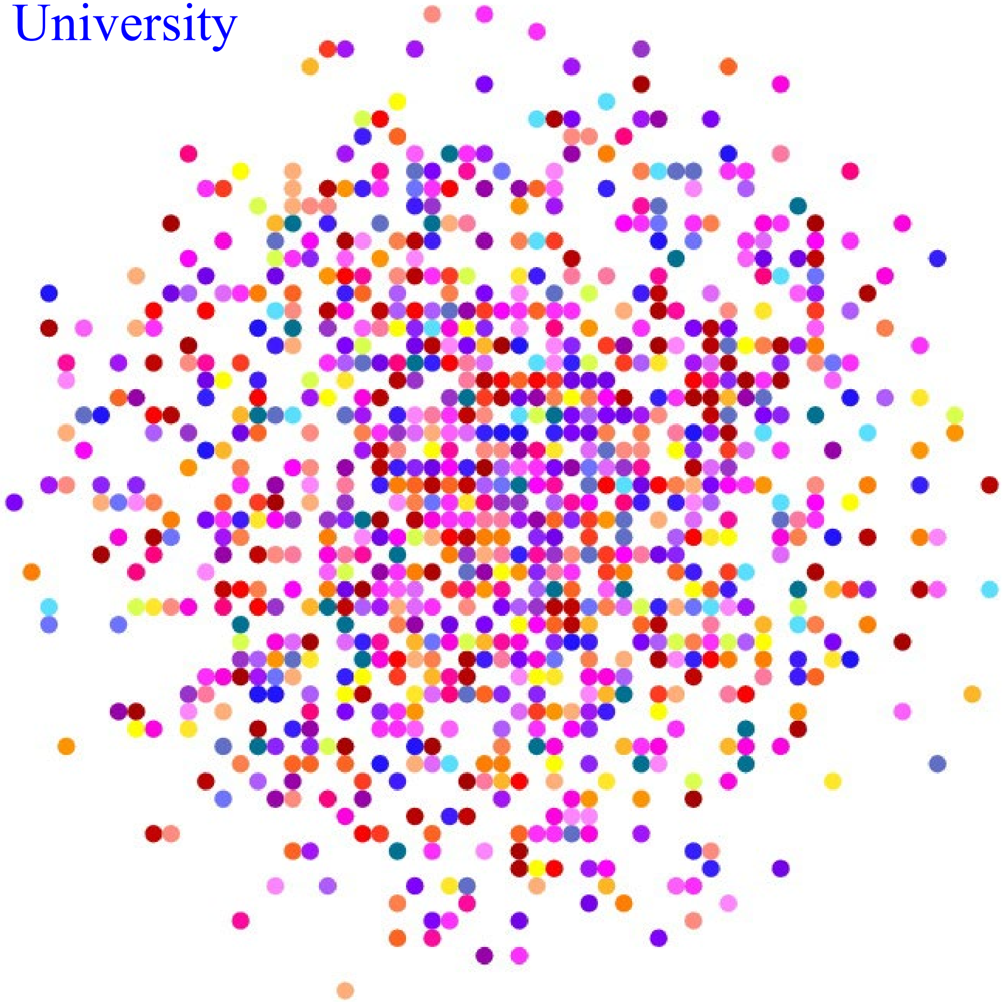
# The Nuclear Shell Model and Beta Decay

Alex Brown

Michigan State University



PHYSICAL REVIEW C **92**, 041301(R) (2015)

# Evaluation of the theoretical nuclear matrix elements for $\beta\beta$ decay of $^{76}$Ge

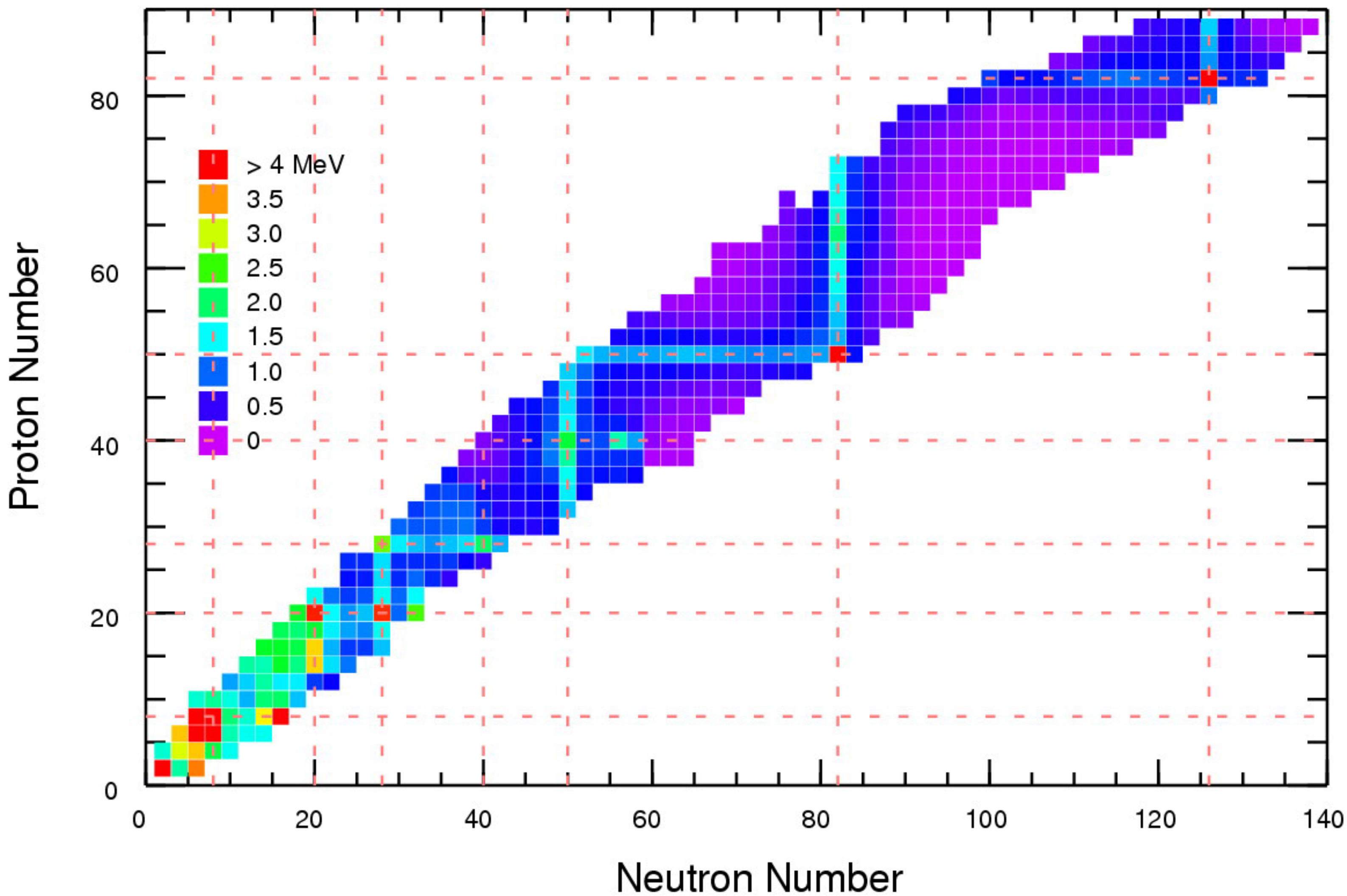
B. A. Brown,[1] D. L. Fang,[2] and M. Horoi[3]

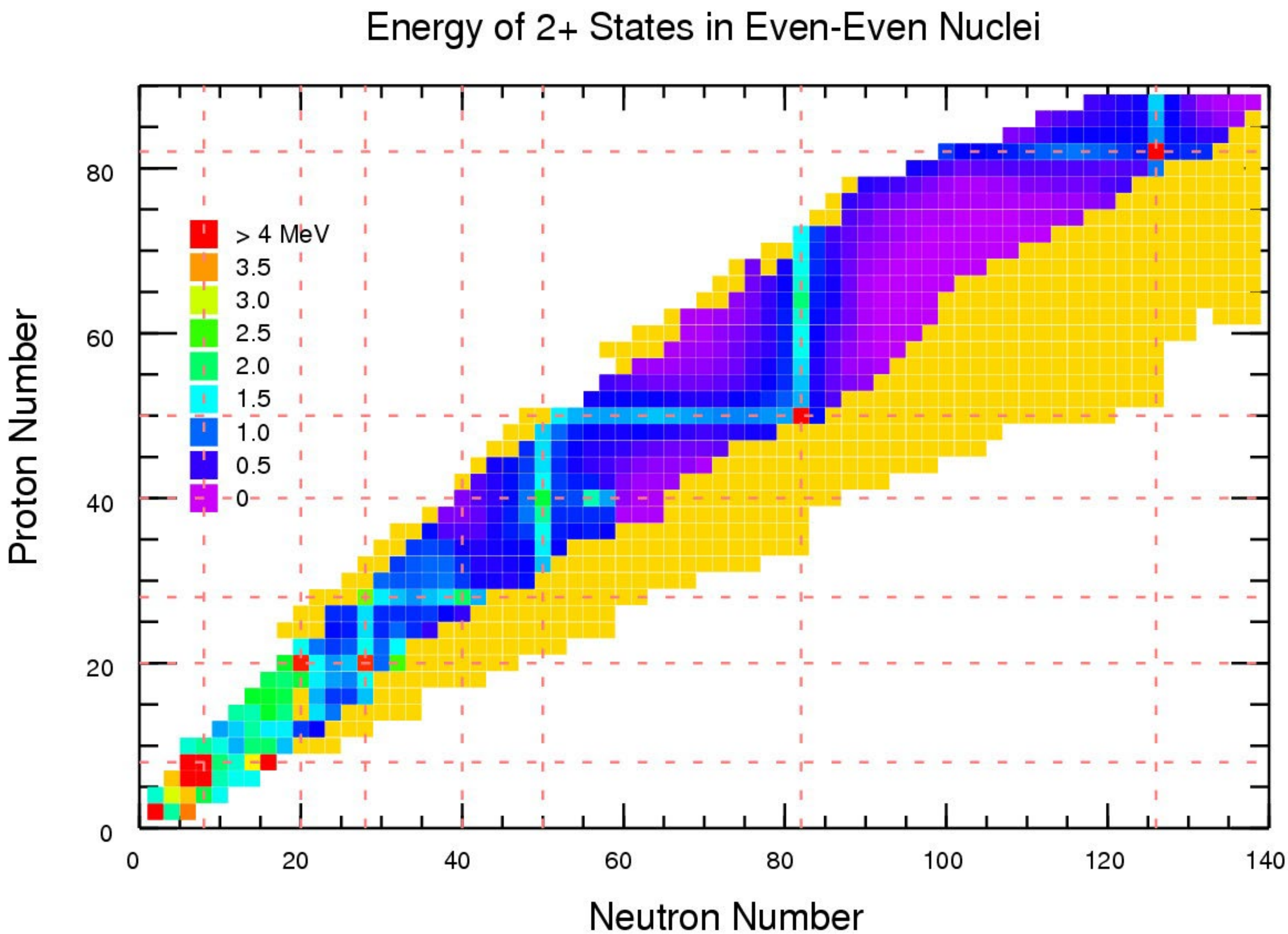
PRL **119,** 192504 (2017) PHYSICAL REVIEW LETTERS week ending 10 NOVEMBER 2017

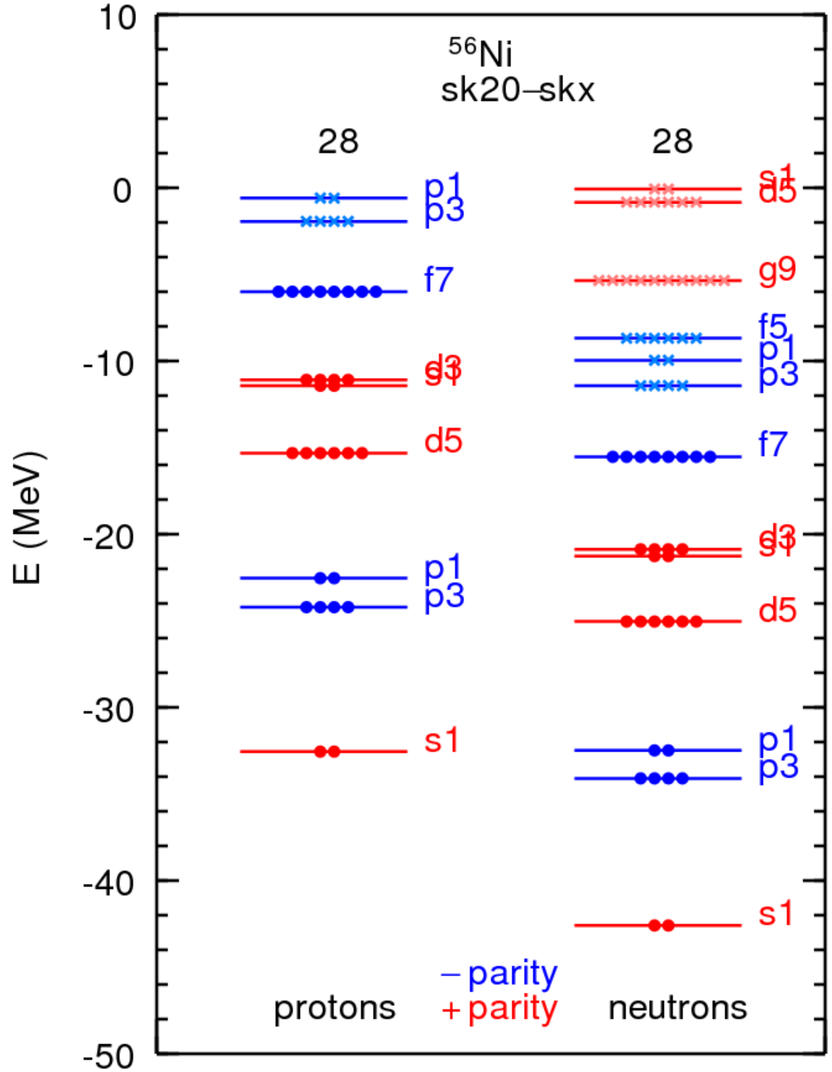
# Nuclear Matrix Elements for Tests of Local Lorentz Invariance Violation

B. A. Brown,[1] G. F. Bertsch,[2] L. M. Robledo,[3] M. V. Romalis,[4] and V. Zelevinsky[1]

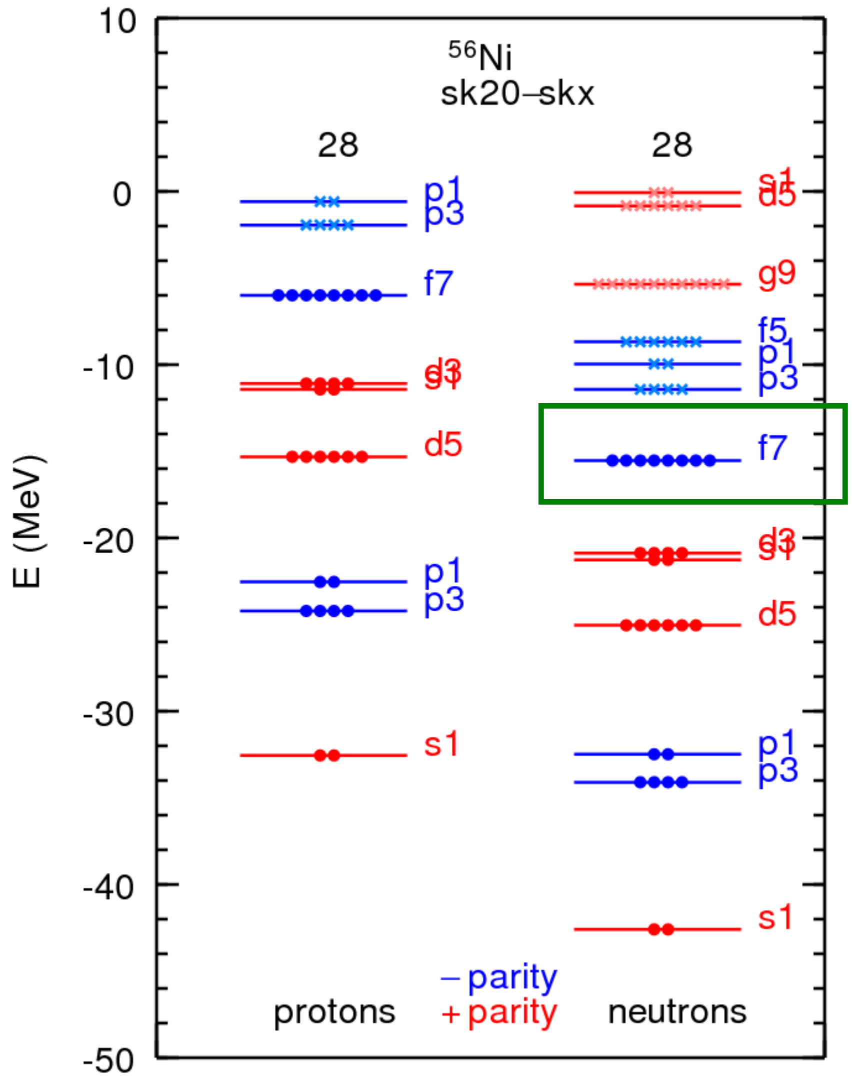
# Energy of 2+ States in Even-Even Nuclei



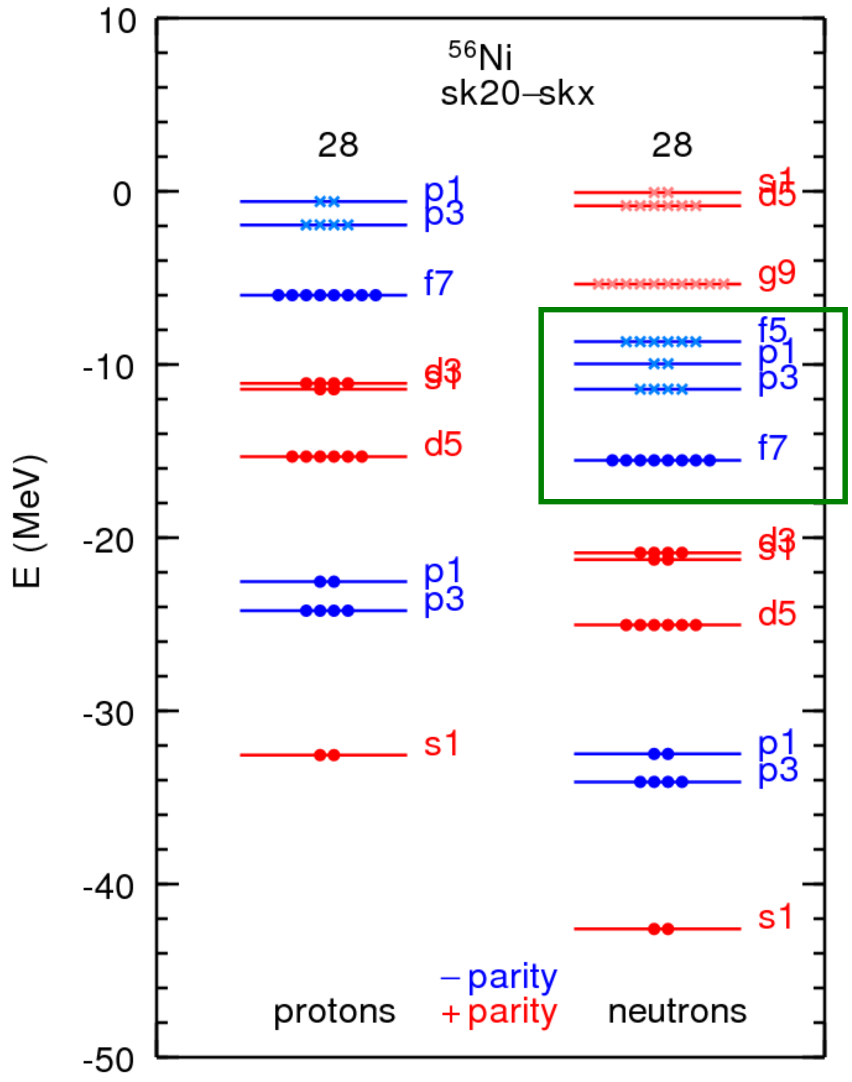

Energy of 2+ States in Even-Even Nuclei
> 4 MeV
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
Proton Number
0
20
40
60
80
Neutron Number
0
20
40
60
80
100
120
140



$^{56}$Ni
sk20–skx
28
28
E (MeV)
10
0
-10
-20
-30
-40
-50
p1
p3
f7
d3
s1
d5
p1
p3
s1
s1
d5
g9
f5
p1
p3
f7
d3
s1
d5
p1
p3
s1
protons
– parity
+ parity
neutrons



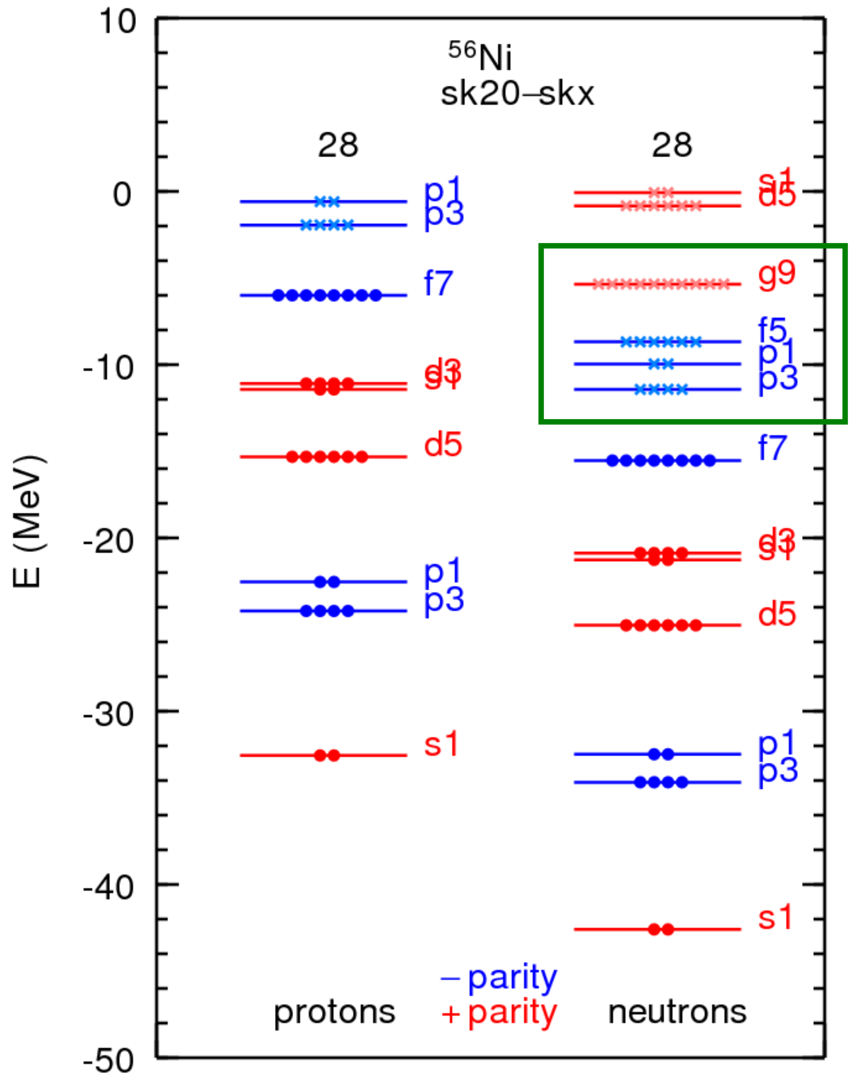

$f_{7/2}$ model space




pf model space

both spin-orbit partners are included

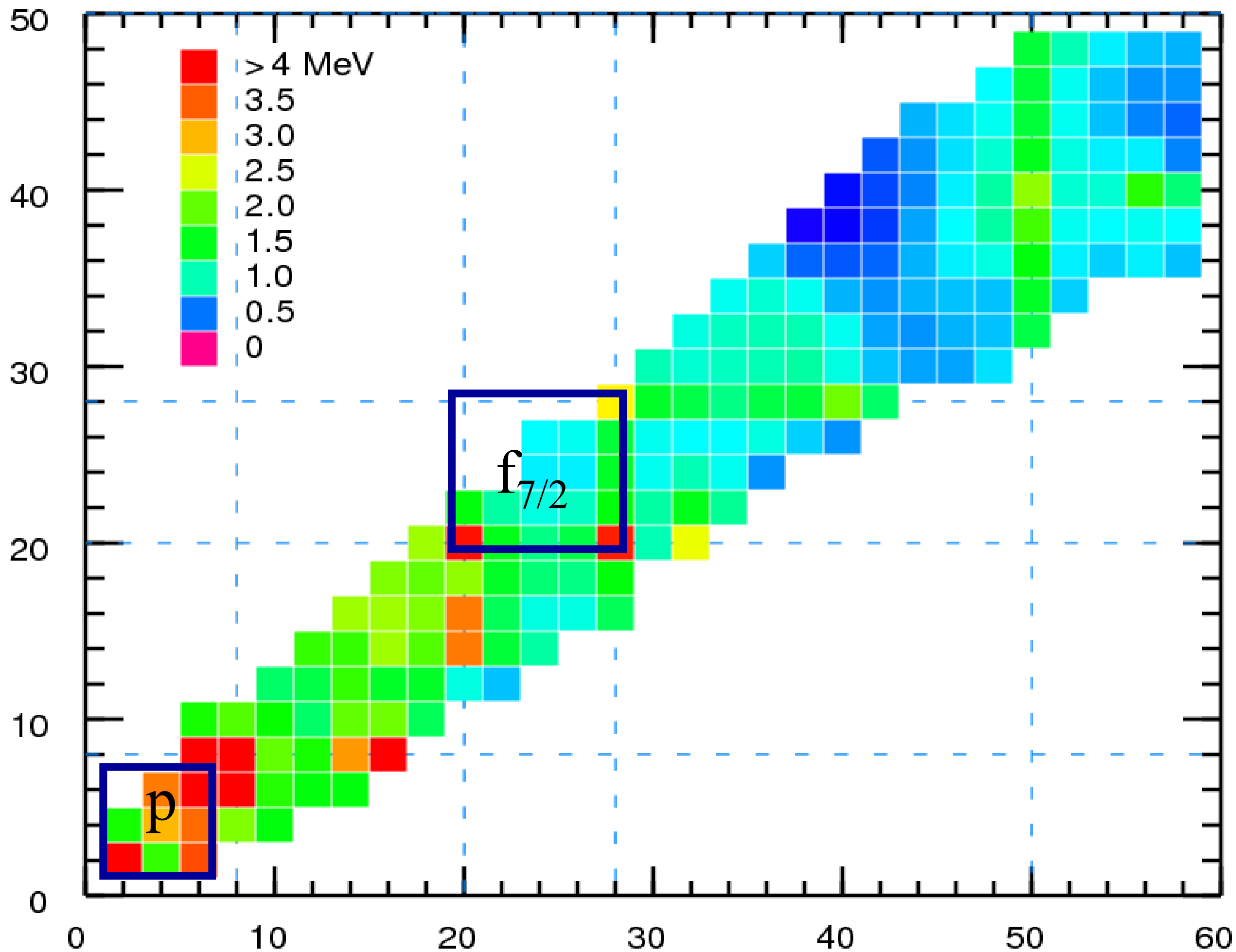
Gamow-Teller Sum rule is full filled in the model space



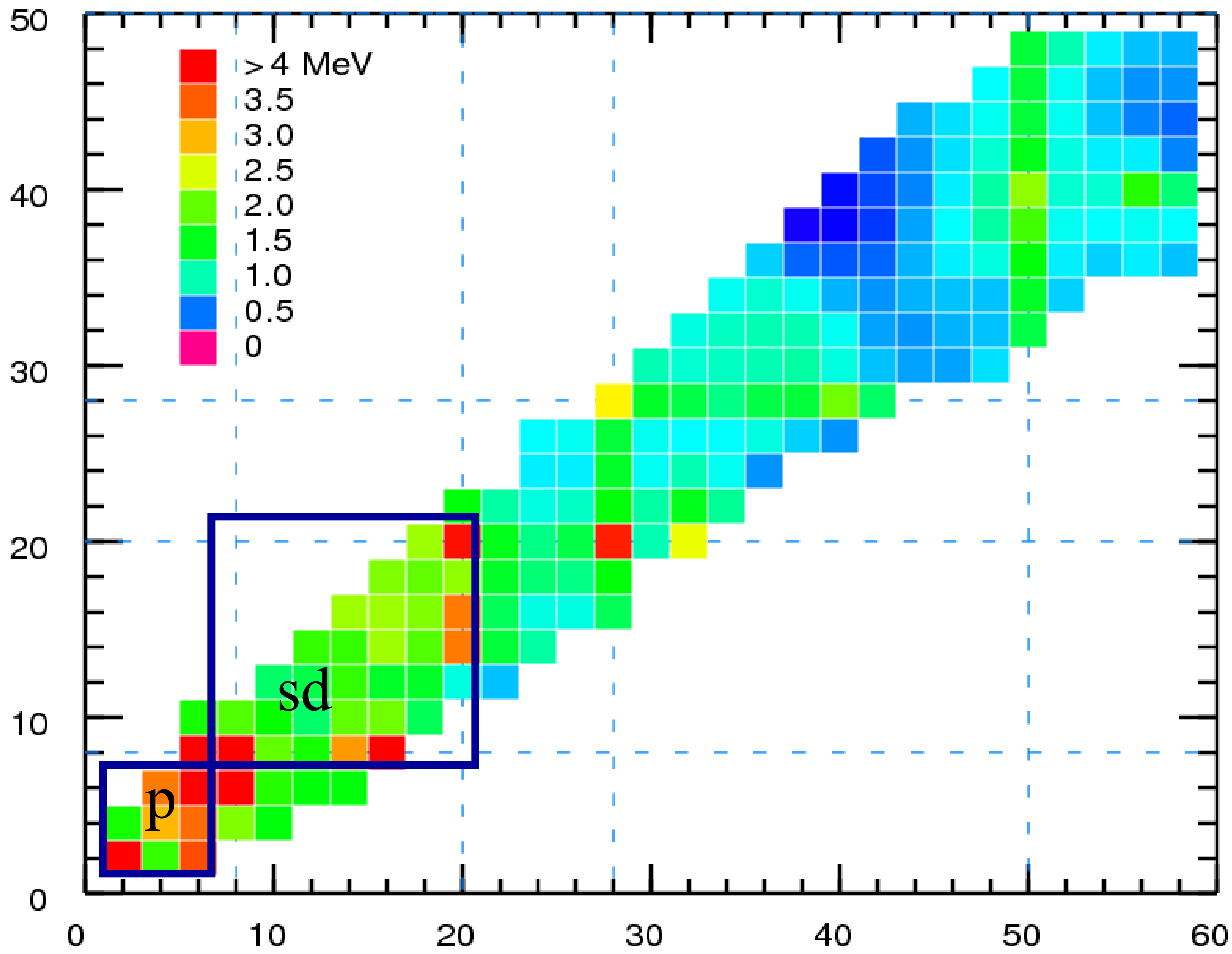

j4 model space

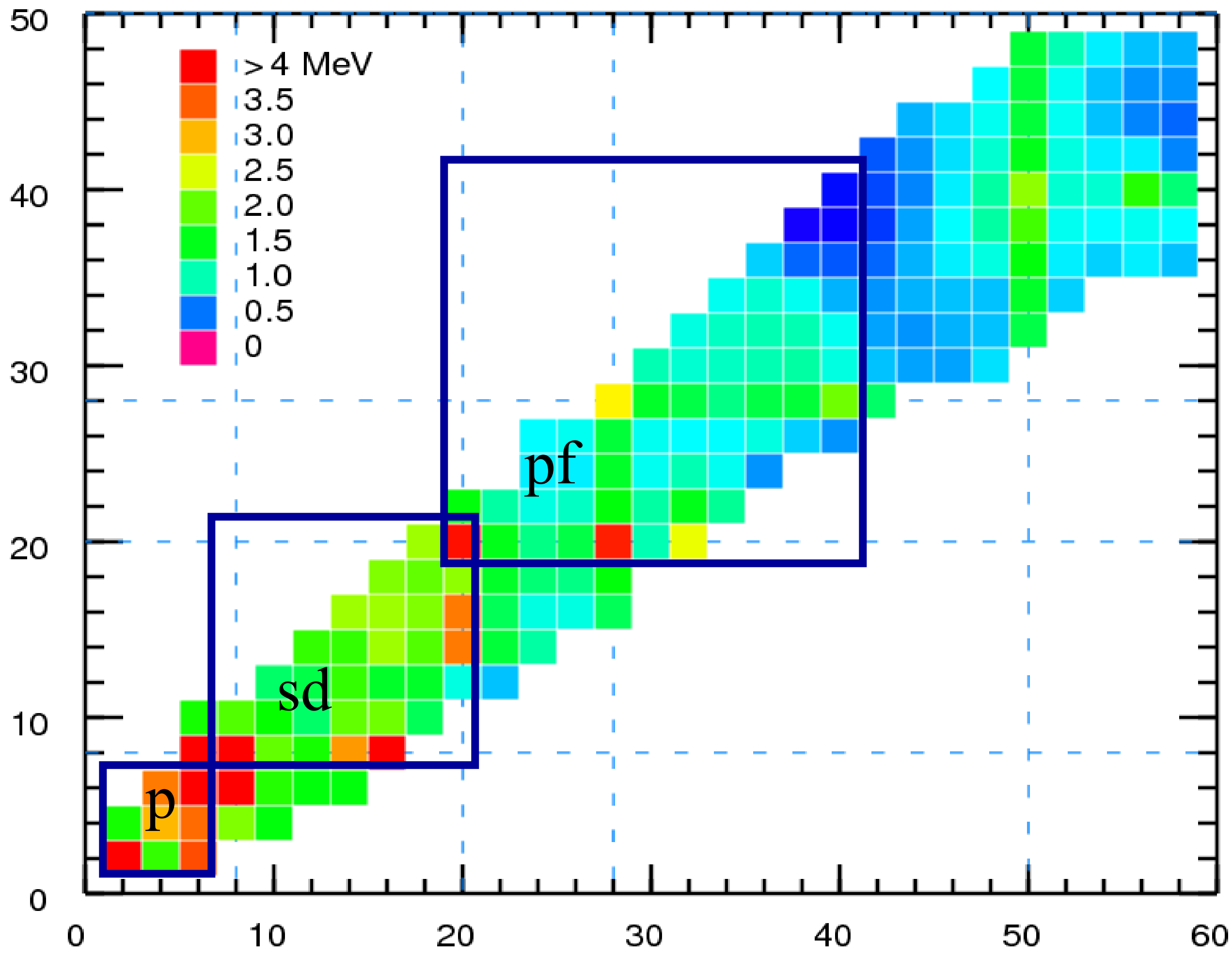
Some spin-orbit partners are missing

# 1950s, 1960s Cohen, Kurath, Talmi, Lawson….

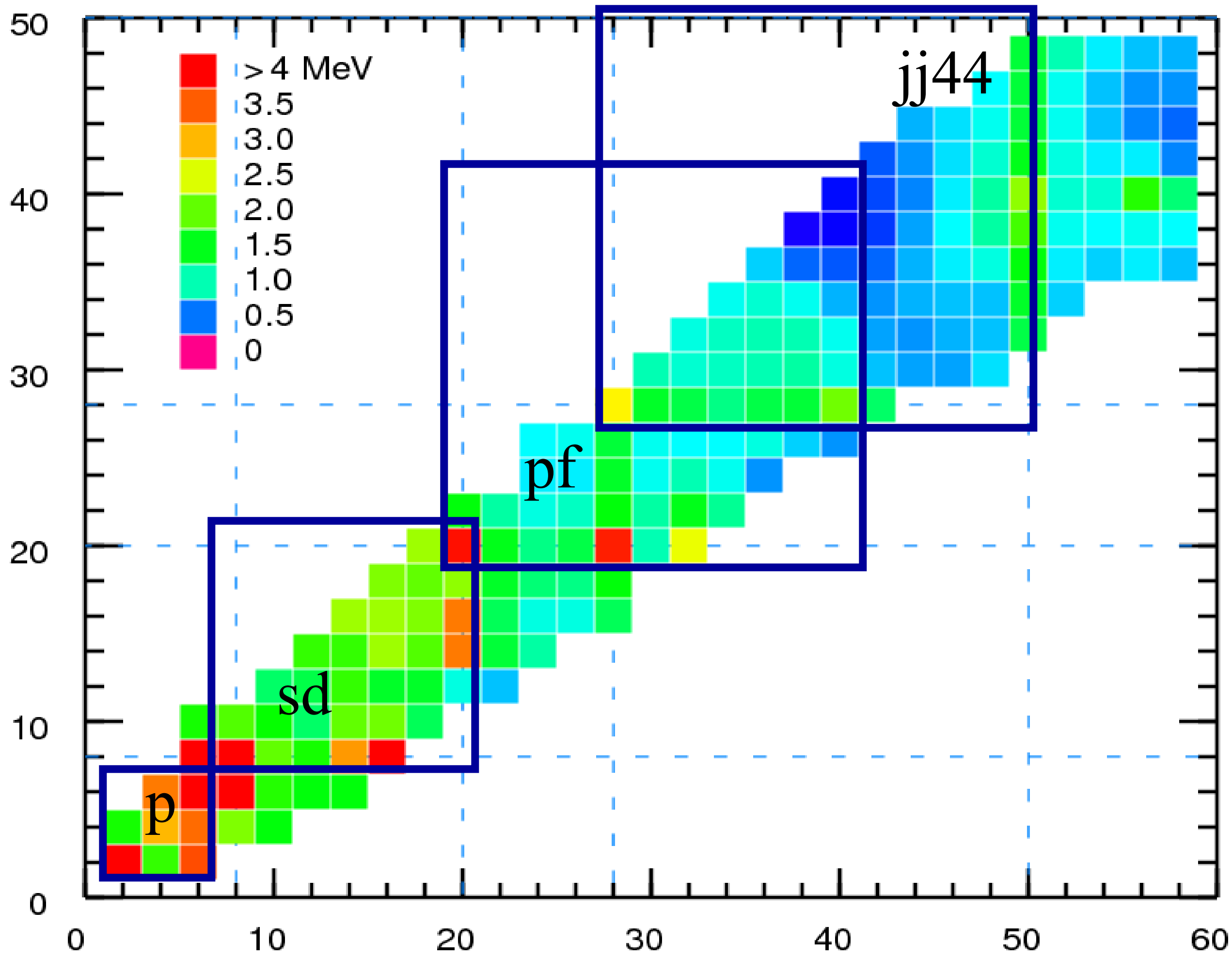



One could understand all details in terms of specific shell-model configurations


> 4 MeV
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
sd
p
0
10
20
30
40
50
60

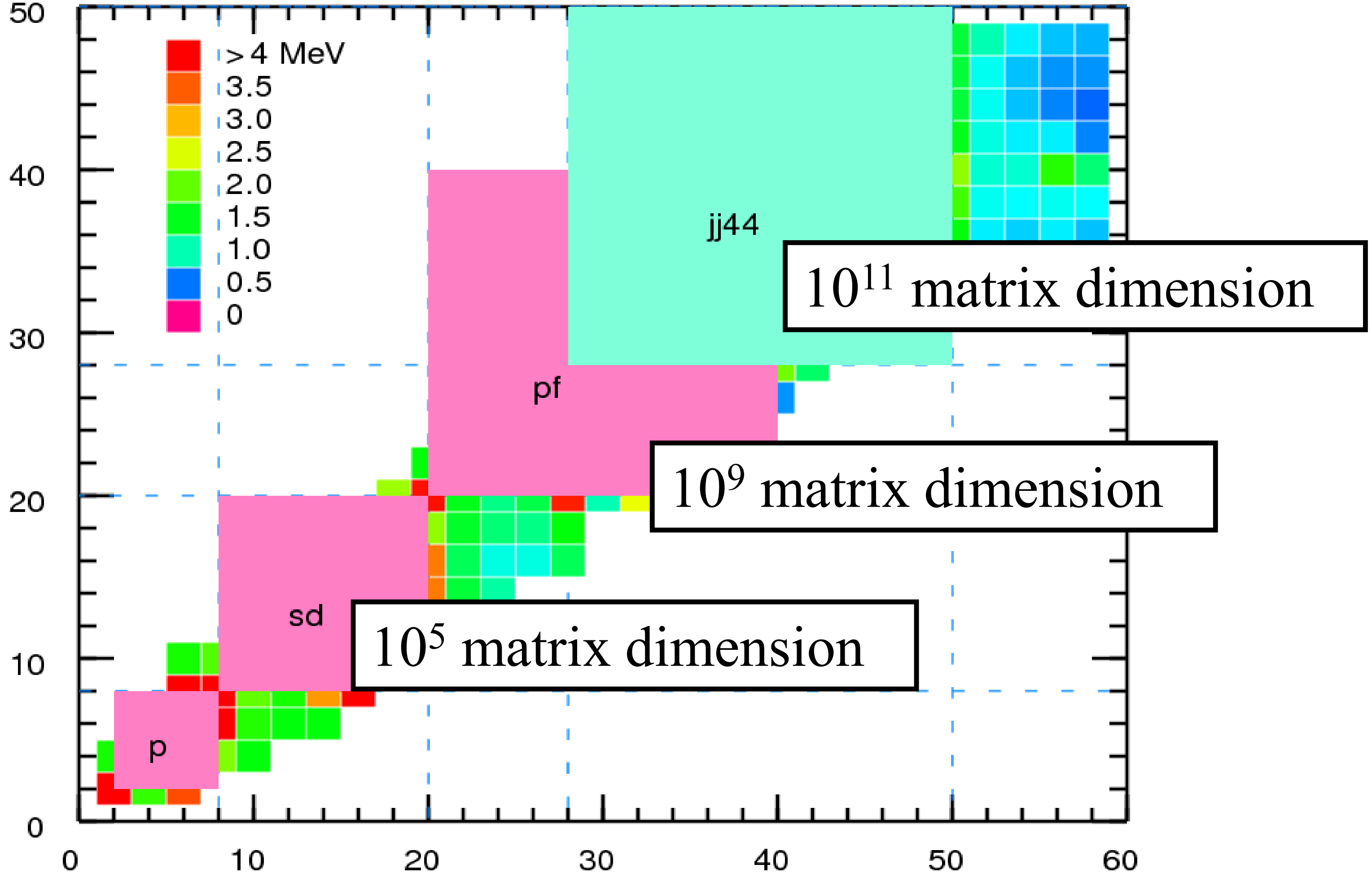


> 4 MeV
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
p
sd
pf
0
10
20
30
40
50
60

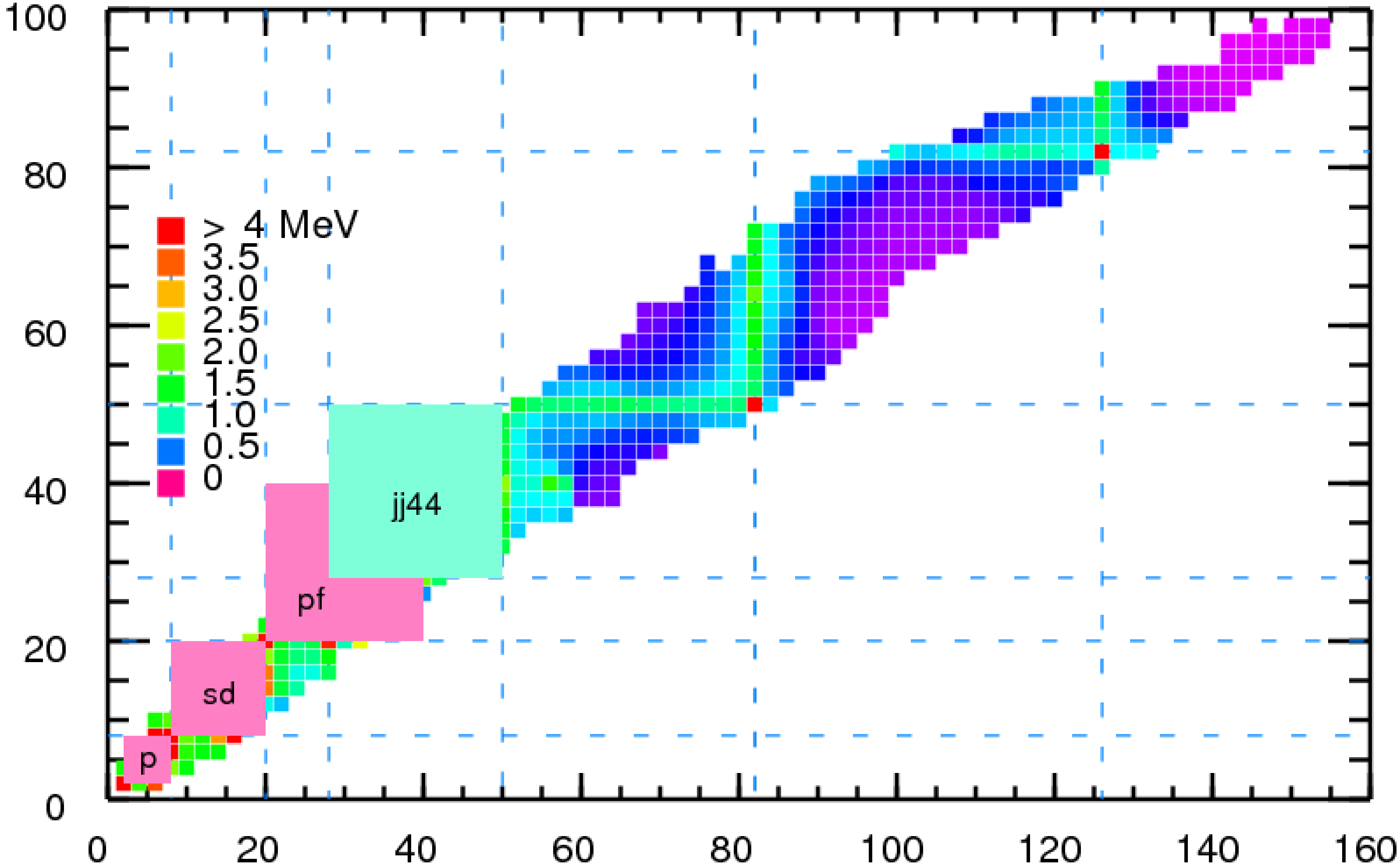


> 4 MeV
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
p
sd
pf
jj44
0
10
20
30
40
50
60

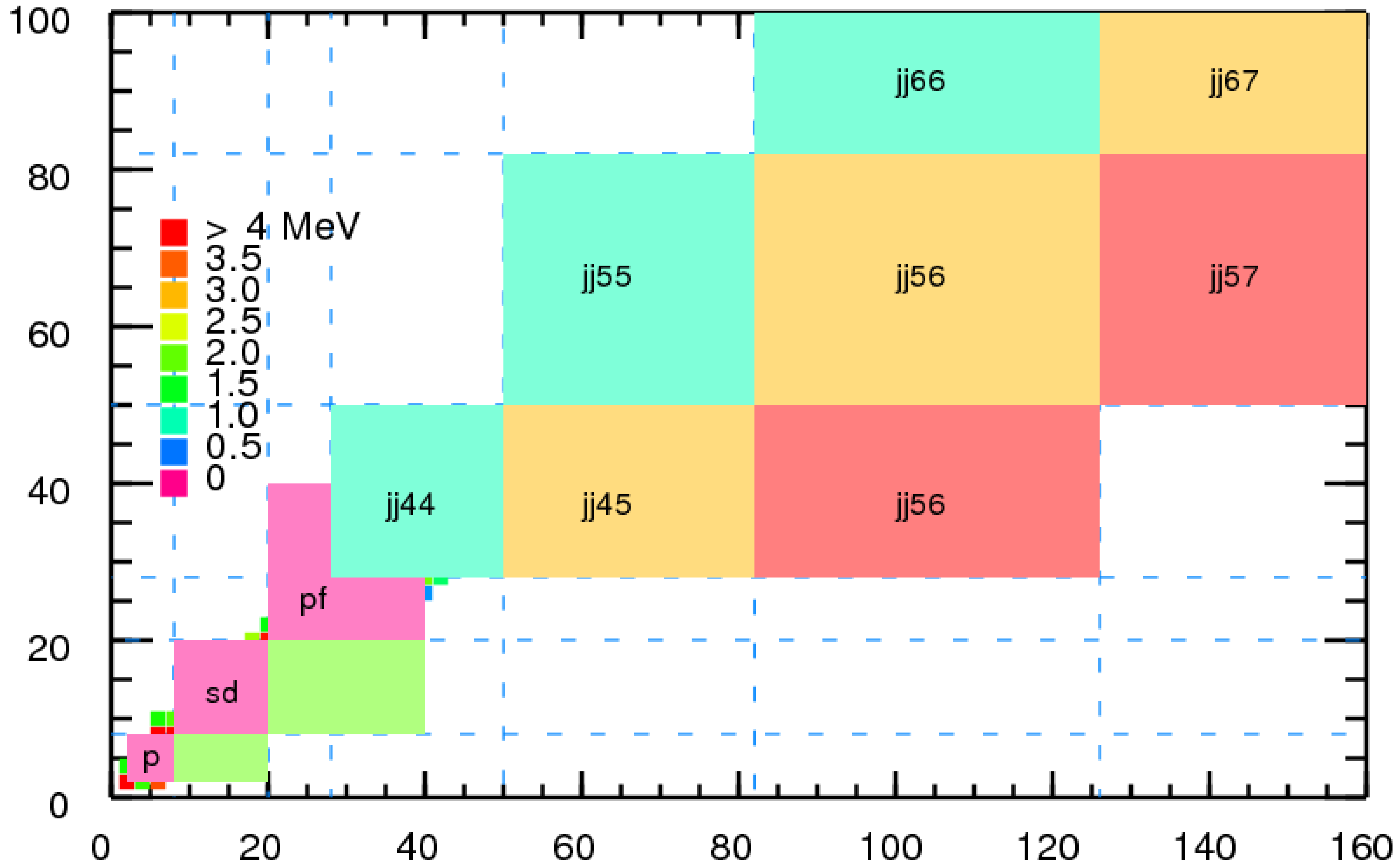
# The computational challenge

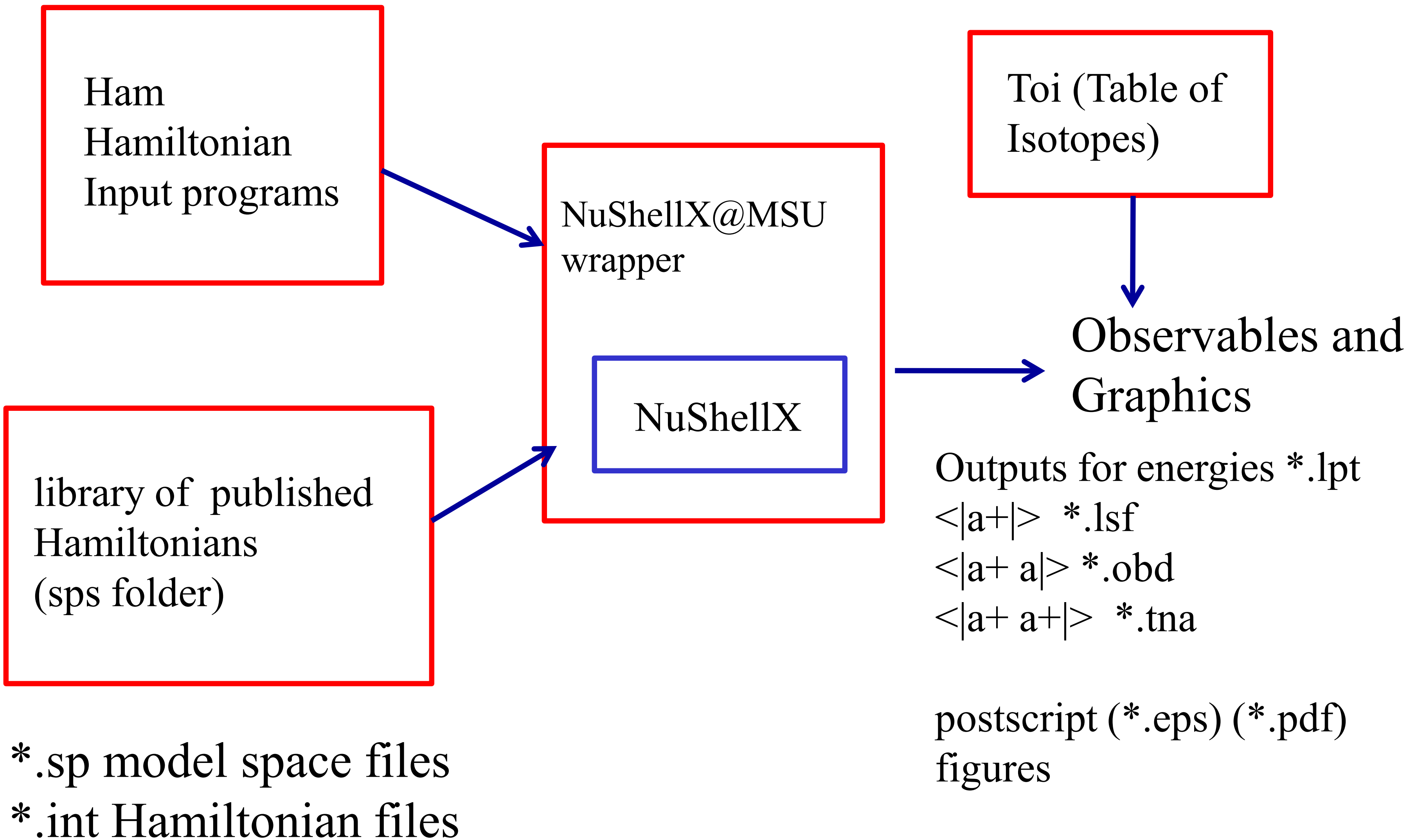



jj44 means $f_{5/2}$, $p_{3/2}$, $p_{1/2}$, $g_{9/2}$ orbits for protons and neutrons




jj44 means $f_{5/2}$, $p_{3/2}$, $p_{1/2}$, $g_{9/2}$ orbits for protons and neutrons


100
80
60
40
20
0
0
20
40
60
80
100
120
140
160
> 4 MeV
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
jj66
jj67
jj55
jj56
jj57
jj44
jj45
jj56
pf
sd
p


Ham
Hamiltonian
Input programs
library of published
Hamiltonians
(sps folder)
NuShellX@MSU
wrapper
NuShellX
Toi (Table of
Isotopes)
Observables and
Graphics
Outputs for energies *.lpt
<|a+|> *.lsf
<|a+ a|> *.obd
<|a+ a+|> *.tna
postscript (*.eps) (*.pdf)
figures
*.sp model space files
*.int Hamiltonian files

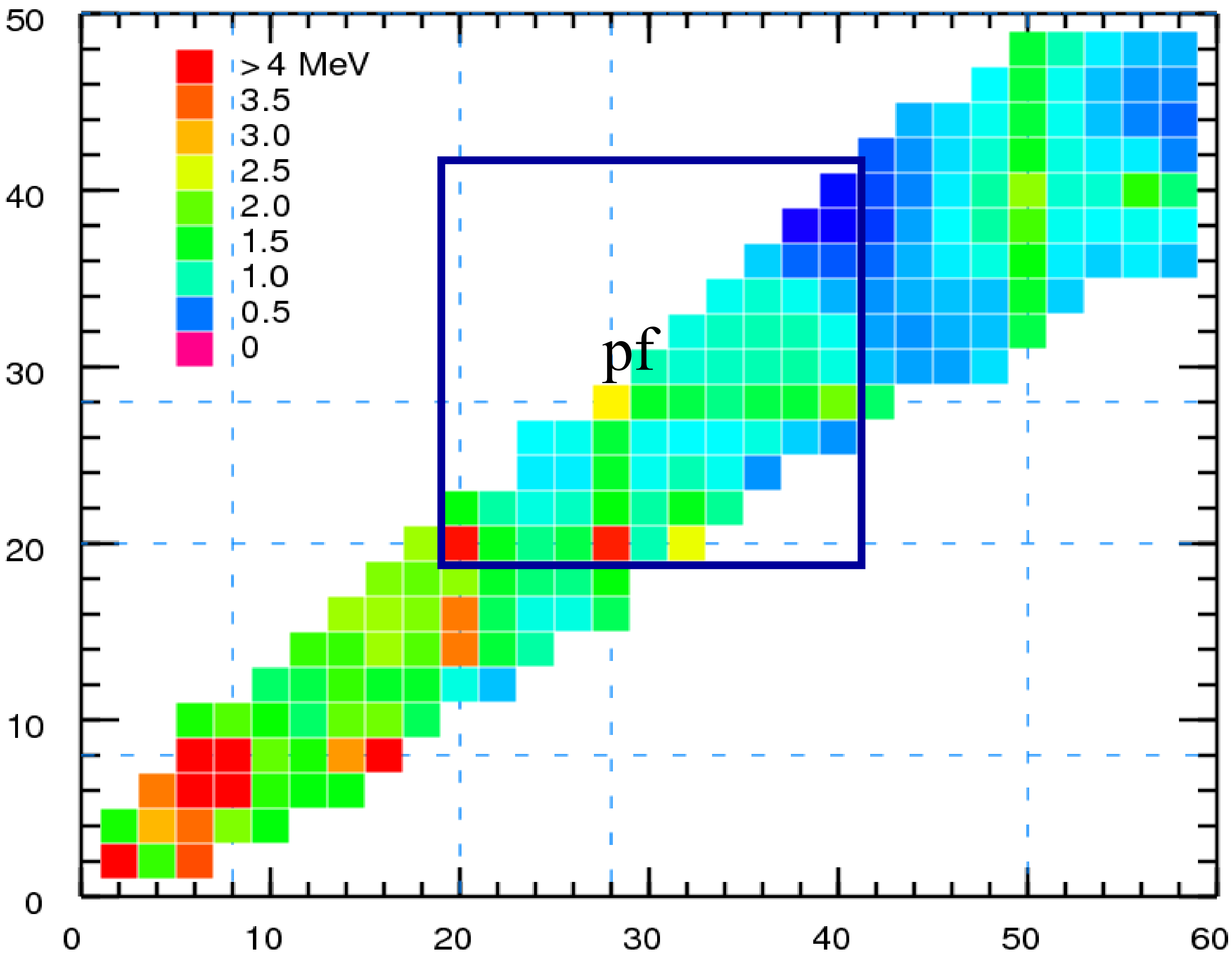
NuShellX (Bill Rae) starts with good-J proton and neutron basis states. Then a good-J pn basis is generated from vector coupling:

$$| [(J_p, \alpha_p) \otimes (J_n, \alpha_n)]J >$$

Fortran95
OpenMP (uses up to about 32 cores)
The Hamiltonian matrix is obtained "on the fly"

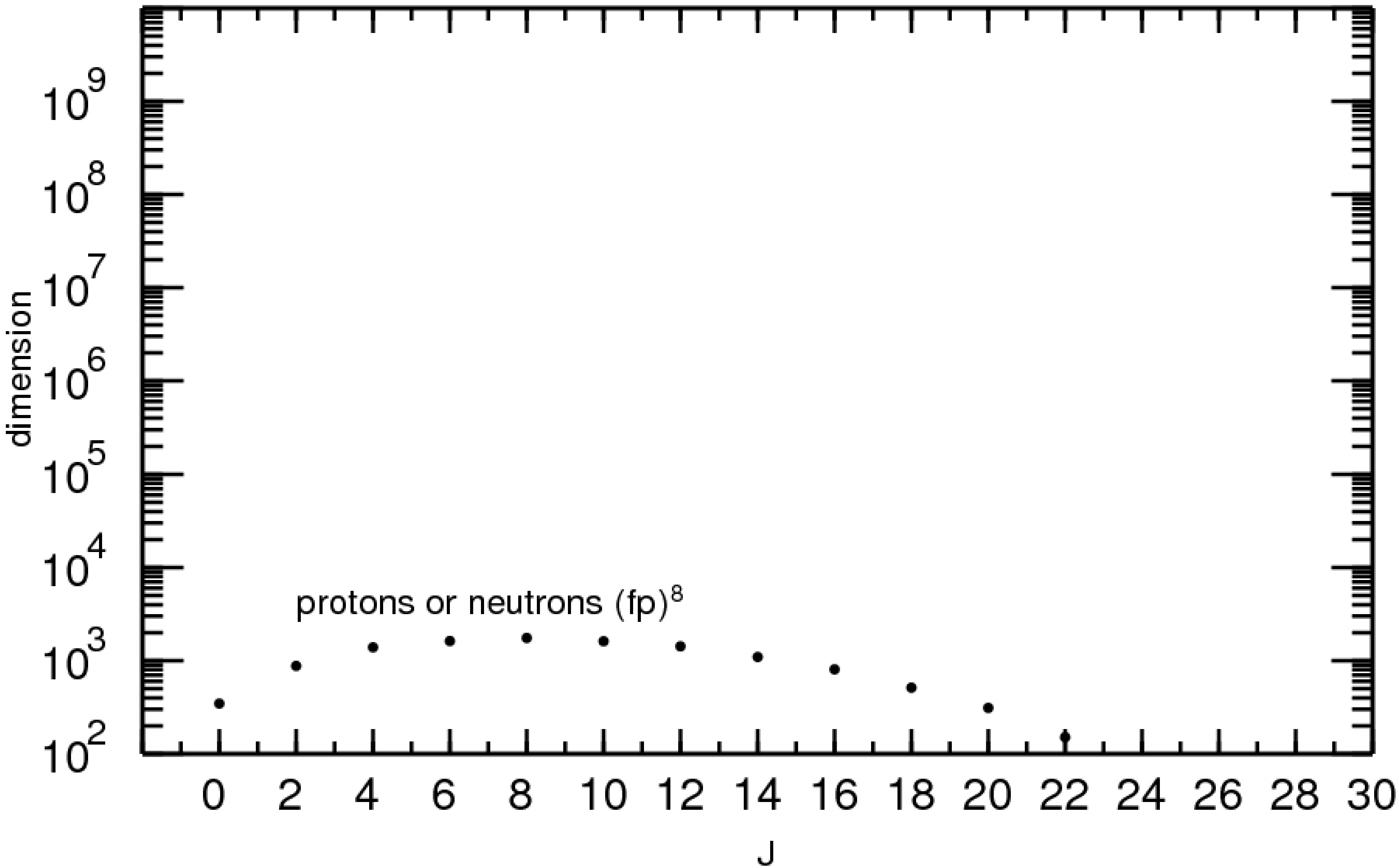

NATHAN, E. Caurier, G. Martinez-Pinedo, F. Nowacki, A. Poves, J. Retamosa and A. P. Zuker, Phys. Rev. C **59**, 2033 (1999). link

J. Toivanen, arXiv:nucl-th/061002v1 9 Oct 2006.

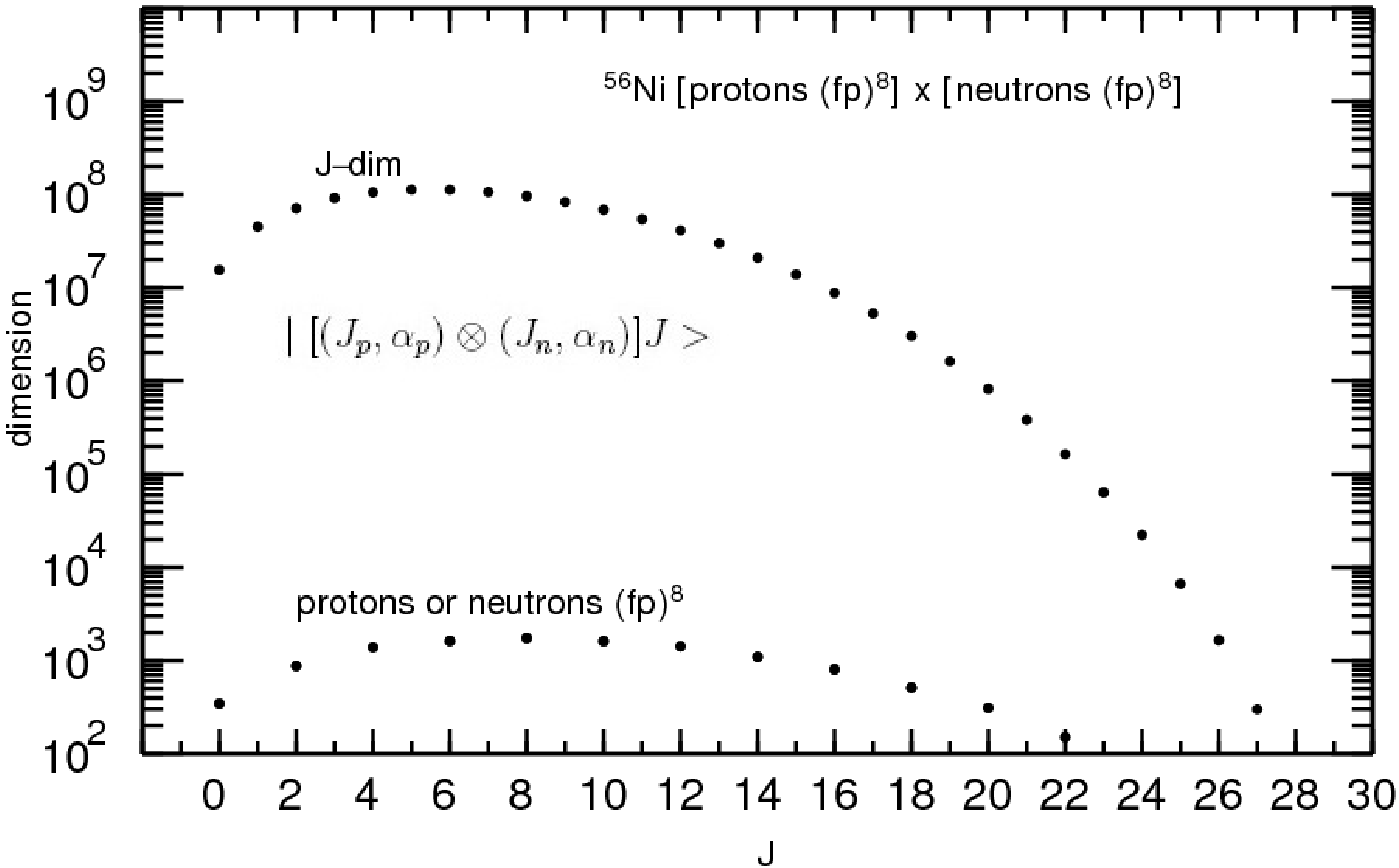


> 4 MeV
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
pf
0
10
20
30
40
50
60

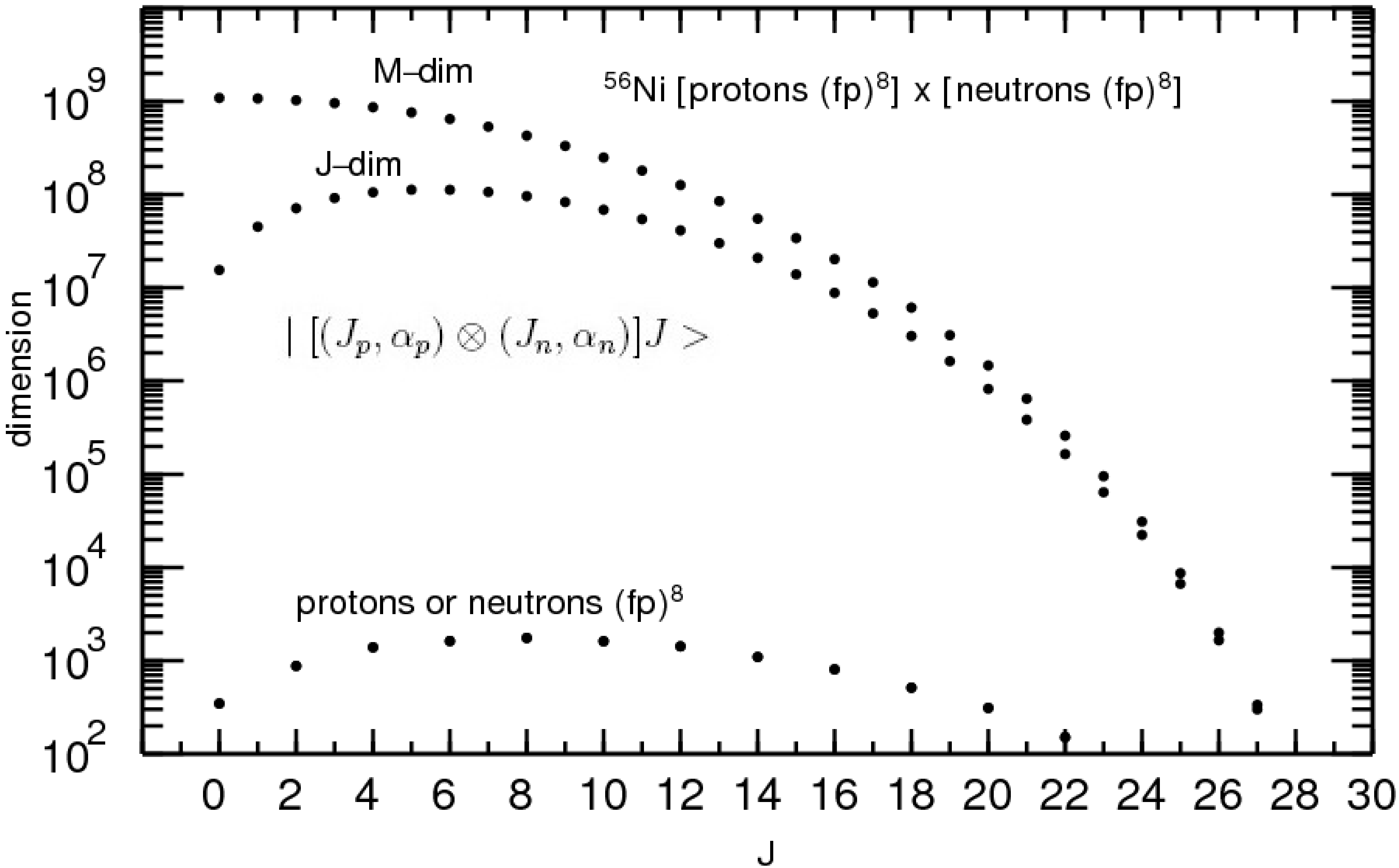
# Example for $^{56}Ni$ in the pf shell



# Example for $^{56}$Ni in the pf shell



# Example for $^{56}$Ni in the pf shell



$$< B_f, J \mid H_{pn} \mid B_i, J >= \sum_{pp'nn'\lambda} F_\lambda(pp'nn')\ \Gamma_\lambda\ \mathrm{RDM}(p_f, p_i, p, p', \lambda)\ \mathrm{RDM}(n_f, n_i, n, n', \lambda)$$

where, for example, $p_f$, stands for labels $(J_{p_f}, \alpha_{p_f})$,

$$\Gamma_\lambda = \left\{ \begin{array}{ccc} J_{p_f} & J_{p_i} & \lambda \\ J_{n_f} & J_{n_i} & \lambda \\ J & J & 0 \end{array} \right\}$$

and RDM are the reduced density matricies:

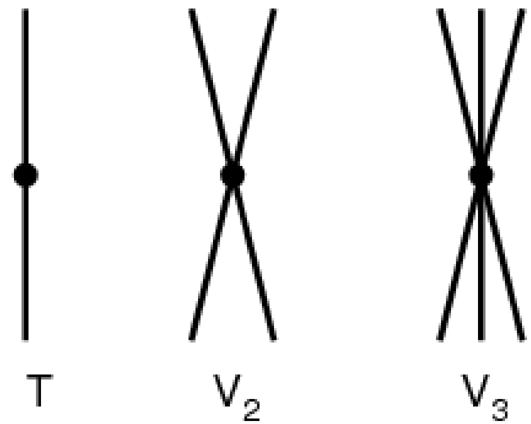
$$\mathrm{RDM}(p_f, p_i, p, p', \lambda) = < [(J_{p_f}, \alpha_{p_f})||[a_p^+ \tilde{a}_{p'}]^\lambda||[(J_{p_i}, \alpha_{p_i}) >$$

and

$$\mathrm{RDM}(n_f, n_i, n, n', \lambda) = < [(J_{n_f}, \alpha_{n_f})||[a_n^+ \tilde{a}_{n'}]^\lambda||[(J_{n_i}, \alpha_{n_i}) >$$

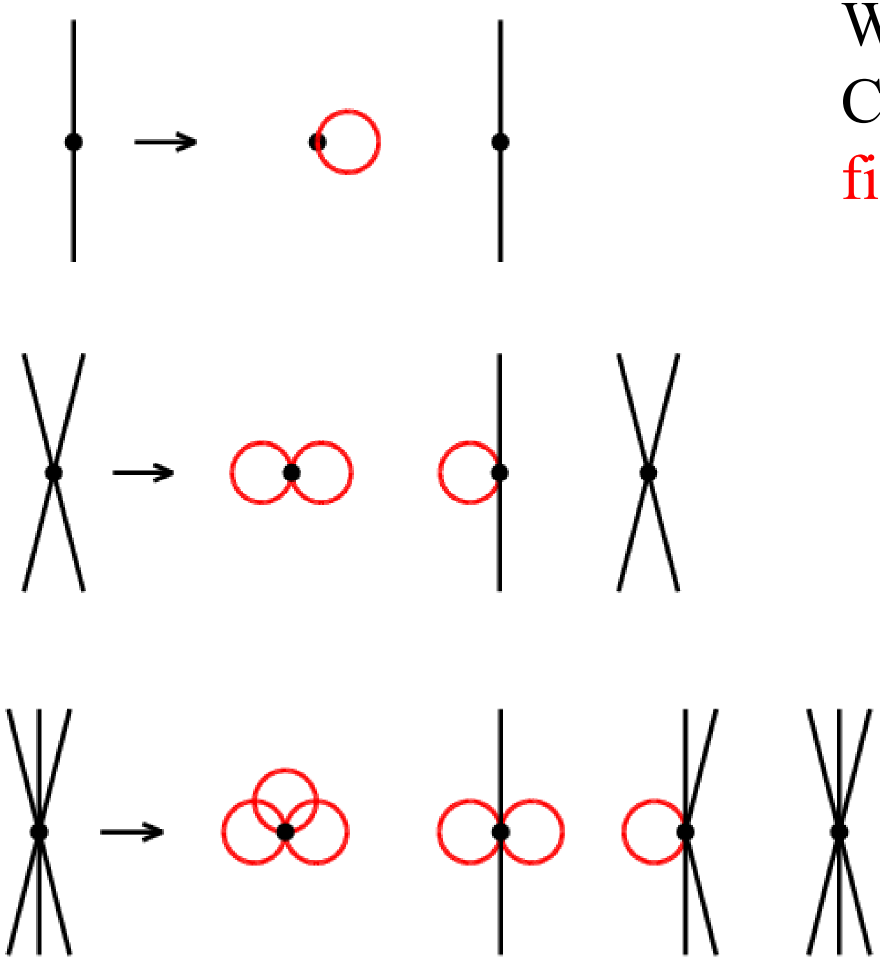
The key is to optimize the sums in this equation for OpenMP and/or MPI

# Hamiltonian



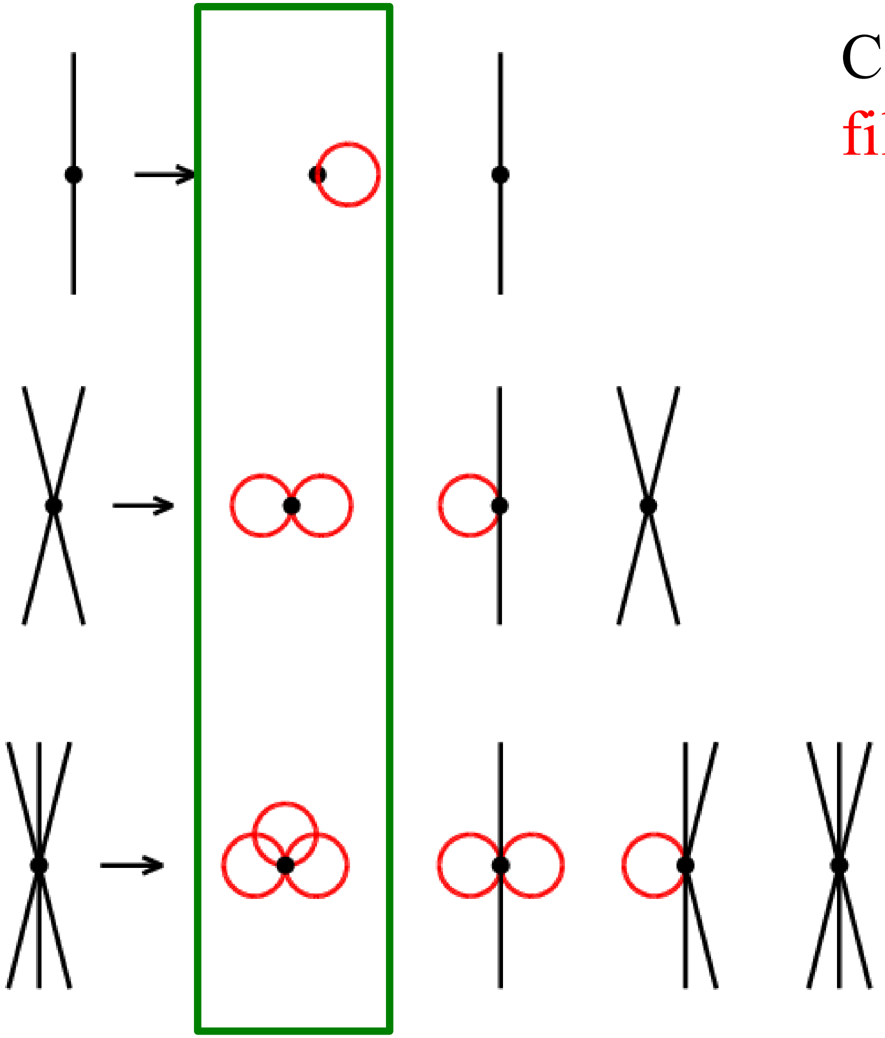
Wick's theorem for a Closed-shell vacuum

filled orbitals



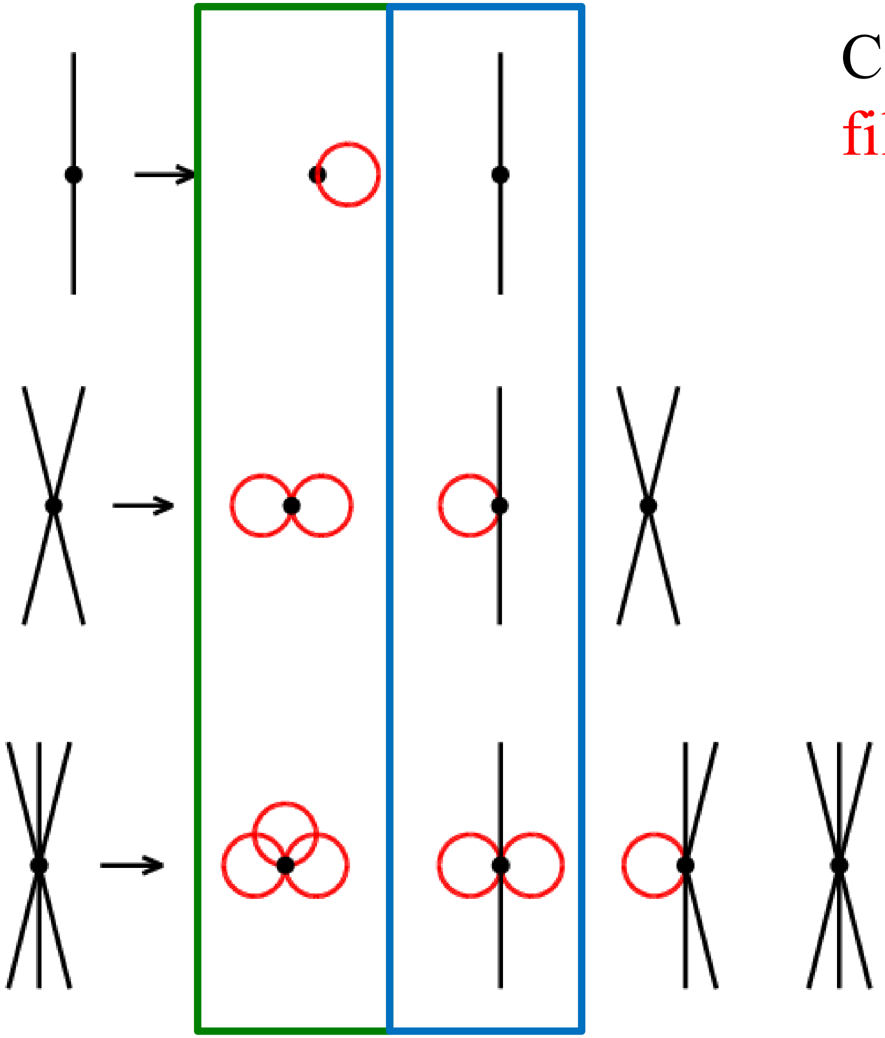
Closed-shell vacuum

filled orbitals



Closed shell
“core” energy

Closed-shell vacuum
filled orbitals

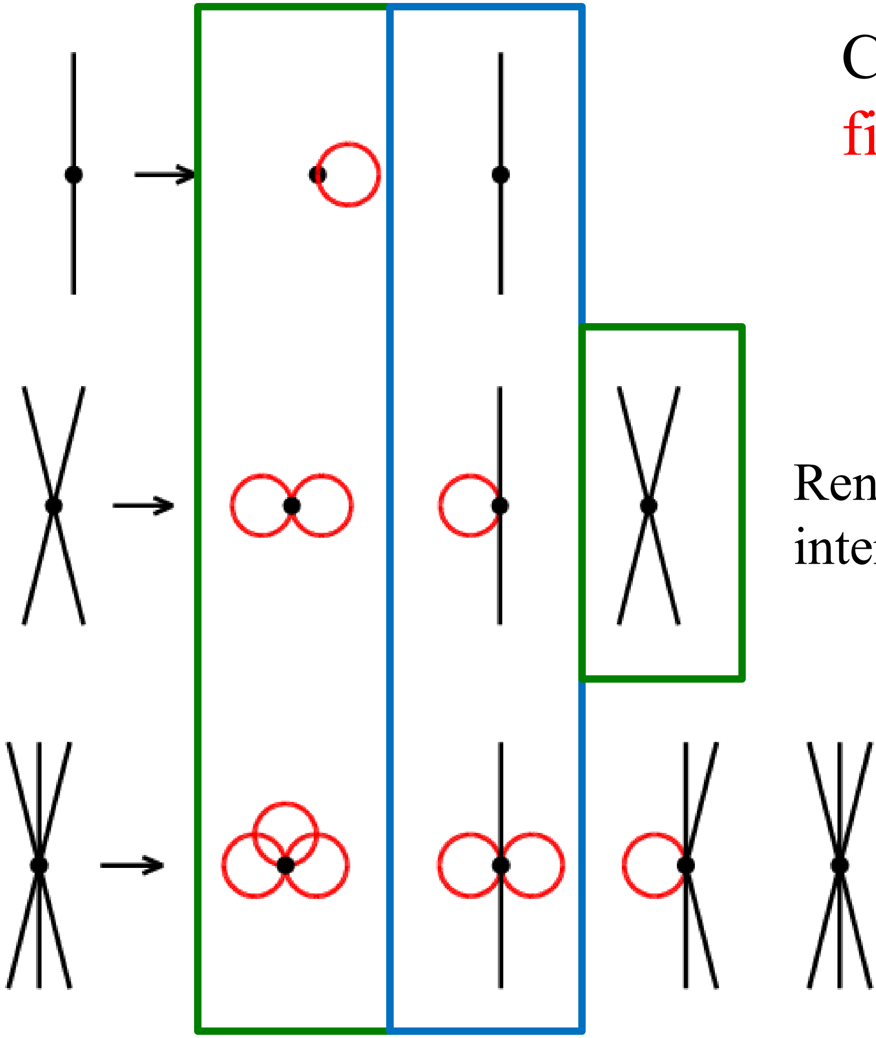


Closed shell
"core" energy

Single particle
energy

From experiment or EDF models

Closed-shell vacuum
filled orbitals

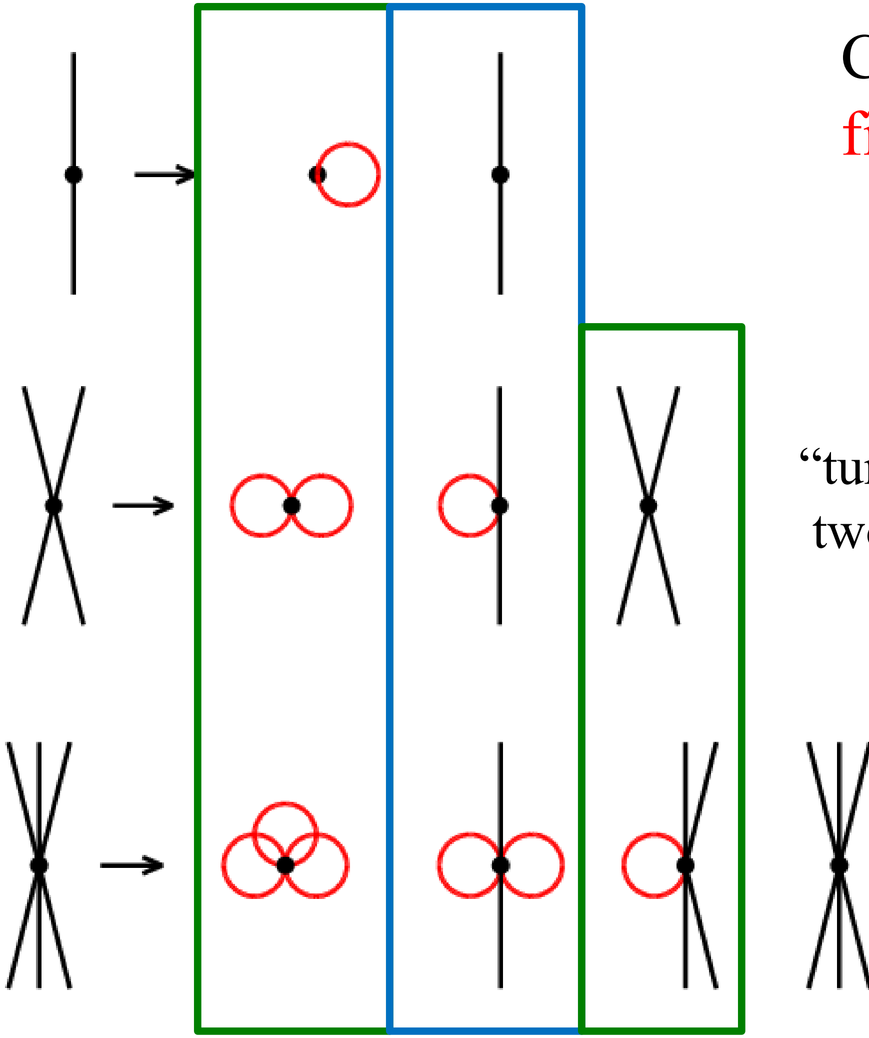


Renormalized nucleon-nucleon interaction

Closed shell "core" energy

Single particle energy

From experiment or EDF models



Closed-shell vacuum

filled orbitals

"tuned" (adjusted) valence two-body matrix elements

Closed shell "core" energy

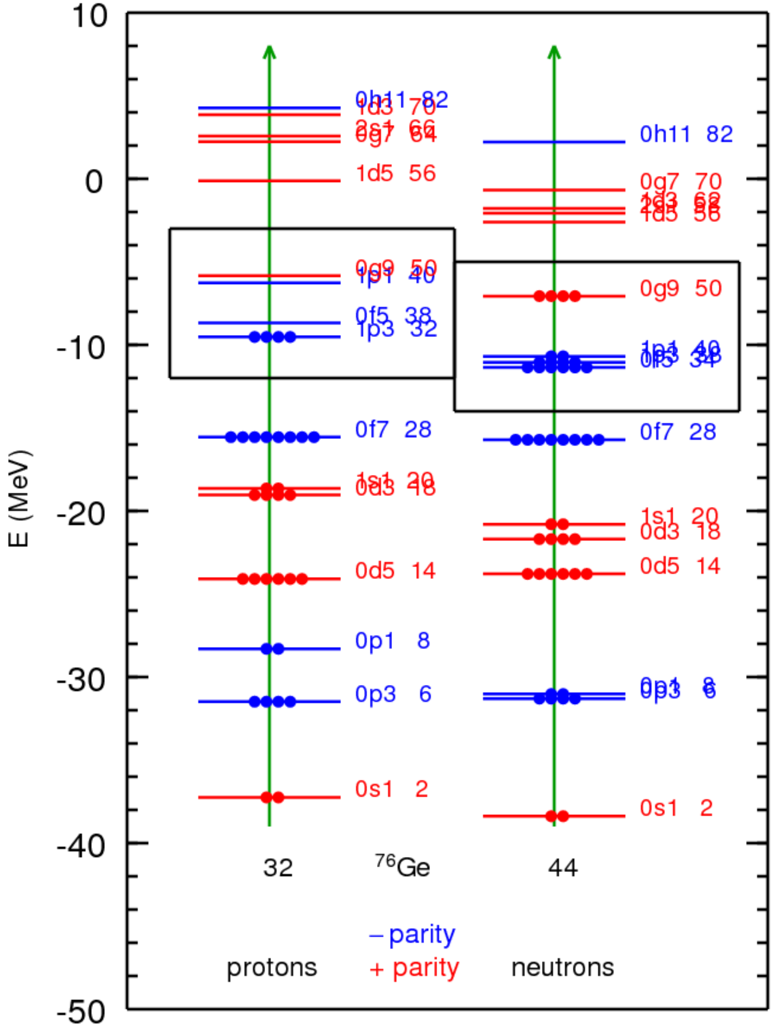
Single particle energy

From experiment or EDF models

# Shell Model Hamiltonians

- Core energy and single-particle energies are taken from experiment (or in their absence some HF-EDF predictions)
- For the two-body part - start with an ab-initio Hamiltonian based on measured NN and then renormalized into the model space.
- Some combinations of two-body matrix elements (10-30) are adjusted to fit energy data – single-valued decomposition
- Hamiltonian is model-space dependent
- Result is that all BE and levels up to about 5 MeV can be reproduced or predicted with an rms deviation of 100-200 keV
- Examples
  - p-shell - Cohen-Kurath, CKI, CKII, CKPOT
  - sd-shell - USD, USDA, USDB
  - pf-shell - FPD6, KB3, KB3G, GPFX1, GPFX1A
  - p-sd-shell (Nhw) - WBP, WBT
  - sd-pf-shell - SDPF-M

# model space




vertical expansion

particle-hole configurations for all orbitals

1) QRPA in

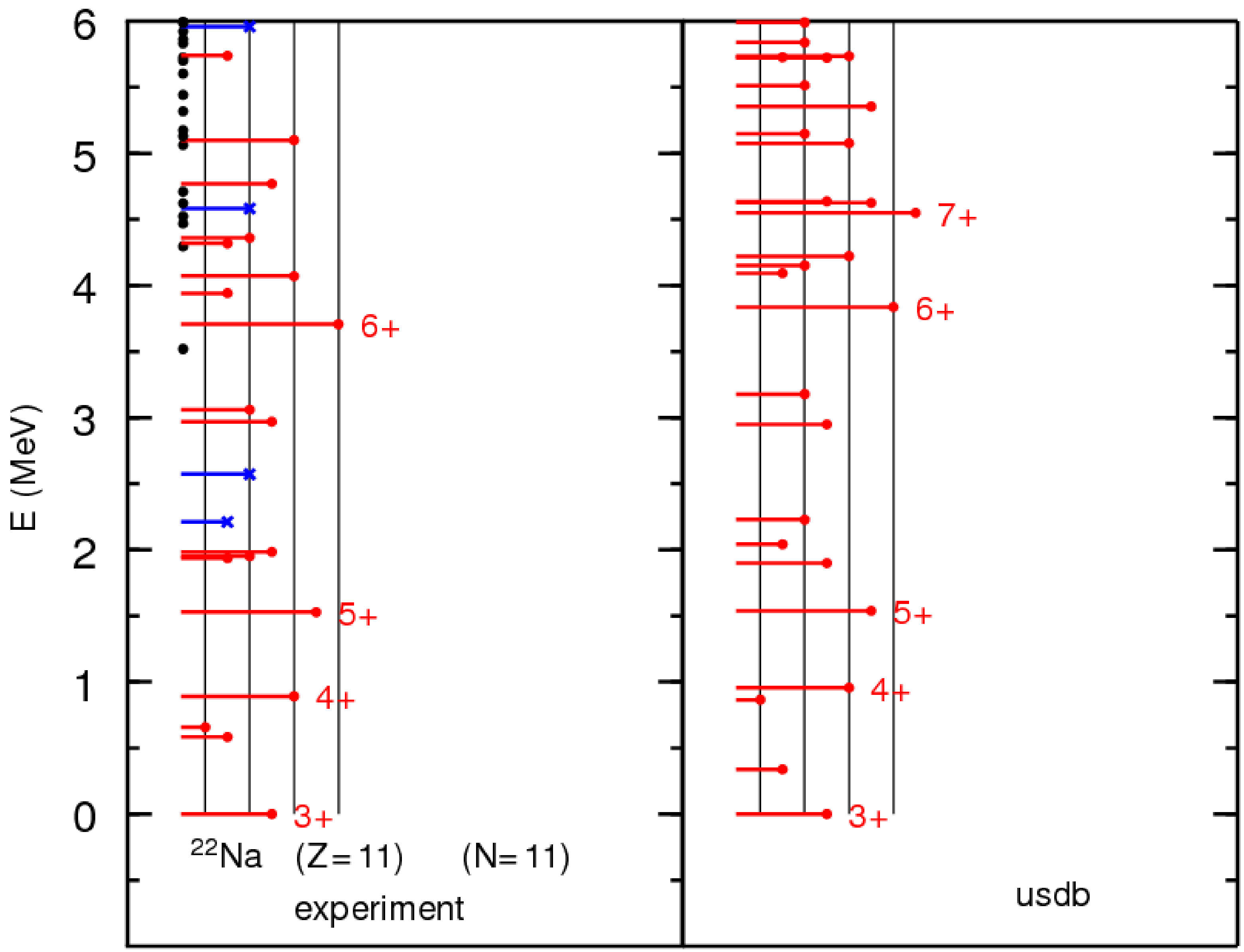
a) jj44 = $(0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2})$
b) fpg = $0f_{7/2}, (0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2})\ 0g_{7/2}$
c) 21 orbits (as on the left)

2) Many-body perturbation theory (MBPT) to include 2 particle-2 hole (2p-2h) excitations to high excitation.

3) $\Delta$ particle admixtures and mesonic exchange currents (MEC)

# The $2_1^+ \rightarrow 3_1^+$ gamma width in $^{22}$Na and second class currents

S. Triambak,[1,2,*] L. Phuthu,[1] A. García,[3] G.C. Harper,[3] J.N. Orce,[1]
D. A. Short,[3] S. P. R. Steininger,[3] A. Diaz Varela,[4] R. Dunlop,[4] D. S. Jamieson,[4]
W. A. Richter,[1] G. C. Ball,[5] P. E. Garrett,[4] C. E. Svensson,[4] and C. Wrede[3,6]


E (MeV)
6
5
4
3
2
1
0
6+
5+
4+
3+
7+
6+
5+
4+
3+
$^{22}$Na (Z= 11) (N= 11)
experiment
usdb

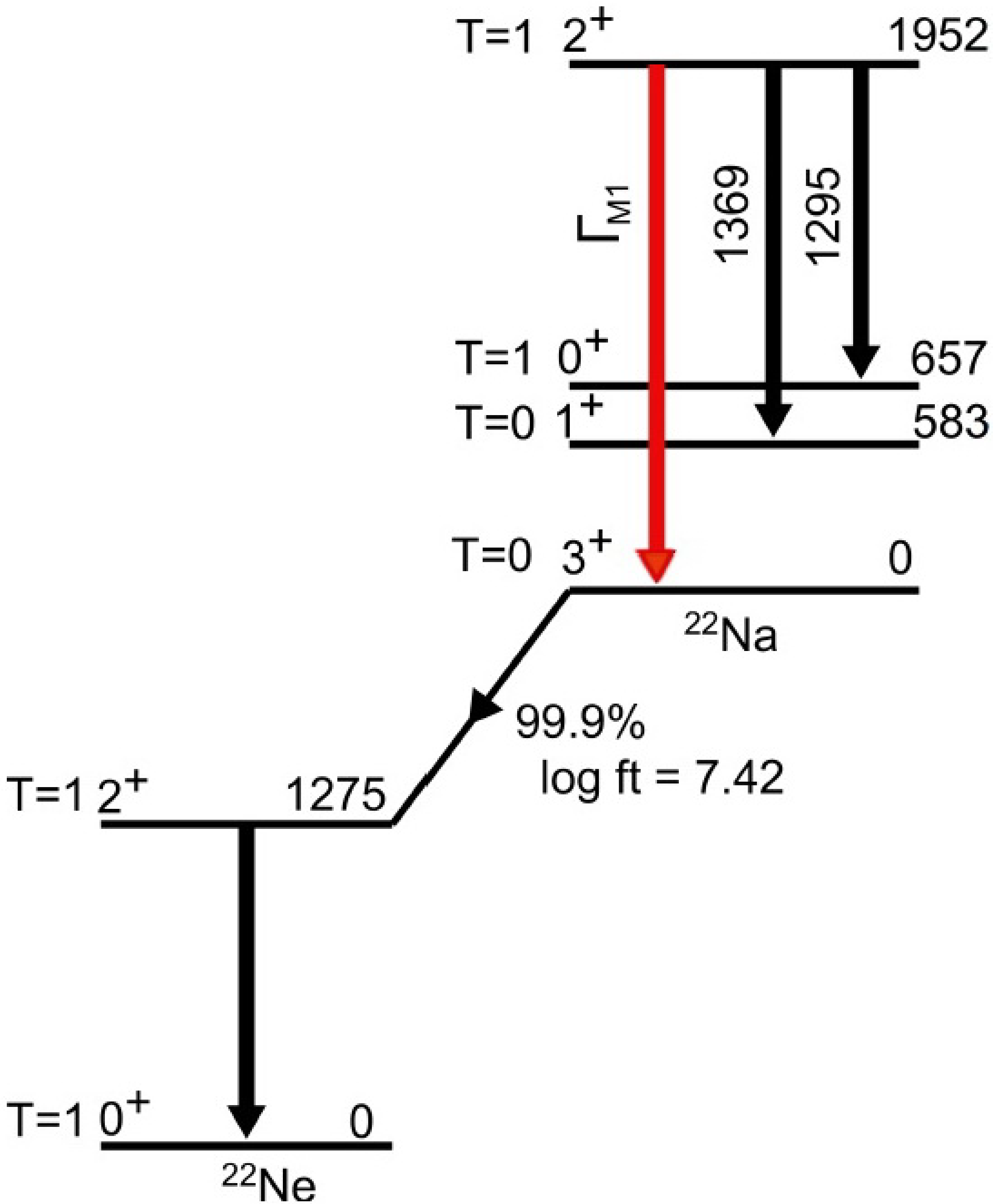

T=1 2+ 1952
$\Gamma_{M1}$
1369
1295
T=1 0+ 657
T=0 1+ 583
T=0 3+ 0
22Na
99.9%
log ft = 7.42
T=1 2+ 1275
T=1 0+ 0
22Ne

TABLE IV. Relative branches obtained for the thre

| $E_\gamma$ (keV) | $E_p = 908$ keV HPGe1 | HPGe2 | USDB | x |
|---|---|---|---|---|
| 1295 | 0.278(69) | 0.280(208) | 0.14 | 0.14 |
| 1369 | 99.291(86) | 99.202(259) | 99.3 | 99.9 |
| 1952 | 0.431(52) | 0.519(156) | 0.56 | 0.0005 |

Branching frac

$$c(q^2) \equiv c_1 + c_2 q^2 + \ldots , \qquad (4)$$

the leading axial-vector form factor $c_1$ can be obtained from the average of several precisely measured corrected $\mathcal{F}t$ values for superallowed Fermi decays [18] and the $ft$ value of $^{22}$Na $\beta$ decay, so that [19]

$$c_1 \simeq \left( \frac{2\mathcal{F}t^{Fermi}}{ft(^{22}\mathrm{Na})} \right)^{1/2} \simeq 0.0153 . \qquad (5)$$

Firestone, McHarris and Holstein [20] performed shell model calculations of recoil-order form factors for $^{22}$Na $\beta$ decay using the impulse approximation and the wavefunctions described in Ref. [12]. The calculations, listed in Table I, yielded higher-order corrections relative to the leading Gamow-Teller term $c_1$ [21]. In light of these calculations, the currently available data present some contradictions if one considers previous measurements of the electron-capture to positron decay branching ratio [20] and the most recent measurement of the $\beta-\gamma$ correlation in $^{22}$Na decay with the Gammasphere array [22]. The

TABLE I. Calculations of higher-order form factors for $^{22}$Na beta decay from Ref. [20]. $A$ is nucleon number and $R$ is nuclear radius.

| Form factor | Calculated value |
|---|---|
| Weak magnetism $b/Ac_1$ | -19 |
| Second-order axial vector $c_2/c_1 R^2$ | -0.37 |
| First-class induced tensor $d/Ac_1$ | -3.2 |

$$\frac{d}{Ac_1} = \frac{1}{4.4}\left[A_{22}10^5 + 0.6\frac{c_2}{c_1 R^2}\right] - \frac{b}{Ac_1}\ , \qquad (6)$$

which yielded $d/Ac_1 = 26(7)$, in strong disagreement with theoretical predictions (Table I). The above conclusion was based on an unpublished determination of the weak magnetism form factor $|b/Ac_1| = 14(4)$ [23] and the assumption that $b$ and $c_1$ have opposite signs, with $c_2$ being a small contribution.

## V. RESULTS AND DISCUSSION

The measured branches of the three $\gamma$ rays of interest are listed in Table IV. With our value for the $1952 \rightarrow 0$ keV branch and the lifetime of the 1952-keV level, $\tau = 11.5(2.9)$ fs [37], we obtain a partial width of

$$\Gamma_\gamma = 2.57(79) \times 10^{-4}\ \text{eV}\ . \tag{8}$$

This value is more precise but not in disagreement with the result reported in Refs. [22, 23]. Making the same assumptions as Ref. [22], namely, that the width in Eq. (7) is the partial width of the $1952 \rightarrow 0$ keV transition and that the relative signs of $b$ and $c_1$ are as predicted by the shell-model calculation, we obtain

$$b/Ac_1 = -8.7(1.1)\ . \tag{9}$$

Inserting this value in Eq. (6) yields

$$d/Ac_1 = 21(6)\ , \tag{10}$$

$$\nu =| \Delta K | - \lambda = \begin{cases} 2 & \text{for } M1 \\ 1 & \text{for } E2 \end{cases} \tag{12}$$

where $\lambda$ is the multipolarity of the transition. Empirically, this implies that the $M1$ strength could be two orders of magnitude more hindered than the $E2$ component [47]. This is at odds with the shell model prediction, but not unexpected, considering that collective excitations are not naturally incorporated in the shell model. Thus, it is likely that the $M1$ component of the transition is much smaller than the one obtained from the branch. Assuming a vanishing isovector $M1$ component and thereby setting $b = 0$, we obtain

$$d/Ac_1 = 12(6) \ , \tag{13}$$

which is roughly consistent with expectations. An alternative scenario is that the $\gamma$ transition is $M1$ dominated, but the relative signs of $b$ and $c_1$ are opposite to that obtained by the shell model. In that case one obtains

$$d/Ac_1 = 3(6) \ . \tag{14}$$

| | exp | USDB | USDA | USD | |
|---|---|---|---|---|---|
| B(M1) 2+ to 1+ | 1.95 | 1.92 | 1.96 | 1.80 | |
| B(M1) 2+ to 3+ | 3.0 | 6.7 | 13.0 | 6.7 | $\times 10^{-3}$ |
| B(GT) 3+ to 2+ | 2.7 | 1.6 | 0.13 | 0.8 | $\times 10^{-4}$ |

USDB M1(b) GT(c) + + for the matrix elements ?????

But < na22 2+ ||F+|| ne22 2+ > = -3.16

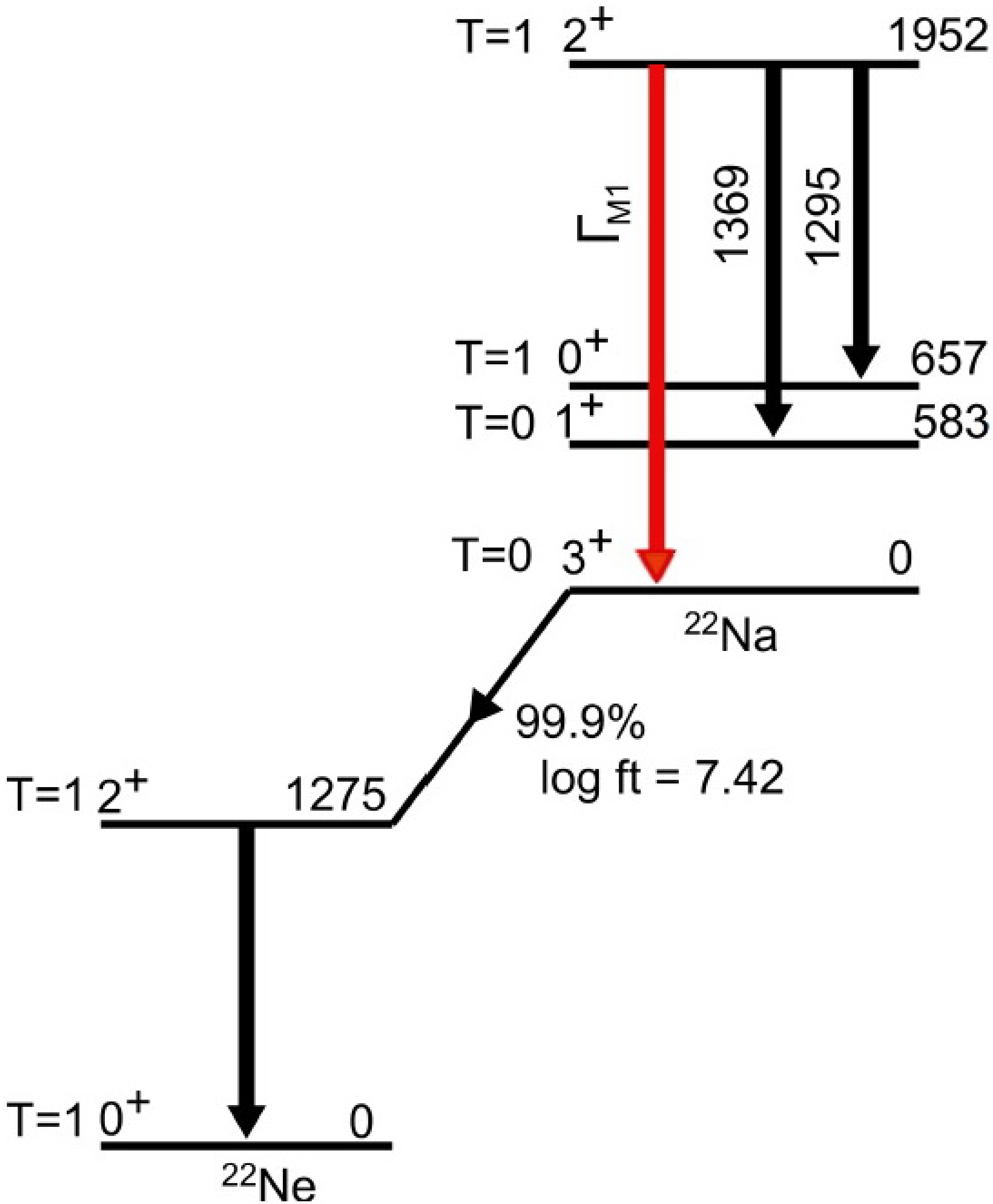
This means the b and c matrix elements have the opposite sign

$$d/Ac_1 = 21(6)$$

| 3+ to 2+ | USDB | USDA | USD |
|---|---|---|---|
| M(s-tau) ($c_1$) | 0.042 | 0.012 | 0.027 |
| M(l-tau) (part of b) | -1.07 | -1.00 | -1.00 |
| M(d-tau) | 0.062 | 0.081 | 0.066 |

Relative phases look robust but s-tau is not very uncertain

so we should look at b/d (not b/c and d/c)


T=1 2+
1952
Γ M1
1369
1295
T=1 0+
657
T=0 1+
583
T=0 3+
0
22Na
99.9%
log ft = 7.42
T=1 2+
1275
T=1 0+
0
22Ne