

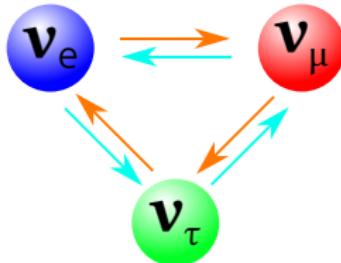
# Neutrino mass physics at lepton colliders

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ACFI workshop,  
July the 19th, 2017, Amherst

# Motivation for sterile neutrinos



Three Generations of Matter (Fermions) spin $\frac{1}{2}$									
	I			II			III		
mass →	2.4 MeV	1.27 GeV	173.2 GeV						
charge →	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{-1}{3}$	$\frac{-1}{3}$	$\frac{-1}{3}$
name →	u up	c charm	t top	d down	s strange	b bottom			
Quarks	Left: $\frac{2}{3}$	Left: $\frac{2}{3}$	Left: $\frac{2}{3}$	Left: $-\frac{1}{3}$	Left: $-\frac{1}{3}$	Left: $-\frac{1}{3}$			
Leptons	$\nu_e$ electron neutrino	$\nu_\mu$ muon neutrino	$\nu_\tau$ tau neutrino	e electron	$\mu$ muon	$\tau$ tau			
Bosons (Forces) spin 1							Z weak force	W weak force	Higgs boson
	91.2 GeV	0	0	0.511 MeV	105.7 MeV	1.777 GeV	80.4 GeV	126 GeV	spin 0
	0	0	0	-1	-1	-1	+1	0	

Shaposhnikov et al.

- ▶ Neutrino oscillations: *at least* two massive light neutrinos.
- ▶ No renormalisable way in the SM therefore;  
⇒ evidence for new physics.
- ▶ Focus: type I seesaw mechanism.

# Type I seesaw

## The “naïve” version:

- ▶ The simplified version:  $(1 \nu_L, 1 \nu_R)$ 
  - ★ Mass matrix  $\sim \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$ , with  $m = y_\nu v_{\text{EW}} \ll M$ .
  - ★ Light neutrino mass:  $m_\nu = \frac{1}{2} \frac{v_{\text{EW}}^2 |y_\nu|^2}{M_R}$ .

## The symmetry protected scenario:

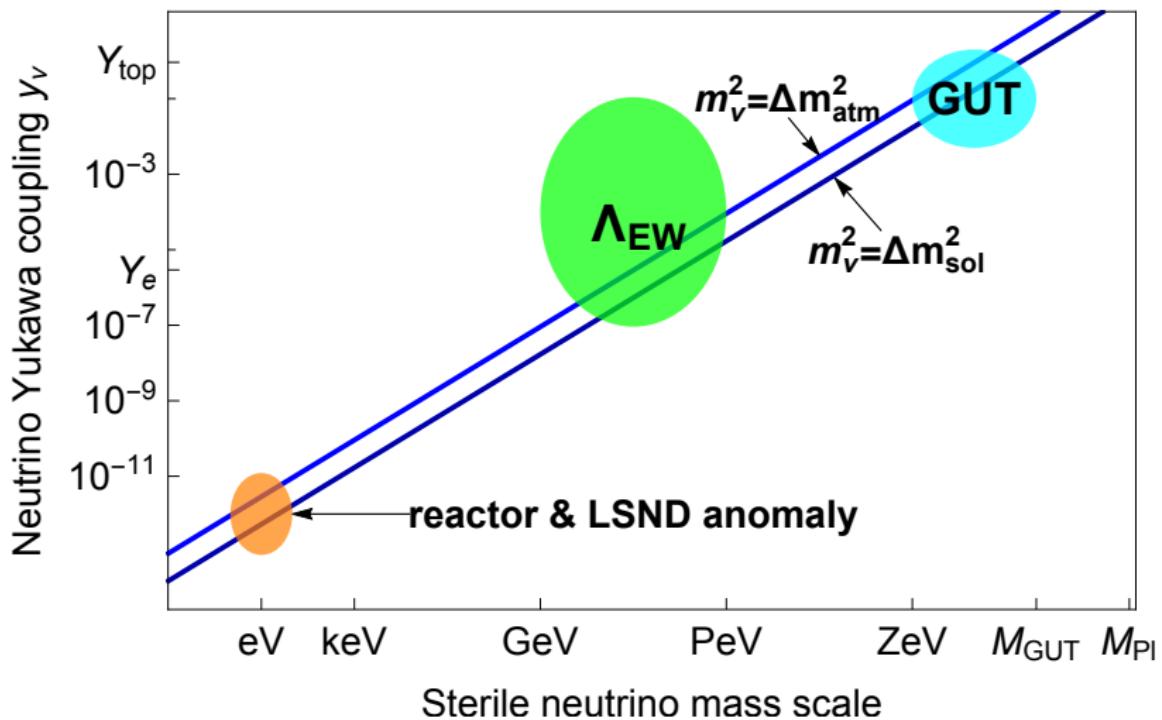
- ▶ Symmetry: for instance lepton number or  $B - L$ .
- ▶ A  $(2 \nu_L, 2 \nu_R)$  example:

$$y_\nu \rightarrow \begin{pmatrix} \mathcal{O}(y_\nu) & 0 \\ \mathcal{O}(y_\nu) & 0 \end{pmatrix}, \quad M \rightarrow \begin{pmatrix} 0 & M_R \\ M_R & \varepsilon \end{pmatrix}$$

$$\Rightarrow m_{\nu_i} = 0 + \varepsilon \frac{v_{\text{EW}}^2 \mathcal{O}(y_\nu^2)}{M_R^2}$$

⇒ Large ratio of  $y_\nu$  and  $M_R$  can be compatible with  $m_{\nu_i}$ .

# The Big Picture



# Symmetry Protected Seesaw Scenario

Benchmark model for future collider studies, defined in Antusch, OF, JHEP 1505 (2015) 053.

Similar to e.g.: Mohapatra, Valle (1986); Shaposhnikov (2007); Gavela, Hambye, Hernandez (2009)

- Collider phenomenology dominated by two sterile neutrinos  $N_i$  with protective symmetry, such that

$$\mathcal{L}_N = -\frac{1}{2} \overline{N_R^1} M (N_R^2)^c - y_{\nu_\alpha} \overline{N_R^1} \tilde{\phi}^\dagger L^\alpha + \text{H.c.}$$

- Further “decoupled” sterile neutrinos may exist.
- Active-sterile mixing:  $\theta_\alpha = y_{\nu_\alpha} \frac{v_{\text{EW}}}{\sqrt{2} M}$ ,  $\theta^2 \equiv \sum_\alpha |\theta_\alpha|^2$
- The leptonic mixing matrix to leading order in  $\theta_\alpha$ :

$$\mathcal{U} = \begin{pmatrix} N_{e1} & N_{e2} & N_{e3} & -\frac{i}{\sqrt{2}}\theta_e & \frac{1}{\sqrt{2}}\theta_e \\ N_{\mu 1} & N_{\mu 2} & N_{\mu 3} & -\frac{i}{\sqrt{2}}\theta_\mu & \frac{1}{\sqrt{2}}\theta_\mu \\ N_{\tau 1} & N_{\tau 2} & N_{\tau 3} & -\frac{i}{\sqrt{2}}\theta_\tau & \frac{1}{\sqrt{2}}\theta_\tau \\ 0 & 0 & 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\theta_e^* & -\theta_\mu^* & -\theta_\tau^* & -\frac{i}{\sqrt{2}} \left(1 - \frac{\theta^2}{2}\right) & \frac{1}{\sqrt{2}} \left(1 - \frac{\theta^2}{2}\right) \end{pmatrix}$$

# Heavy neutrino interactions

- ▶ **Charged current (CC):**

$$j_\mu^\pm = \frac{g}{2} \theta_\alpha \bar{\ell}_\alpha \gamma_\mu (-i N_1 + N_2)$$

- ▶ **Neutral current (NC):**

$$j_\mu^0 = \frac{g}{2 c_W} [\theta^2 \bar{N}_2 \gamma_\mu N_2 + (\bar{\nu}_i \gamma_\mu \xi_{\alpha 1} N_1 + \bar{\nu}_i \gamma_\mu \xi_{\alpha 2} N_2 + \text{H.c.})]$$

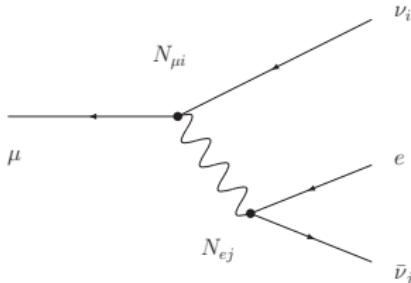
- ▶ Higgs boson **Yukawa** interaction:

$$\mathcal{L}_{\text{Yukawa}} = \sum_{i=1}^3 \xi_{\alpha 2} \frac{\sqrt{2} M}{v_{\text{EW}}} \nu_i \phi^0 (\bar{N}_1 + \bar{N}_2)$$

- ▶ With the mixing parameters:  $\xi_{\alpha 1} = (-i) \mathcal{N}_{\alpha \beta}^* \frac{\theta_\beta}{\sqrt{2}}$ ,  $\xi_{\alpha 2} = i \xi_{\alpha 1}$

# Precision observables in seesaw scenarii

**Input parameters:**  $M_Z$ ,  $\alpha(M_Z)$ ,  $G_F$ .



**The Fermi constant:**

- ▶ Muon decay  $\propto (NN^\dagger)_{ee} (NN^\dagger)_{\mu\mu}$
- ▶ Fermi constant  $G_F \neq$  muon decay constant  $G_\mu$ .
- ▶ Tree-level relation:  $G_F = \frac{G_\mu}{\sqrt{(NN^\dagger)_{ee}(NN^\dagger)_{\mu\mu}}} = \frac{\alpha\pi}{\sqrt{2}s_W^2 c_W^2 m_Z^2}$
- ▶ Analogous: Observables involving weak decays.
- ⇒ Theory prediction for electroweak observables.

# Constraints on PMNS non-unitarity from precision data

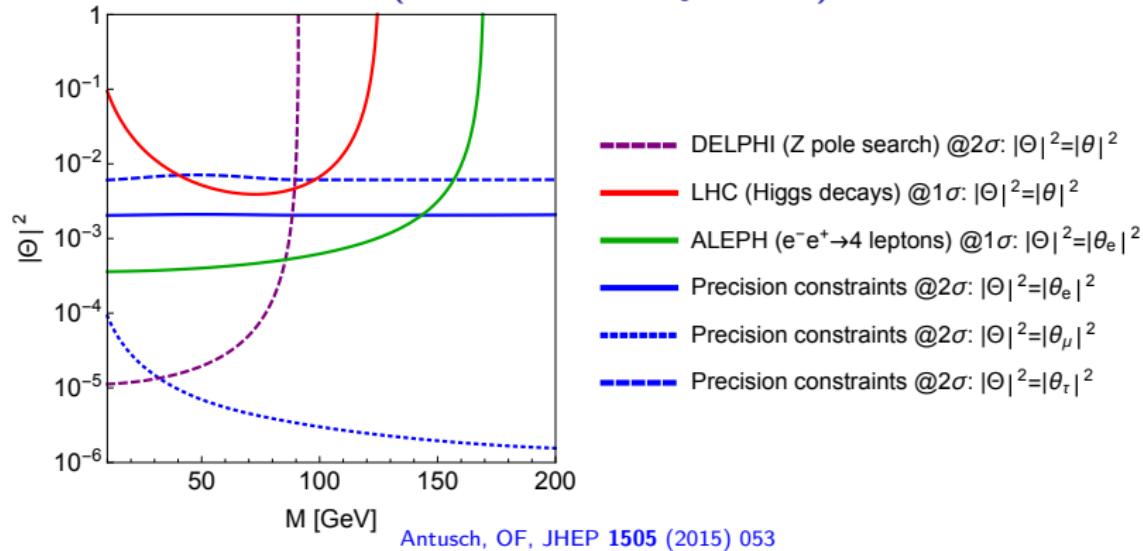
- ▶ Analysis of non-unitarity of the PMNS matrix.
  - ▶ 34 precision observables:  
Electroweak Precision Observables (EWPO), lepton universality, charged lepton flavour violation, CKM unitarity
- Lots of details in the backup - please ask!
- ▶ Highest posterior density intervals at 90% Bayesian C.L.:

$-0.0021 \leq \varepsilon_{ee} \leq -0.0002$	$ \varepsilon_{e\mu}  < 1.0 \times 10^{-5}$
$-0.0004 \leq \varepsilon_{\mu\mu} \leq 0$	$ \varepsilon_{e\tau}  < 2.1 \times 10^{-3}$
$-0.0053 \leq \varepsilon_{\tau\tau} \leq 0$	$ \varepsilon_{\mu\tau}  < 8.0 \times 10^{-4}$

Antusch, OF, JHEP 1410 (2014) 094

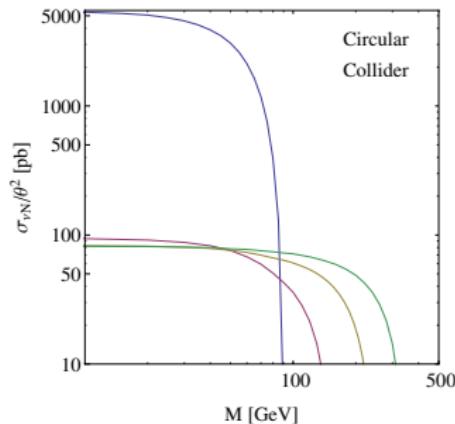
- ★ Non-unitarity parameters:  $\varepsilon_{\alpha\beta} = -\theta_{\alpha}^* \theta_{\beta}$ .
- ★ Weak statistical preference for non-zero mixing for  $\varepsilon_{ee}$ .

# Present Constraints (dominated by LEP)

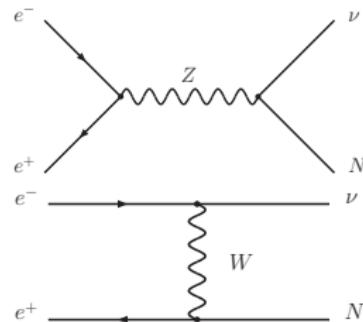


- ▶ Z pole search: limits from  $Z$  branching ratios .  
Abreu et al. Z.Phys. C74 (1997) 57-71
- ▶ Higgs decays: Best constraints from  $h \rightarrow \gamma\gamma$ .
- ▶  $WW$  production cross section:  $\delta\sigma_{\text{SM}}^{WW} = 0.011_{\text{stat}} + 0.007_{\text{syst}}$   
OPAL collaboration, Abbiendi et al. (2007)

# Heavy Neutrino Production in electron-positron collisions

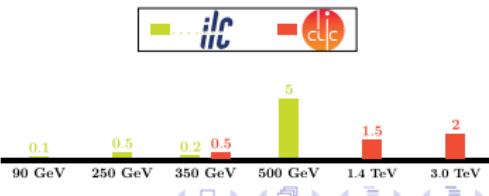
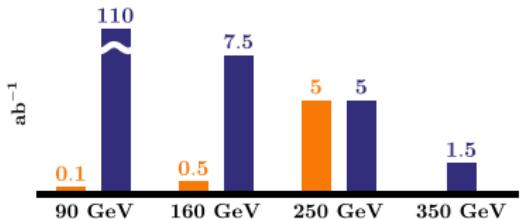
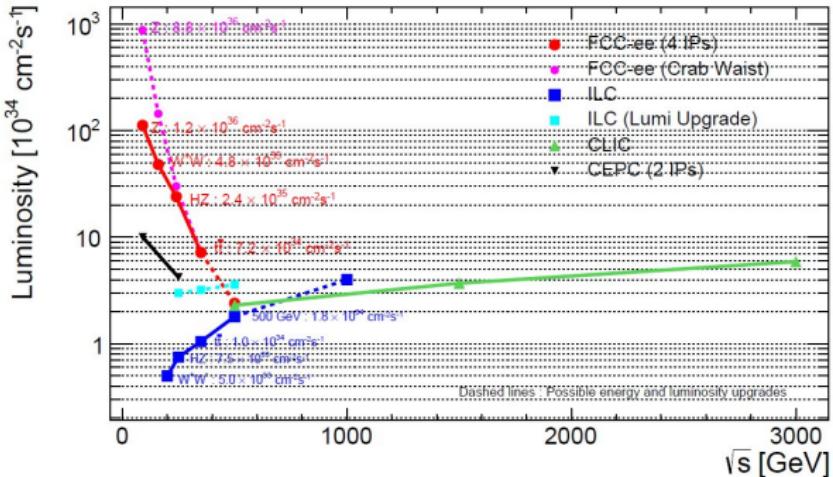


- Z pole run (90 GeV)
- WW threshold run (160 GeV)
- Higgs physics run (250 GeV)
- top threshold run (350 GeV)



- ▶ Z pole: production via s-channel  $Z$ , sensitive to  $|\theta|^2$ .
- ▶ At higher energies: t-channel  $W$ , production sensitive to  $|\theta_e|^2$ .
- ▶ At  $Z$  pole very large instantaneous luminosities are feasible.

# Luminosities at lepton colliders (old)



# Signatures for direct searches

Name	Final State	$ \theta , Z$ pole	$ \theta , \sqrt{s} > m_Z$
lepton-dijet	$\ell_\alpha \nu jj$	$ \theta_\alpha ^2$	$\frac{ \theta_e \theta_\alpha ^2}{\theta^2}$
mixed flavour dilepton	$\ell_\alpha \ell_\beta \nu \nu$	$ \theta_\alpha ^2$	$\frac{ \theta_e \theta_\alpha ^2}{\theta^2}$
same flavour dilepton	$\ell_\alpha \ell_\alpha \nu \nu$	$ \theta ^2$	$ \theta_e ^2$
dijet	$\nu \nu jj$	$ \theta ^2$	$ \theta_e ^2$
invisible	$\nu \nu \nu \nu$	$ \theta ^2$	$ \theta_e ^2$

S. Antusch, E. Cazzato and OF, Int. J. Mod. Phys. A 32 (2017) no.14, 1750078

- ▶ Measurement of LNV not straightforward.
- ▶ The dependency on the active-sterile mixing is determined by the center-of-mass energy, i.e. by the physics run.
- ▶ For masses below  $m_W$  lepton isolation becomes an issue.

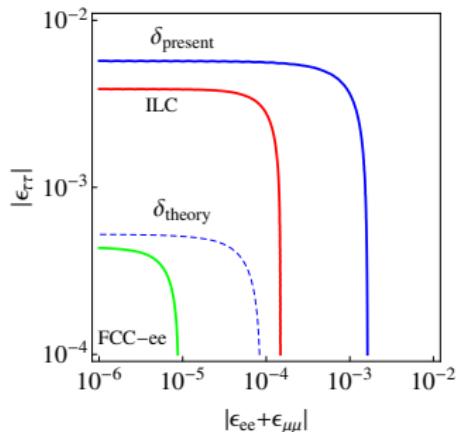
S. Dube, D. Gadkari and A. M. Thalapillil, arXiv:1707.00008 [hep-ph].

# Indirect Signatures: Electroweak precision tests

Observable	LEP precision	from CEPC preCDR
$M_W$ [MeV]	33	3
$\sin^2 \theta_W^{\text{eff}}$	0.07%	0.01%
$R_b$	0.3%	0.08%
$R_c$	0.3%	0.07%
$R_{\text{inv}}$	0.27%	$8.9 \times 10^{-4}$
$R_\ell$	0.1%	0.1%
$\Gamma_\ell$	0.1%	0.1%
$\sigma_h^0$ [nb]	$8.9 \times 10^{-4}$	$1 \times 10^{-4}$

FCC-ee: much more ambitious;

ILC: no strong  $Z$  pole program.



S. Antusch and OF, JHEP 1410 (2014) 094

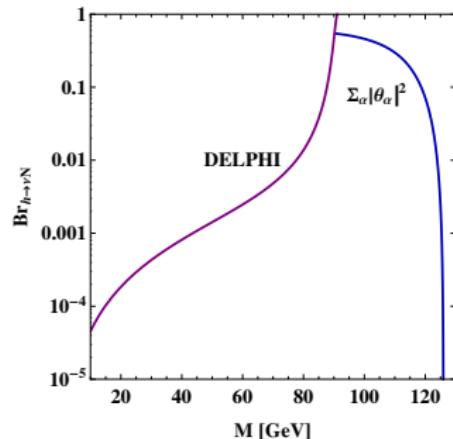
- ▶ Measuring the non-unitarity of the PMNS matrix.
- ▶ Improvement required:  $\delta_{\text{theory}}$  and  $\delta_{\text{syst}}$ .
- ▶ Not included: lepton universality tests of  $W$  decays
- ▶ Not included: rare charged LFV  $\ell$  decays

# Indirect signatures: Higgs boson properties

## Higgs boson branching ratios:

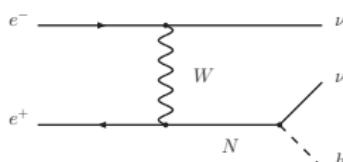
- ▶ New decay channel  $h \rightarrow \nu N$
- ▶ Large branching ratio possible, modified  $\text{Br}_{h \rightarrow \text{SM}}$
- ▶ Precision  $\text{Br}_{h \rightarrow WW} \sim 10^{-3}$

M. Ruan, Nucl. Part. Phys. Proc. 273-275, 857 (2016)



Antusch, OF, JHEP 1505 (2015) 053

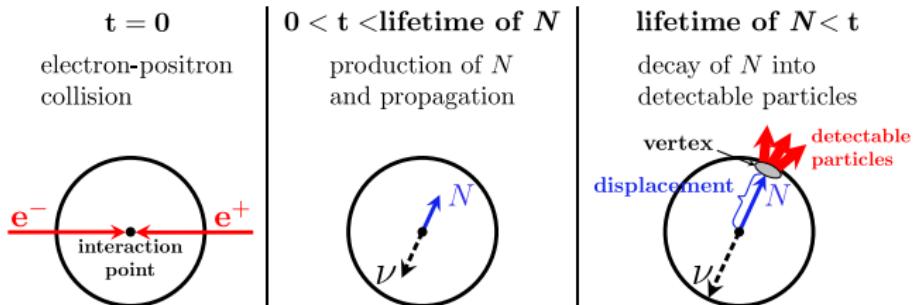
## Higgs production:



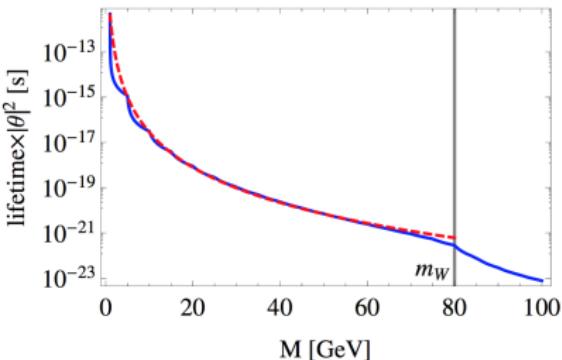
- ▶ Additional production mechanism at high energies.
- ▶ Enhanced mono-Higgs production cross section.

S. Antusch, OF, JHEP 1604 (2016) 189

# Exotic signature: displaced vertices I



- ▶ Lifetime  $\sim \mathcal{O}(1 - 100)$  ps.
- ▶ Assumption: no SM background for displacements  $> 0.1$  mm.
- ▶ Considered ILC detector SiD as benchmark.

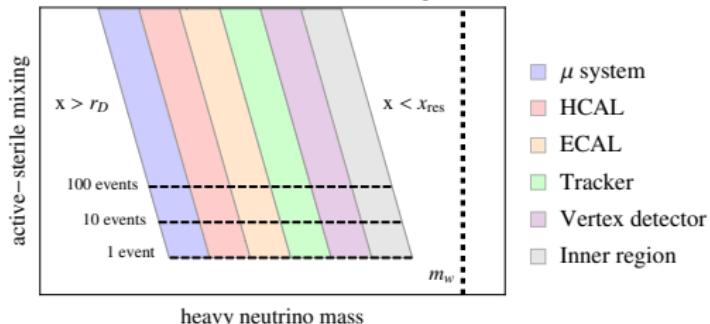


S. Antusch, E. Cazzato and OF, JHEP 1612, 007 (2016)

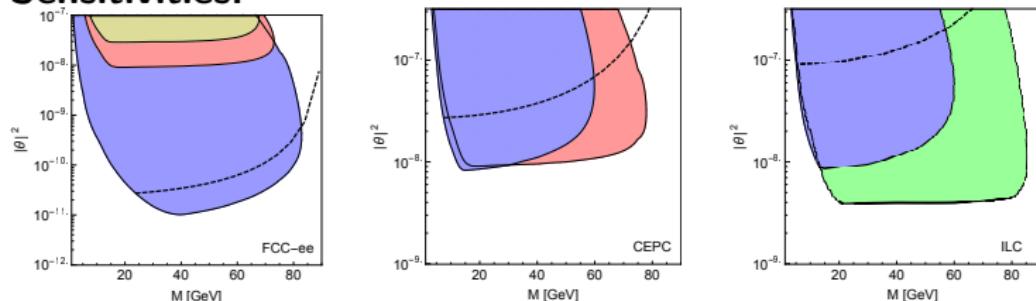
A. Blondel et al., Nucl. Part. Phys. Proc. 273-275 (2016) 1883

# Exotic signature: displaced vertices II

## Schematic of the detector components' sensitivities:

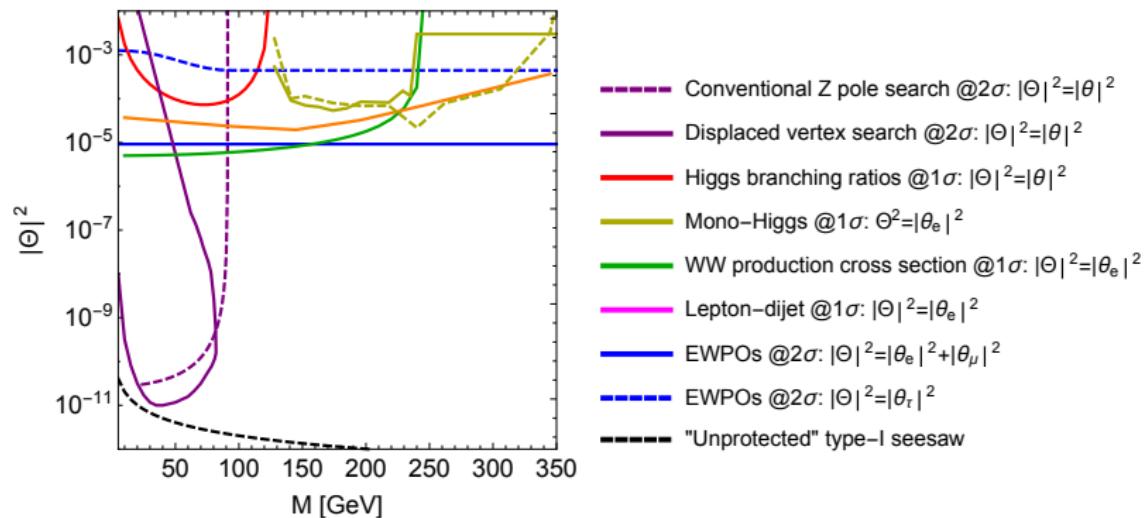


## Sensitivities:



■  $E_{\text{cm}} = m_Z$ ; ■  $E_{\text{cm}} = 250 \text{ GeV}$ ; ■  $E_{\text{cm}} = 350 \text{ GeV}$ ; ■  $E_{\text{cm}} = 500 \text{ GeV}$ ; — Conventional search (95% C.L.)

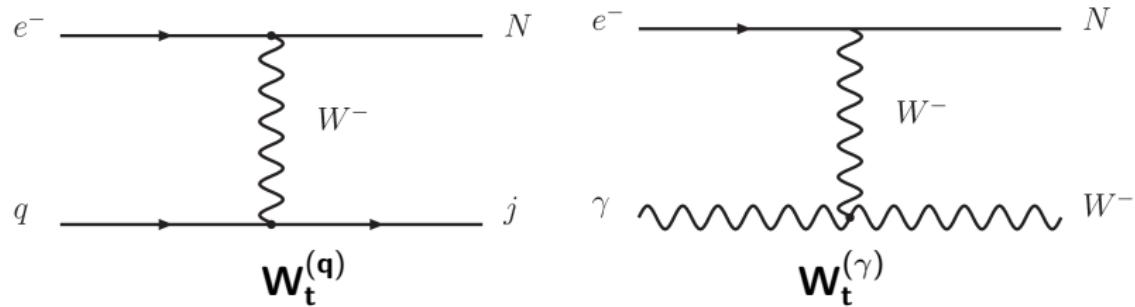
# Summary: FCC-ee sensitivities



- ▶ Displaced vertex searches test  $|\theta|^2 \sim 10^{-11}$  for  $M \leq m_W$ .
- ▶ EWPOs test  $|\theta|^2 \sim 10^{-5}$  up to  $M \sim 60$  TeV with  $\mathcal{O}(1)$  Yukawa couplings.

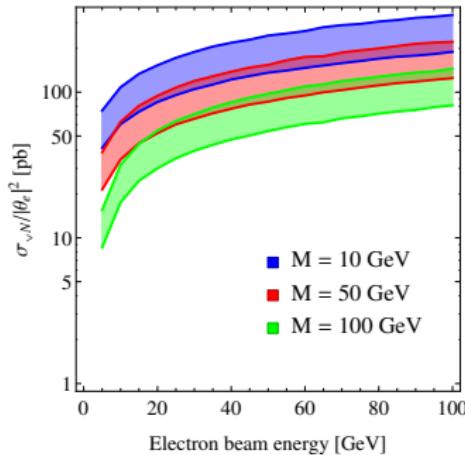
# Promising search strategies at electron-proton colliders

# Heavy neutrino production at electron-proton colliders

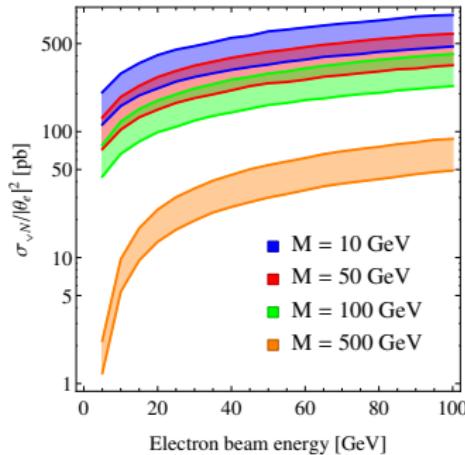


- ▶ Leading order production of heavy neutrino mass eigenstate.
- ▶  $W_t^{(q)}$ : dominant at lower center-of-mass energies.
- ▶  $W_t^{(\gamma)}$ : relevant for larger masses.

# Production cross sections ( $W_t^{(q)}$ )



LHeC



FCC-eh

For 60 GeV as benchmark for the electron beam  $E_e$ :

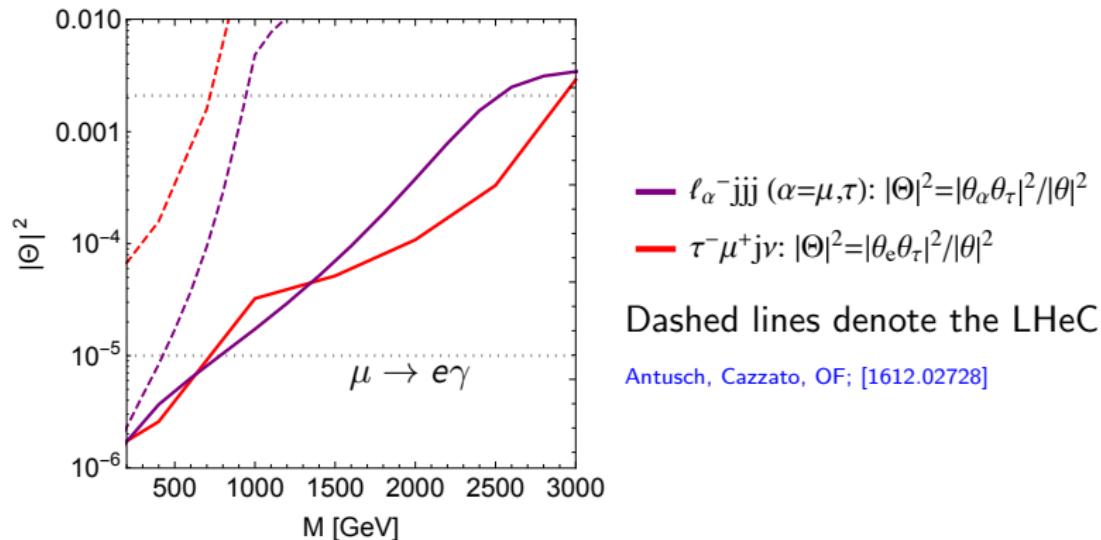
- ▶  $\sigma_{\nu N}$  increase of  $\sim 30\%$  for  $E_e \rightarrow 100$  GeV.
- ▶ Increased by  $\sim 80\%$  when including polarisation.
- ▶ Consider  $1 \text{ ab}^{-1}$  (for FCC-eh and LHeC).

# Signal channels from $\mathbf{W}_t^{(q)}$

Name	Final State	$ \theta_\alpha $ Dependency	LFV
lepton-trijet	$jjj\ell_\alpha^-$	$\frac{ \theta_e \theta_\alpha ^2}{\theta^2}$	✓
jet-dilepton	$j\ell_\alpha^- \ell_\beta^+ \nu$	$\frac{ \theta_e \theta_\alpha ^2 (*)}{\theta^2}$	✓
trijet	$jjj\nu$	$ \theta_e ^2$	✗
monojet	$j\nu\nu\nu$	$ \theta_e ^2$	✗

- ▶ LFV (and LNV) signature for  $\alpha \neq e$ ,  $\beta \neq \alpha$ , and  $\gamma \neq \alpha, \beta$
- ▶ Unambiguous lepton-number-violating final states, e.g.  $e^+ jjj$ .
- ▶ More (and more complex) signatures from  $\mathbf{W}_t^{(\gamma)}$ .

# Lepton-flavour-violating signatures

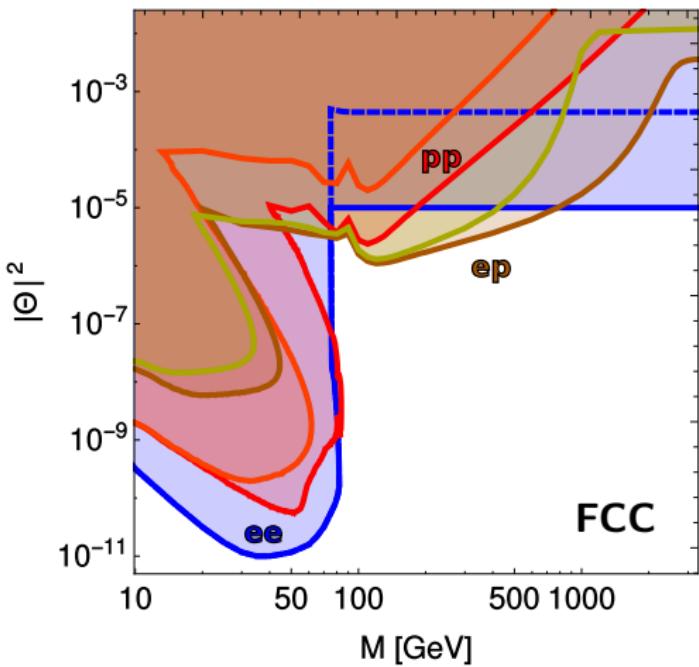


- ▶ Very sensitive tests of combinations  $|\theta_e \theta_\alpha|$ .
- ▶ Upper bounds:  
 $|\theta_e \theta_\mu|$  from  $\mu \rightarrow e\gamma$  (MEG);  $|\theta_e \theta_\tau|$  from precision data.
- ▶ Requires  $|\theta_\alpha| \gtrsim |\theta_e|$  for sizeable branching ratios.

# Sensitivities: summary

At one-sigma confidence level.

ep and pp at parton level



S. Antusch, E. Cazzato, OF; 1612.02728

The combination of ee with  $pp$  and  $ep$  colliders provides complementary tests for the neutrino mass mechanism.

# Summary

- ▶ Present constraints: active-sterile mixing  $|\theta|^2 \leq 10^{-3}$ .
- ▶ Search strategies at lepton colliders:
  - ★ **Direct:** lepton-dijet, dilepton, dijets
  - ★ **Higgs:** production and decay channels
  - ★ **EWPO:** measurement of PMNS non-unitarity
  - ★ **Displaced vertices:** best sensitivity for  $M < m_W$
  - ★ **pp/ep:** LFV dilepton-dijet/lepton-trijet  
          & displaced vertices
- ▶ **Synergy:** The combination of direct and indirect signatures at ee/ep will allow to test model specific predictions.
- ⇒ Testing the origin of neutrino masses.

# Conclusion

- ▶ CEPC CDR writeup this year
- ▶ FCC-ee CDR writeup next year
- ▶ ILC & LHeC seeking additional physics motivation  
(neutrinos NOT included yet)

Extremely important input  
for future collider projects!

**Thank you for your attention.**

## Backup I - EWPO

Experimental results and SM predictions for the EWPO, and the modification\*, to first order in the “non-unitarity” parameters

$\varepsilon_{\alpha\alpha} = \theta_\alpha^* \theta_\beta$ . (formulae for  $M \gg m_Z$ )

Prediction in MUV	SM Prediction	Experiment
$[R_\ell]_{\text{SM}} (1 - 0.15(\varepsilon_{ee} + \varepsilon_{\mu\mu}))$	20.744(11)	20.767(25)
$[R_b]_{\text{SM}} (1 + 0.03(\varepsilon_{ee} + \varepsilon_{\mu\mu}))$	0.21577(4)	0.21629(66)
$[R_c]_{\text{SM}} (1 - 0.06(\varepsilon_{ee} + \varepsilon_{\mu\mu}))$	0.17226(6)	0.1721(30)
$[\sigma_{had}^0]_{\text{SM}} (1 - 0.25(\varepsilon_{ee} + \varepsilon_{\mu\mu}) - 0.27\varepsilon_\tau) / \text{nb}$	41.470(15)	41.541(37)
$[R_{inv}]_{\text{SM}} (1 + 0.75(\varepsilon_{ee} + \varepsilon_{\mu\mu}) + 0.67\varepsilon_\tau)$	5.9723(10)	5.942(16)
$[M_W]_{\text{SM}} (1 - 0.11(\varepsilon_{ee} + \varepsilon_{\mu\mu})) / \text{GeV}$	80.359(11)	80.385(15)
$[\Gamma_{\text{lept}}]_{\text{SM}} (1 - 0.59(\varepsilon_{ee} + \varepsilon_{\mu\mu})) / \text{MeV}$	83.966(12)	83.984(86)
$[(s_{W,\text{eff}}^{\ell,\text{lep}})^2]_{\text{SM}} (1 + 0.71(\varepsilon_{ee} + \varepsilon_{\mu\mu}))$	0.23150(1)	0.23113(21)
$[(s_{W,\text{eff}}^{\ell,\text{had}})^2]_{\text{SM}} (1 + 0.71(\varepsilon_{ee} + \varepsilon_{\mu\mu}))$	0.23150(1)	0.23222(27)

\* Minimal Unitarity Violation scheme: [Antusch et al.; JHEP 0610 \(2006\) 084.](#)

## Backup II - lepton universality

Modification due to sterile neutrinos (formulae for  $M \gg m_Z$ ):

$$R_{\alpha\beta} = \sqrt{\frac{(NN^\dagger)_{\alpha\alpha}}{(NN^\dagger)_{\beta\beta}}} \simeq 1 + \frac{1}{2} (\varepsilon_{\alpha\alpha} - \varepsilon_{\beta\beta}) .$$

	Process	Bound		Process	Bound
$R_{\mu e}^\ell$	$\frac{\Gamma(\tau \rightarrow \nu_\tau \mu \bar{\nu}_\mu)}{\Gamma(\tau \rightarrow \nu_\tau e \bar{\nu}_e)}$	1.0018(14)	$R_{\mu e}^\pi$	$\frac{\Gamma(\pi \rightarrow \mu \bar{\nu}_\mu)}{\Gamma(\pi \rightarrow e \bar{\nu}_e)}$	1.0021(16)
$R_{\tau \mu}^\ell$	$\frac{\Gamma(\tau \rightarrow \nu_\tau e \bar{\nu}_e)}{\Gamma(\mu \rightarrow \nu_\mu e \bar{\nu}_e)}$	1.0006(21)	$R_{\tau \mu}^\pi$	$\frac{\Gamma(\tau \rightarrow \nu_\tau \pi)}{\Gamma(\pi \rightarrow \mu \bar{\nu}_\mu)}$	0.9956(31)
$R_{e \mu}^W$	$\frac{\Gamma(W \rightarrow e \bar{\nu}_e)}{\Gamma(W \rightarrow \mu \bar{\nu}_\mu)}$	1.0085(93)	$R_{\tau \mu}^K$	$\frac{\Gamma(\tau \rightarrow K \nu_\tau)}{\Gamma(K \rightarrow \mu \bar{\nu}_\mu)}$	0.9852(72)
$R_{\tau \mu}^W$	$\frac{\Gamma(W \rightarrow \tau \bar{\nu}_\tau)}{\Gamma(W \rightarrow \mu \bar{\nu}_e)}$	1.032(11)	$R_{\tau e}^K$	$\frac{\Gamma(\tau \rightarrow K \nu_\tau)}{\Gamma(K \rightarrow e \bar{\nu}_e)}$	1.018(42)

## Backup III - CKM unitarity constraint

Current world averages:  $V_{ud} = 0.97427(15)$ ,  $V_{ub} = 0.00351(15)$

$$|V_{ij}^{th}|^2 = |V_{ij}^{exp}|^2(1 + f^{\text{process}}(\varepsilon_{\alpha\alpha})) ,$$

$$|V_{ud}^{th}|^2 = |V_{ud}^{exp,\beta}|^2(NN^\dagger)_{\mu\mu} .$$

For the kaon decay processes we have:

$$|V_{us}^{th}|^2 = |V_{us}^{exp,K \rightarrow e}|^2(NN^\dagger)_{\mu\mu} ,$$

$$|V_{us}^{th}|^2 = |V_{us}^{exp,K \rightarrow \mu}|^2(NN^\dagger)_{ee} .$$

Process	$V_{us}f_+(0)$
$K_L \rightarrow \pi e\nu$	0.2163(6)
$K_L \rightarrow \pi \mu\nu$	0.2166(6)
$K_S \rightarrow \pi e\nu$	0.2155(13)
$K^\pm \rightarrow \pi e\nu$	0.2160(11)
$K^\pm \rightarrow \pi \mu\nu$	0.2158(14)
Average	0.2163(5)

Processes involving tau leptons:

Process	$f^{\text{process}}(\varepsilon)$	$ V_{us} $
$\frac{B(\tau \rightarrow K\nu)}{B(\tau \rightarrow \pi\nu)}$	$\varepsilon_{\mu\mu}$	0.2262(13)
$\tau \rightarrow K\nu$	$\varepsilon_{ee} + \varepsilon_{\mu\mu} - \varepsilon_{\tau\tau}$	0.2214(22)
$\tau \rightarrow \ell, \tau \rightarrow s$	$0.2\varepsilon_{ee} - 0.9\varepsilon_{\mu\mu} - 0.2\varepsilon_{\tau\tau}$	0.2173(22)

## Backup IV - lepton flavour violation

- ▶ Present experimental limits at 90% C.L.:

Process	MUV Prediction	Bound	Constraint on $ \varepsilon_{\alpha\beta} $
$\mu \rightarrow e\gamma$	$2.4 \times 10^{-3}  \varepsilon_{\mu e} ^2$	$5.7 \times 10^{-13}$	$\varepsilon_{\mu e} < 1.5 \times 10^{-5}$
$\tau \rightarrow e\gamma$	$4.3 \times 10^{-4}  \varepsilon_{\tau e} ^2$	$1.5 \times 10^{-8}$	$\varepsilon_{\tau e} < 5.9 \times 10^{-3}$
$\tau \rightarrow \mu\gamma$	$4.1 \times 10^{-4}  \varepsilon_{\tau\mu} ^2$	$1.8 \times 10^{-8}$	$\varepsilon_{\tau\mu} < 6.6 \times 10^{-3}$

- ▶ Estimated sensitivities of planned experiments at 90% C.L.:

Process	MUV Prediction	Bound	Sensitivity
$Br_{\tau e}$	$4.3 \times 10^{-4}  \varepsilon_{\tau e} ^2$	$10^{-9}$	$\varepsilon_{\tau e} \geq 1.5 \times 10^{-3}$
$Br_{\tau \mu}$	$4.1 \times 10^{-4}  \varepsilon_{\tau\mu} ^2$	$10^{-9}$	$\varepsilon_{\tau\mu} \geq 1.6 \times 10^{-3}$
$Br_{\mu eeee}$	$1.8 \times 10^{-5}  \varepsilon_{\mu e} ^2$	$10^{-16}$	$\varepsilon_{\mu e} \geq 2.4 \times 10^{-6}$
$R_{\mu e}^{Ti}$	$1.5 \times 10^{-5}  \varepsilon_{\mu e} ^2$	$2 \times 10^{-18}$	$\varepsilon_{\mu e} \geq 3.6 \times 10^{-7}$

$\Rightarrow R_{\mu e}^{Ti}$  yields a sensitivity to  $m_{\nu_R}$  up to 0.3 PeV.

## Backup V - precision estimates for EWPOs

Observable	ILC	FCC-ee	CEPC	CEPC*
$R_\ell$	0.004	0.001	0.01	0.003*
$R_{inv}$	0.01	0.002	0.012	0.006*
$R_b$	0.0002	0.00002	0.00017	0.00007*
$M_W$ [MeV]	2.5	0.5	0.5	0.5
$s_{eff}^{2,\ell}$	$1.3 \times 10^{-5}$	$1 \times 10^{-6}$	$2.3 \times 10^{-5}$	$3.3 \times 10^{-6}$ *
$\sigma_h^0$ [nb]	0.025	0.0025	n.a.	0.008*
$\Gamma_\ell$ [MeV]	0.042	0.0042	n.a.	0.014*
Reference	1310.6708	1308.6176	Ruan (2014) <sup>†</sup>	scaled*

† Private communication.

\* Assumption: CEPC produces  $10^{11}$   $Z$  bosons, compared to the  $10^{12}$   $Z$  bosons @FCC-ee.

⇒ Uncertainties scaled:  $\delta_{CEPC} = \delta_{FCC-ee} \times \sqrt{10}$ .