Charged Pion Polarizability & Muon g-2

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http://www.physics.umass.edu/acfi/

ACFI-J Lab Workshop
March 2014

Outline

I. *Intro & Motivation: SM & Beyond*

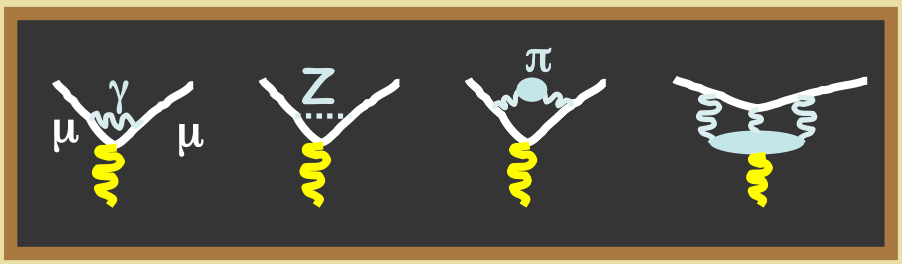
II. Hadronic Light-by-Light: Review & Status

III. Charged Pion Loops revisited

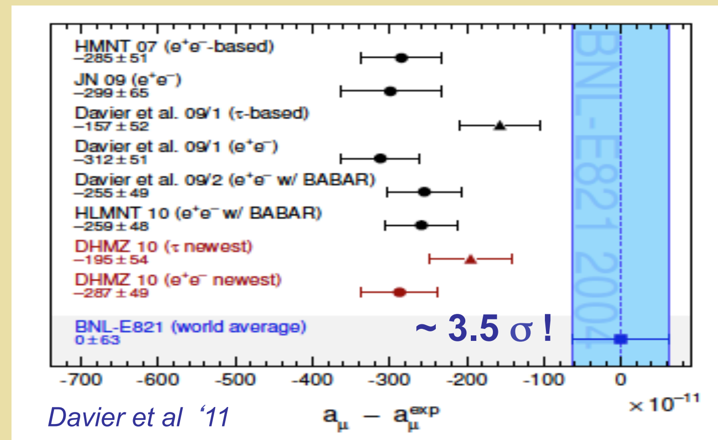
IV. Summary and Outlook

Intro & Motivation: SM & Beyond

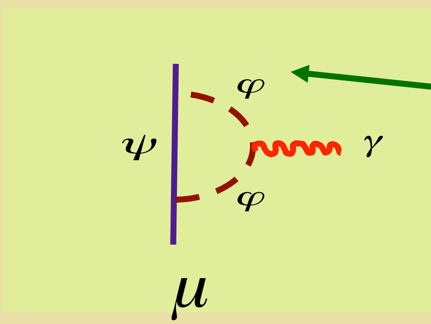
Muon Anomalous Magnetic Moment



QED Weak Had VP Had LbL



SM Loops



- *Smuon (SUSY)*
- *Leptoquark*
- *"Dark photon"*
- *Heavy Z'*
- *Extended scalar sector*

Theory Error Budget

$a_\mu(EW) = 154\ (2) \times 10^{-11}$

$a_\mu(HVP\text{-}LO) = 7015\ (47) \times 10^{-11}$

$a_\mu(HVP\text{-}NLO) = -98\ (1) \times 10^{-11}$

$a_\mu(HLBL) = 116\ (39) \times 10^{-11}$
$105\ (26) \times 10^{-11}$

$\delta a_\mu^{TH} = \pm 53 \times 10^{-11}$

$\delta a_\mu^{EXP} = \pm 63 \times 10^{-11}$ BNL E821

W. Marciano, arXiv: 1001.4528/hep-ph
M. Davier et al, EPJ C 71 (2011) 1515

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$$\boxed{\Delta a_\mu = a_\mu^{EXP} - a_\mu^{TH} = 287\ (83) \times 10^{-11}}$$

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Most challenging

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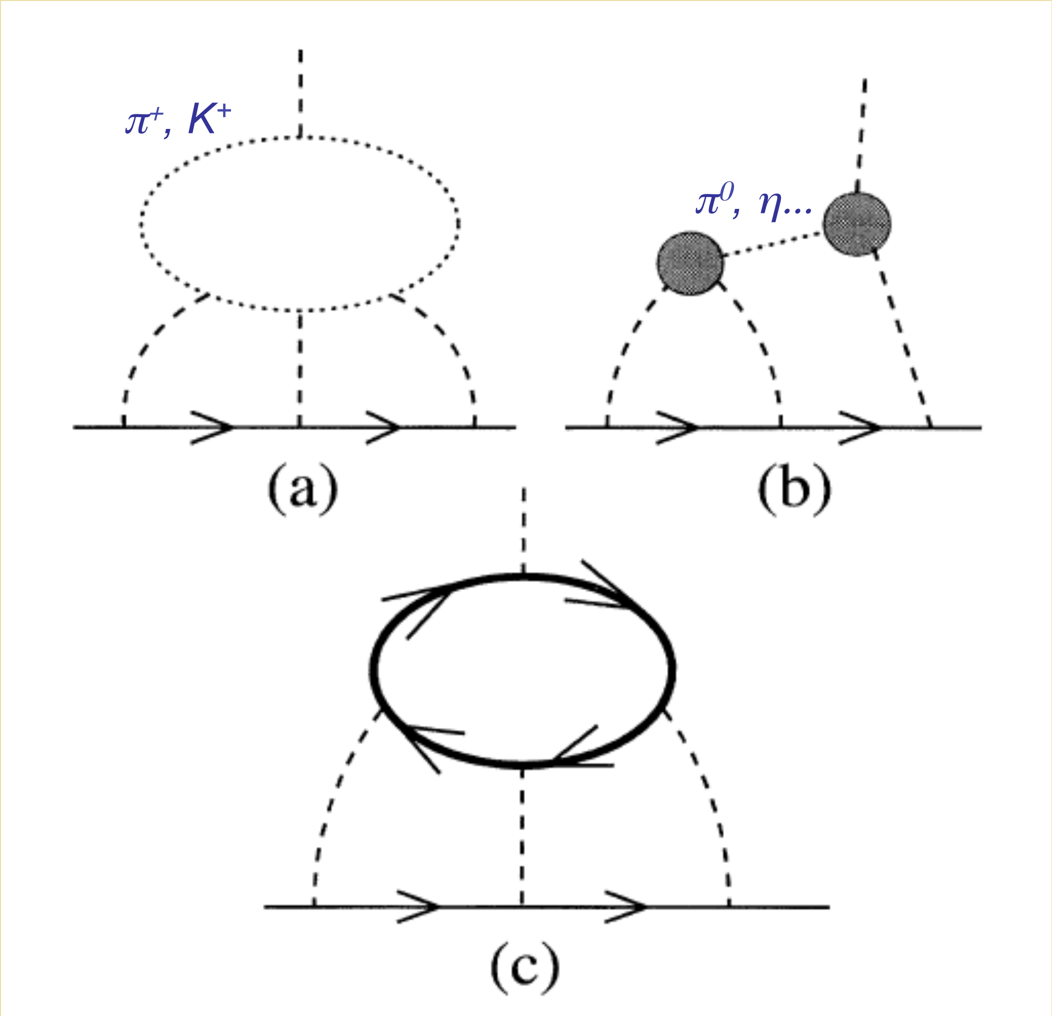
Hadronic Light-by-Light: Review & Status

HLBL Contributions

Pseudoscalar Loops

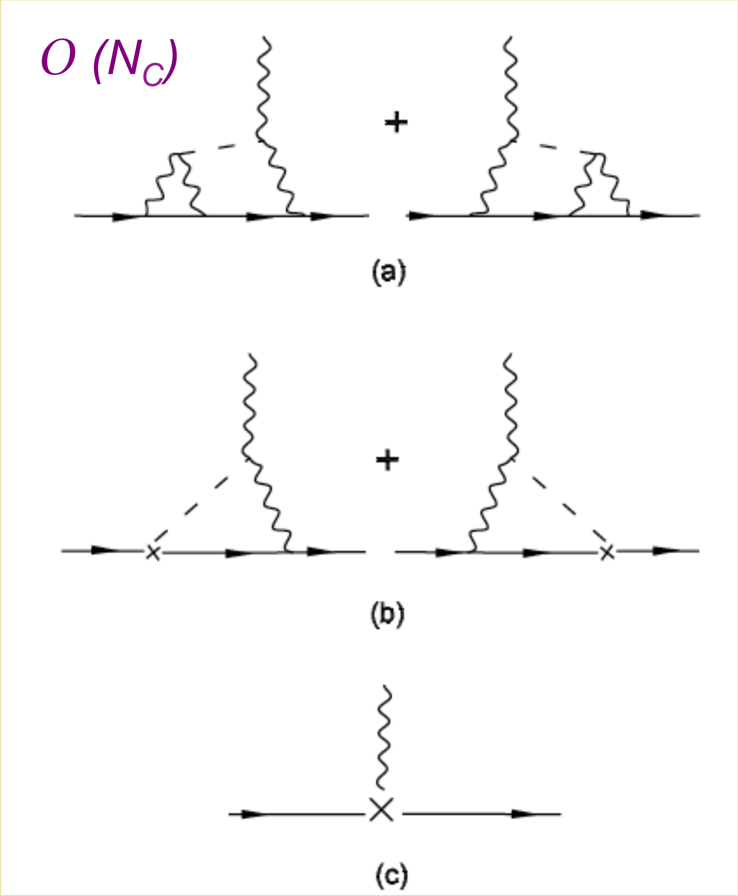
Pseudoscalar Poles

Quark Loops

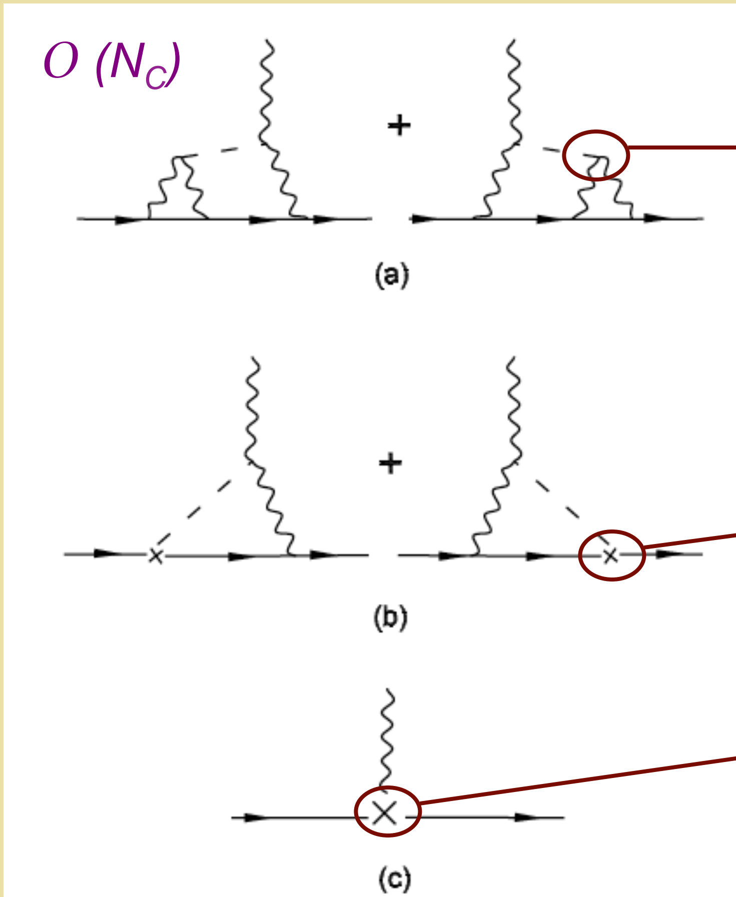


Hayakawa, Kinoshita, Sanda '95

Pseudoscalar Pole Contribution



Pseudoscalar Pole Contribution



$\mathcal{L}_{WZW}$: ln^2 term $\rightarrow$

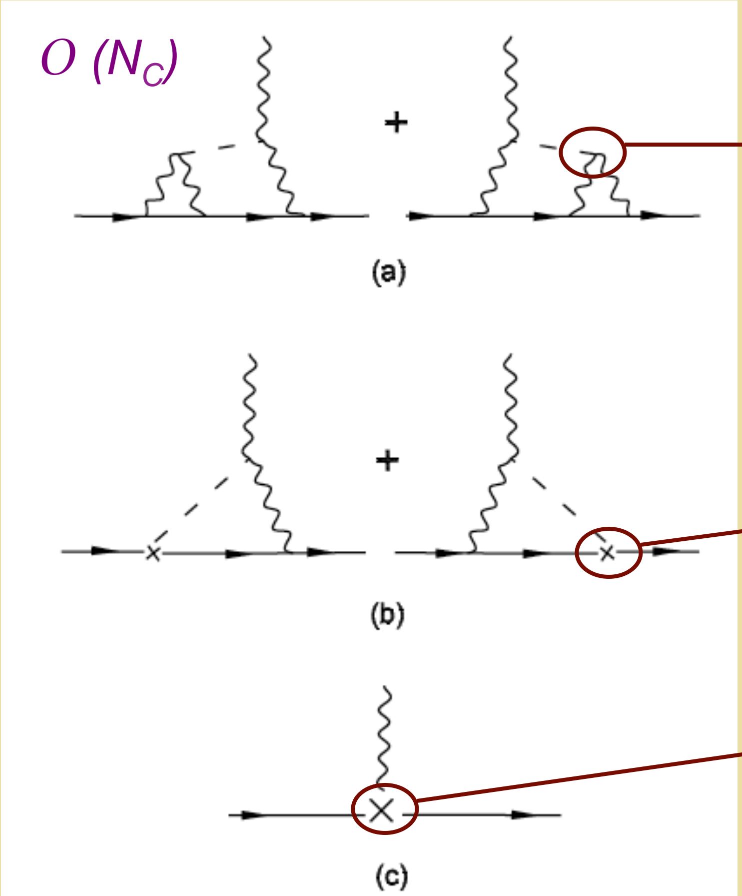
Sign error discovered by Knecht et al

$\mathcal{P} \rightarrow \ell^+\ell^-$: ln term $\rightarrow$

Exp't (MRM, Wise)

Overall LEC $\rightarrow$
Models, lattice QCD…

Pseudoscalar Pole Contribution



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$a_\mu(\chi PT) = (57^{+50}_{-60} + 31\ \mathcal{C}) \times 10^{-11}$

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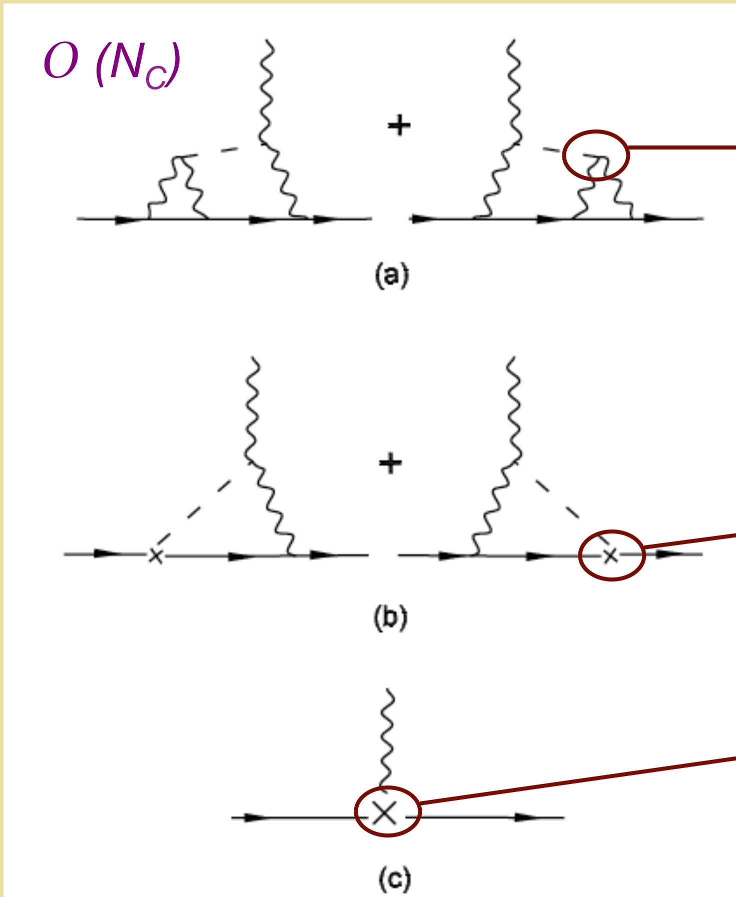

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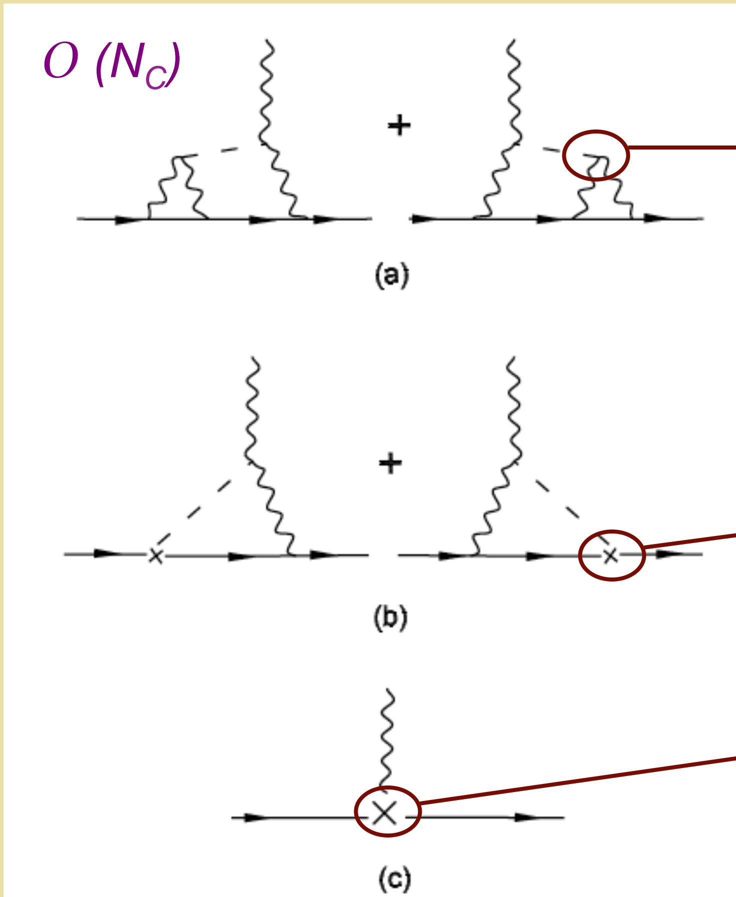
Models, lattice QCD…

Significantly reduced: KTeV '07

$a_\mu(\chi PT) = (57^{+50}_{-60} + 31\ C\,) \times 10^{-11}$

Pseudoscalar Pole Contribution



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$\mathcal{P} \rightarrow \ell^+\ell^-$: ln term $\rightarrow$

Exp't (MRM, Wise)

Overall LEC $\rightarrow$
Models, lattice QCD…

$a_\mu(\chi PT) = (57^{+50}_{-60} + 31\, \mathcal{C}\,) \times 10^{-11}$

Models: $\mathcal{C} \sim 2$

E821: $\mathcal{C} \sim 10$

$\sim 1\sigma$

Representative Models

- *Hidden Local Symmetry (HLS) [1]*
- *Extended NJL (ENJL)/VMD[1,2]*
- *Constituent Chiral Quark Model (CχQM) [3]*
- *AdS/CFT [4]*
- *Dyson-Schwinger [5]*

[1] Hayakawa, Kinoshita, Sanda '95

[2] Bijnens, Pallante, Prades '96

[3] De Rafael '12; Boughezal & Melkinov '11

[4] Hong & Kim '09; Cappiello, Cata, D'Ambrosio '11

[5] Goeke, Fischer, Williams '11, '12

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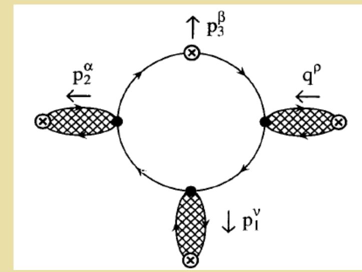

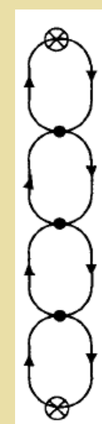

ρ pole

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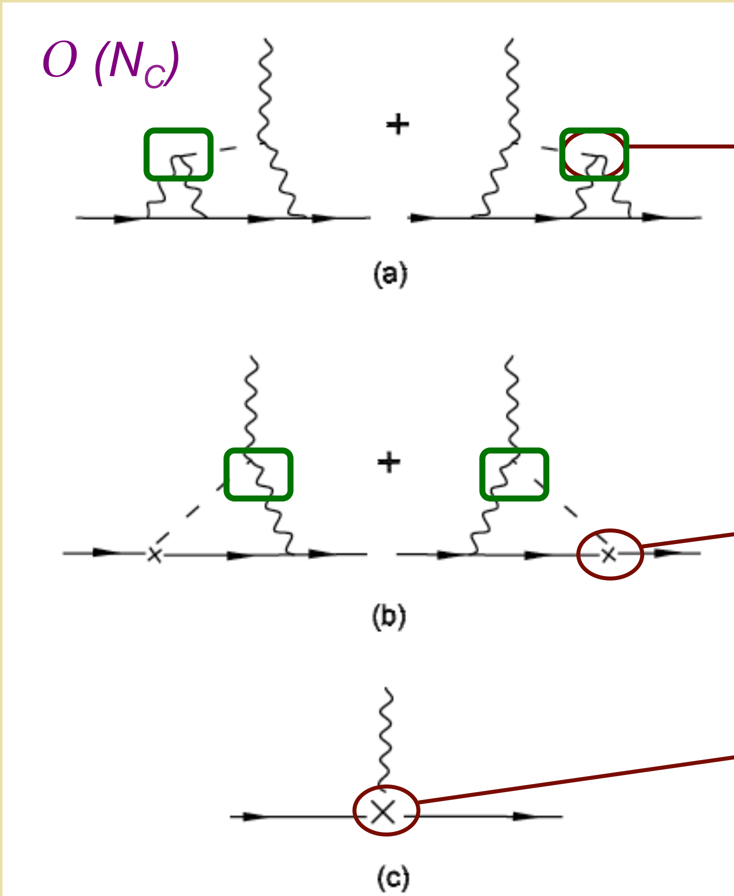
Short Distance Constraints

Vainshtein & Melnikov '04



$\mathcal{L}_{WZW}$: $\ln^2$ term $\rightarrow$

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$\mathcal{P} \rightarrow \ell^+\ell^-$: ln term $\rightarrow$

Exp't (MRM, Wise)

Overall LEC $\rightarrow$
Models, lattice QCD…

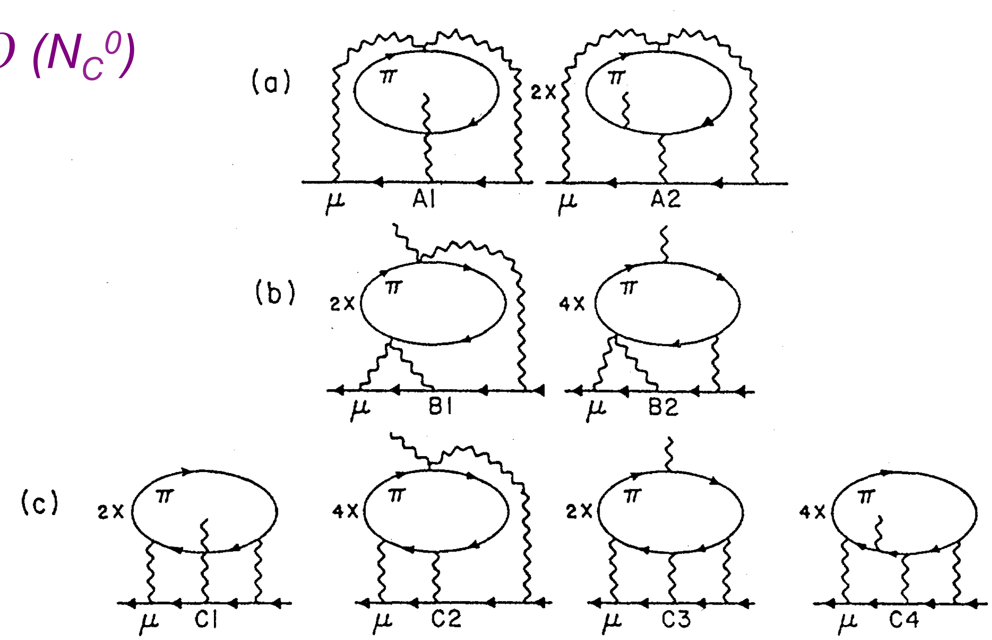
$\Delta\, a_\mu(OPE) = 30 \times 10^{-11}\ (\ \rightarrow \mathcal{C} = +1\ )$

Charged Pion Loops Revisited

Charged Pion Contribution

Kinoshita, Nizic, Okamoto '85 ; Hayakawa, Kinoshita, Sanda '95

$O\ (N_C^0)$



Point-like pions:	$-0.0383\ (19)\ (\alpha/\pi)^3$	=	$-48\ (2)$	$\times 10^{-11}$
Include $F_\pi\ (q^2)$:	$-0.0125\ (19)\ (\alpha/\pi)^3$	=	$-16\ (2)$	$\times 10^{-11}$
"HLS":	$-0.00355\ (12)\ (\alpha/\pi)^3$	=	$-4.5\ (0.2)$	$\times 10^{-11}$
ENJL:	$-0.015\ (4)\ (\alpha/\pi)^3$	=	$-19\ (5)$	$\times 10^{-11}$

Charged Pion Contribution

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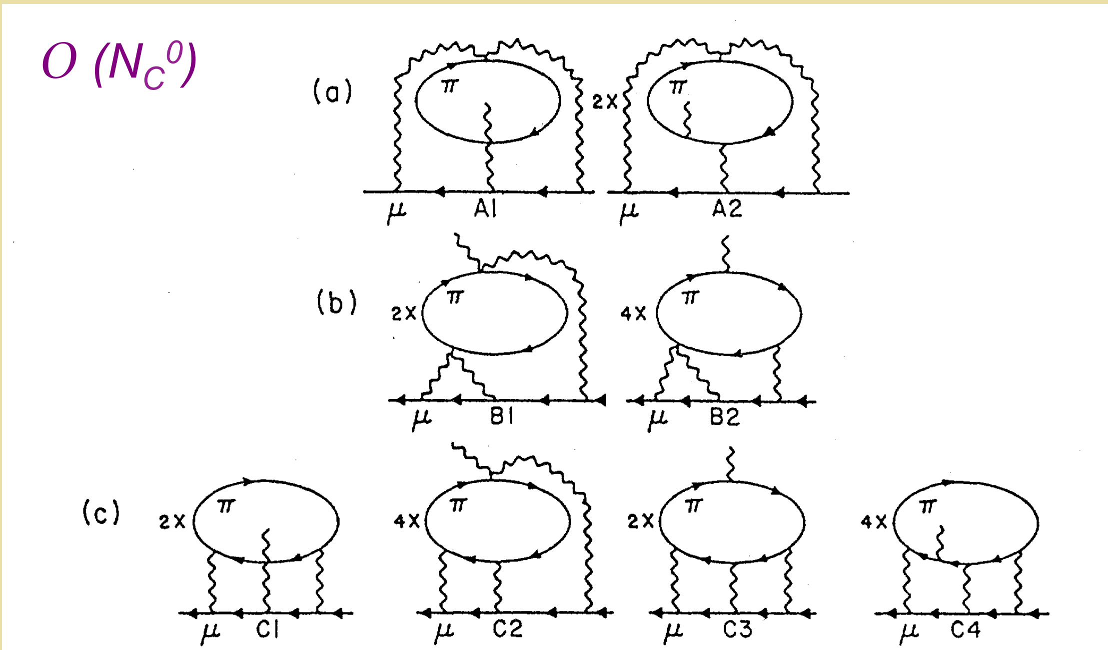

Substantial NLO impact

Point-like pions: $-0.0383\,(19)\,(\alpha/\pi)^3 \quad = \quad -48\,(2) \quad \times 10^{-11}$

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Charged Pion Contribution: χPT

Kevin Engel (Caltech), Hiren Patel (Wisconsin), MRM

Chiral Perturbation Theory

- *Pion: Goldstone boson of spont broken chiral symmetry*

$SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$

$$\boxed{\Sigma = \exp(i\tau^a\ \pi^a / F_\pi)}$$

- *Expand in* p / Λ_χ $\qquad$ $\Lambda_\chi \sim 1$ *GeV*

$$\boxed{\begin{aligned}\mathcal{L}\ \supset\ & ie\alpha_9\ F_{\mu\nu}\,\mathrm{Tr}\ \left(Q\left[D^\mu\Sigma, D^\nu\Sigma^\dagger\right]\right) \\ & +e^2\alpha_{10}\ F^2\ \mathrm{Tr}\left(Q\Sigma Q\Sigma^\dagger\right) \quad ,\end{aligned}}$$

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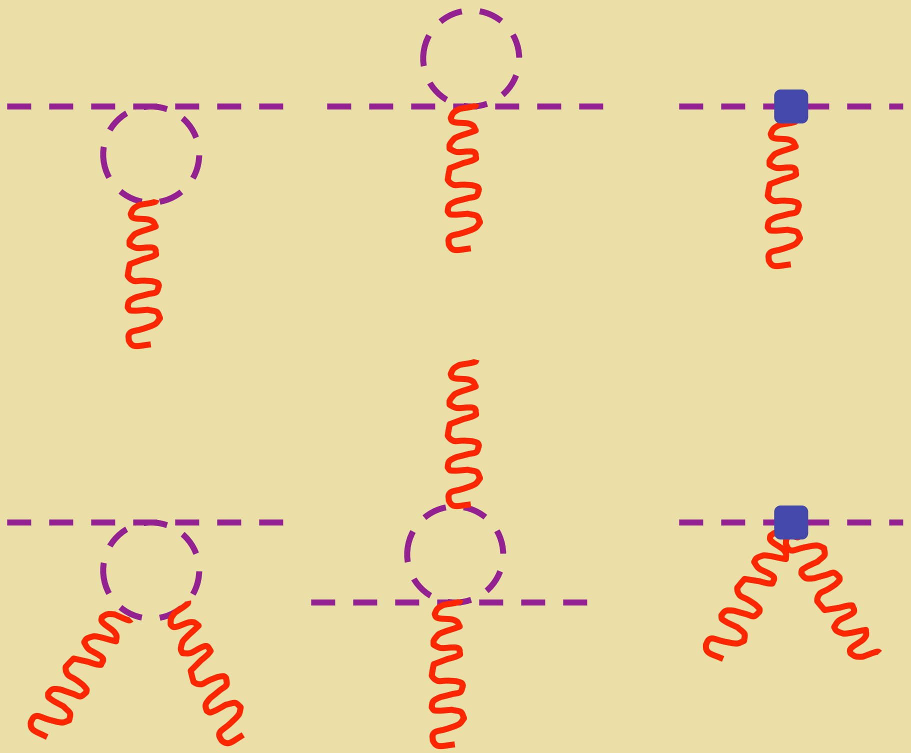
Beyond leading order: subgraphs



Pion charge radius: first non-trivial term in expansion of $F_\pi(q^2)$

$\mathcal{O}(p^4)$ *LEC:* α_9

Pion polarizability: distinct physics from ff

$\mathcal{O}(p^4)$ *LEC:* $\alpha_9 + \alpha_{10}$

+ …

Charged Pion Contribution: χPT

Kevin Engel (Caltech), Hiren Patel (Wisconsin), MRM

*PRD **86**:037502 (2012)*

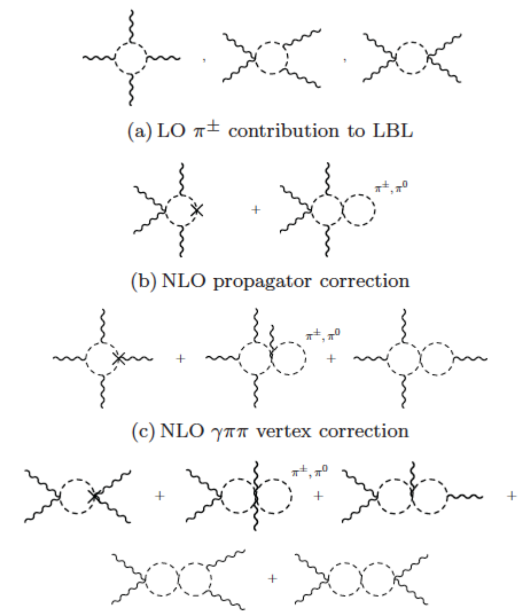
Beyond leading order: embedding subgraphs in full HLBL contribution

d=8 ops

Operator	1 loop χPT	2 loop	VMD
$\mathcal{O}_1^{(8)}$	1/9	$\frac{m_\pi^2}{F_\pi^2}\frac{16}{3}(\alpha_9^r+\alpha_{10}^r)$	0
$\mathcal{O}_2^{(8)}$	1/45	0	0

n	1 loop	2 loop	VMD
1	$\frac{1}{45}$	$\frac{1}{3}\left\{\frac{1}{9}(m_\pi r_\pi)^2+\frac{4}{5}\left(\frac{m_\pi}{F_\pi}\right)^2(\alpha_9^r+\alpha_{10}^r)\right\}$	$\frac{2}{9}\frac{m_\pi^2}{M_V^2}$
2	$\frac{2}{45}$	$\frac{1}{9}\left\{\frac{1}{3}(m_\pi r_\pi)^2+\frac{1}{2}\frac{m_\pi^2}{\Lambda_\chi^2}+\frac{44}{5}\left(\frac{m_\pi}{F_\pi}\right)^2(\alpha_9^r+\alpha_{10}^r)\right\}$	$\frac{2}{9}\frac{m_\pi^2}{M_V^2}$
3	$\frac{2}{315}$	$\frac{1}{135}(m_\pi r_\pi)^2$	$\frac{2}{45}\frac{m_\pi^2}{M_V^2}$
4	$\frac{1}{189}$	$\frac{1}{135}(m_\pi r_\pi)^2$	$\frac{2}{45}\frac{m_\pi^2}{M_V^2}$
5	$\frac{1}{135}$	$\frac{4}{45}\left(\frac{m_\pi}{F_\pi}\right)^2(\alpha_9^r+\alpha_{10}^r)$	0
6	$\frac{1}{315}$	0	0
7	$\frac{1}{945}$	0	0

d=10 ops



(a) LO $\pi^\pm$ contribution to LBL

(b) NLO propagator correction

(c) NLO $\gamma\pi\pi$ vertex correction

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LO: suppressed

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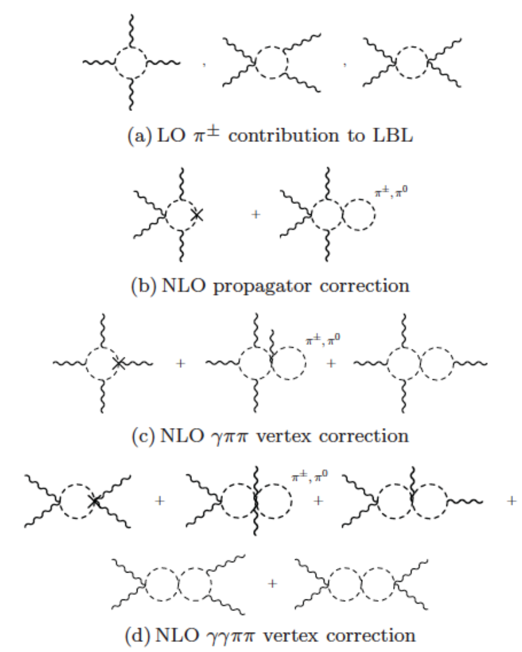

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Pol'bility

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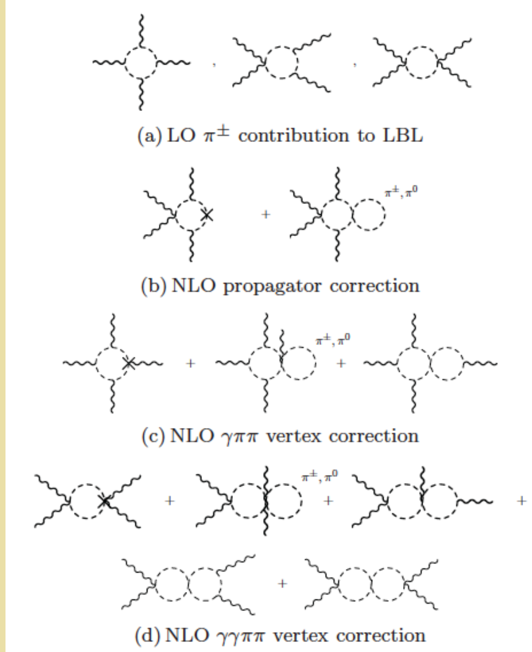
Charge radius



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Currently Omitted

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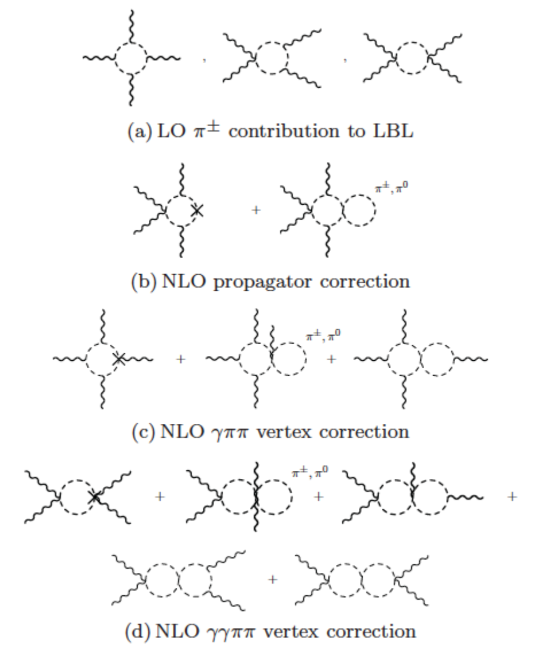

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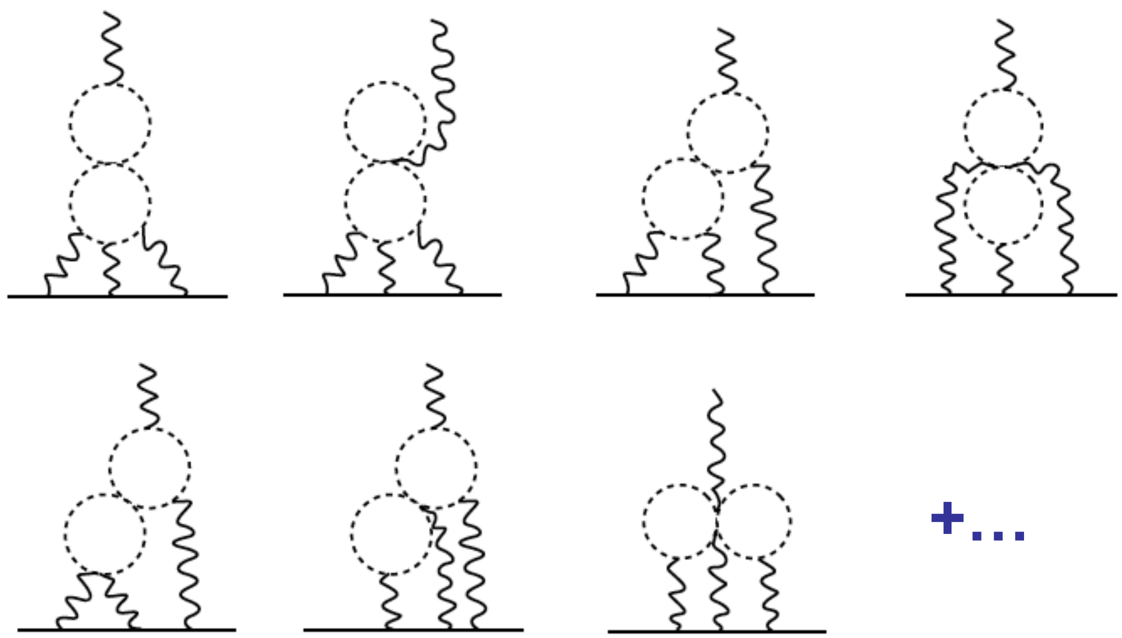
Kevin Engel (Caltech), MRM

Beyond leading order: embedding subgraphs in full HLBL contribution



+…

Charged Pion Contribution: χPT

Kevin Engel (Caltech), MRM

Beyond leading order: embedding subgraphs in full HLBL contribution

Charge radius

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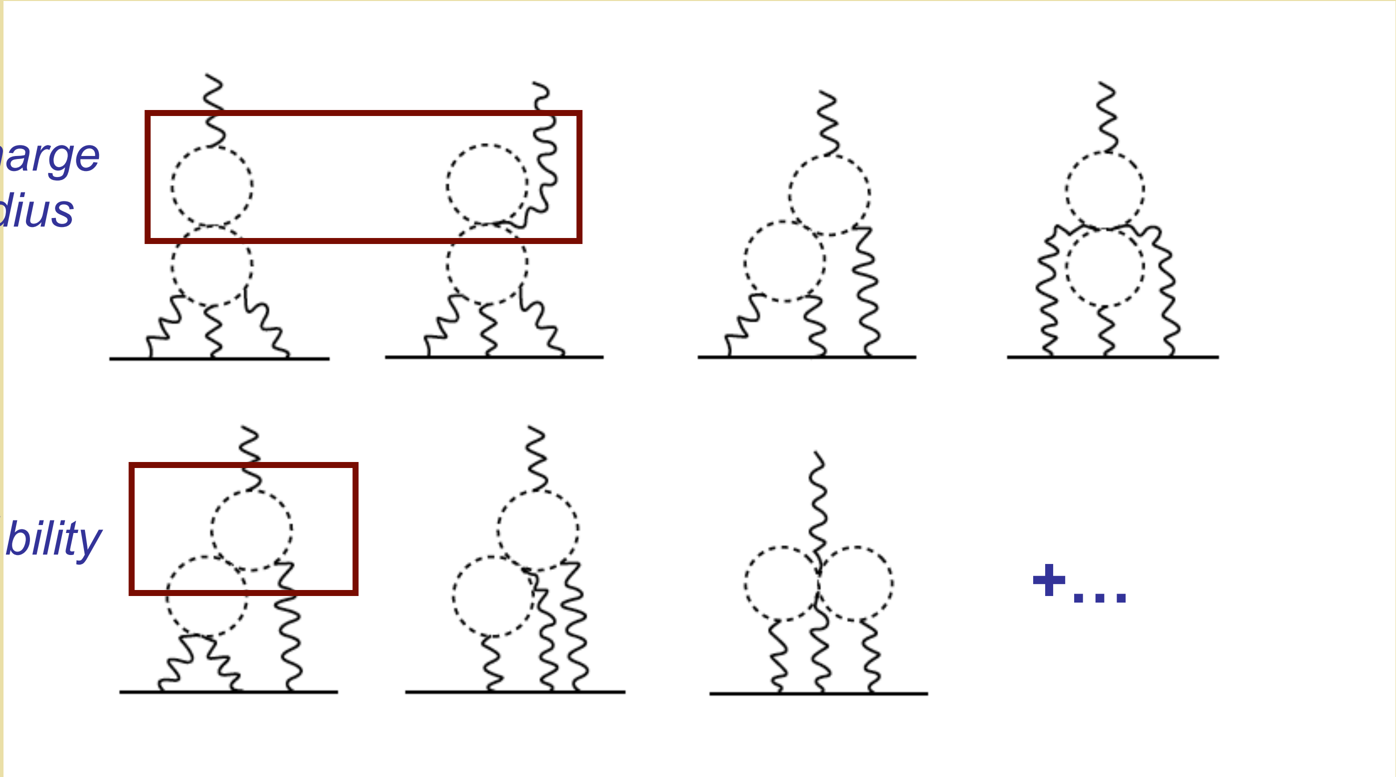

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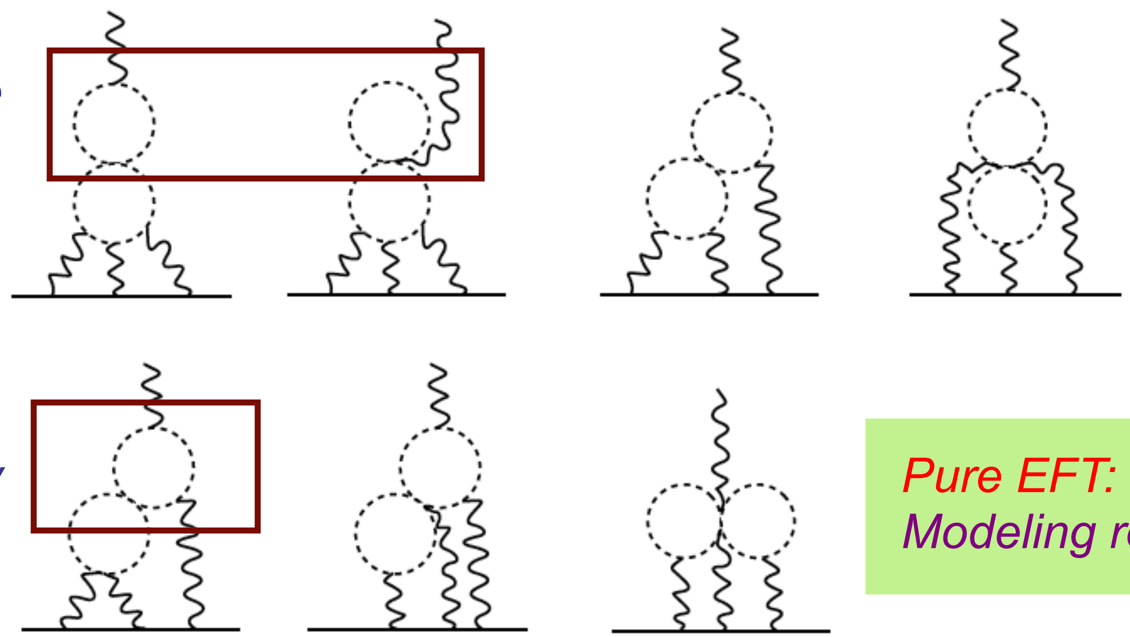

Pure EFT: Divergent $\rightarrow$ Modeling required

Charged Pion Contribution: χPT

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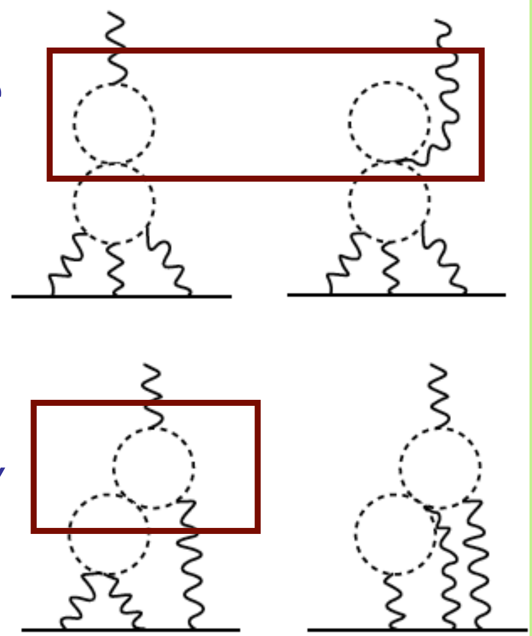

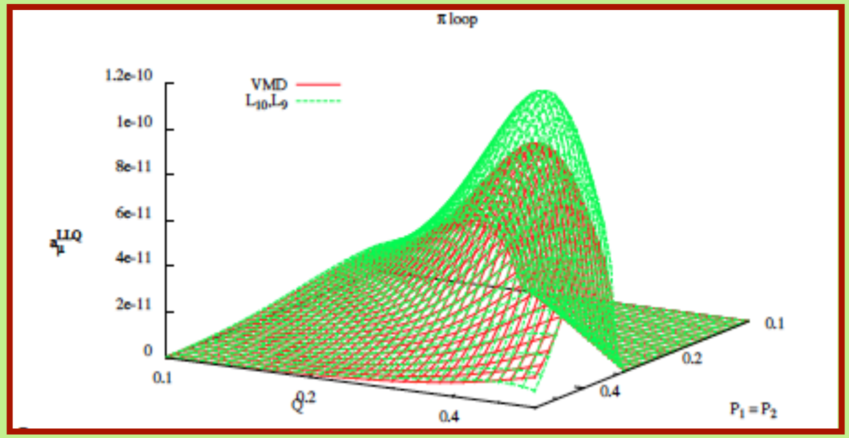

Bijnens & Abyaneh 1208.3548 :
Include $\alpha_9+\alpha_{10}$ to k_{loop} ~ 500 MeV

$\rightarrow$ 10% increase in a_μ (π loop)

Beyond χPT: Modeling High Q^2

Analgous problem: Pseudoscalar EM mass splitting

Donoghue, Holstein, Wyler '93

$$\begin{aligned}\Delta m_\pi^2 &= \delta m_+^2 - \delta m_0^2 \\ &= \frac{e^2}{2}\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2} g^{\mu\nu}[T_{\mu\nu}^+(p,q) - T_{\mu\nu}^0(p,q)]\end{aligned}$$

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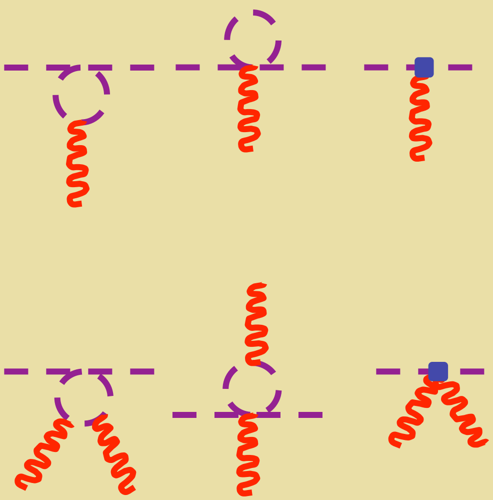

Pure EFT:
Divergent $\rightarrow$ Modeling required

Beyond χPT: Modeling High Q^2

Analgous problem: Pseudoscalar EM mass splitting

Donoghue, Holstein, Wyler '93

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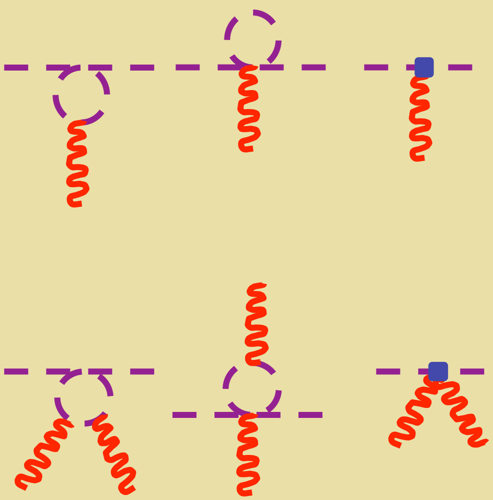
Quark counting rules

$$\boxed{q^2 \to \infty : \qquad T_{\mu\nu}(p,q) \quad \sim \quad \frac{1}{q^2}}$$



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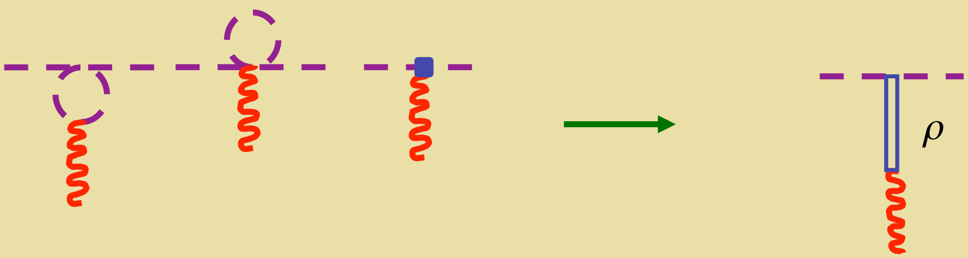
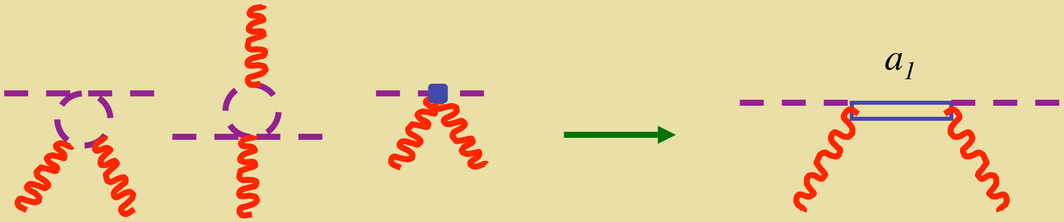


$$\frac{1}{Q^2 + M_V^2} \quad \longrightarrow \quad r_\pi^2 = \frac{6}{M_V^2}$$



$$\frac{1}{Q^2 + M_A^2} \quad \longrightarrow \quad \alpha_9 + \alpha_{10} \sim \frac{1}{M_A^2}$$

Beyond χPT: Modeling High Q^2

Analgous problem: Pseudoscalar EM mass splitting

Donoghue, Holstein, Wyler ‘93

Finite $\Delta\, m_\pi^2$ requires additional form factors

See Donoghue & Perez ‘96

Quark counting rules

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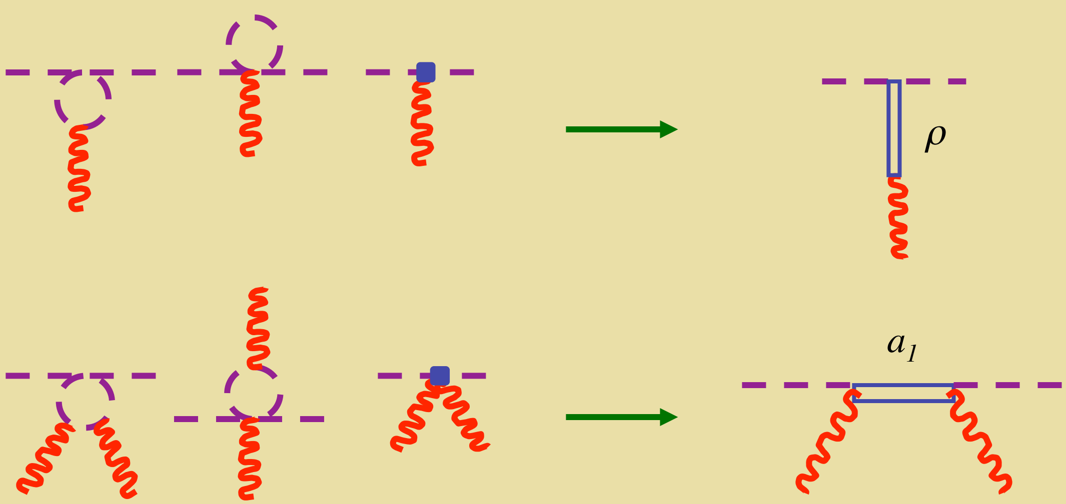

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Beyond χPT: Modeling High Q^2

Analgous problem: Pseudoscalar EM mass splitting

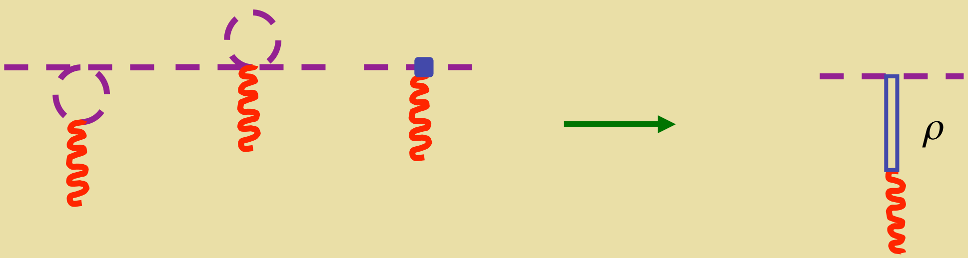
Donoghue, Holstein, Wyler '93

Model: $\Delta m_\pi^2 = 2\, m_\pi \times 5.6$ MeV

Expt: $\Delta m_\pi^2 = 2\, m_\pi \times 4.6$ MeV

Quark counting rules

$$q^2 \to \infty : \qquad T_{\mu\nu}(p,q) \quad \sim \quad \frac{1}{q^2}$$



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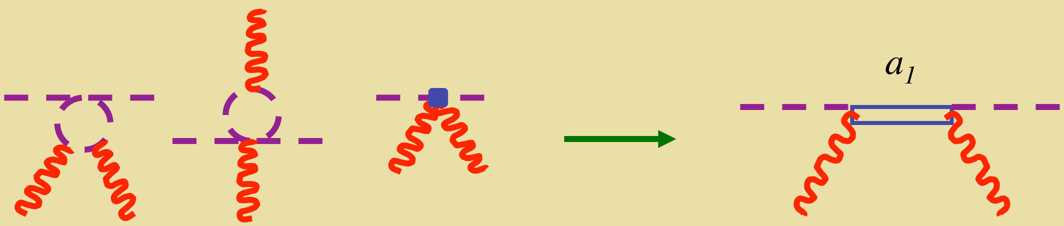


$$\frac{1}{Q^2 + M_A^2} \quad \longrightarrow \quad \alpha_9 + \alpha_{10} \sim \frac{1}{M_A^2}$$

Charged Pion Contribution: Model

Kevin Engel (Caltech), MRM

arXiv: 1309.2225

Interpolating from chiral to high momentum regime:

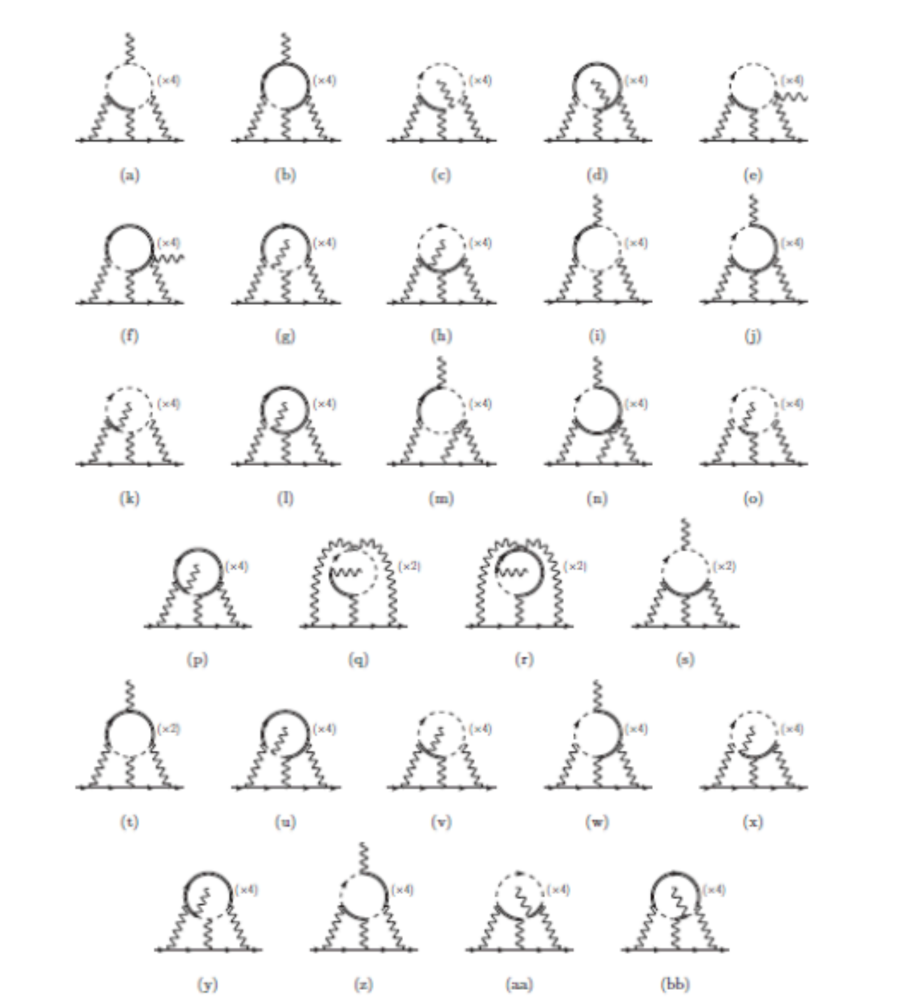
- *Match onto $\mathcal{O}(p^4)$ χPT results in low p regime*
- *Reproduce $1/q^2$ asymptotic behavior for $T_{\mu\nu}$*
- *Incorporate resonance saturation physics*
- *Produce finite a_μ*
- *Reproduce EM Δm_π^2 if possible*

Note: Δm_π^2 receives contribution from

$$\boxed{\mathcal{L} \supset \frac{e^2 C}{F_\pi^2} \mathrm{Tr}\left(Q\Sigma Q\Sigma^\dagger\right)}$$

Charged Pion Contribution: Model

Kevin Engel (Caltech), MRM



+ ρ pole

Charged Pion Contribution: Model

Kevin Engel (Caltech), MRM

arXiv: 1309.2225

Two Models:

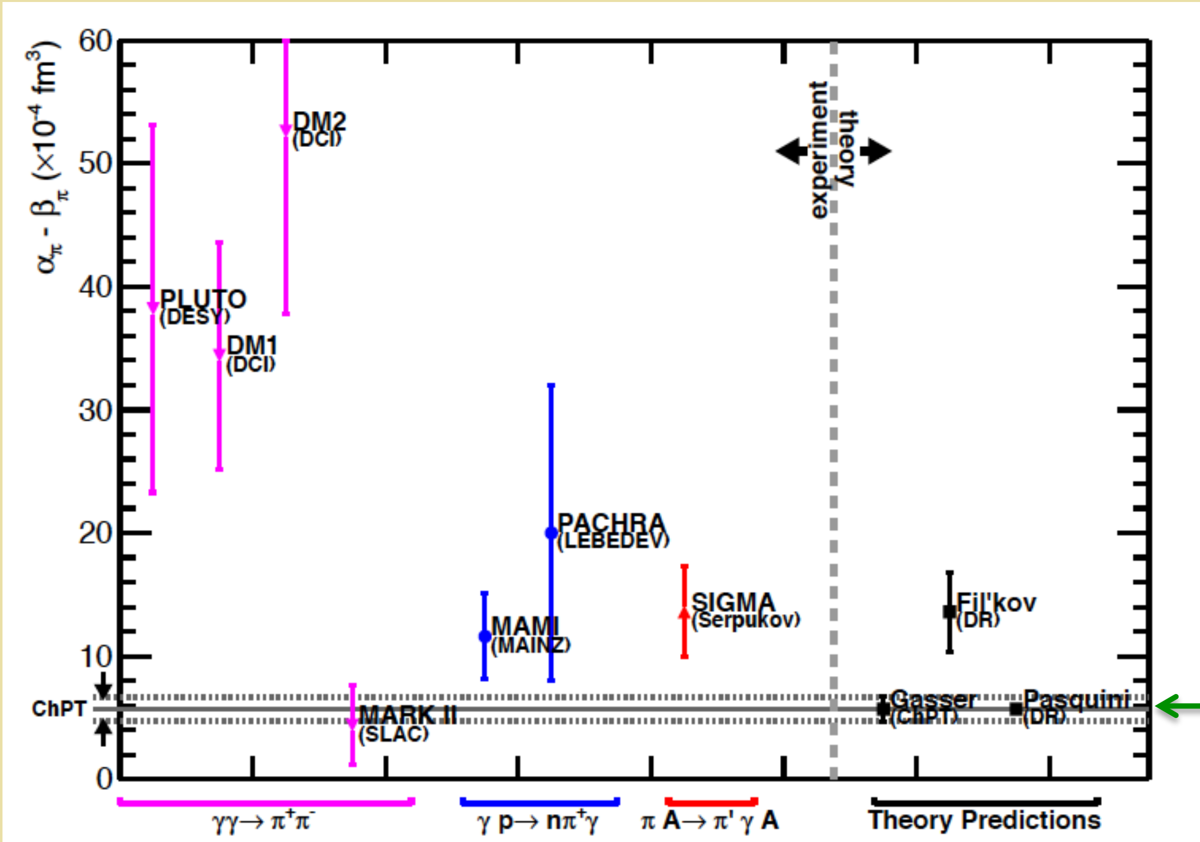
$$\mathcal{L}_I = -\frac{e^2}{4} F_{\mu\nu} \pi^+ \left(\frac{1}{D^2 + M_A^2} \right) F^{\mu\nu} \pi^- + \text{h.c.} + \cdots$$

Finite Δm_π^2 for $M_A^2 = 2\, M_V^2$

$$\mathcal{L}_{II} = -\frac{e^2}{2M_A^2} \pi^+ \pi^- \left[\left(\frac{M_V^2}{\partial^2 + M_V^2} \right) F^{\mu\nu} \right]^2 + \cdots$$

$4\, M_A^2\, (\alpha_9 + \alpha_{10}) = F_A^2$

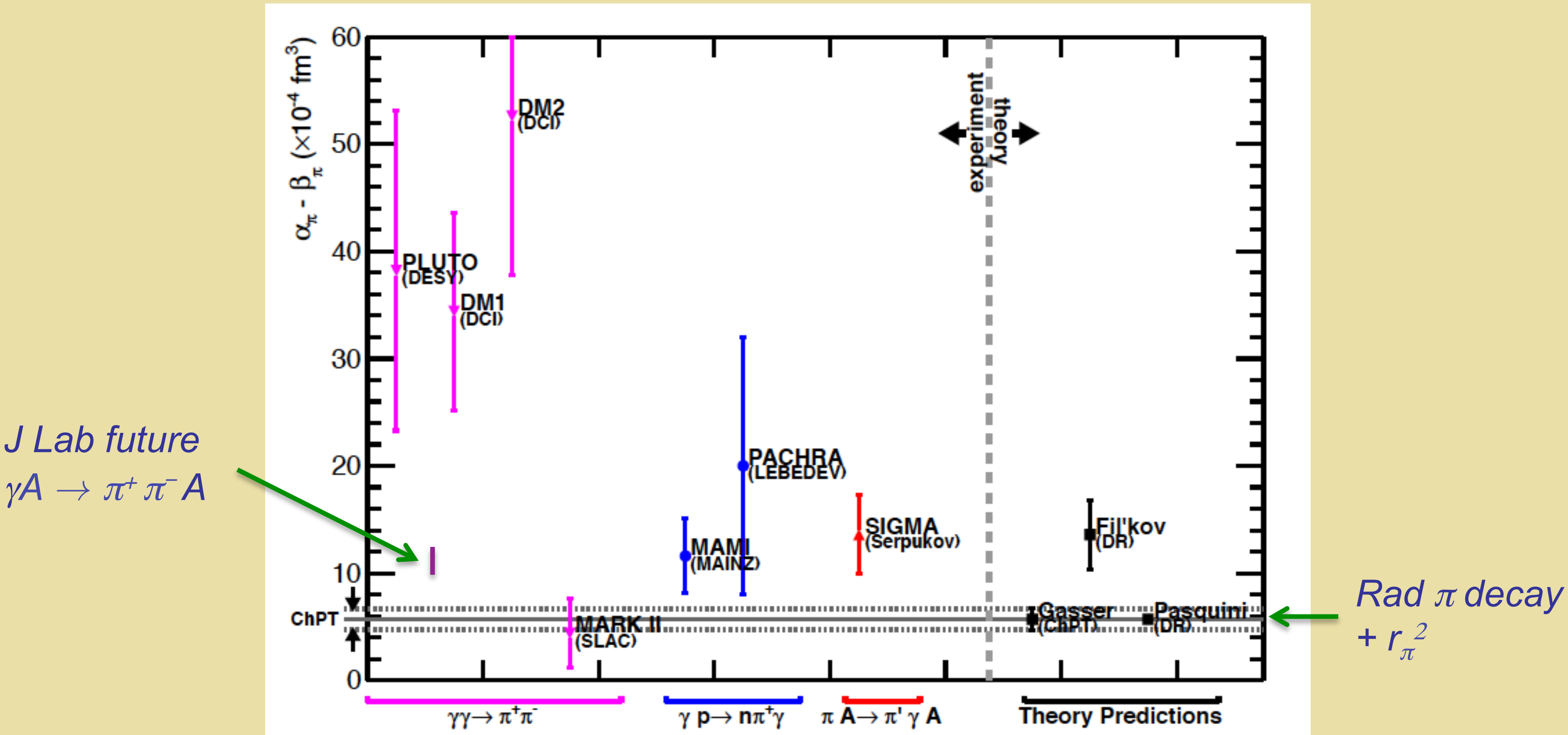
Pion Polarizability Experiment



Rad π decay
+ r_π^2

$$(\alpha_1 - \beta_1)_{\pi^+} = 8\alpha(\alpha_9^r + \alpha_{10}^r)/(F_\pi^2 m_\pi) + \cdots$$

Pion Polarizability Experiment



$$(\alpha_1 - \beta_1)_{\pi^+} = 8\alpha(\alpha_9^r + \alpha_{10}^r)/(F_\pi^2 m_\pi) + \cdots$$

Charged Pion Contribution: Results

Kevin Engel (Caltech), MRM

arXiv: 1309.2225

Approach	$a_\mu^{\pi^+\pi^-} \times 10^{11}$ (a)	$a_\mu^{\pi^+\pi^-} \times 10^{11}$ (b)
LO	-44	-44
HLS	-4.4 (2)	-4.4 (2)
ENJL	-19 (13)	-19(13)
NLO/cut-off	-20 (5)	-24 (5)
Model I	-11	-34
Model II	-40	-71

$(\alpha_9 + \alpha_{10})$: *Rad π decay* $\gamma\, p \to \gamma'\, \pi^+ n$

Recall:

$$(\alpha_1 - \beta_1)_{\pi^+} = 8\alpha(\alpha_9^r + \alpha_{10}^r)/(F_\pi^2 m_\pi) + \cdots$$

Charged Pion Contribution: Results

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$(\alpha_9 + \alpha_{10})$: *Rad π decay* *$\gamma\, p \to \gamma'\, \pi^+ n$*

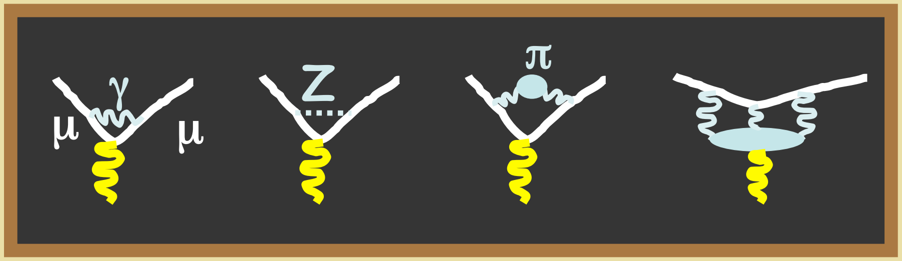
~ 30 x 10^{-11} spread from $(\alpha_9 + \alpha_{10})$
~ 30 x 10^{-11} spread from interpolation (modeling)

Summary & Outlook

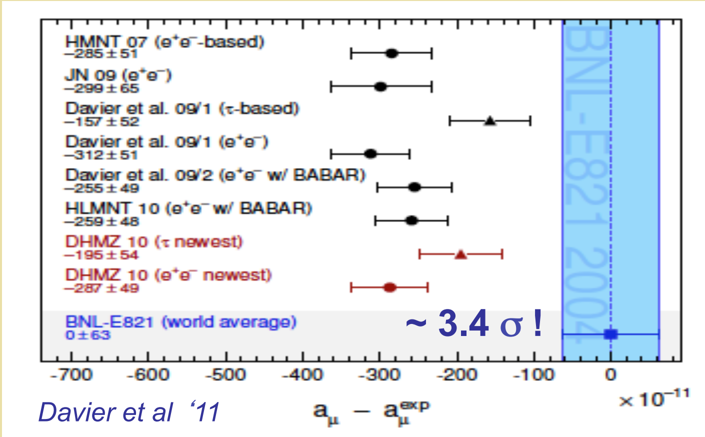
- Charged pion polarizability previously omitted from SM prediction for muon g-2
- Inclusion tends to increase discrepancy between experimental value and SM prediction
- Parametric and modeling uncertainties are significant compared to expected precision of FNAL measurement
- J Lab polarizability measurement would eliminate parameteric uncertainty
- Future challenge: experimental test of interpolation with γ^* momentum

Back Up Slides

Muon Anomalous Magnetic Moment

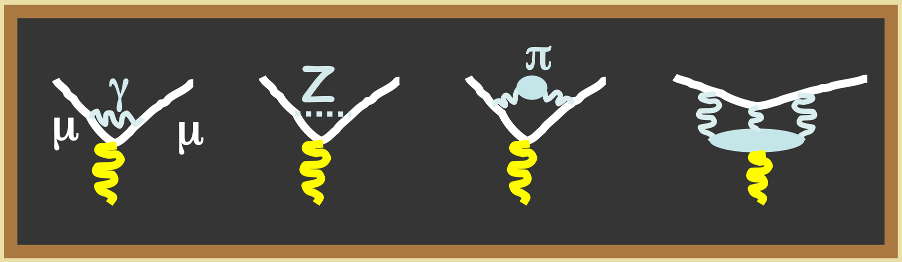


QED Weak Had VP Had LbL



SM Loops

Muon Anomalous Magnetic Moment



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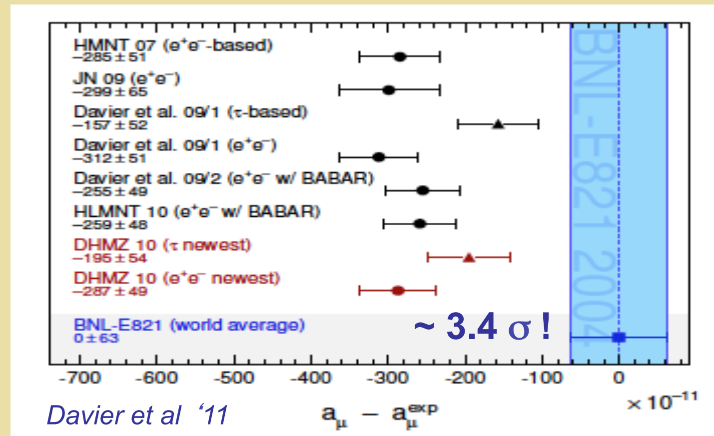

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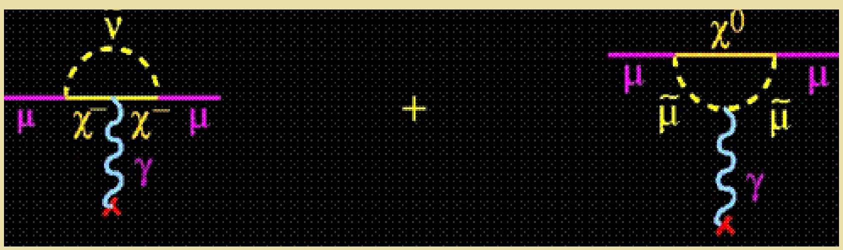
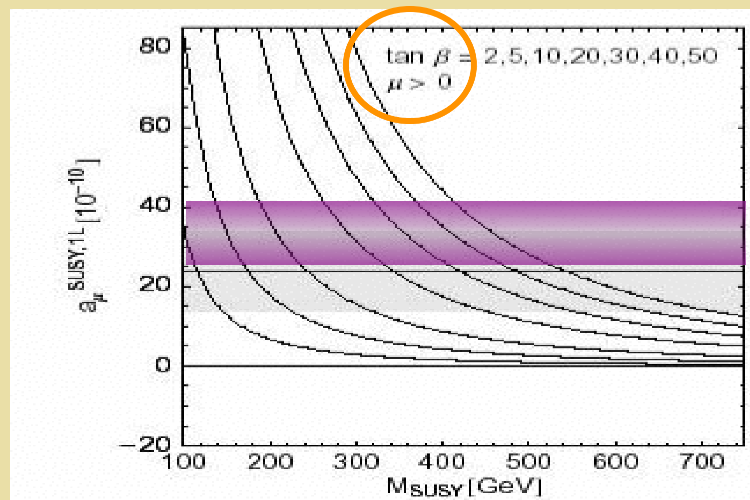
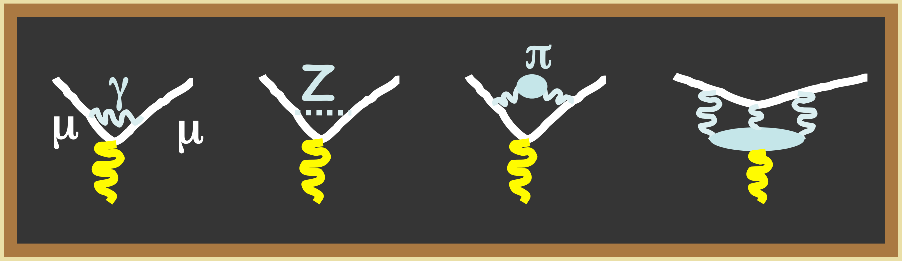


SUSY Loops

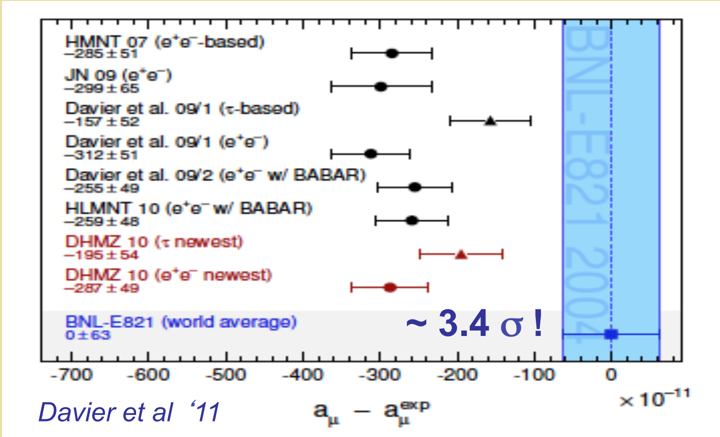


Muon Anomalous Magnetic Moment

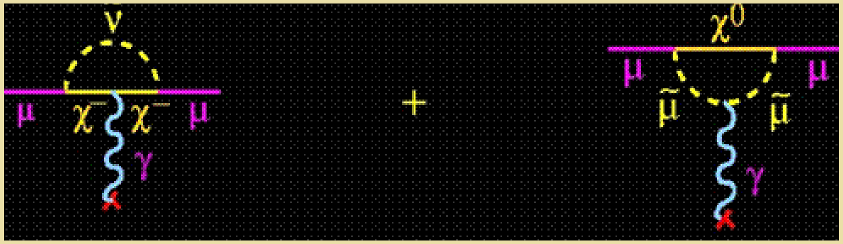


QED
Weak
Had VP
Had LbL

SM Loops



Davier et al '11



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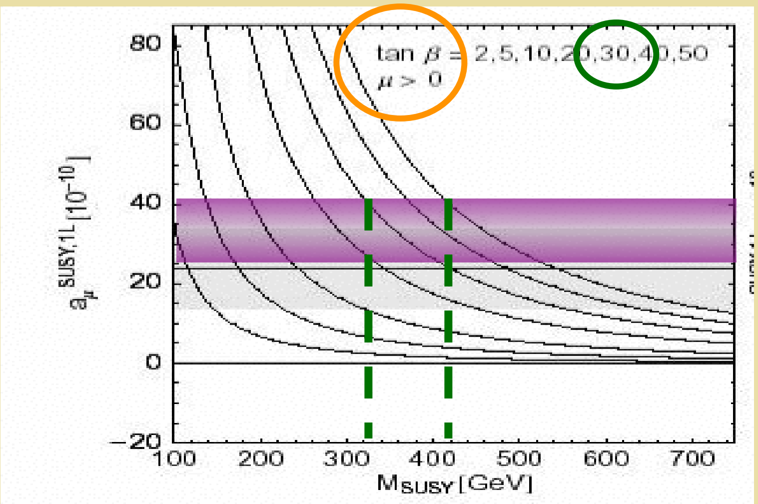

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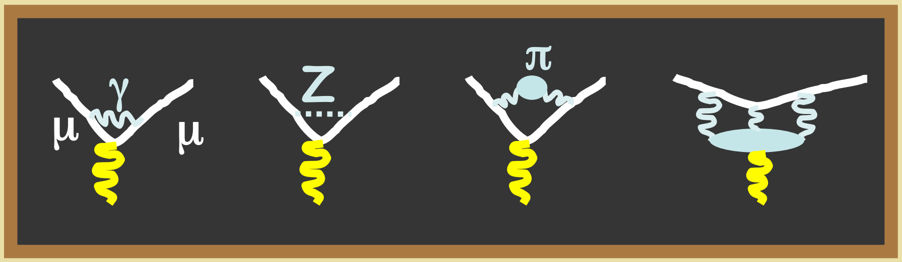

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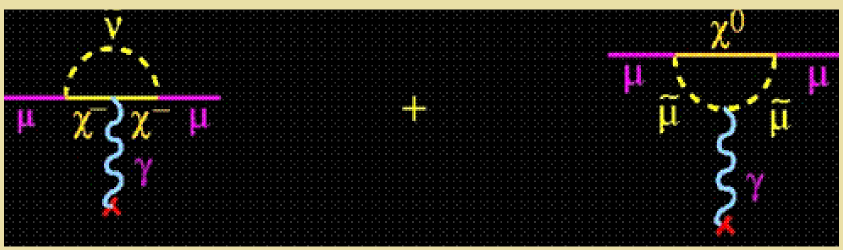

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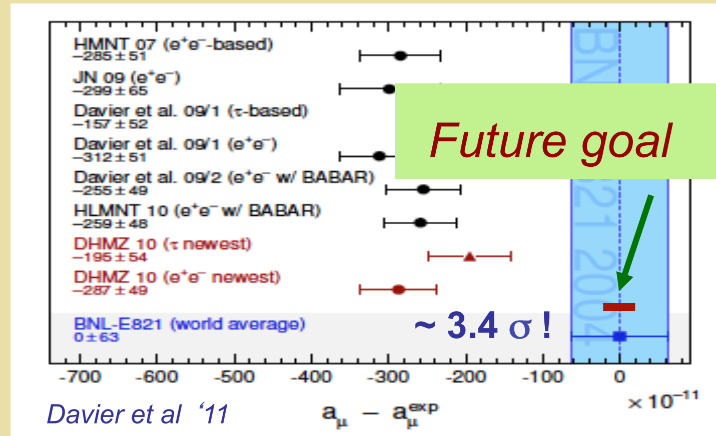

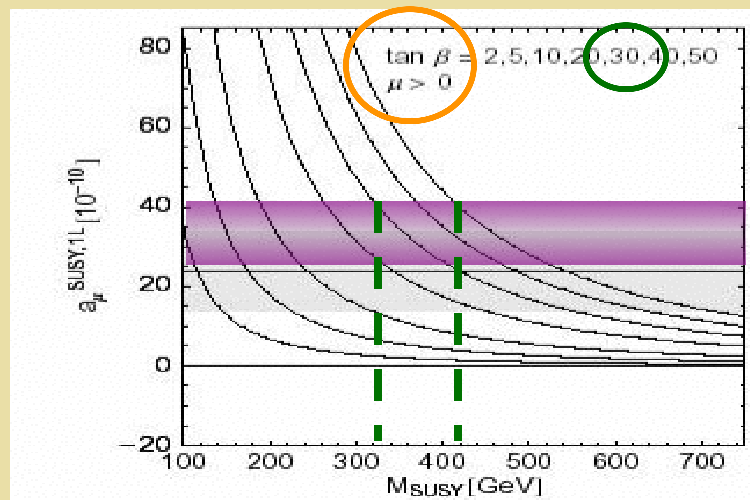

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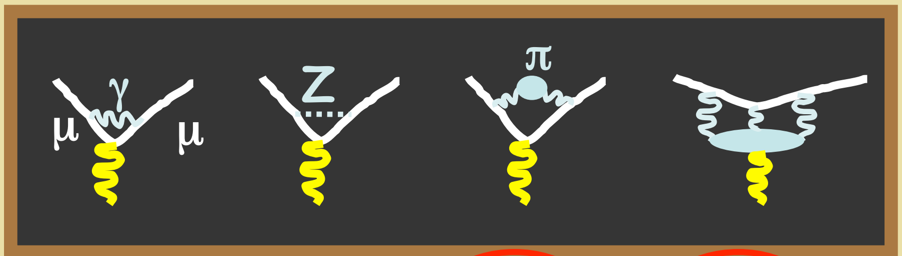

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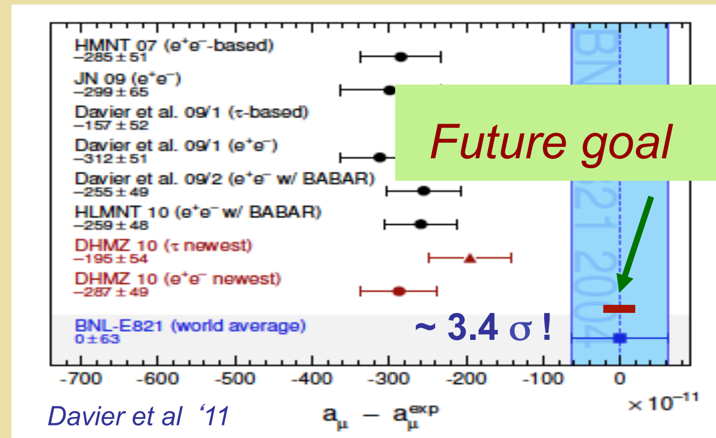

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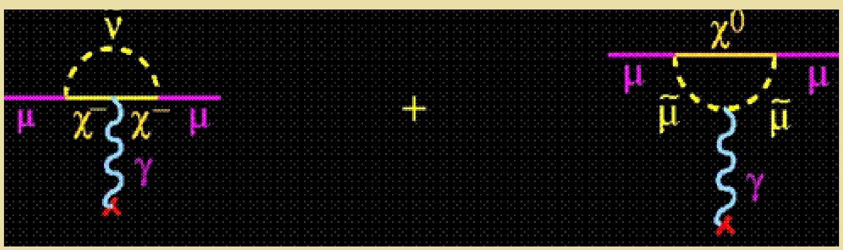

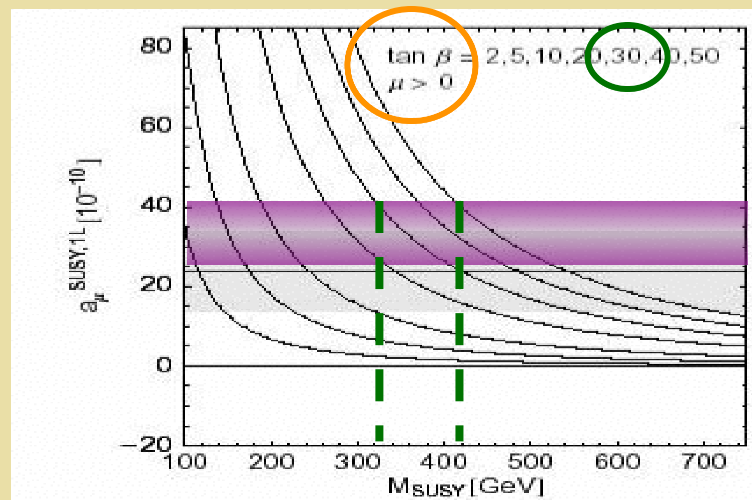

SUSY Loops

HLBL: Compilation

Nyffeler 1001.3970

Contribution	BPP [8]	HKS, HK [9]	KN [10]	MV [11]	BP [5], MdRR [1]	PdRV [6]	N [13], JN [3]
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	–	114±13	99±16
axial vectors	2.5±1.0	1.7±1.7	–	22±5	–	15±10	22±5
scalars	−6.8±2.0	–	–	–	–	−7±7	−7±2
π, K loops	−19±13	−4.5±8.1	–	–	–	−19±19	−19±13
π, K loops +subl. N_C	–	–	–	0±10	–	–	–
quark loops	21±3	9.7±11.1	–	–	–	2.3	21±3
Total	83±32	89.6±15.4	80±40	136±25	110±40	105±26	116±39

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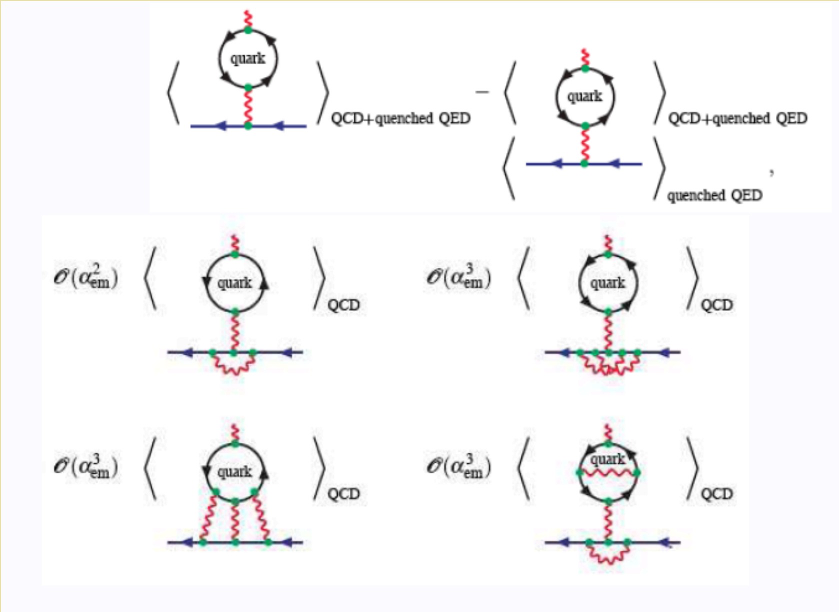
HLS

Lattice QCD

See A. Juttner (B1)

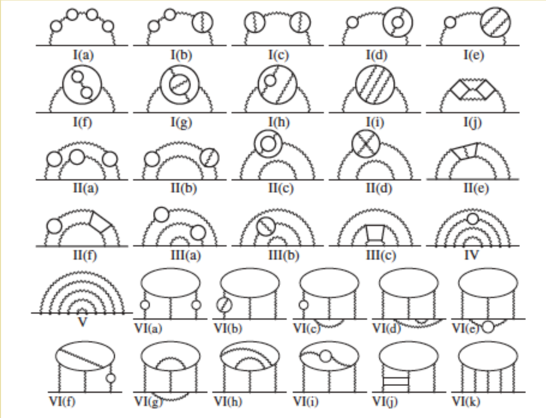
- *Blum, Izubuchi, ….: QED + QCD*



- *Rakow (QCDSF): 4pt function*

Tenth Order QED

Aoyama, Hayakawa, Kinoshita, Nio ‘12



$$a_\mu(\mathrm{QED}) = \sum_{n=1}^{\infty} \left(\frac{\alpha}{\pi}\right)^n a_\mu^{(2n)},$$

TABLE III. Contributions to muon $g-2$ from QED perturbation term $a_\mu^{(2n)}(\alpha/\pi)^n \times 10^{11}$. They are evaluated with two values of the fine-structure constant determined by the Rb experiment and by the electron $g-2$ (a_e).

order	with $\alpha^{-1}(\mathrm{Rb})$	with $\alpha^{-1}(a_e)$
2	116 140 973.318 (77)	116 140 973.213 (30)
4	413 217.6291 (90)	413 217.6284 (89)
6	30 141.902 48 (41)	30 141.902 39 (40)
8	381.008 (19)	381.008 (19)
10	5.0938 (70)	5.0938 (70)
$a_\mu(\mathrm{QED}) \times 10^{11}$	116 584 718.951 (80)	116 584 718.846 (37)