Radiative Corrections in Free Neutron Decays

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Outline

1. Error Budget in First-Row CKM Unitarity
2. Nucleus-Independent RC: Current Status
3. Dispersive Approach: Formalism
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Error Budget in First-Row CKM Unitarity

- An important prediction of SM is that the flavor eigenstates and mass eigenstates of quarks are different and are related by a **unitary** transformation:

$$\psi_{d,f} = \begin{pmatrix} d \\ s \\ b \end{pmatrix}_f = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_m$$

The CKM matrix

- **First-row CKM unitarity** reads:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- A significant deviation from 1 will be a signal for the existence of BSM physics!

Error Budget in First-Row CKM Unitarity

- Current status (PDG 2018):

Contains ambiguity

$$|V_{ud}| = 0.97420(21), \quad |V_{us}| = 0.2243(5), \quad |V_{ub}| = 0.00394(36)$$

which gives: $$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994(5)$$

Negligible

in good agreement with unitarity.

- Breaking down the contribution from each uncertainty:

$$|V_{ud}||\delta V_{ud}| = 0.00020$$
$$|V_{us}||\delta V_{us}| = 0.00011$$

The main source of uncertainty comes from V_{ud} .

Error Budget in First-Row CKM Unitarity

- Current status (PDG 2018):

Contains ambiguity (→ $|V_{us}|$)

$$|V_{ud}| = 0.97420(21), \quad |V_{us}| = 0.2243(5), \quad |V_{ub}| = 0.00394(36)$$

which gives: $$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994(5)$$

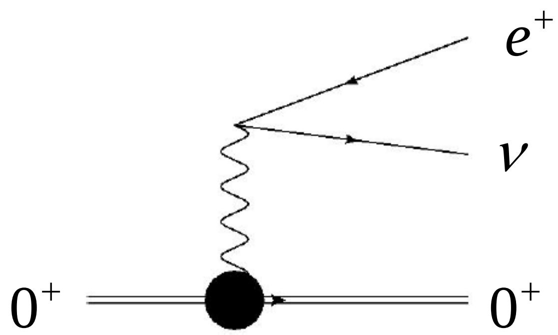
Negligible (→ $|V_{ub}|$)

in good agreement with unitarity.

- But how trustable is this?
  - Experimental uncertainties can be systematically improved.
  - **Theory uncertainty** is the main issue. Need to thoroughly examine how each theoretical uncertainty is assigned.
  - Is there any theory systematics that is not accounted for in the theory uncertainty?

Error Budget in First-Row CKM Unitarity

- Currently the most precise determination of V_{ud} is through **superallowed ($0^+ \rightarrow 0^+$) beta decay** (normalized by muon decay):



Tree-level matrix element completely fixed by isospin symmetry!

Correction occurs only at higher order.

- Experimental results are expressed in terms of the "ft-value": half time + Fermi function correction. One introduces a **nucleus-independent "Ft-value":**

$$\mathcal{F}t = ft(1+\delta'_R)(1+\delta_{NS}-\delta_C)$$

- So that V_{ud} is given by:

$$|V_{ud}|^2 = \frac{2984.432(3)\,\mathrm{s}}{\mathcal{F}t(1+\Delta_R^V)}$$

Error Budget in First-Row CKM Unitarity

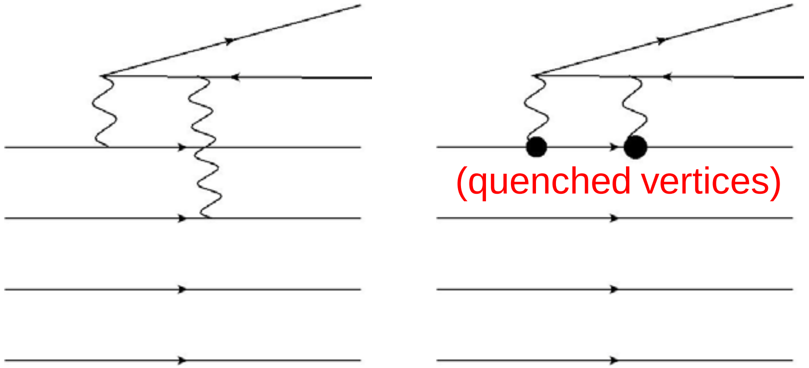
- Meaning of each term:

$$\mathcal{F}t = ft(1+\delta'_R)(1+\delta_{NS}-\delta_C)$$

δ_{NS} : **Nuclear Structure Correction**

δ_C : **Isospin-Breaking Correction**

δ_R': **“Outer” Radiative Correction (RC)**



- Nuclear-structure-related corrections (δ_{NS} and δ_C) have been studied extensively for decades. That, combining with 14 best-measured superallowed beta decays, give:

$$\mathcal{F}t = 3072.27(72)s$$

Hardy and Towner, PRC91,025501 (2015)

or, more recently

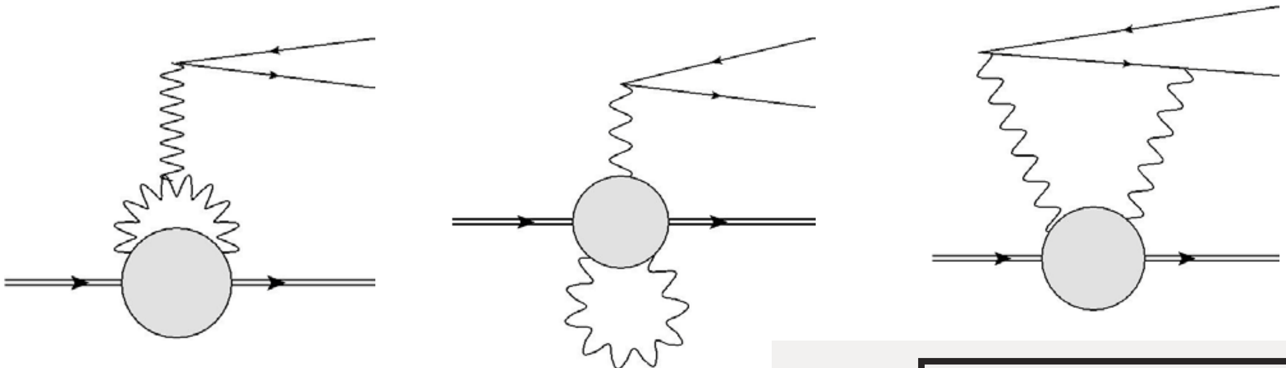
$$\mathcal{F}t = 3072.07(63)s$$

Implied from Czarnecki, Marciano and Sirlin, PRL 120, 202002 (2018)

Error Budget in First-Row CKM Unitarity

- Finally, in the master formula: $|V_{ud}|^2 = \frac{2984.432(3)\,\mathrm{s}}{\mathcal{F}t(1+\Delta_R^V)}$

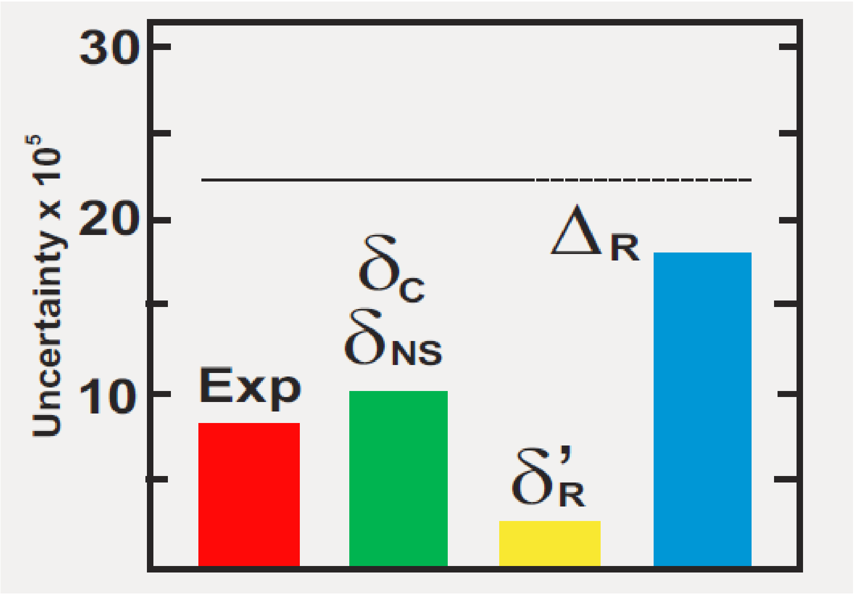
Δ_R^V : **Nucleus-Independent RC** (RC acting on a **single nucleon**)



contributes to the

LARGEST uncertainty

in the determination of V_{ud}!

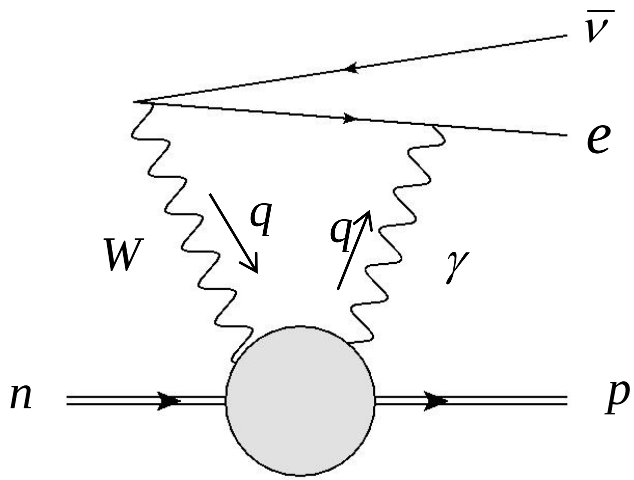


From John Hardy's slides

Nucleus-Independent RC: Current Status

- Most parts in the nucleus-independent RC Δ_R^V (up to 10^{-4}) are either:
  1. Process-independent so they cancel out when taking ratio with muon decay
  2. Exactly calculable through current algebra

     Sirlin, Rev. Mod. Phys 50, 573 (1978)
  3. Depend only the physics in the **UV regime** so that they are **perturbatively calculable**
- The only exception is the **γW-BOX DIAGRAM**:



$$\nu = \frac{p \cdot q}{m_N}$$

which is sensitive to the loop momentum q at **ALL SCALES**!

Nucleus-Independent RC: Current Status

- The part that depends on physics at the hadron scale comes from the **V*A component** of the **EM-weak current product**:

$$\Box_{\gamma W}^{VA} = \frac{\alpha}{8\pi}\int_0^\infty dQ^2 \frac{M_W^2}{M_W^2+Q^2}\int_{-iQ}^{iQ}\frac{d\nu}{\nu}\frac{4(Q^2+\nu^2)^{3/2}}{\pi m_N Q^4}\boxed{T_3(\nu,Q^2)}$$

where the forward Compton tensor is:

$$T_{VA}^{\mu\nu} = \frac{1}{2}\int d^4x e^{iq\cdot x}\left\langle p(\vec{p})\right| T[J_{em}^{\mu}(x)J_{W,A}^{\nu}(0)]\left|n(\vec{p})\right\rangle = \frac{i\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta}{2p\cdot q}\boxed{T_3}$$

- Furthermore, crossing symmetry indicates that one only needs the **ISOSCALAR component of the EM current**.
- It is related to the nucleus-independent RC as:

$$\Delta_R^V = 2\Box_{\gamma W}^{VA} + \ldots$$

Other well-understood terms

Nucleus-Independent RC: Current Status

- State-of-the-art study of box contribution (Marciano and Sirlin, **M&S**):

Marciano and Sirlin, Phys.Rev.Lett. 96 (2006) 032002

- Write the RC as a single variable integral over Q^2, and identify the dominant physics as a function of Q^2.

$$\Box^{VA}_{\gamma W} = \frac{\alpha}{8\pi}\int_0^\infty dQ^2 \frac{M_W^2}{M_W^2+Q^2} F(Q^2)$$

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1. Short distance: **leading OPE + perturbative QCD**

$$F(Q^2)=\frac{1}{Q^2}\left[1-\frac{\alpha_S(Q^2)_{\overline{MS}}}{\pi}-C_2\left(\frac{\alpha_S(Q^2)_{\overline{MS}}}{\pi}\right)^2-C_3\left(\frac{\alpha_S(Q^2)_{\overline{MS}}}{\pi}\right)^3\right] \qquad (1.5\text{GeV})^2 < Q^2 < \infty$$

$$\Box_{\gamma W}^{VA(1)} = 2.14\times 10^{-3}$$ (negligible uncertainty)

C_2 and C_3 taken from existing calculations of pQCD corrections to the polarized Bjorken sum rule.

Larin and Vermaseren, Phys.Lett.B 259 (1991) 345

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- Write the RC as a single variable integral over Q^2, and identify the dominant physics as a function of Q^2.

$$\Box_{\gamma W}^{VA} = \frac{\alpha}{8\pi}\int_0^\infty dQ^2 \frac{M_W^2}{M_W^2+Q^2}F(Q^2)$$

3. Long distance: **Born/elastic contribution** with nucleon EM and axial current dipole FFs:

$$\Box_{\gamma W}^{VA(3)} = 0.96(9)\times 10^{-3} \qquad 0 < Q^2 < (0.823\text{GeV})^2$$

Nucleus-Independent RC: Current Status

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Marciano and Sirlin, Phys.Rev.Lett. 96 (2006) 032002

- Write the RC as a single variable integral over Q^2, and identify the dominant physics as a function of Q^2.

$$\Box_{\gamma W}^{VA} = \frac{\alpha}{8\pi}\int_0^\infty dQ^2 \frac{M_W^2}{M_W^2+Q^2}F(Q^2)$$

2. Intermediate distance: **VMD-inspired interpolating function**

$$F(Q^2) = \frac{-1.490}{Q^2+m_\rho^2} + \frac{6.855}{Q^2+m_A^2} - \frac{4.414}{Q^2+m_{\rho'}^2} \qquad (0.823\text{GeV})^2 < Q^2 < (1.5\text{GeV})^2$$

$$\Box_{\gamma W}^{VA(2)} = 0.16(16)\times 10^{-3}$$

A 100% error is arbitrarily assigned!

All combine to give: $\Delta_R^V(\text{M\&S}) = 0.02361(38)$

Nucleus-Independent RC: Current Status

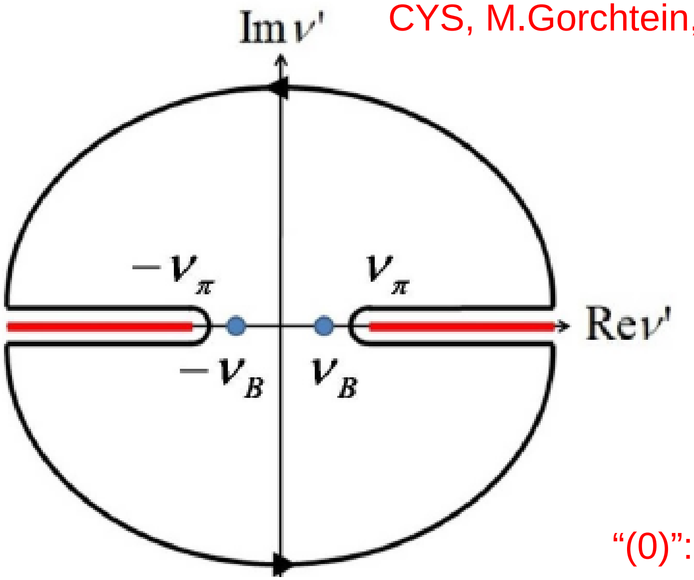
- The "**intermediate distances**" contribution gives rise to most of the theoretical uncertainty:

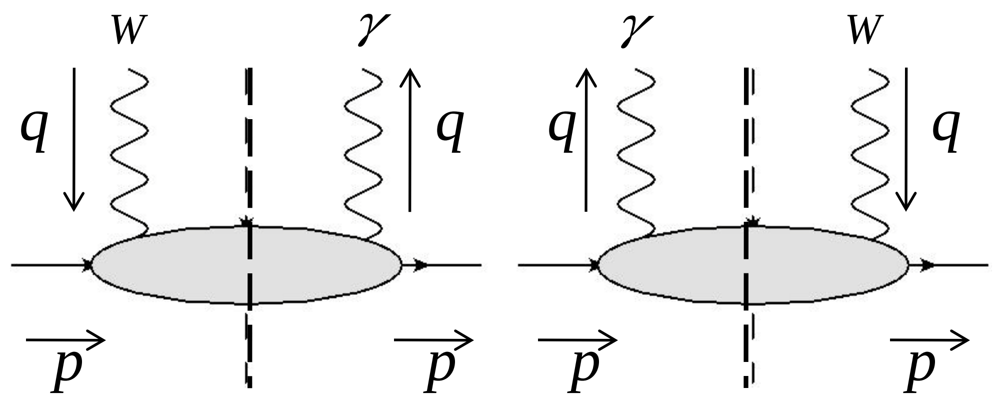
$$\sqrt{\underline{0.16}^2 + 0.09^2} \times 10^{-3} \approx 0.19 \times 10^{-3}$$

- Some limitations in M&S approach:
  - **Conceptually**: separation of physical regions based on just Q^2 is questionable. E.g. how about many-particle virtual state effect at low Q^2? A more consistent separation of regions should also involve the variable $W^2=(p+q)^2$.
  - **Practically**: there is **almost no data input** during the estimation of the "intermediate distance" contribution, which means that its **error bar cannot be reduced** even with future experiments.
- Alternative approach: **dispersion relation**. The main spirit is to express T_3 in terms of **single-current on-shell matrix elements**, such that its uncertainty could be reduced following the **improvement of experimental precision**/**model** !

Dispersive Approach: Formalism

CYS, M.Gorchtein, H.H.Patel and M.J.Ramsey-Musolf, arXiv:1807.10197[hep-ph]





"(0)": with ISOSCALAR EM current

$$T_3^{(0)}(\nu,Q^2) = -\frac{2\nu \mathrm{Res} T_3^{(0)}(\nu_B,Q^2)}{\nu_B^2-\nu^2} - \frac{i\nu}{\pi}\int_{\nu_\pi}^{\infty} d\nu' \frac{\mathrm{Dis}T_3^{(0)}(\nu',Q^2)}{\nu'^2-\nu^2}$$

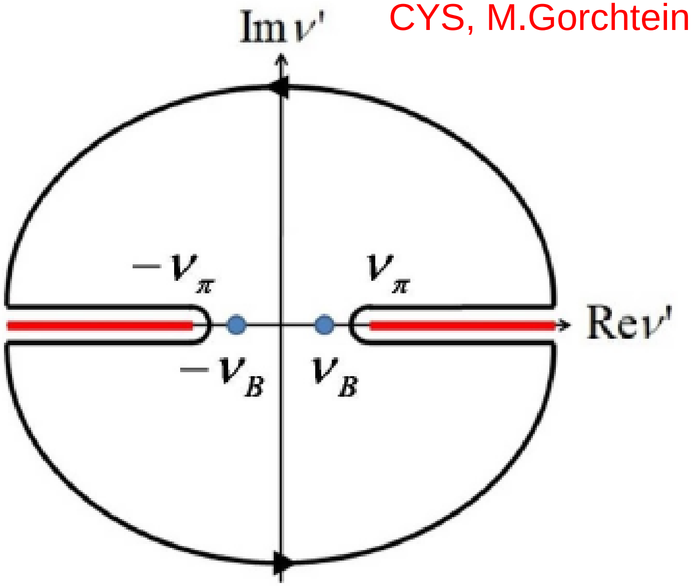
No constant subtraction term since $T_3^{(0)}$ is odd wrt ν

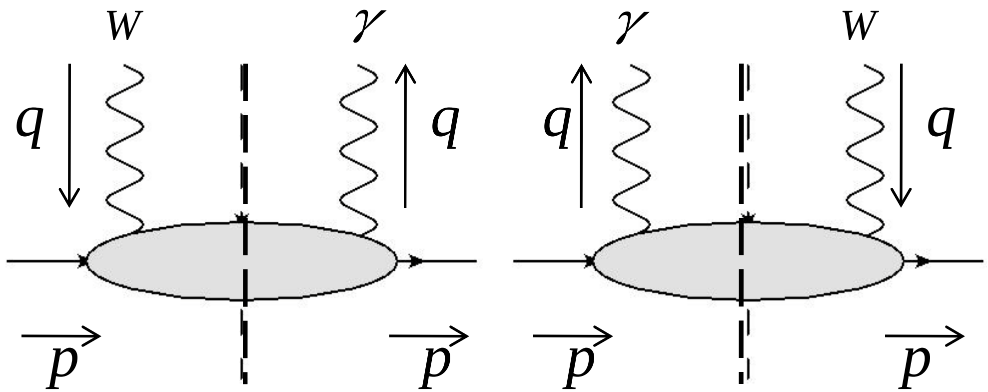
Optical theorem: $\mathrm{Dis}T_3^{(0)}(\nu,Q^2) = 4\pi F_3^{(0)}(\nu,Q^2)$

$$\frac{1}{8\pi}\sum_X (2\pi)^4 \delta^4(p+q-p_X)\langle p|J^\mu_{EM,0}|X\rangle\langle X|(J_W^\nu)_A|n\rangle = \frac{i\varepsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta}{2m_N\nu}F_3^{(0)}(\nu,Q^2)$$

Dispersive Approach: Formalism

CYS, M.Gorchtein, H.H.Patel and M.J.Ramsey-Musolf, arXiv:1807.10197[hep-ph]





Final expression: $$\Box^{VA}_{\gamma W} = \frac{3\alpha}{2\pi}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}M_3^{(0)}(1,Q^2),$$

where $M_3^{(0)}(1,Q^2)$ is the **first Nachtmann moment** of $F_3^{(0)}$:

$$M_3^{(0)}(1,Q^2) = \frac{4}{3}\int_0^1 dx\frac{1+2r}{(1+r)^2}F_3^{(0)}(x,Q^2)$$

$$r = \sqrt{1+4m_N^2 x^2/Q^2}$$

Dispersive Approach: Separation of Regions

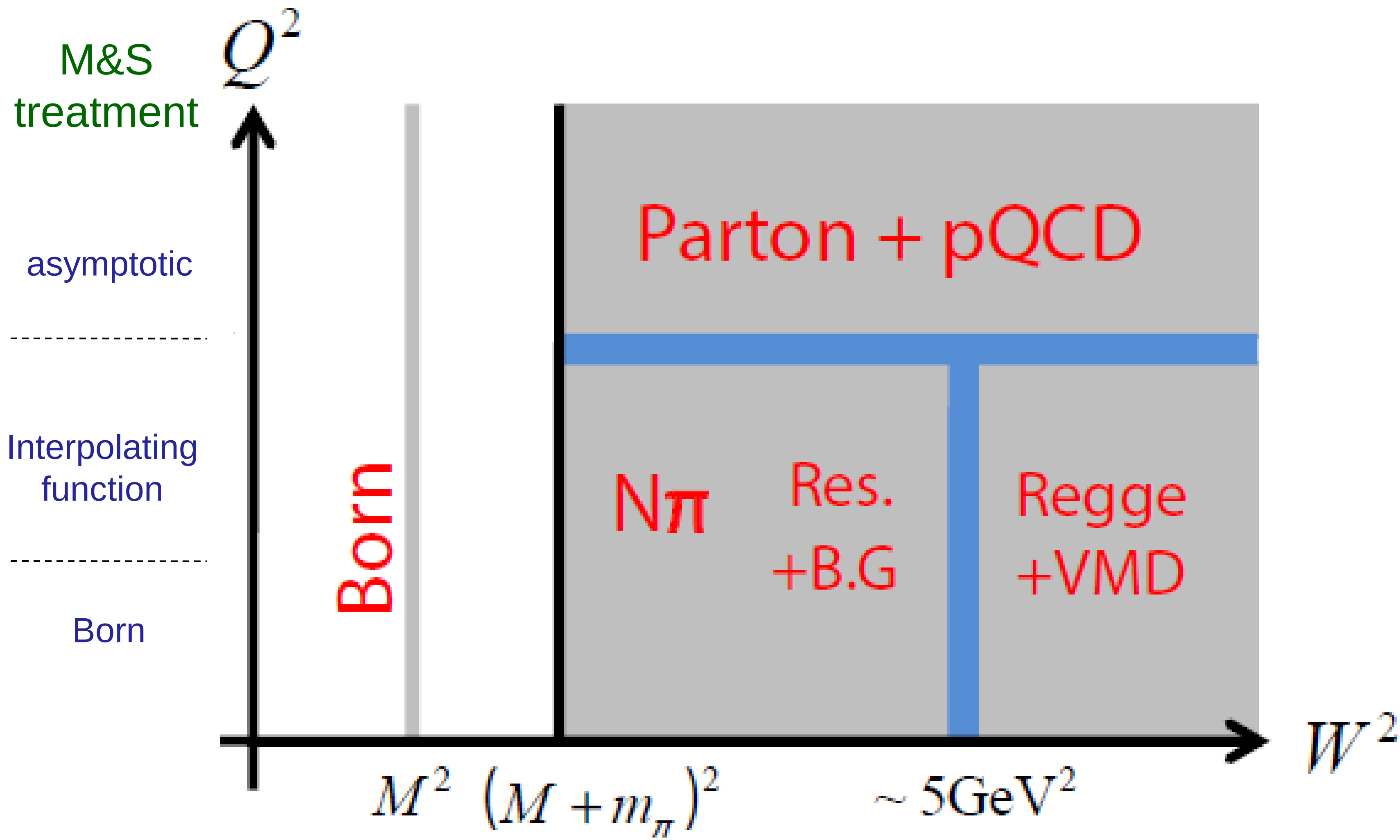
M&S treatment

asymptotic

Interpolating function

Born

Dispersive Approach: Separation of Regions



Dispersive Approach: Separation of Regions

(1) Large Q²-limit (asymptotic region):

$$Q^2 \to \infty,\ \nu > \nu_\pi = \frac{m_\pi^2 + 2m_N m_\pi + Q^2}{2m_N} \to \infty$$

approaches DIS regime; Partonic description is appropriate

The ν–integral is simplified:

$$\square_{\gamma W}^{VA,\text{asym}} = \frac{\alpha}{\pi}\int_{\Lambda^2}^{\infty} dQ^2 \frac{M_W^2}{M_W^2+Q^2}\int_{\nu_\pi}^{\infty} d\nu \frac{1}{m_N \nu}\frac{2\sqrt{\nu^2+Q^2}+\nu}{\left(\sqrt{\nu^2+Q^2}+\nu\right)^2} F_3^{(0)}(\nu,Q^2)$$

$$\to \frac{\alpha}{\pi}\int_{\Lambda^2}^{\infty} dQ^2 \frac{M_W^2}{M_W^2+Q^2}\frac{1}{16Q^2}\int_0^1 dx d_V^n(x) = \frac{\alpha}{8\pi}\int_{\Lambda^2}^{\infty} dQ^2 \frac{M_W^2}{M_W^2+Q^2}\frac{1}{Q^2}$$

Same as the M&S result. PQCD correct is also identical.

Scale at which PDF description becomes valid

Valence d-quark PDF in a neutron

Dispersive Approach: Separation of Regions

(2) Born contribution:

$$\Box_{\gamma W}^{VA,\mathrm{Born}} = -\frac{\alpha}{\pi}\int_0^\infty dQ \frac{2\sqrt{4M^2+Q^2}+Q}{\left(\sqrt{4M^2+Q^2}+Q\right)^2} G_A(Q^2) G_M^S(Q^2)$$

Axial F.F

Isosinglet magnetic Sachs F.F

Using the updated Sachs and axial FF, we obtain:

Z.Ye, J.Arrington, R.J.Hill and G.Lee, Phys.Lett.B777,8 (2018)

B.Bhattacharya, R.J.Hill and G.Paz,Phys.Rev.D84,073006 (2011)

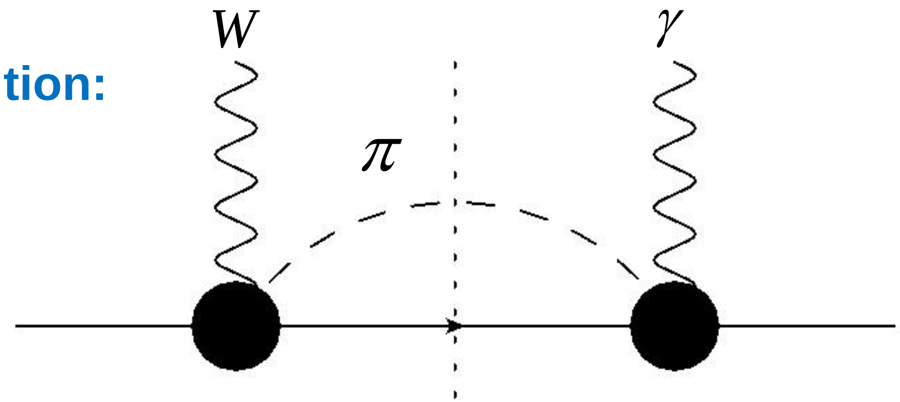
$$\Box_{\gamma W}^{VA,\mathrm{Born}} = 1.05(6)\times 10^{-3}$$

Central value significantly larger than the M&S value 0.96(9)*10^{-3} as we take $Q_{max}=\infty$ instead of cutting it off at some intermediate scale.

Dispersive Approach: Separation of Regions

(3) Nπ contribution:

$$W_\pi^2 < W^2 < \infty$$
$$0 < Q^2 < \Lambda^2$$



Calculable using **baryon chiral perturbation theory at tree level**.

Q-dependence modeled by **electroweak form factor** insertion:

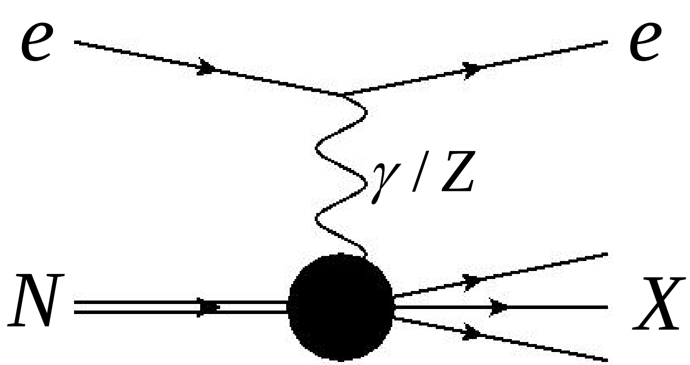
Numerically: with Λ^2=2GeV^2 it gives: $\Box_{\gamma W}^{VA,N\pi} = 4.6 \times 10^{-5}$ Very small!

(4) Resonance contribution:

Only I=1/2 resonances contribute, but their sizes are negligibly small. Resonance parameters taken from: Drechsel, Kamalov and Tiator, EPJA 34 (2007) 69
Lalakulich, Paschos and Piranishvili, PRD 74 (2006) 014009

Connection to Scattering Data

- The biggest problem is the large-W^2 multi-particle contributions, reflected by the large uncertainty in the M&S "interpolating function".
- Dispersion formalism in principle allows input from experimental data. **(I=0)*(I=1) V*A structure function** can in principle be obtained from **parity-odd e-N scattering**:



$$4F_3^{(0)} = F_{3,\gamma Z}^{p} - F_{3,\gamma Z}^{n} \approx 2F_{3,\gamma Z}^{p} - F_{3,\gamma Z}^{D}$$

- Unfortunately, existing data do not cover intermediate region of W^2 and Q^2.

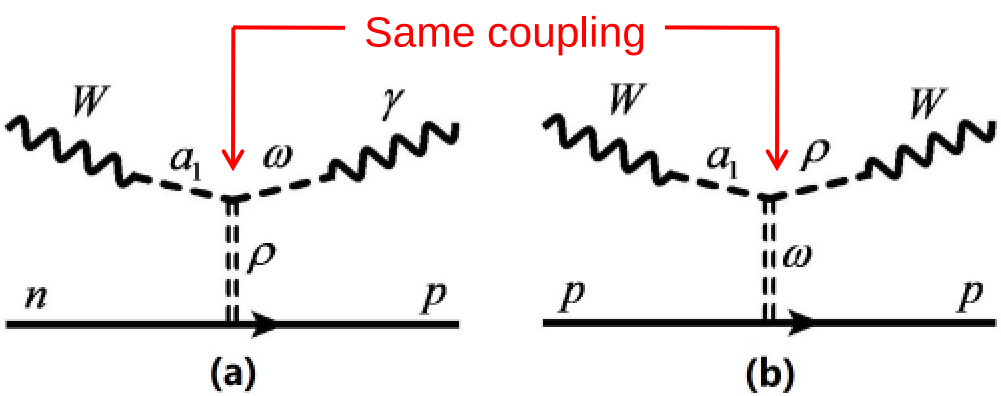
Connection to Scattering Data

- Alternative approach: obtain input from **(I=1)*(I=1) channel: neutrino-proton scattering processes**.
- Strategy: Below asymptotic regime, one has:

(I=1)*(I=1): Born + Nπ + Resonances + Regge

(I=0)*(I=1): Born + Nπ + Resonances + Regge

- The first three terms can be computed separately, while the last term for the two cases are related by simple **Regge-exchange picture**.

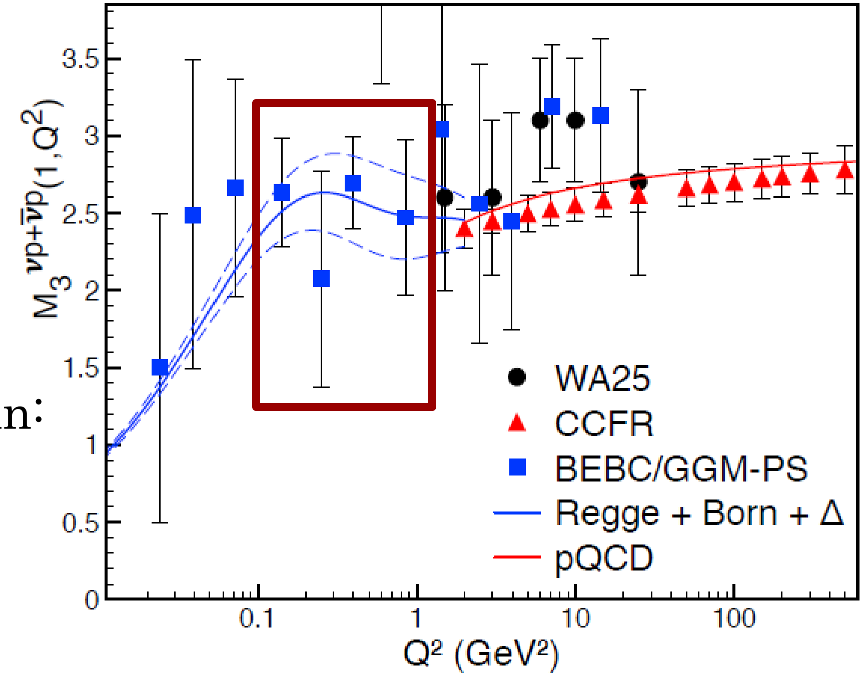


$F_3^{(0)}$ $\qquad$ $F_3^{\nu p+\bar{\nu}p}$

Connection to Scattering Data

- Data exists for the **first Nachtmann moment** of the **P-odd SF for neutrino-proton/antineutrino-proton scattering**, which allows for a fitting to the Regge contribution:

which is equivalent to a fit of the Regge contribution to γW, according to our simple Regge-exchange picture.



- Combining everything, we obtain:

$$\Delta_R^V = 0.02467(22)$$

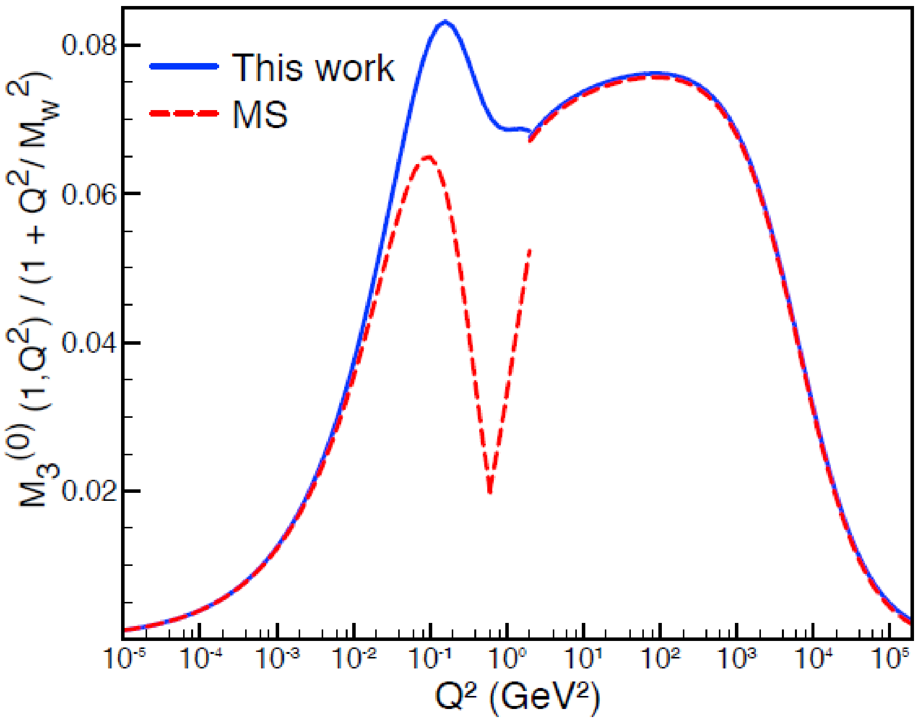
which represents a **significant reduction of theoretical uncertainty** as well as a **substantial upward shift in the central value** of the M&S result:

$$\Delta_R^V(\mathrm{M\,\&\,S}) = 0.02361(38)$$

Connection to Scattering Data

$$\Delta_R^V = \frac{3\alpha}{\pi}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}M_3^{(0)}(1,Q^2)+\ldots$$

Area under the curve measures the RC!



Reasons for the discrepancy:

- The M&S's classification of dominant physics based on only Q^2 misses the contribution from multi-particle intermediate states at low Q^2
- The M&S's choice of boundary conditions for their "interpolating function" leads to a discontinuity at the UV-matching point and a steep fall at intermediate Q^2.
- As a result, the **M&S treatment** leads to a significant **UNDERESTIMATION of the nucleus-independent RC,** and thus an **OVERESTIMATION of V_{ud}**.

Implications and Final Discussions

- Implication of our work to V_{ud} :

$$|V_{ud}| : \quad 0.97420(21) \rightarrow 0.97370(14)$$

provided all experimental results and other theoretical analysis remain unchanged.

- Implication to first-row CKM unitarity:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 :$$

$$0.9994(5) \rightarrow 0.9984(4)$$

An apparent 4σ–deviation from unitarity!

Implications and Final Discussions

- What could have happened?
  - Although M&S definitely underestimated Δ_R^V , did we go too far and overestimated it? (Although our analysis is a data-driven one)
  - Is the higher-order RC effects (i.e.2 loops and above) not calculated correctly?
  - Is the **nuclear-structure correction** analysis (by Hardy & Towner) incomplete? (**see Misha's talk**)
  - Is it due to the experimental V_{us} discrepancy?
  - … or is it a signal of something new?

Summary

1. The **γW-box diagram** uncertainty to V_{ud} constitutes the largest uncertainty in the first-row CKM unitarity. Current Marciano-Sirlin treatment does not allow a systematic improvement of precision.
2. We adopted a **dispersion-relation approach** that allows inputs from experimental data.
3. Utilizing neutrino-proton scattering data, we obtain a reduced uncertainty of V_{ud} but also a significant shift of its central value. This leads to an **apparent 4σ-deviation** from the first-row CKM unitarity.
4. Future **experimental data on e-N scattering** as well as **SM/BSM** calculations in **hadron/nuclear level** are required to address such anomaly.