

Standard Model β spectra

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Motivation

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What's our goal?

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Where to look for it?

Quirky weak interaction!

Introduction

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Where to look for it?

Quirky weak interaction!

How to do this?

Spectrum shape

1. Direct BSM sensitivity
2. Enters into reactor anomaly

Introduction

General Hamiltonian

$$H = \sum_{j=V;A;S;P;T} \hbar \omega_j |j\rangle \langle j| + \sum_j \hbar \omega_j [C_j + C_j^\dagger] |j\rangle \langle j| + \text{h.c.}$$

Introduction

General Hamiltonian

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Questions:

In Standard Model only V-A ! where are the **others**?

Introduction

General Hamiltonian

$$H = \sum_{j=V;A;S;P;T} \left(\frac{1}{2} m_j \dot{\phi}_j^2 + \frac{1}{2} k_j \phi_j^2 \right) + \dots$$

Questions:

In Standard Model only $V-A$! where are the **others**?

QCD influences ! *induced* currents, influenced through **nuclear structure**?

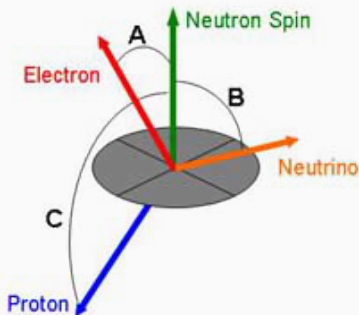
BSM Observables in β decay

Typical BSM searches through correlations

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega} \propto 1 + a \frac{\boldsymbol{\beta}_e \cdot \boldsymbol{\rho}}{E_e E} + b_F \frac{m_e}{E_e} + A \frac{\boldsymbol{\beta}_e \cdot \boldsymbol{h} \boldsymbol{t} \boldsymbol{i}}{E_e} + \dots$$

Measure effective correlations

$$\tilde{X} = \frac{X}{1 + b_F \frac{m_e}{E_e}}$$



BSM Observables in β decay

Typical BSM searches through correlations

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega} \propto 1 + a \frac{p_e \cdot p}{E_e E} + b_F \frac{m_e}{E_e} + A \frac{p_e \cdot \hat{p}}{E_e} + \dots$$

Sensitivity comes from b_F

$$b_F = \frac{1}{1 + \frac{1}{2}} \operatorname{Re} \frac{C_S + C_S^\ell}{C_V} + 2 \operatorname{Re} \frac{C_T + C_T^\ell}{C_A}$$

because it's **linear** in coupling constants

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because it's **linear** in coupling constants

! measure spectrum directly & fit for $1 = E_e$

Beta Spectrum Shape

Exploring the Standard Model and Beyond via the allowed spectrum shape:

$$\frac{dN}{dE_e} \propto 1 + b_{\text{Fierz}} \frac{m_e}{E_e} + b_{\text{WM}} E_e$$

b_{Fierz} : Proportional to scalar (Fermi) and tensor (Gamow-Teller) couplings

b_{WM} : Weak Magnetism (main induced current), poorly known for $A > 60$, forbidden decays

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This requires knowledge of the theoretical spectrum shape to 10^{-3} level!

Beta spectrum shape

Beta Spectrum Shape

3-body decay

$$P(E_e) = (E_0 - E_e)^2 E_e p_e \quad (E_0 - E_e)^2 E_e^2$$

Beta Spectrum Shape

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Beta Spectrum Shape

Active participation of QED, QCD & WI ! Complicated system

Weak Hamiltonian is **modified**

1. Emitted particle immersed in Coulomb field: (electroweak) radiative corrections

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Large scale gap to cross

Quark ! Nucleon ! Nucleus ! Atom ! Molecule

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Whole slew of **approximations** introduced

Standard Model Calculation: Quark

Starting from the Standard Model $SU(2)_L \times U(1)_Y$ EW sector

$$M = \frac{g^2}{8} V_{ud} u \quad (1) \quad \text{and} \quad \frac{g}{q^2} \frac{q \cdot q}{M_W^2} e \quad (1) \quad (5)$$

Standard Model Calculation: Quark

Starting from the Standard Model $SU(2)_L \times U(1)_Y$ EW sector

$$M = \frac{g^2}{8} V_{ud} \bar{u} \gamma^\mu (1 - \gamma_5) d \frac{g}{q^2} \frac{q^\nu q^\rho}{M_W^2} e \gamma_\nu (1 - \gamma_5) \nu_e \quad (1.5)$$

Since $q \ll M_W$, identify Fermi coupling constant

$$G_F = \frac{g^2}{8M_W^2}$$

Moving to the nucleon system, we face

$$\text{hpju } (1 \quad ^5)\text{djni}$$

Moving to the nucleon system, we face

$$\langle p | j_\mu | n \rangle = (1 + \dots)$$

Symmetries to the rescue! CVC & PCAC define nucleon currents

$$V_\mu + A_\mu = g_V(q^2) (1 + \dots)$$

where $g_V(q^2) = 1$ and \dots from the lattice

Standard Model Calculation: Nucleon

Strong interaction introduces extra terms into the vertex

Construct all Lorentz invariants

$$\begin{aligned}
 \langle p | V | j \rangle &= \bar{p} \left[g_V + \frac{g_M}{2M} \not{q} + i \frac{g_S}{2M} \not{q} \right] n \\
 \langle p | A | j \rangle &= \bar{p} \left[g_A \not{h} + \frac{g_T}{2M} \not{q} + i \frac{g_P}{2M} \not{q} \right] \gamma_5 n
 \end{aligned}$$

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 \end{aligned}$$

Introduction of recoil ($q=M$) terms

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 \end{aligned}$$

Introduction of recoil ($q=M$) terms

CVC requires $g_S = 0$ & $g_M = \frac{g_A}{2}$ $n = 4:7$

Nucleus is spherical system multipole decomposition,
elementary particle

Standard Model Calculation: Nucleus

Nucleus is spherical system multipole decomposition,
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Relativistic generalization in **Breit frame**

$$\langle f | jV^0 + A^0 j | i \rangle = \sum_{LM} \langle J_f M_f | (Y_L^M)^{J_f M_f} | J_i M_i \rangle F_L(q^2)$$

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Form factors reduced matrix elements

Require transformation from form factors to matrix elements

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Immediately faced with several issues:

Weak current in strongly bound system?

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- Final state interactions

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Immediately faced with several issues:

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Here the going gets rough **severe approximations**

Weak current in strongly bound system?

! **Impulse approximation**, non-interacting nucleons

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Neglects meson exchange

Nucleon-nucleon interaction present in many-body methods

Weak current in strongly bound system?

! **Impulse approximation**, non-interacting nucleons

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Nucleon-nucleon interaction present in many-body methods

Relativistic nuclear wave functions

! **Non-relativistic** nucleons

expand operator to $\mathcal{O}(v=c)$

Incomplete wave function basis, core polarization

Final state interactions

1. Coulomb interaction

Final state interactions

1. Coulomb interaction

! Fermi function, induced Coulomb terms

Standard Model Calculation: Nucleus

Final state interactions

1. Coulomb interaction

Make several **approximations**

Initial & Final Coulomb potentials are same

Standard Model Calculation: Nucleus

Final state interactions

1. Coulomb interaction

Make several **approximations**

Initial & Final Coulomb potentials are same

Typically neglect intermediate decays

Final state interactions

2. EW Radiative corrections

Final state interactions

2. EW Radiative corrections

+ higher orders, W boxes: see previous talks

Standard Model Calculation: Atom

Must consider total nuclear + atomic Hamiltonian

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Must consider total nuclear + atomic Hamiltonian

Changes

- Available phase space

- Final state interactions

- Opens new decay modes (bound & exchange)

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Must consider total nuclear + atomic Hamiltonian

Changes

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- Opens new decay modes (bound & exchange)

Require atomic wave functions

- Central & static potential

- Sudden approximation

Similar as atomic system, but changes

- Available phase space

- Molecular excitation, ionization

- Recoil correction & distribution

Similar as atomic system, but changes

- Available phase space

- Molecular excitation, ionization

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Enter quantum chemistry

- Born-Oppenheimer approximation

- MOLCAO

Current status

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$$N(W)dW = \frac{G_V^2 V_{ud}^2}{2^3} F_0(Z; W) L_0(Z; W) U(Z; W) R_N(W; W_0; M) \\ Q(Z; W; M) R(W; W_0) S(Z; W) X(Z; W) r(Z; W) \\ C(Z; W) D_C(Z; W; \alpha) D_{FS}(Z; W; \alpha) \\ pW(W_0 - W)^2 dW$$

LH et al., Rev. Mod. Phys. 90 (2018) 015008; 1709.07530

80 years of history, in detail

Item	Effect	Formula	Magnitude
1	Phase space factor	$pW(W_0 - W)^2$	Unity or larger
2	Traditional Fermi function	F_0	
3	Finite size of the nucleus	L_0	
4	Radiative corrections	R	
5	Shape factor	C	$10^{-1}-10^{-2}$
6	Atomic exchange	X	
7	Atomic mismatch	r	

Analytical spectrum shape

Item	Effect	Formula	Magnitude
8	Atomic screening	S	
9	Shake-up	See 7	
10	Shake-o	See 7	
11	Isovector correction	C_I	
12	Recoil Coulomb correction	Q	10^{-3} - 10^{-4}
13	Disuse nuclear surface	U	
14	Nuclear deformation	D_{FS}	
15	Recoiling nucleus	R_N	
16	Molecular screening	S_{Mol}	
17	Molecular exchange	Case by case	

Added/Improved/Didactic

Performance summary

Comparison against numerical results for superallowed & mirror transitions

Agreement is very good

Serves as input for several experiments, C++ code available

Order of magnitude estimates

Nuclear structure sensitivity in shape factor

$$C(Z;W) \approx 1 - \frac{4}{3} \frac{W}{M_N} \frac{b}{A_c} \frac{4^p}{21} \bar{2} \text{ ZWR} \frac{1}{3WMc} (2b + d)$$

Order of magnitude estimates

Nuclear structure sensitivity in shape factor

$$C(Z;W) \approx 1 - \frac{4}{3} \frac{W}{M_N} \frac{b}{A_C} - \frac{4}{21} \frac{p}{2} ZWR - \frac{1}{3WMc} (2b + d)$$

Fill in typical numbers to obtain

Matrix element	Name	Slope (% MeV ⁻¹)
b	Weak Magnetism	0.5
d	Induced Tensor	0.1
	Induced Pseudoscalar	0.1

Weak magnetism is generally more stable than others

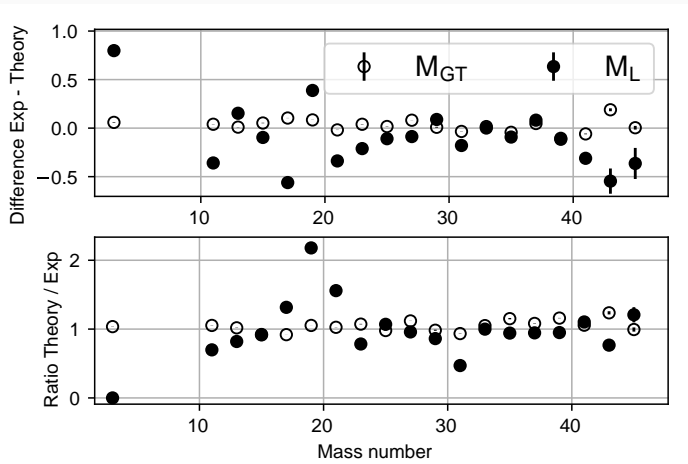
! **essential** to get this right

Weak magnetism

Mirror nuclei have CVC-determined WM

open: $l + 1 = 2$, closed: $l \quad 1 = 2$

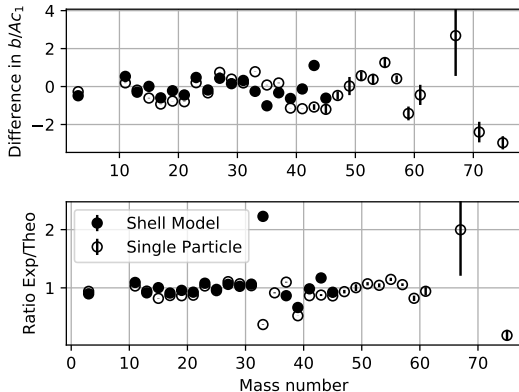
Weak magnetism



'Easy' matrix elements only accurate to 10-20%

Weak magnetism

How does shell model perform right now?



$$\Delta b = Ac = 1 \text{ ! } 0.1\% \text{ MeV}^{-1}$$

Still large discrepancies for $d=Ac$

PHYSICAL REVIEW C **95**, 035501 (2017)

2_1^+ to 3_1^+ γ width in ^{22}Na and second class currents

S. Triambak,^{1,2,*} L. Phuthu,¹ A. García,³ G. C. Harper,³ J. N. Orce,¹ D. A. Short,³ S. P. R. Steinger,³ A. Diaz Varela,⁴
R. Dunlop,⁴ D. S. Jamieson,⁴ W. A. Richter,¹ G. C. Ball,⁵ P. E. Garrett,⁴ C. E. Svensson,⁴ and C. Wrede^{3,6}

$$21(6) \quad d=Ac \quad 3(6)$$

Factor 7 differences depending on shell model results ! killer!

Challenges

At $O(10^{-3})$, nuclear structure is main culprit

Nuclear matrix elements only precise to 10-20%

Generally: large meson exchange corrections on induced currents

Isospin multiplet decays are way to go: WM from CVC, induced tensor = 0

At $O(10^4)$, *everything* breaks

At $O(10^{-4})$, *everything* breaks , but not in the same place!

Low energy: Atomic & Molecular effects (exchange)

Endpoint: Final state interactions, excitations

Radiative corrections: higher order, model dependence

Low Z : recoil corrections to matrix elements

High Z : everything electromagnetic

Spectrum shape measurements are valuable tests for S, T currents

Conclusions

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Theoretical spectrum is *theoretically* valid to few 10^{-4}

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Further, radiative & recoil corrections become bottleneck even for nuclear-structure-favorable transitions