Bubble Wall Velocities

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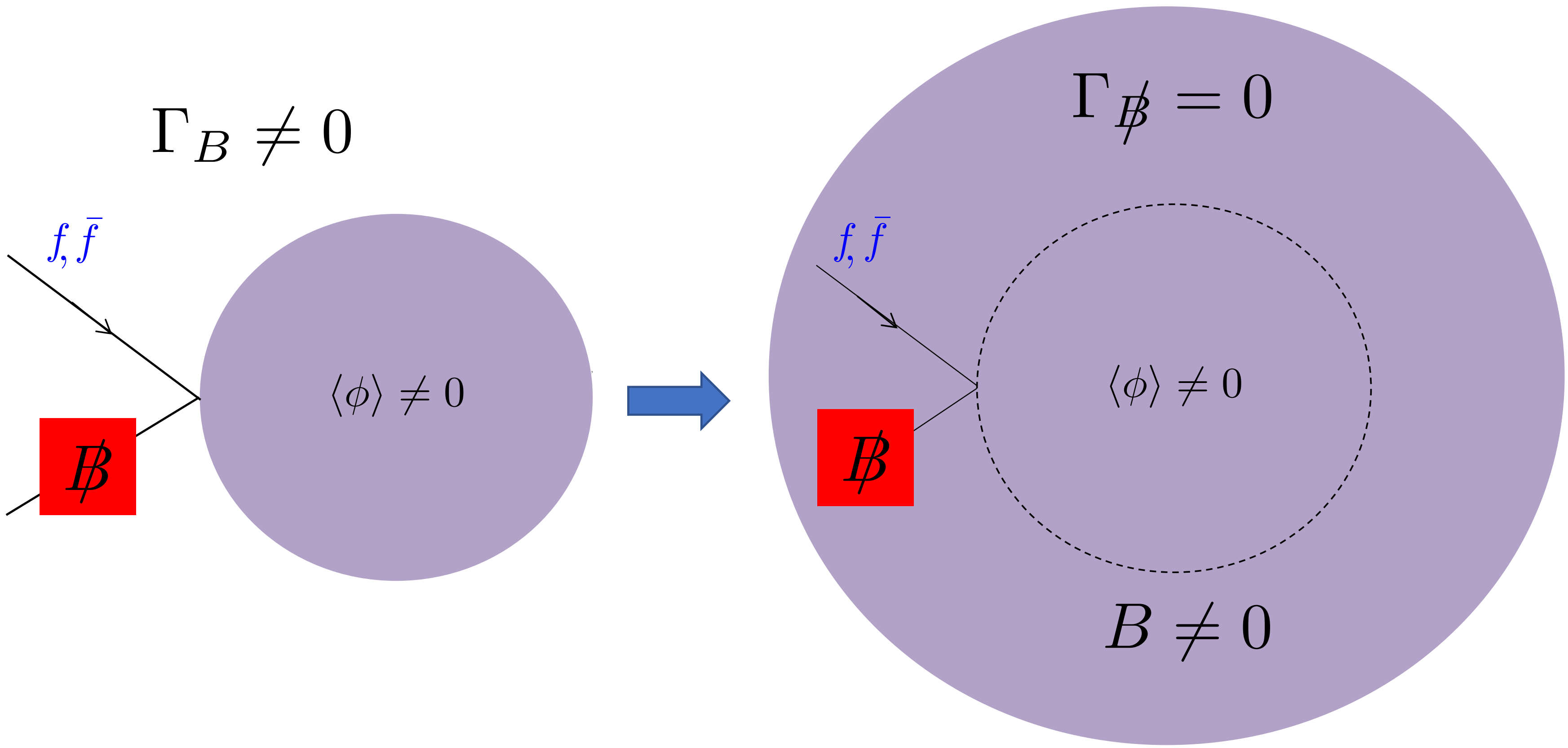
Why should we care about bubble wall dynamics?

Electroweak baryogenesis



Bubble wall catches up with diffusing current, freezing in B

Electroweak baryogenesis

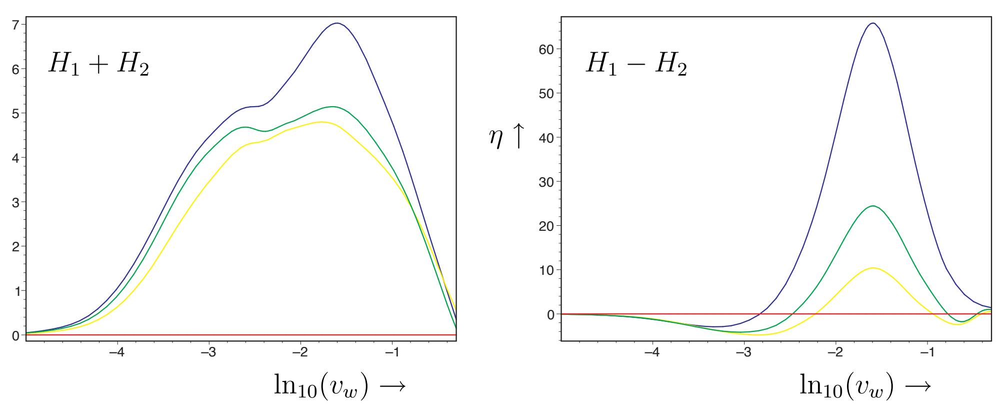
Requiring sufficient B-violation during diffusion requires relatively slow bubble walls

Wall velocities conventionally required to be subsonic: $\mathbf{v_w<0.58}$

(subtle and there are exceptions; see e.g. No, 2011; Caprini + No, 2011; Katz+Riotto, 2016)

Resulting asymmetry can strongly depend on v_w, depending on the form of the primary CP-violating source

Huber et al, 2001



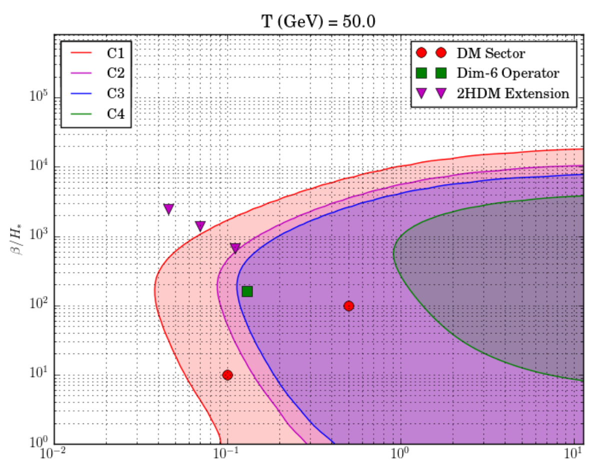
Gravitational waves

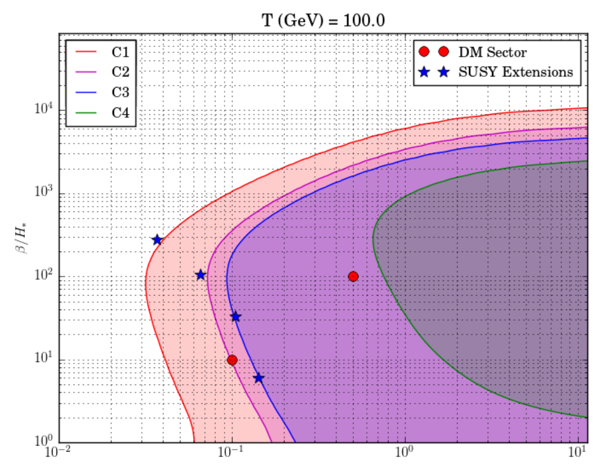
Spectrum, and prospects for detection, depend on the wall velocity

$$h^2\Omega_{\rm env}(f) \;=\; 1.67\times 10^{-5}\left(\frac{H_*}{\beta}\right)^2\left(\frac{\kappa\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\left(\frac{0.11\,v_w^3}{0.42+v_w^2}\right)S_{\rm env}(f)$$

$$\kappa_v \simeq \begin{cases} \alpha\,(0.73+0.083\sqrt{\alpha}+\alpha)^{-1} & v_w\sim 1 \\ v_w^{6/5}6.9\,\alpha\,(1.36-0.037\sqrt{\alpha}+\alpha)^{-1}\,, & v_w\lesssim 0.1 \end{cases}$$

Caprini et al, 2015





Guiding questions

How do we compute the wall velocity?

What are the theoretical challenges involved in these calculations?

Master equation

Scalar field EOM for one scalar D.O.F. at finite T:

$$\Box\phi + V'(\phi) + \sum \frac{dm^2}{d\phi} \int \frac{d^3k}{(2\pi)^3 2E} f(k,z) = 0$$

See e.g. Moore+Prokopec, 1996; Konstandin et al, 2014

T=0 effective potential

Distribution functions of all particles coupled to Higgs

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Rewrite in terms of finite-T effective potential:

$$\Box\phi + V'(\phi, T) + \sum \frac{dm^2}{d\phi} \int \frac{d^3k}{(2\pi)^3 2E} \delta f(k,z) = 0$$

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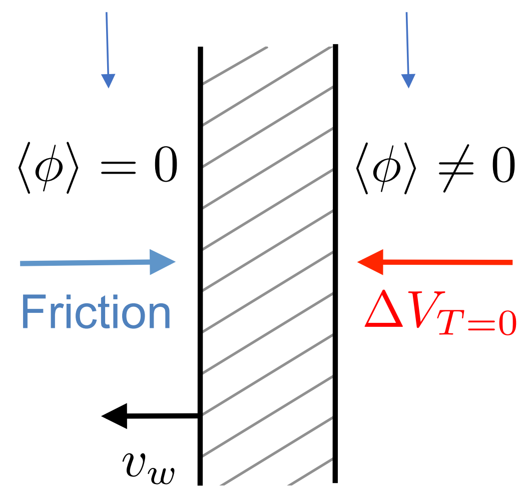
Non-accelerating wall: $\boxed{\Delta V(T) = -\int \sum \frac{dm^2}{d\phi}\int \frac{d^3k}{(2\pi)^3 2E} \delta f(k,z)\phi'(z)\, dz}$

Master equation

Boils down to drawing a free body diagram

$$\boxed{\int \sum \frac{dm^2}{d\phi} \int \frac{d^3k}{(2\pi)^3\, 2E} \delta f(k,z) \phi'(z) dz = -\Delta V(T)}$$



Two questions:

$\exists$ enough friction to stop the wall from accelerating?

If so, what is the terminal velocity and bubble profile satisfying the master equation above?

"Runaway Bubbles"

Is there enough friction to stop the wall from accelerating once it's moving ultra-relativistically? Bodeker + Moore, 2009

$$\Delta V_{\text{vac}} = -\int \sum \frac{dm^2}{d\phi} \int \frac{d^3k}{(2\pi)^3\, 2E} f(k,z)\phi'(z)dz$$

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For $\gamma >> 1$:

Equilibrium distributions

$$\int \sum \frac{dm^2}{d\phi} \int \frac{d^3k}{(2\pi)^3\, 2E} f(k,z)\phi'(z)dz \simeq \sum \left[m_i^2(h_2) - m_i^2(h_1)\right] \int \frac{dm^2}{d\phi} \int \frac{d^3k}{(2\pi)^3\, 2E} f(k)|_{h_1}$$

High-T expansion (m/T<<1)

$$\approx \sum a_i T^2 \left[m_i^2(h_2) - m_i^2(h_1)\right] + \mathcal{O}(1/\gamma^2)$$

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High-T expansion (m/T<<1)

$$\approx \sum a_i T^2 \left[m_i^2(h_2) - m_i^2(h_1)\right] + \mathcal{O}(1/\gamma^2)$$

$$\Delta V_{\rm vac} < -\int \sum \frac{dm^2}{d\phi} \int \frac{d^3k}{(2\pi)^3\, 2E} f(k,z)\phi'(z)dz$$ $\Longrightarrow$ vacuum energy difference overwhelms the friction

"Runaway Bubbles"

$$\Delta V_{\rm vac} + \sum a_i T^2 [m_i^2(h_2) - m_i^2(h_1)] < 0 \Longrightarrow \text{Runaway}$$

Bodeker + Moore, 2009

In the high-T approximation, the difference between vacua of the finite-T effective potential is given by

$$\Delta V(T) \approx \Delta V_{\rm vac} + \sum a_i T^2 \left[m_i^2(h_2) - m_i^2(h_1)\right] - \sum b_i T \left[m_i^3(h_2) - m_i^3(h_1)\right]$$

Runaway condition can be interpreted in terms of finite-T effective potential with no cubic term:

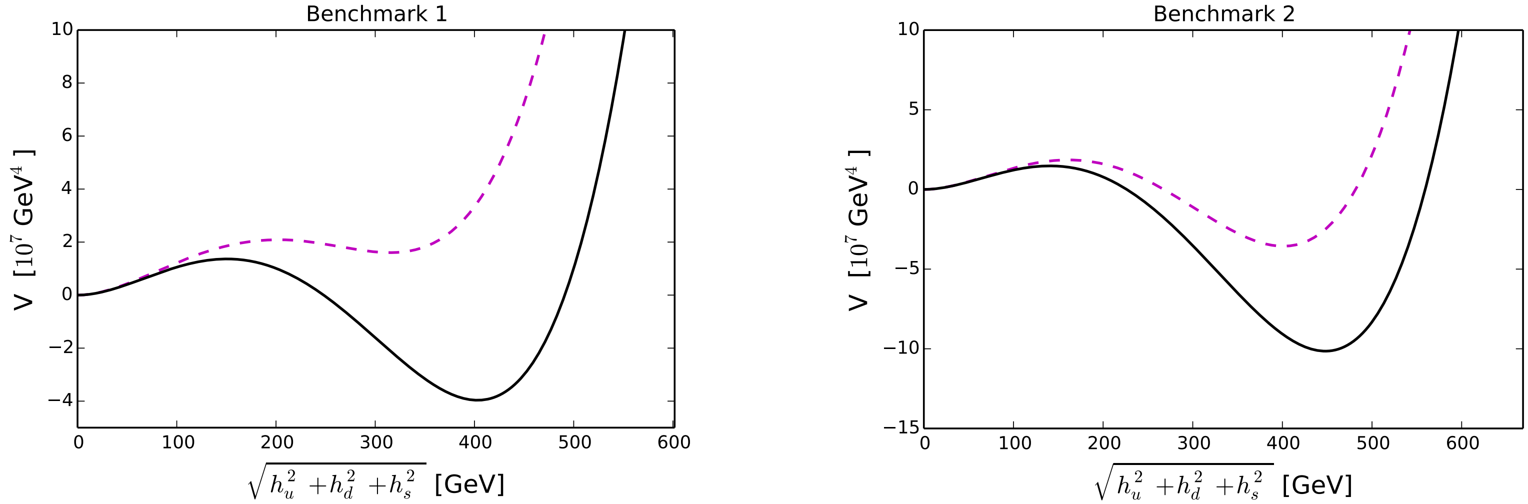
If, after dropping thermal cubic terms, it is energetically favorable to tunnel to the broken phase, the bubble can run away

"Runaway Bubbles"

$$\Delta V_{\rm vac} + \sum a_i T^2 [m_i^2(h_2) - m_i^2(h_1)] < 0 \Longrightarrow \text{Runaway}$$

Bodeker + Moore, 2009

From JK et al, 2014 for the NMSSM



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Some comments

Important (and simple) criterion to check when doing pheno studies

Theories with tree-level cubic terms (e.g. singlet models) are especially susceptible to runaways

Recent progress and outstanding theoretical questions associated with friction in the ultra-relativistic limit: Bodeker+Moore, 2017

-NLO (e.g. 1 $\rightarrow$ 2) processes enhance the friction, tend to prevent runaway! Important for gravitational wave spectra

-Effect is dominated by soft emission (in the wall frame). Full calculation is challenging

-Sphalerons behind the wall?!

Beyond runaway

What about non-relativistic walls? How do we get a number out?

Some references:

Moore + Prokopec, 1995 & 1996
John + Schmidt, 2000
Konstandin et al, 2014
JK, 2016

Beyond runaway

Ultimate goal: find a wall velocity and bubble profile that solves the scalar field EOMs

$$-(1-v_w^2)\phi_i'' + \frac{\partial V(\phi_i)}{\partial \phi_i} + \sum_j \frac{\partial m_j^2(\phi_i)}{\partial \phi_i} \int \frac{d^3p}{(2\pi)^3 2E_j} f_j(p,x) = 0$$

(in the wall frame, neglecting sphericity, assuming stationary solution)

Simplification: look for configurations satisfying constraint equations

$$\int \left[-(1-v_w^2)\vec{\phi}'' + \nabla_\phi V(\phi_i) + \sum_j \nabla_\phi m_j^2(\phi_i) \int \frac{d^3p}{(2\pi)^3 2E_j} f_j(p,x) \right] \cdot \frac{d\vec{\phi}}{dx}\, dx = 0$$

(Vanishing pressure)

$$\int \left[-(1-v_w^2)\vec{\phi}'' + \nabla_\phi V(\phi_i) + \sum_j \nabla_\phi m_j^2(\phi_i) \int \frac{d^3p}{(2\pi)^3 2E_j} f_j(p,x) \right] \cdot \frac{d^2\vec{\phi}}{dx^2}\, dx = 0$$

(Vanishing pressure gradient)

Wall velocities with singlets

Illustrate in the real singlet extension of SM

$$-(1-v_w^2)\phi_i'' + \frac{\partial V(\phi_i)}{\partial \phi_i} + \sum_j \frac{\partial m_j^2(\phi_i)}{\partial \phi_i} \int \frac{d^3p}{(2\pi)^3 2E_j} f_j(p,x) = 0$$

First, write down Boltzmann equations for distributions

$$\frac{d}{dt} f_i \equiv \left(\frac{\partial}{\partial t} + \dot{z}\frac{\partial}{\partial z} + \dot{p_z}\frac{\partial}{\partial p_z} \right) f_i = -C[f]_i$$

$$C[f]_i = \frac{1}{2N_i} \sum_{jmn} \frac{1}{2E_p} \int \frac{d^3k d^3p' d^3k'}{(2\pi)^9 2E_k 2E_{p'} 2E_{k'}} \left| \mathcal{M}_{ij\to mn}(p,k;p',k') \right|^2 (2\pi)^4 \delta(p+k-p'-k') \\ \times \mathcal{P}_{ij\to mn} \left[f_i(p), f_j(k), f_m(p'), f_n(k') \right]$$

Utilize effective kinetic theory for excitations: $E \gg \frac{1}{L_w}$, $p \gtrsim gT$

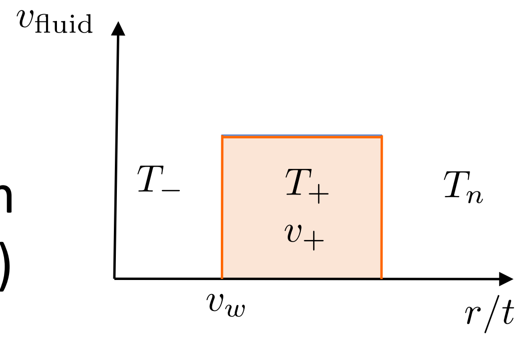
Infrared modes with $p \ll T$ (e.g. for gauge bosons) dealt with separately

Wall velocities with singlets

Classify particles depending on strength of interaction with the condensate:

Top quarks, $SU(2)_L$ gauge bosons, Higgs and singlet fields "feel" the passage of the wall the most; dominate the contribution in EOM

All other particles treated as in local thermal equilibrium at common (space-time dependent) temperature and fluid velocity (determined from bulk properties of the PT: see Jose Miguel's talk!)



Ansatz for relevant distributions:

$$f_a = \left(e^{(E+\delta_a)/T} \pm 1\right)^{-1} \quad , \quad \delta_j = -\mu_j - \frac{E}{T}(\delta T_j + \delta T_{\rm bg}) - p_z(\delta v_j + v_{\rm bg})$$

$(\mu_j/T,\ \delta T_j/T,\ \delta T_{\rm bg}/T,\ \delta v_j,\ v_{\rm bg}\ \ll\ 1)$

Wall velocities with singlets

Take moments of the Boltzmann equations

$$c_2^i \frac{\partial}{\partial t}\mu_i + c_3^i \frac{\partial}{\partial t}(\delta T_i + \delta T_{\rm bg}) + \frac{c_3^i T}{3}\frac{\partial}{\partial z}(\delta v_i + v_{\rm bg}) + \int \frac{d^3p}{(2\pi)^3 T^2} C[f]_i = \frac{c_1^i}{2T}\frac{\partial m_i^2}{\partial t}$$

$$c_3^i \frac{\partial}{\partial t}\mu_i + c_4^i \frac{\partial}{\partial t}(\delta T_i + \delta T_{\rm bg}) + \frac{c_4^i T}{3}\frac{\partial}{\partial z}(\delta v_i + v_{\rm bg}) + \int \frac{E d^3p}{(2\pi)^3 T^3} C[f]_i = \frac{c_2^i}{2T}\frac{\partial m_i^2}{\partial t}$$

$$\frac{c_3^i}{3}\frac{\partial}{\partial z}\mu_i + \frac{c_4^i}{3}\frac{\partial}{\partial z}(\delta T_i + \delta T_{\rm bg}) + \frac{c_4^i T}{3}\frac{\partial}{\partial t}(\delta v_i + v_{\rm bg}) + \int \frac{p_z d^3p}{(2\pi)^3 T^3} C[f]_i = 0$$

$$\sum c_4 \left(\frac{\partial}{\partial t}\delta T_{\rm bg} + \frac{c_4 T}{3}\frac{\partial}{\partial z}v_{\rm bg}\right) + \int \frac{E d^3p}{(2\pi)^3 T^3} C[f]_{\rm bg} = 0$$

$$\sum \frac{c_4}{3}\left(\frac{\partial}{\partial z}\delta T_{\rm bg} + T\frac{\partial}{\partial t}v_{\rm bg}\right) + \int \frac{p_z d^3p}{(2\pi)^3 T^3} C[f]_{\rm bg} = 0$$

Obtain collision terms from interactions of the various species in the plasma

$$\int \frac{d^3p}{(2\pi)^3 T^2} C[f]_i \equiv \sum_j \left(\delta\mu_j \Gamma^i_{\mu 1,j} + \delta T_i \Gamma^i_{T_1,j}\right)$$

$$\int \frac{d^3p}{(2\pi)^3 T^3} E_i C[f]_i \equiv \sum_j \left(\delta\mu_j \Gamma^i_{\mu 2,j} + \delta T_i \Gamma^i_{T_2,j}\right)$$

$$\int \frac{d^3p}{(2\pi)^3 T^4} p_{z,i} C[f]_i \equiv \sum_j \left(\delta v_j \Gamma^i_{v_1,j}\right)$$

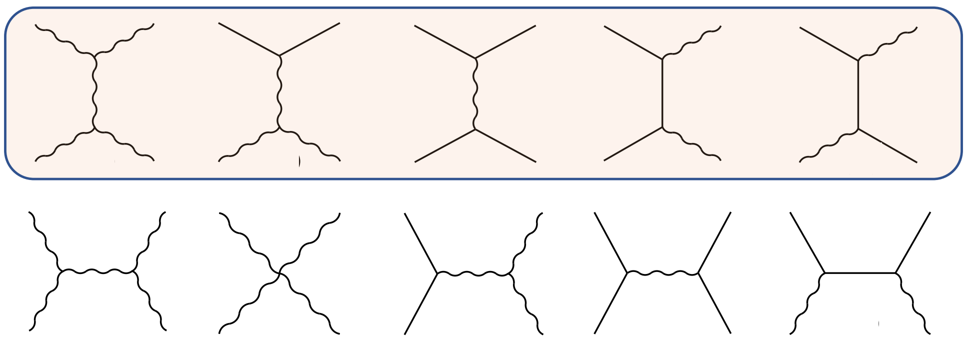
$$\int \frac{d^3p}{(2\pi)^3 T^3} E_i C[f]_{\rm bg} \equiv -\sum_j \left(\delta\mu_j \widetilde{\Gamma}_{\mu 2,j} + \delta T_i \widetilde{\Gamma}_{T_2,j}\right)$$

$$\int \frac{d^3p}{(2\pi)^3 T^4} p_{z,i} C[f]_{\rm bg} \equiv -\sum_j \left(\delta v_j \widetilde{\Gamma}_{v_1,j}\right)$$

Interaction rates

For the top quarks and Higgs bosons, work in a formal leading log, small coupling/high-temperature expansion, including hard thermal loops in internal propagators

t/*u*-channel diagrams lead to IR divergences cut off by thermal masses; yield parametrically "large" logarithms $\sim \log(\#/g^2)$. Only keep these contributions



Leading-log in QCD-like gauge theory

Arnold, Moore, Yaffe, 2000-2003

Systematically drop all terms of $\mathcal{O}(m^2/T^2)$. Then calculate in the gauge basis and treat external modes as massless

Resum the hard thermal loops for the *t*/*u*-channel propagators

Keep only processes of $\mathcal{O}(\alpha_s^2)$, $\mathcal{O}(\alpha_s\alpha_t)$, $\mathcal{O}(\alpha_t^2)$

Interaction rates

Leading log vacuum matrix elements

Process	$|\mathcal{M}|^2_{\text{tot}}$	Internal Propagator
$\underline{\mathcal{O}(g_3^4)}$:		
$t\bar{t} \leftrightarrow gg$:	$\frac{128}{3} g_3^4 \left(\frac{u}{t} + \frac{t}{u}\right)$	t
$tg \leftrightarrow tg$:	$-\frac{128}{3} g_3^4 \frac{s}{u} + 96 g_3^4 \frac{s^2+u^2}{t^2}$	g, t
$tq(\bar{q}) \leftrightarrow tq(\bar{q})$:	$160 g_3^4 \frac{u^2+s^2}{t^2}$	g
$\underline{\mathcal{O}(y_t^2 g_3^2)}$:		
$t\bar{t} \leftrightarrow hg,\ \phi^0 g$:	$8 y_t^2 g_3^2 \left(\frac{u}{t} + \frac{t}{u}\right)$	t
$t\bar{b} \leftrightarrow \phi^+ g$:	$8 y_t^2 g_3^2 \left(\frac{u}{t} + \frac{t}{u}\right)$	$t,\ b$
$tg \leftrightarrow th,\ t\phi^0$:	$-8 y_t^2 g_3^2 \frac{s}{t}$	t
$tg \leftrightarrow b\phi^+$:	$-8 y_t^2 g_3^2 \frac{s}{t}$	b
$t\phi^- \leftrightarrow bg$:	$-8 y_t^2 g_3^2 \frac{s}{t}$	t
$\underline{\mathcal{O}(y_t^4)}$:		
$t\bar{t} \leftrightarrow hh,\ \phi^0\phi^0$:	$\frac{3}{2} y_t^4 \left(\frac{u}{t} + \frac{t}{u}\right)$	t
$t\bar{t} \leftrightarrow \phi^+\phi^-$:	$3 y_t^4 \frac{u}{t}$	b
$t\bar{t} \leftrightarrow h\phi^0$:	$\frac{3}{2} y_t^4 \left(\frac{u}{t} + \frac{t}{u}\right)$	t
$t\bar{b} \leftrightarrow h\phi^+,\ \phi^0\phi^+$:	$\frac{3}{2} y_t^4 \frac{u}{t}$	t
$th,\ t\phi^0 \leftrightarrow ht,\ \phi^0 t$:	$-\frac{3}{2} y_t^4 \frac{s}{t}$	t
$t\phi^- \leftrightarrow hb,\ \phi^0 b$:	$\frac{3}{2} y_t^4 \frac{u}{t}$	t
$t\phi^+ \leftrightarrow \phi^+ t$:	$3 y_t^4 \frac{u}{t}$	b

Hard thermal loop dressing:

$$\frac{u}{t} \simeq -\frac{s}{t} \to \frac{4\,\mathrm{Re}(p\cdot\widetilde{q}\,k\cdot\widetilde{q}^* + s\widetilde{q}\cdot\widetilde{q}^*)}{|\widetilde{q}\cdot\widetilde{q}|^2}$$

$$\widetilde{q}^\mu = p^\mu - p'^\mu - \Sigma^\mu(p-p')$$

$$\Sigma^0(q) = \frac{m_f^2(T)}{2\,|\mathbf{q}|}\left(\log\frac{|\mathbf{q}|+q^0}{|\mathbf{q}|-q^0} - i\pi\right)$$

$$\boldsymbol{\Sigma}(q) = \frac{-m_f^2(T)\,\widehat{\mathbf{q}}}{|\mathbf{q}|}\left(1 + i\pi - \frac{q^0}{2\,|\mathbf{q}|}\log\frac{|\mathbf{q}|+q^0}{|\mathbf{q}|-q^0}\right)$$

$$\frac{s^2+u^2}{t^2} \to \frac{1}{2}\left(1 + \left|D_{\mu\nu}(p-p')(p+p')^\mu(k+k')^\nu\right|^2\right)$$

$$D_{00}(q) = \frac{-1}{|\mathbf{q}|^2 + \Pi_{00}(q,T)}$$

$$D_{ij}(q) = \frac{\delta_{ij} - \widehat{\mathbf{q}}_i\widehat{\mathbf{q}}_j}{q^2 + \Pi_T(q,T)}$$

$$D_{i0}(q) = D_{i0}(q) = 0.$$

$$\Pi_{00}(q) = m_{\mathrm{D}}^2(T)\left(1 - \frac{q^0}{2\,|\mathbf{q}|}\log\frac{|\mathbf{q}|+q^0}{|\mathbf{q}|-q^0} + \frac{i\pi q^0}{2\,|\mathbf{q}|}\right)$$

$$\Pi_T(q) = m_{\mathrm{D}}^2(T)\left[\frac{q^0}{2\,|\mathbf{q}|} + \frac{q^0 q^2}{4\,|\mathbf{q}|^3}\left(\log\frac{|\mathbf{q}|+q^0}{|\mathbf{q}|-q^0} - i\pi\right]\right)$$

Interaction rates

Results for the tops and Higgses

$$\Gamma^h_{\mu 1,h} \simeq (1.1\times 10^{-3} g_3^2 y_t^2 + 6.0\times 10^{-4} y_t^4)T$$
$$\Gamma^h_{T1,h} \simeq \Gamma^h_{\mu 2,h} \simeq (2.5\times 10^{-3} g_3^2 y_t^2 + 1.4\times 10^{-3} y_t^4)T$$
$$\Gamma^h_{T2,h} \simeq (8.6\times 10^{-3} g_3^2 y_t^2 + 4.8\times 10^{-3} y_t^4)T$$
$$\Gamma^h_{v,h} \simeq (3.5\times 10^{-3} g_3^2 y_t^2 + 1.8\times 10^{-3} y_t^4)T,$$

$$\Gamma^t_{\mu 1,t} \simeq (5.0\times 10^{-4} g_3^4 + 5.8\times 10^{-4} g_3^2 y_t^2 + 1.5\times 10^{-4} y_t^4)T$$
$$\Gamma^t_{T1,t} \simeq \Gamma^t_{\mu 2,t} \simeq (1.2\times 10^{-3} g_3^4 + 1.4\times 10^{-3} g_3^2 y_t^2 + 3.6\times 10^{-4} y_t^4)T$$
$$\Gamma^t_{T2,t} \simeq (1.1\times 10^{-2} g_3^4 + 4.6\times 10^{-3} g_3^2 y_t^2 + 1.1\times 10^{-3} y_t^4)T$$
$$\Gamma^t_{v,t} \simeq (2.0\times 10^{-2} g_3^4 + 1.7\times 10^{-3} g_3^2 y_t^2 + 4.3\times 10^{-4} y_t^4)T,$$

…

These then enter the system of Boltzmann equations. Solved by Green's function techniques for general profile

$$A_{lk}\frac{d}{dx}\delta_k + \Gamma_{lk}\delta_k = F_l \quad \longrightarrow \quad \boxed{\delta_i(x) = \chi_{ij}\int_{-\infty}^{\infty}\left[\chi^{-1}A^{-1}\mathbf{F}(y)\right]_j G_j(x,y)dy}$$

$$\boldsymbol{\delta}^{\mathrm{T}} \equiv (\delta\mu_t, \delta T_t, \delta v_t, \delta\mu_h, \delta T_h, \delta v_h) \qquad \left(A^{-1}\Gamma\right)_{ij}\chi_{jk} = \chi_{ik}\lambda_k$$

$$\mathbf{F}(x)^{\mathrm{T}} \equiv \frac{v_w}{2T}\left(c_1^t\frac{dm_t^2(\phi_h)}{dx},\ c_2^t\frac{dm_t^2(\phi_h)}{dx},\ 0,\ c_1^h(x)\frac{dm_h^2(\phi_h,\phi_s)}{dx},\ c_2^h\frac{dm_h^2(\phi_h,\phi_s)}{dx},\ 0\right)$$

$$G_i(x,y) = \mathrm{sgn}(\lambda_i)e^{-\lambda_i(x-y)}\Theta\left[\mathrm{sgn}(\lambda_i)(x-y)\right]$$

Interaction rates

Beware the uncertainties! Different leading log prescriptions lead to factors of ~ 1– 10 difference in these rates → $\mathcal{O}(100\%)$ effects in v$_w$

Consider e.g. $t\bar{t} \to gg$ contribution to $\Gamma^t_{\mu 1,t}$:

HTL numerical result vs. Analytic result with thermal mass insertion

$$\Delta\Gamma^t_{\mu 1,t} \approx 1.1 \times 10^{-3} T$$

$$\Delta\Gamma^t_{\mu 1,t} \simeq \frac{16\alpha_s^2}{9\pi^3} \times \frac{9\zeta_2^2}{16} \log \frac{9T^2}{m_q^2} T \approx 3.8 \times 10^{-3} T$$

Factor of ~4 difference, formally at the same order in the LL expansion

Why? Because the logarithms are not numerically very large!

Above result requires evaluating the angular integral

$$\int d\cos\theta \frac{1}{2} \log\left(\frac{2\,|\mathbf{p}|\,|\mathbf{k}|\,(1-\cos\theta)}{m_t^2}\right) = -1 + \log\frac{4\,|\mathbf{p}|\,|\mathbf{k}|}{m_t^2}$$

Difference between dropping the constant piece or not accounts for most of the discrepancy

Singlet friction

In the high-T approximation, the singlet scalar interaction rates are suppressed at leading log order relative to e.g. tops and Higgs

Approximate collision term as small and drop from the Boltzmann equations. They can then be solved exactly:

$$\int \frac{d^3p}{(2\pi)^3 2E} \delta f_s(p,x) = v_w \int \frac{d^3p}{(2\pi)^3 2E} \frac{e^{E_p/T}}{\left(e^{E_p/T} \pm 1\right)^2} \frac{\mathcal{Q}(p_z)}{T}$$

$$\mathcal{Q}(p_z) = \begin{cases} \sqrt{p_z^2 + m_s(\phi_h, \phi_s, T)^2} - p_z, & p_z > -\sqrt{m_s^0(T)^2 - m_s(\phi_h, \phi_s, T)^2} \\ -\sqrt{p_z^2 + m_s(\phi_h, \phi_s, T)^2 - m_s^0(T)^2} - p_z, & p_z < -\sqrt{m_s^0(T)^2 - m_s(\phi_h, \phi_s, T)^2} \end{cases}$$

Full leading order result can go beyond this approximation. Likely overestimates friction.

Gauge bosons

Two prescriptions:

Gauge invariant (approximate) treatment – drop thermal cubic term and gauge boson friction term

Include both the cubic term and the gauge boson friction contribution

Corresponding friction dominated by highly infrared modes; treat semi-classically Moore, 2000

$$\frac{\pi m_{D,W}^2(T)}{8p}\frac{df_W(p,T)}{dt} = -[p^2 + m_W^2(\phi_h)]f_W(p,T) + \mathcal{N}$$

Exact solution

$$\Longrightarrow \quad \frac{dm_W^2(\phi_h)}{d\phi_h}\int \frac{d^3p}{(2\pi)^3 2E}\delta f_W(p,x) = v_w \frac{3T}{32\pi} m_{D,W}^2(T)\frac{\phi_h'(x)}{\phi_h(x)^2},\ \Theta(x - x_*)$$

$$x_* \text{ solves } m_W[\phi_h(x_*)] = 1/L_h$$

Putting it all together...

Putting these pieces into the EOM (and dropping the gauge piece for now) we get:

$$\boxed{\begin{aligned} -(1-v_w^2)\phi_i'' + \frac{\partial V(\phi_i,T)}{\partial \phi_i} + \sum_j \frac{\partial m_j^2(\phi_i)}{\partial \phi_i}\frac{T}{2}\left[c_1^j \delta\mu_j + c_2^j(\delta T_j + \delta T_{bg})\right] \\ + \frac{\partial m_s^2(\phi_i)}{\partial \phi_i}\int \frac{d^3p}{(2\pi)^3 2E}\delta f_s(x,p) = 0 \end{aligned}}$$

B.C.'s: $\phi_{h,s}(x \to \mp\infty) = \phi_{h,s;\pm}(T_+)$, $\phi'_{h,s}(x \to \pm\infty) = 0$

Need to find profile, $\vec{\phi}(x)$, and v_w such that the EOM or, more weakly, our constraints are satisfied:

$$\int dx \; (\text{E.O.M.}) \cdot \frac{d\vec{\phi}}{dx} = 0 \qquad \int dx \; (\text{E.O.M.}) \cdot \frac{d^2\vec{\phi}}{dx^2} = 0$$

How??

Solving for the profile

This is a set of integro-differential equations for $\vec{\phi}(x)$: perturbations at a given point determined by integral involving profile

Strategy: consider simpler case of constant friction term $\propto d\vec{\phi}/dx$

$$-(1-v_w^2)\frac{d^2\mathbf{\Phi}}{dx^2}+\nabla_\phi V(\mathbf{\Phi},T)+\mathcal{F}\frac{d\mathbf{\Phi}}{dx}=0$$

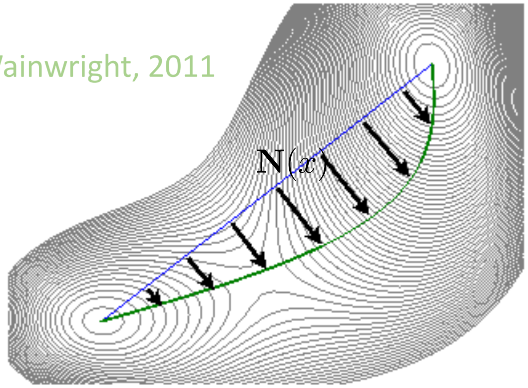
Much simpler: looks almost like the Euclidean EOMs for the bounce!
Can solve this via path deformations JK et al, 2014

Starting from initial guess, deform the path to minimize "normal force", $\mathbf{N}(x)$

Corresponds to friction only parallel to the path in field space

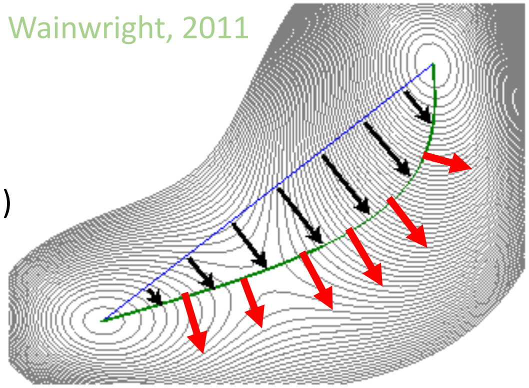
Result well fit by kink: $\phi_i(x)=\frac{\phi_i^0}{2}\left(1+\tanh\frac{x-\delta_i}{L_i}\right)$

Wainwright, 2011



Solving for the profile

How do things change when including the full friction term? Friction is no longer purely parallel to the trajectory satisfying the constant friction EOM



Fortunate simplification: perpendicular friction typically perturbatively small! (Can check validity a posteriori)

$$\nabla_\phi V(\mathbf{\Phi}, T=0) \cdot \frac{d\mathbf{\Phi}(s)}{ds} \sim -\sum \frac{dm(\mathbf{\Phi})^2}{d\mathbf{\Phi}(s)} \int \frac{d^3p}{(2\pi)^3 2E} f_0(p,T) \cdot \frac{d\mathbf{\Phi}(s)}{ds}$$

Parallel: $$\left\{-(1-v_w^2)\frac{d^2\mathbf{\Phi}}{dx^2} + \sum \frac{dm(\mathbf{\Phi})^2}{d\mathbf{\Phi}} \int \frac{d^3p}{(2\pi)^3 2E} \delta f(p,T) + \mathcal{O}(\delta f^2)\right\} \cdot \frac{d\mathbf{\Phi}(s)}{ds} \sim 0$$

Friction contribution dominates

Perpendicular: $$\left\{-(1-v_w^2)\frac{d^2\mathbf{\Phi}}{dx^2} + \nabla_\phi V(T=0) + \sum \frac{dm(\mathbf{\Phi})^2}{d\mathbf{\Phi}} \int \frac{d^3p}{(2\pi)^3 2E} f_0(p,T) + \mathcal{O}(\delta f)\right\} \cdot \frac{d^2\mathbf{\Phi}(s)}{ds^2} \sim 0$$

If we neglect this subdominant piece, only effect of going to full friction term is rescaling of profile: $L_{h,s} \to a L_{h,s}, \quad \delta_s \to a\delta_s$

A prescription

Suggests the following prescription:

1. Compute nucleation temperature and initial profile

2. Solve for constant friction profile. Fit to tanh

3. Solve hydrodynamic equations to determine T near the wall (see Jose Miguel's talk)

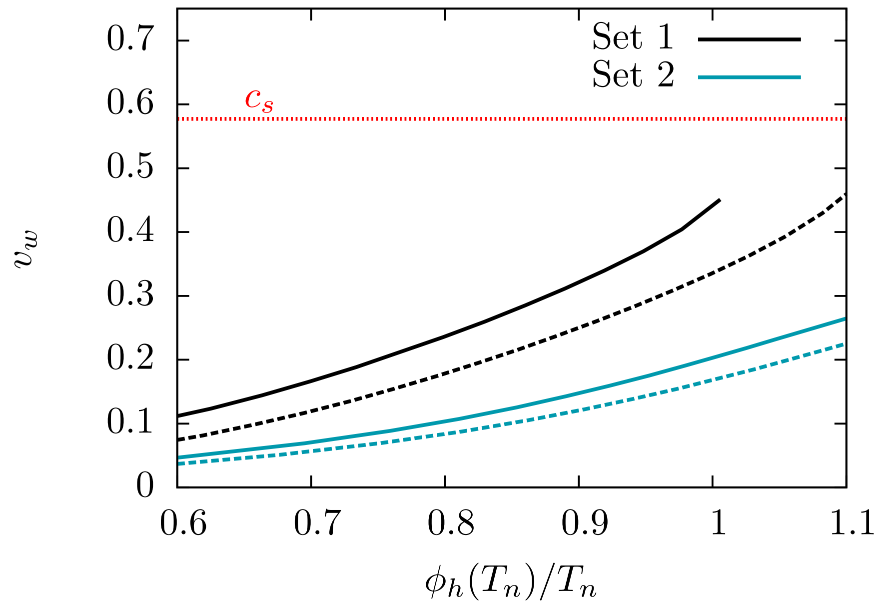
4. Vary values of v_w , a. For each pair, solve for perturbations

5. Inset perturbations and profile into EOM and impose constraints

Values of v_w , a approximately solving full set of EOMs and Boltzmann equations correspond to those satisfying the constraints.

Results

Walls move quickly



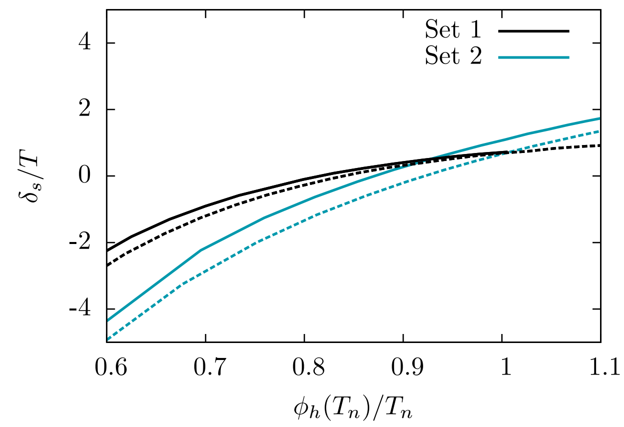
For strong enough phase transitions, no subsonic solutions

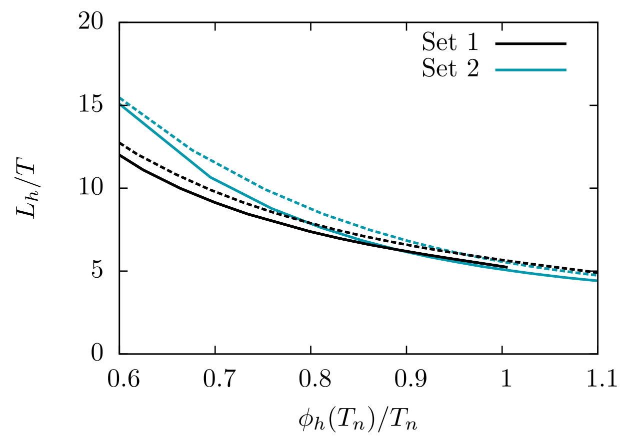
Likely optimistic friction estimate (rates can be larger, singlet friction neglected)

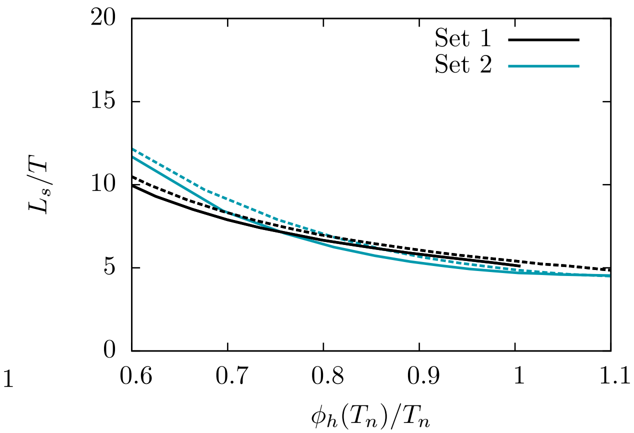
Results

Walls move quickly and they're thin

$$\phi_i(x) = \frac{\phi_i^0}{2}\left(1 + \tanh\frac{x - \delta_i}{L_i}\right)$$







Extending to other models

Don't always have to compute wall velocity from scratch. Can match onto existing results for models with similar features

$$\phi_i'' - \frac{\partial V(\phi, T)}{\partial \phi_i} = \eta_i v_w \gamma \frac{\phi_i^2}{T} \phi_i'$$

Phenomenological friction terms; matched to microphysical calculation. Can have more complicated parametric dependence

See e.g. Huber + Sopena, 2013; Megevand, 2013l; Konstandin et al, 2014…

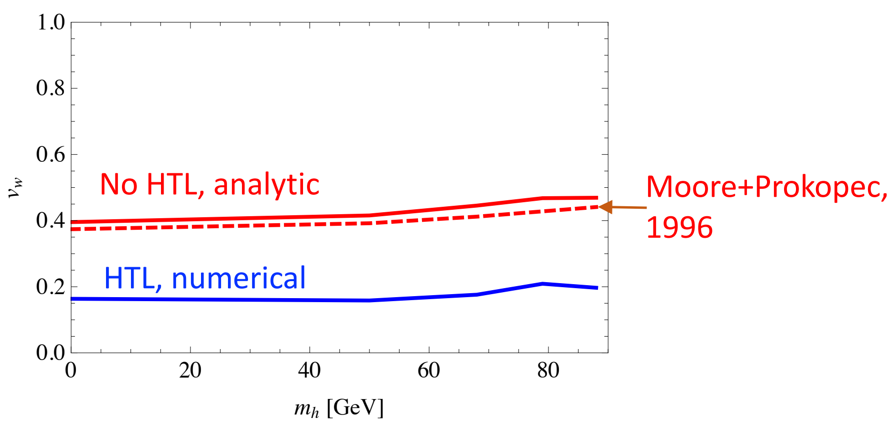
This is fortunate, but results only as good as the underlying microphysical calculation you are matching onto

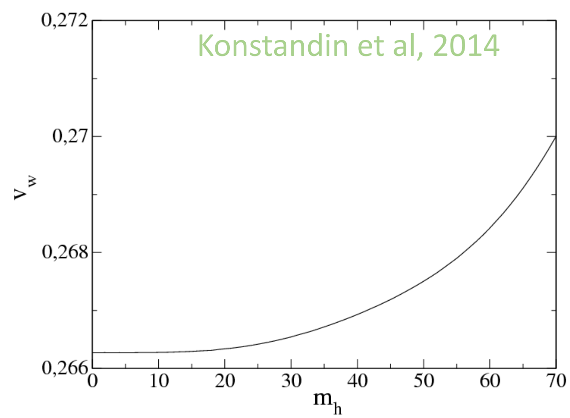
Closing thoughts

Lots of theoretical uncertainties associated with approximations in microphysical calculation of wall velocities

- -Simplified hydro
- -Fluid approximation
- -Interaction rates (kinetic theory, high-T/leading log expansion)
- -Free passage for singlet

These lead to large (~100%) uncertainties on v_w





Many opportunities for improving these calculations

Closing thoughts

Zeroth order determination of whether to expect fast/"runaway" bubbles can be obtained using Bodeker-Moore criterion. Probably sufficient for gravitational wave spectrum, but no analog for checking whether $v_w < c_s$.

Phenomenological approaches allow one to bootstrap using existing results (if model doesn't look too different). Still, this approach is at most as accurate as the underlying microphysical calculation.

Good idea to check robustness of baryon asymmetry calculation WRT to v_w (and don't use the MSSM predicted value!).

Ultimately, sharp predictions for the baryon asymmetry will likely require improving these microphysical calculations.