

$0\nu\beta\beta$ decay NMEs with the generator coordinate method

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Generator Coordinate Method (GCM)

1. GCM 2. Correlations 3. Multi-shell GCM 4. Summary

Generator Coordinate Method: an approach that treats large-amplitude fluctuations, which is essential for nuclei that cannot be approximated by a single mean field.

How it works:

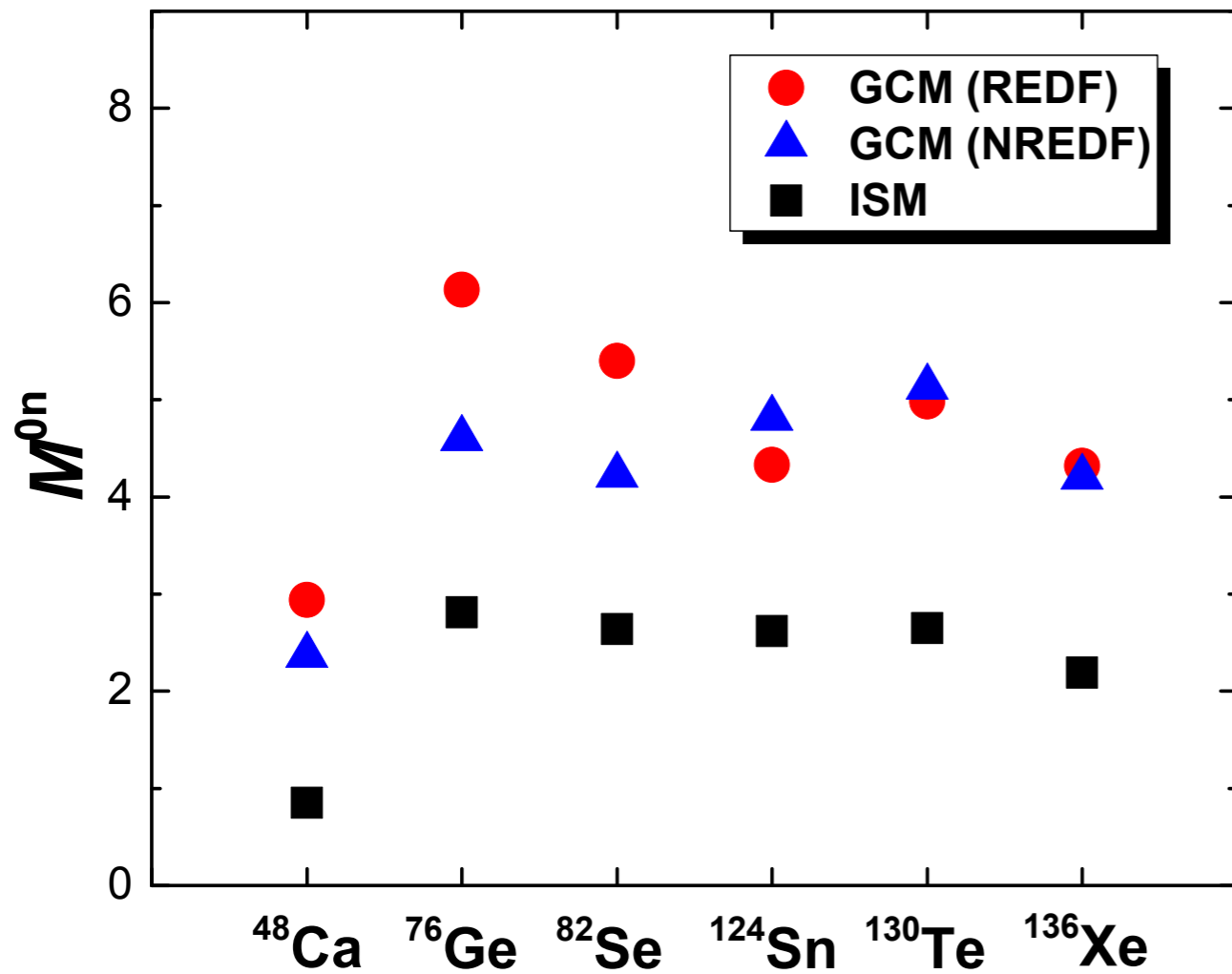
Construct a set of mean-field states by constraining coordinates, e.g., quadrupole moment. Then diagonalize Hamiltonian in space of symmetry-restored nonorthogonal vacua with different amounts of quadrupole deformation.

GCM based on EDF has been applied to double-beta decay, however...

Comparison between GCM and SM

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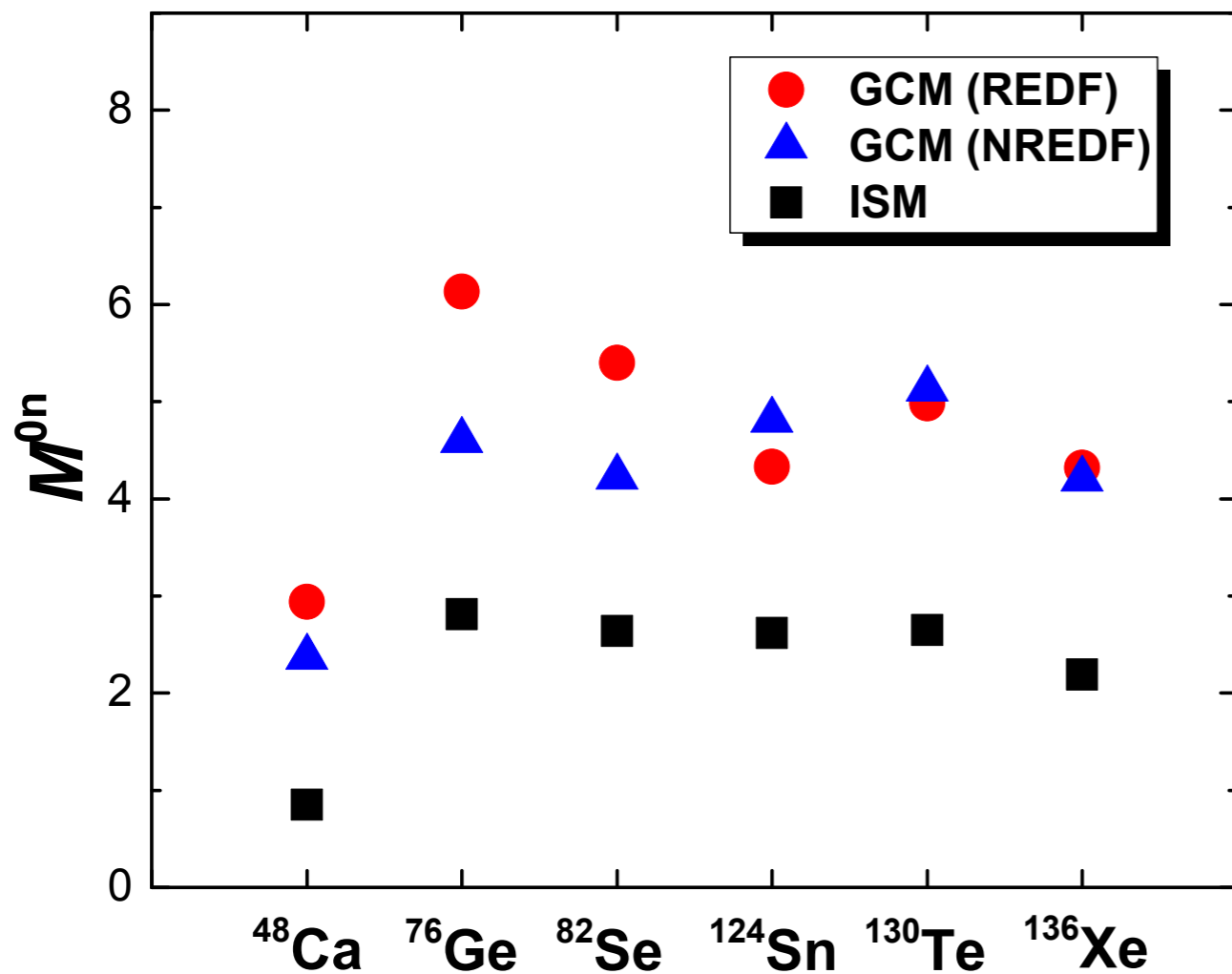
Current results with EDF-based GCM



Comparison between GCM and SM

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Current results with EDF-based GCM



Both the shell model and the EDF-based GCM could be missing important physics.

The discrepancy may be because:

- The GCM omits correlations.
- The shell model omits many single-particle levels

Our long-term goal is to combine the virtues of both frameworks through an EDF-based or *ab-initio* GCM that includes all the important shell model correlations and a large single-particle space.

To get closer to the ultimate goal:

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We can use SM Hamiltonian in the GCM

Our short-term goal is more modest: a shell-model Hamiltonian-based GCM in one and two (and possibly more) shells.

At a minimum, we can use these as a first step in the MR-IMSRG (see J. M. Yao's talk).

Our Current Procedure

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① Using a shell-model Hamiltonian

② HFB states $|\Phi(q)\rangle$ with multipole constraints q .

We are trying to include all possible collective correlations.

③ Angular momentum and particle number projection

$$|JMK; NZ; q\rangle = \hat{P}_{MK}^J \hat{P}^N \hat{P}^Z |\Phi(q)\rangle$$

④ Configuration mixing within GCM:

$$|\Psi_{NZ\sigma}^J\rangle = \sum_{K,q} f_{\sigma}^{JK}(q) |JMK; NZ; q\rangle$$

$$\sum_{K',q'} \{ \mathcal{H}_{KK'}^J(q; q') - E_{\sigma}^J \mathcal{N}_{KK'}^J(q; q') \} f_{\sigma}^{JK'}(q') = 0 \quad \longrightarrow \quad f_{\sigma}^{JK}(q)$$

$$M_{\xi}^{0\nu\beta\beta} = \langle \Psi_{N_f Z_f}^{J=0} | \hat{O}_{\xi}^{0\nu\beta\beta} | \Psi_{N_i Z_i}^{J=0} \rangle$$

Level 1 GCM: Axial shape and pn pairing fluctuation

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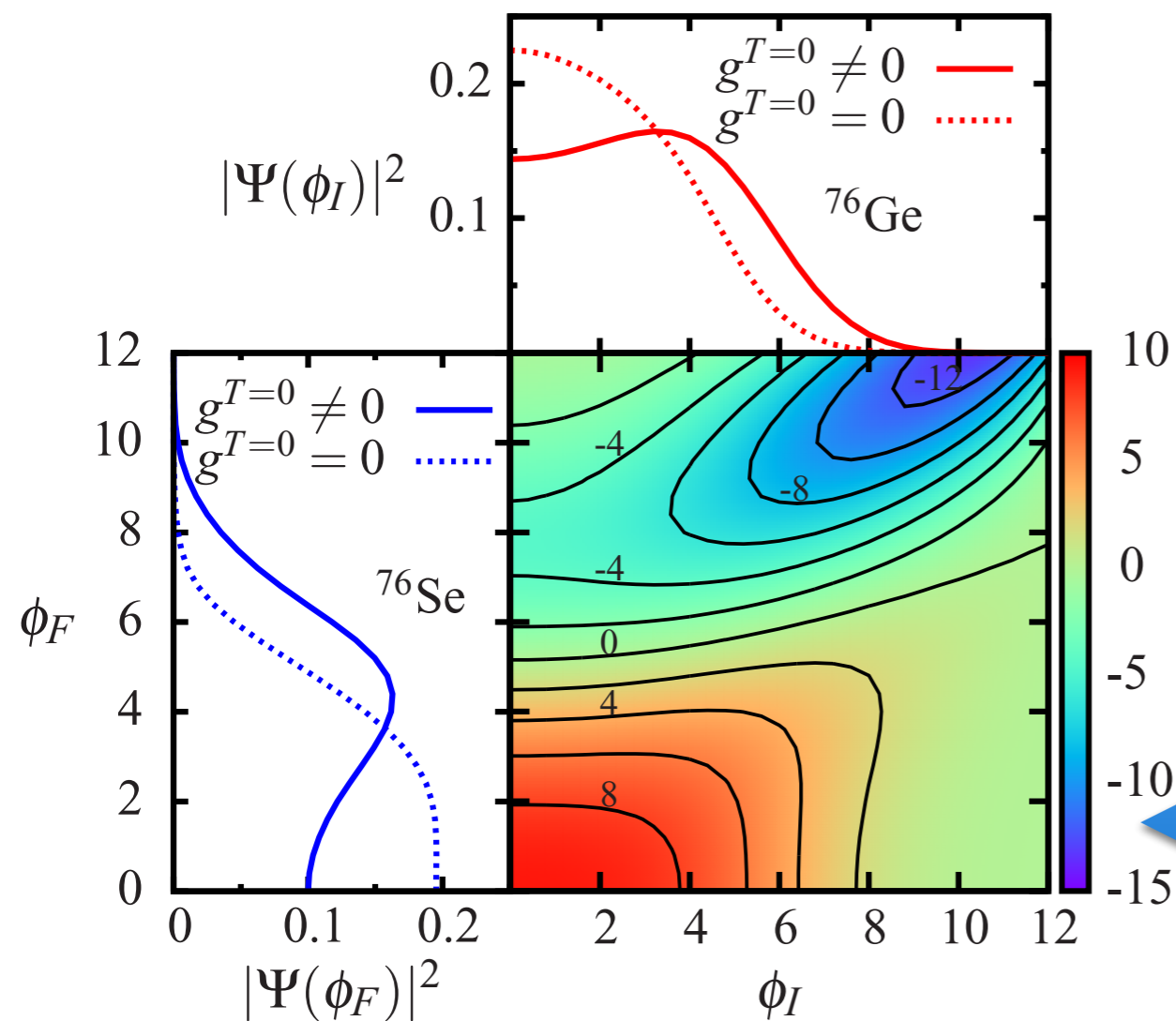
$$H' = H - \lambda_Z N_Z - \lambda_N N_N - \lambda_0 Q_{20} - \frac{\lambda_P}{2} (P_0 + P_0^\dagger)$$

isoscalar pn pairing constrained

ϕ is the isoscalar pairing amplitude

$$\phi = \langle P_0 + P_0^\dagger \rangle / 2$$

$$P_0^\dagger = \frac{1}{\sqrt{2}} \sum_l \hat{l} [c_l^\dagger c_l^\dagger]_{M_S=0}^{L=0, S=1, T=0}$$



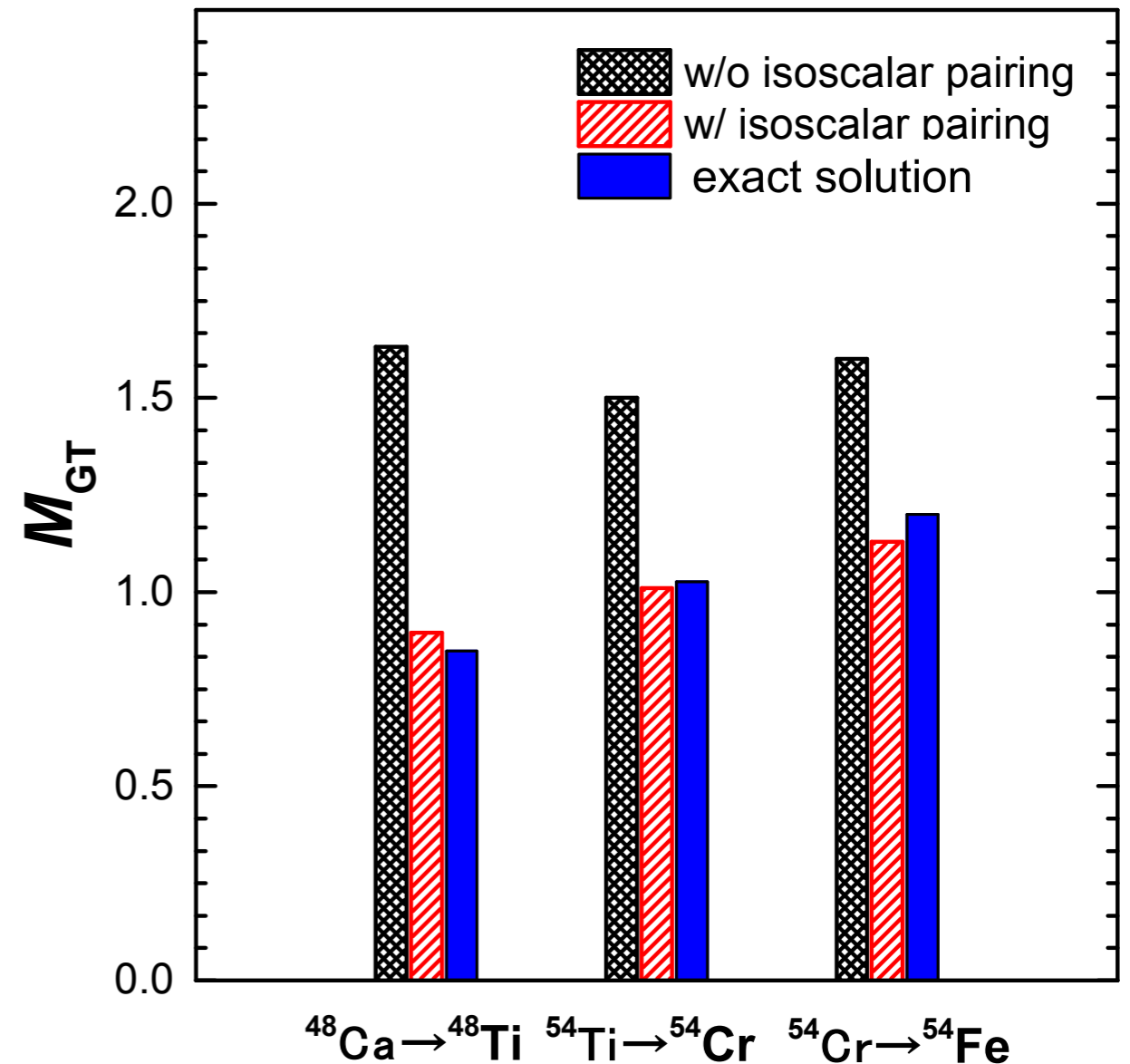
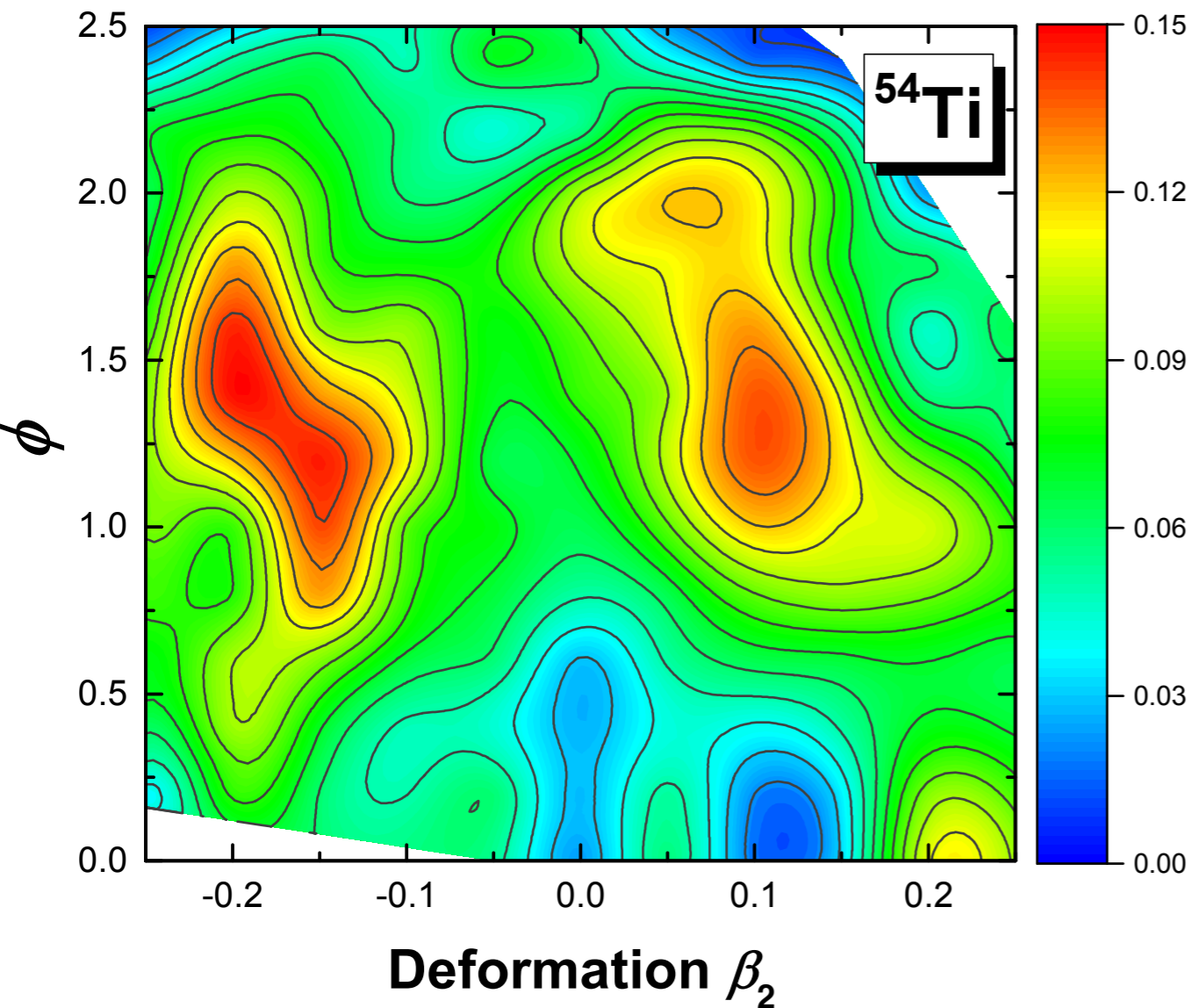
The wave functions are pushed into a region with large isoscalar pairing amplitude.

→ **reduce the $0\nu\beta\beta$ NMEs.**

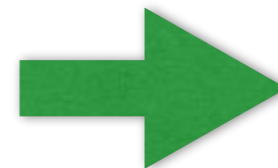
Level 1 GCM: Axial shape and pn pairing fluctuation

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We use the KB3G interaction for *the pf* shell



collective wave function shows peaks at nonzero isoscalar-pairing amplitude.



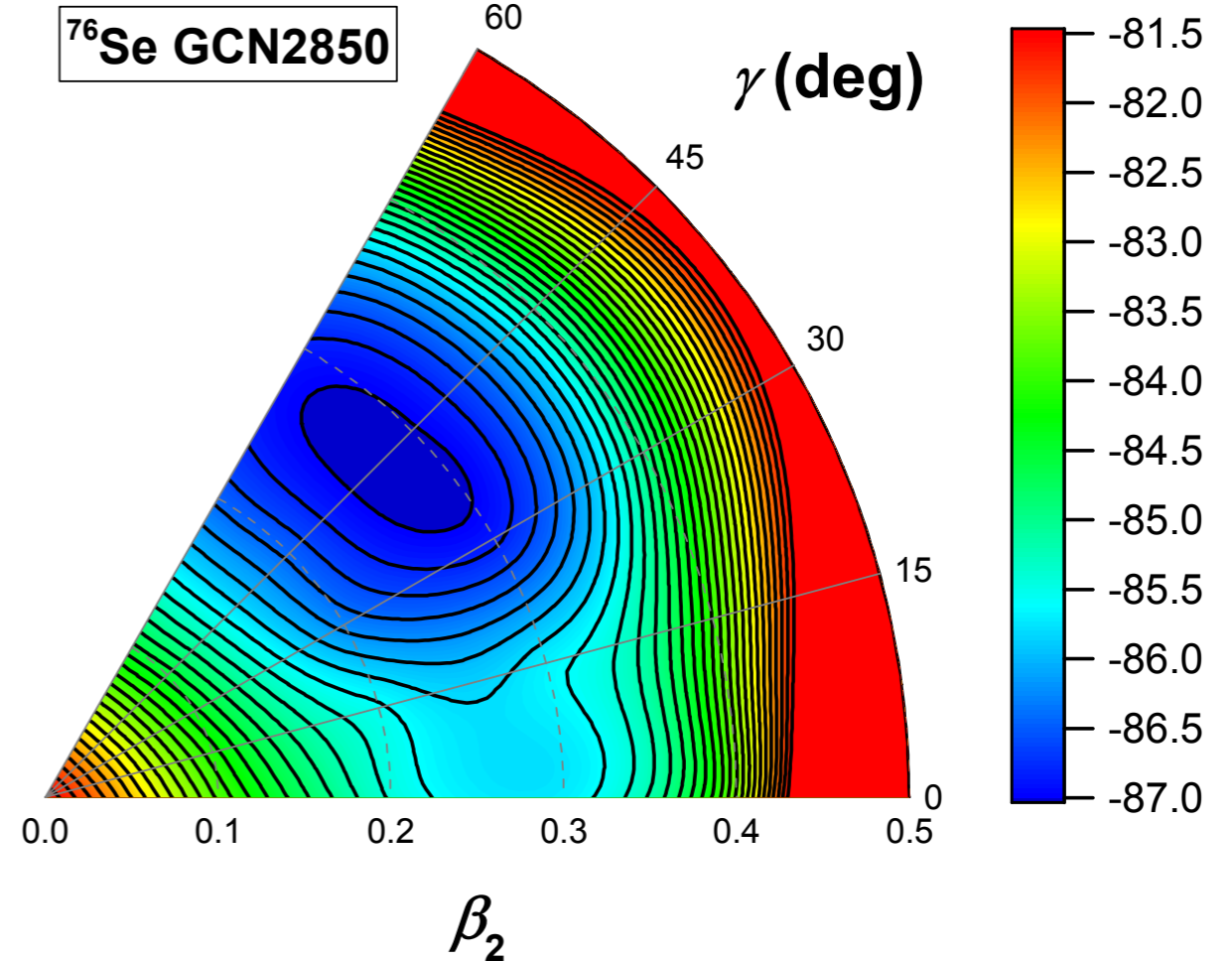
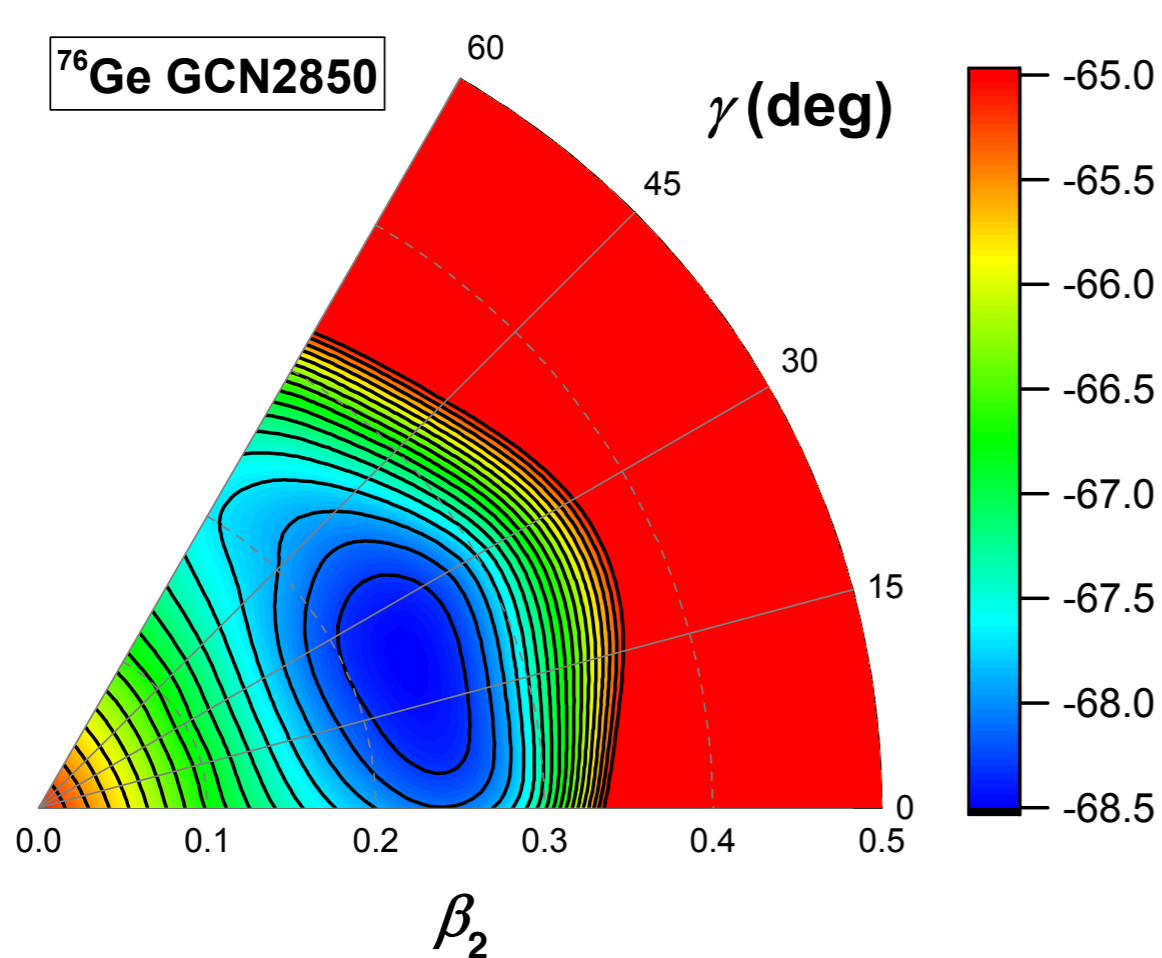
Reduction of NME

Level 2 GCM: Triaxial deformation

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$$H' = H - \lambda_Z N_Z - \lambda_N N_N - \lambda_0 Q_{20} - \frac{\lambda_P}{2} (P_0 + P_0^\dagger) - \lambda_2 Q_{22}$$

triaxial deformation constrained



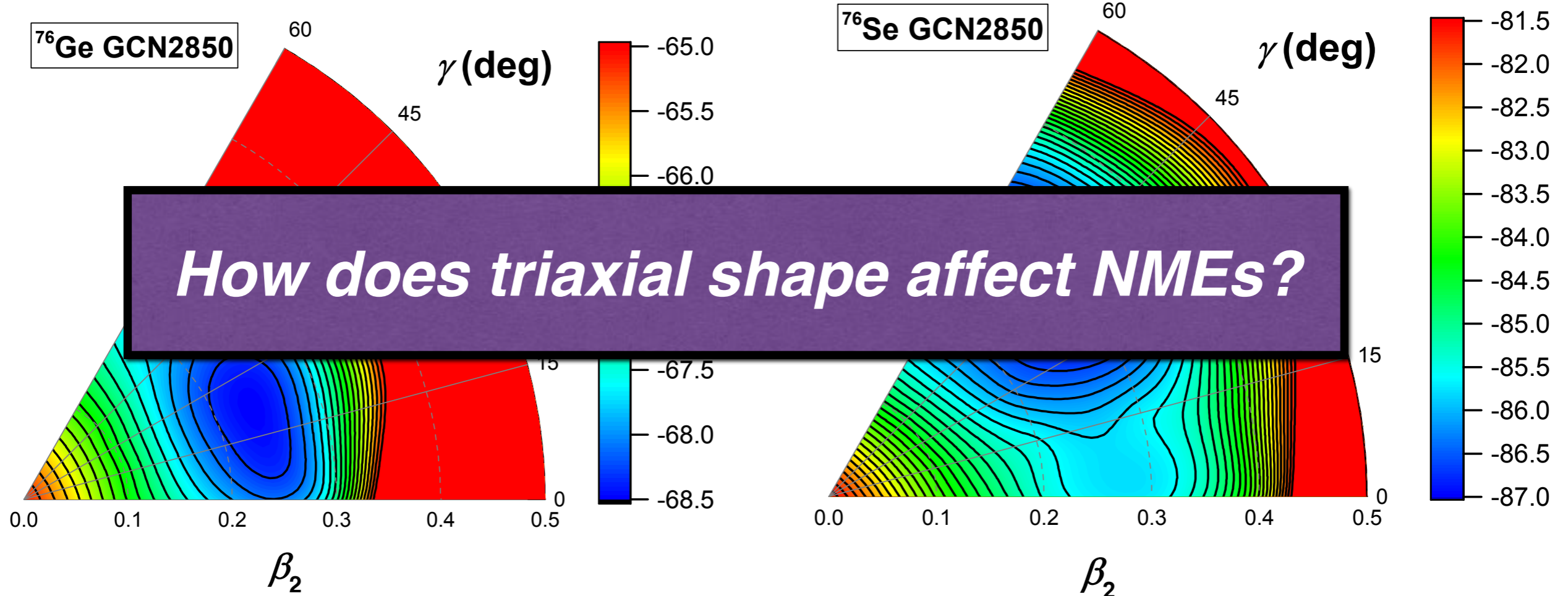
With GCN2850 or JUN45 interaction, projected potential energy surfaces for ⁷⁶Ge and ⁷⁶Se give minima with triaxial deformation.

Level 2 GCM: Triaxial deformation

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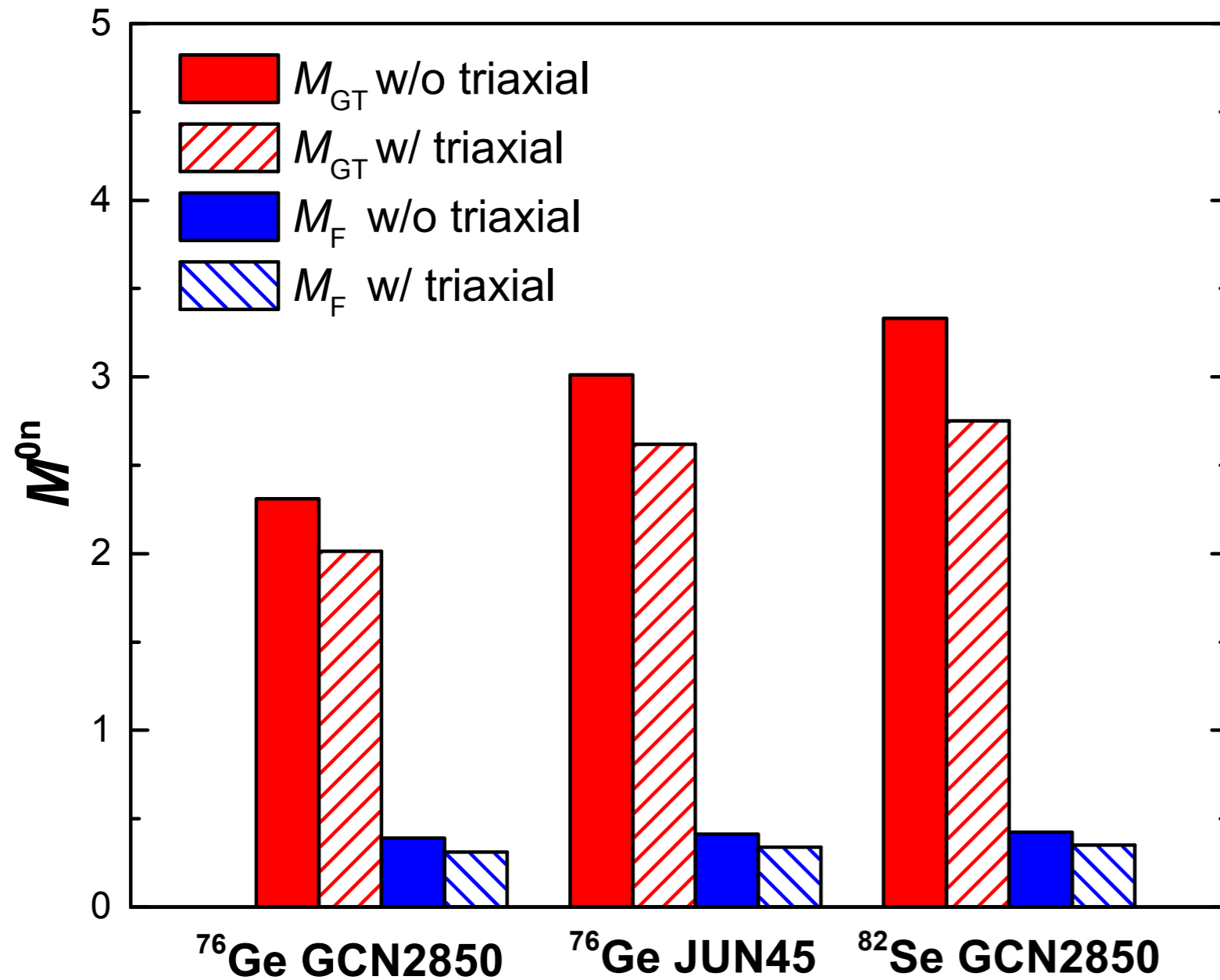
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Level 2 GCM: triaxial deformation

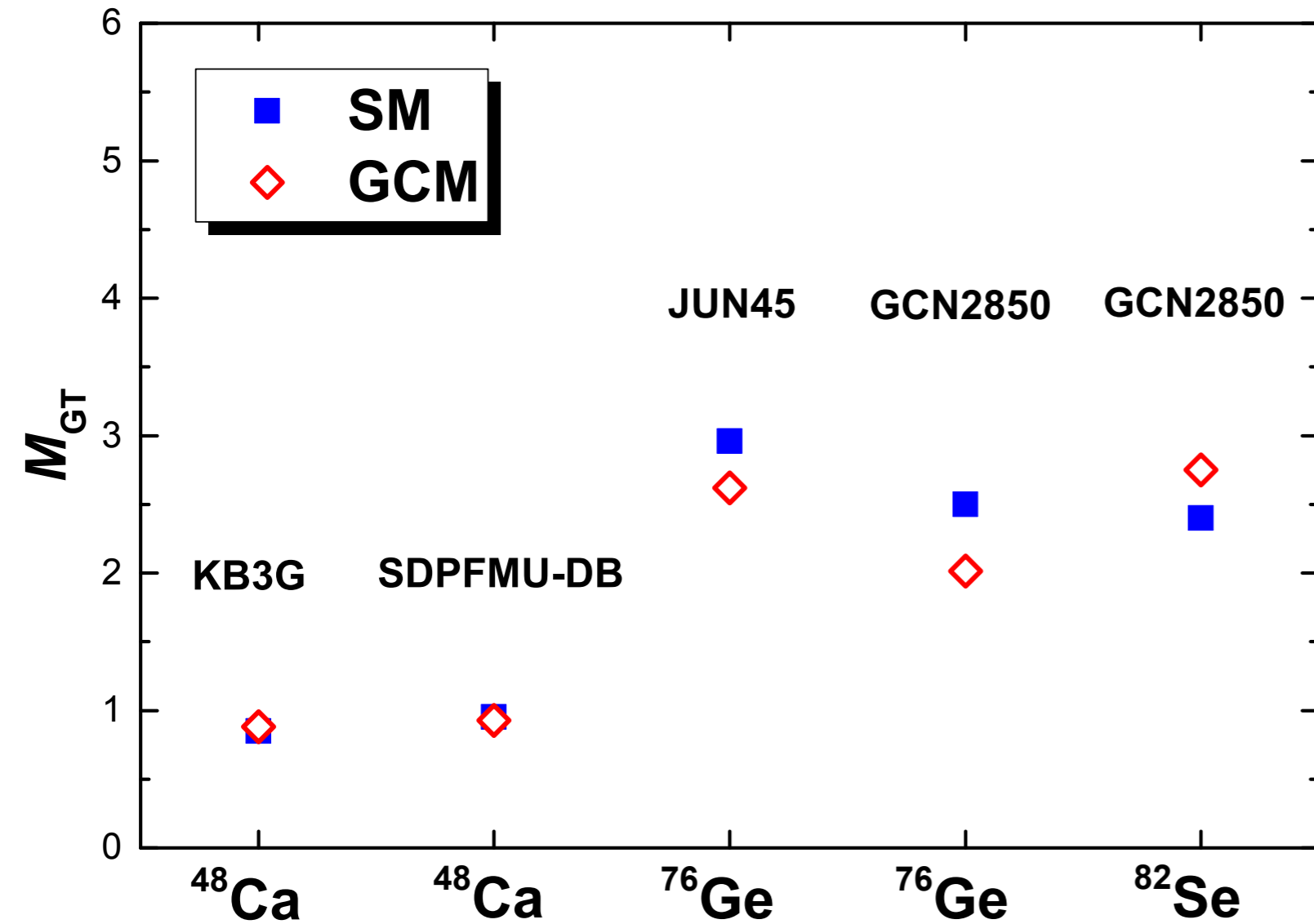
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15%~20% reduction for both GT and Fermi part of NME if triaxial shape fluctuation is included.

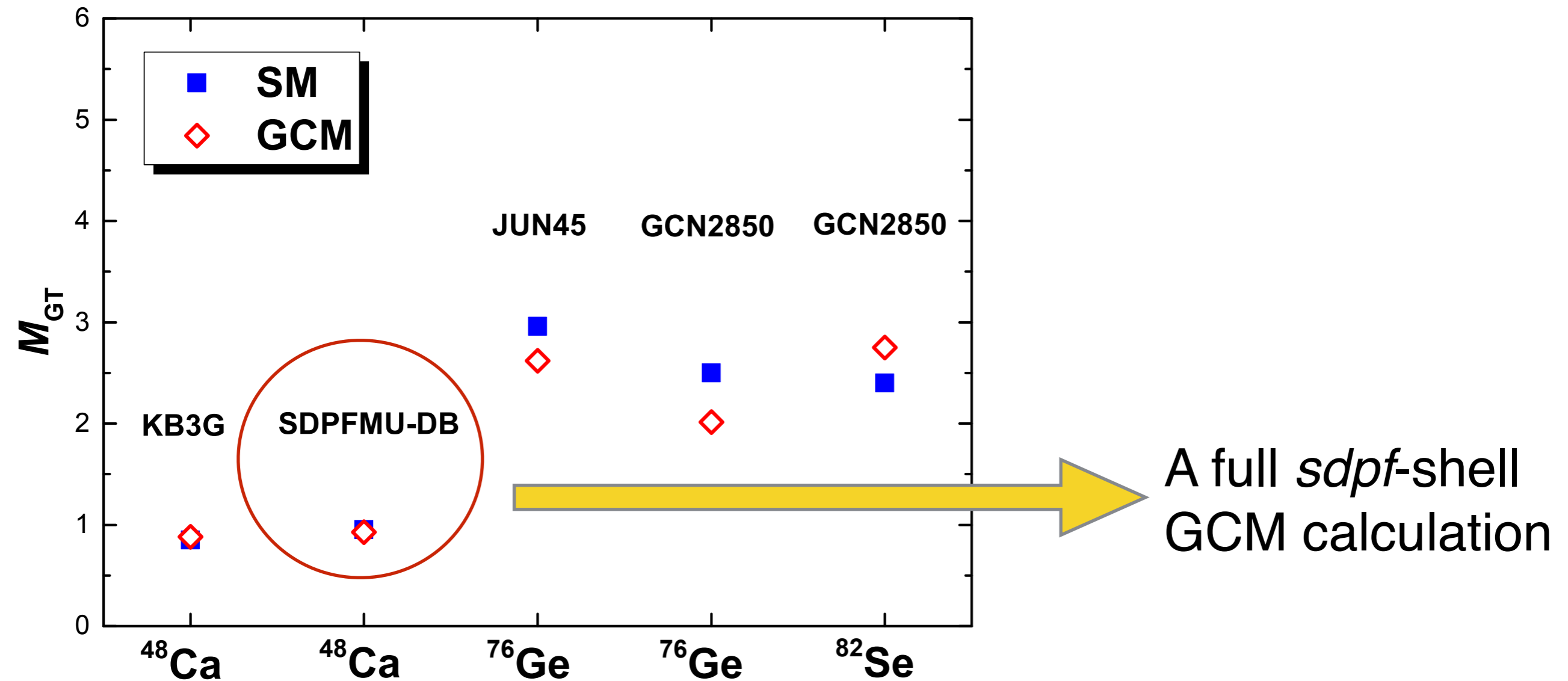
Benchmarking: $0\nu\beta\beta$ NMEs given by GCM and SM

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Benchmarking: $0\nu\beta\beta$ NMEs given by GCM and SM

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The NMEs given by SM and GCM are in good agreement, indicating that **the GCM captures most important valence-shell correlations.**

Multi-shell GCM

1. GCM 2. Correlations 3. Multi-shell GCM 4. Summary

- In principle, effective *pf*sdg-shell interaction based on chiral EFT can be calculated by many-body perturbation theory (MBPT), similarity renormalization group (SRG) or couple cluster (CC).
- We employ two effective *pf*sdg-shell interactions calculated by MBPT, which are provided by J. D. Holt.

***pf*sdg-1**: 3N forces normal ordered with respect to ^{40}Ca

***pf*sdg-2**: 3N forces normal ordered with respect to ^{56}Ni

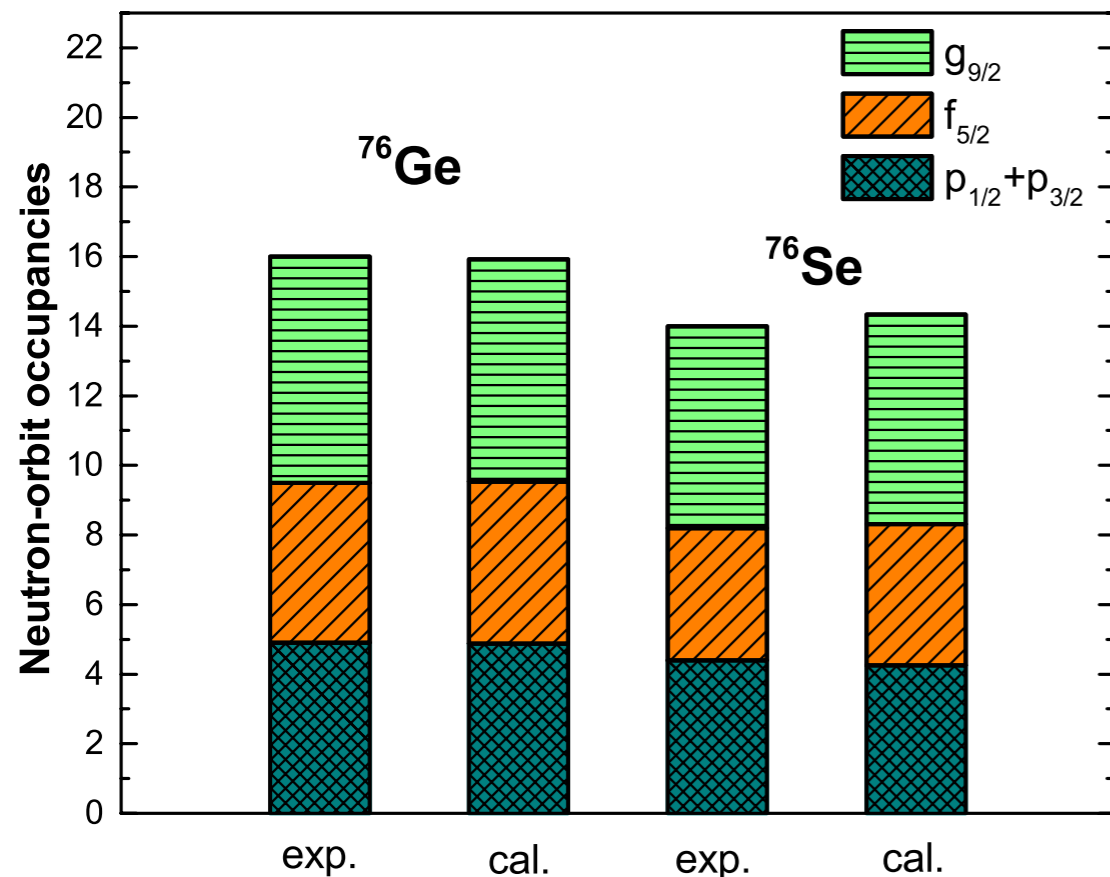
Computing Usage:

- Our calculation within *pf*5g9 shell used about 15K CPU hours, including axial shape, triaxial shape, and isoscalar pairing as coordinates.
- Extension to *pf*sdg shell will increase time by a factor of 25, because of the increased number of orbits.

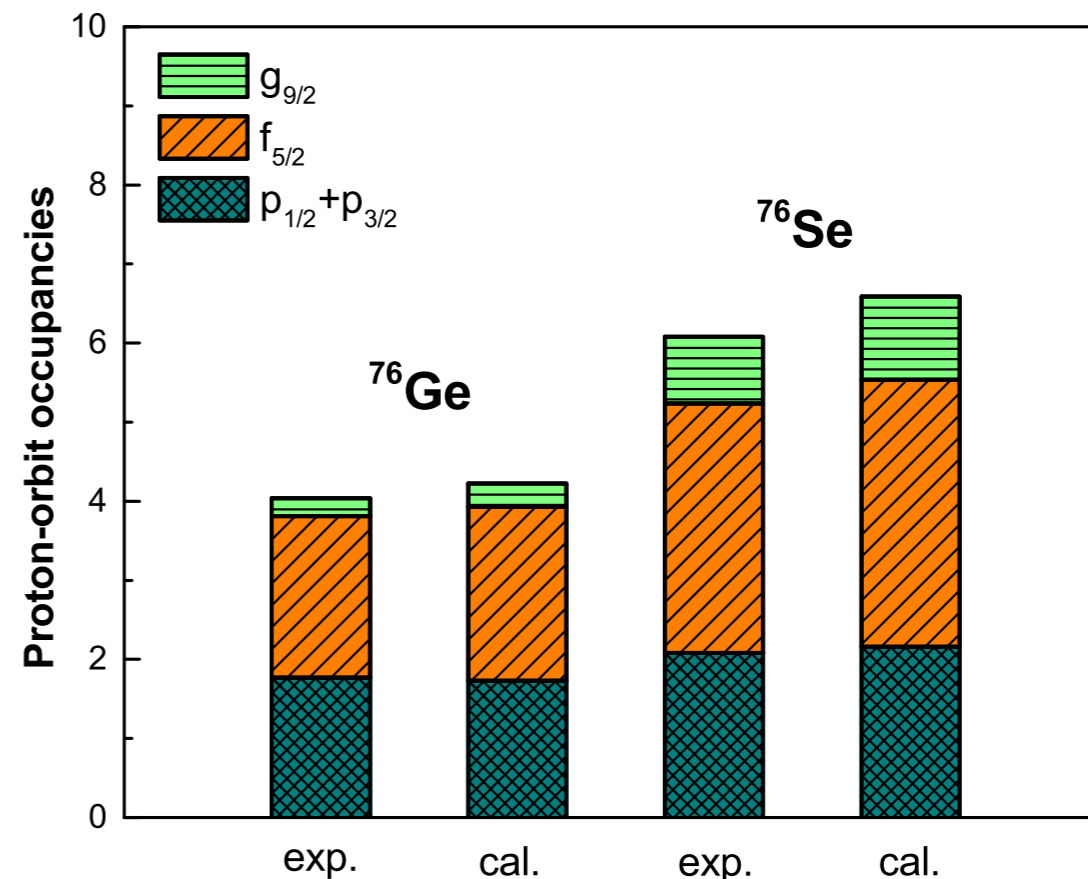
Multi-shell GCM: SPEs optimization

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Neutron-orbit occupancies



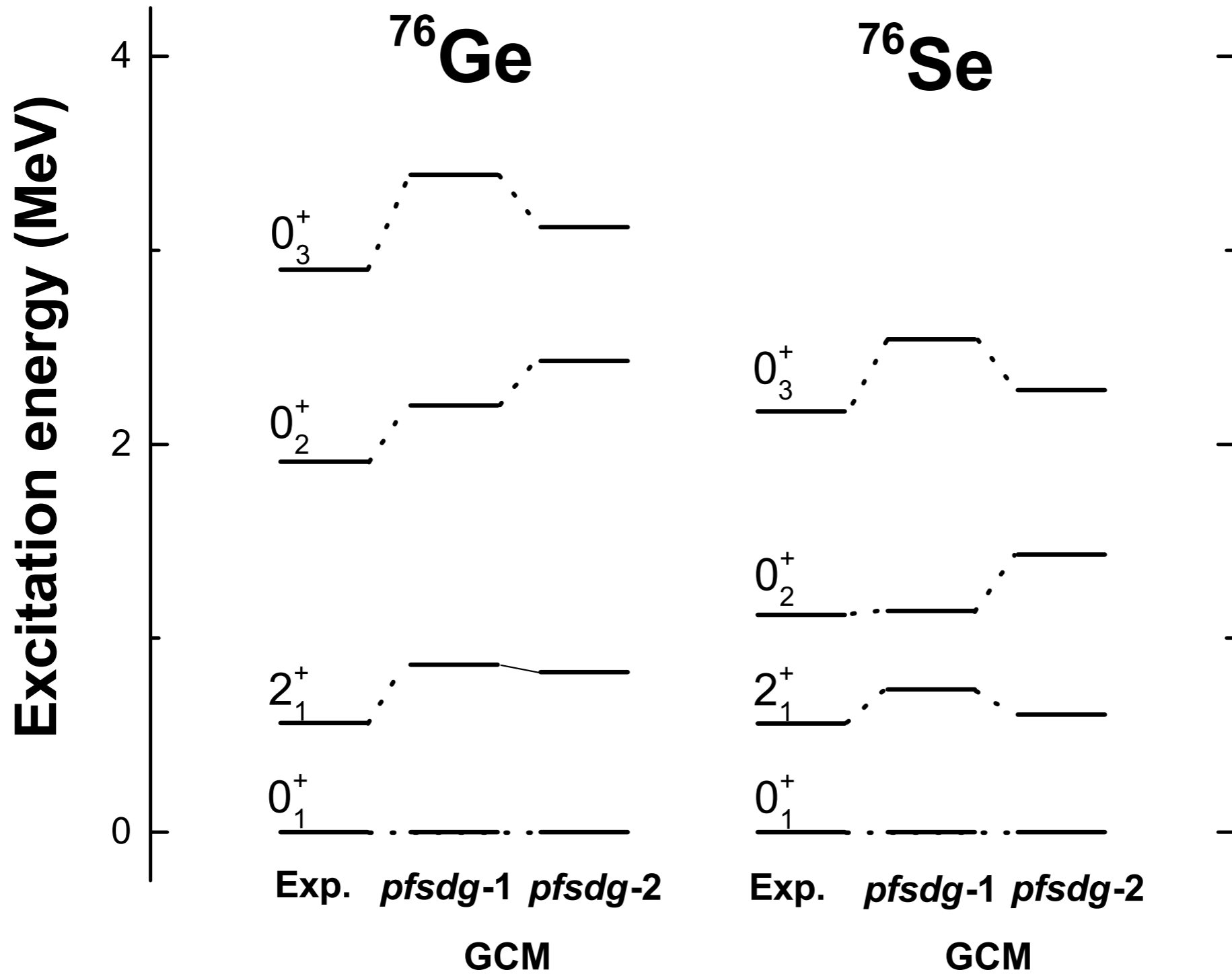
Proton-orbit occupancies



We optimize the single-particle energies for *pfsg*-shell interactions by fitting the measured occupancies of valence neutron and proton orbits.

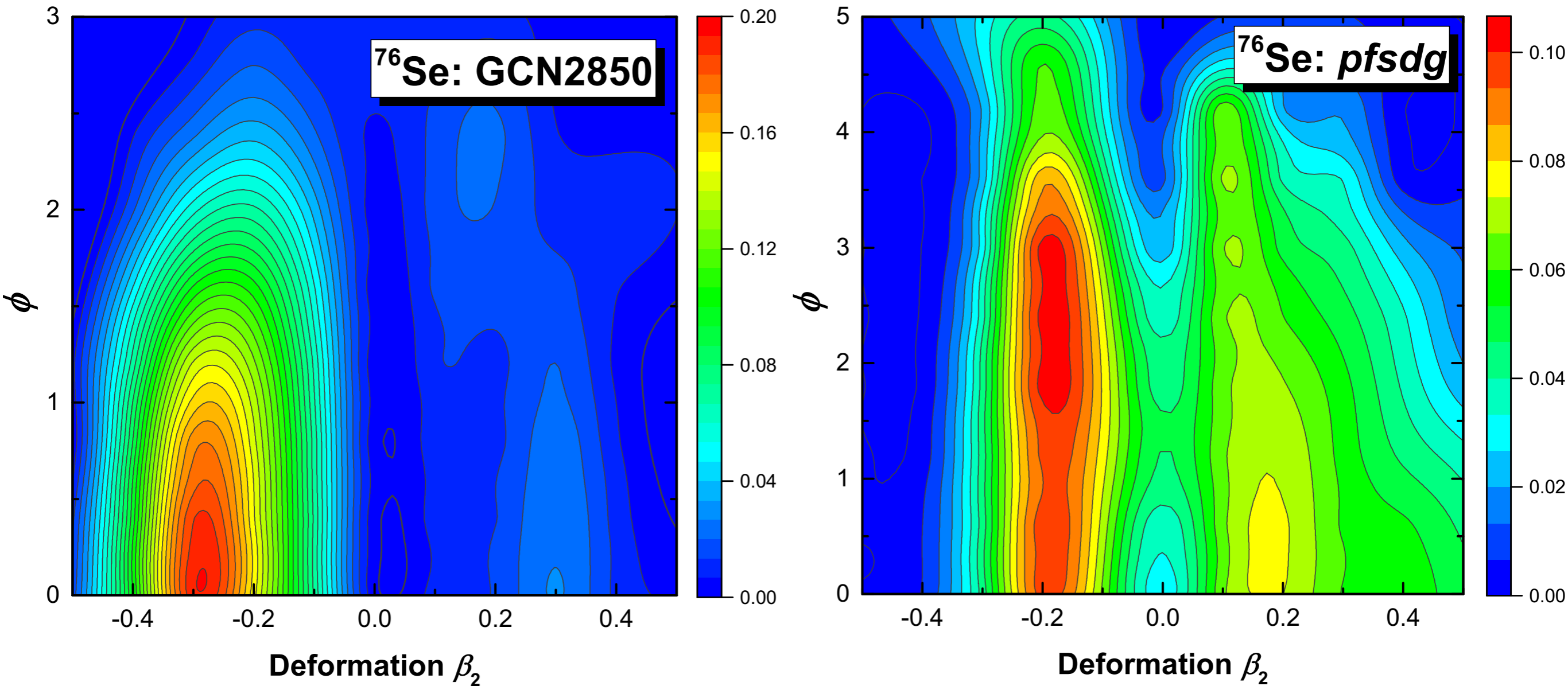
Multi-shell GCM: low-lying spectra

1. GCM 2. Correlations 3. Multi-shell GCM 4. Summary



Multi-shell GCM: collective wave function

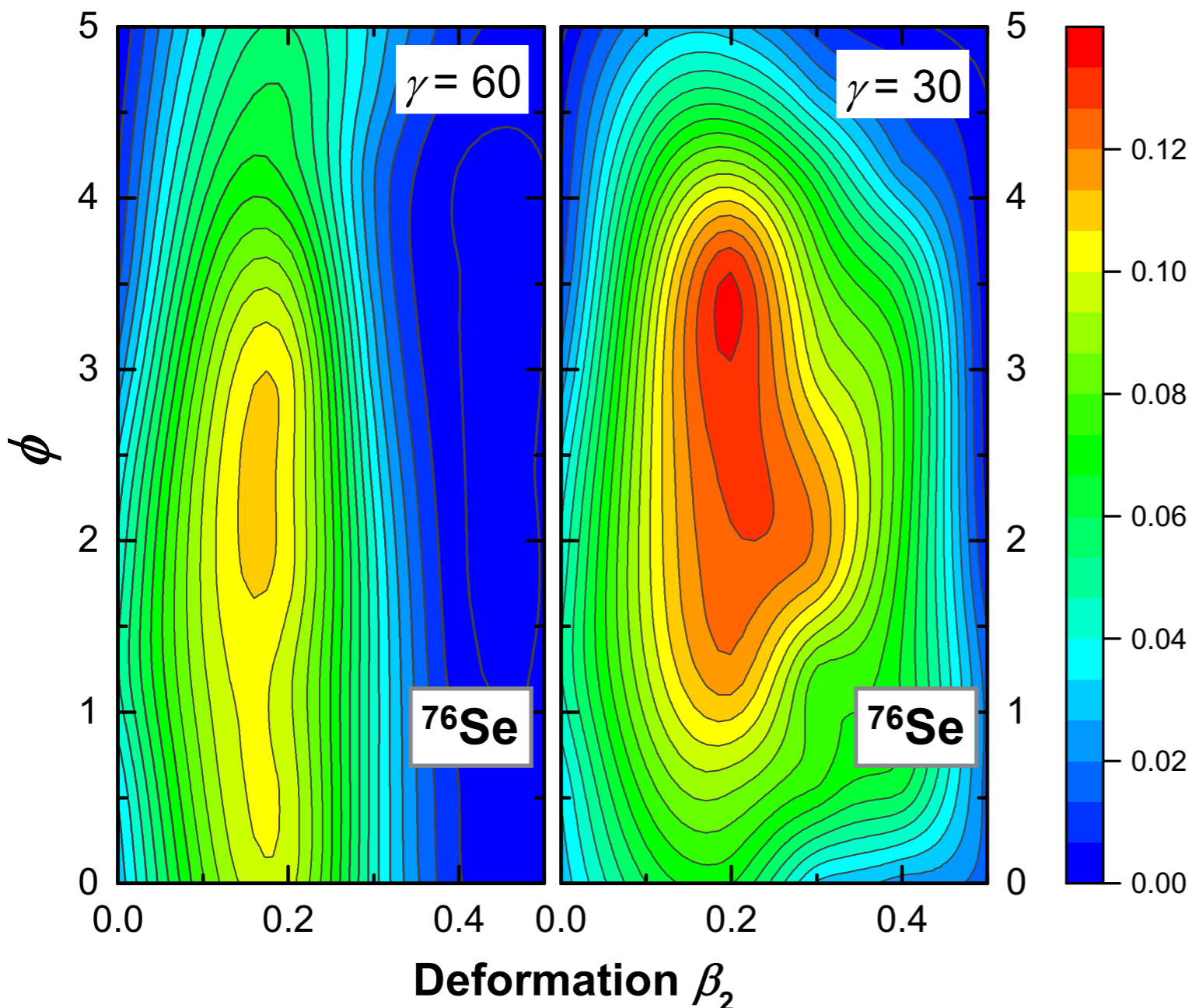
1. GCM 2. Correlations 3. Multi-shell GCM 4. Summary



- Larger model space: larger isoscalar pairing in *pfsdg*-shell calculation
- How does triaxial shape influence NMEs?

Multi-shell GCM: triaxial deformation

1. GCM 2. Correlations 3. Multi-shell GCM 4. Summary

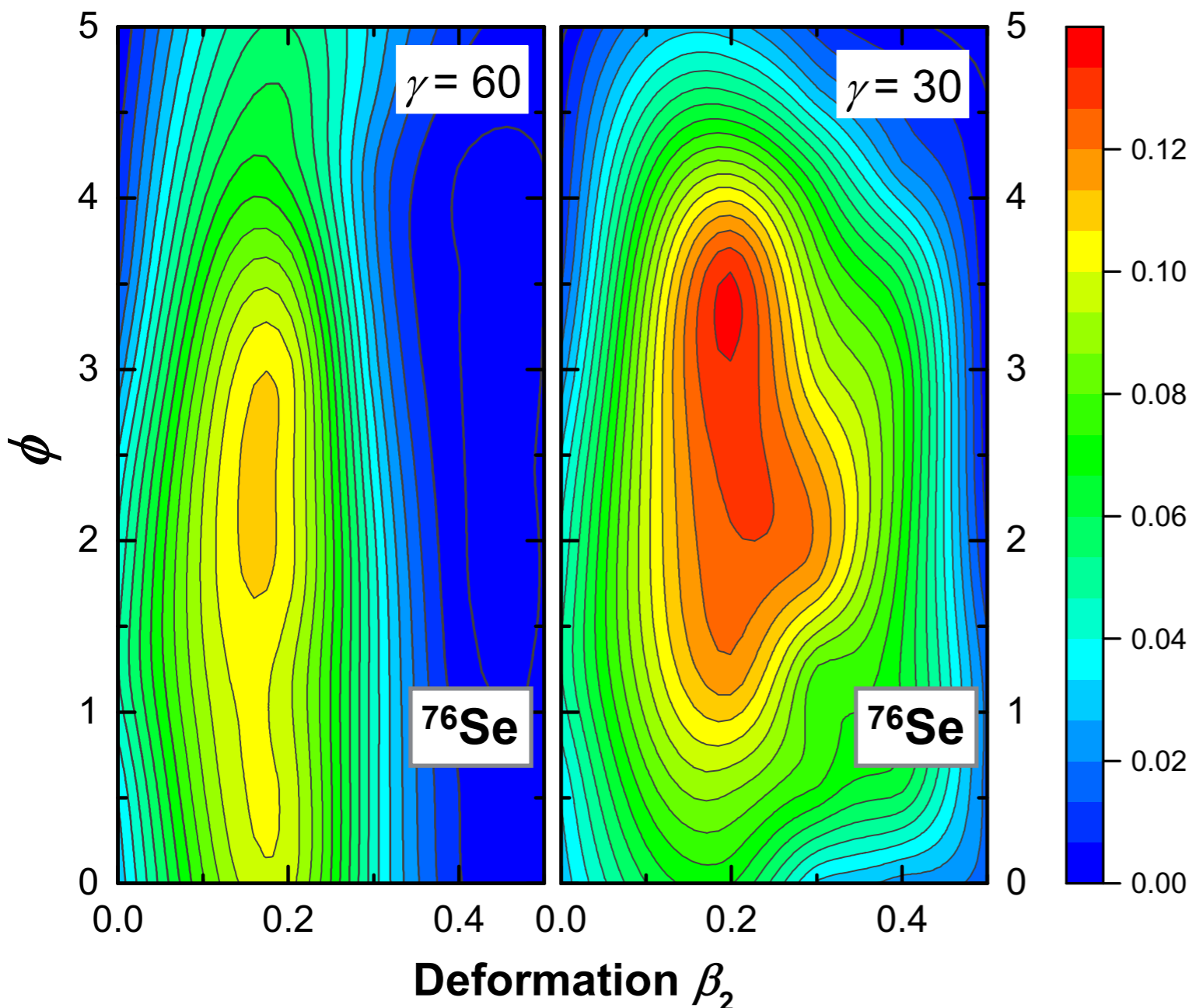


With triaxially deformed configurations, the wave functions:

- ① are pushed to the region with larger isoscalar pn pairing.
- ② spread widely to the region with larger deformation

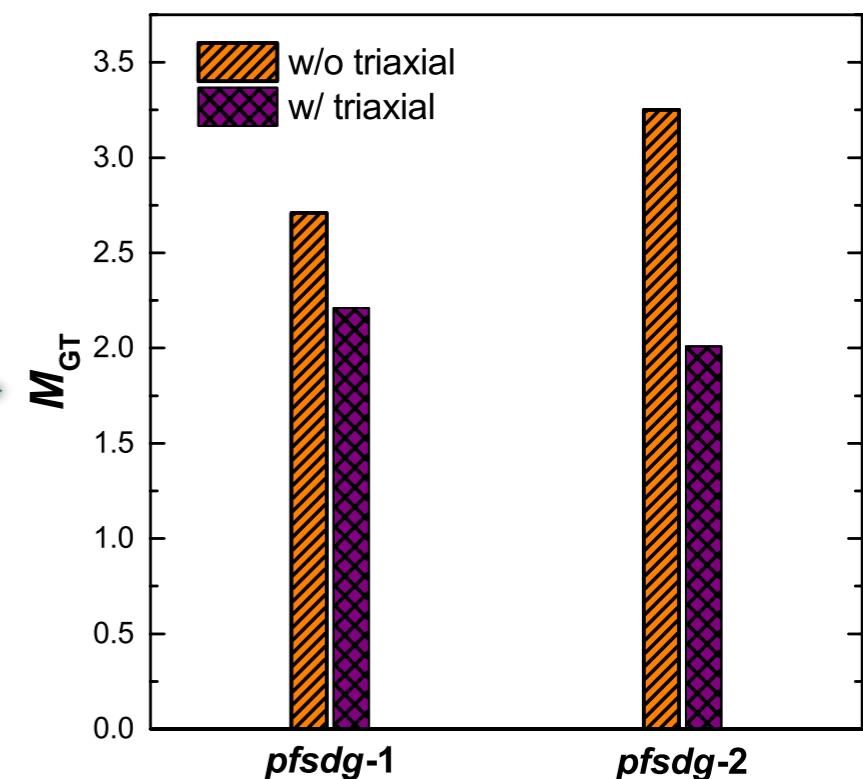
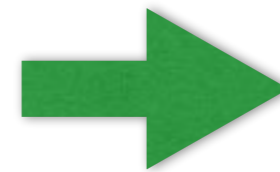
Multi-shell GCM: triaxial deformation

1. GCM 2. Correlations 3. Multi-shell GCM 4. Summary



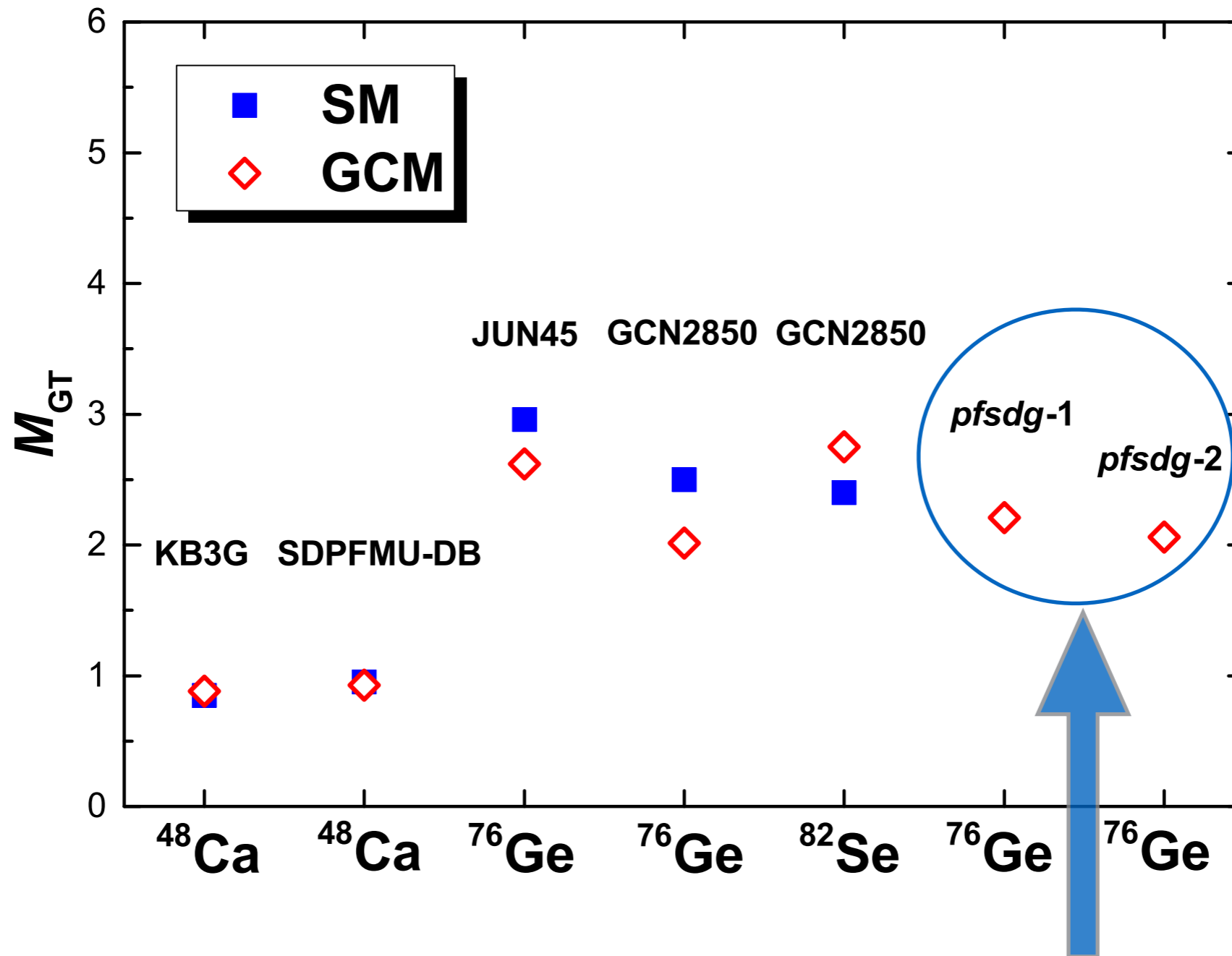
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Multi-shell GCM

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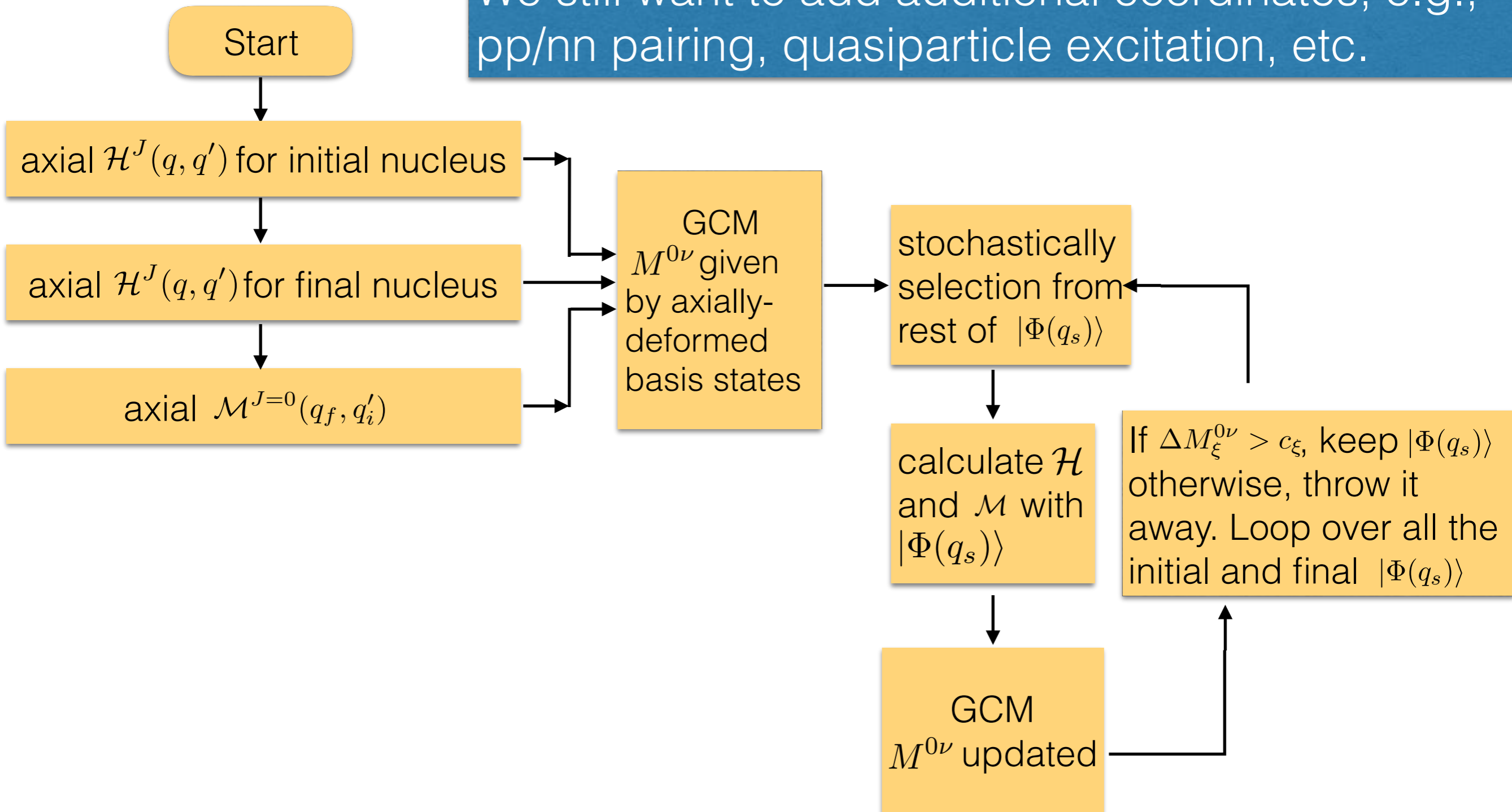


full *pf*sdg-shell GCM calculations

A relatively simple strategy for stochastic basis selection

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We still want to add additional coordinates, e.g., pp/nn pairing, quasiparticle excitation, etc.



Summary

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- We are trying to combine the virtues of the shell model and EDF calculations by including all collective correlations in the GCM.
- Tests against exact solutions in one shell indicate that we indeed have all important valence-space correlations.
- Calculation has been extended to two major shell (e.g., *pf**sdg* shell) model space, which is out of scope of the conventional SM. Including triaxially deformed configurations significantly affect the calculated NMEs.
- To speed up the two-shell calculation, stochastic selection of basis states is under construction, and we are looking for more efficient methods.

Summary

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Collaborators:

- Jonathan Engel, UNC
- Jiangming Yao, UNC
- Mihai Horoi, CMU
- Jason Holt, TRIUMF
- Javier Menendez, University of Tokyo
- Nobuo Hinohara, University of Tsukuba

**Thank you for your
attention!**