Quantum-induced non-local actions for general relativity

1) Non-local actions in general

2) Quantum corrections in GR

3) Cosmology – singularity avoidance?

4) Black hole structure

Past work with Basem El-Menoufi
Ongoing work with Basem, Leandro Beviláqua and Russell Phelan
(see also Basem's talk for his independent work)

AMHERST CENTER FOR FUNDAMENTAL INTERACTIONS
Physics at the interface: Energy, Intensity, and Cosmic frontiers
University of Massachusetts Amherst

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Basic message:

We are used to the local derivative/energy expansion in GR

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots \right\}$$

and we know that quantum corrections generate R^2 terms

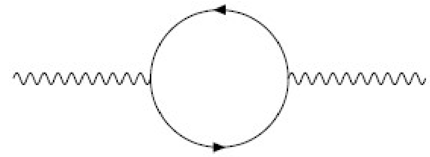
$$\Delta \mathcal{L}_0^{(1)} = \frac{1}{8\pi^2} \frac{1}{\epsilon} \left\{ \frac{1}{120} R^2 + \frac{7}{20} R_{\mu\nu} R^{\mu\nu} \right\}$$

but **real** quantum content of GR is a non-local action:

$$\begin{aligned} S_{tot} = \int d^4x \, \sqrt{g} \, \frac{2}{\kappa^2} R & \\ & + \left[\bar{\alpha} R \log\left(\nabla^2/\Lambda_1^2\right) R + \bar{\beta} C_{\mu\nu\alpha\beta} \log(\nabla^2/\Lambda_2^2) C^{\mu\nu\alpha\beta} \right. \\ & \left. + \bar{\gamma} \left(R_{\mu\nu\alpha\beta} \log\left(\nabla^2\right) R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} \log\left(\nabla^2\right) R^{\mu\nu} + R \log\left(\nabla^2\right) R \right) \right] + ... \end{aligned}$$

What is impact/role of non-local action?

Example: Non local action for massless QED:



Vacuum polarization contains divergences but also log q^2
Integrate out massless matter field and write effective action:

$$S = \int d^4x \ -\frac{1}{4}F_{\mu\nu}\left[\frac{1}{e^2(\mu)} - b_i \ln(\Box/\mu^2)\right] F^{\mu\nu} \qquad +\mathcal{O}(F^4)$$

Displays running of charge

$$b = \frac{1}{12\pi}$$

Really implies a non-local effective action:

$$\langle x| \ln\left(\frac{\Box}{\mu^2}\right)|y\rangle \equiv L(x-y) = \int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot(x-y)} \ln\left(\frac{-q^2}{\mu^2}\right)$$

Connection: Running and non-local effects

As usual, μ can be removed from

$$\frac{1}{e^2(\mu)} = b \log \Lambda^2/\mu^2$$

<u>**The running is kinematic**</u>:

$$S = \int d^4x \; -\frac{1}{4} F_{\mu\nu} \left[-b_i \ln(\Box/\Lambda^2) \right] F^{\mu\nu}$$

Deser Duff Isham

Aside: QED Trace anomaly:

Tree Lagrangian has no scale

$$A_\mu(x) \to \lambda A_\mu(\lambda x),\ \psi(x) \to \lambda^{3/2}\psi(\lambda x),\ \phi(x) \to \lambda\phi(\lambda x)$$

Such that $J^\mu_{\rm scale} = x_\nu \theta^{\mu\nu}$, $\partial_\mu J^\mu_{\rm scale} = \theta^\nu{}_\nu = 0$

But loops introduce scale dependence in the derivatives

$$S = \int d^4x \ -\frac{1}{4}F_{\mu\nu}\left[\frac{1}{e^2(\mu)} - b_i \ln(\Box/\mu^2)\right] F^{\mu\nu}$$

Now $L(x-y) \to \lambda^{-4}\left(L(x-y) - \ln\lambda^2 \delta^4(x-y)\right)$

$$\partial_\mu J^\mu_{\rm scale} = \theta^\nu{}_\nu = \frac{\partial \hat{\mathcal{L}}_\lambda}{\delta\lambda}|_{\lambda=1} = \frac{b}{4}F_{\rho\sigma}F^{\rho\sigma} \qquad \hat{\mathcal{L}}_\lambda = \lambda^{-4}\mathcal{L}[\lambda A(\lambda x)]$$

Anomaly not derivable from any local Lagrangian,

-but does come from a non-local action

- IR property, independent of any renormalization scheme

Another example: Chiral perturbation theory

Calculate all one-loop processes at once: (Gasser, Leutwyler)

$$D = D_0 + \delta ,$$
$$\delta = \{\hat{\Gamma}^\mu , \partial_\mu\} + \hat{\Gamma}^\mu \hat{\Gamma}_\mu + \bar{\sigma} ,$$

$$Z_{\text{one loop}} = \tfrac{1}{2} i \ln \det D_0 + \tfrac{1}{4} i \,\mathrm{Tr}\,(D_0^{-1}\delta) - \tfrac{1}{4} i \,\mathrm{Tr}\,(D_0^{-1}\delta D_0^{-1}\delta) + \cdots ,$$

Nonlocal action:

$$Z_u = \sum_{P,Q} \int dx\, dy\, [\{(\partial_{\mu\nu} - g_{\mu\nu}\Box\} M^r(x-y) - g_{\mu\nu} L(x-y)\} \hat{\Gamma}^\mu_{PQ}(x) \hat{\Gamma}^\nu_{QP}(y)$$
$$- \partial_\mu K(x-y) \hat{\Gamma}^\mu_{PQ}(x) \bar{\sigma}_{QP}(y) + \tfrac{1}{4} J^r(x-y) \bar{\sigma}_{PQ}(x) \bar{\sigma}_{QP}(y)] . \quad (8.13)$$

$$\bar{J}(s) = (32\pi^2)^{-1} \left\{ 2 + \frac{\Delta}{s} \ln \frac{M_Q^2}{M_P^2} - \frac{\Sigma}{\Delta} \ln \frac{M_Q^2}{M_P^2} - \frac{\nu}{s} \ln \frac{(s+\nu)^2 - \Delta^2}{(s-\nu)^2 - \Delta^2} \right\} ,$$
$$\nu^2 = s^2 + M_P^4 + M_Q^4 - 2s(M_P^2 + M_Q^2) - 2M_P^2 M_Q^2 ,$$
$$\Sigma = M_P^2 + M_Q^2 , \qquad \Delta = M_P^2 - M_Q^2 .$$

Expand in powers of the field

- get all the one-loop predictions (up to ϕ^6)

Now, back to QED example – lets add gravity:

Perturbatively:

$$\Gamma^{(1)}[A,h] = -\frac{1}{2}\int d^4x\, h^{\mu\nu}\left[b_f \log\left(\frac{\Box}{\mu^2}\right) T^{cl}_{\mu\nu} - \frac{b_f}{2}\frac{1}{\Box}\tilde{T}^f_{\mu\nu}\right]$$

with the classical term

**

$$T_{\mu\nu} = -F_{\mu\sigma}F^{\sigma}_{\nu} + \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}$$

and

$$\tilde{T}^f_{\mu\nu} = -F_{\alpha\beta}\partial_\mu\partial_\nu F^{\alpha\beta} - \frac{1}{2}\eta_{\mu\nu}\partial_\lambda F_{\alpha\beta}\partial^\lambda F^{\alpha\beta}$$

Consistent with scale and conformal anomalies

Lets make this covariant

The problem of covariant $\ln \nabla^2$

Expect

$$\frac{b_i}{4}\int d^4x\ \eta^{\mu\alpha}\eta^{\nu\beta}F_{\mu\nu}\ln\left(\Box/\mu^2\right)F_{\alpha\beta} \to \frac{b_i}{4}\int d^4x\sqrt{-g}\ g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}\ln\left(\nabla^2/\mu^2\right)F_{\alpha\beta}$$

But $\ln \nabla^2$ is not uniquely defined!

1) Single propagator version:

$$\ln\left(\nabla^2/\mu^2\right) = -\int_0^\infty dm^2\left[\frac{1}{\nabla^2+m^2} - \frac{1}{\mu^2+m^2}\right]$$

2) Double propagator version:

Osborn-Erdmenger

$$\frac{i}{16\pi^2} < x|\log\nabla^2|y >= \Delta_F^2(x-y) - \frac{i}{16\pi^2}\left[\frac{1}{\epsilon} - \gamma - \log 4\pi\right]\delta^4(x-y)$$

Both versions have IR singularities not found in direct calculation

#1 is $1/\lambda^2$ and #2 is $\ln\lambda$

For example, with single propagator version:

$$\int d^4x\sqrt{g}\,F^{\alpha\beta}\ln\left(\frac{\nabla^2}{\mu^2}\right)F_{\alpha\beta} = \int d^4x\,\left[F^{\alpha\beta}\ln\left(\Box/\mu^2\right)F_{\alpha\beta} + h_{\mu\nu}\left(\mathcal{O}_1^{\mu\nu} + \mathcal{O}_2^{\mu\nu}\right)\right]$$

where

$$\mathcal{O}_1^{\mu\nu} = \frac{1}{2}\eta^{\mu\nu}F^{\alpha\beta}\ln(\Box/\mu^2)F_{\alpha\beta} - 2F^{\mu}_{\ \alpha}\log(\Box/\mu^2)F^{\nu\alpha}$$

$$\mathcal{O}_2^{\mu\nu} = \partial^\mu\partial^\nu F_{\alpha\beta}\frac{1}{\Box}F^{\alpha\beta} + \partial^\mu\partial^\nu F^{\alpha\beta}\frac{1}{\Box}F_{\alpha\beta} - \eta^{\mu\nu}\partial_\lambda F_{\alpha\beta}\frac{1}{\Box}\partial^\lambda F^{\alpha\beta}$$

Unphysical
- 1/(photon mass/momentum) !!

These terms show no relation to what was found by calculation!

Covariant action (for specific $\ln \nabla^2$):

Real
(Riegert)

$$
\begin{aligned}
\Gamma_{log} = &\frac{b_i}{4} \int d^4x \sqrt{g} \Big\{ F_{\alpha\beta} \ln\left(\nabla^2/\mu^2\right) F^{\alpha\beta} - \frac{1}{3} F_{\alpha\beta} F^{\alpha\beta} \frac{1}{\nabla^2} R \\
&+ 4R^{\mu\nu} \frac{1}{\nabla^2} \Big[\log(\nabla^2) \left(-F_{\mu\sigma} F_{\nu}{}^{\sigma} + \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right) \\
&+ F_{\mu\sigma} \log(\nabla^2) F_{\nu}{}^{\sigma} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} \log(\nabla^2) F^{\alpha\beta} \Big] \\
&+ \frac{1}{3} R F_{\alpha\beta} \frac{1}{\nabla^2} F^{\alpha\beta} - C^{\alpha}{}_{\beta\mu\nu} F_{\alpha}{}^{\beta} \frac{1}{\nabla^2} F^{\mu\nu} \Big\}
\end{aligned}
$$

Compensation

Matching procedure used here:

- nonlinear completion
- matching general covariant form to perturbative result

<u>**Now on to General Relativity**</u>:

Important for quantum GR to get beyond scattering amplitudes

Pioneered by Barvinsky, Vilkovisky and collaborators

<u>Non-local curvature exapansion:</u>

Second order in the curvature $R \ln \nabla^2 R$

Third order in the curvature $R^2 \frac{1}{\nabla^2} R$

This is very different from the local derivative expansion,
but is required to capture known quantum effects

Calculable by non-local heat kernel or by matching to PT

Second order is simple – tied to renormalization:

Barvinsky, Vilkovisky, Avrimidi

Perturbative running is contained in the R^2 terms

$$S_4 = \int d^4x \sqrt{g}\left[c_1(\mu)R^2 + c_2(\mu)R_{\mu\nu}R^{\mu\nu}\right]$$
$$+ \left[\bar{\alpha}R\log\left(\nabla^2/\mu^2\right)R + \bar{\beta}C_{\mu\nu\alpha\beta}\log(\nabla^2/\mu^2)C^{\mu\nu\alpha\beta}\right.$$
$$\left.+ \bar{\gamma}\left(R_{\mu\nu\alpha\beta}\log\left(\nabla^2\right)R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}\log\left(\nabla^2\right)R^{\mu\nu} + R\log\left(\nabla^2\right)R\right)\right] \quad +\mathcal{O}(R^3)$$

Again running can all be packaged in non-local terms:

$$S_{tot} = \int d^4x \sqrt{g}\,\frac{2}{\kappa^2}R$$
$$+ \left[\bar{\alpha}R\log\left(\nabla^2/\Lambda_1^2\right)R + \bar{\beta}C_{\mu\nu\alpha\beta}\log(\nabla^2/\Lambda_2^2)C^{\mu\nu\alpha\beta}\right.$$
$$\left.+ \bar{\gamma}\left(R_{\mu\nu\alpha\beta}\log\left(\nabla^2\right)R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}\log\left(\nabla^2\right)R^{\mu\nu} + R\log\left(\nabla^2\right)R\right)\right] + \ldots.$$

	α	β	γ	$\bar{\alpha}$	$\bar{\beta}$	$\bar{\gamma}$
Scalar	$5(6\xi-1)^2$	-2	2	$5(6\xi-1)^2$	3	-1
Fermion	-5	8	7	0	18	-11
Vector	-50	176	-26	0	36	-62
Graviton	430	-1444	424	90	126	298

Coefficients of different fields. All numbers should be divided by $11520\pi^2$

Comment on non-local basis:

need three terms

$$\left(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + RR\right)$$ is a total derivative

$$\left(R_{\mu\nu\alpha\beta}\log\left(\Box\right)R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}\log\left(\Box\right)R^{\mu\nu} + R\log\left(\Box\right)R\right)$$ is not

Calculationally simplest basis:

$$S_{QL} = \int d^4x\sqrt{g}\left(\alpha R\log\left(\frac{\Box}{\mu_\alpha^2}\right)R + \beta R_{\mu\nu}\log\left(\frac{\Box}{\mu_\beta^2}\right)R^{\mu\nu} + \gamma R_{\mu\nu\alpha\beta}\log\left(\frac{\Box}{\mu_\gamma^2}\right)R^{\mu\nu\alpha\beta}\right)$$

Conceptually better basis:

$$S_{QL} = \int d^4x\,\sqrt{g}\,\Big[\bar{\alpha}R\log\left(\frac{\Box}{\mu_1^2}\right)R + \bar{\beta}C_{\mu\nu\alpha\beta}\log\left(\frac{\Box}{\mu_2^2}\right)C^{\mu\nu\alpha\beta} + \bar{\gamma}\big(R_{\mu\nu\alpha\beta}\log\left(\Box\right)R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}\log\left(\Box\right)R^{\mu\nu} + R\log\left(\Box\right)R\big)\Big]\,.$$

$$C_{\mu\nu\alpha\beta} = R_{\mu\nu\alpha\beta} - \frac{1}{2}\left(g_{\mu\alpha}R_{\nu\beta} - g_{\mu\beta}R_{\nu\alpha} + g_{\nu\alpha}R_{\mu\beta} - g_{\nu\beta}R_{\mu\alpha}\right) + \frac{1}{6}R\left(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}\right)$$

Last term has no scale dependence
Second term (Weyl tensor) vanishes in FLRW
First term vanishes for conformal fields

Third order in curvature is a mess:

BV+

Tied to definition of ln ∇^2

-BV use the "single propagator" version

- lots of spurious compensation terms

But many different "real" terms also

- arises from massless triangle diagram
- permutations of $\frac{1}{\nabla^2}$ and $\frac{(\ln \nabla^2)}{\nabla^2}$

Real IR singularities in general

- recall Passarino-Veltman reduction
- Fourth order in curvature is the box diagram (worse)

194 pages of dense results, such as these:

$$
\begin{aligned}
&\int dx\, g^{1/2}\,\mathrm{tr}\,\hat{a}_4(x,x) = \int dx\, g^{1/2}\,\mathrm{tr}\,\{\frac{\Box_2{}^2}{120}\hat{P}_1\hat{P}_2 + \frac{\Box_2{}^2}{1260}\hat{P}_1 R_2 + \frac{\Box_2{}^2}{1680}\hat{\mathcal{R}}_{1\mu\nu}\hat{\mathcal{R}}_2^{\mu\nu} \\
&+ \frac{\Box_2{}^2}{15120}R_{1\,\mu\nu}R_2^{\mu\nu}\hat{1} + \frac{\Box_3}{24}\hat{P}_1\hat{P}_2\hat{P}_3 - \frac{\Box_3}{630}\hat{\mathcal{R}}_1{}^{\mu}{}_{\alpha}\hat{\mathcal{R}}_2{}^{\alpha}{}_{\beta}\hat{\mathcal{R}}_3{}^{\beta}{}_{\mu} \\
&+ \Big(\frac{\Box_1}{180} + \frac{\Box_2}{180} + \frac{\Box_3}{90}\Big)\hat{\mathcal{R}}_1^{\mu\nu}\hat{\mathcal{R}}_{2\,\mu\nu}\hat{P}_3 + \Big(\frac{\Box_1}{7560} - \frac{\Box_3}{15120}\Big)R_1 R_2\hat{P}_3 \\
&+ \Big(\frac{\Box_1}{1680} + \frac{\Box_1{}^2}{1680\Box_2} + \frac{\Box_3}{2520} + \frac{\Box_1\Box_3}{1680\Box_2} - \frac{\Box_3{}^2}{336\Box_2} + \frac{\Box_3{}^3}{1120\Box_1\Box_2}\Big)R_1^{\mu\nu}R_{2\,\mu\nu}\hat{P}_3 \\
&+ \frac{\Box_3}{720}\hat{P}_1\hat{P}_2 R_3 + \Big(\frac{13\Box_1}{30240} - \frac{\Box_3}{15120}\Big)R_1\hat{\mathcal{R}}_2^{\mu\nu}\hat{\mathcal{R}}_{3\,\mu\nu} \\
&+ \Big(\frac{\Box_1}{840} + \frac{\Box_3}{210} + \frac{\Box_2\Box_3}{210\Box_1} + \frac{\Box_3{}^2}{210\Box_1}\Big)R_1^{\alpha\beta}\hat{\mathcal{R}}_{2\,\alpha}{}^{\mu}\hat{\mathcal{R}}_{3\,\beta\mu} \\
&+ \Big(\frac{\Box_1{}^2}{25200\Box_3} + \frac{\Box_1\Box_2}{50400\Box_3} - \frac{\Box_3}{25200} + \frac{\Box_3{}^3}{50400\Box_1\Box_2}\Big)R_1 R_2 R_3\hat{1} \\
&+ \Big(-\frac{\Box_1{}^2}{9450\Box_3} - \frac{\Box_1\Box_2}{18900\Box_3} - \frac{\Box_3}{12600} + \frac{\Box_3{}^3}{12600\Box_1\Box_2}\Big)R^{\mu}_{1\,\alpha}R^{\alpha}_{2\,\beta}R^{\beta}_{3\,\mu}\hat{1} \\
&+ \Big(\frac{\Box_1}{151200} - \frac{\Box_1{}^2}{151200\Box_2} + \frac{\Box_3}{25200} + \frac{\Box_1\Box_3}{18900\Box_2} - \frac{13\Box_3{}^2}{151200\Box_2} \\
&+ \frac{\Box_3{}^3}{50400\Box_1\Box_2}\Big)R_1^{\mu\nu}R_{2\,\mu\nu}R_3\hat{1} + \frac{1}{252}\hat{\mathcal{R}}_1^{\alpha\beta}\nabla^{\mu}\hat{\mathcal{R}}_{2\mu\alpha}\nabla^{\nu}\hat{\mathcal{R}}_{3\nu\beta} \\
&+ \frac{1}{60}\hat{\mathcal{R}}_1^{\mu\nu}\nabla_{\mu}\hat{P}_2\nabla_{\nu}\hat{P}_3 + \frac{1}{180}\nabla_{\mu}\hat{\mathcal{R}}_1^{\mu\alpha}\nabla^{\nu}\hat{\mathcal{R}}_{2\,\nu\alpha}\hat{P}_3 - \frac{1}{1890}R_1^{\mu\nu}\nabla_{\mu}R_2\nabla_{\nu}\hat{P}_3 \\
&+ \Big(\frac{1}{630} + \frac{\Box_1}{420\Box_2} + \frac{\Box_3}{210\Box_2} - \frac{\Box_3{}^2}{280\Box_1\Box_2}\Big)\nabla^{\mu}R_1^{\nu\alpha}\nabla_{\nu}R_{2\,\mu\alpha}\hat{P}_3 \\
&+ \frac{1}{180}R_1^{\mu\nu}\nabla_{\mu}\nabla_{\nu}\hat{P}_2\hat{P}_3 + \Big(\frac{1}{1260} + \frac{\Box_3}{105\Box_1}\Big)R_{1\,\alpha\beta}\nabla_{\mu}\hat{\mathcal{R}}_2^{\mu\alpha}\nabla_{\nu}\hat{\mathcal{R}}_3^{\nu\beta} \\
&+ \Big(-\frac{1}{1260} - \frac{\Box_3}{210\Box_1}\Big)R_1^{\alpha\beta}\nabla_{\alpha}\hat{\mathcal{R}}_2^{\mu\nu}\nabla_{\beta}\hat{\mathcal{R}}_{3\,\mu\nu} - \frac{1}{7560}R_1\nabla_{\alpha}\hat{\mathcal{R}}_2^{\alpha\mu}\nabla^{\beta}\hat{\mathcal{R}}_{3\,\beta\mu} \\
&+ \Big(\frac{1}{630} + \frac{\Box_2}{105\Box_1} + \frac{\Box_3}{105\Box_1}\Big)R_1^{\mu\nu}\nabla_{\mu}\nabla_{\lambda}\hat{\mathcal{R}}_2^{\lambda\alpha}\hat{\mathcal{R}}_{3\,\alpha\nu} + \Big(\frac{1}{226800} - \frac{\Box_1}{8400\Box_2} \\
&- \frac{\Box_1{}^2}{10080\Box_2\Box_3} + \frac{\Box_3}{25200\Box_1} - \frac{\Box_3}{25200\Box_2} - \frac{\Box_3{}^2}{25200\Box_1\Box_2}\Big)R_1^{\alpha\beta}\nabla_{\alpha}R_2\nabla_{\beta}R_3\hat{1} \\
&+ \Big(-\frac{\Box_1}{37800\Box_2} + \frac{\Box_3}{5400\Box_2} - \frac{\Box_3{}^2}{12600\Box_1\Box_2}\Big)\nabla^{\mu}R_1^{\nu\alpha}\nabla_{\nu}R_{2\,\mu\alpha}R_3\hat{1} \\
&+ \Big(-\frac{1}{9450} - \frac{\Box_1}{12600\Box_2} + \frac{\Box_1{}^2}{8400\Box_2\Box_3} - \frac{\Box_3}{6300\Box_2}\Big)R_1^{\mu\nu}\nabla_{\mu}R_2^{\alpha\beta}\nabla_{\nu}R_{3\,\alpha\beta}\hat{1} \\
&+ \Big(-\frac{1}{3150} - \frac{\Box_1}{9450\Box_2} - \frac{\Box_1{}^2}{6300\Box_2\Box_3} - \frac{\Box_3}{3150\Box_1} + \frac{\Box_3}{9450\Box_2} + \frac{\Box_3{}^2}{3150\Box_1\Box_2}\Big) \\
&\times R_1^{\mu\nu}\nabla_{\alpha}R_{2\,\beta\mu}\nabla^{\beta}R^{\alpha}_{3\,\nu}\hat{1} + \Big(\frac{1}{420\Box_2} + \frac{\Box_3}{280\Box_1\Box_2}\Big)\nabla_{\alpha}\nabla_{\beta}R_1^{\mu\nu}\nabla_{\mu}\nabla_{\nu}R_2^{\alpha\beta}\hat{P}_3 \\
&+ \Big(-\frac{1}{3780\Box_2} - \frac{1}{6300\Box_3} - \frac{\Box_1}{4200\Box_2\Box_3} + \frac{\Box_3}{25200\Box_1\Box_2}\Big)\nabla_{\alpha}\nabla_{\beta}R_1^{\mu\nu}\nabla_{\mu}\nabla_{\nu}R_2^{\alpha\beta}R_3\hat{1}
\end{aligned}
$$

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$$
\begin{aligned}
&\Gamma_{12}(-\Box_1,-\Box_2,-\Box_3) = \Gamma(-\Box_1,-\Box_2,-\Box_3)\frac{1}{D^3}\Big(-2\Box_1{}^5 \\
&+ 4\Box_1{}^4\Box_2 - 4\Box_1{}^2\Box_2{}^3 + 2\Box_1\Box_2{}^4 + 4\Box_1{}^4\Box_3 \\
&- 24\Box_1{}^3\Box_2\Box_3 + 12\Box_1{}^2\Box_2{}^2\Box_3 + 8\Box_1\Box_2{}^3\Box_3 + 12\Box_1{}^2\Box_2\Box_3{}^2 \\
&- 20\Box_1\Box_2{}^2\Box_3{}^2 - 4\Box_1{}^2\Box_3{}^3 + 8\Box_1\Box_2\Box_3{}^3 + 2\Box_1\Box_3{}^4\Big) \\
&+ \frac{\ln(\Box_1/\Box_2)}{9D^3\Box_2\Box_3}\Big(-2\Box_1{}^5\Box_2 + 10\Box_1{}^4\Box_2{}^2 - 20\Box_1{}^3\Box_2{}^3 \\
&+ 20\Box_1{}^2\Box_2{}^4 - 10\Box_1\Box_2{}^5 + 2\Box_2{}^6 - \Box_1{}^5\Box_3 - 21\Box_1{}^4\Box_2\Box_3 \\
&- 6\Box_1{}^3\Box_2{}^2\Box_3 + 66\Box_1{}^2\Box_2{}^3\Box_3 - 25\Box_1\Box_2{}^4\Box_3 - 13\Box_2{}^5\Box_3 \\
&+ 5\Box_1{}^4\Box_3{}^2 + 36\Box_1{}^3\Box_2\Box_3{}^2 - 162\Box_1{}^2\Box_2{}^2\Box_3{}^2 - 36\Box_1\Box_2{}^3\Box_3{}^2
\end{aligned}
$$

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$$
\begin{aligned}
&+ 29\Box_2{}^4\Box_3{}^2 - 10\Box_1{}^3\Box_3{}^3 - 6\Box_1{}^2\Box_2\Box_3{}^3 + 78\Box_1\Box_2{}^2\Box_3{}^3 \\
&- 30\Box_2{}^3\Box_3{}^3 + 10\Box_1{}^2\Box_3{}^4 - 2\Box_1\Box_2\Box_3{}^4 + 16\Box_2{}^2\Box_3{}^4 \\
&- 5\Box_1\Box_3{}^5 - 5\Box_2\Box_3{}^5 + \Box_3{}^6\Big) \\
&+ \frac{\ln(\Box_1/\Box_3)}{9D^3\Box_2\Box_3}\Big(-\Box_1{}^5\Box_2 + 5\Box_1{}^4\Box_2{}^2 - 10\Box_1{}^3\Box_2{}^3 \\
&+ 10\Box_1{}^2\Box_2{}^4 - 5\Box_1\Box_2{}^5 + \Box_2{}^6 - 2\Box_1{}^5\Box_3 \\
&- 21\Box_1{}^4\Box_2\Box_3 + 36\Box_1{}^3\Box_2{}^2\Box_3 - 6\Box_1{}^2\Box_2{}^3\Box_3 - 2\Box_1\Box_2{}^4\Box_3 \\
&- 5\Box_2{}^5\Box_3 + 10\Box_1{}^4\Box_3{}^2 - 6\Box_1{}^3\Box_2\Box_3{}^2 - 162\Box_1{}^2\Box_2{}^2\Box_3{}^2 \\
&+ 78\Box_1\Box_2{}^3\Box_3{}^2 + 16\Box_2{}^4\Box_3{}^2 - 20\Box_1{}^3\Box_3{}^3 + 66\Box_1{}^2\Box_2\Box_3{}^3 \\
&- 36\Box_1\Box_2{}^2\Box_3{}^3 - 30\Box_2{}^3\Box_3{}^3 + 20\Box_1{}^2\Box_3{}^4 - 25\Box_1\Box_2\Box_3{}^4 \\
&+ 29\Box_2{}^2\Box_3{}^4 - 10\Box_1\Box_3{}^5 - 13\Box_2\Box_3{}^5 + 2\Box_3{}^6\Big) \\
&+ \frac{\ln(\Box_2/\Box_3)}{9D^3\Box_2\Box_3}\Big(\Box_1{}^5\Box_2 - 5\Box_1{}^4\Box_2{}^2 + 10\Box_1{}^3\Box_2{}^3 \\
&- 10\Box_1{}^2\Box_2{}^4 + 5\Box_1\Box_2{}^5 - \Box_2{}^6 - \Box_1{}^5\Box_3 \\
&+ 42\Box_1{}^3\Box_2{}^2\Box_3 - 72\Box_1{}^2\Box_2{}^3\Box_3 + 23\Box_1\Box_2{}^4\Box_3 + 8\Box_2{}^5\Box_3 \\
&+ 5\Box_1{}^4\Box_3{}^2 - 42\Box_1{}^3\Box_2\Box_3{}^2 + 114\Box_1\Box_2{}^3\Box_3{}^2 - 13\Box_2{}^4\Box_3{}^2 \\
&- 10\Box_1{}^3\Box_3{}^3 + 72\Box_1{}^2\Box_2\Box_3{}^3 - 114\Box_1\Box_2{}^2\Box_3{}^3 + 10\Box_1{}^2\Box_3{}^4 \\
&- 23\Box_1\Box_2\Box_3{}^4 + 13\Box_2{}^2\Box_3{}^4 - 5\Box_1\Box_3{}^5 - 8\Box_2\Box_3{}^5 + \Box_3{}^6\Big) \\
&+ \frac{\ln(\Box_1/\Box_2)}{(\Box_1-\Box_2)}\frac{1}{3\Box_3} \\
&+ \frac{\ln(\Box_1/\Box_3)}{(\Box_1-\Box_3)}\frac{1}{3\Box_2} \\
&+ \frac{1}{3D^2}\Big(16\Box_1{}^2 - 12\Box_1\Box_2 - 4\Box_2{}^2 - 12\Box_1\Box_3 + 8\Box_2\Box_3 - 4\Box_3{}^2\Big),
\end{aligned}
$$

The physics of the second order non-local action

1) Hints of singularity avoidance

- FLRW equations become non-local in time
- perturbative treatment indicates singularity avoidance
- can be made more theoretically controlled by large N

2) Black hole structure

- Schwd. solution no longer solves non-local vacuum equations
- dimensional analysis reveals nature of non-local curvature expansion

Cosmology

FLRW equations become non-local in time
   - use Schwinger-Keldish for correct BC

Hawking Penrose assumptions no longer valid
   - seem to avoid *some* singularities

Key points/caveats:

- working to second order in curvature
- Use flat ln ∇^2- difference is higher order in curvature
- Perturbative treatment – classical behavior as source
    - this approximation is being explored now
- Effects near but below Planck scale – control by N
    - $E \sim \frac{M_P}{\sqrt{N}}$

<u>Non-local FLRW equations</u>:

Quantum memory

$$\frac{3a\dot{a}^2}{8\pi} + N_s \left[6(\sqrt{a}\,\ddot{a})_t \int dt'\, L(t-t')\mathcal{R}_1 + 6\left(\frac{\dot{a}^2}{\sqrt{a}}\right) \int dt'\, L(t-t')\mathcal{R}_2 + 12(\sqrt{a}\dot{a})_t \int dt'\, L(t-t')\frac{d\mathcal{R}_3}{dt'} \right] = a^3\rho$$

with

$$\mathcal{R}_1 = -\sqrt{a}\ddot{a}(6\alpha + 2\beta + 2\gamma) - \frac{\dot{a}^2}{\sqrt{a}}(6\alpha + \beta)$$

$$\mathcal{R}_2 = -\sqrt{a}\ddot{a}(12\alpha + \beta - 2\gamma) - \frac{\dot{a}^2}{\sqrt{a}}(12\alpha + 5\beta - 6\gamma)$$

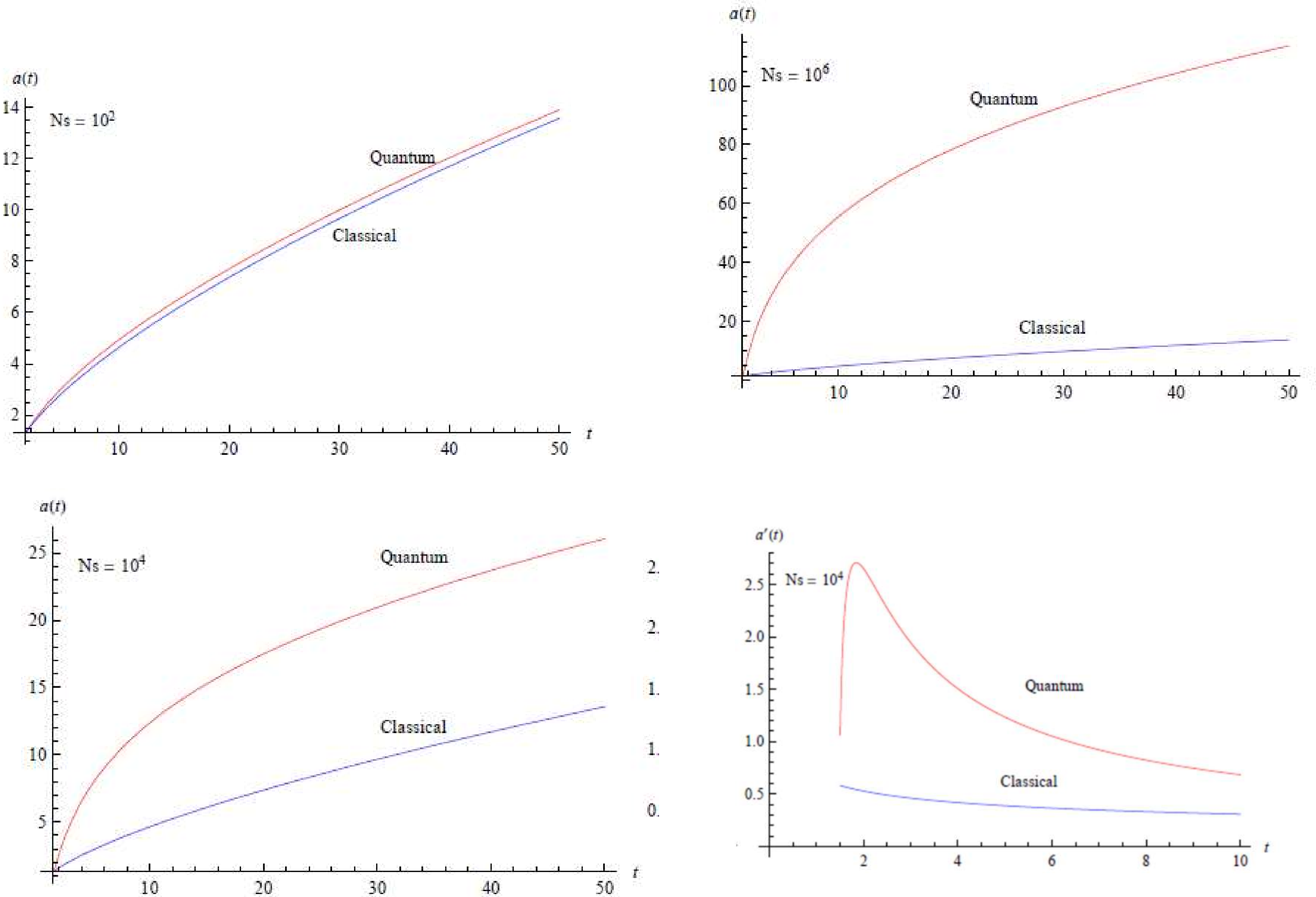
$$\mathcal{R}_3 = \sqrt{a}\ddot{a}(6\alpha + 2\beta + 2\gamma) + \frac{\dot{a}^2}{\sqrt{a}}(6\alpha + \beta)$$

and the time-dependent weight:

$$L(t-t') = \lim_{\epsilon\to 0}\left[\frac{\theta(t-t'-\epsilon)}{t-t'} + \delta(t-t')\log(\mu_R\,\epsilon)\right]$$

For scalars: $\alpha = \dfrac{1}{2304\pi^2} \quad \beta = \dfrac{-1}{5760\pi^2}, \quad \gamma = \dfrac{1}{5760\pi^2}$

Emergence of classical behavior:



Collapsing universe – singularity avoidance

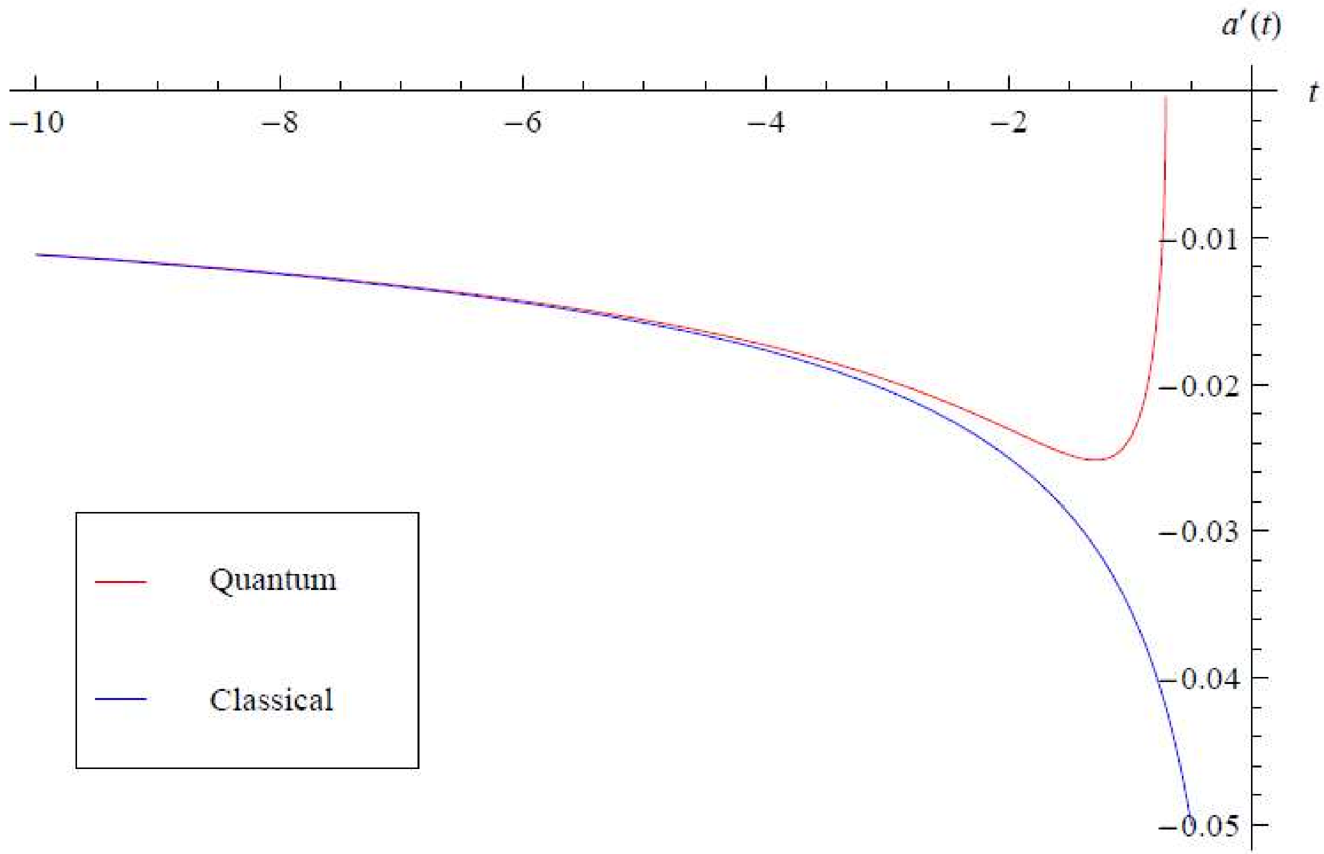


FIG. 12: Collapsing radiation-filled universe with gravitons only considered.

No free parameters in this result

With all the standard model fields:

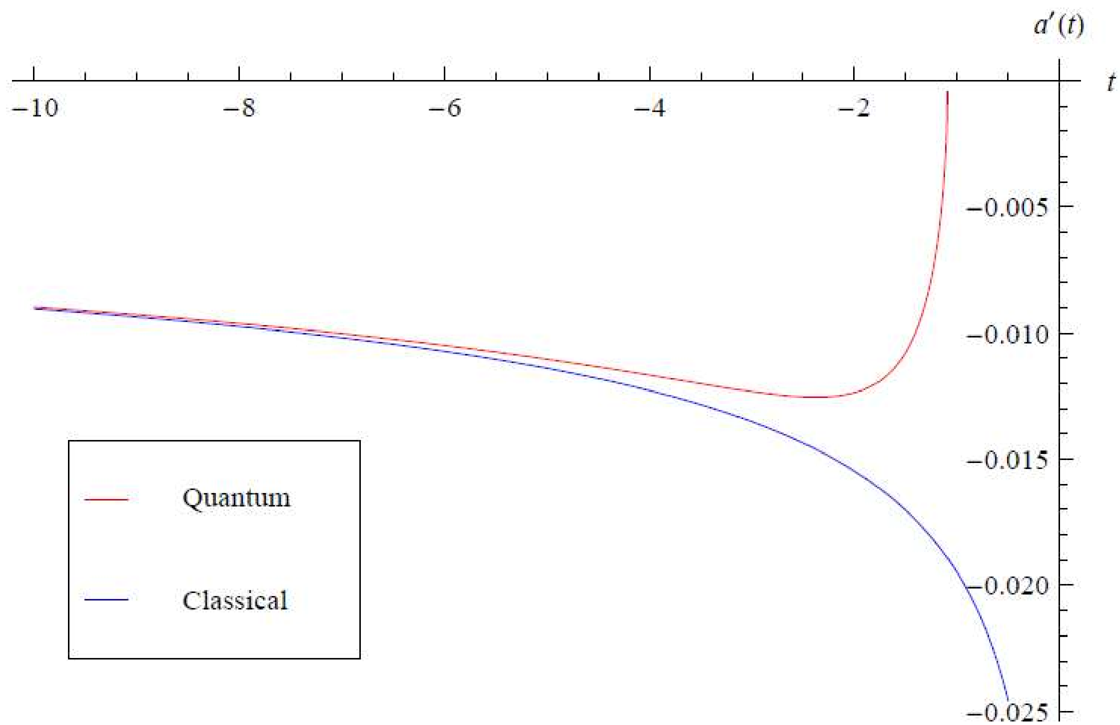
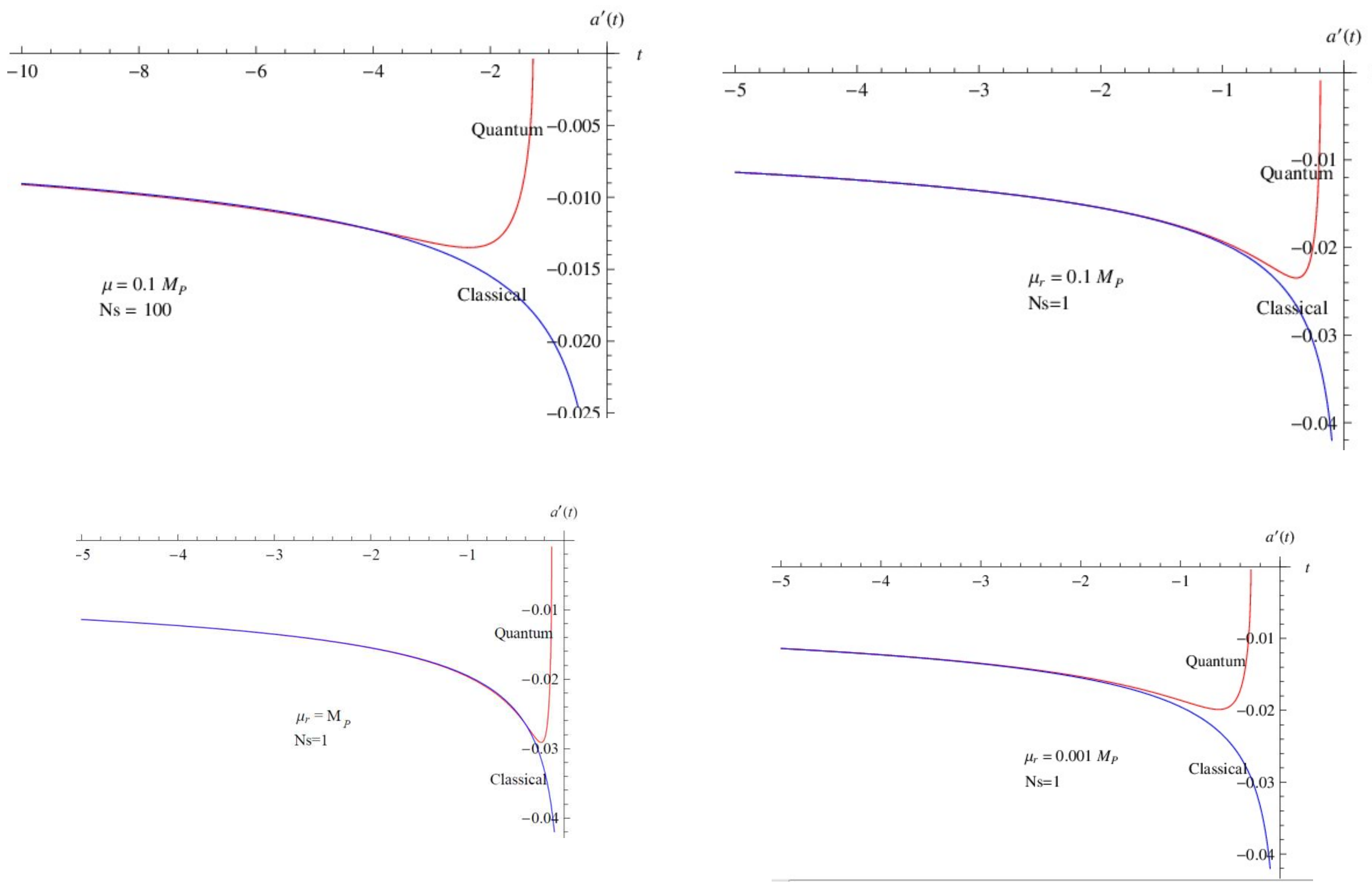
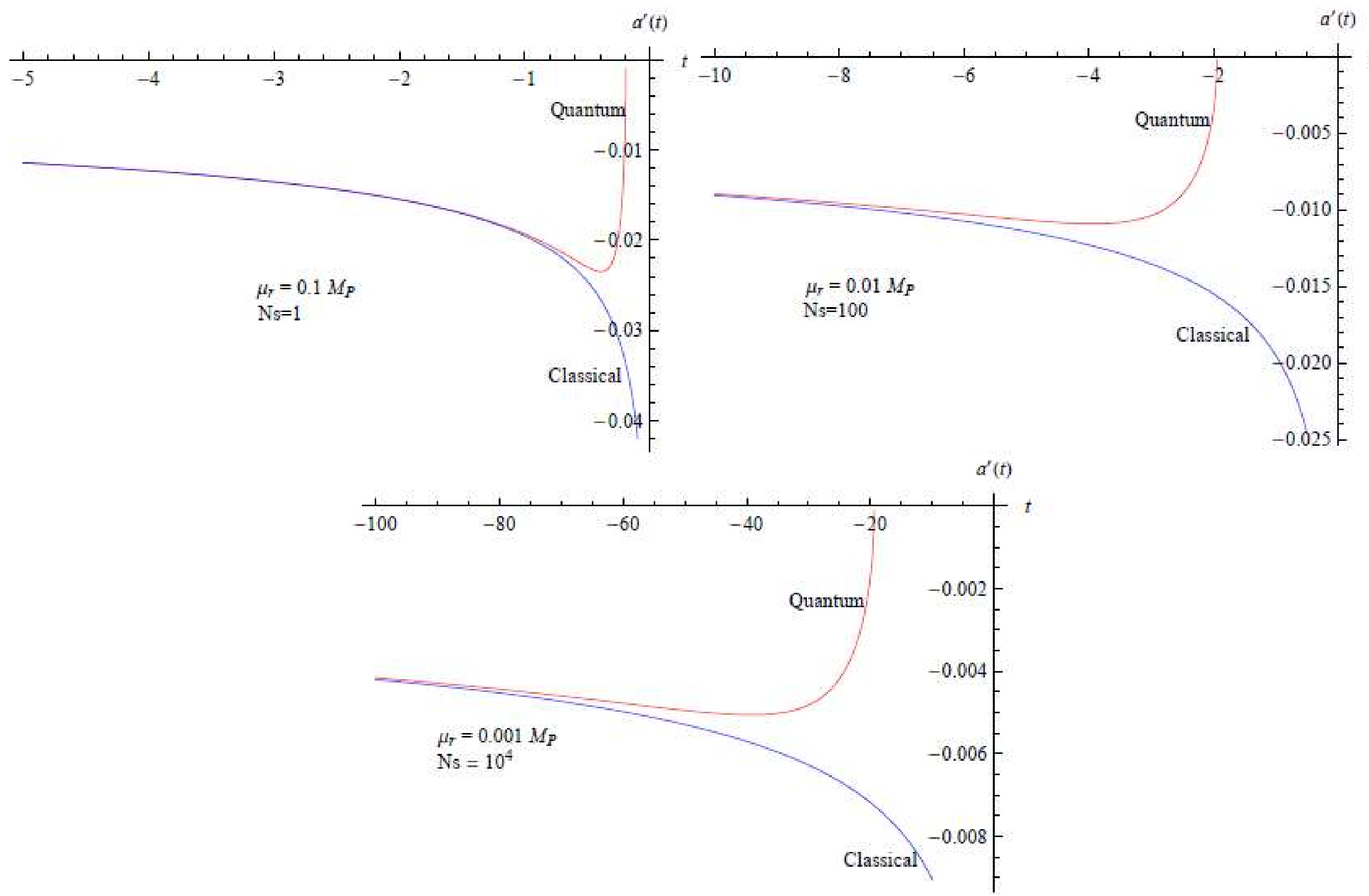


FIG. 11: Collapsing dust-filled universe with the Standard Model particles and a conformally coupled Higgs. The result is purely non-local and hence independent of any scale μ_R.

Collapsing phase – singularity avoidance

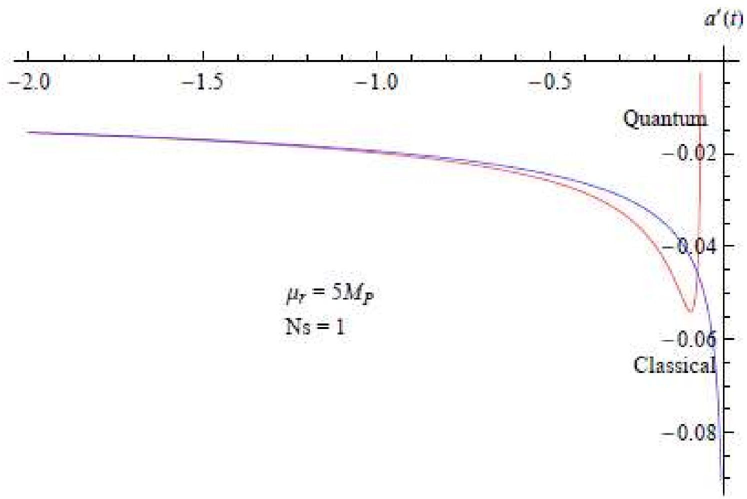


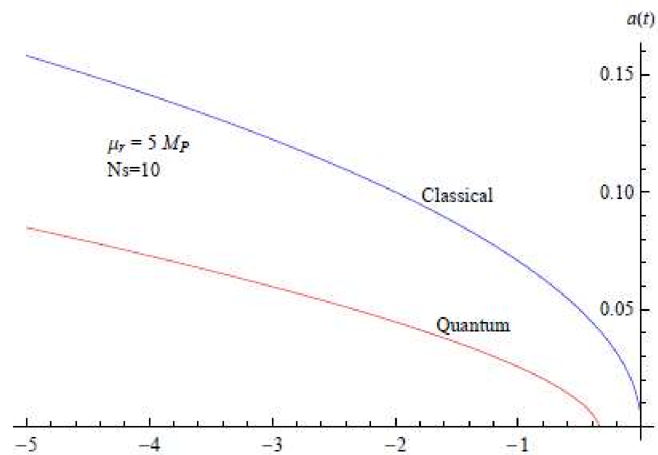
Physics does scale like $M_P/\sqrt{N}$



But there are some cases where singularity is not overcome:

Note: a(t), not a'(t)





Local terms overwhelm non-local effect:

Comments:

We have not followed bounce (yet)

Updated evolution study underway
- should be more reliable than P.T.

Large N can be used to argue for control

In progress

Black holes:

For local effective action, Schwd. is still solution at R^2

- all changes to local EoM are proportional to R or $R_{\mu\nu}$
- new solutions also (Holdom, Stelle Lu Pope)
    - finite at origin, no horizon, double horizon….

But Schwd is <u>no longer a solution</u> to the quantum action

First correction due uniquely to $R_{\mu\nu\alpha\beta}\ \ln \nabla^2\ R^{\mu\nu\alpha\beta}$

Will be independent of the local terms in the action

-scale independent

Treating non-local terms as a perturbation

Correction to Schld.

Easy to illustrate difference between local and non-local curvature expansion

Appear to be well defined so far

Perhaps preparation for more ambitious calculation

Dimensional analysis:

$$Riem. \sim \frac{GM}{r^3}$$

Non-local curvature expansion breaks down near horizon

- compare $R \ln \nabla^2 R$ to $R^2 \frac{1}{\nabla^2} R$

$$\log \Box Riem. \sim \frac{GM}{r^3}$$

$$\frac{1}{\Box} R = \int d^4y D(x-y) R(y) \sim \int d^3y \frac{1}{4\pi|\mathbf{x}-\mathbf{y}|} R(y) \sim \frac{GM}{r}$$

- "third order in curvature" is subdominant at large distance but not near horizon

Local curvature expansion can be well defined there

$$R \sim \frac{1}{(GM)^2}$$

Nature of quantum correction to vacuum solution:

In Kerr-Schild coordinates

$$g_{\mu\nu} = \eta_{\mu\nu} - \phi(r) r_\mu r_\nu = \eta_{\mu\nu} - \frac{GM}{r} r_\mu r_\nu \qquad \text{with} \qquad r_\mu = (1, \hat{r})$$

Modified vacuum equations:

$$R_{\mu\nu} + G(\alpha \Box \log \Box\, R + \beta R \log \Box\, R)_{\mu\nu} = 0$$

Pert. Treatment:

$$\nabla^2 \phi + \alpha \frac{G^2 M}{r^5} + \beta \frac{G^3 M^2}{r^6}$$

$$\phi = \frac{GM}{r} + \delta\phi \qquad\qquad \delta\phi = \frac{GM}{r}\left[\alpha \frac{G}{r^2} + \beta \frac{G}{r^2}\frac{GM}{r}\right]$$

Again, ordering breaks down at horizon but correction behaves

$$\lim_{r \to r_h} \delta\phi \sim \frac{\alpha + \beta}{GM^2}$$

Comments

These corrections are not optional

Perhaps can use large N for more control
- but low curvature region only gravitons and photons

Can we use this non-perturbatively?
- self consistent solutions

Signs will be interesting/extrapolation to small mass

Summary:

Non-local actions capture the quantum impact of massless particles in GR

These are very poorly understood

Even useful representation of log ∇^2 for general coordinates is not known (see Basem for K-S)

Cosmology application hints at singularity avoidance

Black hole structure will be modified.