OLD AND NEW RESULTS FOR HADRONIC-LIGHT-BY-LIGHT

Johan Bijnens

Lund University

bijnens@thep.lu.se
http://www.thep.lu.se/∼bijnens
http://www.thep.lu.se/∼bijnens/chpt.html

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Overview

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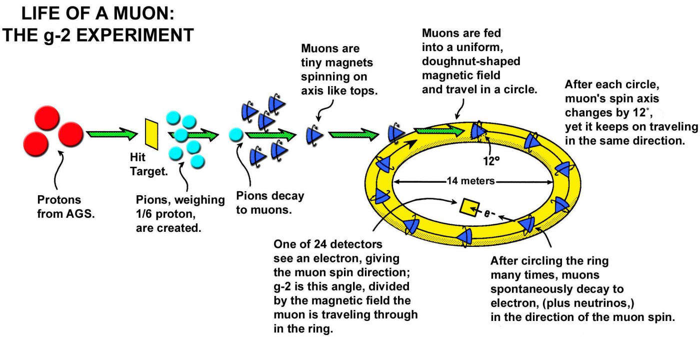
Literature

- Final experimental paper:
  G. W. Bennett *et al.* [Muon G-2 Collaboration], "Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL," Phys. Rev. D **73** (2006) 072003 [hep-ex/0602035].
- Review 1:
  F. J. M. Farley and Y. K. Semertzidis, "The 47 years of muon g-2," Prog. Part. Nucl. Phys. **52** (2004) .
- Review 2:
  J. P. Miller, E. de Rafael and B. L. Roberts, "Muon (g-2): Experiment and theory," Rept. Prog. Phys. **70** (2007) 795 [hep-ph/0703049].
- Review 3:
  F. Jegerlehner and A. Nyffeler, "The Muon g-2," Phys. Rept. **477** (2009) 1 [arXiv:0902.3360 [hep-ph]].

Literature

- Lectures:
  M. Knecht, Lect. Notes Phys. **629** (2004) 37 [hep-ph/0307239].
- "Final" HLBL number:
  - J. Bijnens and J. Prades, Mod. Phys. Lett. A **22** (2007) 767 [hep-ph/0702170].
  - J. Prades, E. de Rafael and A. Vainshtein, "Hadronic Light-by-Light Scattering Contribution to the Muon Anomalous Magnetic Moment," (Advanced series on directions in high energy physics. 20) [arXiv:0901.0306 [hep-ph]].
- New stuff here:
  JB, Mehran Zahiri Abyaneh, Johan Relefors
  HLbL pion loop contribution
  arXiv:1208.3548, arXiv:1208.2554, arXiv:1308.2575 and to be published

Muon $g-2$: measurement



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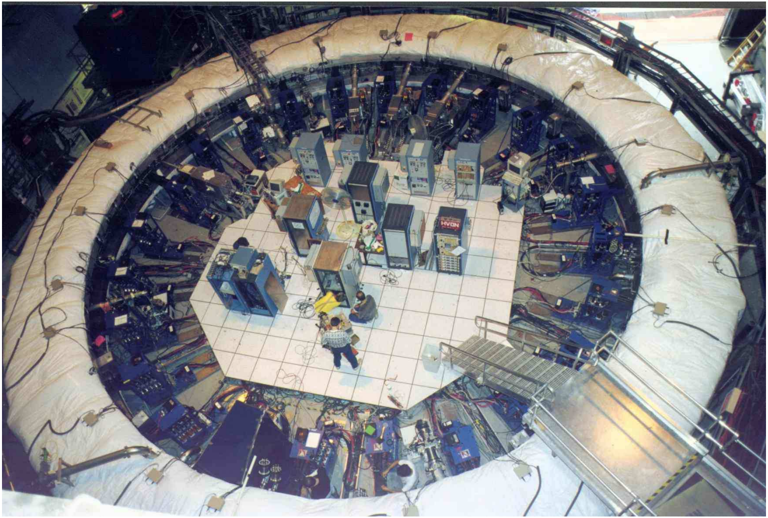

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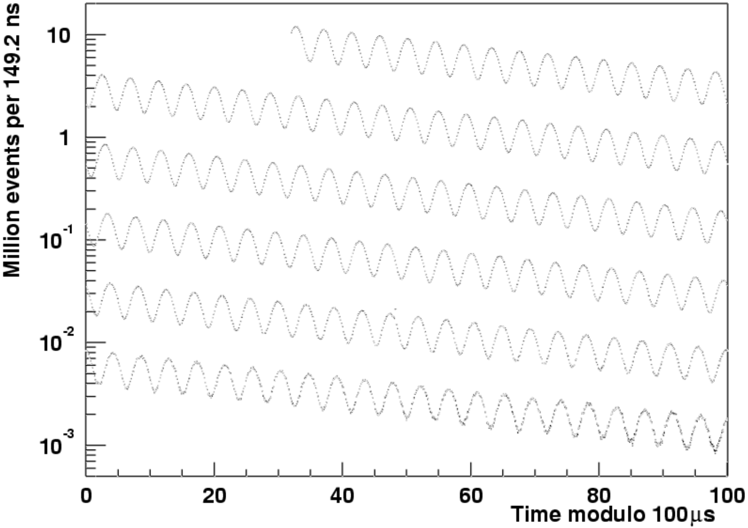

Phys.Rev. D73 (2006) 072003 [hep-ex/0602035]

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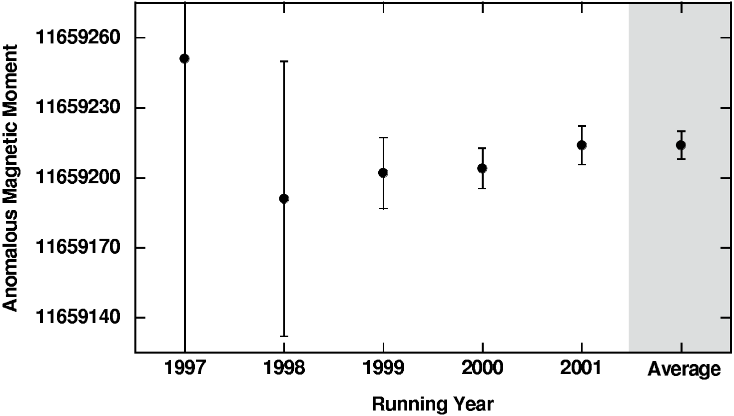

Phys.Rev. D73 (2006) 072003 [hep-ex/0602035]

2001: μ^-, others μ^+

Muon $g-2$: overview

- in terms of the anomaly $a_\mu = (g-2)/2$
- Data dominated by BNL E821 (statistics)(systematic)
  $a_{\mu^+}^{\mathrm{exp}} = 11659204(6)(5) \times 10^{-10}$
  $a_{\mu^-}^{\mathrm{exp}} = 11659215(8)(3) \times 10^{-10}$
  $a_{\mu}^{\mathrm{exp}} = 11659208.9(5.4)(3.3) \times 10^{-10}$
- Theory is off somewhat (electroweak)(LO had)(HO had)
  $a_{\mu}^{\mathrm{SM}} = 11659180.2(0.2)(4.2)(2.6) \times 10^{-10}$
- $\Delta a_\mu = a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = 28.7(6.3)(4.9) \times 10^{-10}$ (PDG)
- E821 goes to Fermilab, expect factor of four in precision
- Note: g agrees to $3\ 10^{-9}$ with theory
- Many BSM models CAN predict a value in this range (often a lot more or a lot less)
- Numbers taken from PDG2012, see references there

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Summary of Muon $g-2$ contributions

	$10^{10} a_\mu$	
exp	11 659 208.9	6.3
theory	11 659 180.2	5.0
QED	11 658 471.8	0.0
EW	15.4	0.2
LO Had	692.3	4.2
HO HVP	-9.8	0.1
HLbL	10.5	2.6
difference	28.7	8.1

- Error on LO had
  all e^+e^- based OK
  τ based 2 σ
- Error on HLbL
- Errors added quadratically
- 3.5 σ
- Difference:
  4% of LO Had
  270% of HLbL
  1% of leptonic LbL

Generic SUSY: $12.3 \times 10^{-10} \left(\frac{100\ \mathrm{GeV}}{M_{SUSY}}\right)^2 \tan\beta$

$M_{SUSY} \approx 66\ \mathrm{GeV}\sqrt{\tan\beta}$

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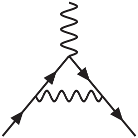
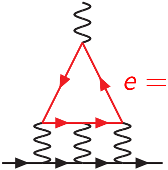
Muon $g-2$: QED

$$a_\mu^{\text{QED}} = \frac{\alpha}{2\pi} + 0.765857410(27)\left(\frac{\alpha}{\pi}\right)^2 + 24.05050964(43)\left(\frac{\alpha}{\pi}\right)^3 + 130.8055(80)\left(\frac{\alpha}{\pi}\right)^4 + 663(20)\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

- First three loops known analytically
- four-loops fully done numerically
- Five loops estimate
- Kinoshita, Laporta, Remiddi, Schwinger,...
- α fixed from the electron $g-2$: $\alpha = 1/137.035999084(51)$
- $a_\mu^{\text{QED}} = 11658471.809(0.015) \times 10^{-10}$
- Light-by-light surprisingly large: 2670×10^{-10}

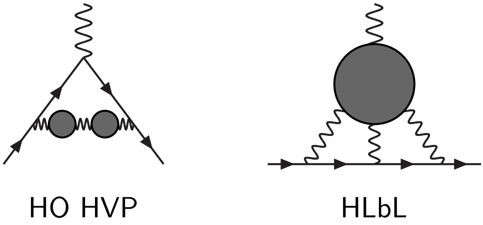




$e = 20.95, \mu = 0.37, \tau = 0.002$

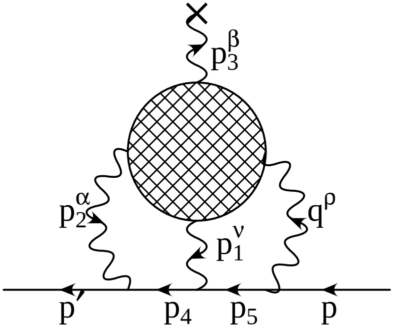
Muon $g-2$: HO hadronic

- Two main types of contributions



- HO HVP is like LO Had, can be derived from $e^+e^- \to$hadrons. $a_\mu^{\mathrm{HO\ HVP}} = -9.84(0.06) \times 10^{-10}$
- HLbL is the real problem: best estimate now: $a_\mu^{\mathrm{HLbL}} = 10.5(2.6) \times 10^{-10}$
- Note that the sum is very small: but not an indication of the error

HLbL: the main object to calculate



- Muon line and photons: well known
- The blob: fill in with hadrons/QCD
- Trouble: low and high energy very mixed
- Double counting needs to be avoided: hadron exchanges versus quarks

A separation proposal: a start

E. de Rafael, "Hadronic contributions to the muon g-2 and low-energy QCD," Phys. Lett. **B322** (1994) 239-246. [hep-ph/9311316].

- Use ChPT p counting and large N_c
- p^4, order 1: pion-loop
- p^8, order N_c: quark-loop and heavier meson exchanges
- p^6, order N_c: pion exchange

Does not fully solve the problem
only short-distance part of quark-loop is really p^8
but it's a start

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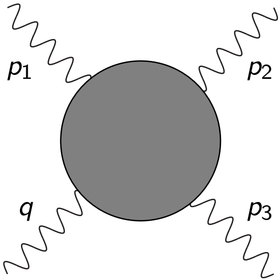
Implemented by two groups in the 1990s:

- Hayakawa, Kinoshita, Sanda: meson models, pion loop using hidden local symmetry, quark-loop with VMD, calculation in Minkowski space
- JB, Pallante, Prades: Try using as much as possible a consistent model-approach, ENJL, calculation in Euclidean space

General properties

$\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)$ $=$ 

Actually we really need $\left.\frac{\delta \Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)}{\delta p_{3\lambda}}\right|_{p_3=0}$

General properties

$\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)$:

- In general 138 Lorentz structures (but only 28 contribute to $g-2$)
- Using $q_\rho \Pi^{\rho\nu\alpha\beta} = p_{1\nu}\Pi^{\rho\nu\alpha\beta} = p_{2\alpha}\Pi^{\rho\nu\alpha\beta} = p_{3\beta}\Pi^{\rho\nu\alpha\beta} = 0$ 43 gauge invariant structures
- Bose symmetry relates some of them
- All depend on p_1^2, p_2^2 and q^2, but before derivative and $p_3 \to 0$ also $p_3^2, p_1 \cdot p_2, p_1 \cdot p_3$
- Compare HVP: one function, one variable
- General calculation from experiment difficult to see how
- In four photon measurement: lepton contribution

General properties

$\int \frac{\mathrm{d}^4 p_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 p_2}{(2\pi)^4}$ plus loops inside the hadronic part

- 8 dimensional integral, three trivial,
- 5 remain: $p_1^2, p_2^2, p_1 \cdot p_2, p_1 \cdot p_\mu, p_2 \cdot p_\mu$
- Rotate to Euclidean space:
  - Easier separation of long and short-distance
  - Artefacts (confinement) in models smeared out.
- More recent: can do two more using Gegenbauer techniques Knecht-Nyffeler, Jegerlehner-Nyffeler,JB–Zahiri-Abyaneh–Relefors
- P_1^2, P_2^2 and Q^2 remain
- study $a_\mu^{\mathrm{X}} = \int dl_{P_1} dl_{P_2} a_\mu^{\mathrm{XLL}} = \int dl_{P_1} dl_{P_2} dl_Q a_\mu^{\mathrm{XLLQ}}$ $l_P = \ln\left(P/\mathrm{GeV}\right)$, to see where the contributions are
- Study the dependence on the cut-off for the photons

General properties

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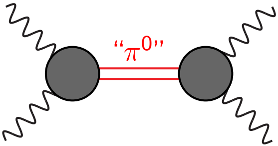
π^0 exchange



- “π^0”$= 1/(p^2 - m_\pi^2)$
- The blobs need to be modelled, and in e.g. ENJL contain corrections also to the $1/(p^2 - m_\pi^2)$
- Pointlike has a logarithmic divergence
- Numbers π^0, but also η, η'

π^0 exchange

- BPP: $a_\mu^{\pi^0} = 5.9(0.9) \times 10^{-10}$
- Nonlocal quark model: $a_\mu^{\pi^0} = 6.27 \times 10^{-10}$

  A. E. Dorokhov, W. Broniowski, Phys.Rev.**D78** (2008)073011. [0805.0760]
- DSE model: $a_\mu^{\pi^0} = 5.75 \times 10^{-10}$

  Goecke, Fischer and Williams, Phys.Rev.D**83**(2011)094006[1012.3886]
- LMD+V: $a_\mu^{\pi^0} = (5.8 - 6.3) \times 10^{-10}$

  M. Knecht, A. Nyffeler, Phys. Rev. **D65**(2002)073034, [hep-ph/0111058]
- Formfactor inspired by AdS/QCD: $a_\mu^{\pi^0} = 6.54 \times 10^{-10}$

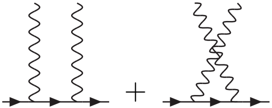
  Cappiello, Cata and D'Ambrosio, Phys.Rev.D**83**(2011)093006 [1009.1161]
- Chiral Quark Model: $a_\mu^{\pi^0} = 6.8 \times 10^{-10}$

  D. Greynat and E. de Rafael, JHEP **1207** (2012) 020 [1204.3029].
- Constraint via magnetic susceptibility: $a_\mu^{\pi^0} = 7.2 \times 10^{-10}$

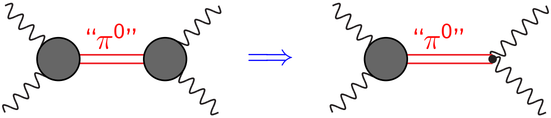
  A. Nyffeler, Phys. Rev. D **79** (2009) 073012 [0901.1172].
- All in reasonable agreement

MV short-distance: π^0 exchange

- K. Melnikov, A. Vainshtein, Hadronic light-by-light scattering contribution to the muon anomalous magnetic moment revisited, Phys. Rev. **D70** (2004) 113006. [hep-ph/0312226]
- take $P_1^2 \approx P_2^2 \gg Q^2$: Leading term in OPE of two vector currents is proportional to axial current
- $\Pi^{\rho\nu\alpha\beta} \propto \frac{P_\rho}{P_1^2} \langle 0 | T \left(J_{A\nu} J_{V\alpha} J_{V\beta} \right) | 0 \rangle$
- J_A comes from



- AVV triangle anomaly: extra info
- Implemented via setting one blob = 1



- $a_\mu^{\pi^0} = 7.7 \times 10^{-10}$

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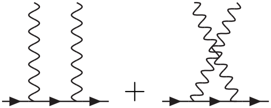
π^0 exchange

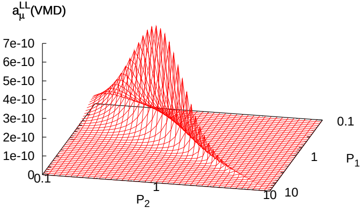
- The pointlike vertex implements shortdistance part, not only π^0-exchange
- 

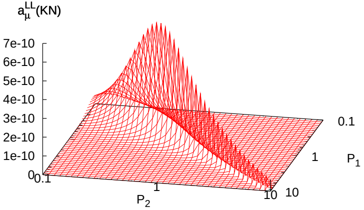
  Are these part of the quark-loop? See also in

  Dorokhov,Broniowski, Phys.Rev. D78(2008)07301
- BPP quarkloop + π^0-exchange $\approx$ MV π^0-exchange

π^0 exchange

- Which momentum regimes important studied: JB and J. Prades, Mod. Phys. Lett. A **22** (2007) 767 [hep-ph/0702170]
- $a_\mu = \int dl_1 dl_2 a_\mu^{LL}$ with $l_i = \log(P_i/GeV)$





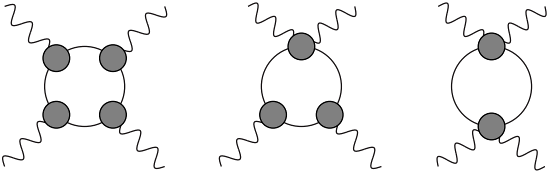
Which momentum regions do what:
volume under the plot $\propto a_\mu$

Pseudoscalar exchange

- Point-like VMD: π^0 η and η' give 5.58, 1.38, 1.04.
- Models that include $U(1)_A$ breaking give similar ratios
- Pure large N_c models use this ratio
- The MV argument should give some enhancement over the full VMD like models
- Total pseudo-scalar exchange is about $a_\mu^{PS} = 8 - 10 \times 10^{-10}$
- AdS/QCD estimate (includes excited pseudo-scalars) $a_\mu^{PS} = 10.7 \times 10^{-10}$

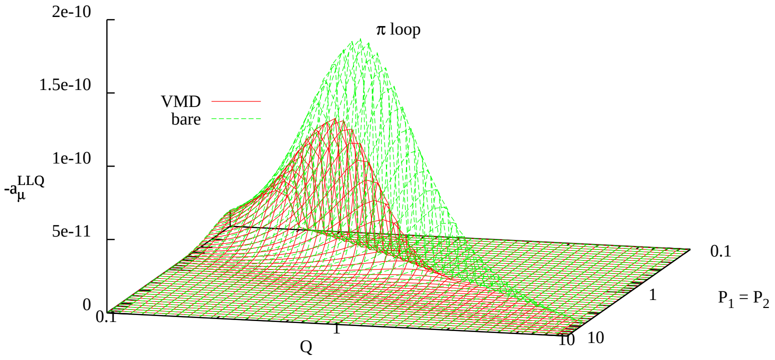
  D. K. Hong and D. Kim, Phys. Lett. B **680** (2009) 480 [0904.4042]

π-loop



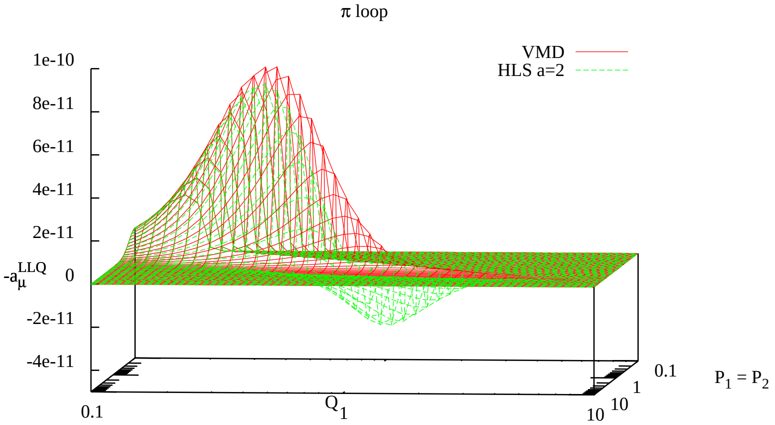
- A bare π-loop (sQED) give about $-4 \cdot 10^{-10}$
- The $\pi\pi\gamma^*$ vertex is always done using VMD
- $\pi\pi\gamma^*\gamma^*$ vertex two choices:
  - Hidden local symmetry model: only one γ has VMD
  - Full VMD
  - Both are chirally symmetric
  - The HLS model used has problems with π^+-π^0 mass difference (due to not having an a_1)
- Final numbers quite different: -0.45 and -1.9 $(\times 10^{-10})$
- For BPP stopped at 1 GeV but within 10% of higher Λ

π loop: Bare vs VMD



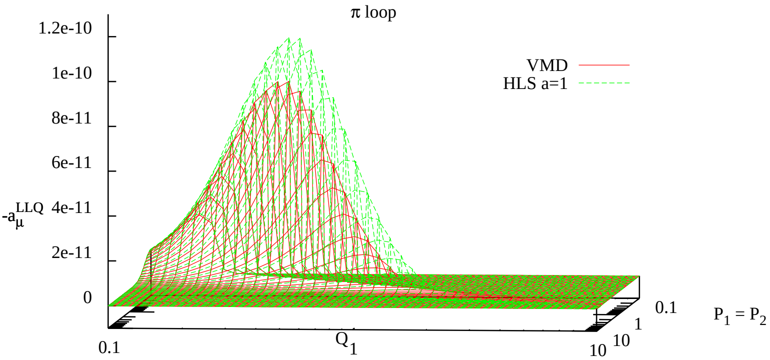
- plotted a_μ^{LLQ} for $P_1 = P_2$
- $a_\mu = \int dl_{P_1} dl_{P_2} dl_Q \, a_\mu^{LLQ}$
- $l_Q = \log(Q/1 \text{ GeV})$

π loop: VMD vs HLS



Usual HLS, $a = 2$

π loop: VMD vs HLS



HLS with $a = 1$, satisfies more short-distance constraints

π loop

- $\pi\pi\gamma^*\gamma^*$ for $q_1^2 = q_2^2$ has a short-distance constraint from the OPE as well.
- HLS does not satisfy it
- full VMD does: so probably better estimate
- Ramsey-Musolf suggested to do pure ChPT for the π loop
  K. T. Engel, H. H. Patel and M. J. Ramsey-Musolf, "Hadronic light-by-light scattering and the pion polarizability," Phys. Rev. D **86** (2012) 037502 [arXiv:1201.0809 [hep-ph]].
- So far ChPT at p^4 done for four-point function in limit $p_1, p_2, q \ll m_\pi$ (Euler-Heisenberg plus next order)
- Polarizability ($L_9 + L_{10}$) up to 10%, charge radius 30%
- Both HLS and VMD have charge radius effect but not polarizability

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π loop: L_9, L_{10}

- ChPT for muon $g-2$ at order p^6 is not powercounting finite so no prediction for a_μ exists.
- But can be used to study the low momentum end of the integral over P_1, P_2, Q
- The four-photon amplitude is finite still at two-loop order (counterterms start at order p^8)
- Add L_9 and L_{10} vertices to the bare pion loop JB-Zahiri-Abyaneh
- Program the Euler-Heisenberg plus NLO result of Ramsey-Musolf et al. into our programs for a_μ
- Bare pion-loop and L_9, L_{10} part in limit $p_1, p_2, q \ll m_\pi$ agree with Euler-Heisenberg plus next order analytically

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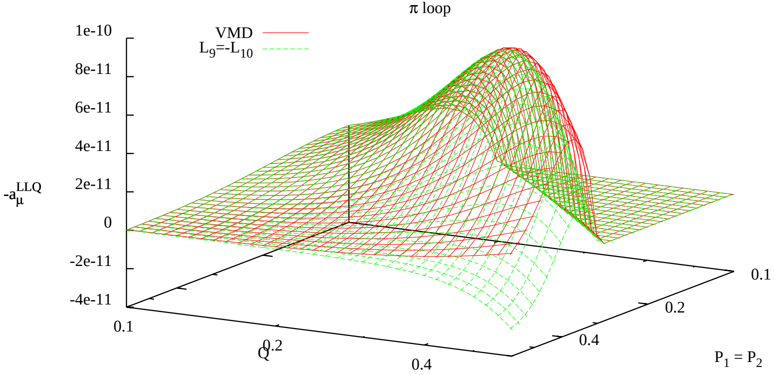
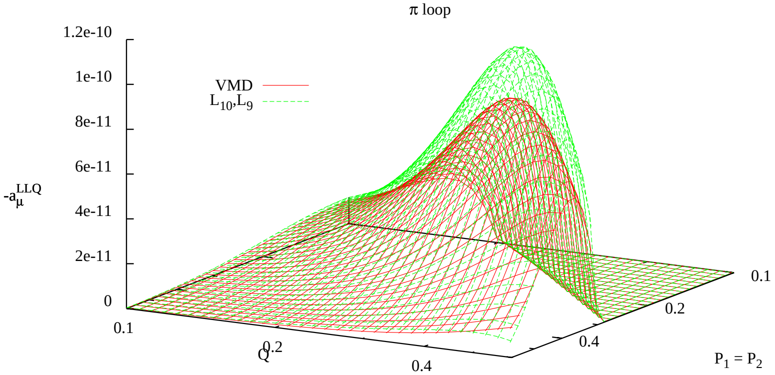
π loop: VMD vs charge radius



low scale, charge radius effect well reproduced

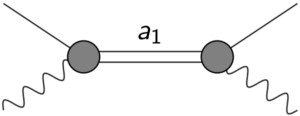
π loop: VMD vs L_9 and L_{10}



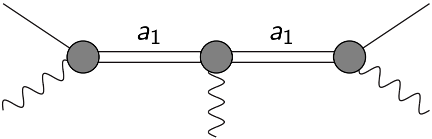
- $L_9 + L_{10} \neq 0$ gives an enhancement of 10-15%
- To do it fully need to get a model: include a_1

Include a_1

- $L_9 + L_{10}$ effect is from



- But to get gauge invariance correctly need



Include a_1

- Consistency problem: full a_1-loop?
- Treat a_1 and ρ classical and π quantum: there must be a π that closes the loop
  Argument: integrate out ρ and a_1 classically, then do pion loops with the resulting Lagrangian
- To avoid problems: representation without a_1-π mixing
- Check for curiosity what happens if we add a_1-loop

Include a_1

- Use antisymmetric vector representation for a_1 and ρ
- Fields $A_{\mu\nu}$, $V_{\mu\nu}$ (nonets)
- Kinetic terms: $-\frac{1}{2}\left\langle \nabla^\lambda V_{\lambda\mu}\nabla_\nu V^{\nu\mu} - \frac{1}{2}V_{\mu\nu}V^{\mu\nu}\right\rangle$
  $-\frac{1}{2}\left\langle \nabla^\lambda A_{\lambda\mu}\nabla_\nu A^{\nu\mu} - \frac{1}{2}A_{\mu\nu}A^{\mu\nu}\right\rangle$
- Terms that give contributions to the L_i^r:

  $\frac{F_V}{2\sqrt{2}}\left\langle f_{+\mu\nu}V^{\mu\nu}\right\rangle + \frac{iG_V}{\sqrt{2}}\left\langle V^{\mu\nu}u_\mu u_\nu\right\rangle + \frac{F_A}{2\sqrt{2}}\left\langle f_{-\mu\nu}A^{\mu\nu}\right\rangle$
- $L_9 = \frac{F_V G_V}{2M_V^2}$, $L_{10} = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2}$
- Weinberg sum rules: (Chiral limit)

  $F_V^2 = F_A^2 + F_\pi^2$ $\qquad$ $F_V^2 M_V^2 = F_A^2 M_A^2$
- VMD for $\pi\pi\gamma$: $\qquad$ $F_V G_V = F_\pi^2$

$V_{\mu\nu}$ only

- $\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)$ is not finite (but was also not finite for HLS)
- But $\left.\frac{\delta\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)}{\delta p_{3\lambda}}\right|_{p_3=0}$ also not finite (but was finite for HLS)
- Derivative one finite for $G_V = F_V/2$
- Surprise: $g-2$ identical to HLS with $a = \frac{F_V^2}{F_\pi^2}$
- Yes I know, different representations are identical BUT they do differ in higher order terms and even in what is higher order
- Same comments as for HLS numerics

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$V_{\mu\nu}$ and $A_{\mu\nu}$

- Add a_1
- Calculate a lot
- $\left.\frac{\delta\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)}{\delta p_{3\lambda}}\right|_{p_3=0}$ finite for:
  - $G_V = F_V = 0$ and $F_A^2 = -2F_\pi^2$
  - If adding full a_1-loop $G_V = F_V = 0$ and $F_A^2 = -F_\pi^2$
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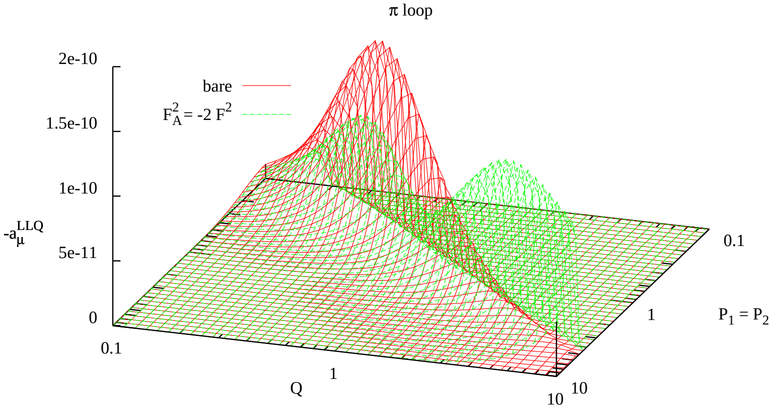
$V_{\mu\nu}$ and $A_{\mu\nu}$

- Start by adding $\rho a_1 \pi$ vertices
- $\lambda_1 \langle [V^{\mu\nu}, A_{\mu\nu}] \chi_- \rangle + \lambda_2 \langle [V^{\mu\nu}, A_{\nu\alpha}] h_\mu{}^\nu \rangle$
  $+\lambda_3 \langle i [\nabla^\mu V_{\mu\nu}, A_{\nu\alpha}] u_\alpha \rangle + \lambda_4 \langle i [\nabla_\alpha V_{\mu\nu}, A_{\alpha\nu}] u^\mu \rangle$
  $+\lambda_5 \langle i [\nabla^\alpha V_{\mu\nu}, A_{\mu\nu}] u_\alpha \rangle + \lambda_6 \langle i [V^{\mu\nu}, A_{\mu\nu}] f_-{}^\alpha{}_\nu \rangle$
  $+\lambda_7 \langle i V_{\mu\nu} A^{\mu\rho} A^\nu{}_\rho \rangle$
- All lowest dimensional vertices of their respective type
- Not all independent, there are three relations
- Follow from the constraints on $V_{\mu\nu}$ and $A_{\mu\nu}$ (thanks to Stefan Leupold)

$V_{\mu\nu}$ and $A_{\mu\nu}$: big disappointment

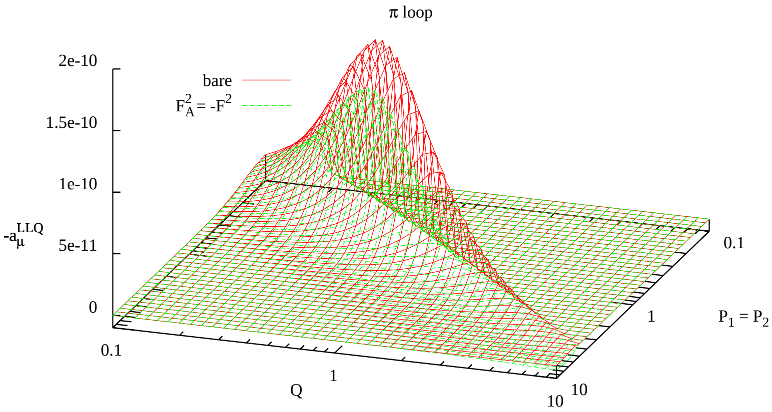
- Work a whole lot
- $\left.\frac{\delta\Pi^{\rho\nu\alpha\beta}(p_1,p_2,p_3)}{\delta p_{3\lambda}}\right|_{p_3=0}$ not obviously finite
- Work a lot more
- Prove that $\left.\frac{\delta\Pi^{\rho\nu\alpha\beta}(p_1,p_2,p_3)}{\delta p_{3\lambda}}\right|_{p_3=0}$ finite, only same solutions as before
- Try the combination that show up in $g-2$ only
- Work a lot
- Again, only same solutions as before
- Small loophole left: after the integration for $g-2$ could be finite but many funny functions of m_π, m_μ, M_V and M_A show up.

π loop: add a_1 and $F_A^2 = -2F_\pi^2$



- Lowers at low energies, $L_9 + L_{10} < 0$ here
- funny peak at a_1 mass

π loop: add a_1 and $F_A^2 = -F_\pi^2$ plus a_1-loop



- Lowers at low energies, $L_9 + L_{10} < 0$ here
- funny peak at a_1 mass canceled
- Still unphysical case

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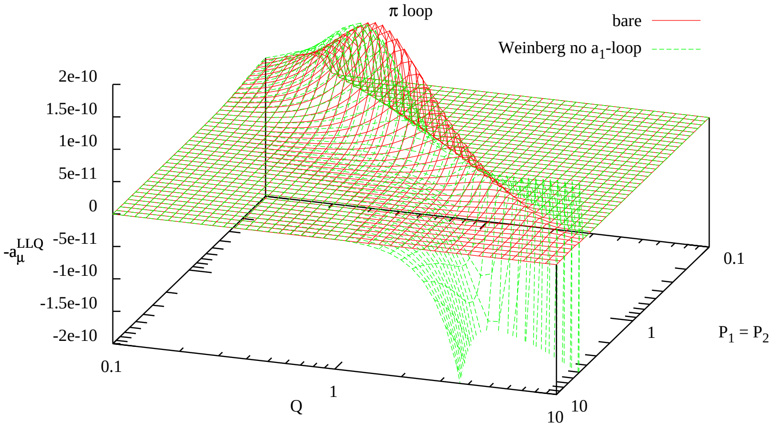
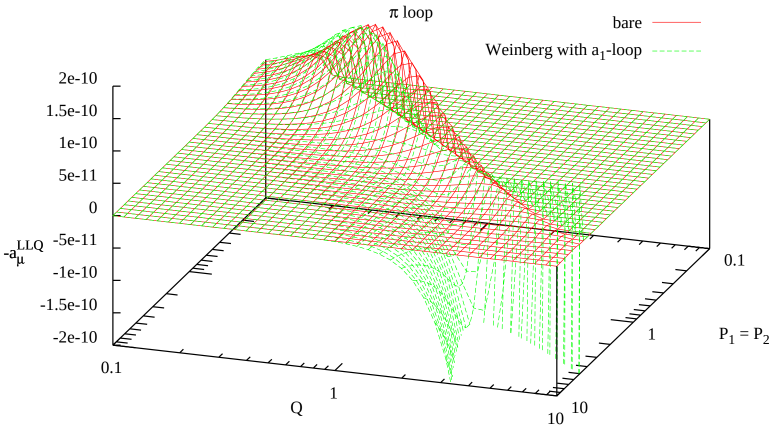
a_1-loop: cases with good L_9 and L_{10}



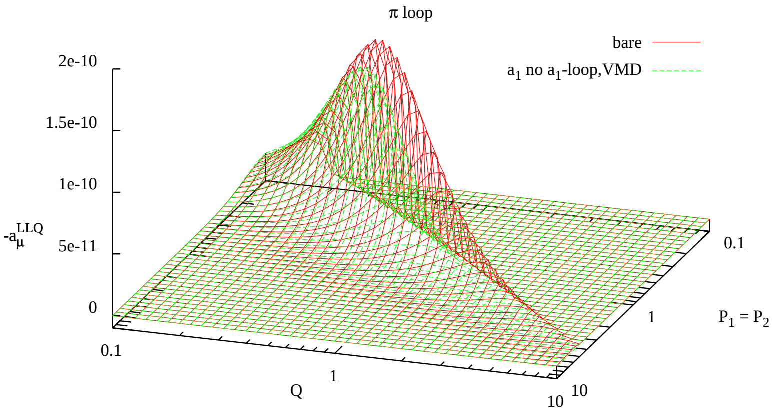
- Add F_V, G_V and F_A
- Fix values by Weinberg sum rules and VMD in $\gamma^*\pi\pi$
- no a_1-loop

a_1-loop: cases with good L_9 and L_{10}



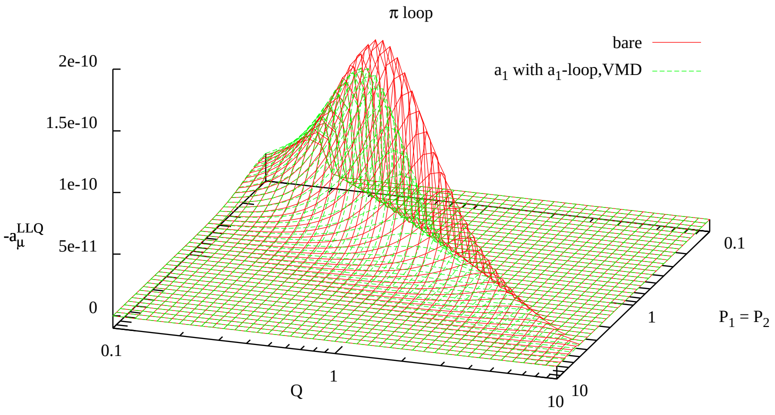
- Add F_V, G_V and F_A
- Fix values by Weinberg sum rules and VMD in $\gamma^*\pi\pi$
- With a_1-loop (is different plot!!)

a_1-loop: cases with good L_9 and L_{10}



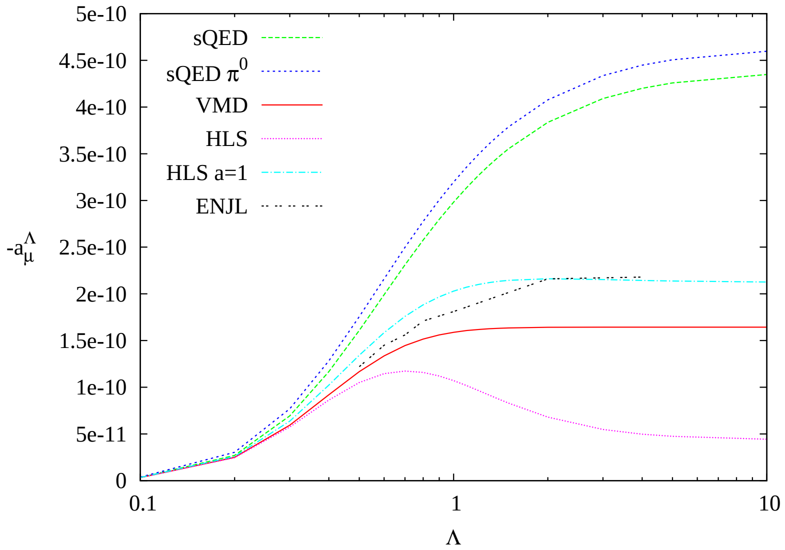
- Add a_1 with $F_A^2 = +F_\pi^2$
- Add the full VMD as done earlier for the bare pion loop

a_1-loop: cases with good L_9 and L_{10}



- Add a_1 with $F_A^2 = +F_\pi^2$ and a_1-loop
- Add the full VMD as done earlier for the bare pion loop

Integration results



$P_1, P_2, Q \leq \Lambda$

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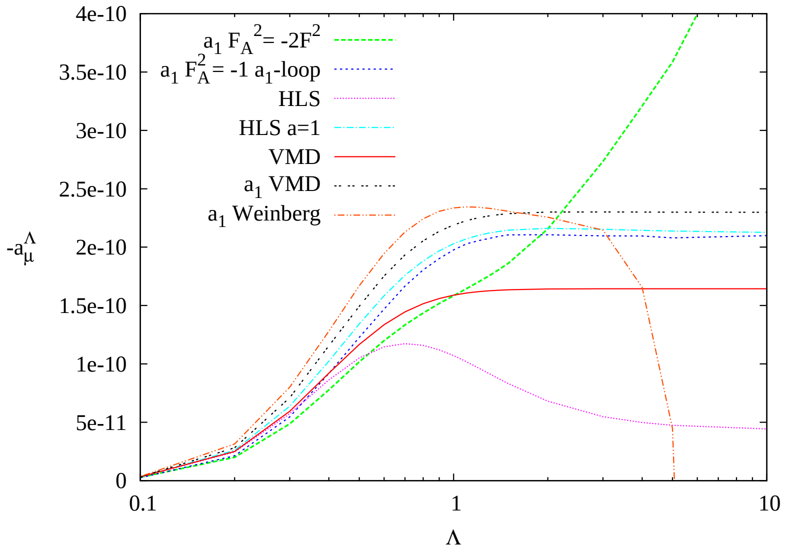
Integration results



$P_1, P_2, Q \leq \Lambda$

Integration results with a_1

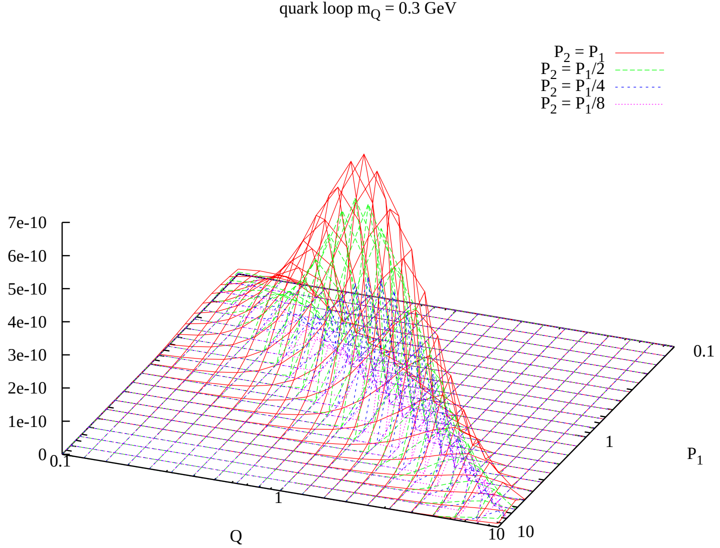
- Problem: get high energy behaviour good enough
- But all models with reasonable L_9 and L_{10} fall way inside the error quoted earlier $(-1.9 \pm 1.3)\ 10^{-10}$
- Tentative conclusion: Use hadrons only below about 1 GeV: $a_\mu^{\pi-\mathrm{loop}} = (-2.0 \pm 0.5)\ 10^{-10}$
- Note that Engel and Ramsey-Musolf, arXiv:1309.2225 is a bit more pessimistic quoting numbers from $(-1.1 \text{ to } -7.1)\ 10^{-10}$

Pure quark loop

Cut-off Λ (GeV)	$a_\mu \times 10^7$ Electron Loop	$a_\mu \times 10^9$ Muon Loop	$a_\mu \times 10^9$ Constituent Quark Loop
0.5	2.41(8)	2.41(3)	0.395(4)
0.7	2.60(10)	3.09(7)	0.705(9)
1.0	2.59(7)	3.76(9)	1.10(2)
2.0	2.60(6)	4.54(9)	1.81(5)
4.0	2.75(9)	4.60(11)	2.27(7)
8.0	2.57(6)	4.84(13)	2.58(7)
Known Results	2.6252(4)	4.65	2.37(16)

- M_Q : 300 MeV
- now known fully analytically
- Us: 5+(3-1) integrals extra are Feynman parameters
- Slow convergence:
  - electron: all at 500 MeV
  - Muon: only half at 500 MeV, at 1 GeV still 20% missing
  - 300 MeV quark: at 2 GeV still 25% missing

Pure quark loop: momentum area



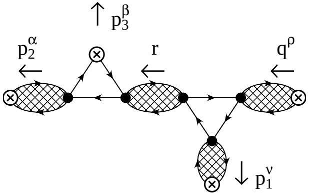
Most from $P_1 \approx P_2 \approx Q$, sizable large momentum part

ENJL quark-loop

Cut-off Λ GeV	$a_\mu \times 10^{10}$ VMD	$a_\mu \times 10^{10}$ ENJL	$a_\mu \times 10^{10}$ masscut	$a_\mu \times 10^{10}$ sum ENJL+masscut
0.5	0.48	0.78	2.46	3.2
0.7	0.72	1.14	1.13	2.3
1.0	0.87	1.44	0.59	2.0
2.0	0.98	1.78	0.13	1.9
4.0	0.98	1.98	0.03	2.0
8.0	0.98	2.00	.005	2.0

- Very stable
- ENJL cuts off slower than pure VMD
- masscut: $M_Q = \Lambda$ to have short-distance and no problem with momentum regions
- Quite stable in region 1-4 GeV

ENJL: scalar



- $$\Pi^{\rho\nu\alpha\beta} = \overline{\Pi}^{VVS}_{ab}(p_1,r) g_S \left(1+g_S\Pi^S(r)\right)\overline{\Pi}^{SVV}_{cd}(p_2,p_3)\mathcal{V}^{abcd\rho\nu\alpha\beta} + \text{permutations}$$
- $g_S\left(1+g_S\Pi_S\right) = \frac{g_A(r^2)(2M_Q)^2}{2f^2(r^2)}\frac{1}{M_S^2(r^2)-r^2}$
- $\mathcal{V}^{abcd\rho\nu\alpha\beta}$: ENJL VMD legs
- In ENJL only scalar+quark-loop properly chiral invariant

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ENJL: scalar/QL

Cut-off Λ GeV	$a_\mu \times 10^{10}$ Quark-loop VMD	$a_\mu \times 10^{10}$ Quark-loop ENJL	$a_\mu \times 10^{10}$ Scalar Exchange
0.5	0.48	0.78	-0.22
0.7	0.72	1.14	-0.46
1.0	0.87	1.44	-0.60
2.0	0.98	1.78	-0.68
4.0	0.98	1.98	-0.68
8.0	0.98	2.00	-0.68

- ENJL only scalar+quark-loop properly chiral invariant
- Note: ENJL+scalar (BPP) $\approx$ Quark-loop VMD (HKS)
- $M_S \approx 620$ MeV certainly an overestimate for real scalars
- If scalar is σ: related to pion loop part?
- quark-loop: $a_\mu^{ql} \approx 1 \times 10^{-10}$ bare $a_\mu^{ql} = 2.37 \times 10^{-9}$

Quark loop DSE

- DSE model: $a_\mu^{ql} = 13.6(5.9) \times 10^{-10}$ T. Goecke, C. S. Fischer and R. Williams, Phys. Rev. D **83** (2011) 094006 [arXiv:1012.3886 [hep-ph]]
- Not a full calculation (yet) but includes an estimate of some of the missing parts
- a lot larger than bare quark loop with constituent mass
- I am puzzled: this DSE model (Maris-Roberts) does reproduces a lot of low-energy phenomenology. I would have guessed that it would give numbers very similar to ENJL.
- Can one find something in between full DSE and ENJL that is easier to handle?
- Error found in calculation, still not finalized: preliminary $a_\mu^{ql} = 10.7(0.2) \times 10^{-10}$ T. Goecke, C. S. Fischer and R. Williams, arXiv:1210.1759

Other quark loop

- de Rafael-Greynat 1210.3029 $(7.6 - 8.9)\ 10^{-10}$
- Boughezal-Melnikov 1104.4510 $(11.8 - 14.8)\ 10^{-10}$
- Masjuan-Vanderhaeghen 1212.0357 $(7.6 - 12.5)\ 10^{-10}$
- Various interpretations: the full calculation or not
- All (even DSE) have in common that a low quark mass is used for a large part of the integration range

Axial-vector exchange exchange

Cut-off Λ (GeV)	$a_\mu \times 10^{10}$ from Axial-Vector Exchange $\mathcal{O}(N_c)$
0.5	0.05(0.01)
0.7	0.07(0.01)
1.0	0.13(0.01)
2.0	0.24(0.02)
4.0	0.59(0.07)

There is some pseudo-scalar exchange piece here as well, off-shell not quite clear what is what.

- $a_\mu^{\mathrm{axial}} = 0.6 \times 10^{-10}$
- MV: short distance enhancement + mixing (both enhance about the same)
  $a_\mu^{\mathrm{axial}} = 2.2 \times 10^{-10}$

Summary: ENJL vc PdRV

	BPP	PdRV arXiv:0901.0306
quark-loop	$(2.1 \pm 0.3) \cdot 10^{-10}$	—
pseudo-scalar	$(8.5 \pm 1.3) \cdot 10^{-10}$	$(11.4 \pm 1.3) \cdot 10^{-10}$
axial-vector	$(0.25 \pm 0.1) \cdot 10^{-10}$	$(1.5 \pm 1.0) \cdot 10^{-10}$
scalar	$(-0.68 \pm 0.2) \cdot 10^{-10}$	$(-0.7 \pm 0.7) \cdot 10^{-10}$
πK-loop	$(-1.9 \pm 1.3) \cdot 10^{-10}$	$(-1.9 \pm 1.9) \cdot 10^{-10}$
errors	linearly	quadratically
sum	$(8.3 \pm 3.2) \cdot 10^{-10}$	$(10.5 \pm 2.6) \cdot 10^{-10}$

What can we do more?

- The ENJL model can certainly be improved:
  - Chiral nonlocal quark-model (like nonlocal ENJL): so far only π^0-exchange done
  - DSE: π^0-exchange similar to everyone else, quark-loop very different, looking forward to final results
- More resonances models should be tried, AdS/QCD is one approach, RχT (Valencia *et al.*) possible,...
- Note short-distance matching must be done in many channels, there are theorems JB,Gamiz,Lipartia,Prades that with only a few resonances this requires compromises
- π-loop: HLS smaller than double VMD (understood) models with ρ and a_1: difficulties with infinities

What can we do more?

- Constraints from experiment:

  J. Bijnens and F. Persson, hep-ph/hep-ph/0106130
  Studying three formfactors $P\gamma^*\gamma^*$ in $P \to \ell^+\ell^-\ell'^+\ell'^-$, $e^+e^- \to e^+e^- P$ exact tree level and for $g-2$ (but beware sign):
  - Conclusion: possible but VERY difficult
  - Two γ^* off-shell not so important for our choice of form-factor
- All information on hadrons and 1-2-3-4 off-shell photons is welcome: constrain the models
- More short-distance constraints: MV, Nyffeler integrate with all contributions, not just π^0-exchange
- Need a new overall evaluation with consistent approach.
- Lattice has done first steps
- Some tentative steps from dispersion theory Pauk-Vanderhaeghen

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BNL magnet has moved to Fermilab

Goal $\pm 1.6\ 10^{-10}$

Credit: Brookhaven National Laboratory

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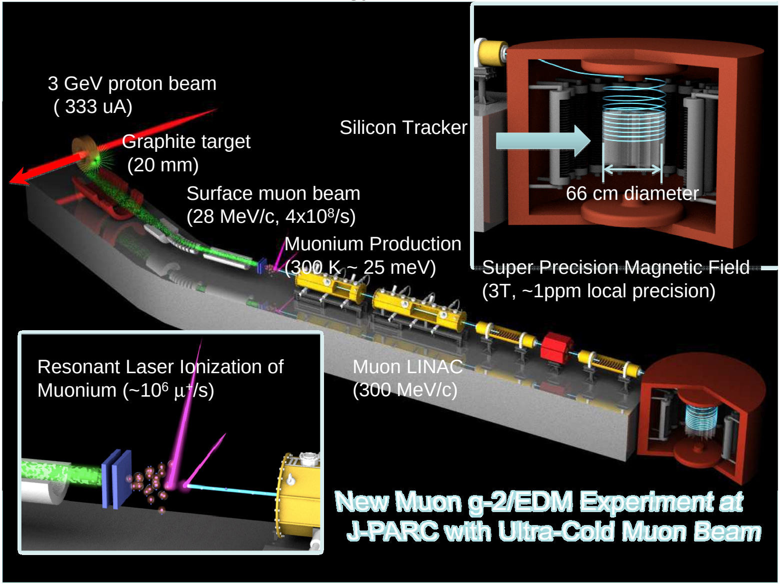
JPARC with a very different method

Ultracold muons at low energy (Credit: JPARC)



Summary of Muon $g-2$ contributions

	$10^{10} a_\mu$	
exp	11 659 208.9	6.3
theory	11 659 180.2	5.0
QED	11 658 471.8	0.0
EW	15.4	0.2
LO Had	692.3	4.2
HO HVP	-9.8	0.1
HLbL	10.5	2.6
difference	28.7	8.1

- Error on LO had
  all e^+e^- based OK
  τ based 2 σ
- Error on HLbL
- Errors added quadratically
- 3.5 σ
- Difference:
  4% of LO Had
  270% of HLbL
  1% of leptonic LbL

Generic SUSY: $12.3 \times 10^{-10} \left(\frac{100\ \mathrm{GeV}}{M_{SUSY}}\right)^2 \tan\beta$

$M_{SUSY} \approx 66\ \mathrm{GeV}\sqrt{\tan\beta}$

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