Standard Model β spectra

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IKS, KU Leuven, Belgium

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Three basic questions:

What's our goal?

Understand Standard Model & Go Beyond

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Where to look for it?

Quirky weak interaction!

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Where to look for it?

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How to do this?

β Spectrum shape

1. Direct BSM sensitivity
2. Enters into reactor anomaly

Introduction

General Hamiltonian

$$\mathcal{H} = \sum_{j=V,A,S,P,T} \langle f|\, \mathcal{O}_j \,|i\rangle \, \langle e|\, \mathcal{O}_j[C_j + C_j^{'}\gamma_5] \,|\nu\rangle + h.c.$$

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In Standard Model only V-A $\rightarrow$ where are the **others**?

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Questions:

In Standard Model only V-A $\rightarrow$ where are the **others**?

QCD influences $\rightarrow$ *induced* currents, influenced through **nuclear structure**?

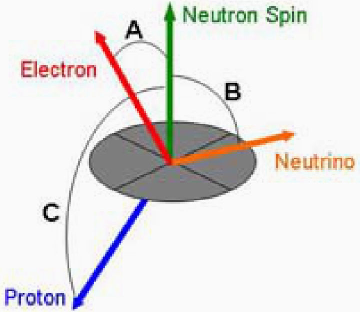
BSM Observables in β decay

Typical BSM searches through correlations

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} \propto 1 + a_{\beta\nu} \frac{\vec{p_e} \cdot \vec{p_\nu}}{E_e E_\nu} + b_F \frac{m_e}{E_e} + A \frac{\vec{p_e}}{E_e} \langle \vec{I} \rangle + \dots$$

Measure effective correlations

$$\tilde{X} = \frac{X}{1 + b_F \langle \frac{m_e}{E_e} \rangle}$$



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Sensitivity comes from b_F

$$b_F = \pm\frac{1}{1+\rho^2}\left[\mathrm{Re}\left(\frac{C_S + C_S'}{C_V}\right) + \rho^2\mathrm{Re}\left(\frac{C_T + C_T'}{C_A}\right)\right]$$

because it's linear in coupling constants

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because it's linear in coupling constants

$\rightarrow$ measure β spectrum directly & fit for $1/E_e$

Beta Spectrum Shape

Exploring the Standard Model and Beyond via the allowed β spectrum shape:

$$\frac{dN}{dE_e} \propto 1 + b_{\text{Fierz}} \frac{m_e}{E_e} + b_{WM} E_e$$

b_{Fierz}: Proportional to scalar (Fermi) and tensor (Gamow-Teller) couplings

b_{WM}: Weak Magnetism (main induced current), poorly known for $A > 60$, forbidden decays

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This requires knowledge of the theoretical spectrum shape to $\leq \mathbf{10^{-3}}$ level!

Beta spectrum shape

Beta Spectrum Shape

3-body decay

$$P(E_e) = (E_0 - E_e)^2 E_e p_e \approx (E_0 - E_e)^2 E_e^2$$

Beta Spectrum Shape

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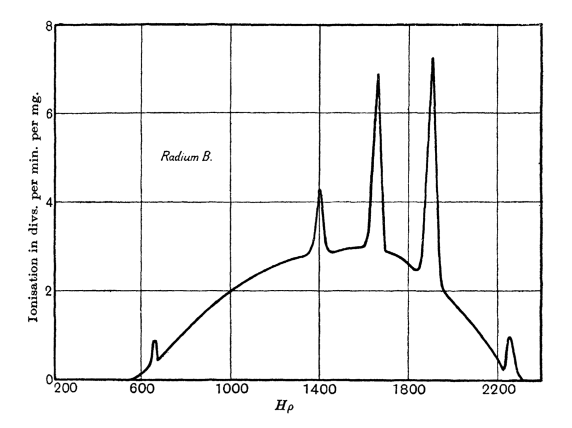

Ellis & Chadwick, 1914

Beta Spectrum Shape

Active participation of QED, QCD & WI $\rightarrow$ Complicated system

Weak Hamiltonian is modified

1. Emitted β particle immersed in Coulomb field: (electroweak) radiative corrections

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Large scale gap to cross

Quark $\rightarrow$ Nucleon $\rightarrow$ Nucleus $\rightarrow$ Atom $\rightarrow$ Molecule

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Whole slew of approximations introduced

Standard Model Calculation: Quark

Starting from the Standard Model $SU(2)_L \times U(1)_Y$ EW sector

$$\mathcal{M} = \frac{g^2}{8} V_{ud} \bar{u} \gamma^\mu (1 - \gamma^5) d \frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \bar{e} \gamma^\nu (1 - \gamma^5) \nu$$

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Since $q \ll M_W$, identify Fermi coupling constant

$$G_F = \frac{g^2}{8 M_W^2}$$

Standard Model Calculation: Nucleon

Moving to the nucleon system, we face

$$\langle p|\bar{u}\gamma^{\mu}(1-\gamma^{5})d|n\rangle$$

Standard Model Calculation: Nucleon

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$$\langle p|\bar{u}\gamma^\mu(1-\gamma^5)d|n\rangle$$

Symmetries to the rescue! CVC & PCAC define new *nucleon* currents

$$V^\mu + A^\mu \approx g_V(q^2)\gamma^\mu(1-\lambda\gamma^5)$$

where $g_V(q^2) \approx 1$ and λ from the lattice

Standard Model Calculation: Nucleon

Strong interaction introduces extra terms into the vertex $\rightarrow$ Construct all Lorentz invariants

$$\langle p|V^{\mu}|n\rangle = \bar{p}\left[g_V\gamma^{\mu} + \frac{g_M - g_V}{2M}\sigma^{\mu\nu}q_{\nu} + i\frac{g_S}{2M}q^{\mu}\right]n$$

$$\langle p|A^{\mu}|n\rangle = \bar{p}\left[g_A\gamma^{\mu}\gamma^5 + \frac{g_T}{2M}\sigma^{\mu\nu}q_{\nu}\gamma^5 + i\frac{g_P}{2M}q^{\mu}\gamma^5\right]n$$

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Introduction of recoil ($\sim q/M$) terms

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Introduction of recoil ($\sim q/M$) terms

CVC requires $g_S = 0$ & $g_M = \mu_p^{an} - \mu_n = 4.7$

Standard Model Calculation: Nucleus

Nucleus is spherical system $\rightarrow$ multipole decomposition,
elementary particle

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Relativistic generalization in Breit frame

$$\langle f|V^0 + A^0|i\rangle \propto \sum_{LM} (-)^{J_f - M_f} \begin{pmatrix} J_f & L & J_i \\ -M_f & M & M_i \end{pmatrix} (Y_L^M)^* F_L(q^2)$$

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Form factors $\sim$ reduced matrix elements

Standard Model Calculation: Nucleus

Require transformation from form factors to matrix elements

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Immediately faced with several issues:

- Weak current in strongly bound system?

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- Final state interactions

Here the going gets rough $\rightarrow$ severe approximations

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Weak current in strongly bound system?

$\rightarrow$ Impulse approximation, non-interacting nucleons

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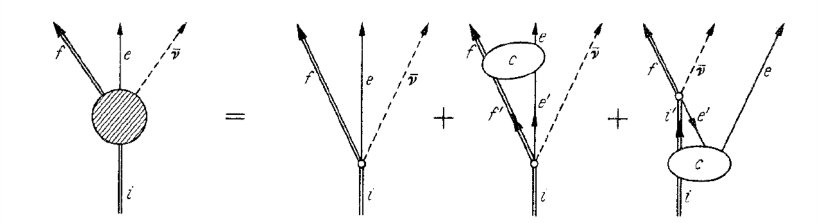
Relativistic nuclear wave functions
$\rightarrow$ Non-relativistic nucleons

- expand operator to $\mathcal{O}(v/c)$
- Incomplete wave function basis, core polarization

Standard Model Calculation: Nucleus

Final state interactions

1. Coulomb interaction



Standard Model Calculation: Nucleus

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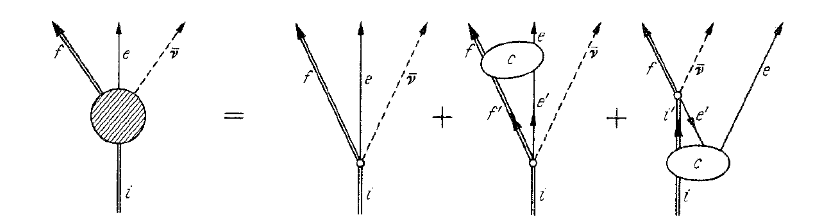


→ Fermi function, *induced Coulomb* terms

Standard Model Calculation: Nucleus

Final state interactions

1. Coulomb interaction

Make several approximations

- Initial & Final Coulomb potentials are same

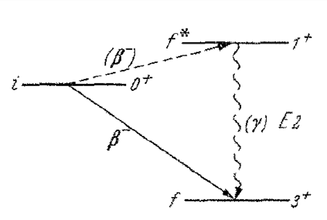
Standard Model Calculation: Nucleus

Final state interactions

1. Coulomb interaction

Make several approximations

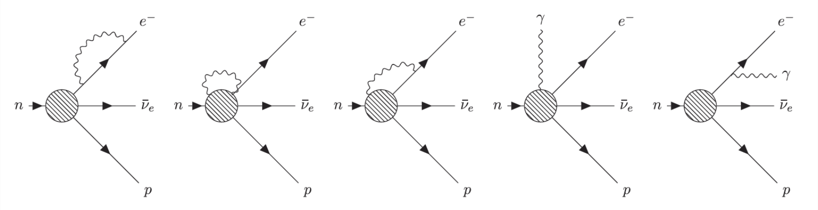
- Initial & Final Coulomb potentials are same
- Typically neglect intermediate decays



Standard Model Calculation: Nucleus

Final state interactions

2. EW Radiative corrections



Standard Model Calculation: Nucleus

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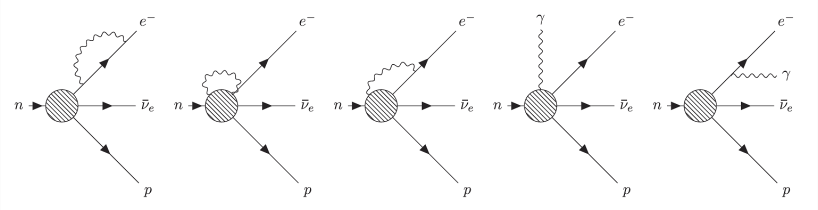

+ higher orders, γW boxes: see previous talks

Standard Model Calculation: Atom

Must consider total nuclear + atomic Hamiltonian

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Changes

- Available phase space
- Final state interactions
- Opens new decay modes (bound & exchange)

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Must consider total nuclear + atomic Hamiltonian

Changes

- Available phase space
- Final state interactions
- Opens new decay modes (bound & exchange)

Require *atomic* wave functions

- Central & static potential
- Sudden approximation

Standard Model Calculation: Molecule

Similar as atomic system, but changes

- Available phase space
- Molecular excitation, ionization
- Recoil correction & distribution

Standard Model Calculation: Molecule

Similar as atomic system, but changes

- Available phase space
- Molecular excitation, ionization
- Recoil correction & distribution

Enter quantum chemistry

- Born-Oppenheimer approximation
- MOLCAO

Current status

Beta Spectrum Shape

Active participation of QED, QCD & WI $\rightarrow$ Complicated system

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$$\begin{aligned} N(W)dW = \frac{G_V^2 V_{ud}^2}{2\pi^3}\ & F_0(Z,W)\ L_0(Z,W)\ U(Z,W)\ R_N(W,W_0,M) \\ \times\ & Q(Z,W,M)\ R(W,W_0)\ S(Z,W)\ X(Z,W)\ r(Z,W) \\ \times\ & C(Z,W)\ D_C(Z,W,\beta_2)\ D_{\mathrm{FS}}(Z,W,\beta_2) \\ \times\ & pW(W_0-W)^2\ dW \end{aligned}$$

LH *et al.*, Rev. Mod. Phys. 90 (2018) 015008; 1709.07530

Analytical β spectrum shape

80 years of history, in detail

Item	Effect	Formula	Magnitude
1	Phase space factor	$pW(W_0 - W)^2$	Unity or larger
2	Traditional Fermi function	F_0	
3	Finite size of the nucleus	L_0	10^{-1}-10^{-2}
4	Radiative corrections	R	
5	Shape factor	C	
6	Atomic exchange	X	
7	Atomic mismatch	r	

Added/Improved/Didactic

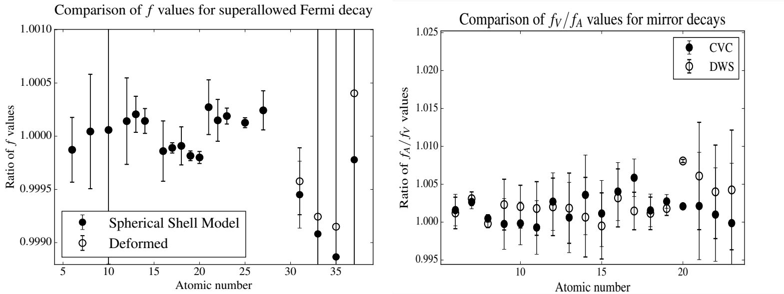
Analytical β spectrum shape

Item	Effect	Formula	Magnitude
8	Atomic screening	S	
9	Shake-up	See 7	
10	Shake-off	See 7	
11	Isovector correction	C_I	
12	Recoil Coulomb correction	Q	
13	Diffuse nuclear surface	U	10^{-3}-10^{-4}
14	Nuclear deformation	D_{FS}	
15	Recoiling nucleus	R_N	
16	Molecular screening	ΔS_{Mol}	
17	Molecular exchange	Case by case	

Added/Improved/Didactic

Performance summary

Comparison against numerical results for superallowed & mirror transitions



Agreement is very good

Serves as input for several experiments, C++ code available

L. H. *et al.*, 1803.00525, `github.com/leenderthayen/BSG`

Order of magnitude estimates

Nuclear structure sensitivity in shape factor

$$C(Z,W) \sim 1 \pm \frac{4}{3}\frac{W}{M_N}\frac{\boldsymbol{b}}{Ac} \pm \frac{4\sqrt{2}}{21}\alpha ZWR\boldsymbol{\Lambda} - \frac{1}{3WMc}(\pm 2\boldsymbol{b} + \boldsymbol{d})$$

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Fill in typical numbers to obtain

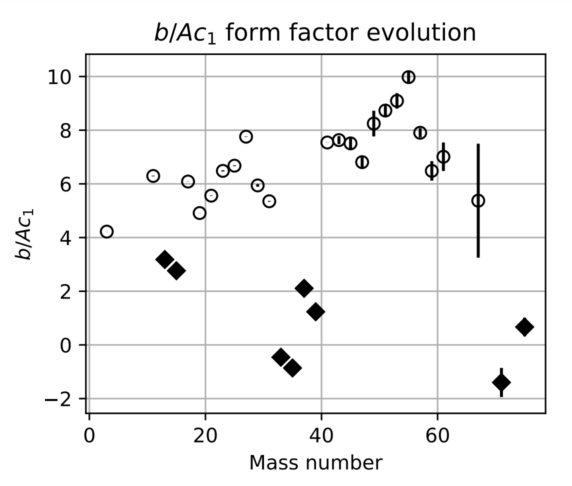
Matrix element	Name	Slope (% MeV^{-1})
b	Weak Magnetism	0.5
d	Induced Tensor	0.1
Λ	Induced Pseudoscalar	0.1

Weak magnetism is generally more stable than others

$\rightarrow$ essential to get this right

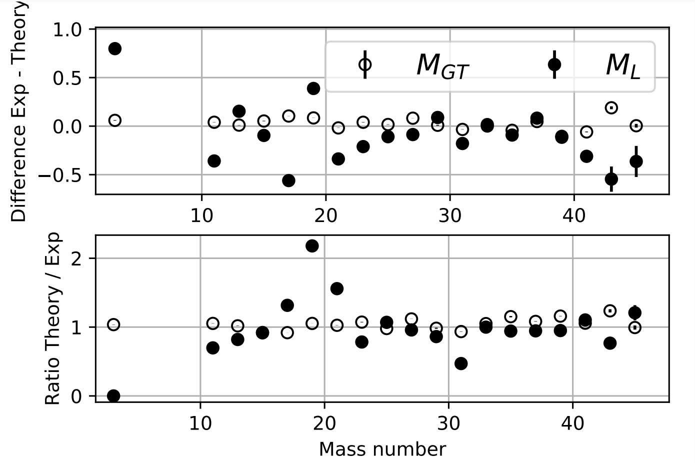
Weak magnetism

Mirror nuclei have CVC-determined WM



open: $l + 1/2$, closed: $l - 1/2$

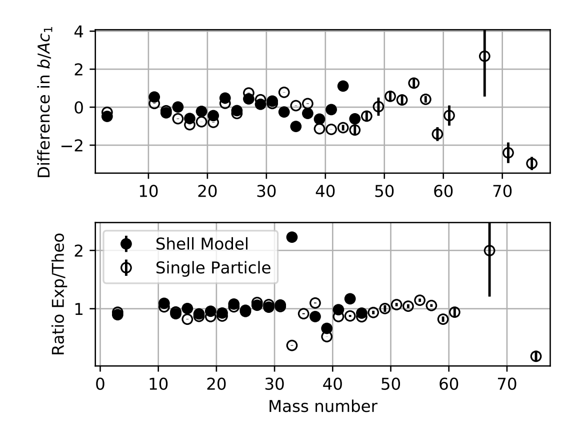
Weak magnetism



'Easy' matrix elements only accurate to 10-20%

Weak magnetism

How does shell model perform right now?



$\Delta b/Ac = 1 \rightarrow 0.1\%\ \mathrm{MeV}^{-1}$

Induced tensor

Still large discrepancies for d/Ac

PHYSICAL REVIEW C **95**, 035501 (2017)

2_1^+ to 3_1^+ γ width in ^{22}Na and second class currents

S. Triambak,[1,2,*] L. Phuthu,[1] A. García,[3] G. C. Harper,[3] J. N. Orce,[1] D. A. Short,[3] S. P. R. Steininger,[3] A. Diaz Varela,[4] R. Dunlop,[4] D. S. Jamieson,[4] W. A. Richter,[1] G. C. Ball,[5] P. E. Garrett,[4] C. E. Svensson,[4] and C. Wrede[3,6]

$$21(6) \geq d/Ac \geq 3(6)$$

Factor 7 differences depending on shell model results $\rightarrow$ killer!

Challenges

Challenges

At $\mathcal{O}(10^{-3})$, nuclear structure is main culprit

- Nuclear matrix elements only precise to 10-20%
- Generally: large meson exchange corrections on induced currents
- Isospin multiplet decays are way to go: WM from CVC, induced tensor $= 0$

Challenges

At $\leq \mathcal{O}(10^{-4})$, *everything* breaks

Challenges

At $\leq \mathcal{O}(10^{-4})$, *everything* breaks , but not in the same place!

- Low energy: Atomic & Molecular effects (exchange)
- Endpoint: Final state interactions, excitations
- Radiative corrections: higher order, model dependence
- Low Z: recoil corrections to matrix elements
- High Z: everything electromagnetic

Conclusions

Spectrum shape measurements are valuable tests for S, T currents

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Theoretical spectrum is *theoretically* valid to few 10^{-4}

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Nuclear structure generally is main current generation bottleneck

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Further, radiative & recoil corrections become bottleneck even for nuclear-structure-favorable transitions