Population Annealing for Spin Glasses

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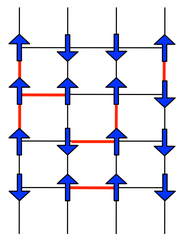
Model

This work is on the Edwards Anderson (EA) spin glass which is an Ising system with random ferromagnetic and antiferromagnetic bonds.

$$H = \sum_{\langle i,j \rangle} J_{i,j} s_i s_j,$$

$\langle i,j \rangle \Rightarrow$ sum over nearest neighbors

$J_{i,j}$ = gaussian random variable



A theoretically important quantity in spin glasses is the overlap between two independent spin configurations with the same set of bonds. The overlap is defined as

$$q = \frac{1}{N} \sum_{i=1}^{N} s_i^1 s_i^2,$$

where s_i^j is the i'th spin of configuration j. When the overlap is sampled at a temperature, it results in a distribution, $P_J(q)$, which is unique for each set of bonds. When averaged over bond configurations, we get

$$P(q) = [P_J(q)]_D \,,$$

where $[...]_D$ denotes an average over bond configurations. In fact, q is the order parameter, and $P(q)$ helps to describe the glass transition.

Characteristics and Open Problems

Due to their random bond structure, spin glasses have no long-range magnetic correlations, and at high temperatures they behave as paramagnets. At low temperatures a more complex "spin glass" phase appears. This phase is characterized by a rough free energy landscape which has several local minima.

Problems:

- After the glass transition, dynamics become stuck and the system becomes numerically difficult
- Finding the ground state of a disorder configuration is an NP-hard optimization problem
- The thermodynamic limit is not understood analytically
- Computationally limited to systems of size $N \sim 10^3$
  $\Rightarrow$ still not understood numerically

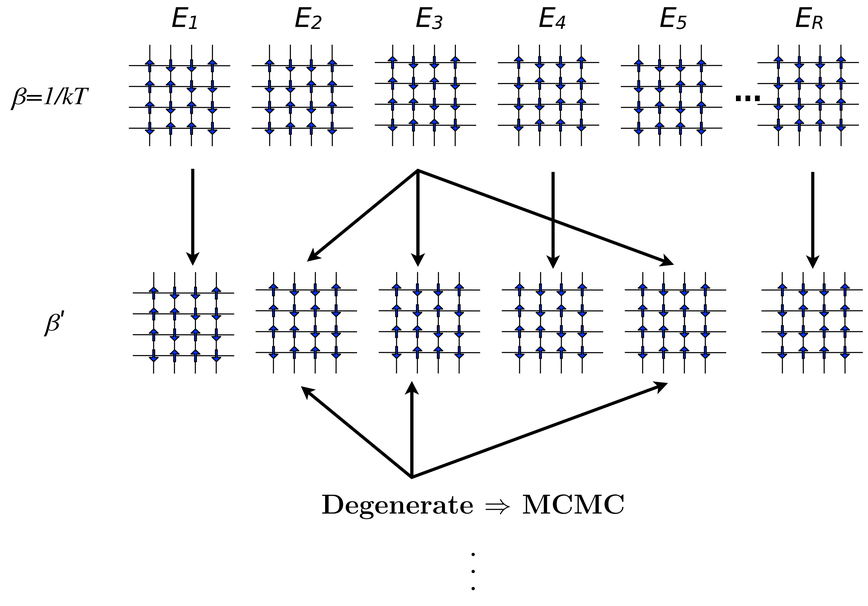
Population Annealing

Population annealing is an algorithm designed to efficiently sample the equilibrium statistics of a spin glass at low temperatures for a single set of bonds. The algorithm's aim is to simulate a population of configurations which is slowly cooled and is always kept in thermal equilibrium.

Protocol:

1. Initialize R independent configurations at inverse temperature $\beta = 0$
2. Thermalize with Markov Chain Monte Carlo (MCMC)
3. Decrease temperature (the population is **out** of equilibrium with respect to new temperature)
4. Resample the configurations according to their Boltzmann weight (now they're in equilibrium, but some are degenerate/correlated)
5. De-correlate and thermalize with MCMC

Graphically:

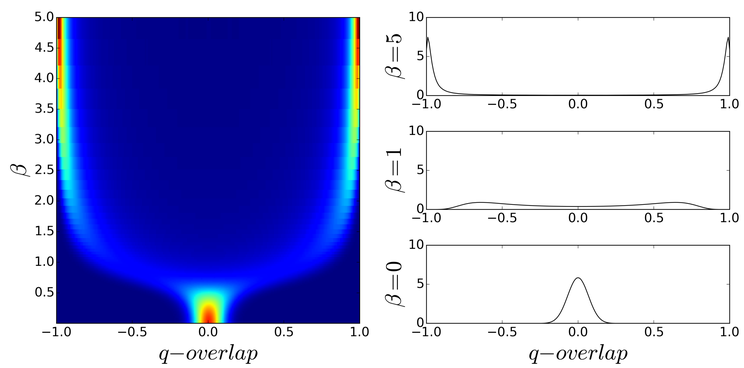


Advantages:

- Large population is able to probe large area of configuration space
- System is always in thermal equilibrium
- Allows sampling of estimators over large range of β values
- Easy to calculate q, $P(q)$, and free energy
- Multiple simulations can be easily combined
- Systematic and statistical errors are easily estimated

Results

Because an entire population is simulated in parallel, population annealing is particularly well suited to measuring the q overlap distribution. Below we show results for the 3D Edwards Anderson spin glass with $N = 6^3$ spins.



The behavior of the distribution of overlaps is particularly important in determining the behavior in the thermodynamic limit.

Our work is ongoing and includes further optimizing population annealing, conducting large scale simulations, and applying population annealing to other frustrated systems.

References

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Acknowledgements

This work was supported by NSF Grant DMR-1507506.