Dark Continuum in the Spectral Function of the Resonant Fermi Polaron

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Summary

- controlled numerical study of the ground state **spectral function** of the **resonant Fermi polaron** in 3d
- establish the existence of a **"dark continuum"** — a region of anomalously low spectral weight between the narrow polaron peak and the broad maximum at positive energy
- the dark continuum develops around $k_F a \lesssim 1$, i.e. in the absence of a small interaction-related parameter
- detailed analysis of what features of the spectrum can be recovered reliably (polaron peak, dark continuum) and what features are inaccessible (width and structure of the higher-frequency part of the spectrum)

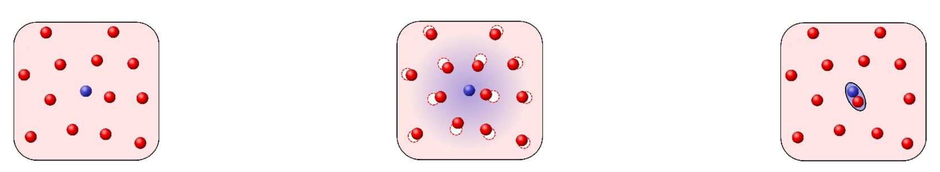
[1] O. Goulko, A. S. Mishchenko, N. Prokof'ev, B. Svistunov, Phys. Rev. A **94**, 051605(R) (2016), eprint arXiv:1603.06963

[2] O. Goulko, A. S. Mishchenko, L. Pollet, N. Prokof'ev, B. Svistunov, Phys. Rev. B **95**, 014102 (2017), eprint arXiv:1609.01260

The Fermi Polaron

A **single spin** $\downarrow$ fermion interacting with a **sea of spin** $\uparrow$ fermions

- extremal case of an imbalanced Fermi mixture
- impurity problem
- direct realization of a quasiparticle

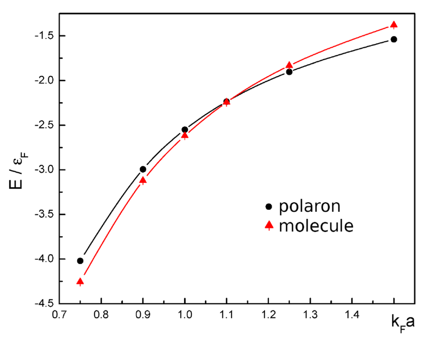


weak attraction: mean-field of the medium | stronger attraction: polaron "dressed" with cloud of spin $\uparrow$ atoms | strongest attraction: molecular bound state

image credit: Schirotzek, Wu, Sommer, Zwierlein, PRL**102**, 230402 (2009)

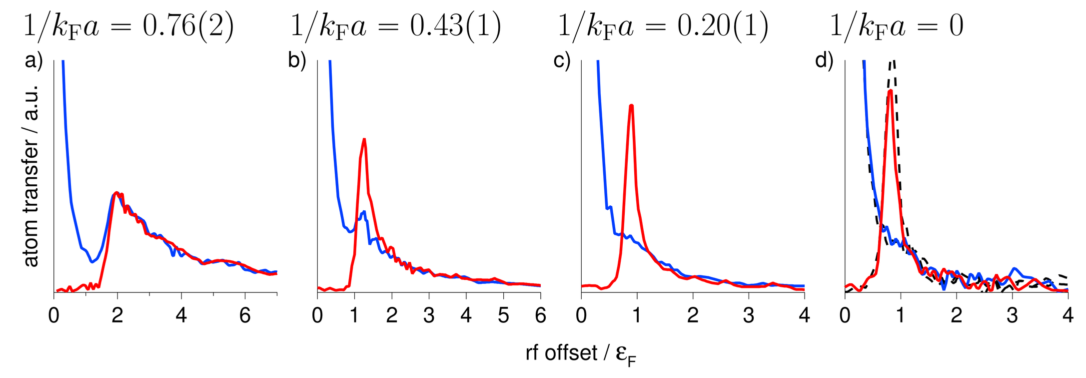
Interesting questions:

- ground state energy and Z-factor,
- polaron to molecule transition,
- polaron effective mass,
- unequal masses, trimers,
- spectral function, kinetics, . . .



[Prokof'ev & Svistunov, PRB77, 125101 and 020408 (2008)]

Experiments measure the **spectral function** via rf-spectroscopy



[Schirotzek, Wu, Sommer and Zwierlein, PRL**102**, 230402 (2009)]

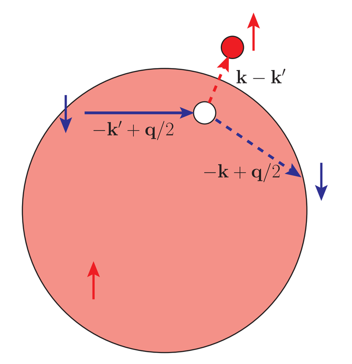
Our goals:

- **Quantitative** and **controlled** calculation of the polaron spectral function
- Rigorous interpretation of experimental spectral functions

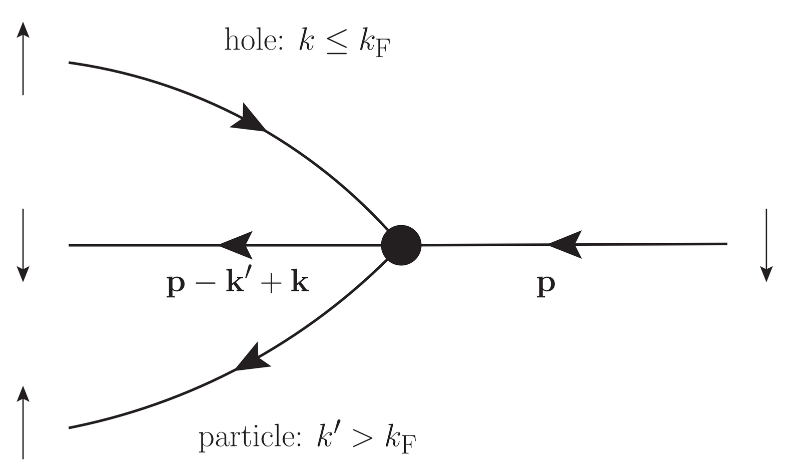
Model

$$H=\sum_{\mathbf{k},\sigma=\uparrow,\downarrow}\epsilon_{\mathbf{k}\sigma}c^\dagger_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}+g_0\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}}c^\dagger_{\mathbf{k}+\mathbf{q}/2,\uparrow}c^\dagger_{-\mathbf{k}+\mathbf{q}/2,\downarrow}c_{-\mathbf{k}'+\mathbf{q}/2,\downarrow}c_{\mathbf{k}'+\mathbf{q}/2,\uparrow}$$

- zero temperature
- 3D
- attractive interaction
- $m_\uparrow = m_\downarrow$
- short-ranged potential



Spin $\downarrow$ impurity excites **particle-hole pairs** via momentum exchange with the spin $\uparrow$ Fermi sea



Spectral function

- gives the system's response to the impurity creation operator $c^\dagger_{\mathbf{k}\downarrow}$
- probability of adding or removing a state with given momentum at energy cost ω

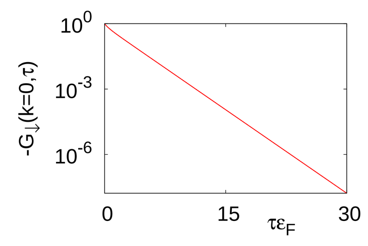
From full impurity Green's function:

$$-G_\downarrow(\mathbf{k},\tau)=\theta(\tau)\langle c_{\mathbf{k}\downarrow}(\tau)c^\dagger_{\mathbf{k}\downarrow}(0)\rangle=\int_0^\infty A_\downarrow(\mathbf{k},\omega)e^{-\omega\tau}d\omega$$

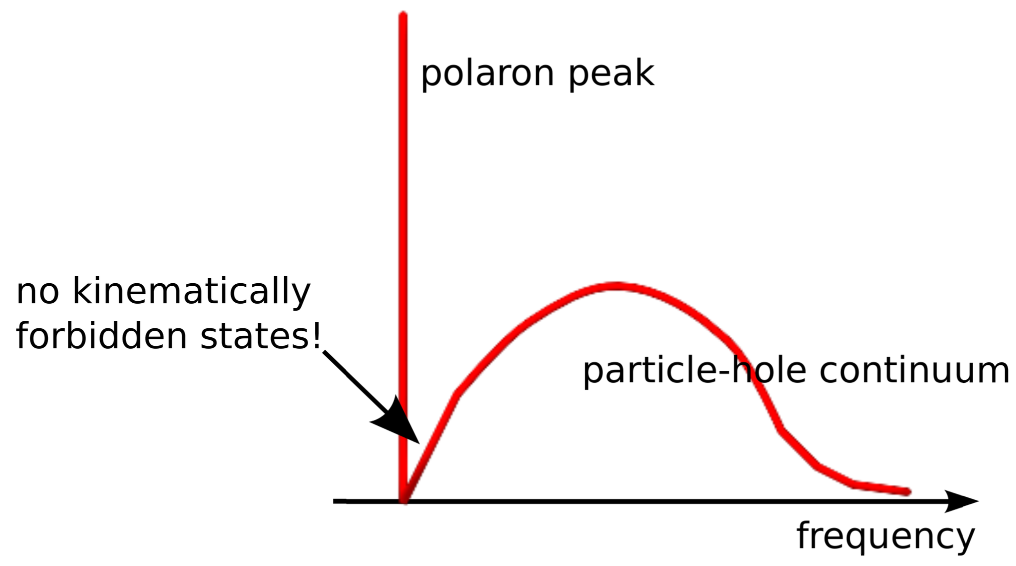
Asymptotic behavior:

$-G_\downarrow(\mathbf{0},\tau\to\infty)\to Ze^{-(E_p-\mu_\downarrow)\tau}$

$\Rightarrow$ polaron peak in spectrum $Z\delta(\omega+\mu_\downarrow-E_p)$

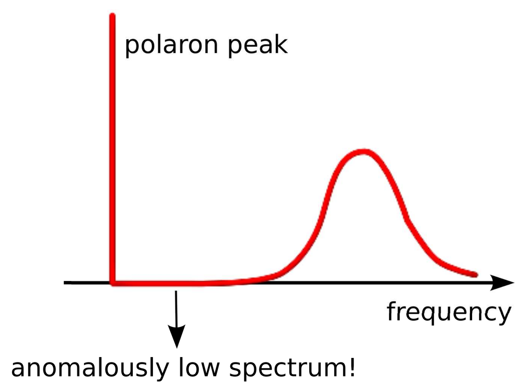


What we expect:



(matrix element controls the slope of the spectral function)

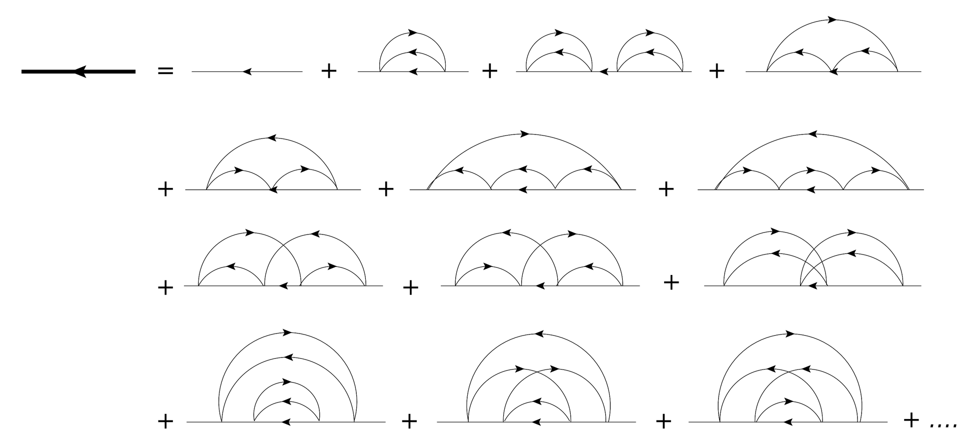
Instead for $k_F a \lesssim 1$ we find:



- spectral weight after the polaron peak strongly suppressed
- this happens despite the absence of a small interaction-related parameter!

Diagrammatic Monte Carlo

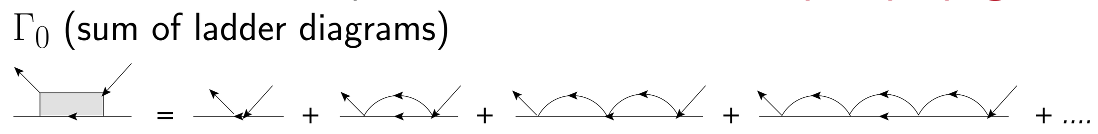
Expansion in the free Green's function and coupling constant:



- excitation of multiple particle-hole pairs
- impurity exchanges momentum with particles and holes

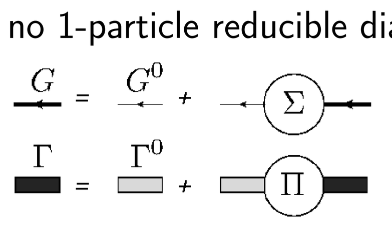
Factorially growing number of diagrams!

- Partial summation: replace interaction vertex with **pair propagator** Γ_0 (sum of ladder diagrams)



- Sample **self-energy**: no 1-particle reducible diagrams

Dyson equation:



- **Boldification**: expand in terms of the full Green's function (and full Γ-function) [Prokof'ev and Svistunov, PRB77, 125101 and 020408 (2008)]

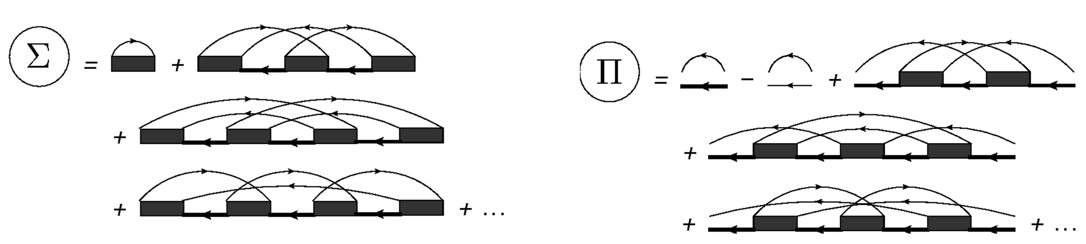
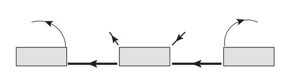


image credit Vlietinck, Ryckebusch, Van Houcke, PRB87, 115133 (2013)

Worm trick: introduce artificial elements to simplify updates



- weight of configurations with worms can be adjusted to make updates maximally efficient
- free worm ends are a convenient intermediate stage between different complex configurations

$\Rightarrow$ chains of several small (local) updates more efficient than attempting large (global) updates

Analytic continuation

discretisation: $A(\omega)=\sum_{n=1}^N A(\omega_n)\delta(\omega-\omega_n)$ $\Rightarrow$ matrix equation

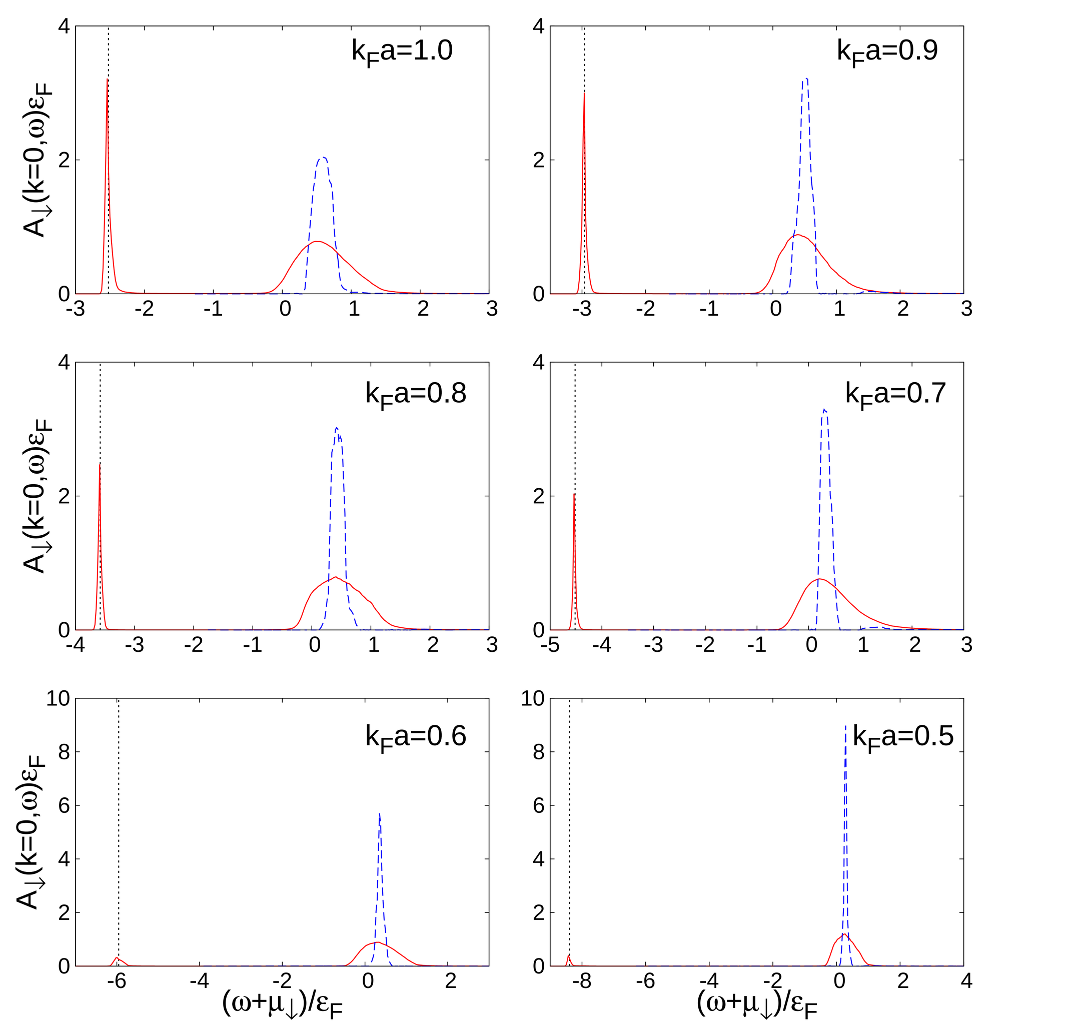
$$-G(\tau)=\int_0^\infty e^{-\omega\tau}A(\omega)d\omega\leftrightarrow -G_m=\sum_{n=1}^N e^{-\omega_n\tau_m}A(\omega_n)$$

infinite number of solutions satisfying this equation within errors of G_m

Stochastic Optimization Method

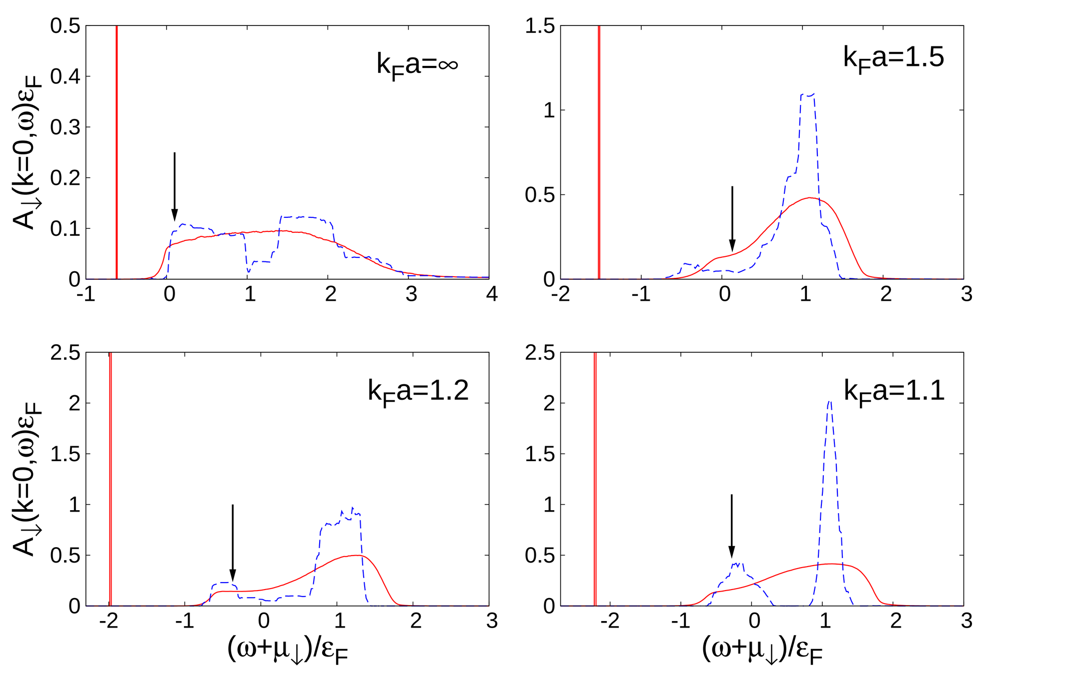
- no default model, no prior except normalization and positivity
- optimize deviation measure: $D[\tilde{A}]=\sum_{m=1}^M\left|\frac{G_m-\tilde{G}_m}{\delta G_m}\right|$
- linear combinations of good solutions are also good solutions
- "smoothest" linear combination of good solutions?
  the solution can be **ambiguous**
  $\Rightarrow$ use target function to force the solution through a set of points
  $\Rightarrow$ check if any given feature can be distored in this way

Polaron spectral function in the molecule regime ($k=0$)

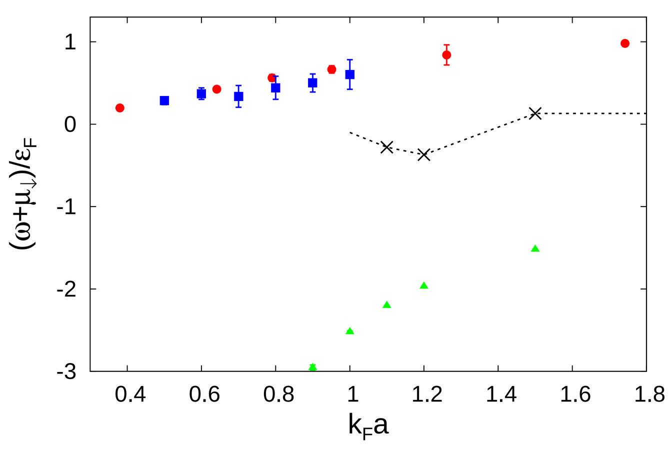


- The maximally smooth spectrum (red solid lines) is broad, but there are solutions consistent with a narrow peak (blue dashed lines)
  $\Rightarrow$ the width of the second peak cannot be unambiguously established!
- The polaron peak is consistent with a δ-function (black short-dashed lines) for all values of $k_F a$ studied
  $\Rightarrow$ polaron is a well-defined quasiparticle even on the molecule side
- with decreasing $k_F a$ the spectral weight of the polaron peak (Z-factor) diminishes and the peak moves to more negative frequencies
- second peak nearly constant in energy
  $\Rightarrow$ gap-like region of very low spectral weight between peaks
  $\Rightarrow$ kinematically, these states should get excited, but remain strongly suppressed
  $\Rightarrow$ without a small parameter at $k_F a\sim 1$, the weight of the few-body continuum should be comparable to the Z-factor

Polaron spectral function in the polaron regime ($k=0$)



- The maximally smooth spectrum (red solid lines) is a broad peak with preceding plateau, but there are solutions with two separate peaks (blue dashed lines)!
- Either the shoulder or the small intermediate peak must be present
- The highest frequency after which the spectrum must be larger than zero (arrow) is much lower than the position of the major second peak, in contrast to the molecule side.



■ Second peak maximum in the molecule regime
● Second peak maximum experiment [Scazza et al., arXiv:1609.09817]
× maximal frequency for the onset of the positive spectrum
▲ polaron peak energy