

Gravitational aspects of binary dynamics: waveform models and their systematics

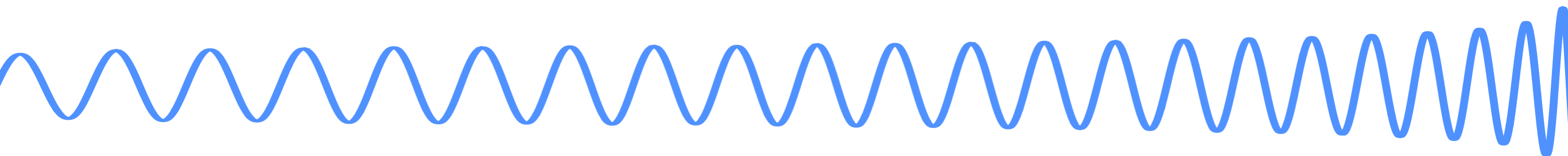
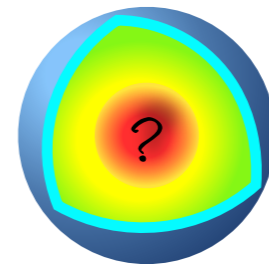
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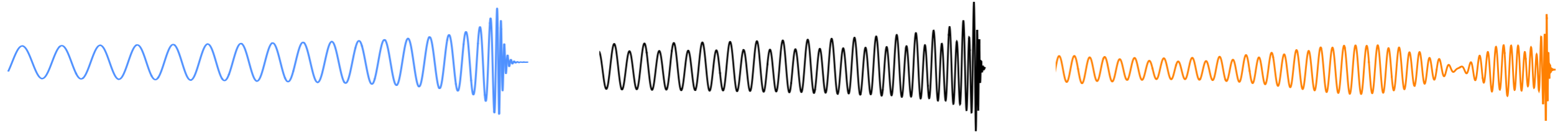
Overview

- Gravitational waves (GWs) now accessible as probes of dense matter
 - For binary systems:
interpreting the data requires detailed **theoretical** understanding & accurate modeling
 - Brief glimpse of challenges, approaches, and status of models that include matter effects
- Outlook: unique scientific prospects with future measurements but only with significant further advances in modeling



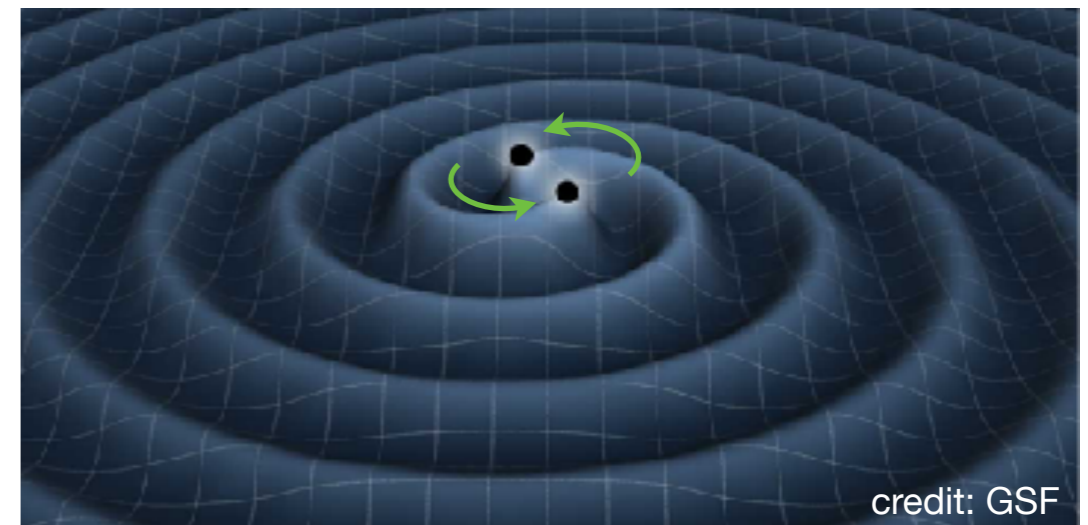
GW measurements (binary systems)

- ▶ Details of the waveforms encode fundamental source properties (masses, spins, ...)



- ▶ Measurements cross-correlate millions of **template models** with the data to determine the source parameters
- ▶ Computation of template waveforms is very challenging:

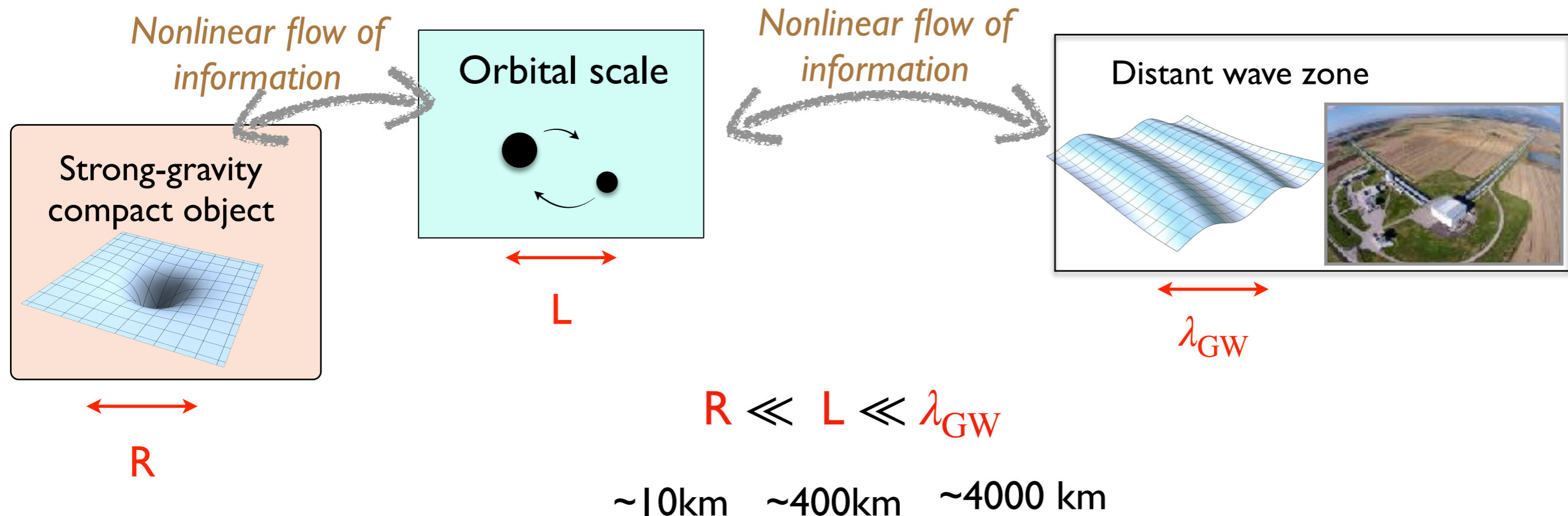
Must solve the **nonlinear Einstein Field equations** coupled with the **matter equations of motion** for the dynamical spacetime



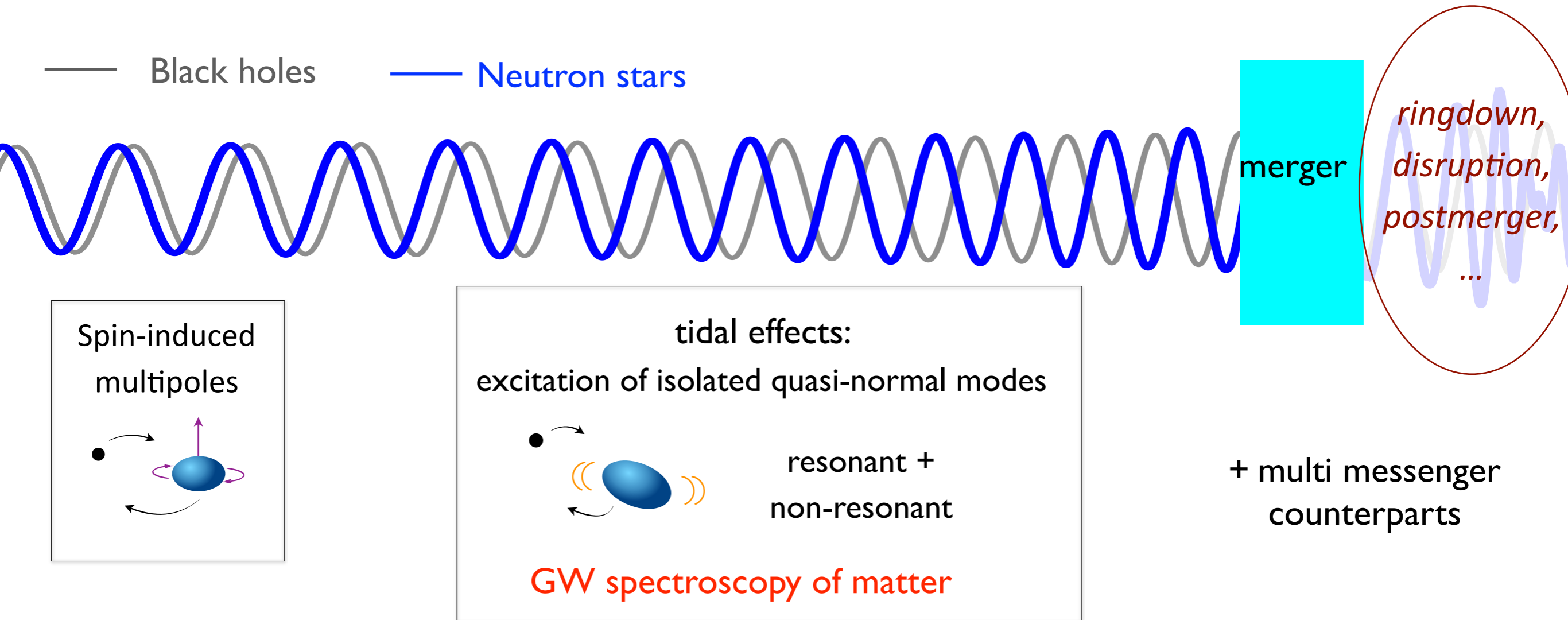
Approaches to computing waveforms

- Numerical relativity simulations: access to complex merger regimes ... limited in parameter space ... sometimes difficult to identify fundamental physics parameters based solely on numerical outputs
- When **different physics dominate at different scales**:
 - **tapestry** of approximation schemes in different patches of spacetime

Example for comparable-mass inspirals:



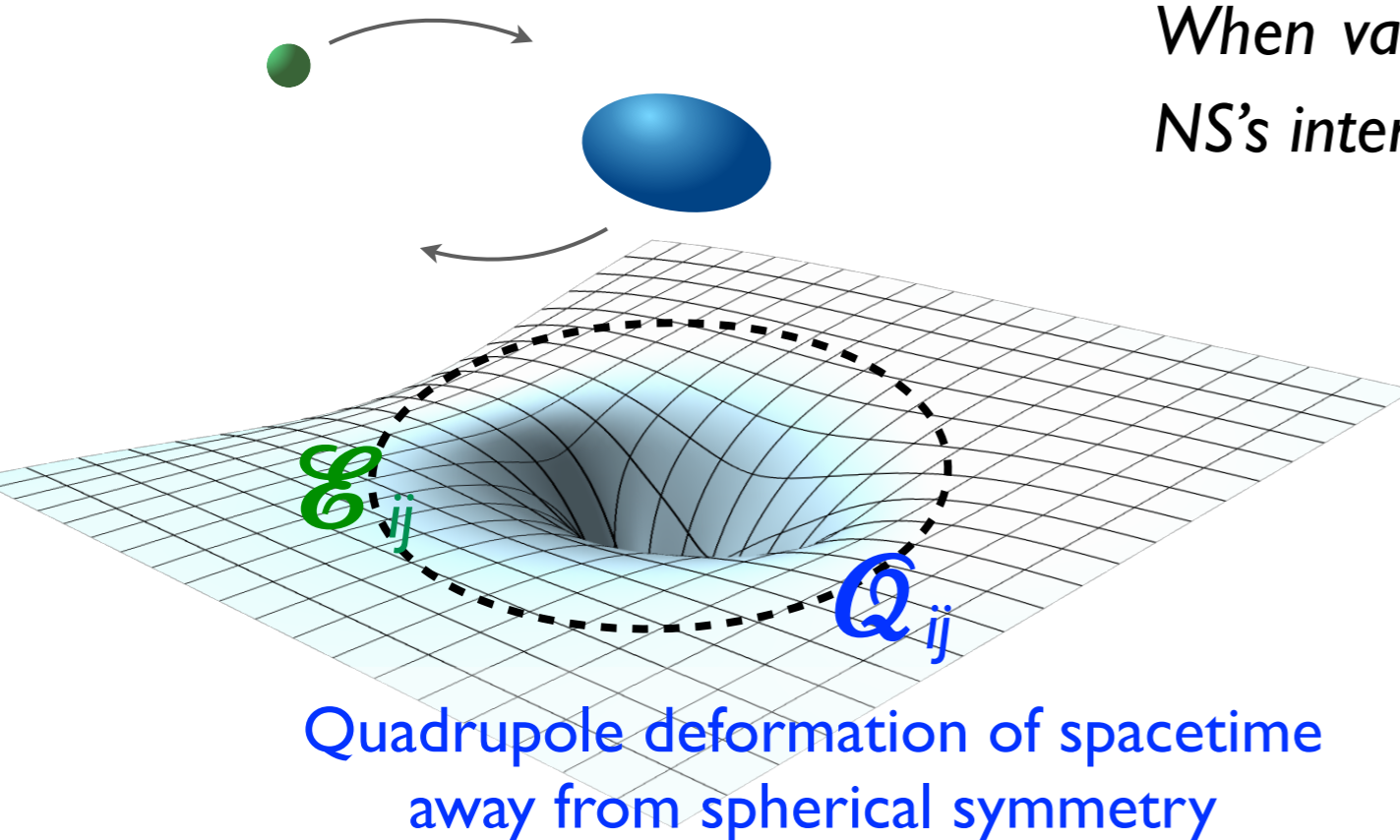
Main GW signatures of matter



Inspiral: small effects but clean and cumulative, accessible with current detectors

Dominant tidal effects

- In a binary: tidal field $\mathcal{E}_{ij} = R_{0i0j}$ due to spacetime curvature from companion



When variations in tidal field are much faster than NS's internal timescales (adiabatic limit):

Induced deformation:

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

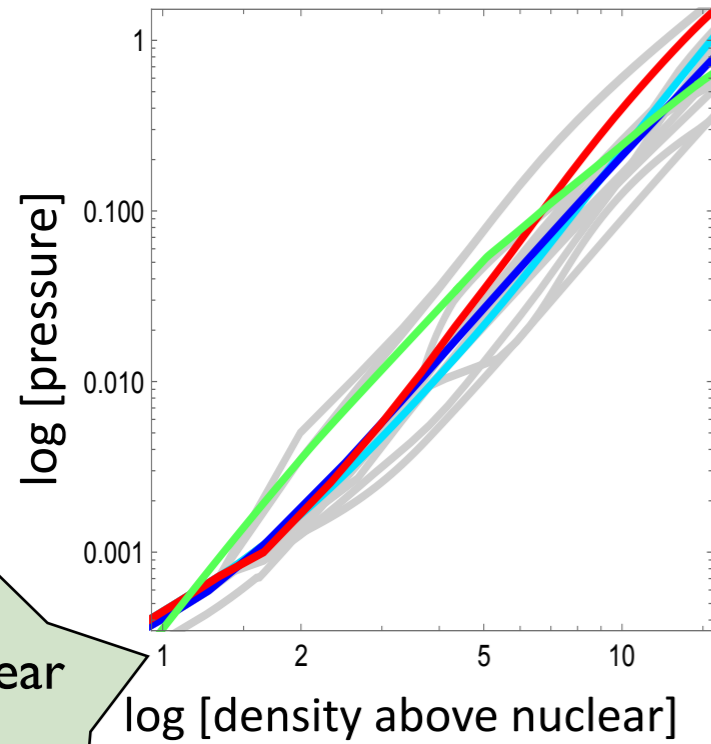
tidal deformability parameter

=0 for a black hole

Similarly for higher multipoles

Properties of NS matter reflected in tidal deformability

NS matter models
(equations of state EoS)



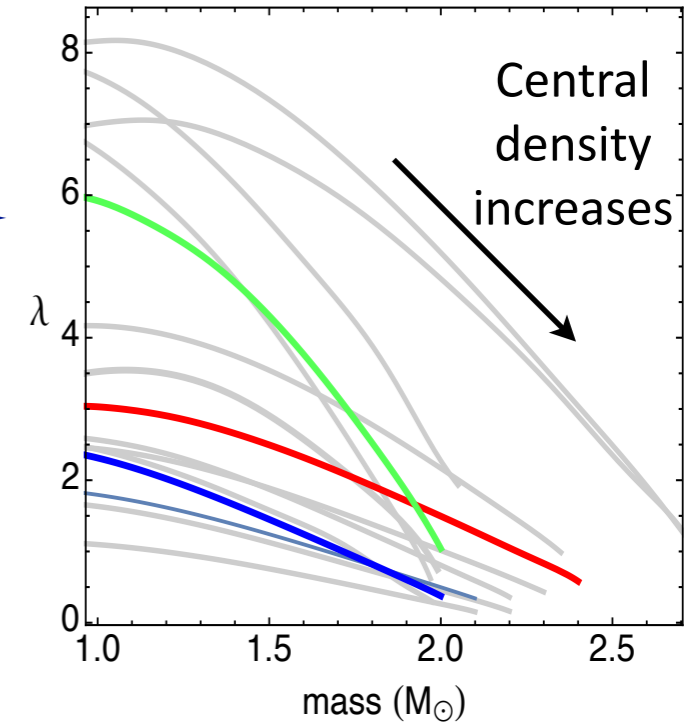
Perturbative QCD

Nuclear EFT, ...

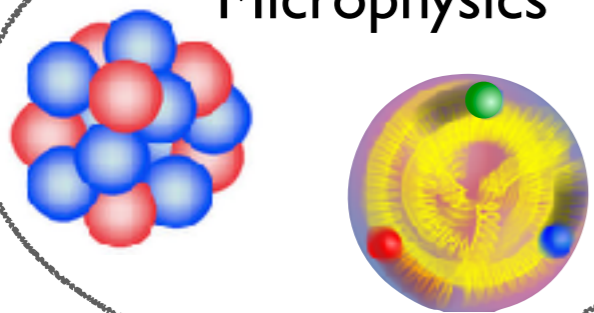
*Einstein field equations &
Matter equations of motion*

Stationary perturbations to equilibrium
or wave scattering

Tidal deformability λ vs. mass



Microphysics

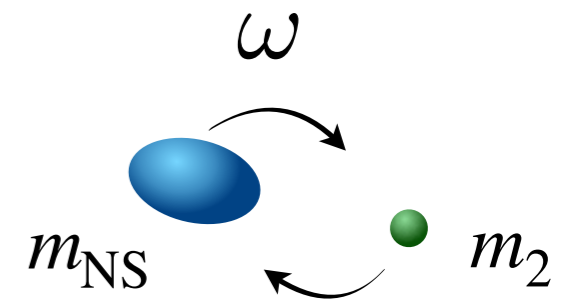


Main influence on GWs

- **Energy** goes into deforming the NS:

$$E \sim E_{\text{orbit}} + \frac{1}{4} Q \varepsilon$$

$$Q = -\lambda \varepsilon$$



$$M = m_{\text{NS}} + m_2$$

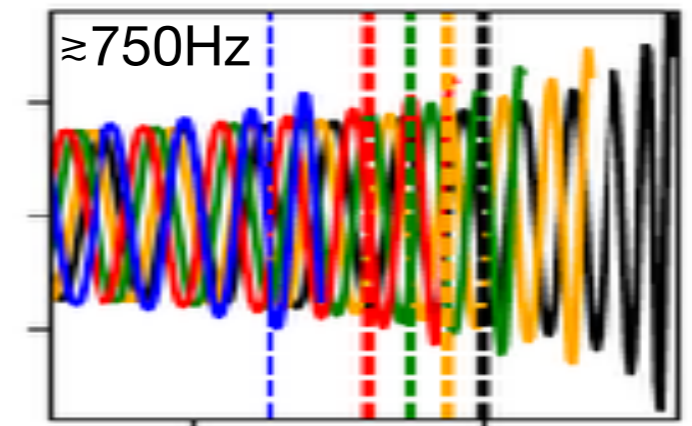
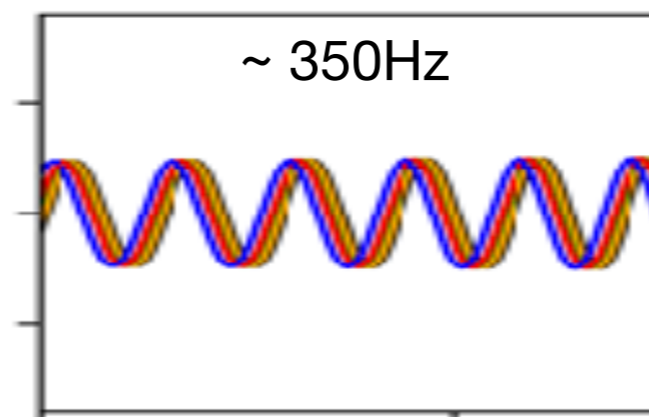
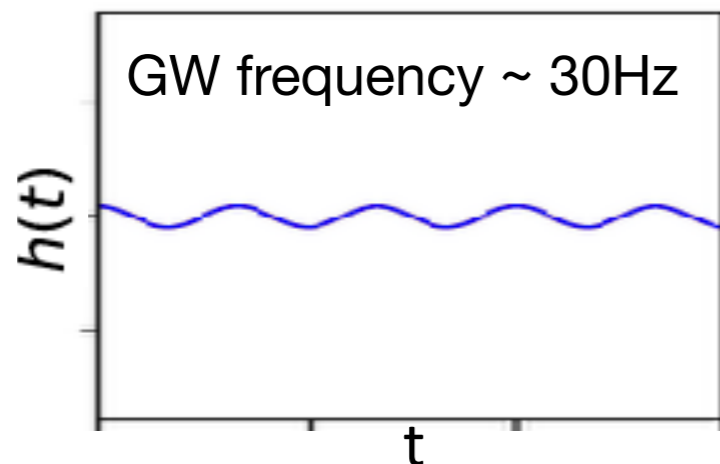
- moving multipoles contribute to **gravitational radiation**

$$\dot{E}_{\text{GW}} \sim \left[\frac{d^3}{dt^3} (Q_{\text{orbit}} + Q) \right]^2$$

- approx. **GW phase evolution** from energy balance:

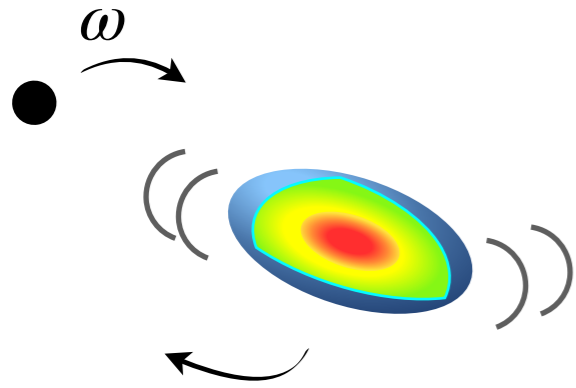
$$\Delta \phi_{\text{GW}}^{\text{tidal}} \sim \lambda \frac{(M\omega)^{10/3}}{M^5}$$

Examples
for different
EoSs
aligned at
30 Hz



Dashed lines: 1kHz

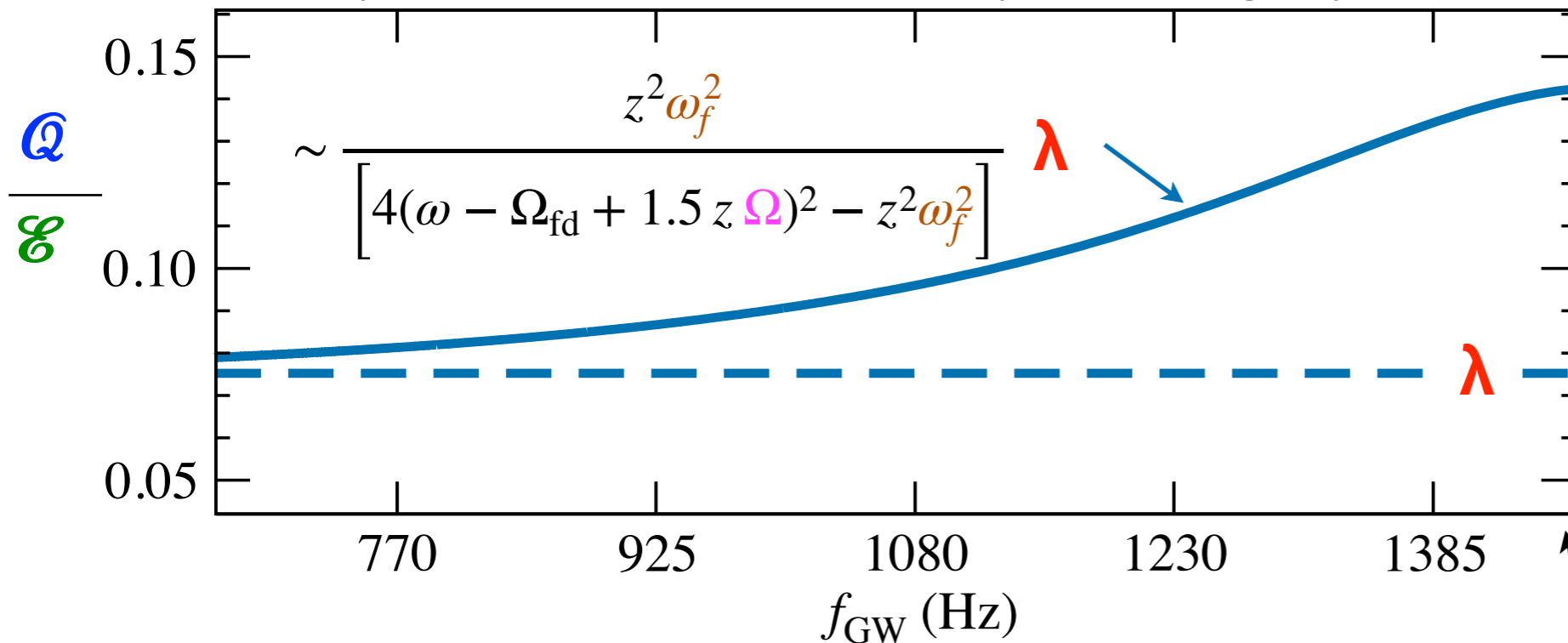
More realistic descriptions of tidal effects



- NSs have a rich spectrum of quasi-normal **oscillation modes**
- For some: **tidal excitations** in inspirals
- **Spectroscopy** of NS interiors, possible EM flares

- Even **non-resonant** excitations can lead to **significant** effects, e.g. fundamental modes with ω_f

Example fundamental-mode tidal response during inspiral (NSBH)



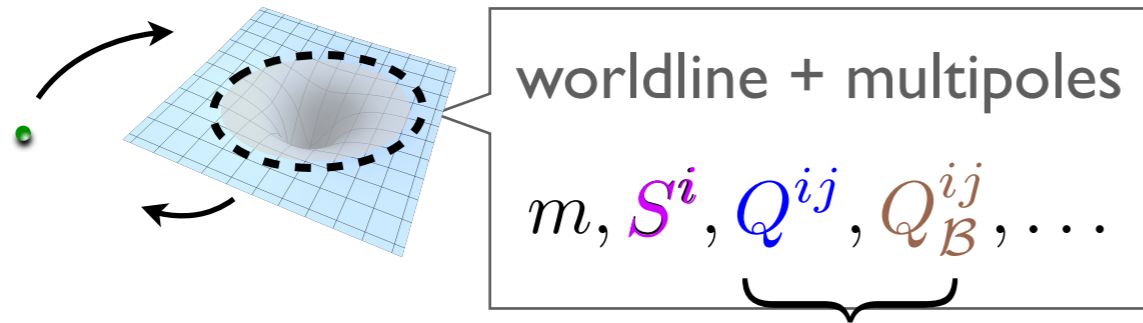
Response also impacted by:

- relativistic **redshift** z
- **frame dragging** Ω_{fd}
(GR, companion spin)
- **NS's spin** Ω

End of inspiral

More realistic description of matter effects on the dynamics

spacetime near the NS viewed on the orbital scale:



Gravitomagnetic modes

tidally induced mass & (matter contributions to) current multipoles

Effective action describing the binary dynamics:

Internal dynamics of the multipoles (modes)

$$S \approx S_{pp} + \int d\tau \left[-\frac{z}{2} Q_{ij} \mathcal{E}_{ij} - \frac{z}{2} \dot{Q}_{ij}^{\mathcal{B}} \mathcal{B}_{ij} + L^{\text{Coriolis}} + L^{\text{FD}} + L^{\text{OSC}} + \dots \right]$$

point-particle part

multipoles interact with companion's spacetime curvature

Effects of NS's spin on tidal response & restoring force for gravitomagnetic modes

Q's angular momentum interacts with orbital angular momentum & companion's spin

More realistic couplings of matter to orbital dynamics

Multipoles behave as harmonic oscillators:

$$L^{\text{osc}} \approx \sum_n \frac{z}{4\lambda_{(n)} z^2 \omega_{(n)}^2} \frac{dQ_{ij}^{(n)}}{d\tau} \frac{dQ_{ij}^{(n)}}{d\tau} - \frac{z}{4\lambda_{(n)}} Q_{ij}^{(n)} Q_{ij}^{(n)} + \dots$$

$$+ \frac{3z}{32(\sigma_{\text{stat}} - \sigma_{\text{irrot}})} \frac{d\dot{Q}_{ij}^{\mathcal{B}}}{d\tau} \frac{d\dot{Q}_{ij}^{\mathcal{B}}}{d\tau} + \frac{2z\sigma_{\text{stat}}}{3} \mathcal{B}_{ij} \mathcal{B}_{ij} + \dots$$

Different quadrupolar modes contribute, dominated by fundamental modes + similarly for higher multipoles
 subdominant but important for future GW detections

two different magnetic tidal deformabilities

$$S \approx S_{pp} + \int d\tau \left[-\frac{z}{2} Q_{ij} \mathcal{E}_{ij} - \frac{z}{2} \dot{Q}_{ij}^{\mathcal{B}} \mathcal{B}_{ij} + L^{\text{Coriolis}} + L^{\text{FD}} + L^{\text{osc}} + \dots \right]$$

point-particle part

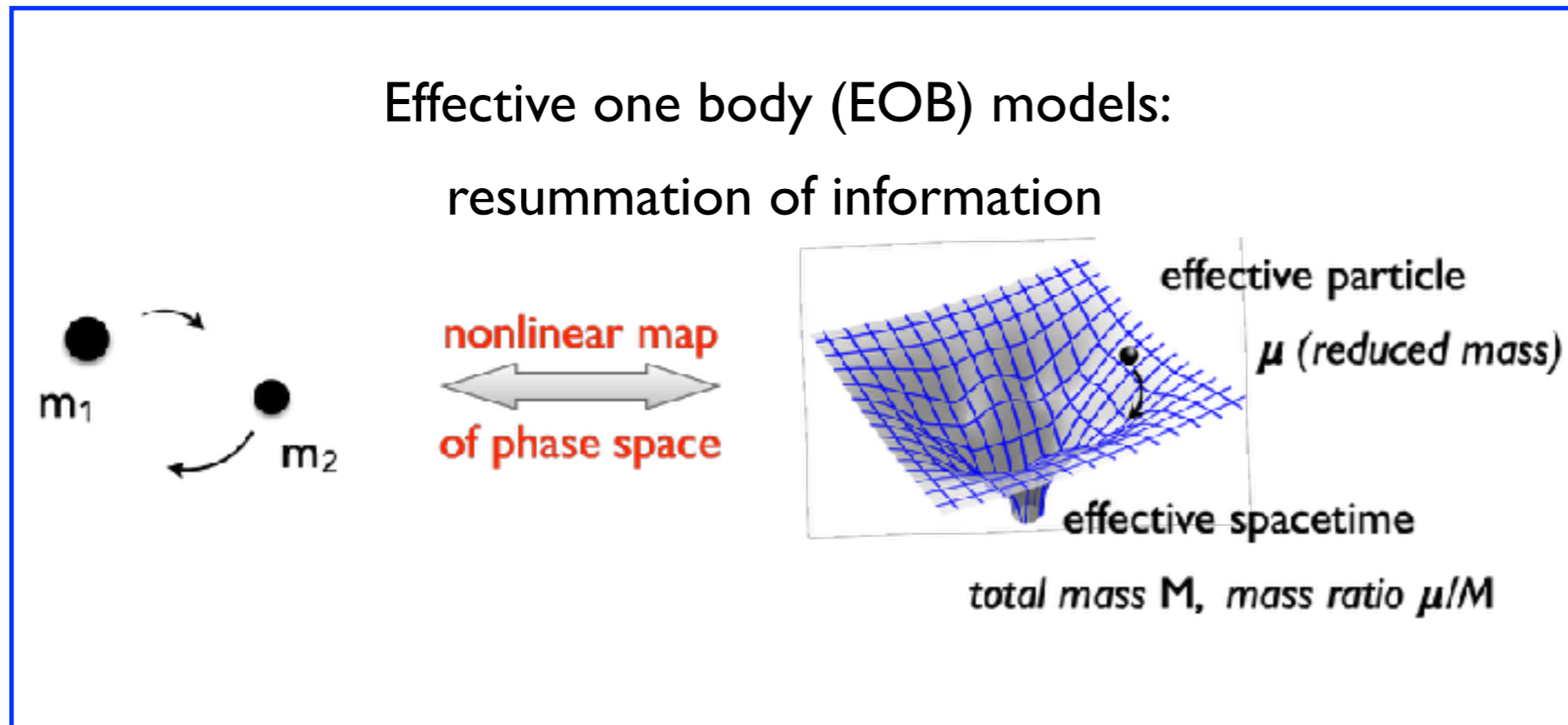
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Effects of NS's spin on tidal response & restoring force for gravitomagnetic modes

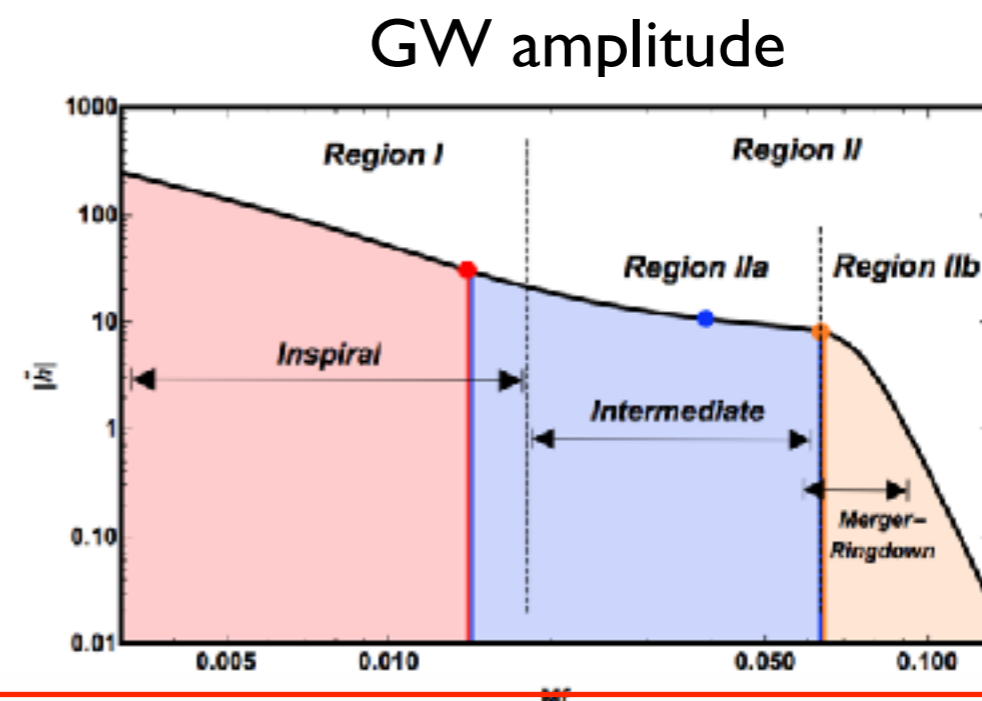
Q's angular momentum interacts with orbital angular momentum & companion's spin

Finite size effects included in models for data analysis

Matter effects on top of full black hole baseline models



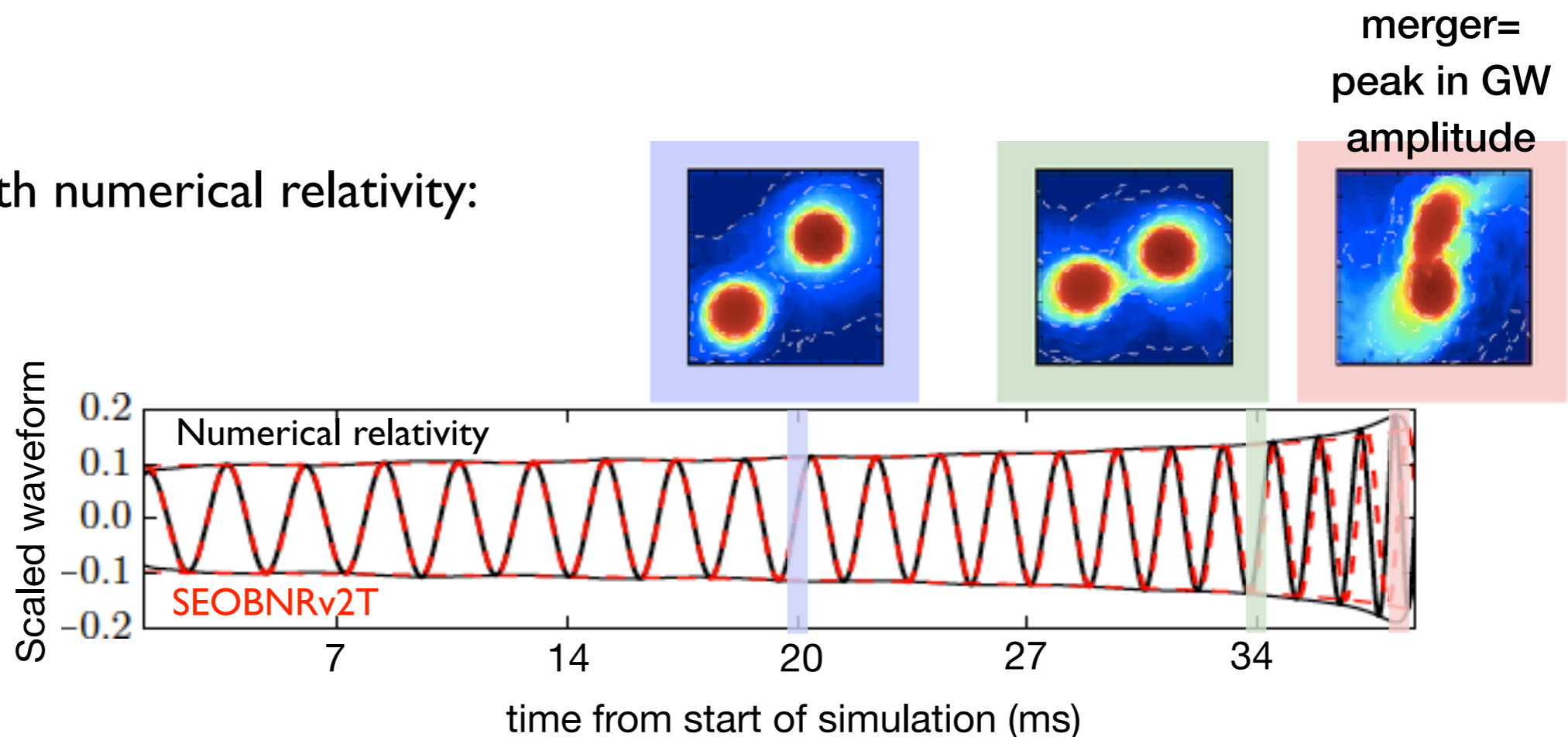
Phenomenological (Phenom)
models:
closed-form expressions for
frequency-domain GWs



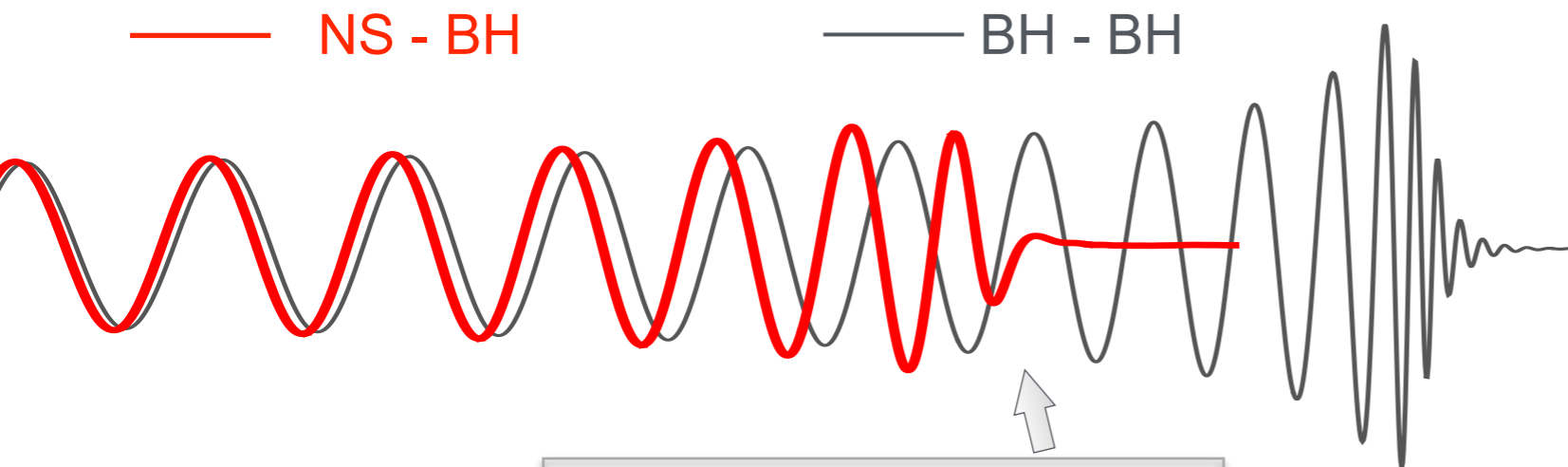
Matter effects in models for data analysis

- For inspirals: variety of physics & assumptions, e.g. **some but not all of the models**
 - Rely on quasi-universal relations used to reduce matter parameters to λ_1, λ_2
 - Are calibrated to numerical relativity
 - Include some dynamical tidal effects
 - ...

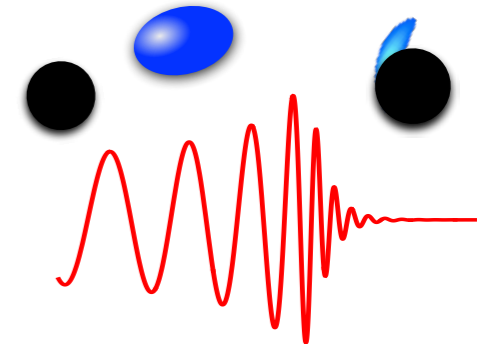
Comparison with numerical relativity:



GW signal from NS-black hole (BH) binaries



- alternative outcome: plunge



tidal disruption

Two diagrams showing tidal disruption. The left one shows a black circle (black hole) and a blue/green/red elongated shape (neutron star) being stretched. The right one shows the black hole and the neutron star after the event, with the neutron star having been disrupted.

Key signature:
'shutoff' in GW
amplitude

Shutoff frequency depends on
parameters
(mass ratio, spins, EoS, ...)

+ Multimessenger counterparts

Modeling uncertainties

sources of inaccuracies:

- Same ‘physics’, different modeling **choices**

(e.g. EOB, Phenom families, various tidal models)

- Missing **physics**

a lot is missing ... but also not *all* micro details are probed by GW measurements

- **For events measured to date:**

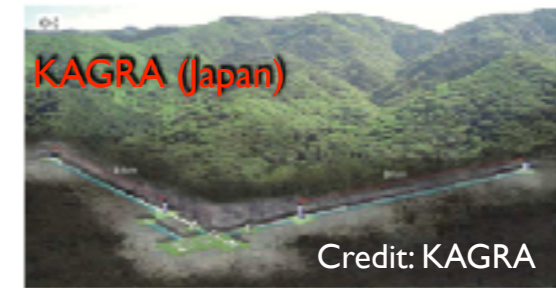
impact of modeling uncertainties still **subdominant** compared to statistical errors,

as far as could be determined, but **starting to become noticeable** in some cases

Abbot+2020, GWTC-2 (Appendix on waveform systematics)

Near-term future prospects

next observing run O4: LIGO/Virgo near/reaching design sensitivity

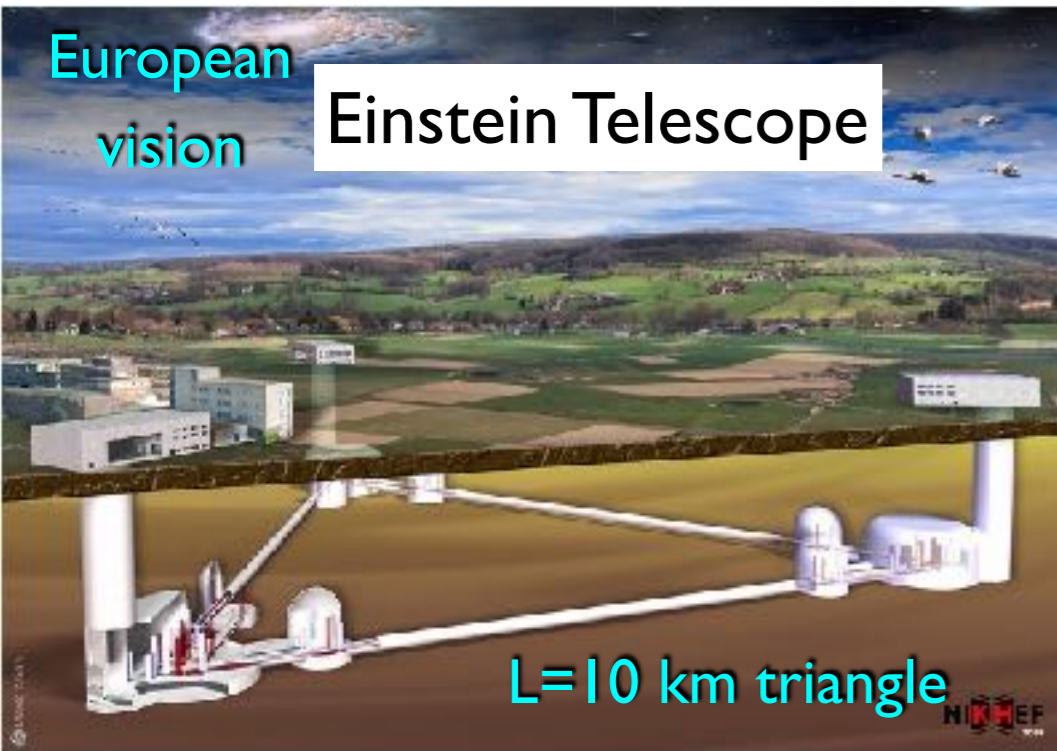


Further upgrades scheduled

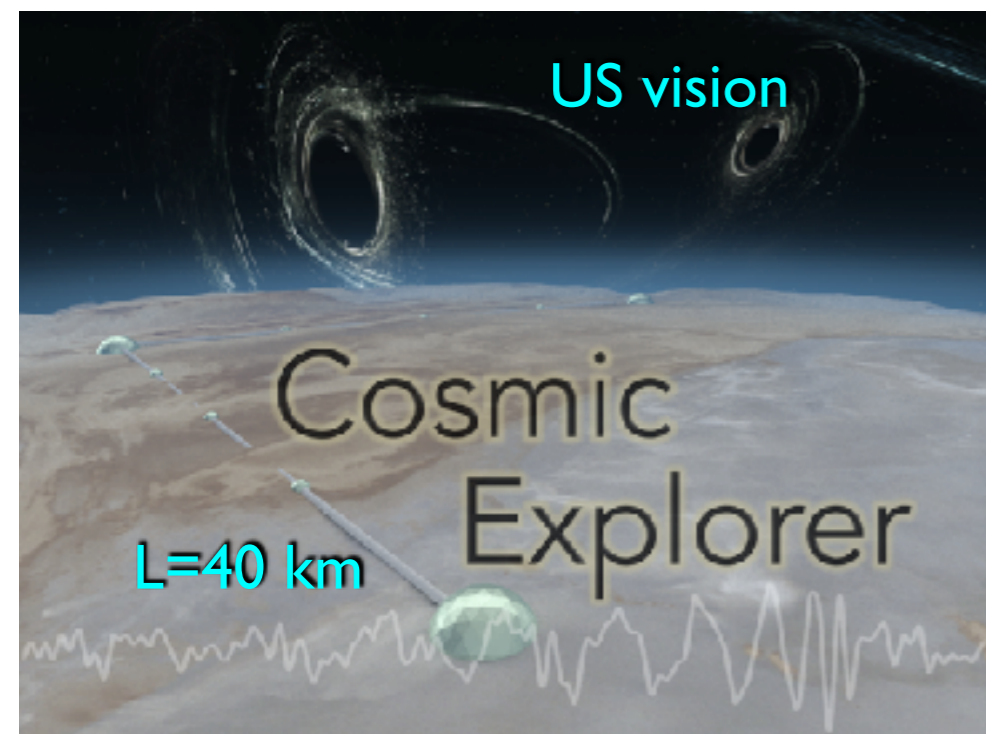


- More accurate measurements of nearby sources
- greater number & diversity of events

Plans for next-generation detectors moving ahead



- Prototype being built in Maastricht



$O(100\ 000)$ binary merger detections per year

High precision studies of nearby sources

Wider frequency range:

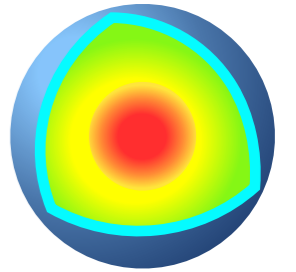
- Low-frequency:
 - tidal resonances, ...
 - infer 'initial state' & 'control parameters' of the collision
 - Early alerts for EM
- High frequency:
 - measure tidal disruptions
 - Probe NS mergers & beyond

A few examples of remaining theoretical challenges

- high-**accuracy** and **efficient** waveforms over wide **parameter space**
- Inspirals/tidal disruptions: more **matter** effects & **relativistic** corrections, **higher GW modes**, arbitrary **spins**, role of **beyond zero-temperature, equilibrium matter?**, **eccentricity**, ...
- Deeper understanding of complex **NS-NS merger** regimes with more **realistic physics**, robustly quantify **numerical errors**,
- **degeneracies** (e.g. modified gravity, dark matter), ...
-



Summary and outlook



- GWs are new **probes of NS physics**: clean gravitational channel of information
- Exciting near- & longer-term prospects with **larger & more precise datasets**
- Simultaneous **advances in modeling** are **essential** to fully realize the **science potential**, reduce biases in measurements and interpretation
- Significant recent progress on understanding, methods, modeling of relevant phenomena but much work remains
- Synergy of theoretical approaches (different **analytical** approximations + **numerical** relativity) important
- Interdisciplinary cooperation needed on **connections** and fundamental **inputs**

