LIBERATION ON THE WALLS IN GAUGE THEORIES AND ANTI-FERROMAGNETS

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North Carolina State University

Erich Poppitz, Mohamed Anber, TS Phys.Rev. D92 (2015) 2, 021701

and with Anders Sandvik, Hui Shao and M. Unsal — In progress

Recent Developments in Semiclassical Probes of Quantum Field Theories — UMass Amherst 2016

INTRODUCTION:
The vacuum structure of gauge theories

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The vacuum structure 1st pass

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- In pure gauge theorie one global symmetry is center symmetry and is unbroaken
- No other global symmetries, no other order parameters
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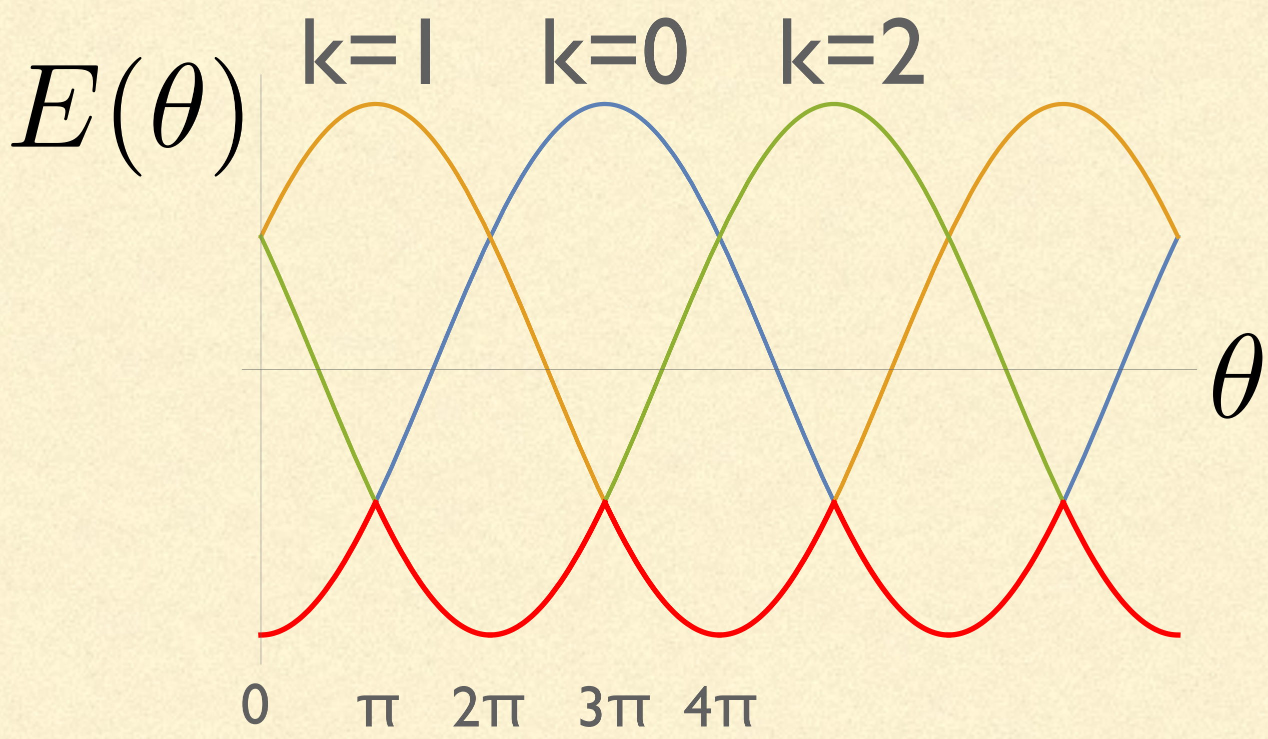
$$\Rightarrow E(\theta) = N^2 f\left(\frac{\theta + 2\pi k}{N}\right)$$

N=3



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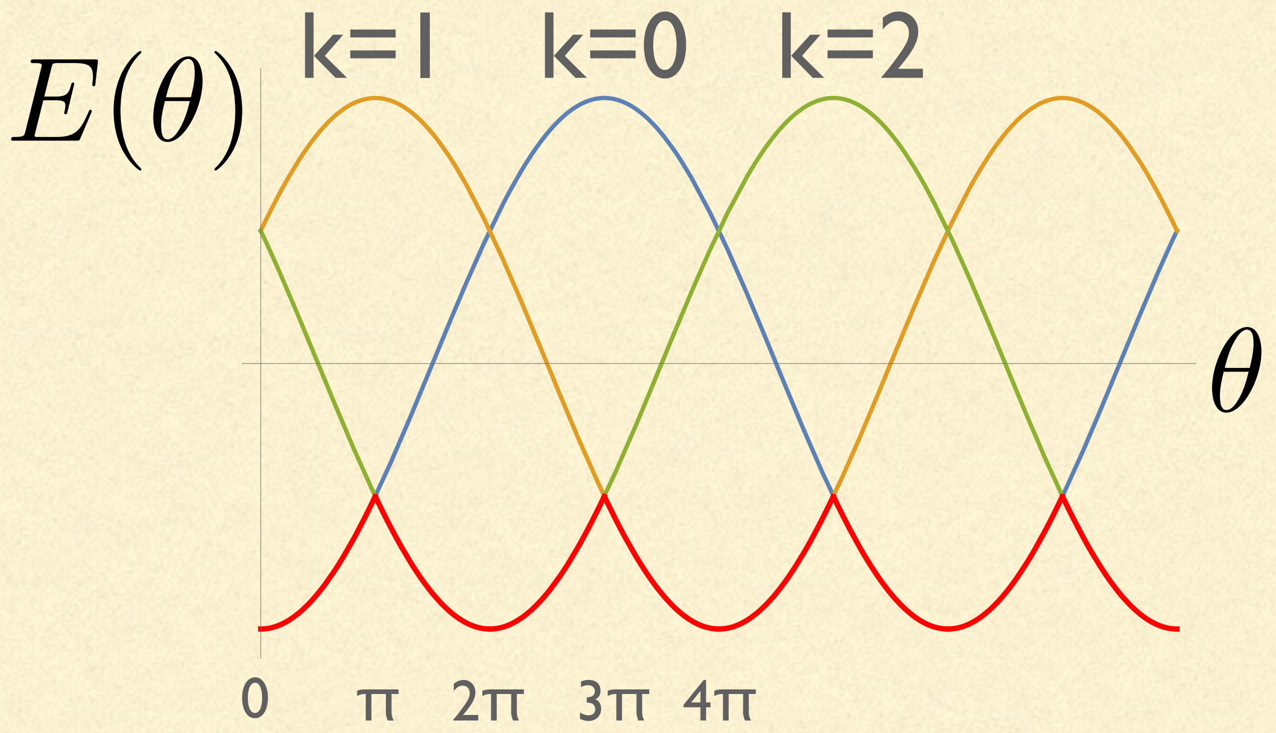

So pure Yang-Mills has multiple vacua labeled by k, but they are non-degenerate except at $\theta=(2k+1)\pi$

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$$\mathcal{L} = \frac{1}{2g^2}\left(\mathrm{tr}F^2 + \sum_{I=1}^{n_f} \bar{\lambda}_I \sigma^\mu D_\mu \lambda_I\right)$$

λ_I -Weyl fermion in adjoint rep.

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Z_{nf} parti belongs to $SU(n_f)$ so the symmetry is $SU(n_f) \times Z_{2N}$

$$\langle \lambda_I \lambda^I \rangle \neq 0 \Rightarrow SU(n_f) \times Z_{2N} \to SO(n_f) \times Z_2$$

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Therefore since the coset $Z_{2N}/Z_2 = Z_N$ the theory has N isolated degenerate vacua

In Super YM the same as before, except there is no continuous symmetries, only descrete

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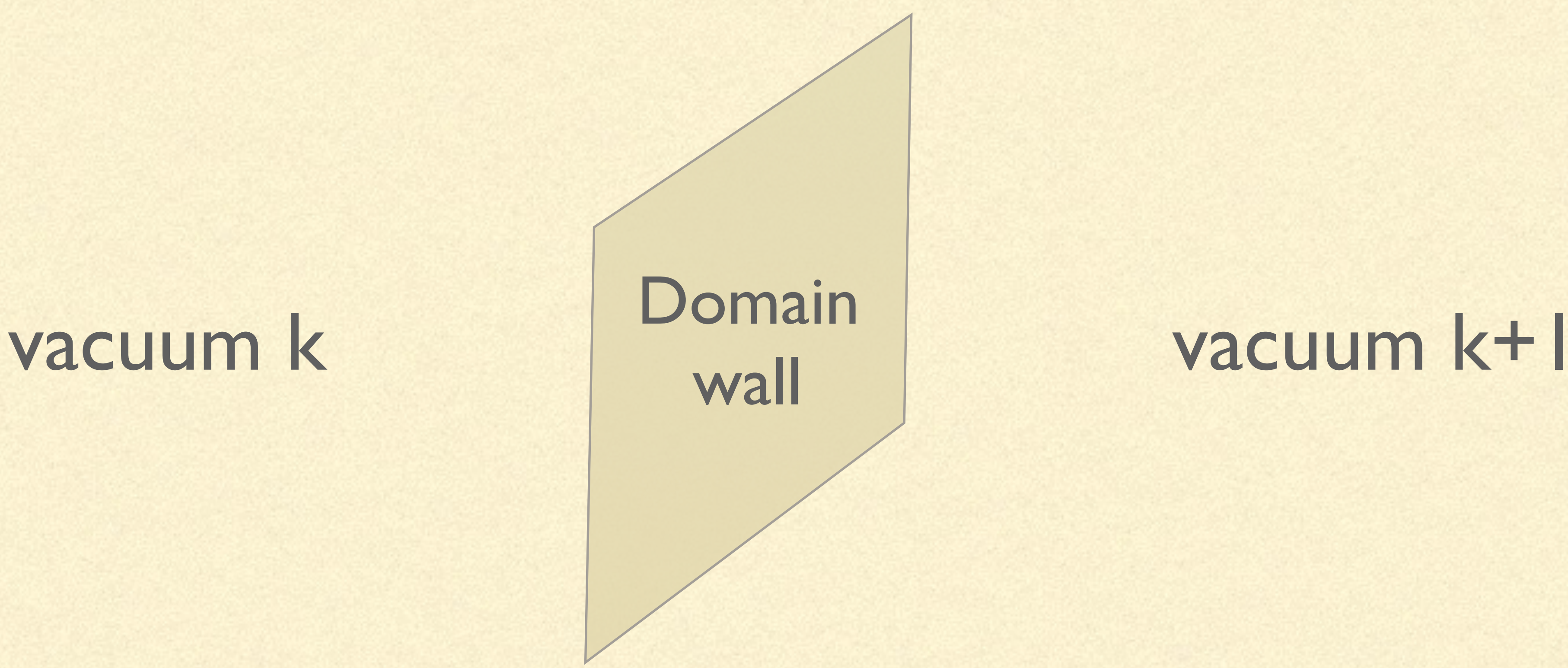


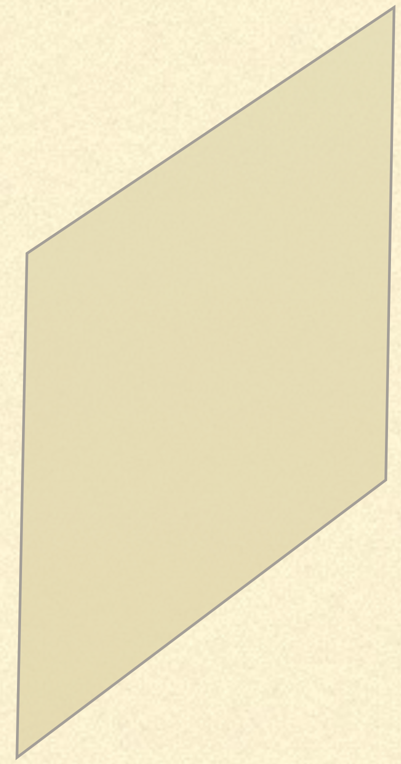
In $\mathcal{N}$=2 Super-Yang mills softly broken to $\mathcal{N}$=1

an entire microscopic picture is known

For SU(2) the picture is: there are two degenerate vacua

Monopoles condense

(e,m)=(0,1)



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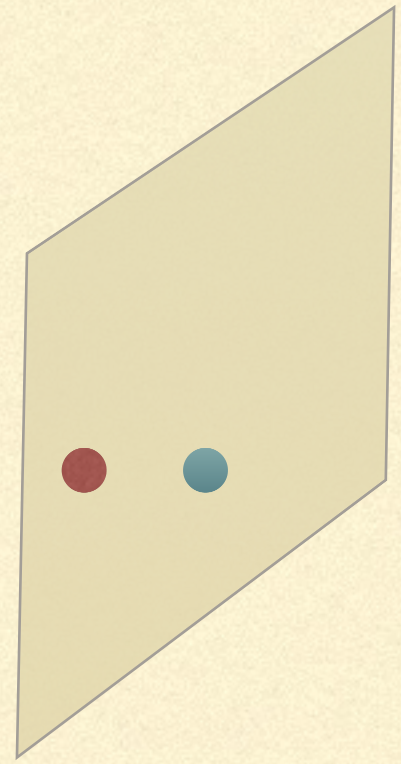

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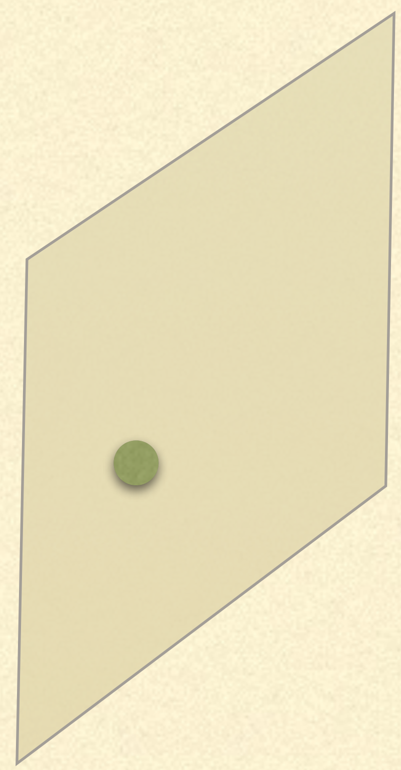

So although there are no objects with unit fundamental charge, there is an excitation on the wall supporting unit fundamental charges

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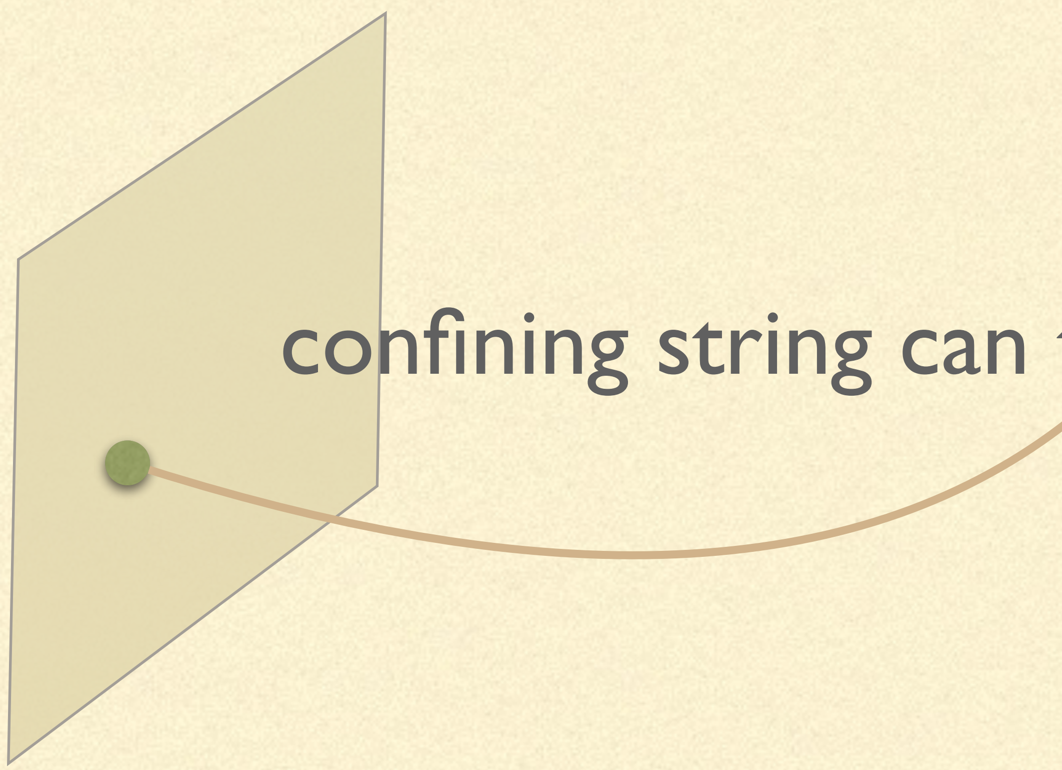

Due to S.-J. Rey 1998

Explored by Witten in M-theory construction of N=1 SYM

- For $\mathcal{N}$<2 no such statements can be made rigorous
- The problem comes from the fact that monopoles while a feature of 4D $\mathcal{N}$=2 theory, are very elusive in $\mathcal{N}$=1 theories and theories with no supersymmetries (they require gauge fixing, assumptions of abelian dominance, etc.)
- The potential implications in non-supersymmetric theories were mostly ignored.

SO HOW TO STUDY THESE THEORIES?

- Non-abelian gauge theories in 4D do not have a small, tunable dimensionless parameter
- There is a prescription by M. Unsal on how to analytically study confining phenomena in 4D
- The prescription involves compacifying one direction in a way that prevents confinement/deconfinement transition
- The theory obtains a dimensionless parameter $L\Lambda$ which can be made arbitrarily small
- It turns out that the theory is completely analytically calculable with semi-classical methods for $L\Lambda \ll 1$
- Note that this is NOT thermal compactification. In fact the thermal theory is not analytically tractable.
- Also note that this is not a 3D YM theory.

$$\mathcal{S} = \frac{1}{2g^2}\int d^4x\, \mathrm{tr} F_{\mu\nu}^2 \to \frac{L}{g^2}\int d^3 \mathrm{tr}\left(F_{ij}^2 + (D_i A_0)^2\right)$$

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I will focus on SU(2) here for simplicity

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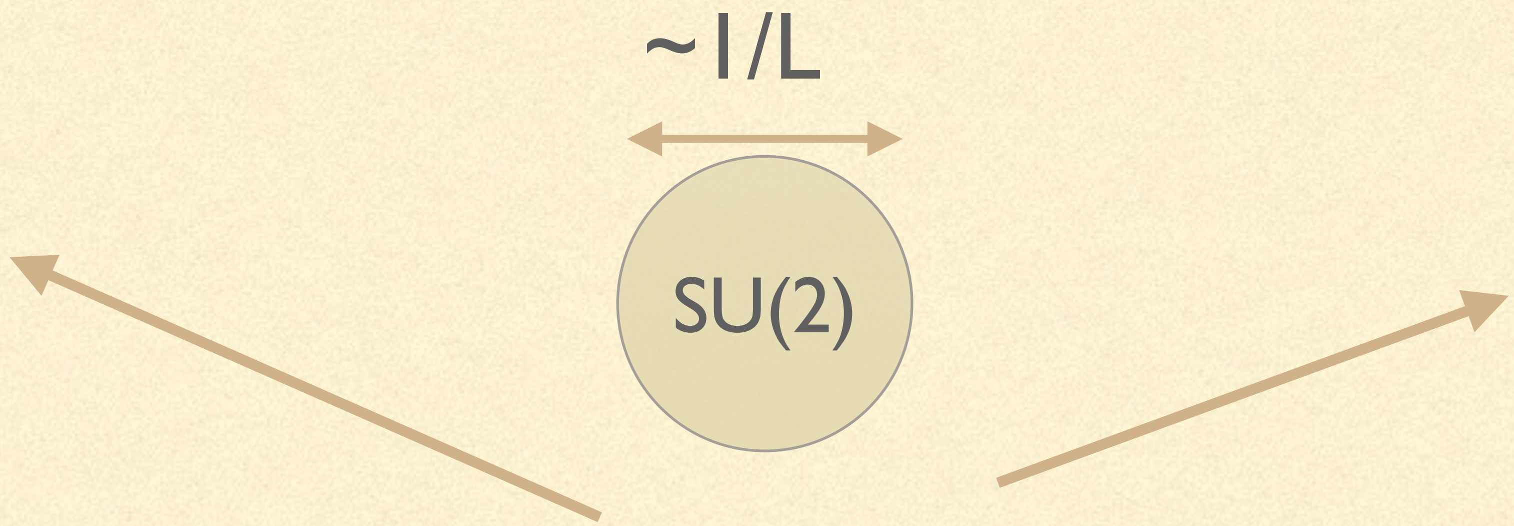

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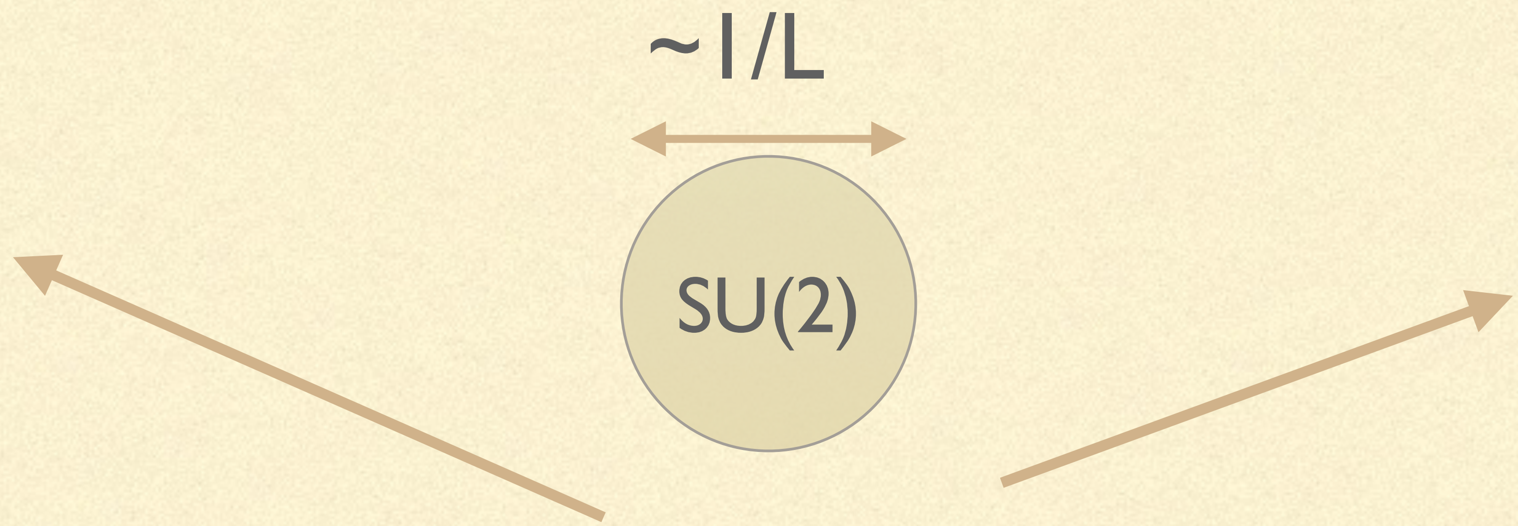

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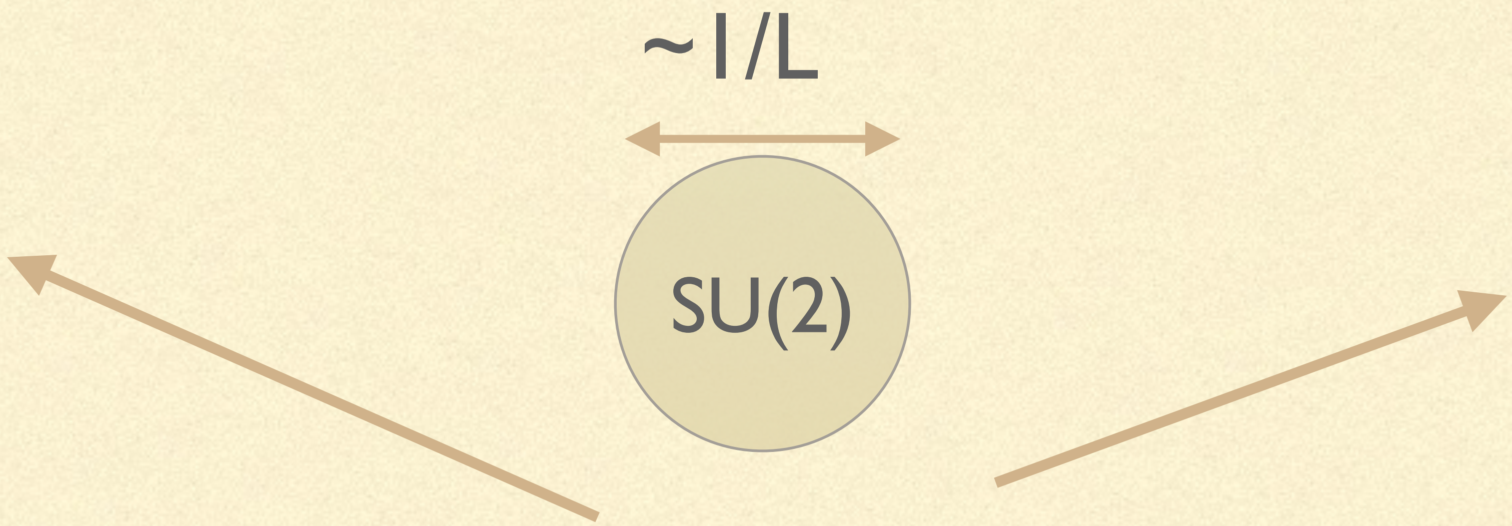

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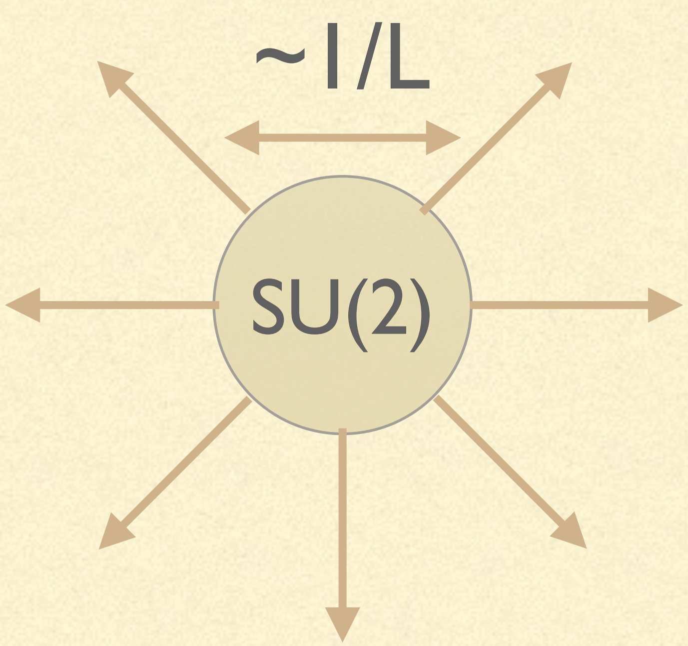
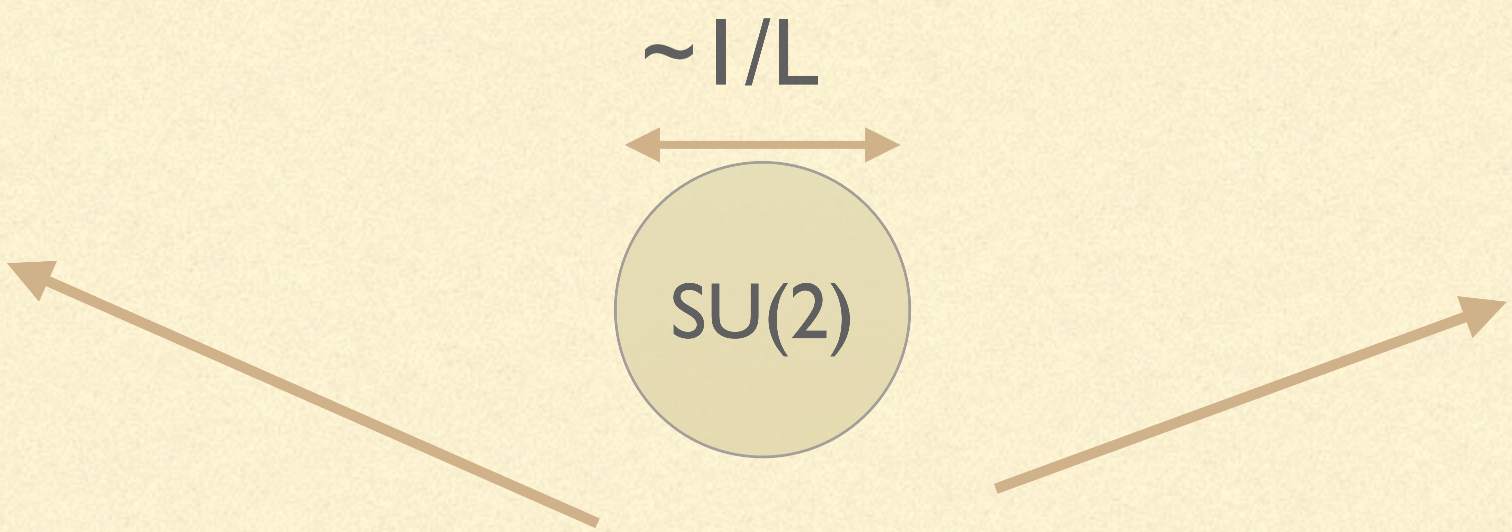


$$\vec{B} \sim \frac{\hat{r}}{r^2}$$



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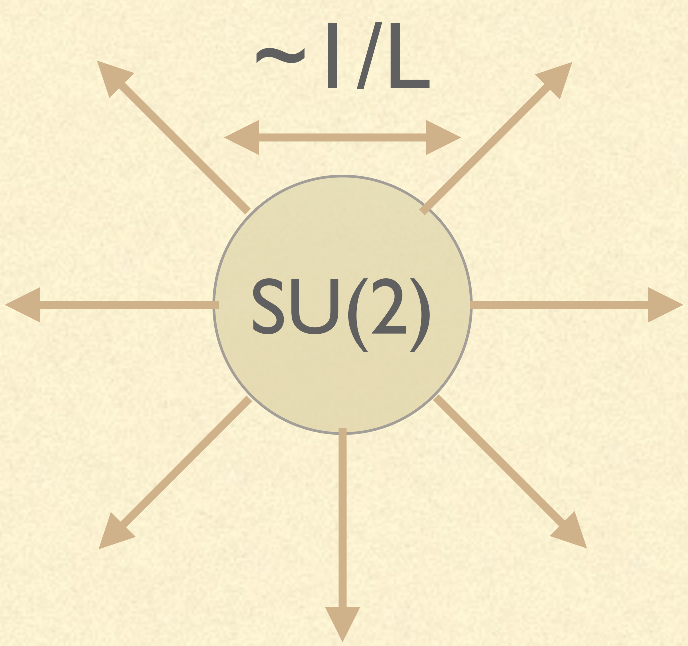

U(1) magnetic monopoles

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- This is the idea of resurgent trans-series construction (Unsal/ Dunne et. al.)

The effective action at L/Λ<<1:

$$\mathcal{L} = \frac{L}{g^2(L)}\mathcal{F}_{ij}^2$$ + monopoles

$i = 0, 1, 2$

time space

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$$\mathcal{L} = \frac{g^2(L)}{2L(2\pi)^2} \left[(\partial_i \sigma)^2 - m^2 \cos \sigma \right]$$

Massgap

Due to monopole(-instantons)

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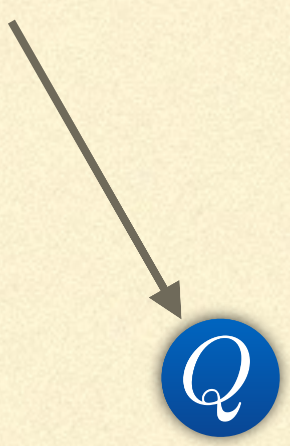

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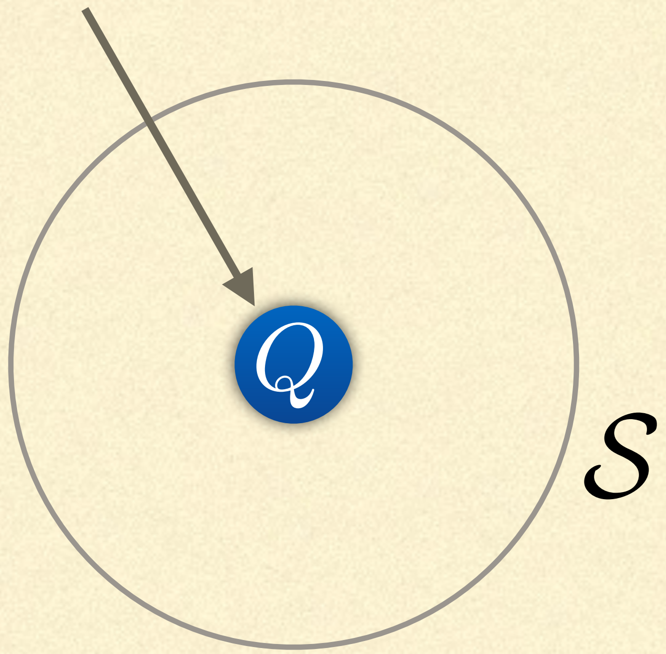

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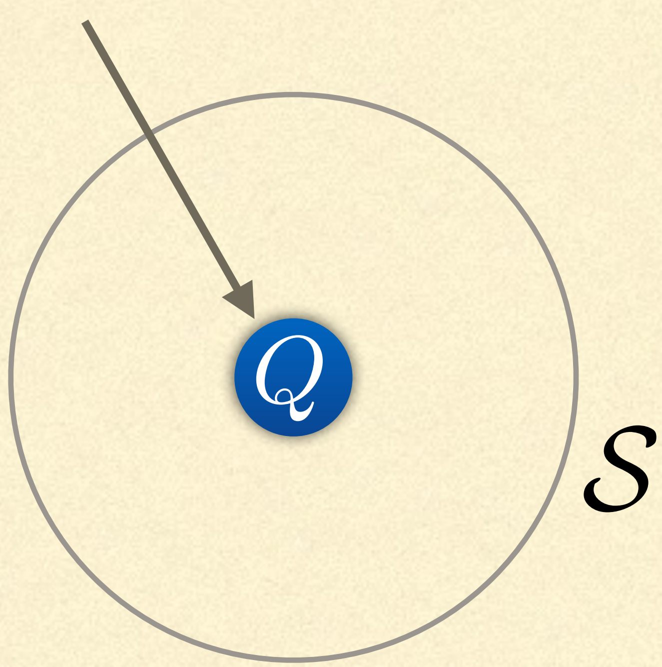

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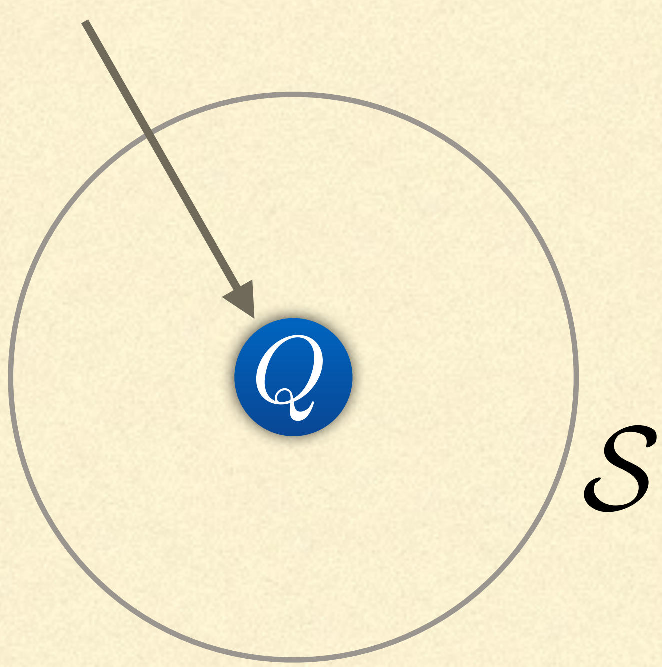

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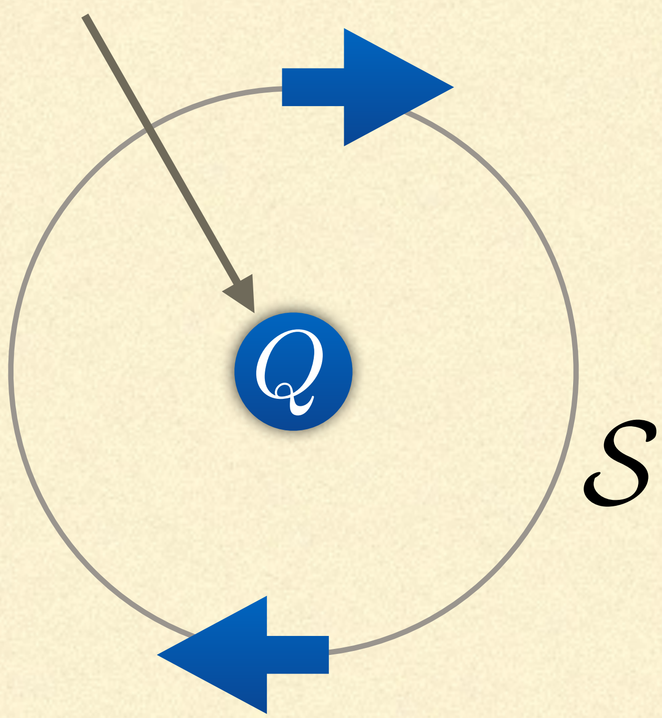

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σ—COMPACT SCALAR $$\oint_{\mathcal{S}} dx^i\partial_i\sigma = 2\pi Q$$

σ Winds Q times around the source

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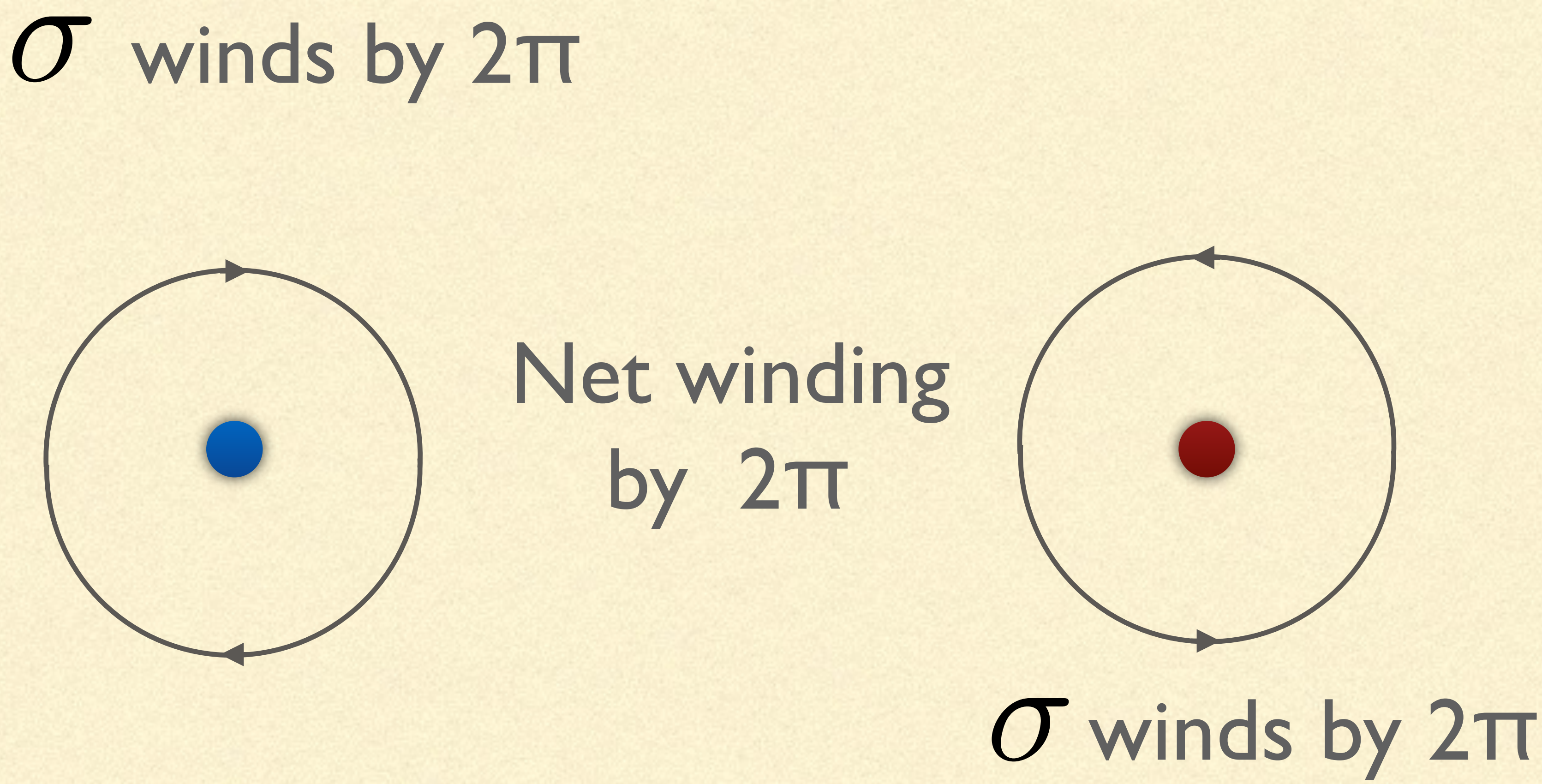
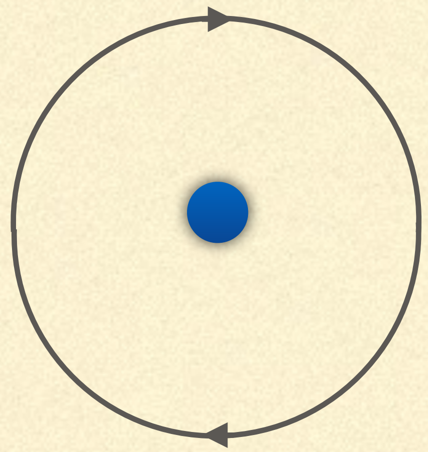


CONFINING STRINGS

σ winds by 2π

$V(\sigma) \propto -\cos(\sigma)$



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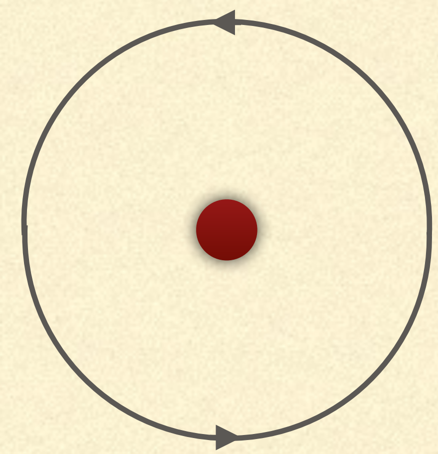
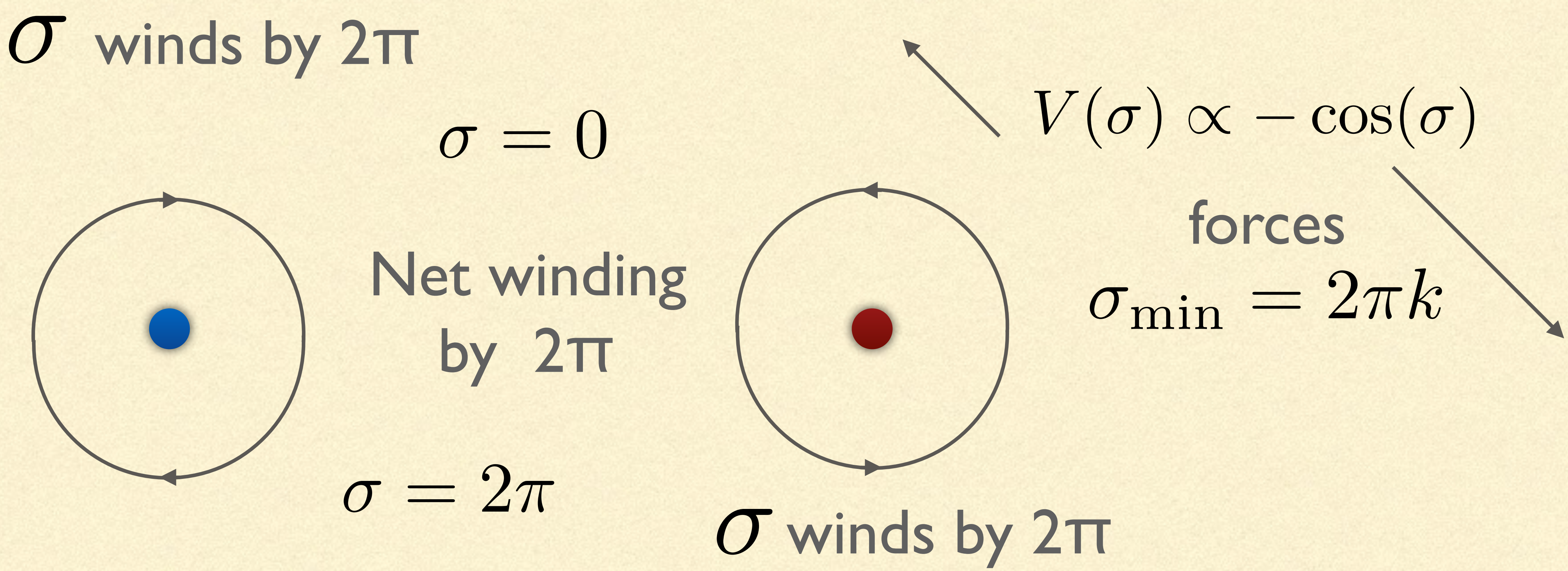


forces $\sigma_{\min} = 2\pi k$

σ winds by 2π

CONFINING STRINGS



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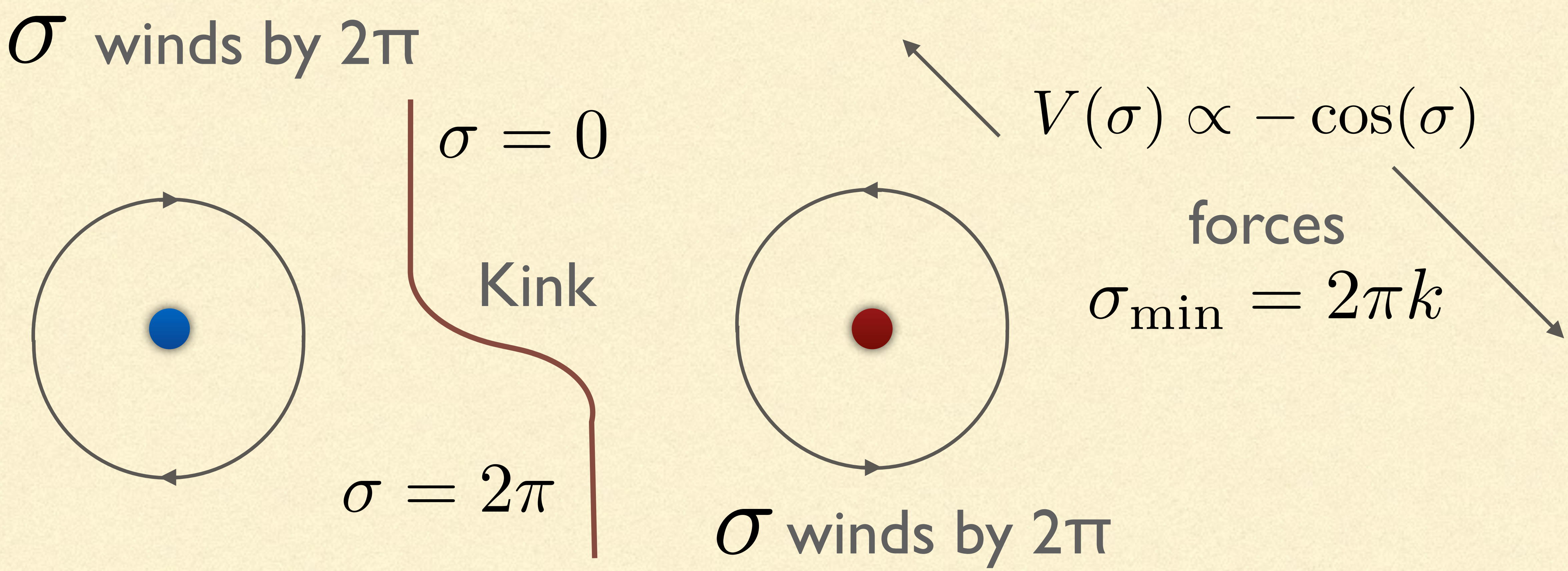

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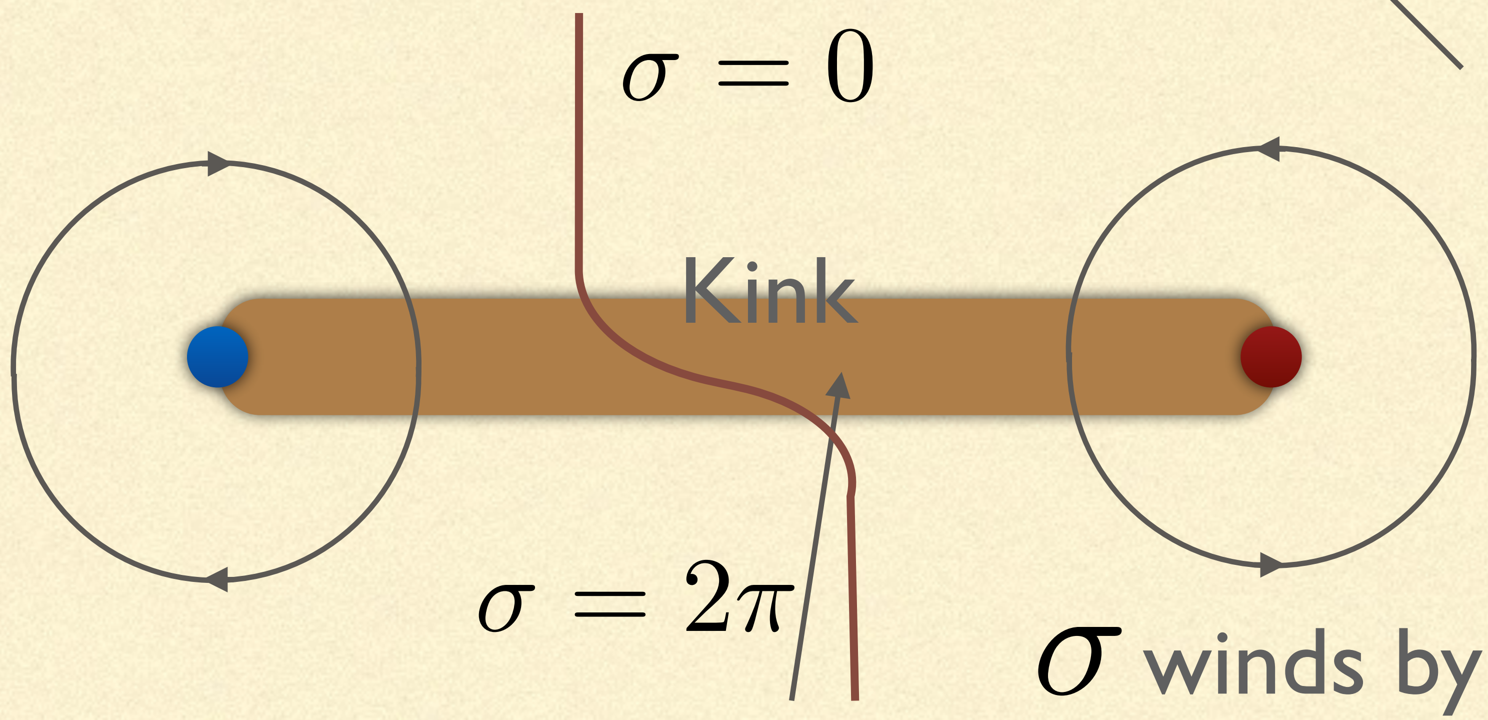

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forces

$\sigma_{\min} = 2\pi k$

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winding is localized on the string

CONFINING STRINGS

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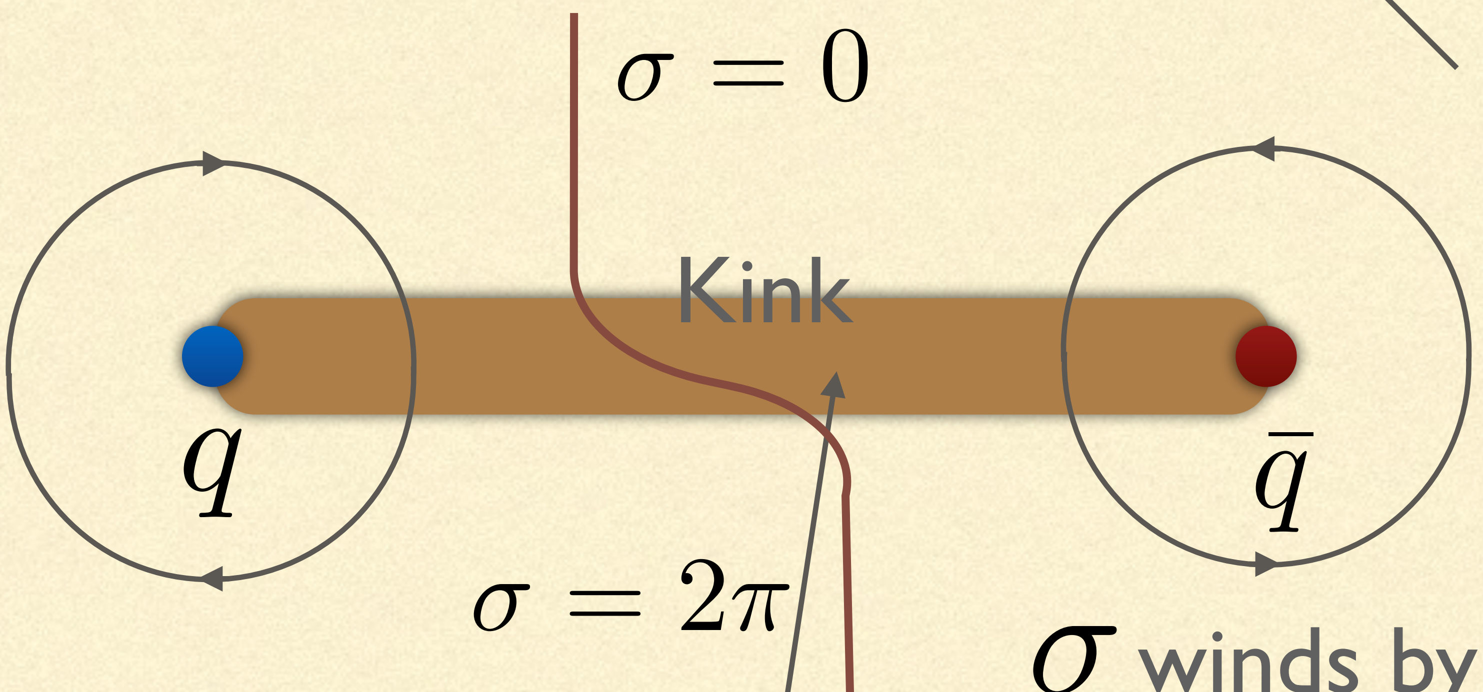

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winding is localized on the string

Thickness of the string ~
scale of the density of monopoles

4D $Q_{top}=1$ instantons

monopole with $Q_{top}=1/2$

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anti-monopole with $Q_{top}=1/2$

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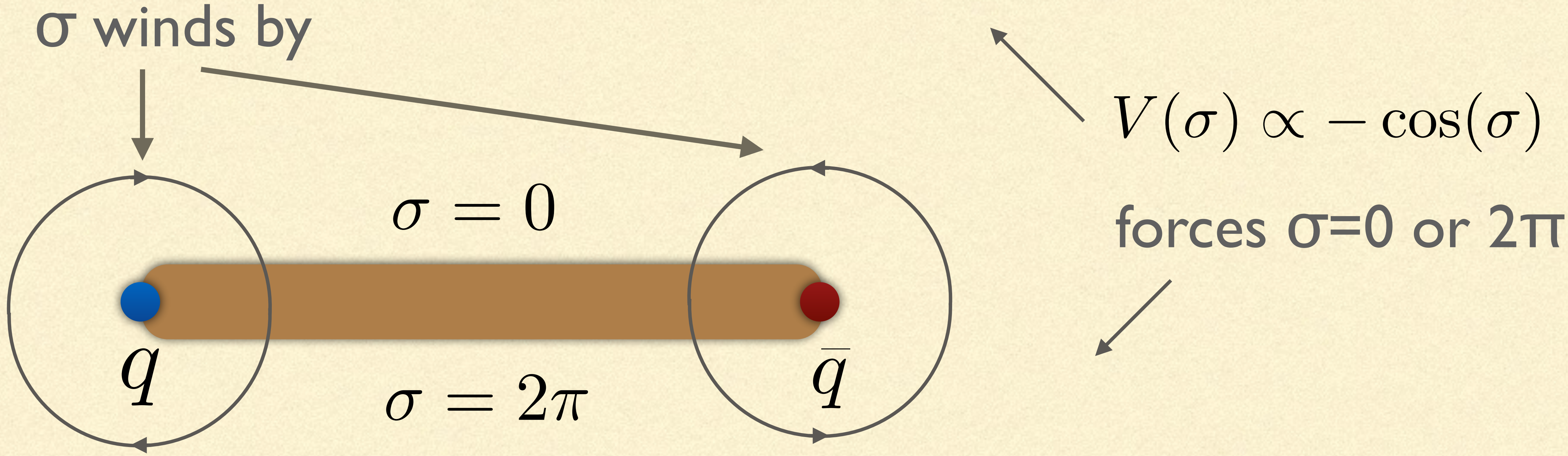
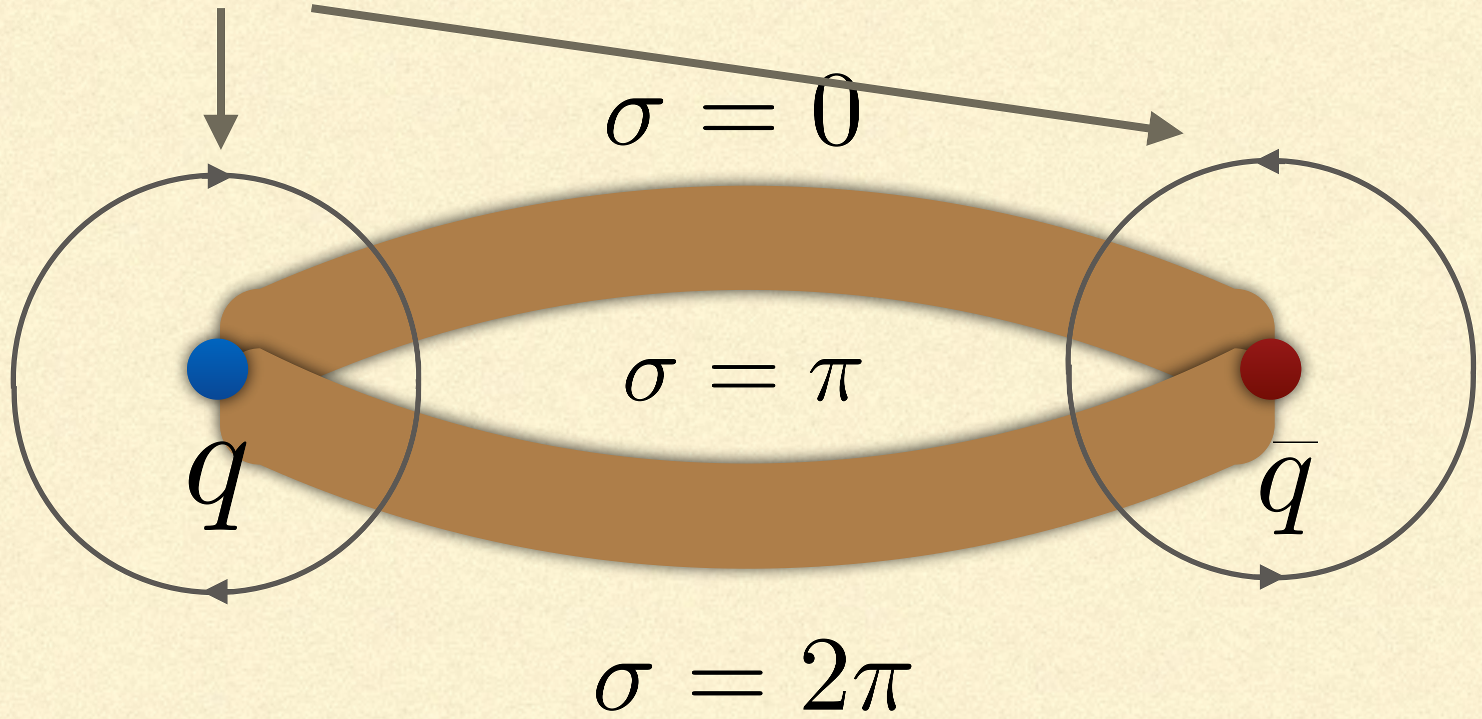
CONFINING STRINGS



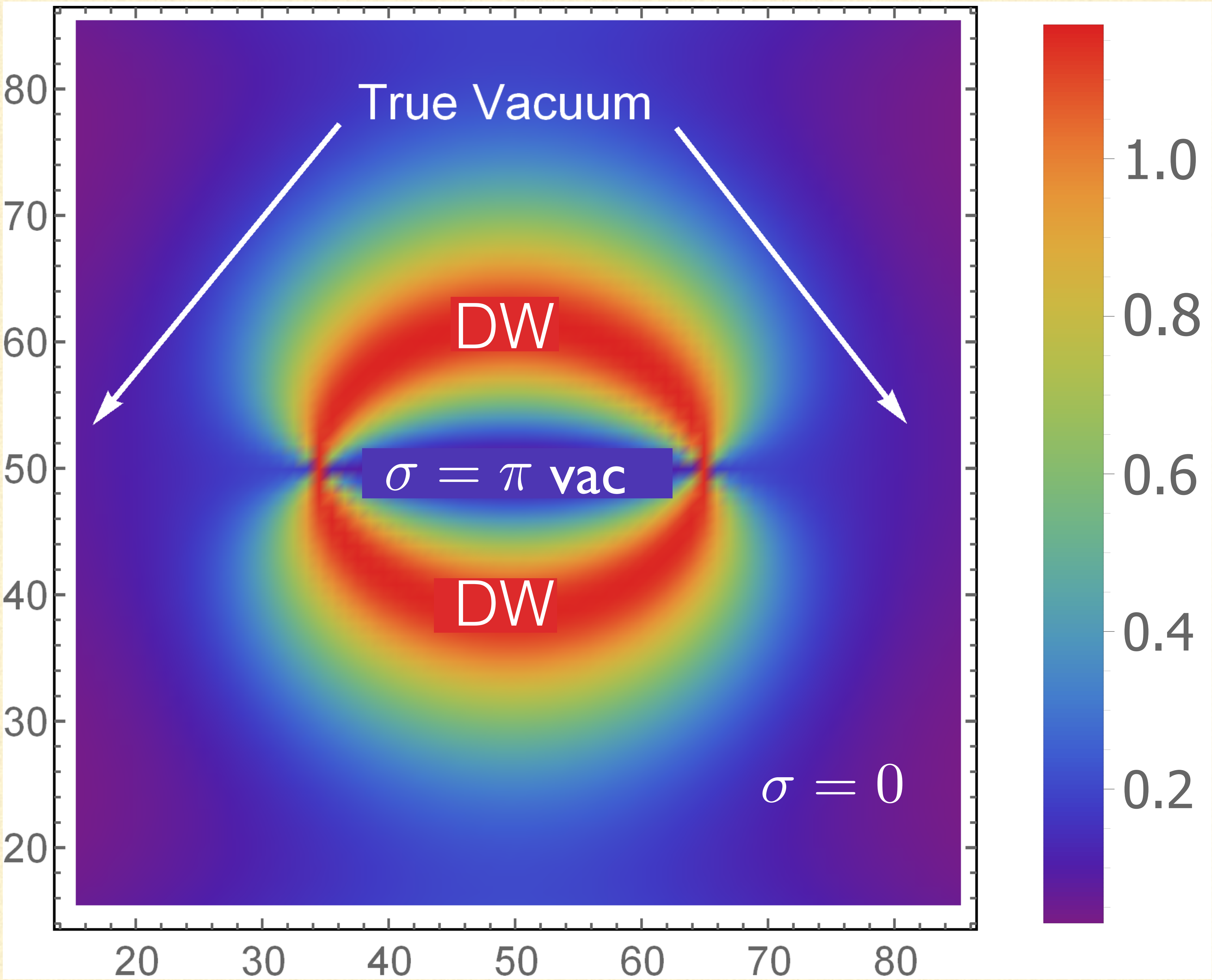
CONFINING STRINGS

σ winds by



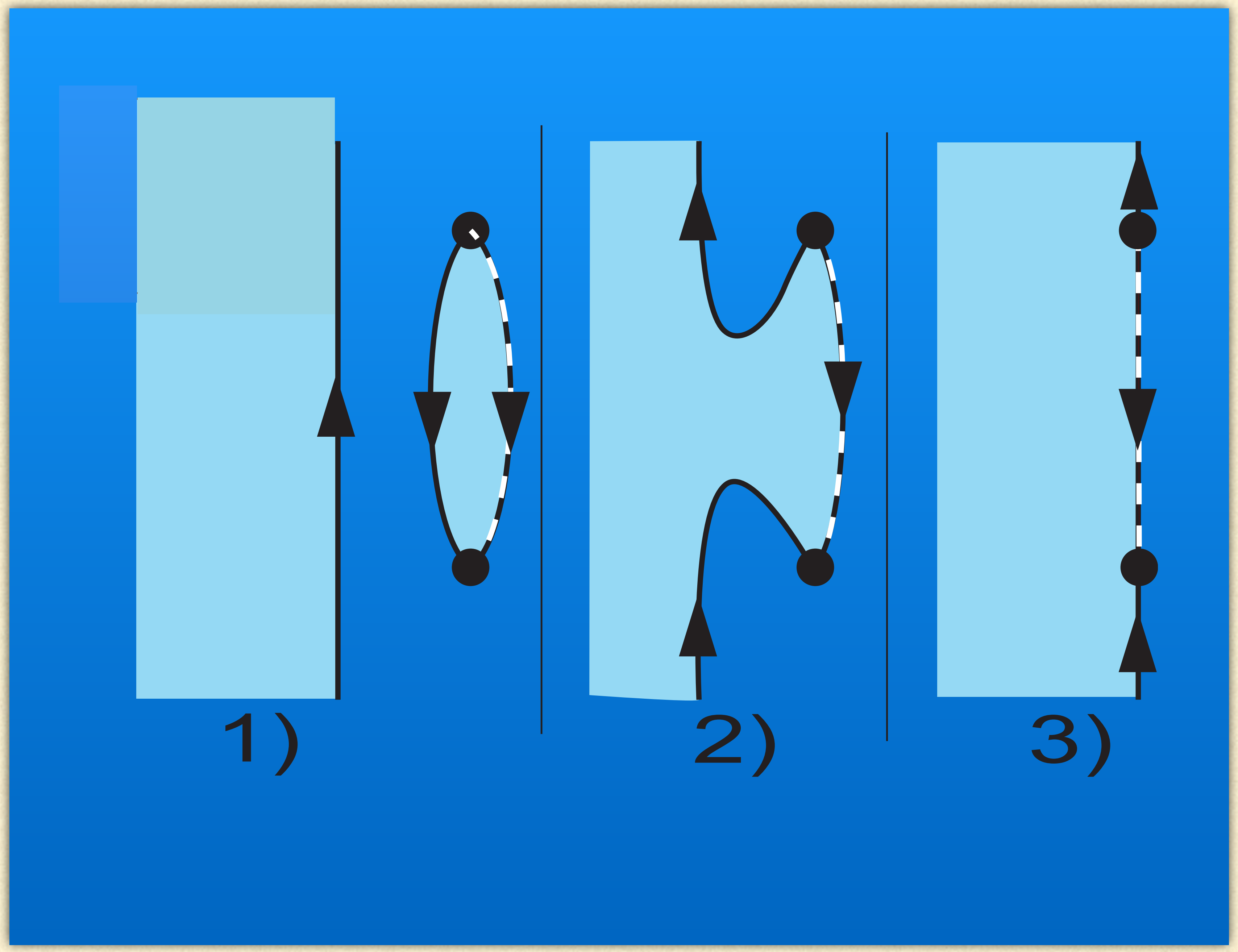
$$\mathcal{V}_{eff} = (\dots)\cos(2\sigma)$$

forces σ=0,π

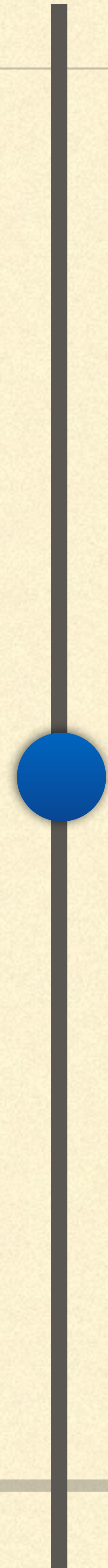


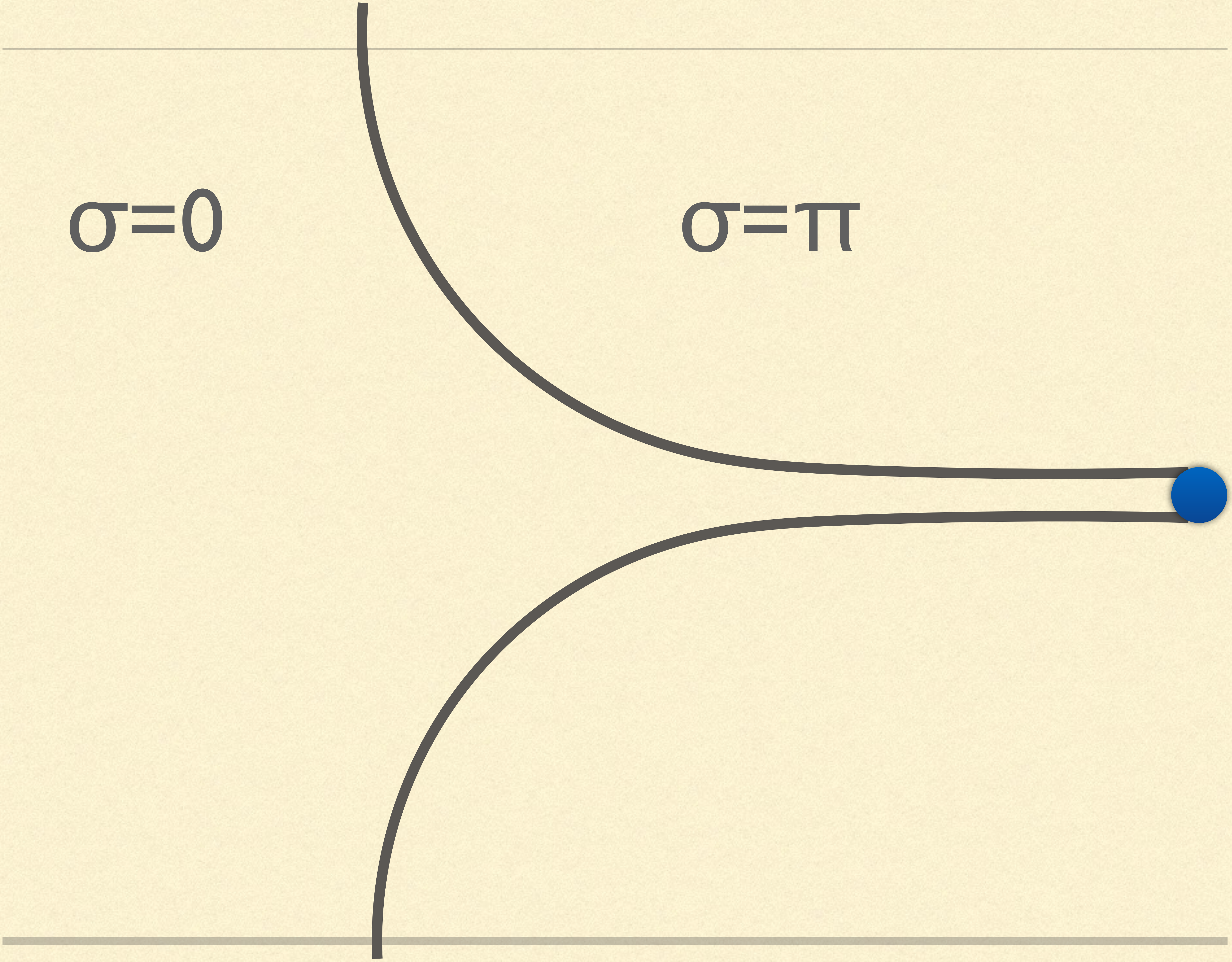
True Vacuum
DW
$\sigma = \pi$ vac
DW
$\sigma = 0$
80
70
60
50
40
30
20
20
30
40
50
60
70
80
1.0
0.8
0.6
0.4
0.2

LIBERATION OF QUARKS ON THE WALL

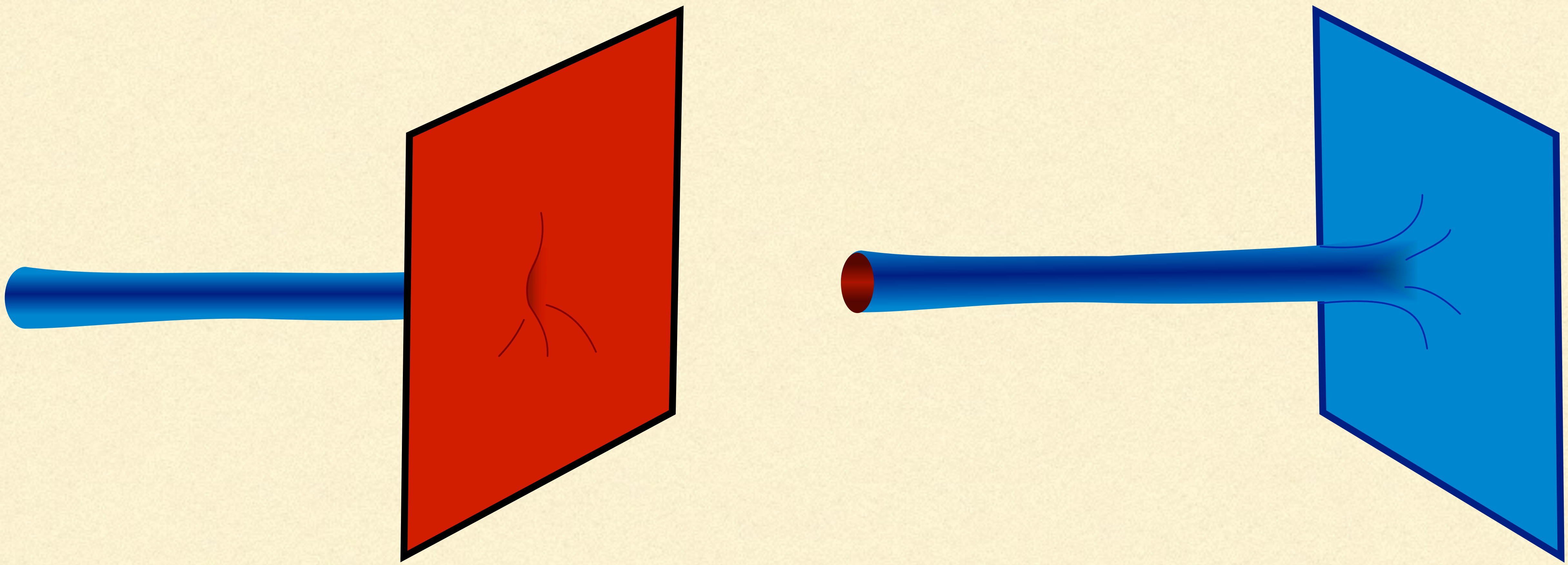


$\sigma=0$ $\sigma=\pi$




σ=0
σ=π

SPECULATION ABOUT 4D



vacuum 1

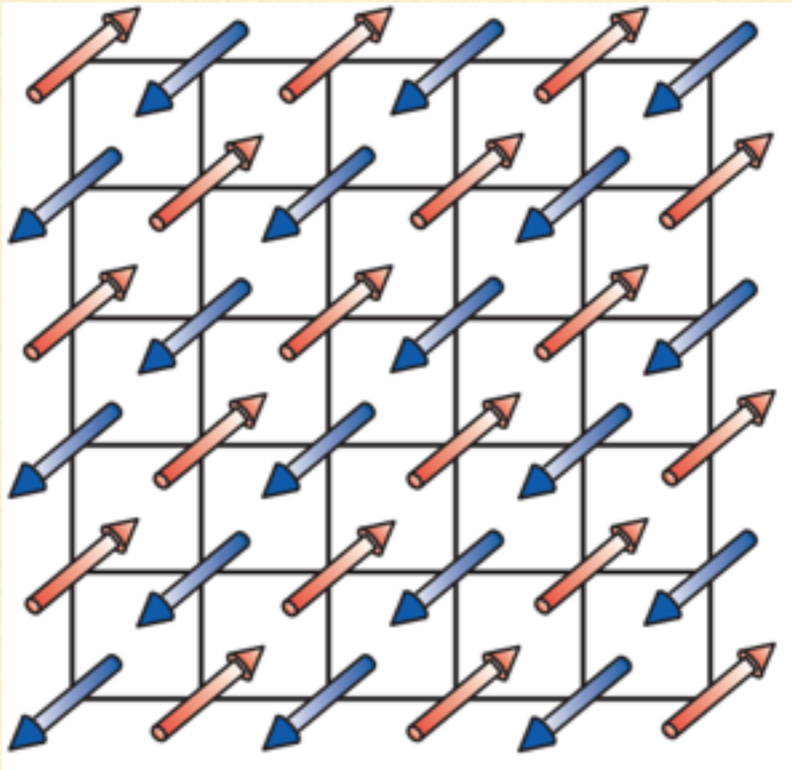
vacuum 2

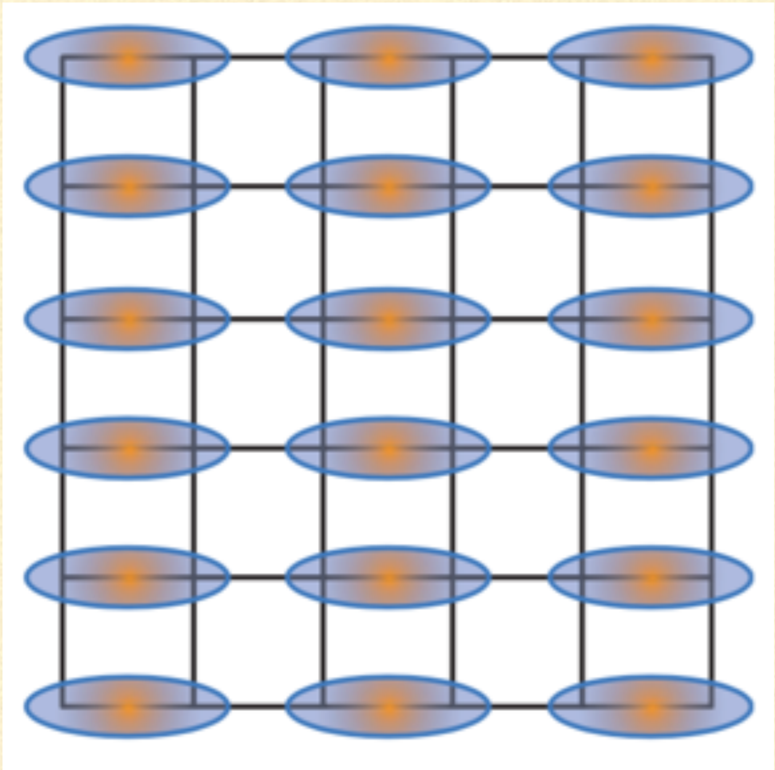
SPIN ANTI-FERROMAGNETS AND VALENCE BOND SOLIDS

(in progress: Anders Sandvik, Hui Shao and Mithat Unsal)

Neel state
— ferromagnetic order

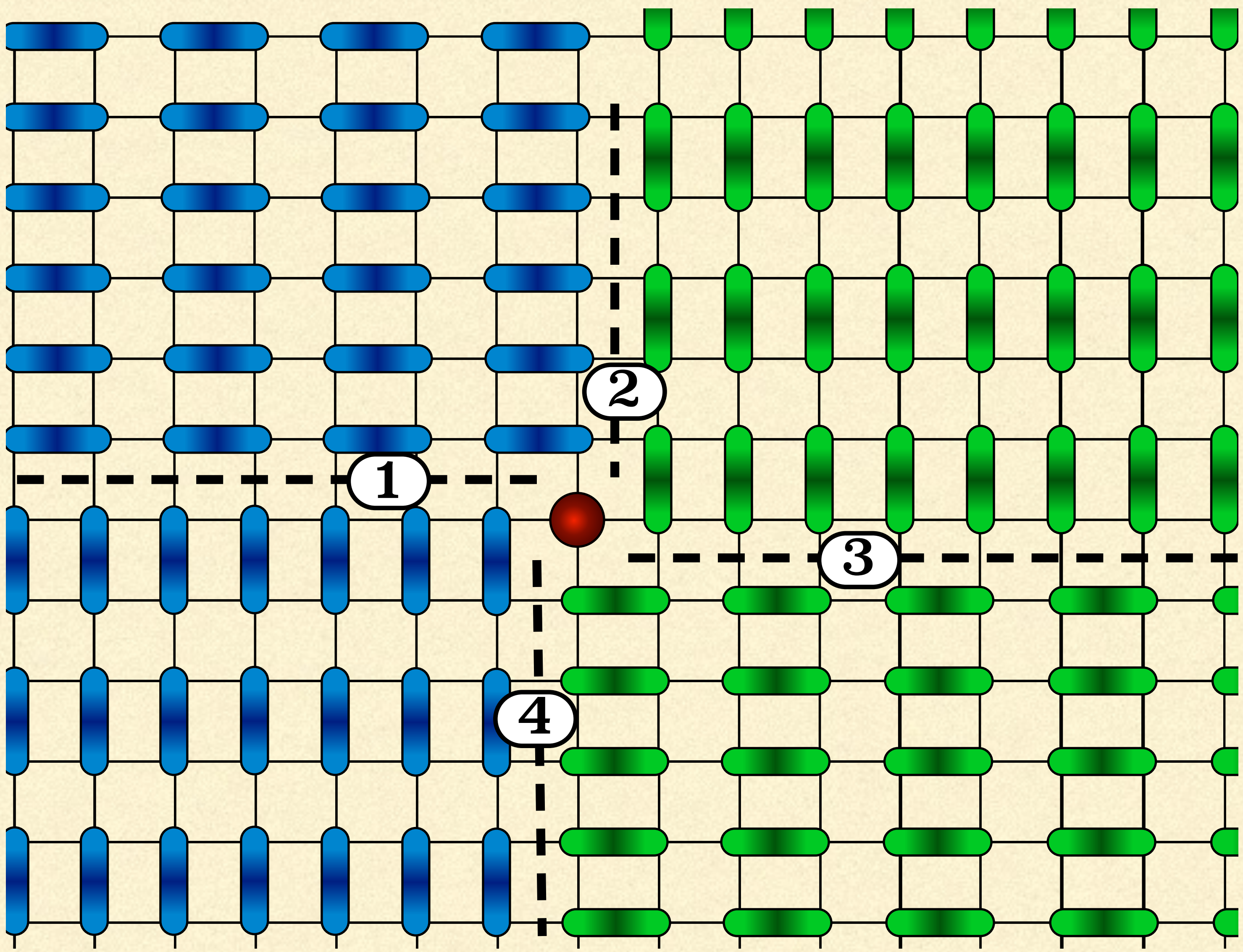
Valence Bond Solid
singlets—dimer have long range crystaline order



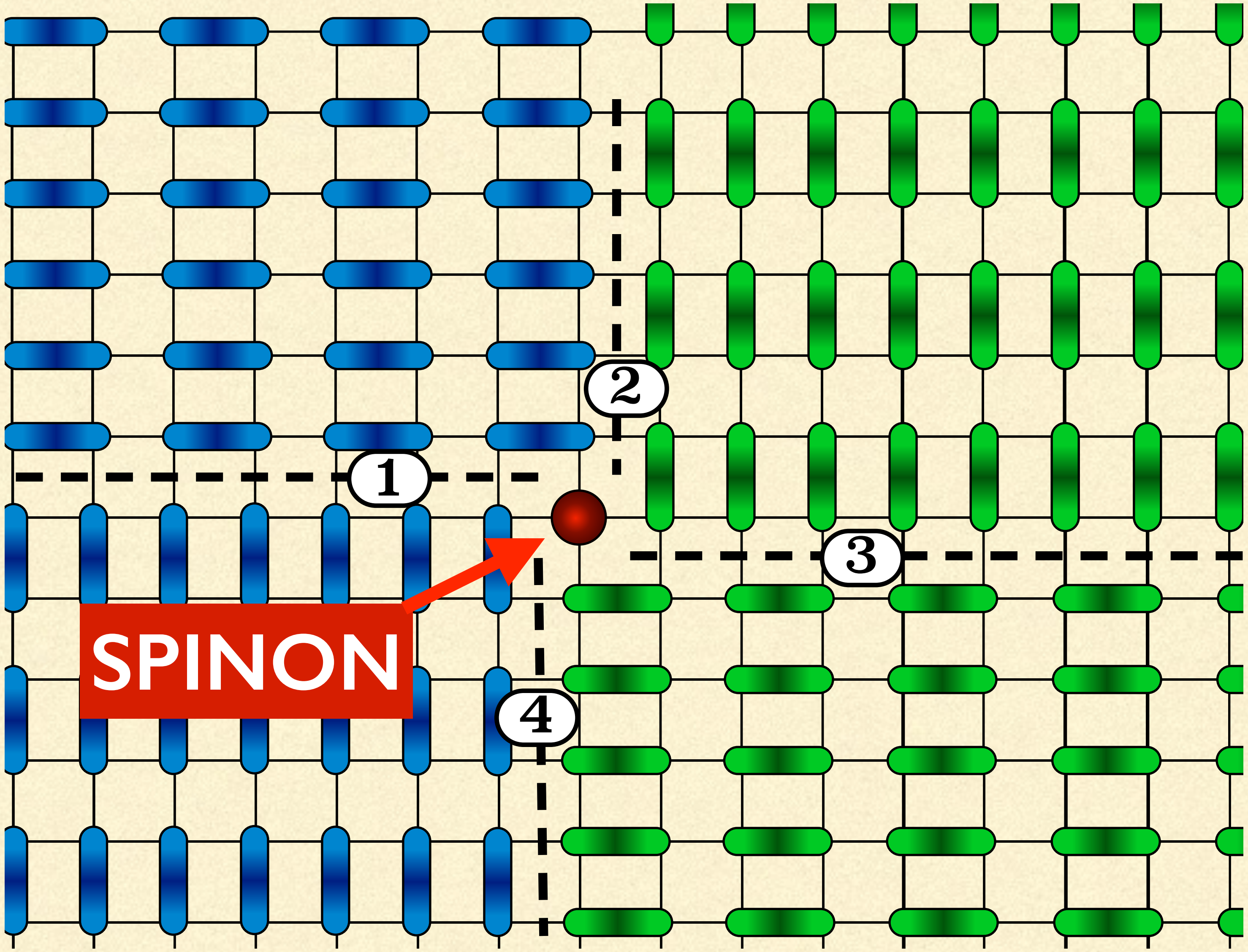


pictures from
Kaul, Melko, Sandvik Annu.Rev.Cond.Matt.Phys.4(1)179 (2013)

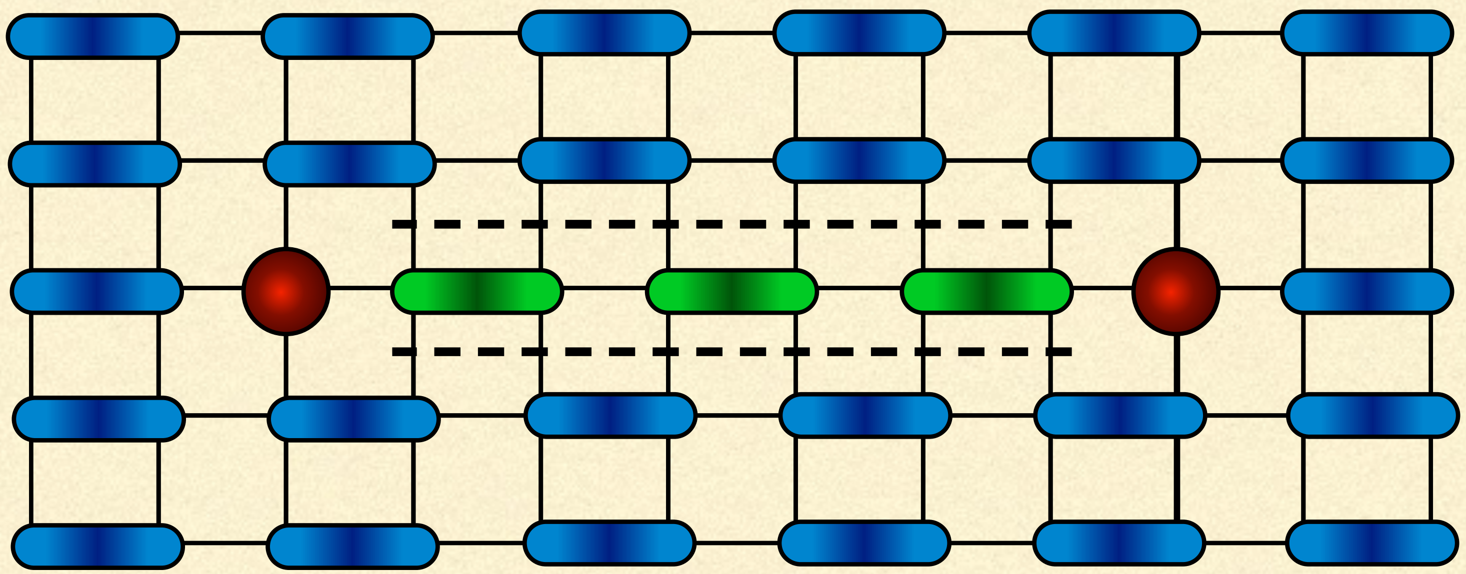
VALENCE BOND SOLID VACUA



VALENCE BOND SOLID VACUA



UNPAIRED SPINS ARE CONFINED



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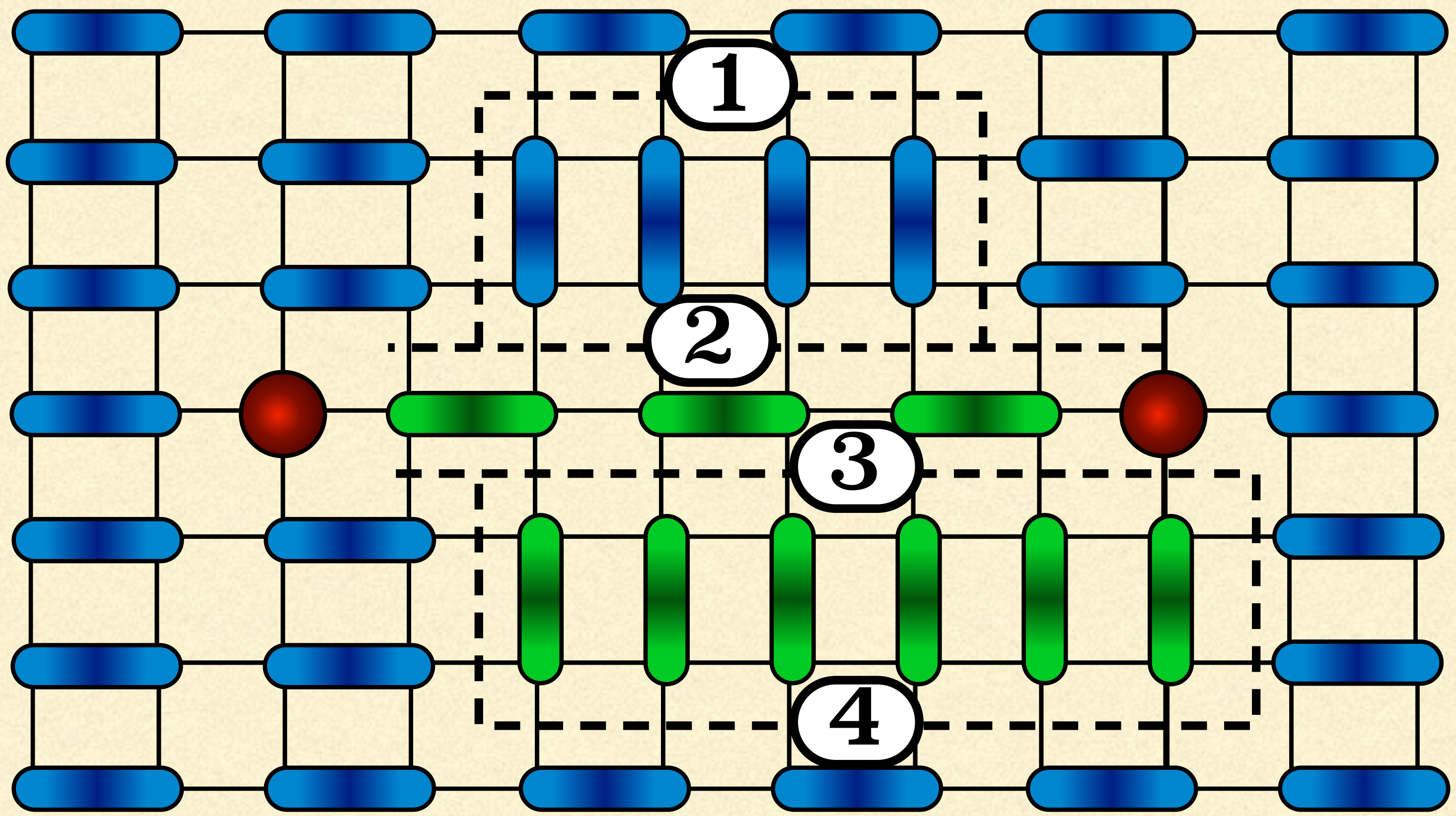

THE J-Q MODEL

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— Generically in the Neel state

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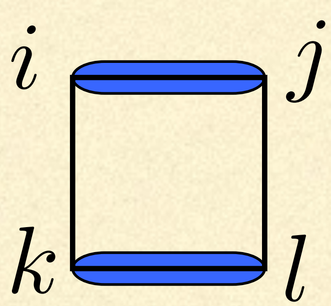
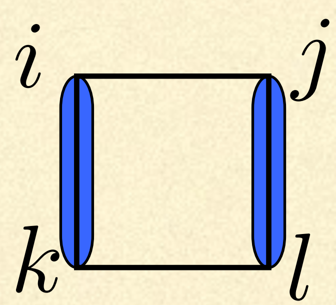
$$H_{JQ} = J \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j - Q_x \sum_{\langle ij \rangle_x \langle kl \rangle_x} P_{ij} P_{kl} - Q_y \sum_{\langle ik \rangle_y \langle jl \rangle_y} P_{ij} P_{kl}$$



Q_x



Q_y

$$P_{ij} = \boldsymbol{S}_i \cdot \boldsymbol{S}_j - 1/4$$ —Singlet projector

THE J-Q MODEL

$$H = J \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$ — minimal quantum anti-ferromagnet

— Generically in the Neel state

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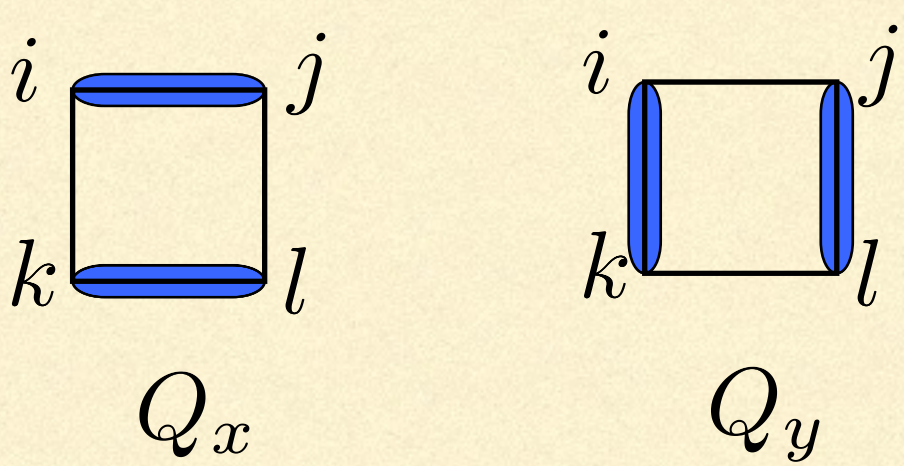
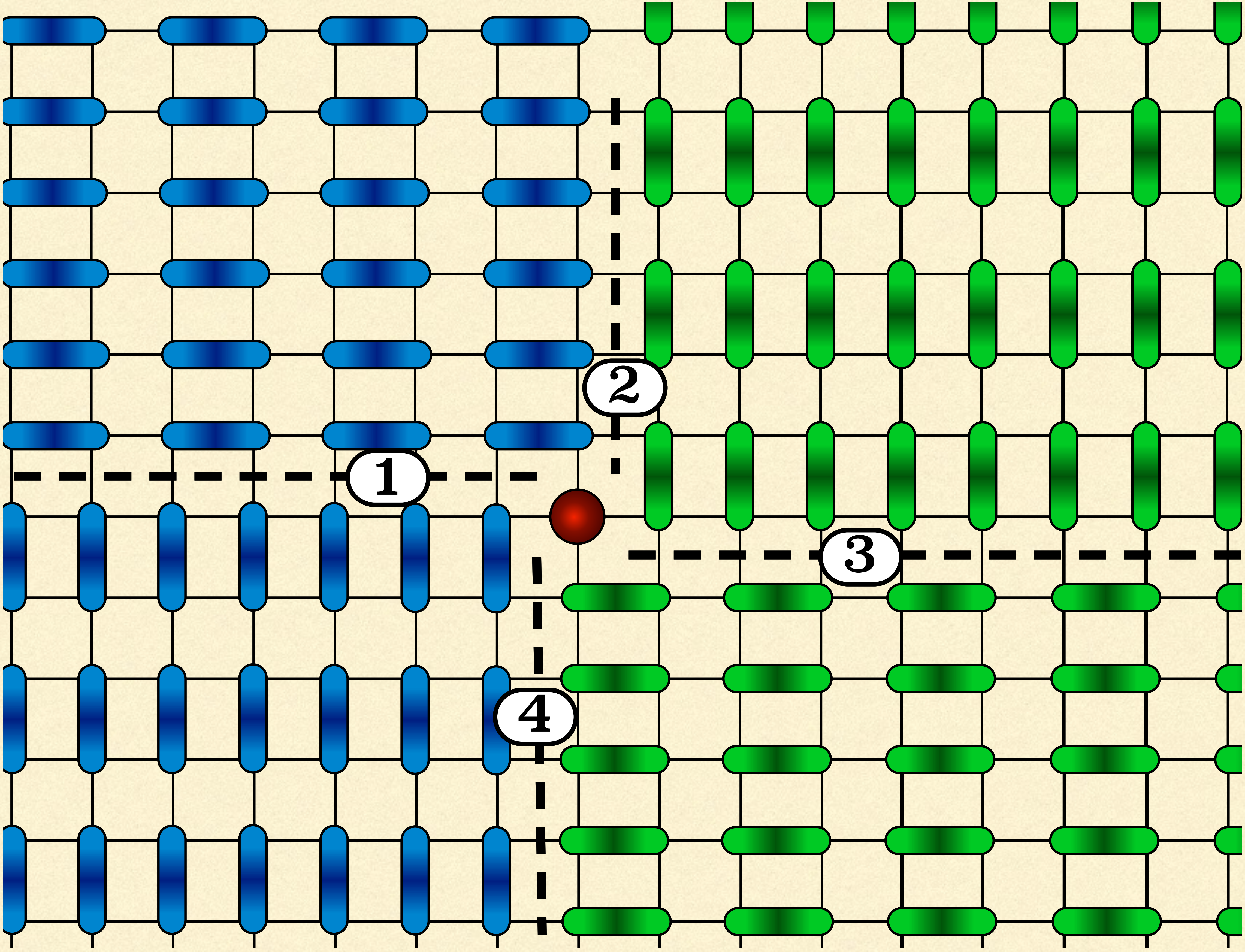
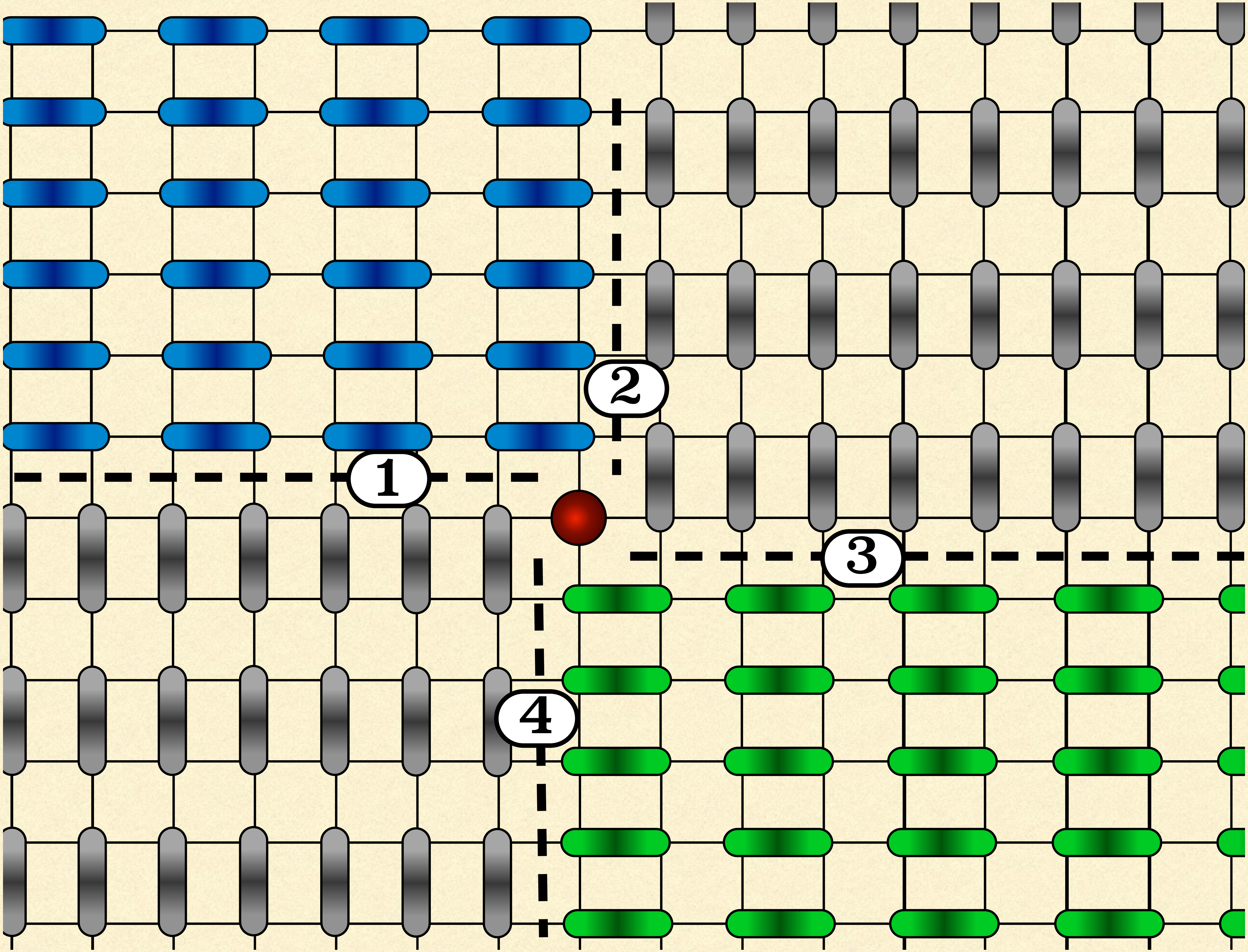


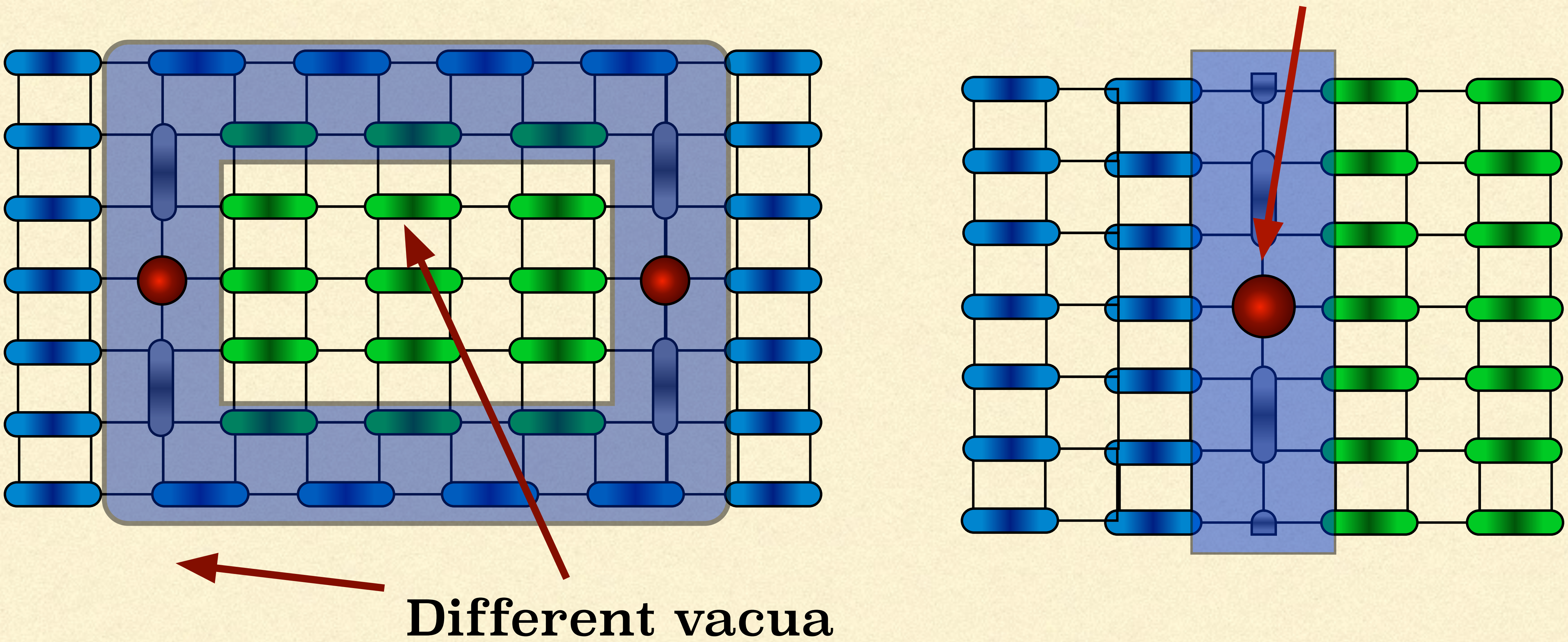
$$P_{ij} = \boldsymbol{S}_i \cdot \boldsymbol{S}_j - 1/4$$ —Singlet projector

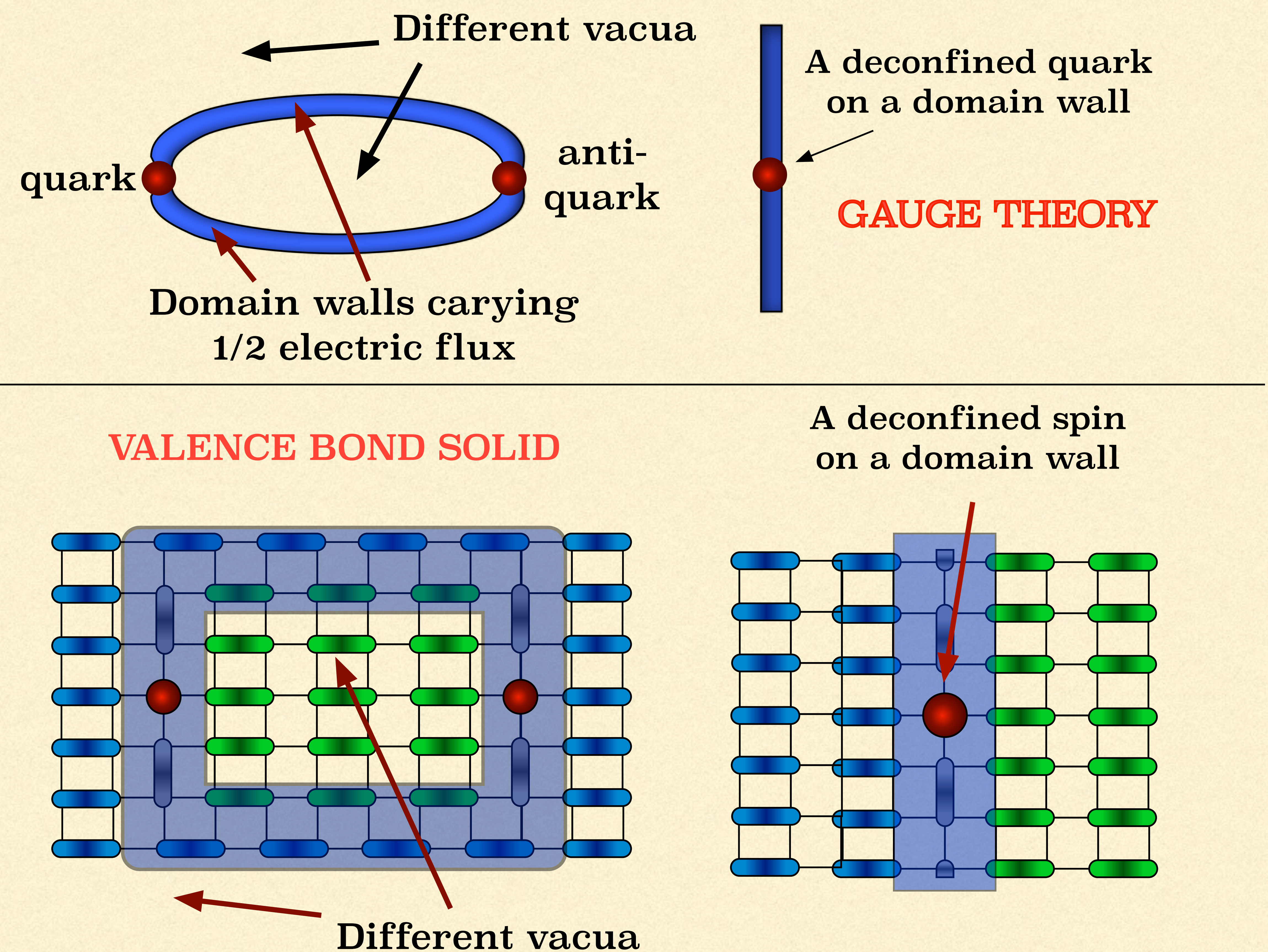
Q-terms introduce singlet attractions

If Qx=Qy: 4 vacua, otherwise only 2

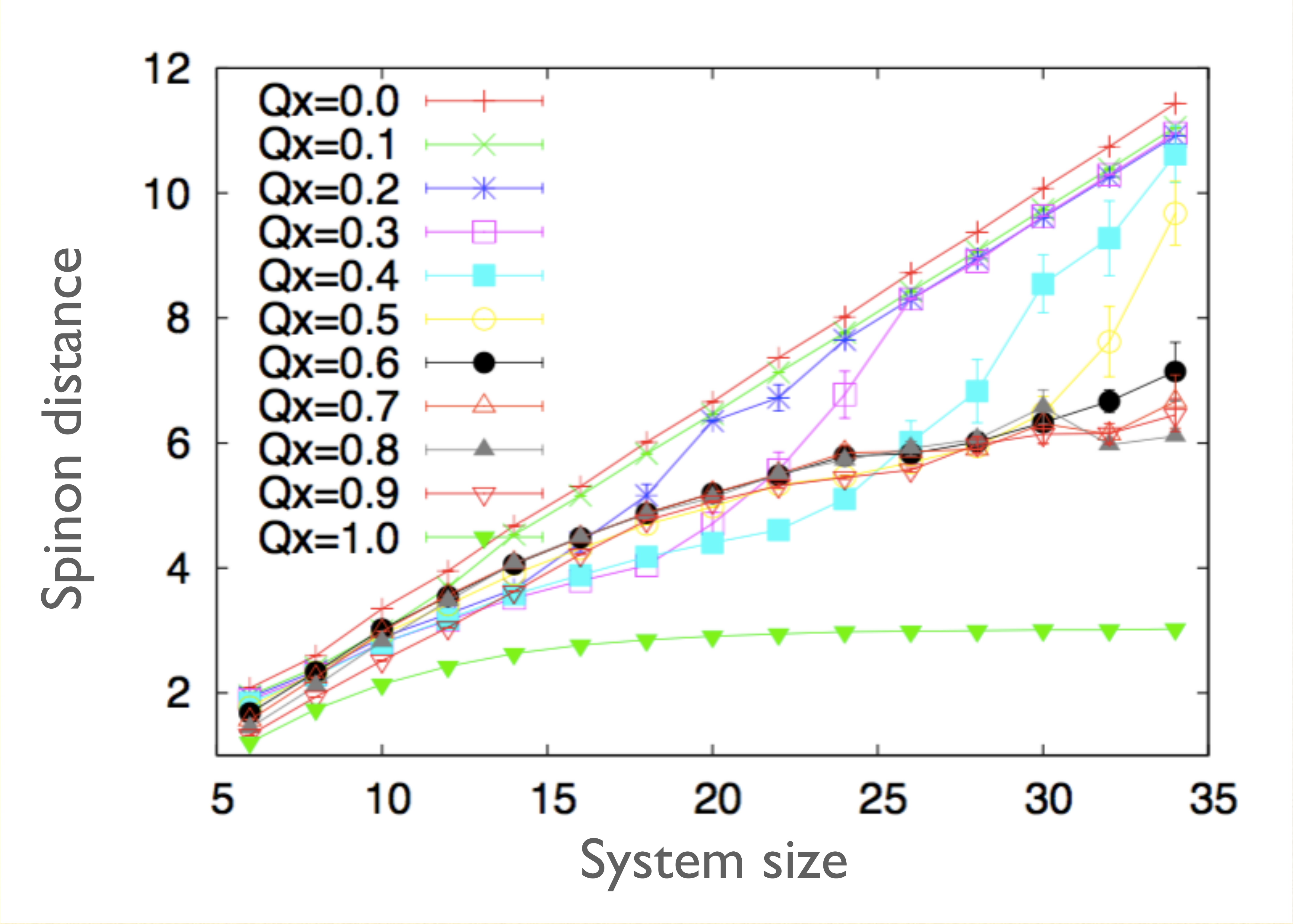

1
2
3
4


1
2
3
4


Different vacua


Different vacua
quark
anti-
quark
Domain walls carying
1/2 electric flux
A deconfined quark
on a domain wall
GAUGE THEORY
VALENCE BOND SOLID
A deconfined spin
on a domain wall
Different vacua

J-Q model with a domain wall along y-direction with J=0, Qy=1



ANTI-FERROMAGNET IN CONTINUUM

In continuum

$$\mathcal{S}_E = \frac{S}{4}\int d^2x\, dt\ \left[\frac{1}{v_s}(\partial_{\boldsymbol{x}}\boldsymbol{n})^2 + v_s(\partial_t\boldsymbol{n})^2\right] + (\text{Berry phase})$$

Valid for large S where finite differences can be approximated well by derivatives

ANTI-FERROMAGNET IN CONTINUUM

$$H = J \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \ldots$$

$$\boldsymbol{S}_i = \eta_i S \, \boldsymbol{n}_i \ , \qquad \eta_i = \pm 1\text{—staggered phase}$$

In continuum

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HEDGEHOGS AS MONOPOLES

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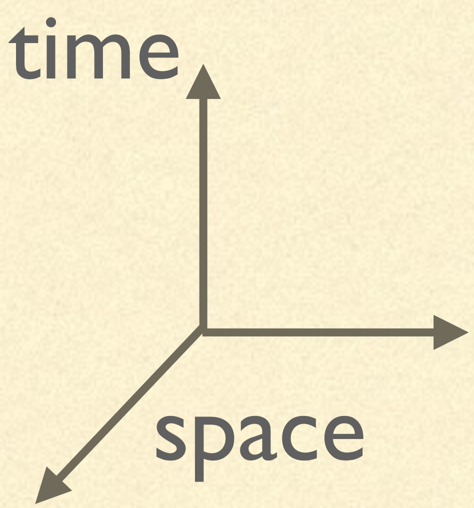
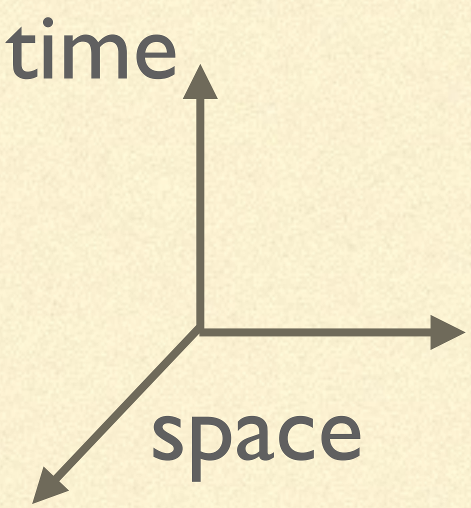
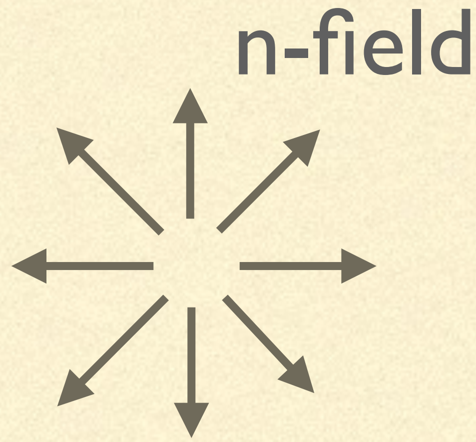


HEDGEHOGS AS MONOPOLES

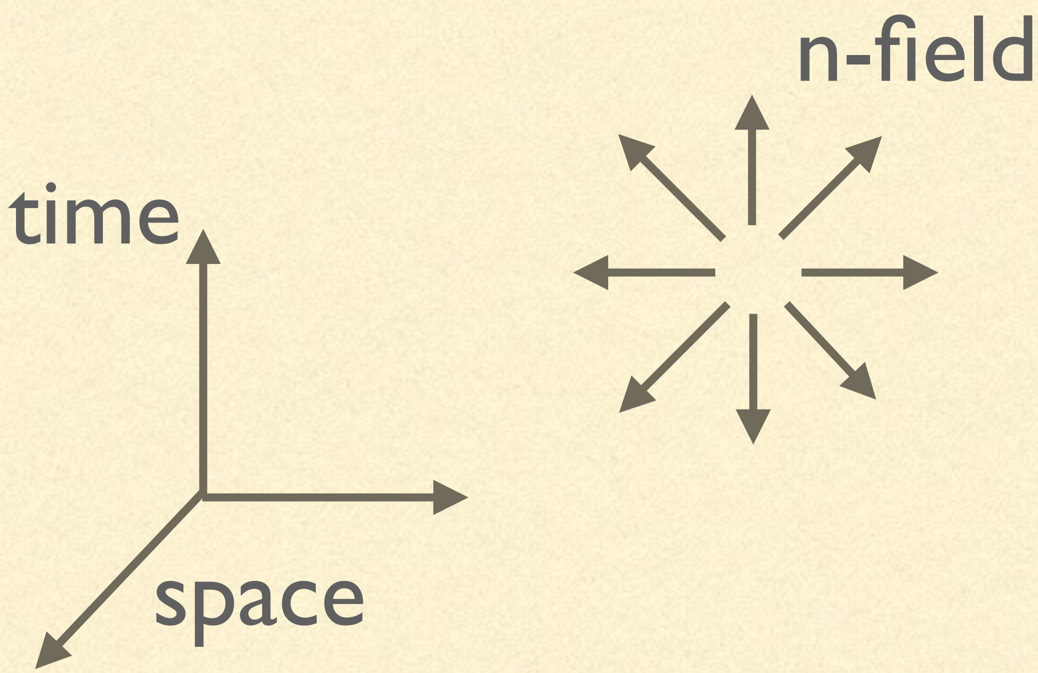




space-time hedgehog

- singular in continuum
- possible because of the underlying lattice

HEDGEHOGS AS MONOPOLES



space-time hedgehog
- singular in continuum
- possible because of the underlying lattice

1	$e^{\frac{i\pi}{2}}$
$e^{-\frac{i\pi}{2}}$	$e^{i\pi}$

The hedgehogs have different Berry phases depending on the the sub-lattice they belong to (Haldane 1988)

ALTERNATIVE DESCRIPTION OF SPIN

$$\boldsymbol{n}_i = u_i^\dagger \boldsymbol{\sigma} u_i \ , \qquad \boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$$

u_i — SU(2) doublet at each site with $u_i^\dagger u_i = 1$

$u_i \to e^{i\phi_i} u_i$ — Local symmetry, i.e. gauge symmetry

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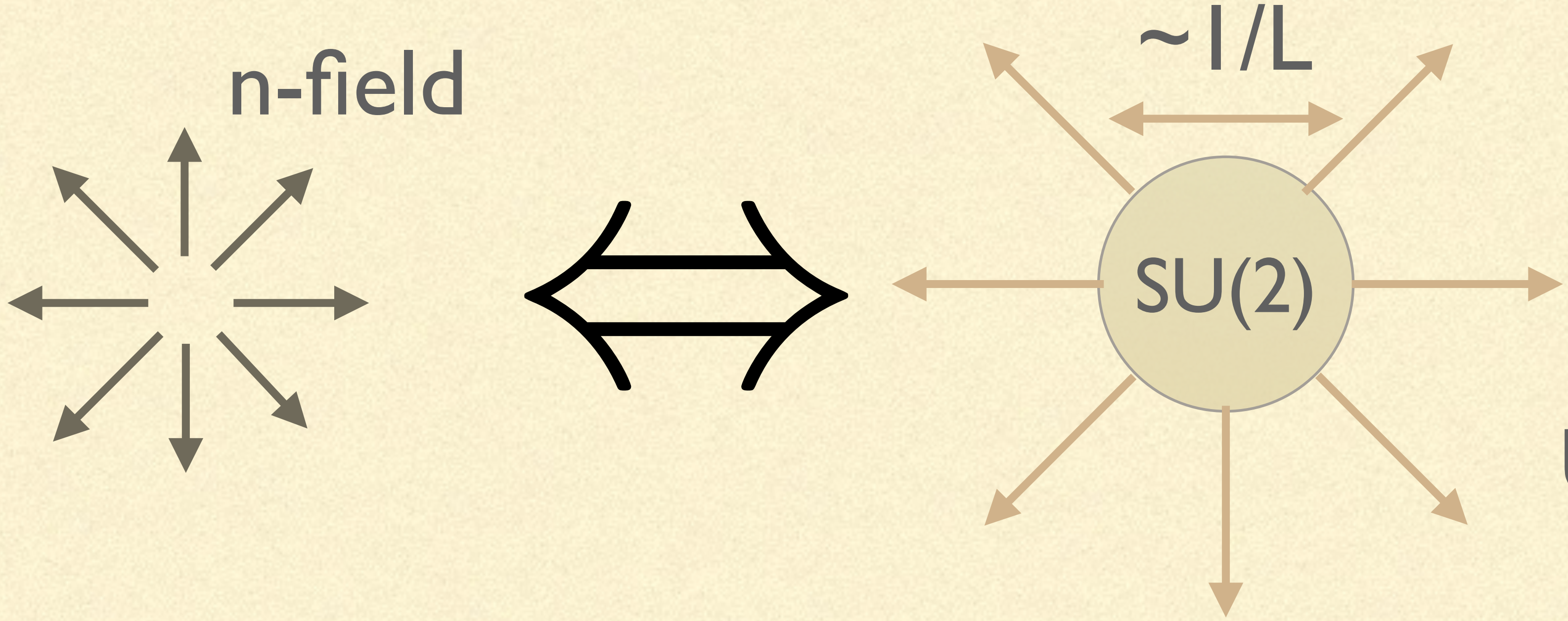

$\vec{B} \sim \frac{\hat{r}}{r^2}$

U(1) magnetic monopoles

1	$e^{\frac{i\pi}{2}}$
$e^{-\frac{i\pi}{2}}$	$e^{i\pi}$

$$\begin{array}{|c|c|} \hline 1 & e^{\frac{i\pi}{2}} \\ \hline e^{-\frac{i\pi}{2}} & e^{i\pi} \\ \hline \end{array} \times e^{+i\sigma}$$

Berry phases cancel each other and the first allowed term is cos(4σ)

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Berry phases cancel each other and the first allowed term is $\cos(4\sigma)$

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1	$e^{-\frac{i\pi}{2}}$

$$\times e^{+i(\sigma-\pi/2)}$$

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Berry phases cancel each other and the first allowed term is cos(4σ)

Under the Q-deformation only π rotations are a symmetry, so cos(2σ) is allowed.

Phases:

- Neel
  - u-field condenses breaking SU(2) symmetry
  - u-condensate acts like a Higgs field and the effective theory is that of goldstones
- VBS
  - u-field is massive and can be integrated out
  - Effective theory is the 3D U(1) gauge theory with monopoles
  - Confining phase
  - Monopoles (hedgehogs) couple to Berry phases which interfere allowing only multiple monopole events to contribute.
  - The different Qx and Qy terms allow for $\cos(2\sigma)$ but not for $\cos(\sigma)$ term.
  - Effective action is the same as in gauge theories

CONCLUSION

- Gauge theories with multiple vacua have an incredibly curious confining string structure
- They generically exhibit features that suggest strings are made out of domain walls
- Immediate consequences are: liberation of charges on the domain walls, strings ending on domain walls, charges are intersections of domain walls.
- Not a supersymmetric phenomena, as is folklore.
  - QCD(adj) — confining with N vacua (discrete chiral symmetry breaking).
  - $\theta=\pi$ — confining with 2 vacua (CP symmetry breaking)
- In non-degenerate vacua, there may be residual effects from this considerations (i.e. strings are composed out of Witten k-vacua)
- Spin-antiferromagnets in the VBS phase exhibit the same phenomenon
- Domain walls as spin-guides?
- So far deconfined spinons were only proposed in at a critical point between the Neel and the VBS state, but it may be possible to achieve this on the domain wall without being at the critical point.