

# Dynamics of a two-step Electroweak Phase Transition

May 2, 2014

in Collaboration with  
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**ACFI Higgs Portal Workshop**

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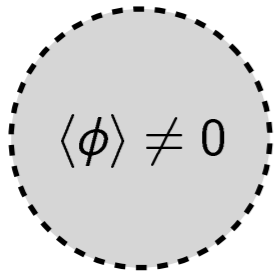
MAX-PLANCK-GESELLSCHAFT




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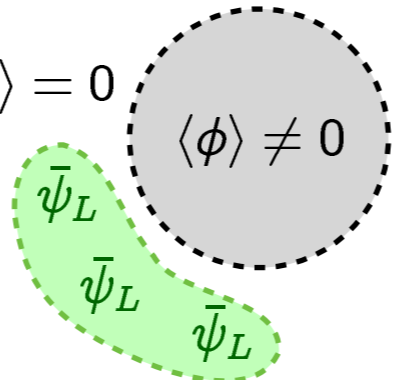
# Electroweak Baryogenesis and Sakharov's Criteria

 First order electroweak phase transition


$\langle \phi \rangle = 0$    $\langle \phi \rangle \neq 0$

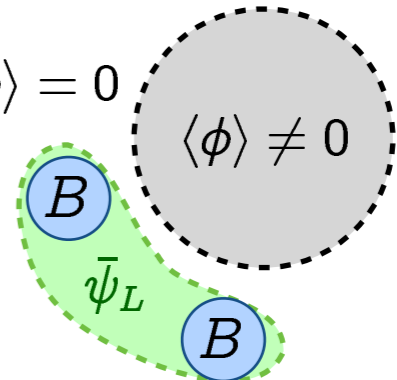
via bubble nucleation

 Generation of particle/antiparticle asymmetry

$\langle \phi \rangle = 0$    $\langle \phi \rangle \neq 0$

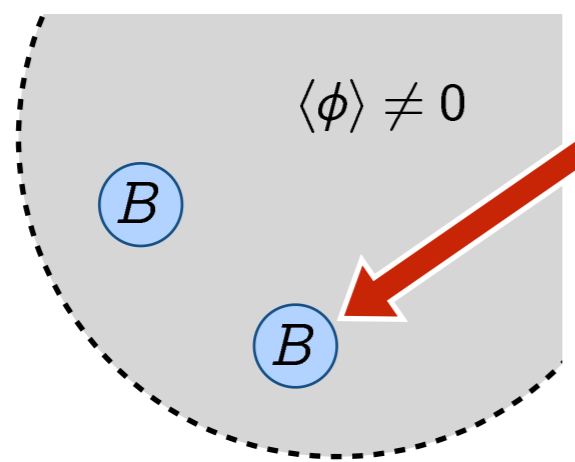
$\bar{\psi}_L$

 Generation of baryon asymmetry

$\langle \phi \rangle = 0$    $\langle \phi \rangle \neq 0$

$B$

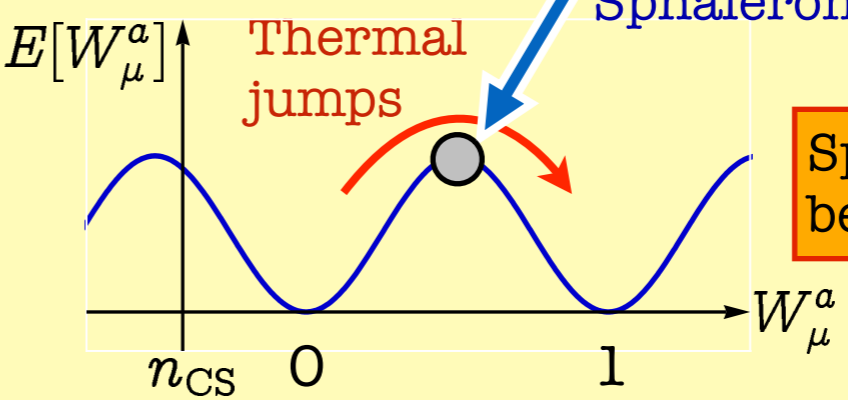
baryons captured, and preserved.

  $\langle \phi \rangle \neq 0$

$B$

$B$

B+L-violating EW Sphalerons convert baryons back to anti-leptons.

  $E[W_\mu^a]$

Thermal jumps

"EW Sphaleron"

$W_\mu^a$

$n_{CS}$  0 1

Sphaleron proc. must be quenched!

# Electroweak Baryogenesis

$e^{-\beta H}$  Fir  
pha

$\langle \phi \rangle =$

via b

Kinetic theory: sphaleron rate related to its mass (energy)

$$\Gamma_{\text{sph.}} \sim (gT)^4 e^{-E_{\text{sph.}}/T}$$

Sphaleron mass dependent on Higgs field value inside bubble

$$E_{\text{sph.}} \sim 4\pi \langle \phi \rangle / g$$

At phase transition, need ratio to be large.

$$\frac{\langle \phi \rangle}{T_c} \gtrsim 1$$

Baryon number preservation criterion on strength of phase transition.

## This talk (outline):

set C, CP-violation aside

### 1. New Strategy to strengthen phase transition

Two-step phase transition

H.Patel, M.J. Ramsey-Musolf, PRD 88 (2013), 035013

Connection to colliders

P. Fileviez Pérez, H.Patel, M.J. Ramsey-Musolf, K. Wang. PRD 79 (2009), 055024

### 2. Gauge dependence

**Problem:**  
**This is gauge dependent**

H.Patel, M.J. Ramsey-Musolf, JHEP 1107 (2011), 029

aryon

oc. must

# Previous Strategies

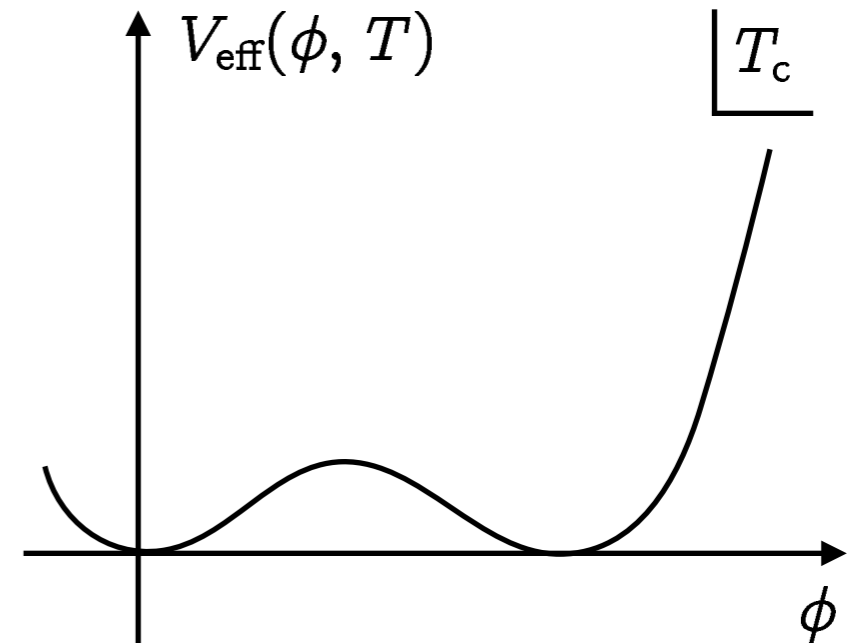
(to strengthen phase transition)

Central quantity of interest: Effective Potential

$$V_{\text{eff}}(\phi, T) = D(T^2 - T_0^2) \phi^2 - E(\xi)T \phi^3 + \frac{\bar{\lambda}}{4} \phi^4 + \dots$$

Condition from requiring quenched sphalerons:

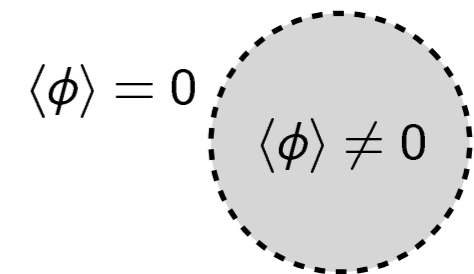
$$1 \lesssim \frac{\langle \phi \rangle(\xi)}{T_c} = \frac{2E(\xi)}{\bar{\lambda}} \left\{ \begin{array}{l} \text{related to model} \\ \text{parameters} \end{array} \right.$$



Tune **parameters**, or **add new fields** (DoF) to model to:

Make  $E(\xi)$  bigger.

Make  $\bar{\lambda}$  smaller.



In general, very difficult.

# Previous Strategies

(to strengthen phase transition)

Extend model with extra scalar degrees of freedom.

Effective potential a function of multiple order parameters.

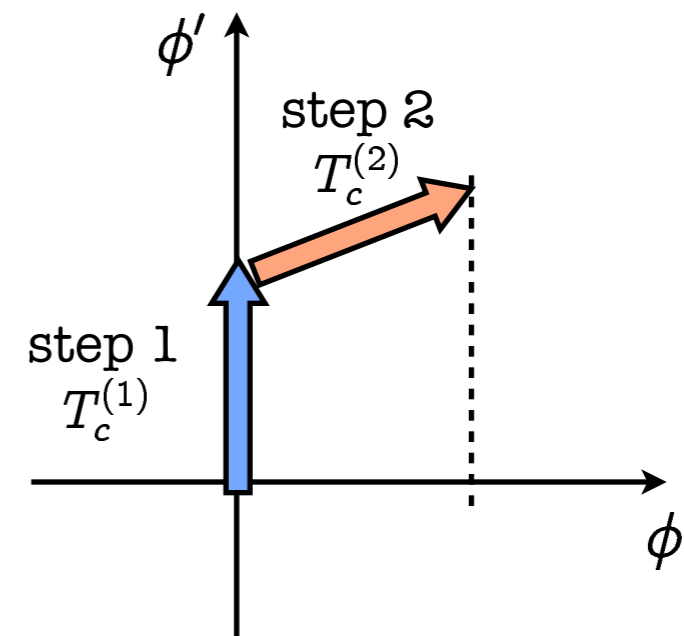
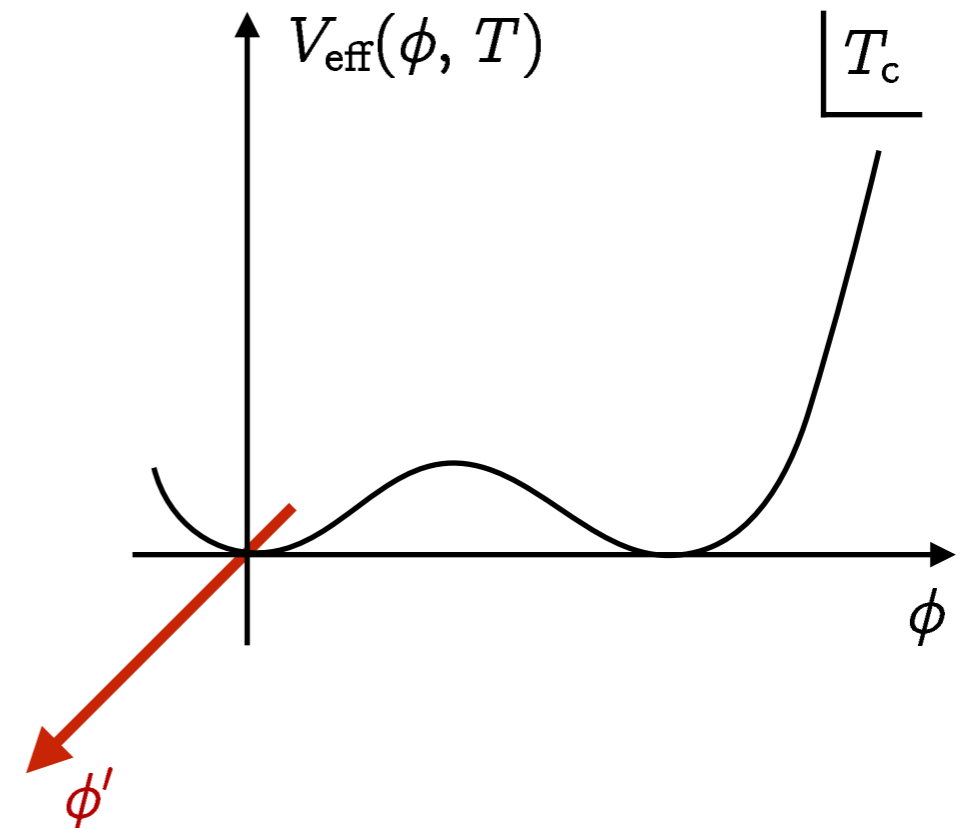
In regions of parameter space, structure of free energy is such that there could be a multi-step phase transition.

If extra degrees of freedom are SM-gauge singlets, EW sphaleron not affected in essential way,

Condition on phase transition strength

$$\frac{\langle \phi \rangle}{T_c^{(2)}} \gtrsim 1$$

Applied only on final step.



If extra scalar degrees of freedom carry gauge quantum numbers,

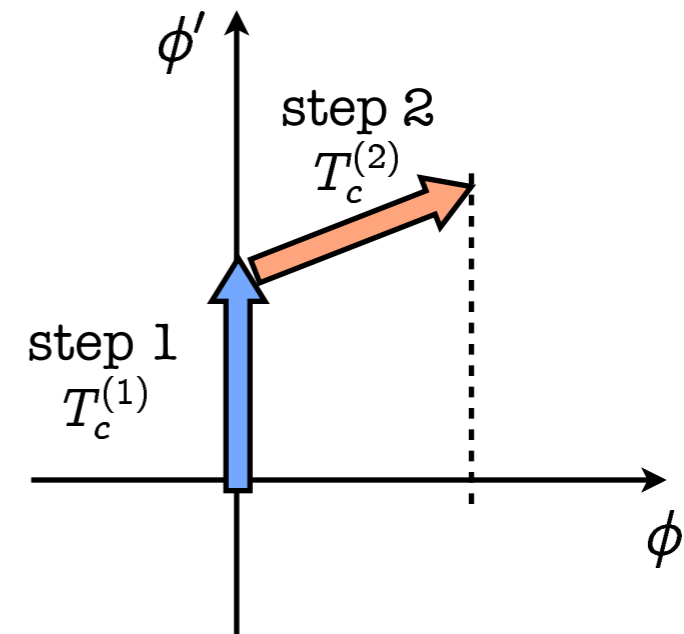
Sphalerons would couple to scalar field, phase transitions induced by these could influence them.

(model-builder's POV)

In this setup, it may be easier to generate a strong first order phase transition at **step 1**.

underlying parameters  
controlling this step  
are largely unconstrained.

(but possibly measured at LHC)



# $\Sigma$ SM – Formulation

P. Fileviez Pérez, H. Patel,  
M.J. Ramsey-Musolf, K. Wang.  
PRD 79 (2009), 055024

## Scalar Field Content:

Higgs doublet

$$H \equiv (1, 2, \frac{1}{2}) = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(H^0 + i\phi^0) \end{pmatrix}$$

SU(2) real triplet

$$\Sigma = (1, 3, 0) = \vec{T} \cdot \vec{\Sigma} = \frac{1}{2} \begin{pmatrix} \Sigma^0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -\Sigma^0 \end{pmatrix}$$

## Couplings: (renormalizable)

Fermiophobic – incompatible hypercharge

Gauge-couplings: couples to W, Z and EM field

Scalar potential:

$$V(H, \Sigma) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^4$$

Standard model

new particle  
mass + self coupling

$$-\frac{1}{2}\mu_\Sigma^2 \vec{\Sigma}^2 + \frac{b_4}{4}(\vec{\Sigma}^2)^2$$

Higgs portal interaction

$$+a_1 H^\dagger \vec{\Sigma} H + \frac{a_2}{2} H^\dagger H \vec{\Sigma}^2$$

Four unmeasured  
parameters  $\mu_\Sigma^2, b_4, a_1, a_2$



# Phenomenological Constraints

In general, potential permits VEVs for both  $\Sigma$  and  $H$ .

Triplet VEV contributes to  $W$  mass (but not  $Z$ )

$$\mathcal{L}_{\text{kin.}} = (D_\mu H)^\dagger (D^\mu H) + \text{Tr}(D_\mu \Sigma)^2$$

$$\longrightarrow \frac{g^2}{4} (v^2 + \underbrace{4\langle \Sigma \rangle^2}_{\text{W mass}}) W_\mu^+ W^{\mu-} + \frac{1}{2} \frac{v^2}{4} (g^2 + g'^2) Z_\mu Z^\mu$$

W mass
Z mass

SM relation: weak charged and neutral current rates upset

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2 \theta_w} \quad \begin{array}{c} \text{SM (L.O.)} \\ = 1 \end{array} \quad \begin{array}{c} \Sigma \text{SM} \\ \longrightarrow 1 + \underbrace{\frac{4\langle \Sigma \rangle^2}{v^2}} \end{array}$$

Experimentally, SM relation satisfied to high prec.

$$\rho_{\text{exp}} = \delta_{\text{SM}} + \delta\rho \quad \delta\rho = 0.0004_{-0.0004}^{+0.0003}$$

(95% conf.)

Translates to bound:  
 $\langle \Sigma \rangle \leq 3 \text{ GeV}$



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 $\langle \Sigma \rangle \leq 3 \text{ GeV}$

Easy and natural explanation of smallness:

$\langle \Sigma \rangle$  depends linearly on  $a_1$  (for small values):

$$\langle \Sigma \rangle \sim \mathcal{O}(a_1) \quad \Rightarrow \quad a_1 \text{ is } \mathcal{O}(3 \text{ GeV}) \quad (1\% \text{ EW scale})$$

Technically natural, but make simplifying assumption:  $a_1 = 0$

$$V(H, \Sigma) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^4 \quad \left[ -\frac{1}{2} \mu_\Sigma^2 \vec{\Sigma}^2 + \frac{b_4}{4} (\vec{\Sigma}^2)^2 \right] \quad \left[ \cancel{+ a_1 H^\dagger \vec{\Sigma} H} + \frac{a_2}{2} H^\dagger H \vec{\Sigma}^2 \right]$$

Three unmeasured parameters  $\mu_\Sigma^2, b_4, \cancel{a_1}, a_2$

Potential is now  $SO(3)$  symmetric and has  $Z_2$  symmetry:  $\vec{\Sigma} \rightarrow -\vec{\Sigma}$

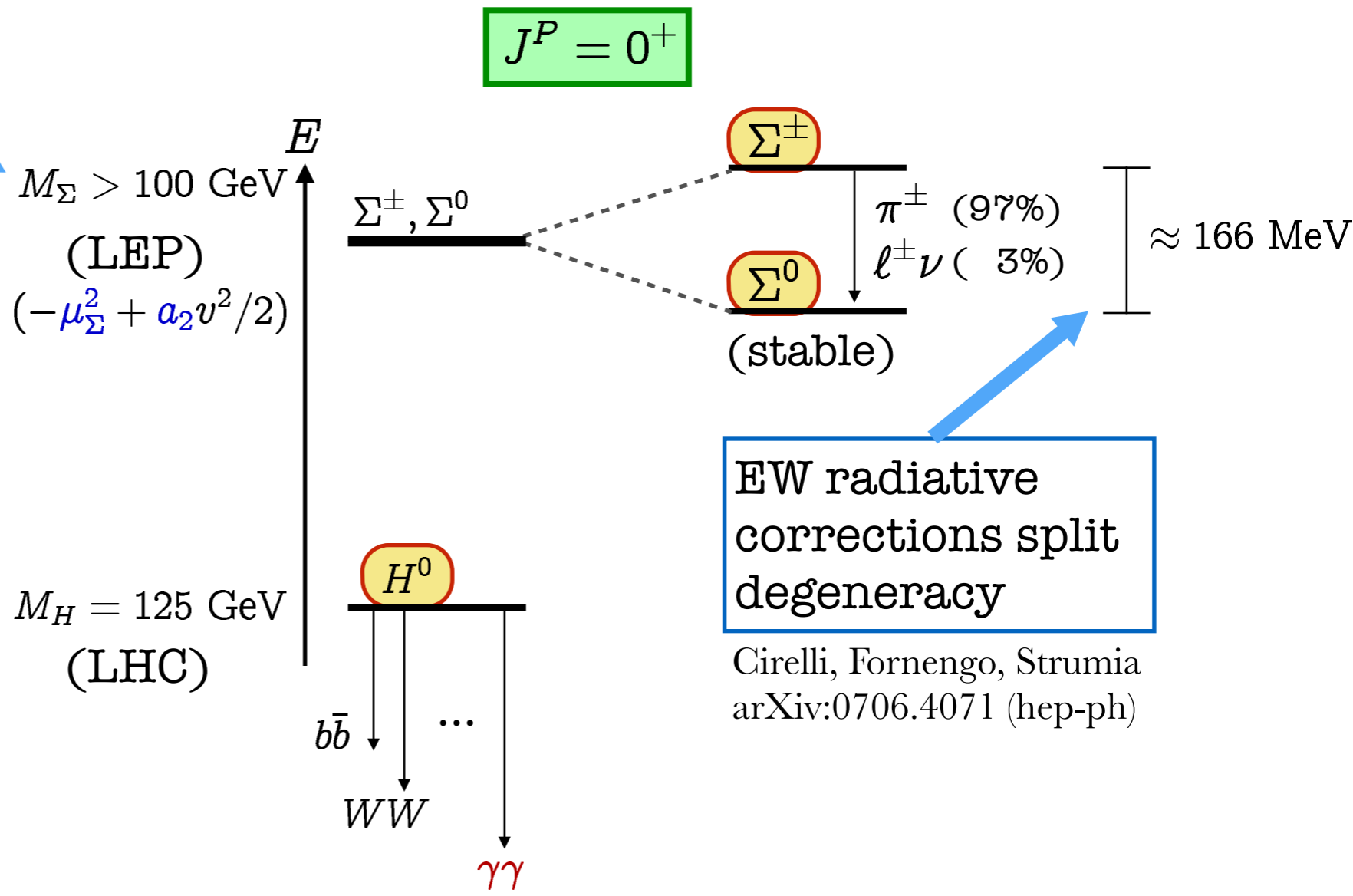
# Particle Spectrum

$$V(H, \Sigma) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^4 - \frac{1}{2} \mu_\Sigma^2 \vec{\Sigma}^2 + \frac{b_4}{4} (\vec{\Sigma}^2)^2 + \frac{a_2}{2} H^\dagger H \vec{\Sigma}^2$$

Three new scalar states

Fix  $a_1 = 0 \Rightarrow \langle \Sigma \rangle = 0$   
 $\Rightarrow$  no  $H^0 / \Sigma^0$  mixing

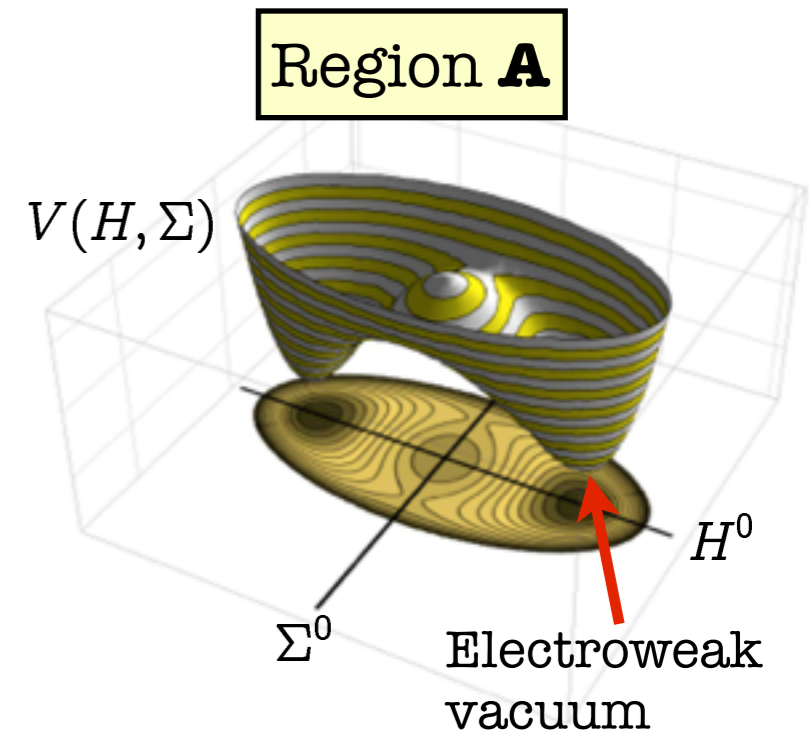
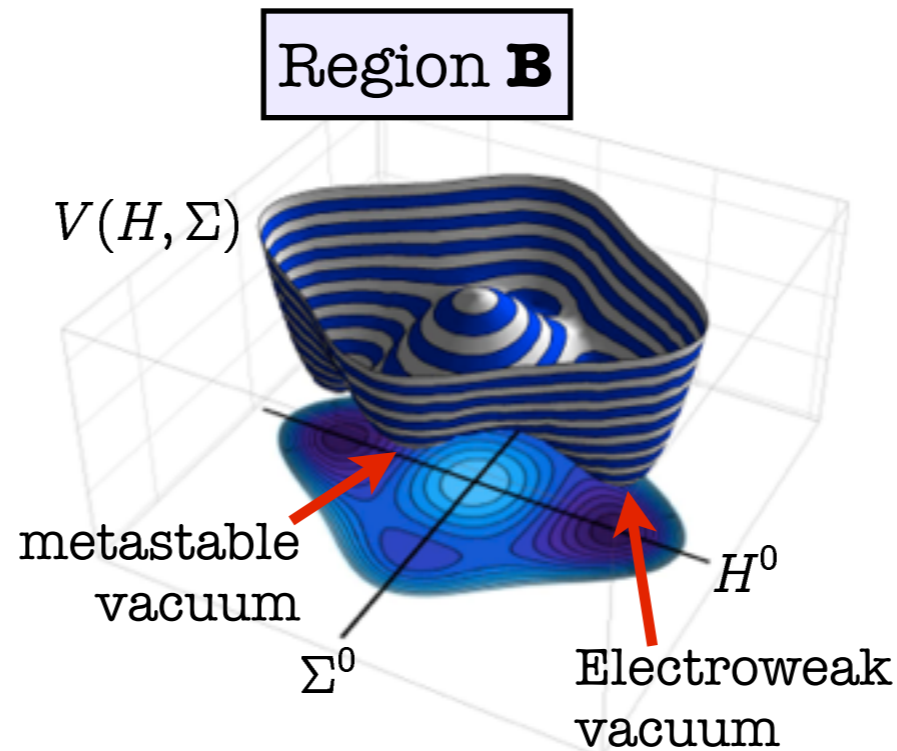
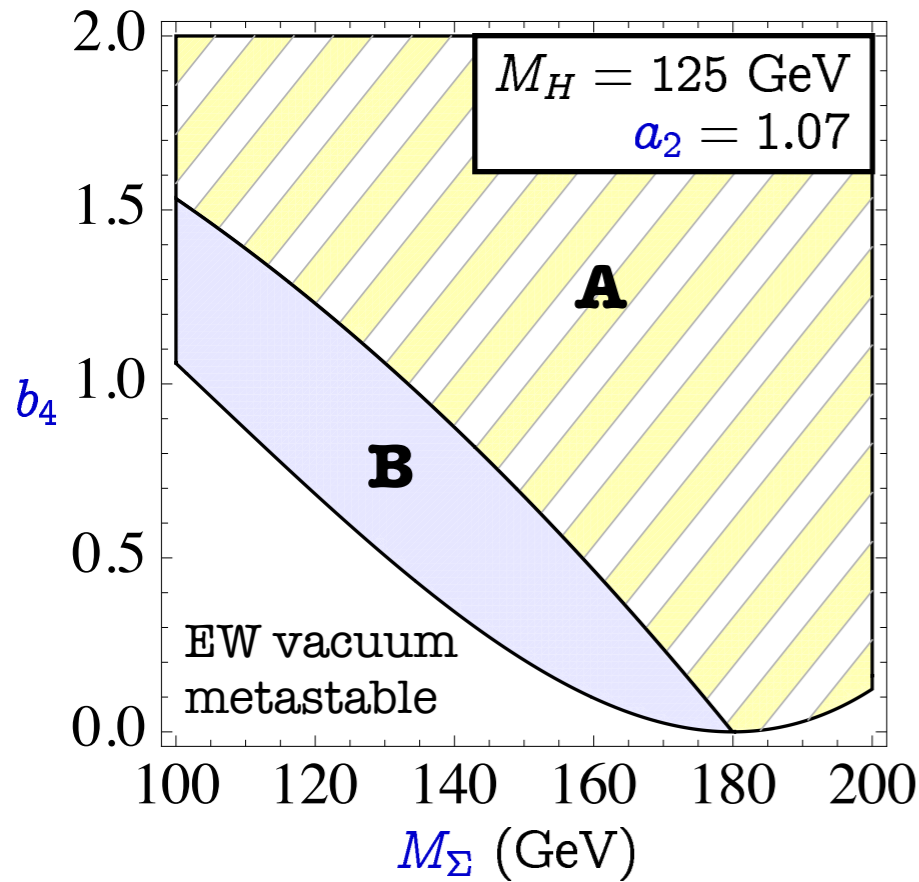
$\Sigma^\pm$  long-lived ( $c\tau \sim 6$  cm)  
 $\Sigma^0$  is stable DM cand  
 Modified  $\Gamma(H^0 \rightarrow \gamma\gamma)$  mode



# Zero-temperature Vacuum Structure

H.Patel, M.J. Ramsey-Musolf,  
PRD 88 (2013), 035013

Pattern of phase transition influenced by zero-T vacuum structure

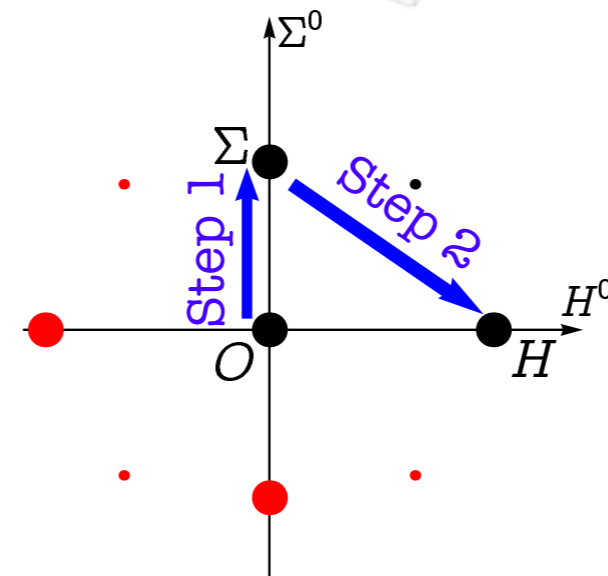


model potential

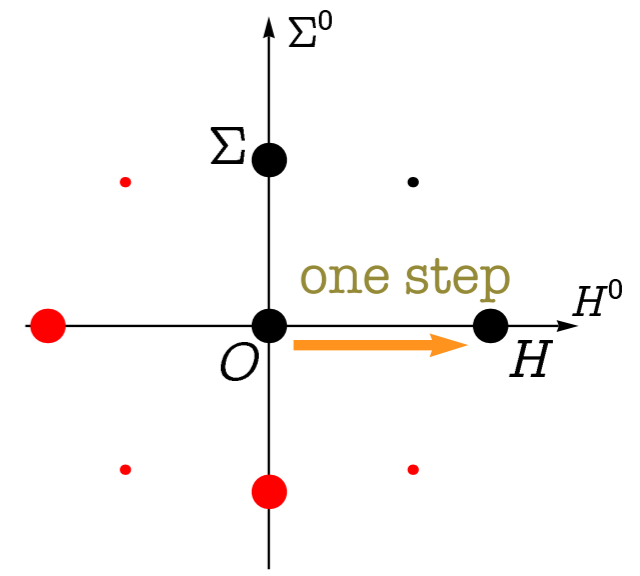
$$V(H, \Sigma) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^4 - \frac{1}{2} \mu_\Sigma^2 \vec{\Sigma}^2 + \frac{b_4}{4} (\vec{\Sigma}^2)^2 + \frac{a_2}{2} H^\dagger H \vec{\Sigma}^2$$

Finite temperature:

Baryon asymmetry generation  
in first step



- $O : SU(2)_L \times U(1)_Y$
- $\Sigma : U(1)_L \times U(1)_Y$
- $H : U(1)_{em}$



- $O : SU(2)_L \times U(1)_Y$
- $H : U(1)_{em}$

# 't Hooft–Polyakov Monopoles

Peculiar feature: Sigma phase resembles Glashow-Salam model of EW interactions  
(no weak-neutral currents)

't Hooft and Polyakov showed stable **magnetic monopole solution**.

=> early universe populated by monopoles

=> subsequently wiped out after 2nd phase transition to EW phase.

Rubakov effect: scattering with Fermions **violates B+L exactly like sphalerons**.

In addition to sphaleron processes, monopoles would also wipeout baryon asymmetry

But to what extent?

Depends on monopole concentration:

1. Kibble mechanism

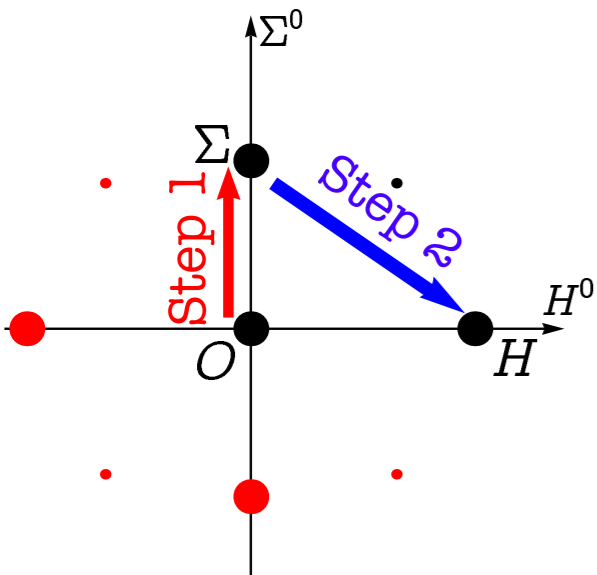
2. Thermal production (**Dominant**)

(monopole-antimonopole pair-production)

$$\left( \begin{array}{l} \text{equil. monopole} \\ \text{number density} \end{array} \right) \sim e^{-m_M(T)/T}$$

Bigger  $\langle \Sigma \rangle \Rightarrow$  Higher mass  $\Rightarrow$  Lower concentration

# Baryon preservation

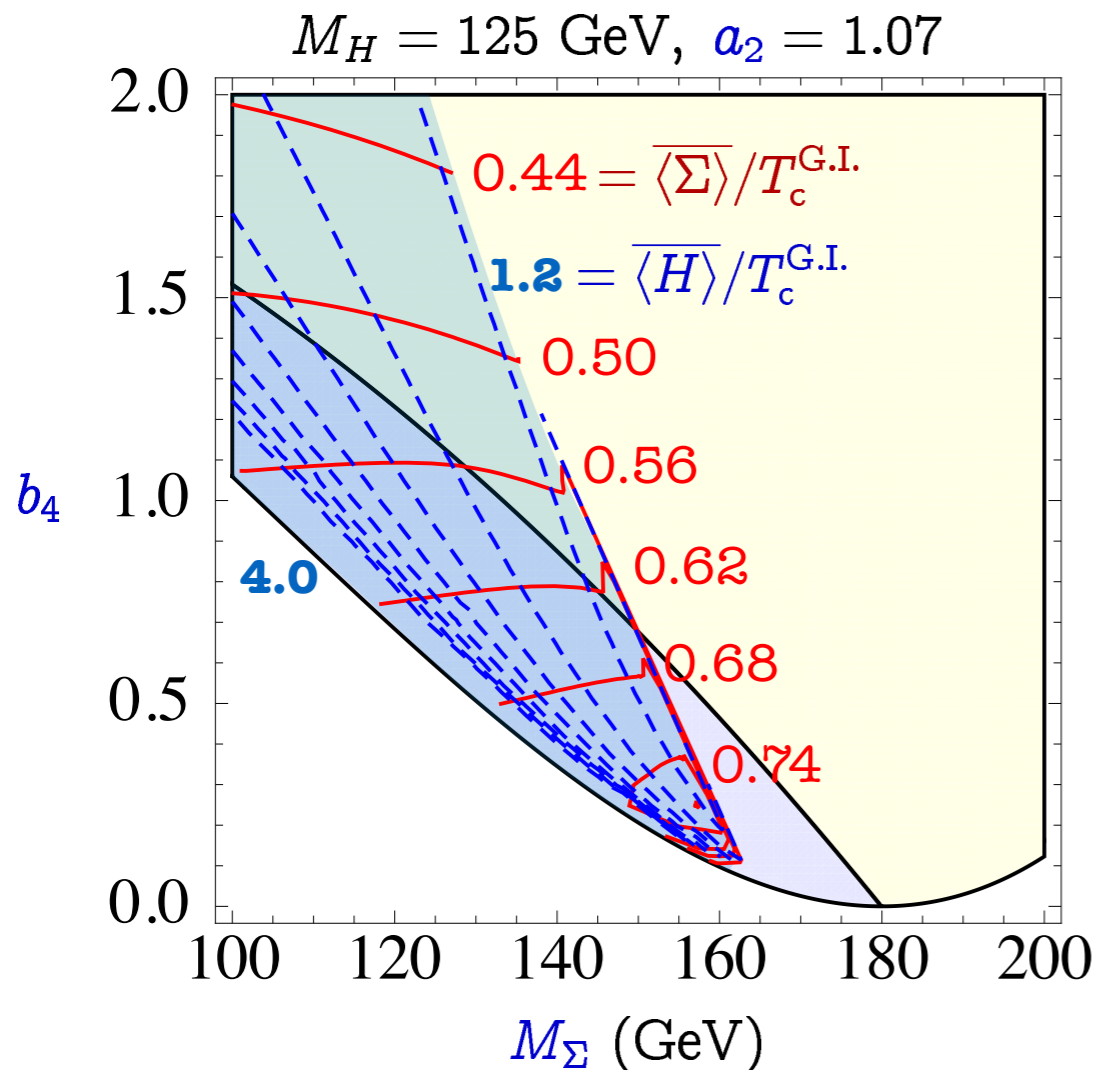


model potential

$$V(H, \Sigma) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^4 - \frac{1}{2} \mu_\Sigma^2 \vec{\Sigma}^2 + \frac{b_4}{4} (\vec{\Sigma}^2)^2 + \frac{a_2}{2} H^\dagger H \vec{\Sigma}^2$$

- Step 1:**  $\vec{\Sigma}$  couples to  $W, Z$  fields:
- Sphalerons rates suppressed
  - Monopole density suppressed

stronger **Step 1**  $\Rightarrow$  greater suppression



Qualitatively: (gauge-dep) **Step 1**

$$V_{\text{eff}}(\phi, T) = D(T^2 - T_0^2) \phi^2 - ET \phi^3 + \frac{b_4}{4} \phi^4 + \dots$$

Smaller  $b_4$  leads to stronger transition:

$$\frac{\langle \Sigma \rangle}{T_c} = \frac{2E}{b_4}$$

**Step 2:**  $H$  couples to  $W, Z$  fields:

- SM EW Klinkhamer-Manton Sphalerons rates suppressed

always **sufficiently** strong



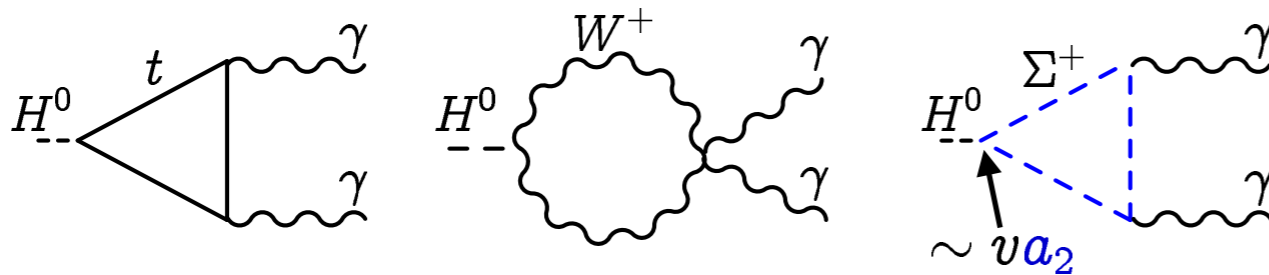
# Modified Higgs Decay

Currently, most sensitive to  $\Gamma(H^0 \rightarrow \gamma\gamma)$

Higgs-portal coupling  $a_2$  adds new contribution to amplitude

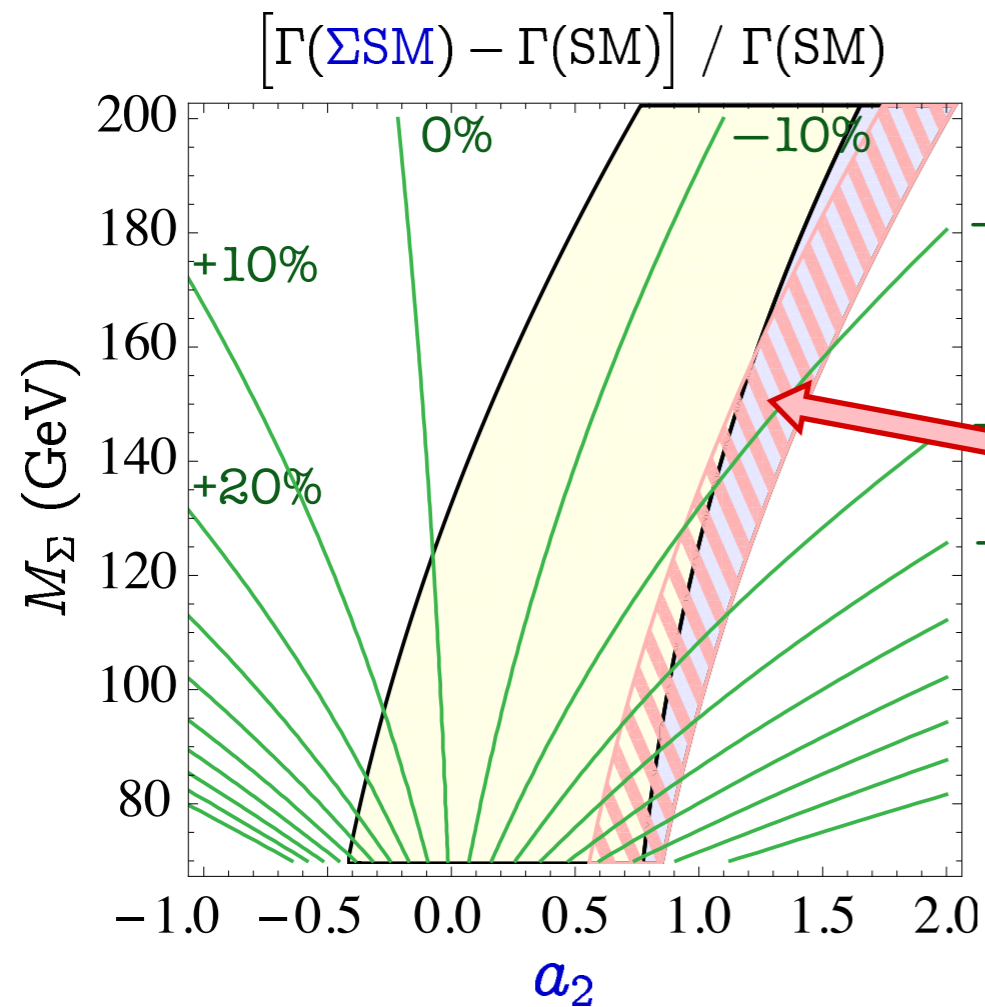
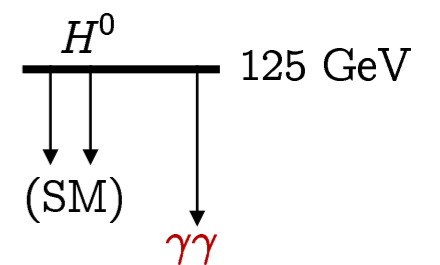
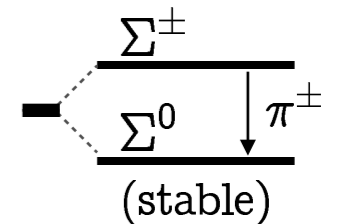
model potential

$$V(H, \Sigma) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^4 - \frac{1}{2} \mu_\Sigma^2 \vec{\Sigma}^2 + \frac{b_4}{4} (\vec{\Sigma}^2)^2 + \frac{a_2}{2} H^\dagger H \vec{\Sigma}^2$$



$$J^P = 0^+$$

$\Delta M_\Sigma \approx 166 \text{ MeV}$



Positive  $a_2$  suppresses  $\gamma\gamma$  mode.

Negative  $a_2$  enhances  $\gamma\gamma$  mode.

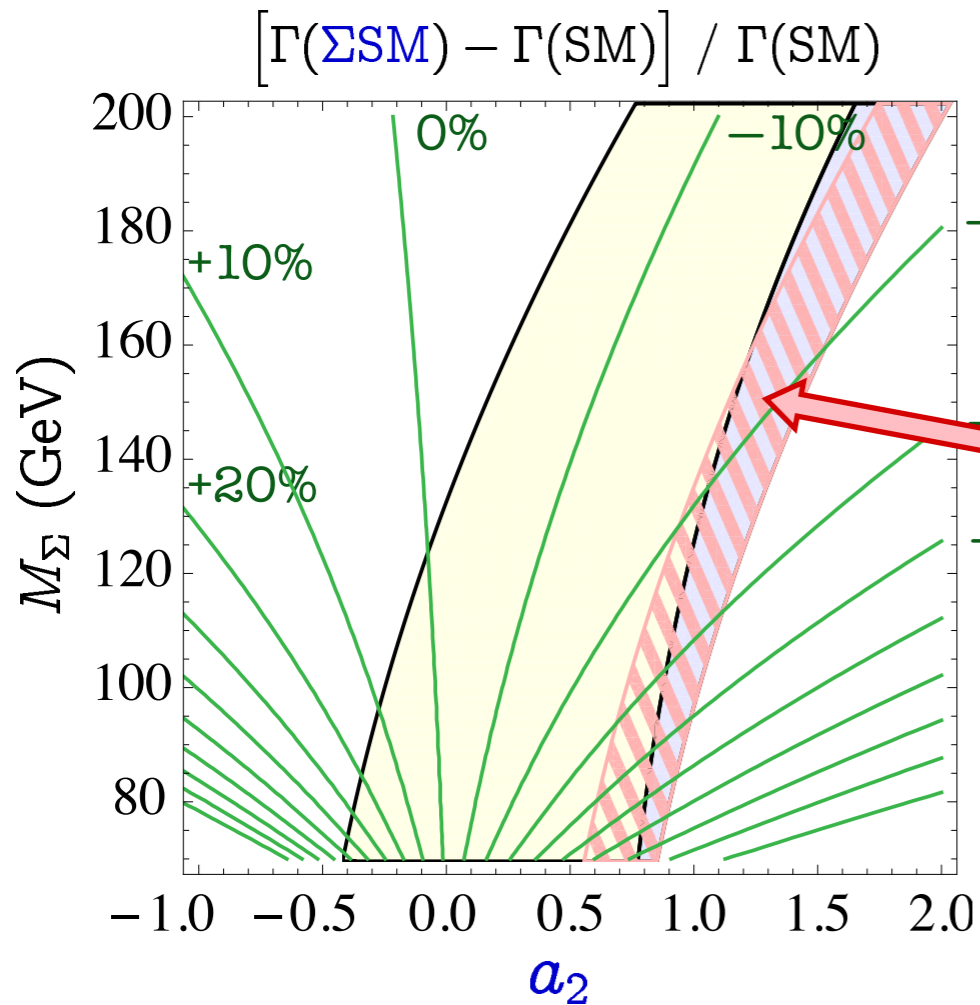
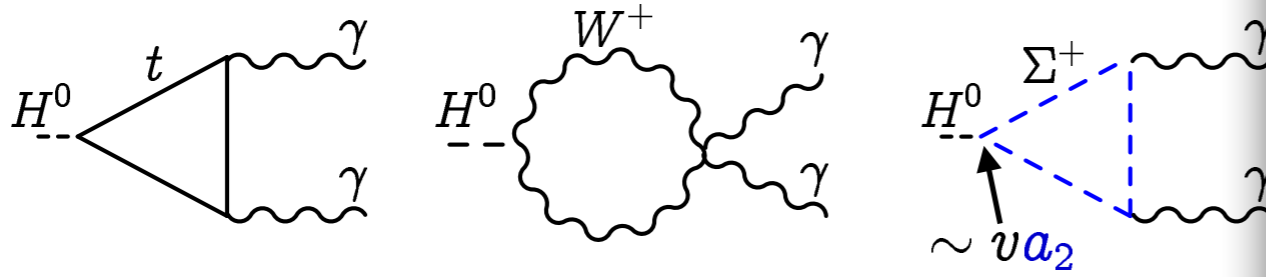
( $b_4$  indep.)

**Two-step EWPT** occurs for positive  $a_2$ .

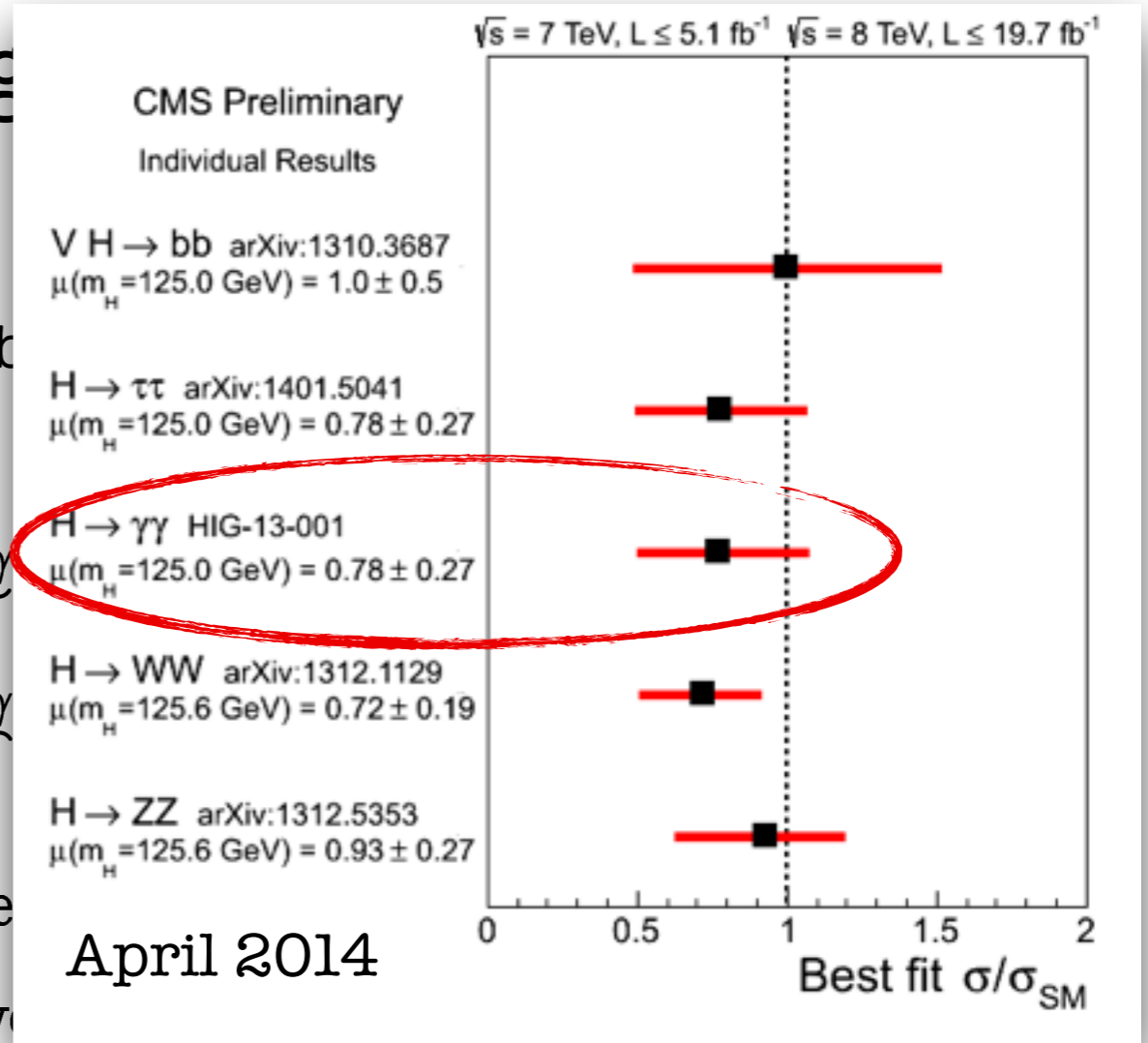
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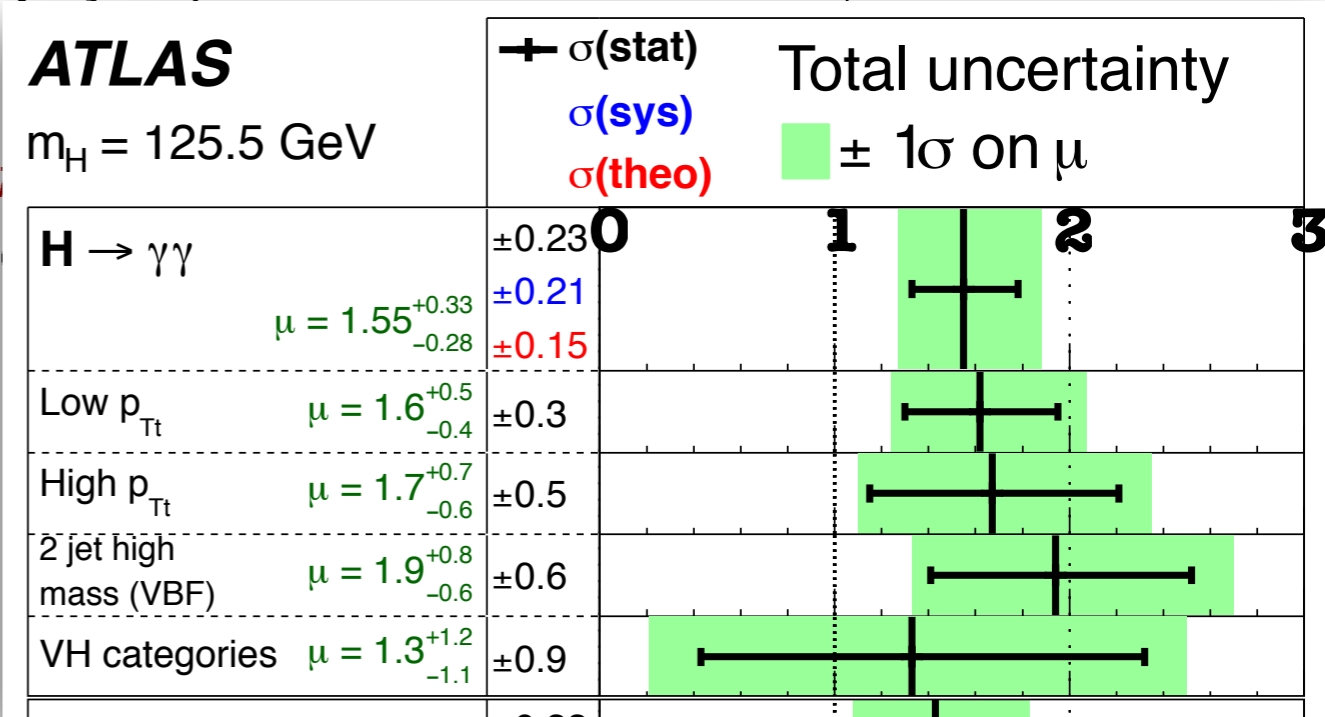


Hiren Patel



Positive  
Negative

$(b_4)$   
 $T_V$   
OC

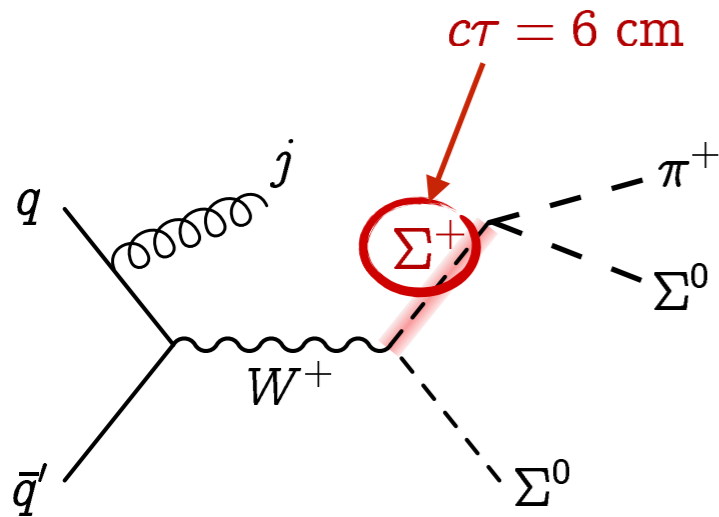




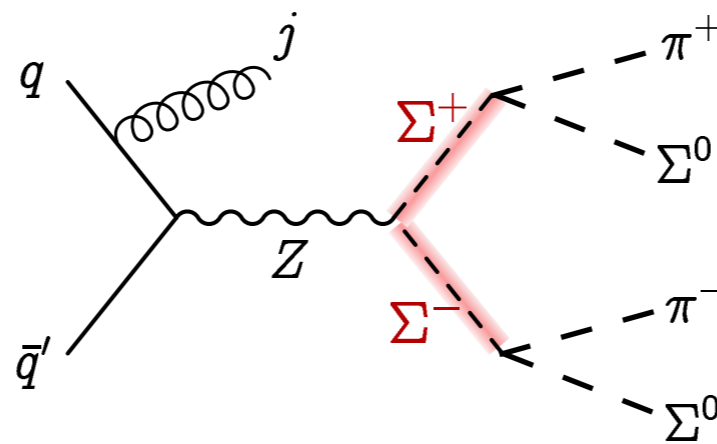
# LHC Production: $\Sigma^\pm$

Potential has  $Z_2$  symmetry:  
associated production of  $\Sigma$ .

Distinctive LHC signature



1 charged track  
missing  $E_T$

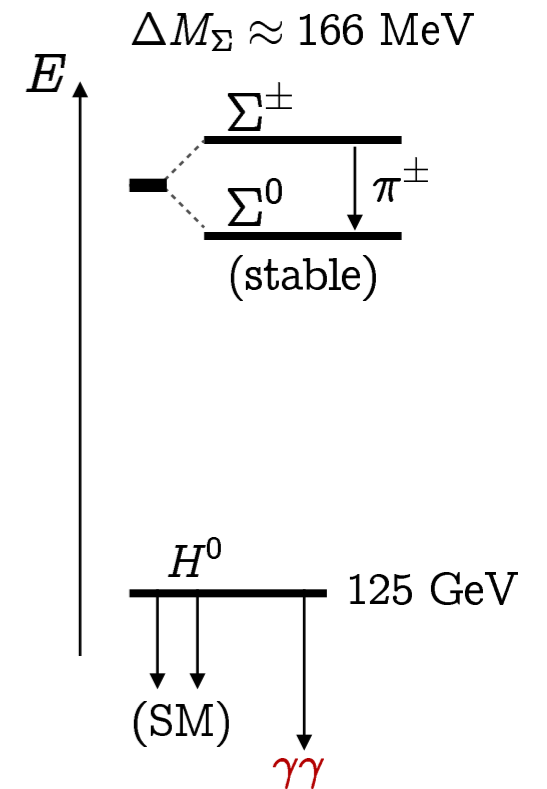


2 charged tracks  
missing  $E_T$

model potential

$$V(H, \Sigma) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^4 - \frac{1}{2} \mu_\Sigma^2 \vec{\Sigma}^2 + \frac{b_4}{4} (\vec{\Sigma}^2)^2 + \frac{a_2}{2} H^\dagger H \vec{\Sigma}^2$$

$$J^P = 0^+$$



Production cross section:

$$M_\Sigma = 200 \text{ GeV}$$

$$\sigma_{\text{prod}} \approx 10 \text{ fb}$$

$$M_\Sigma = 1 \text{ TeV}$$

$$\sigma_{\text{prod}} \approx 10^{-2} \text{ fb}$$

# $\Sigma^0$ as a CDM candidate

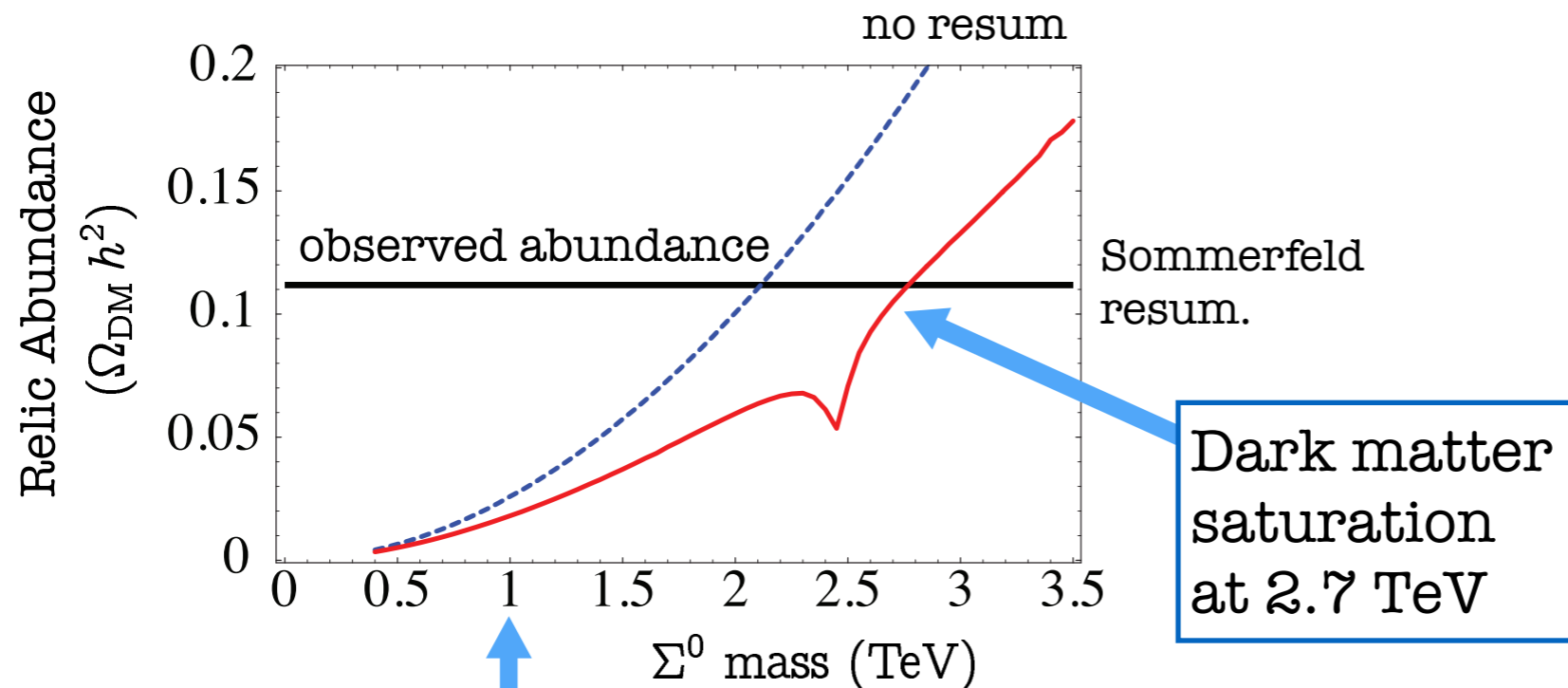
Annihilation channels:

$$\Sigma^0 \Sigma^0 \longrightarrow W^+ W^-, \gamma\gamma, \dots$$

M. Cirelli, A. Strumia, M. Tamburini.  
Nucl. Phys. **B787**, 152 (2007)

model potential

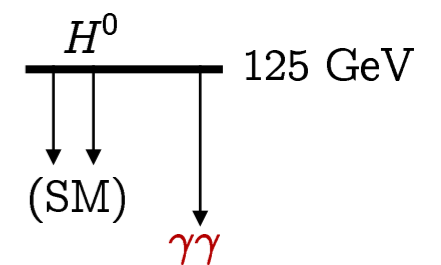
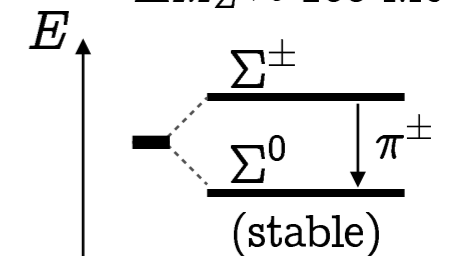
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14 TeV LHC:  $M_\Sigma = 1$  TeV,  
production  $\sigma \approx 10^{-2}$  fb

$$J^P = 0^+$$

$$\Delta M_\Sigma \approx 166 \text{ MeV}$$



Kinetic theory: sphaleron rate related to its mass (energy)

$$\Gamma_{\text{sph.}} \sim (gT)^4 e^{-E_{\text{sph.}}/T}$$

Sphaleron mass dependent on Higgs field value inside bubble

$$E_{\text{sph.}} \sim 4\pi\langle\phi\rangle/g$$

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$$\frac{\langle\phi\rangle}{T_c} \gtrsim 1$$

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**Problem:**

**This is gauge dependent**

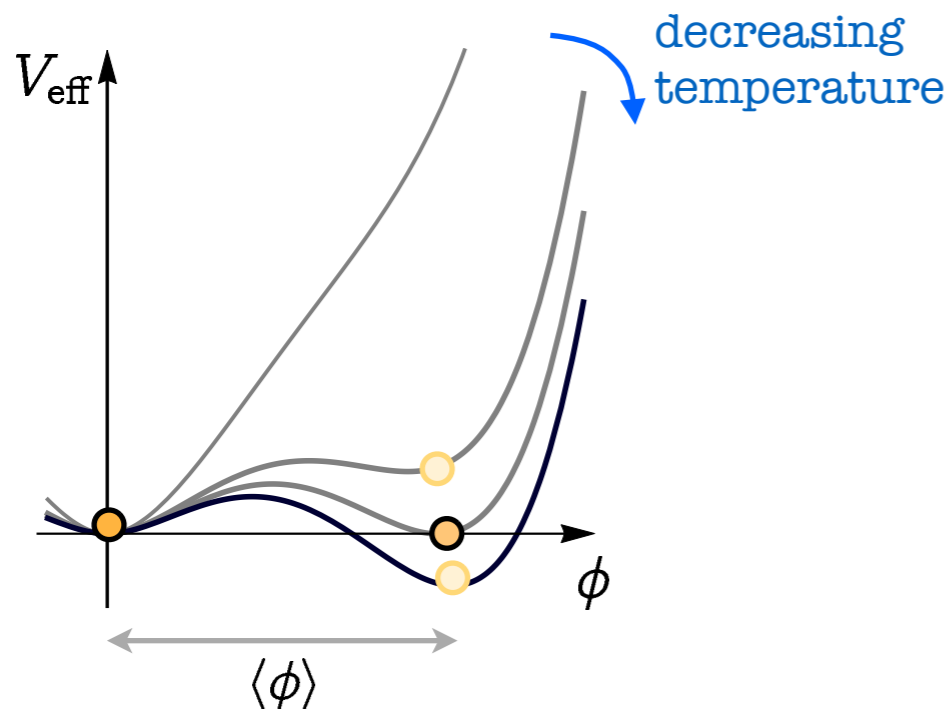
H.Patel, M.J. Ramsey-Musolf, JHEP 1107 (2011), 029

# (standard) Computation of $\frac{\langle \phi \rangle}{T_c}$

1. Track evolution of minima in  $V_{\text{eff}}$  as a function of temperature.
2. Numerically solve **minimization** and **degeneracy** condition equations:

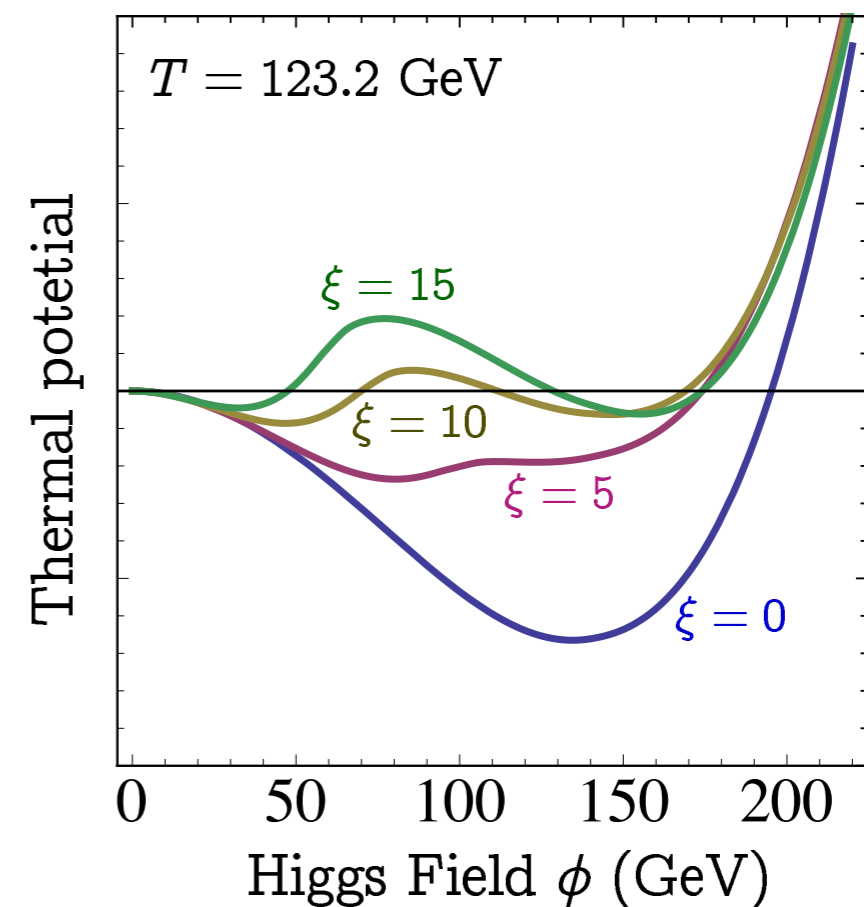
**1**  $\frac{\partial}{\partial \phi} V_{\text{eff}}(\phi_{\text{min}}, T_c) = 0$

**2**  $V_{\text{eff}}(0, T_c) = V_{\text{eff}}(\phi_{\text{min}}, T_c)$



In a gauge theory, the effective potential is gauge dependent.

## Standard Model



Computed  $T_c$  and  $\langle \phi \rangle$  depends on gauge parameter

# Diagnosis & Resolution I

Determination of  $T_c$  (or  $T_N$ )  $\rightarrow$   $\frac{\langle \phi \rangle}{T_c} \gtrsim 1$

*~Resolution~*

*~Diagnosis~*

- Nielsen identity  $\Rightarrow T_c$  gauge-independent
- valid order-by-order in loop-expansion
- But, numerical solution to minimization condition

$$1. \quad V'_{\text{eff}}(\phi_{\text{min}}, T_c) = 0$$

$$2. \quad V_{\text{eff}}(0, T_c) = V_{\text{eff}}(\phi_{\text{min}}, T_c)$$

leads to inconsistent truncation in loop-expansion!

## “h-bar Expansion method”

Minimize by an inversion of series

*counts # of loops*

$$\begin{cases} V(\phi, T) = V_0 + \hbar V_1 + \hbar^2 V_2 + \dots \\ \phi_{\text{min}} = \phi_0 + \hbar \phi_1 + \hbar^2 \phi_2 + \dots \end{cases}$$

Equation for ea. power of  $\hbar$ ; yields  $\phi_{\text{min}}$ .

Subs. into each side;

$$V(\phi_{\text{min}}, T) = V_0(\phi_0) + \hbar V_1(\phi_0, T) + \hbar^2 \left[ V_2(\phi_0, T, \xi) - \frac{1}{2} \phi_1^2(\xi) \frac{\partial^2 V_0}{\partial \phi^2} \Big|_{\phi_0} \right] + \dots$$

# Diagnosis & Resolution I

Determination of  $T_c$  (or  $T_N$ )  $\rightarrow$   $\frac{\langle \phi \rangle}{T_c}$

*~Diagnosis~*

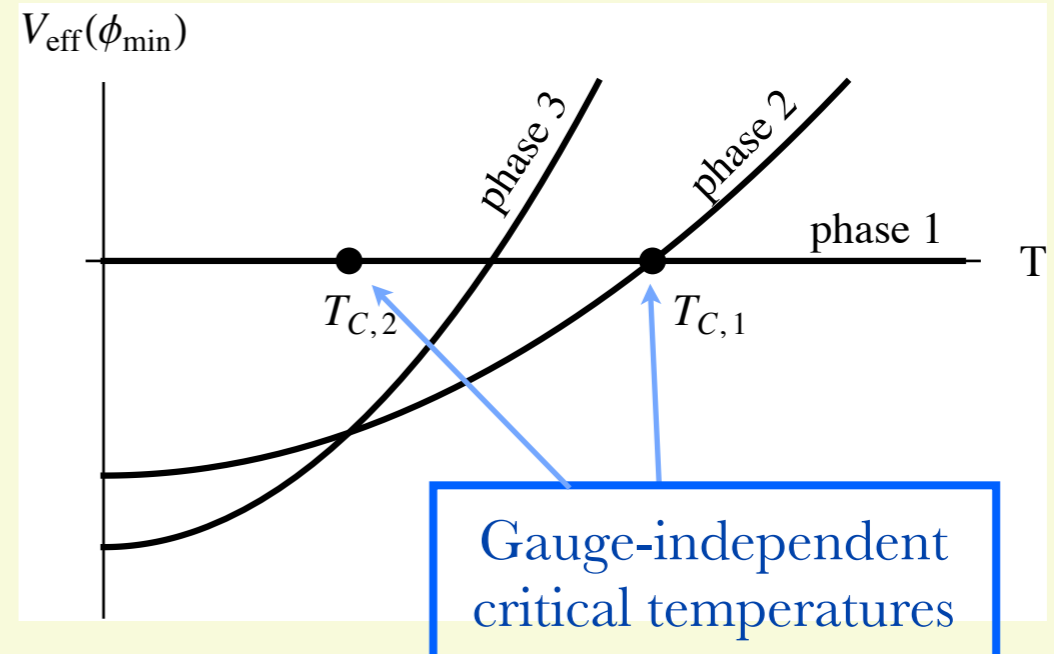
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leads to inconsistent truncation in loop-expansion!

Expression gives gauge independent minima of the effective potential.



consistent with Nielsen identity.  
(explicitly checked at 1-loop)

$$+\hbar^2 \left[ V_2(\phi_0, T, \xi) - \frac{1}{2} \phi_1^2(\xi) \frac{\partial V_0}{\partial \phi^2} \Big|_{\phi_0} \right] + \dots$$

# Diagnosis & Resolution II

Determination of  $\langle \phi \rangle$ .

$$\frac{\langle \phi \rangle}{T_c} \gtrsim 1$$

## Bottom line

Gauge-invariant baryon number preservation criterion:

$$\frac{\langle \phi \rangle}{T_c} \longrightarrow \frac{\bar{v}(T_c^{\text{G.I.}})}{T_c^{\text{G.I.}}}$$

1. Use gauge invariant sphaleron scale

2. Determine  $T_c$  gauge-invariantly

*~Resolution~*

1. Compute sphaleron energy based on gauge-invariant  $\mathcal{O}(T^2)$  effective action.

$$\Gamma(T) \sim -\frac{1}{4} F_{ij}^a F_{ij}^a + (D\phi)^2 + V_0(\phi) + \alpha \phi^2 T^2$$

2. Extract gauge-invariant scale from  $\Gamma(T)$ .

$$\bar{v}(T) = v_0 \sqrt{1 - T^2/T_0^2}$$