^* Y^T \right] \mathcal{M}}{8\pi} \), \( \mathcal{M} = \frac{M_1 + M_2}{2} \), \( \gamma = \frac{\mathcal{M}}{T} \).

- Derivative \( (d/d\gamma) \) negligible if eigenvalues of this equation are larger than Hubble rate \( H \). In strong washout, this condition is (typically) met by presence of \( \Gamma \) – even in mass-degenerate case.

- Time-independent, effective decay asymmetry:

\[
\epsilon_{ab} = \frac{g_{ab}}{8\pi} \left( \frac{1}{[YY]^2_{14} + [YY]^2_{12}} + \frac{1}{[YY]^2_{24} + [YY]^2_{22}} \right) \frac{\mathcal{M}^2 (\mathcal{M}_1^2 - \mathcal{M}_2^2)}{(M_1^2 - M_2^2)^2 + R}
\]

\[
\epsilon_{ab} = -\frac{i}{2} \left( y_1^T [Y^* Y^T + Y^* Y^T]_{22} - y_2^T [Y^* Y^T + Y^* Y^T]_{12} \right)_{16} - y_1^T [Y^* Y^T + Y^* Y^T]_{22} - y_1^T [Y^* Y^T + Y^* Y^T]_{12} \right)_{16}
\]

\[
R = \frac{\mathcal{M}^4}{64\pi^2} \frac{([YY]^2_{14} + [YY]^2_{12})^2}{[YY]^2_{14} [YY]^2_{12}} \left( \text{Im} [YY]^2_{14} \right)^2 + \text{det} [YY]^2_{14}
\]

- Regulator \( R \) different from what has been proposed in other phenomenological calculations [Pilaftsis (1997); Anisimov, Broncano, Plümacher (2005)].

- There is no universal form for the regulator – but there is one that applies to strong washout (wide parametric range).
(Typical) Numerical Example with Phenomenologically Viable Neutrino Parameters and $\Delta M \ll \Gamma$

Time-independent effective decay asymmetry and following solution for the flavoured asymmetries.

Time-dependent effective decay asymmetry and flavoured asymmetries (full solution).

- Effective decay asymmetry applies throughout type-I seesaw with 2RHNs [there may (should) be examples with 3 RHNs, where one has to resort to the full solution, which is no problem, given the present methods].

[BG, Gautier, Klaric (2014)]
Resonant Leptogenesis: Other Recent Developments

- Importance of flavour effects for low-scale resonant Leptogenesis (low $T$, where a naïve treatment would suggest that flavour correlations are irrelevant) → even though the flavour correlations are suppressed by SM Yukawa interactions, they are important to protect part of the initial decay asymmetries from washout. [Dev, Millington, Pilaftsis, Teresi (2014)]

- A detailed study on the thermal mass corrections to the RHNs – albeit small, this needs to be understood because this can have impact on the degeneracy [Hohenegger, Kartavtsev (2014)]. → Talk on Tuesday.
3.5 Leptogenesis from RHN Oscillations

Finite Temperature Cuts

- When $T \gg M$, the standard cuts are strongly suppressed.
- However, the rates including extra radiation that we discussed above lead to new kinematically viable cuts (cf. Thermal production of RHNs):

  $\begin{align*}
  (\ell^-)^{-1} & = \delta + \sum \frac{\mathcal{H}_{\ell^-}}{M} + \ldots \\
  \mathcal{H}_{\ell^-} & = \ldots + \ldots
  \end{align*}$

- RHNs are produced far from equilibrium (underabundant) and start to oscillate [Akhmedov, Rubakov & Smirnov (1998)].
Leptogenesis from GeV (or TeV) Scale RHNs Requires no Mass Degeneracy

Source of flavoured asymmetries, including thermal cuts:

\[
S_{\alpha\beta} = \sum_{i,j} \frac{32}{M_{ij}^2 - M_{ii}^2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\sqrt{k^2 + M_{ii}^2}}
\]

\[
\times \left\{ \ln \left[ Y_{ii}^+ \left( Y Y^c \right)_{ij} Y_{ij} \right] \frac{1}{M_{ij}^2 + 2k^2} \left( \sum_{\alpha} w_{\alpha}^2 + \sum_{\beta} u_{\beta}^2 \right) - 4/\sqrt{k^2 + M_{ii}^2} \sum_{\alpha} w_{\alpha} k \sum_{\alpha} u_{\alpha} \right\} \delta_{ii} \delta_{ij} (k^2)
\]

(Standard) lepton number violating contribution proportional to \(M^2/\Delta M^2\) → need \(\Delta M^2 << M^2\) for large enhancement.

Lepton number conserving (but flavour violating) contribution proportional to \(T^2/\Delta M^2\) → large enhancement for \(\Delta M^2 << T^2\) → no mass degeneracy required.

Leptogenesis is viable with non-degenerate (in mass) RHNs of GeV-scale mass [Drewes, BG (2012)] and for masses \(>5 \times 10^3\) GeV [BG (2014)]. → Possibility of direct detection. Talk by M. Drewes on Tuesday.

\[
\delta m_H^2 = - \sum_{i,a} \frac{1}{16 \pi^2} \left( m^2 + M_i^2 \log \frac{M_i^2}{\Lambda^2} \right)
\]

\(\leftrightarrow M_i \lesssim 10^3\) GeV

Application to classically conformal models [Khoze, Ro (2013)].
3.6 Asymmetry from Mixing/Oscillations of Charged States

- Wave-function corrections lead to off-diagonal correlations, that in turn induce asymmetries in diagonal charge densities.

- Off-diagonal correlations are a non-equilibrium effect → can occur for SM leptons from loop that propagates nonequilibrium RHNs:

\[
\delta \rho_{ab} = \sum_i \frac{\delta \psi_i^\dagger \delta \psi_i}{\sum_i \delta \psi_i^\dagger \delta \psi_i} = \frac{\sum_i \gamma_i^\dagger \gamma_i}{\sum_i \gamma_i^\dagger \gamma_i} \\
B_0 \approx 10^{-2} T^2 \\
B_\phi \approx 1.7 \times 10^{-3} T^2 \\
\Rightarrow \text{max. enhancement:} \quad \frac{T^2}{B_0} \left( \frac{\gamma_i^\dagger \gamma_i}{B_\phi} \right)
\]

\[
\left( \frac{\gamma_i^\dagger \gamma_i}{B_\phi} \right)_{ab} = \delta_{ab} + \sum_i \gamma_i^\dagger \gamma_i
\]

\[
M_1 = 4 \times 10^9 \text{ GeV}
\]

- Third RHN decouples → strong washout
- freezeout occurs around \( z = 10 \)
- reheat temperature as low as \( 5 \times 10^8 \text{ GeV} \) possible.

- Source overlooked so far in type-I seesaw. Can also generate asymmetries when using mixing Higgs doublets [BG (2012)] or other charged states that occur replicatedly → opportunities for model building.
Summary & Conclusions

- Instead of taking the detour via $S$-matrix elements, it is conceptually simpler (and more systematic and hence safer for the derivations) to directly calculate the time-evolution of the quantum state that carries the baryon-/lepton-asymmetry of the Universe in the CTP formalism.

- Once the formalism is set up, the main challenges are the same in traditional and CTP approaches: loop & phase-space integrals in thermal environments (cf. production of RHNs and rate for flavour decoherence).

- Theory benefit: systematic realisation of Sakharov's out-of equilibrium condition within a field theoretical calculation (in particular respecting unitarity $\Rightarrow$ no RIS subtraction “by hand” necessary).

- Phenomenological ideas/calculations triggered by CTP techniques:
  - method for resonant Leptogenesis that applies for all parameters,
  - flavour decoherence of charged leptons during Leptogenesis,
  - improved understanding & extension of the viable parameter space for Leptogenesis from oscillations of sterile neutrinos $\Rightarrow$ direct detection in type-I seesaw framework possible?
  - Leptogenesis from mixing/oscillations of charged states.
\[ J^m \cdot F_m = \sum_{w=1}^{w=3} \text{ number of generations} \]

\[ J^m \cdot B_0 = \frac{2me^3}{32\pi^2} \left( W^a_{\mu\nu} \tilde{\omega}^{\mu\nu} + g_1 B_{\mu\nu} \tilde{B}^{\mu\nu} \right) \]