Making the Electroweak Phase Transition (Theoretically) Strong

Welcome and Theory Overview

April 6, 2017

ACFI Workshop

Hiren Patel
hhpatel@umass.edu

UMASS AMHERST

AMHERST CENTER FOR FUNDAMENTAL INTERACTIONS

Workshop Goals

1. How can we get more accurate estimates of physical quantities related to the EWPT in a tractable way for BSM scenarios?

   a) Effective potential/critical temperatures: V_{eff}, T_C
   b) Bubble nucleation: $\phi_c(r)$, Γ_{nuc}, T_N, α, v_{wall}, L_{wall}…
   c) Sphaleron processes: $\phi(r)/A_{\mathrm{sph}}(r)$, E_{sph}, Γ_{sph}, …
   d) Ultimately, Y_B, and Ω_{GW}
   …

2. Can we make the theoretical level of precision comparable to that of experimental/observational cosmology?

3. Can we reliably assign errors to these estimates?

4. Compare perturbation theory to fully non-perturbative lattice simulations. (benchmark models + parameters)

Outline

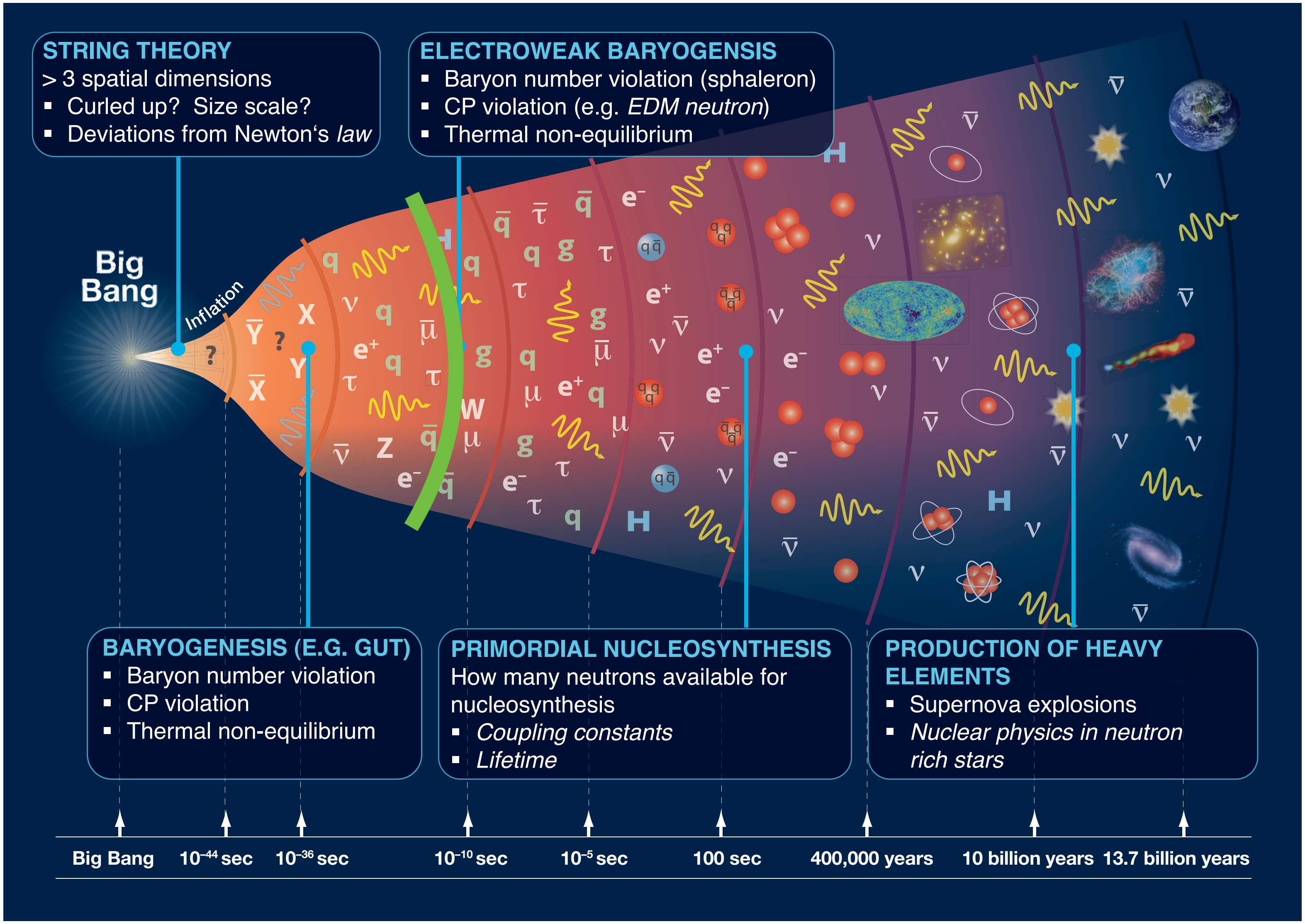
Electroweak Phase Transition

Context
Motivation
Overview of methods

Open theoretical problems (partial list)

1. Thermal potential
   - Spurious imaginary parts
   - Gauge dependence
2. Bubble nucleation rates
   - How to calculate them consistently

Cosmic History



Stephan Paul
arXiv:1205.2451

Electroweak Phase Transition

LHC: 2012 Discovery of SM Higgs-like particle

measurement of mass $m_H = 126$ GeV $\Longrightarrow$

First principles determination of cosmic history through 10^{-10} s.

Numerical simulations suggest a crossover

K. Kajantie, et al. PRL 77 (1996) 2887,
F. Karsch, et al. NPPS 53 (1997) 623,
Y. Aoki et al., PRD 56 (1997) 3860
M. Gurtler et al., PRD 56 (1997) 3888

Electroweak Phase Transition

Why do we care:

Two big physics motivations -

1. A strong 1st order EWPT satisfies Sakharov's out-of-equilibrium criteria for baryogenesis

2. A strong 1st order EWPT generates gravity waves, possibly observable in next gen. gravity wave detectors.

Also, for intellectual curiosity:

3. Detailed understanding of pattern of EW symmetry breaking in the early universe.

Electroweak Phase Transition

The standard model Higgs field alone cannot generate a 1st order EWPT.

$\longrightarrow$ A strong motivation for BSM

Questions for model builders and phenomenologists:

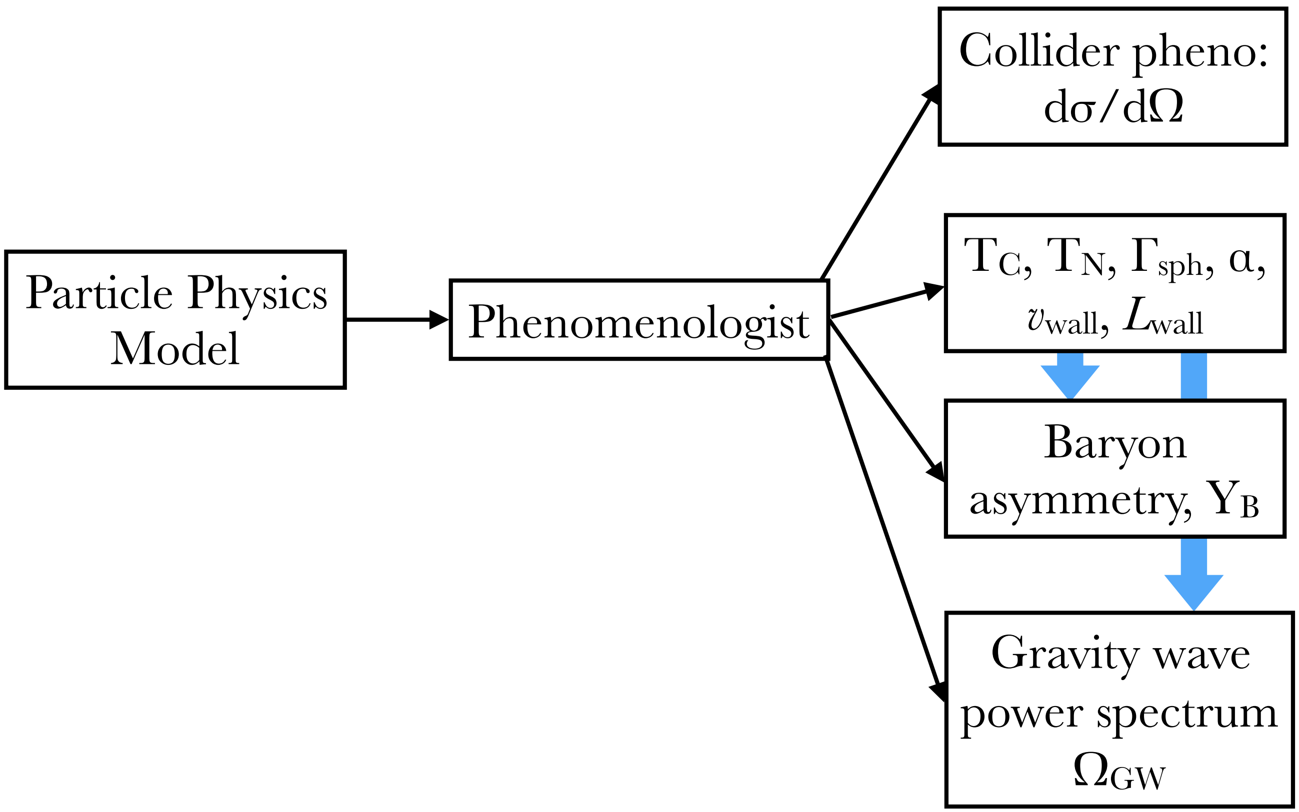
1. Are there fundamental scalars other than the Higgs, and what BSM scenarios can generate a 1st order EWPT?
2. What are the experimental signatures of these scenarios?
3. What are their implications to other theoretical problems (neutrino mass, hierarchy, dark matter, …)?

Electroweak Phase Transition

How these questions are answered:

<u>Adequate Precision?</u>

experiment + obs. cosmology | theory



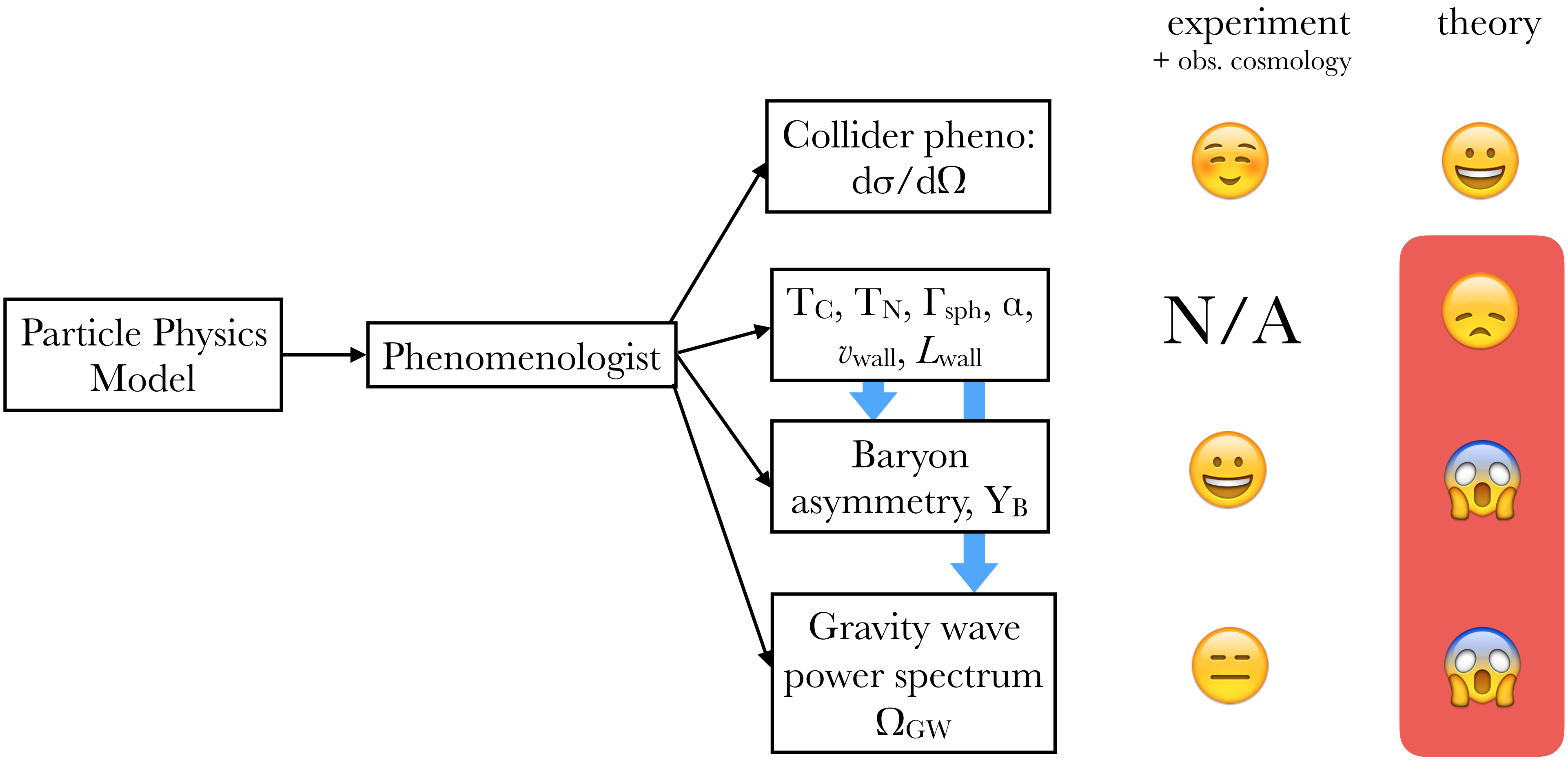
Issue:

Theoretical precision is not competitive with observational cosmology

Electroweak Phase Transition

How these questions are answered:

Adequate Precision?



Issue:

Theoretical precision is not competitive with observational cosmology

Methods of Analysis

- The calculation of V_{eff}, T_C, $\phi_{\text{bubb}}(r)$, Γ_{nuc}, T_N, α, v_{wall}, L_{wall}, E_{sph}, Γ_{sph}, and Y_B, Ω_{GW} are notoriously difficult.
- Many methods: can be put on a spectrum.

Numerical ⟷ **Analytical**

- Fully dynamical 4D numerical simulation
- Partial 4D numerical simulation
- Analytic High-T EFT + 3D numerical simulation
- Analytic V_{eff} + numerical T_C, T_N, Γ_{sph}, …
- Exact analytic evaluation of Γ_{eff}

Methods of Analysis

- The calculation of V_{eff}, T_C, $\phi_{\mathrm{bubb}}(r)$, Γ_{nuc}, T_N, α, v_{wall}, L_{wall}, E_{sph}, Γ_{sph}, and Y_B, Ω_{GW} are notoriously difficult.
- Many methods: can be put on a spectrum.
- Each method of analysis has some degree of approximation

Fully dynamical 4D numerical simulation

Partial 4D numerical simulation

Analytic High-T EFT + 3D numerical simulation

Analytic V_{eff} + numerical T_C, T_N, Γ_{sph}, …

Exact analytic evaluation of Γ_{eff}

Numerical ⟷ **Analytical**

Level of approx.

<u>Statistical</u>

<u>Parametric</u>,
Lattice spacing…

(see D. Weir's talk)

(see T. Tenkanen's talk)

<u>Parametric</u>

$O(g^4)$,
$O(m^2/T^2)$,
μ/T,
ξ-dep.

(remainder of talk…)

Equilibrium Effective Potential

To ultimately obtain Y_B and Ω_{GW}, need <u>dynamical quantities</u>:
a) Bubble nucleation: $\phi_{bubb}(r)$, Γ_{nuc}, T_N, α, v_{wall}, L_{wall}, …
b) Sphaleron processes: $\phi_{sph}(r)/A_{sph}(r)$, E_{sph}, Γ_{sph}, …

For a very rough analytic determination of the strength of phase transition, focus on two quantities:

1. Critical temperature T_C, and
2. discontinuity in order parameter ϕ_C

These are equilibrium quantities, requiring the calculation of the **thermal effective potential** V_{eff}.

Effective Potential

There are two problems associated with the thermal effective potential.

1. The potential (as naively calculated) has spurious imaginary parts. **(see D. Curtin's talk)**

2. The effective potential is gauge dependent.

Effective Potential: Imaginary Part

The effective potential is traditionally calculated in perturbation theory:

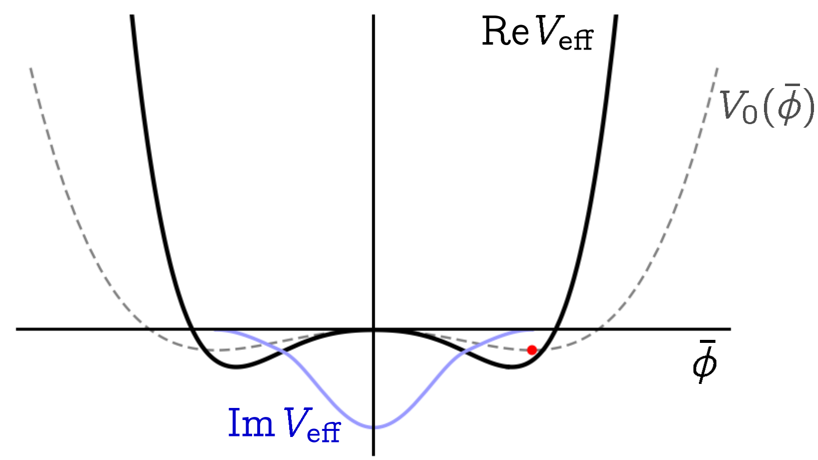
$$V_{\text{eff}}(\phi, T) = V_{\text{tree}} + V_{\text{1-loop}} + \ldots$$

$$V_{\text{1-loop}}(\phi, T) = \sum_i \left\{ \underbrace{\frac{\lambda_i}{4(4\pi)^2}\left[m^2(\phi)\right]^2 \ln\left(\frac{m^2(\phi)}{\mu_R}\right)}_{\text{Zero temperature part}} + \underbrace{\frac{\lambda_i T^2}{2\pi^2}\int_0^\infty dx\, x^2 \ln(1 \mp e^{-\sqrt{x^2+m^2(\phi)/T^2}})}_{\text{Finite temperature part}} \right\}$$

sum over all species, i

At T=0, if symmetry is broken at tree level (as in SM), Im(V$_{\text{eff}}$)≠0, near origin (due to logarithm).

Lee+Weinberg, due to QM instability



At very high T, no instabilities should arise, and <u>we must</u> have: Im(V$_{\text{eff}}$)=0

But this is not necessarily satisfied

Breakdown of PT: requires resummation
(see D. Curtin's talk)

Hiren Patel

Effective Potential: Gauge Depedence

- Partition function of system is given by the trace of the density operator.

$$Z = \mathrm{Tr}[e^{-\beta \hat{H}}]$$

Partition function

Sum over states

- In a gauge theory, only physical states must be summed over.

- Done by fixing a gauge following method of Faddeev and Popov (1967).

$$Z = \mathrm{Tr}[e^{-\beta \hat{H}(\xi)}]$$

gauge-parameter

- Thermal potential becomes gauge-dependent

$$V_{\text{eff}}(\phi, T) = -k_B T \ln Z(\xi)$$

- Nielsen (1975) showed that extrema of V_{eff} are gauge independent, although the Higgs condensate $\langle\phi\rangle$ is not.

- Straightforward extraction of T_C seems to be gauge dependent.

even though the it only requires minima of V_{eff}

h-bar Expansion

H.Patel, M.J. Ramsey-Musolf, JHEP 1107 (2011), 029

A possible resolution:

Key is to extremize the potential while maintaining consistency with expansion parameter **h-bar**.

Insert $\hbar^{-1}$ here

$$Z[j] = \int \mathcal{D}\Phi \mathcal{D}A \, e^{-\frac{1}{\hbar}(S_E[\Phi]+j\Phi)}$$

($\hbar$ counts # of loops)

$$V_{\text{eff}} = V_0 + \hbar V_1 + \hbar^2 V_2 + \ldots$$

$$\phi_{\text{min}} = \phi_0 + \hbar \phi_1 + \hbar^2 \phi_2 + \ldots$$

Solve $V'_{\text{eff}}\big|_{\phi_{\text{min}}} = 0$ consistently order by order

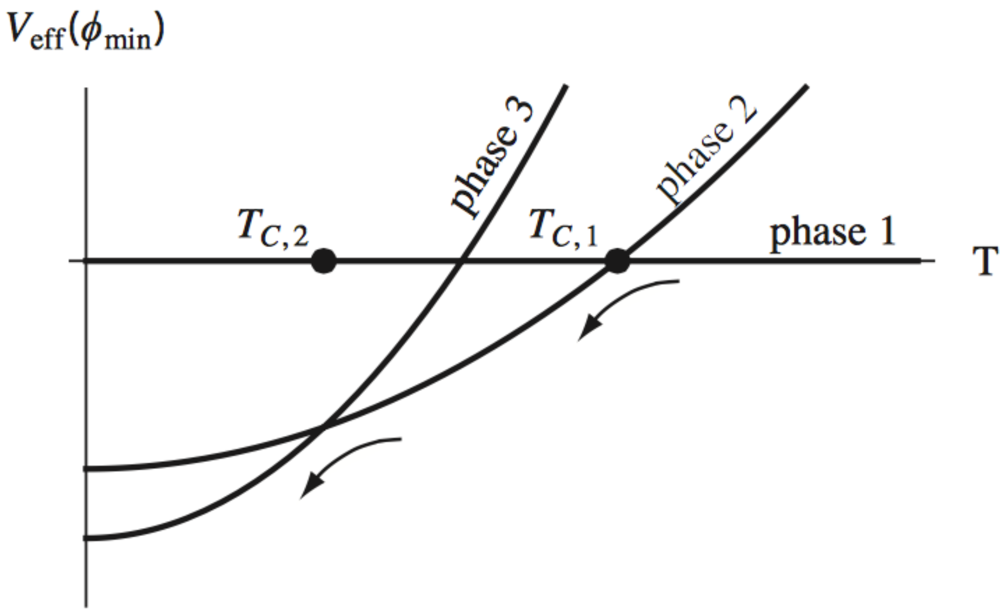
$$V_{\text{eff}}(\phi_{\text{min}}) = V_0(\phi_0) + \hbar\, V_1(\phi_0) + \hbar^2 \left(V_2(\phi_0) - \frac{1}{2} \frac{V_1'(\phi_0)^2}{V_0''(\phi_0)} \right) + \ldots$$

h-bar Expansion

H.Patel, M.J. Ramsey-Musolf, JHEP 1107 (2011), 029

$$V_{\text{eff}}(\phi_{\text{min}}) = V_0(\phi_0) + \hbar\, V_1(\phi_0) + \hbar^2 \left(V_2(\phi_0) - \frac{1}{2} \frac{V_1'(\phi_0)^2}{V_0''(\phi_0)} \right) + \ldots$$

At finite temperature, this gives the thermal energy of the phases of the system as a function of T.



Degeneracy condition satisfied at the intersections, yielding the critical temperature.

h-bar Expansion

H.Patel, M.J. Ramsey-Musolf, JHEP 1107 (2011), 029

$$V_{\text{eff}}(\phi_{\text{min}}) = V_0(\phi_0) + \hbar\, V_1(\phi_0) + \hbar^2\left(V_2(\phi_0) - \frac{1}{2}\frac{V_1'(\phi_0)^2}{V_0''(\phi_0)}\right) + \ldots$$

Advantages

1. Can be proven to be strictly gauge-independent (Critical temperature)

2. Numerically straightforward to implement

3. Can be applied to radiatively induced phase transitions

but…

h-bar Expansion

H.Patel, M.J. Ramsey-Musolf, JHEP 1107 (2011), 029

$$V_{\rm eff}(\phi_{\rm min}) = V_0(\phi_0) + \hbar\, V_1(\phi_0) + \hbar^2\left(V_2(\phi_0) - \frac{1}{2}\frac{V_1'(\phi_0)^2}{V_0''(\phi_0)}\right) + \ldots$$

Open research problems

1. Tends to underestimate T_C. The backreaction of thermal bath on condensate is delayed to O(h^2) (slow convergence)

2. Incompatible with naive methods of resummation.

3. If there is no solution at zeroth order (solution generated radiatively/thermally), then it will be missed.

Nielsen identity does not require h-bar as the power counting scheme. Any consistent power counting scheme would work.

- Are there power counting schemes that have better convergence, can capture more solutions?
- How do they compare to numerical lattice results?

Variant on h-bar expansion

A. Andreassen, W. Frost, M.D. Schwartz,
PRD 91, 016009 (2015)

Applicable to the Coleman-Weinberg mechanism (zero T)

$\lambda \sim e^4$

1. Insert $\hbar^{-1}$ here... ...and $\hbar$ here

$$Z[j] = \int \mathcal{D}\Phi \mathcal{D}A \, e^{-\frac{1}{\hbar}(S_E[\Phi]+j\Phi)}\,; \qquad V(\Phi) = \hbar\lambda|\Phi|^4$$

($\hbar$ no longer counts # of loops)

2. In this case perturbation theory is reorganized,

Nothing at zeroth order

contains tree-level and **part** of one-loop

contains **rest** of one-loop and **part** of two loop + (daisy contributions)

$$V_{\text{eff}} = 0 + \hbar V_1 + \hbar^2 V_2 + \ldots$$

$$\phi_{\min} = \phi_0 + \underbrace{\hbar\phi_1 + \hbar^2\phi_2}_{\text{Quantum corrections}} + \ldots$$

Derived by minimizing V_1. **(Starting point)**

$$V_{\text{eff}}(\phi_{\min}) = (e^{-\frac{32\pi^2\lambda}{3e^4}+\frac{2}{3}} + \ldots)\mu^4\left[\hbar\frac{3e^4}{128\pi^2} + \hbar^2\frac{e^6}{(16\pi^2)^2}\left(\frac{71}{6} - \frac{62}{3}\ln e + 10\ln^2 e\right)\right]$$

Variant on h-bar expansion

A. Andreassen, W. Frost, M.D. Schwartz,
PRD 91, 016009 (2015)

Applicable to the Coleman-Weinberg mechanism (zero T)

$\lambda \sim e^4$

1. Insert $\hbar^{-1}$ here... ...and $\hbar$ here

$$Z[j] = \int \mathcal{D}\Phi \mathcal{D}A \, e^{-\frac{1}{\hbar}(S_E[\Phi]+j\Phi)} \, ; \qquad V(\Phi) = \hbar\lambda|\Phi|^4$$

($\hbar$ no longer counts # of loops)

2. In this case perturbation theory is reorganized,

Nothing at zeroth order

contains tree-level and **part** of one-loop

contains **rest** of one-loop and **part** of two loop + (daisy contributions)

$$V_{\text{eff}} = 0 + \hbar V_1 + \hbar^2 V_2 + \ldots$$

$$\phi_{\min} = \phi_0 + \underbrace{\hbar\phi_1 + \hbar^2\phi_2}_{\text{Quantum corrections}} + \ldots$$

Derived by minimizing V_1.
(Starting point)

- **Can scheme(s) like this be applied at finite T?**
- **How well does it perform numerically?**

$$V_{\text{eff}}(\phi_{\min}) = (e^{-\frac{32\pi^2\lambda}{3e^4}+\frac{2}{3}} + \ldots)\mu^4\left[\hbar\frac{3e^4}{128\pi^2} + \hbar^2\frac{e^6}{(16\pi^2)^2}\left(\frac{71}{6} - \frac{62}{3}\ln e + 10\ln^2 e\right)\right]$$

Bubble Nucleation Rates

If we want to be more accurate, we need quantities related to bubble nucleation/expansion ($\phi_{\text{bubb}}(\text{r})$, Γ_{nuc}, T_N, v_{wall}, L, …)

(see talks by
P. Millington, and
J. Kozaczuk)

I'm just going to touch upon some of the more acute problems.

The standard analytic method is by following Langer's formalism

Ann Phys 41, 108 (1967)

Similar methods for sphaleron rate and tunneling out of metastable vacuum.

Bubble Nucleation Rates

Conventionally, calculated in the high T approximation (integrate out *only* the heavy Matsubara modes):

$\boxed{\text{assumes } T_N \gg m^2\text{:}}$

$$V_{\text{eff}}(\phi, T) = (m^2 + \alpha T^2)\,\phi^2 - \eta\phi^3 + \frac{\bar{\lambda}}{4}\,\phi^4 + \ldots$$

leading temperature dependence

barrier at tree-level

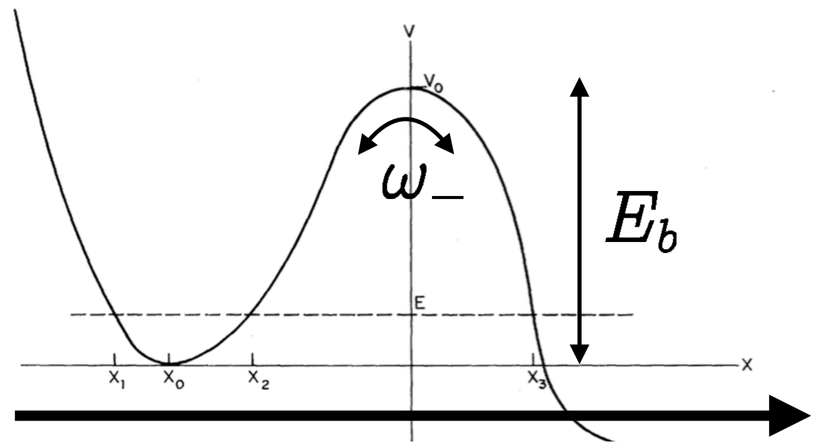
$$\Gamma = e^{-E_b/T}\left[\frac{\omega_-}{2\pi}\left(\frac{E_b}{2\pi T}\right)\underbrace{\left|\frac{\det'\mathcal{O}(\phi_b)}{\det\,\mathcal{O}(0)}\right|^{-1/2}\cdots}_{\text{Fluctuation determinants}}\right]$$

Nucleation rate/ unit time unit vol.

Critical bubble energy

Fluctuation determinants

Difficult calculation (usually omitted)



dynamical coordinate

I. Affleck
PRL 46, 388

T_N: $\Gamma\, H^{-3} \sim H$

Bubble Nucleation Rates

Questions we need answers to:

1. If $T_N \sim m$, the high T EFT becomes invalid.
   (How bad is it if the EFT is used?)
   What analytic alternatives are there to evaluate the rate?

2. If there is no tree level barrier, a solution does not exist.
   How can the rate be calculated consistently?

   (see talks by P. Millington)

   (Integrating out the Matsubara zero modes and computing the critical bubble on top of the effective potential is inconsistent)

3. Even with a tree-level barrier, what is the error is induced by neglecting the fluctuation determinant?

Conclusions

-There are many physical quantities we want to calculate: many are very hard to do:

V_{eff}, T_C, $\phi_c(r)$, Γ_{nuc}, T_N, α, v_{wall}, L_{wall}, $\phi(r)/A_{\text{sph}}(r)$, E_{sph}, Γ_{sph}, Y_B, and Ω_{GW}

But fortunately, there is a wide array of available tools to calculate them.

- How consistent and accurate are these tools? Can we make the theoretical level of precision comparable to that of experimental/observational cosmology?

-Recently there has been a lot of talk about probing the electroweak phase transition at next generation experiments. **This makes these questions all the more important.** We need robust calculations to guide our experimentalists friends.