P-Conserving, C- and T-violation in Effective Field Theory

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Time-reversal Tests in Nuclear and Hadronic Processes workshop, ACFI,
7 November 2014

Outline

1. Some Background
2. Effective PCTV Operators
3. Numerical estimations of the dim-5 contribution to T-violating experimental observables
4. Conclusion

1. Some Background

Andrei Sakharov

I need:

- Baryon number violation
- C and CP-violation
- Interaction out of thermal equilibrium

Assuming CPT:
T-violation → CP-violation

CP-violation → **C-conservation, P-violation**

OR

C-violation, P-conservation

- Standard Model T-violation only comes from the complex phase in the CKM-matrix.
- Characterized by Jarlskog invariant:

  $J \approx 3E\text{-}5$. $\therefore$ SM background is small!
- C-even P-odd: tightly constrained by EDM searches.
- C-odd P-even: less constrained by experiments. One reason: particles/ particle systems with definite C-parity are few and difficult to prepare in large quantities.

Limits on C-even, P-odd observables: EDM

Particle	Current Upper Bound on EDM (e cm)
Electron	8.7E-29
Mercury	3.1E-29
Proton	7.9E-25
Neutron	2.9E-26

And a lot more!

Limits on C-odd observables: neutral ϕ -decays

C-Odd Decay Channel	Decay Width (eV)
$\pi \rightarrow 3\gamma$	< 2.4E-7
$\eta \rightarrow \pi^0 \gamma$	< 1E-1
$\eta \rightarrow 2\pi^0 \gamma$	< 7E-1
$\eta \rightarrow 3\pi^0 \gamma$	< 8E-2
$\eta \rightarrow 3\gamma$	< 2.1E-2
$\eta \rightarrow \pi^0 e^+e^-$	< 5E-2
$\eta \rightarrow \pi^0 \mu^+\mu^-$	< 7E-3

($C=(-1)^{l+s}$ for L^+L^- system)

(PDG 2014)

- Another C-oven, P-even observable: the "D-coefficient" in the polarized neutron β -decay

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = \frac{(G_F V_{ud})^2}{(2\pi)^5}(1+3g_A^2)\,|\,\vec{p}_e\,|\,E_e E_\nu^2 (1 + a\frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + \hat{s}_n \cdot (A\frac{\vec{p}_e}{E_e} + B\frac{\vec{p}_\nu}{E_\nu} + D\frac{\vec{p}_e \times \vec{p}_\nu}{E_e E_\nu}))$$

Ando, McGovern and Sato, **Phys.Lett. B677 (2009) 109-115**

- Current experimental limit:

$$D = (-0.96 \pm 1.89 \pm 1.01) \times 10^{-4}$$

Mumm et al, **Phys.Rev.Lett. 107 (2011) 102301**

(strictly speaking the D-coefficient could be a function of the energies of the outgoing particles)

2. Effective PCTV operators

- EFT analysis: write down higher-order PCTV operators that consist of SM fields.
- PCTV interaction cannot arise via tree-level boson exchange in a renormalizable gauge theory (Herczeg, **Hyperfine Interact, 75, 127 (1992)**)
- Lowest-dimension **flavor-conserving** PCTV operators have dimension 7. (Conti and Khriplovich, **Phys.Rev.Lett. 68 (1992) 3262-3265**)

$$\mathcal{O}_7^{ff'} = C_7^{ff'} \bar{\psi}_f \overleftrightarrow{D}_\mu \gamma_5 \psi_f \bar{\psi}_{f'} \gamma^\mu \gamma_5 \psi_{f'} ,$$

$$\mathcal{O}_7^{\gamma g} = C_7^{\gamma g} \bar{\psi} \sigma_{\mu\nu} \lambda^a \psi F^{\mu\lambda} G_\lambda^{a\nu} ,$$

$$\mathcal{O}_7^{\gamma Z} = C_7^{\gamma Z} \bar{\psi} \sigma_{\mu\nu} \psi F^{\mu\lambda} Z_\lambda^\nu ,$$

Ramsey-Musolf, **Phys.Rev.Lett. 83 (1999) 3997-4000**

Direct vs Indirect Probe

- Direct probe: Look for PCTV-observables. E.g. a PCTV 4-quark operator:

$$\hat{O}_{4q} = i\bar{q}_1\gamma_5\sigma^{\mu\nu}(\overleftarrow{D}_\nu + \overrightarrow{D}_\nu)q_1\bar{q}_2\gamma_\mu\gamma_5 q_2$$

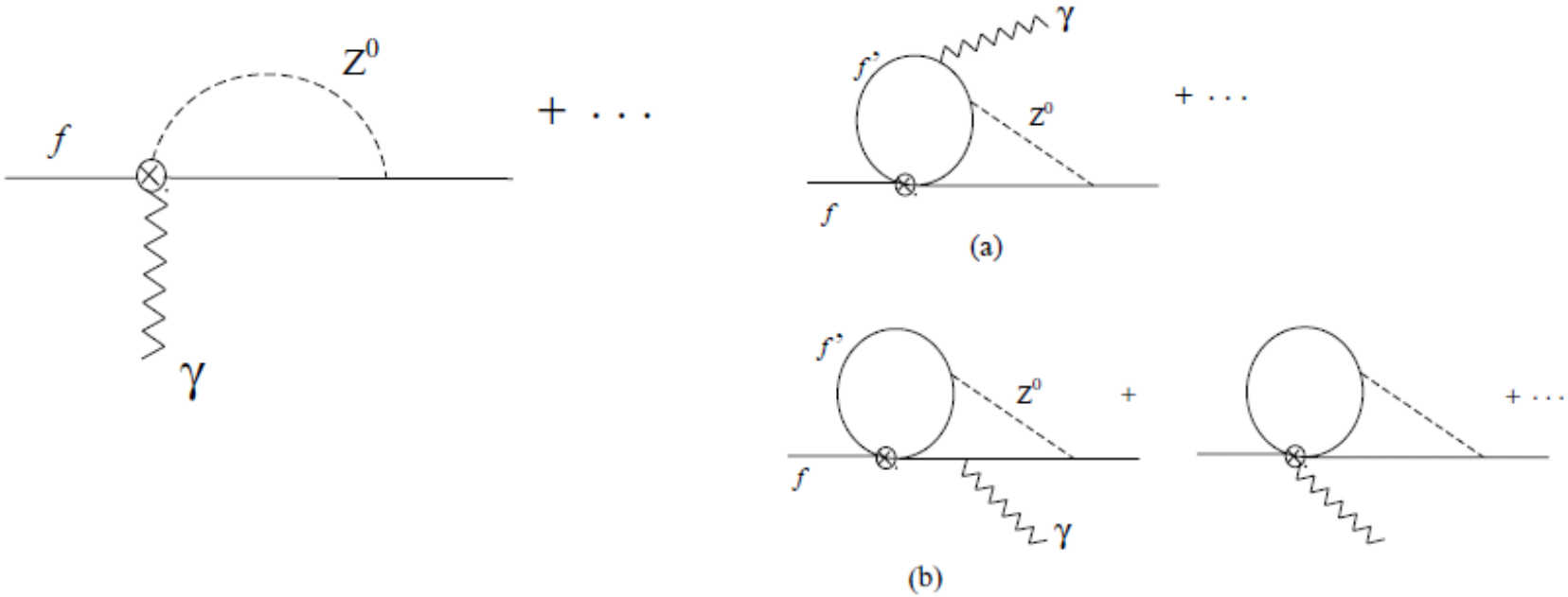
could directly generate a PCTV nucleon-nucleon interaction, or generate a long-range ρ NN-operator:

$$\hat{O}_{\rho NN} = i\bar{N}\sigma^{\mu\nu}(\tau^-\partial_\nu\rho_\mu^+ - \tau^+\partial_\nu\rho_\mu^-)N$$

which then generate a PCTV nucleon-nucleon interaction.

(Question: how about operators like: $\bar{p}\gamma_5 n\pi^+ - \bar{n}\gamma_5 p\pi^-$?)

- Indirect probe: the PCTV operator can generate PVTV observables (such as EDMs) via electroweak loop-corrections.



Ramsey-Musolf, **Phys.Rev.Lett. 83 (1999) 3997-4000**

Indirect probes usually set more stringent bounds because PVTV observables are more constrained experimentally.

- How about flavor non-diagonal operators?

Consider a “pseudo-Chern-Simons” (pCS) type of interaction coming from gauging the axial anomaly of QCD under $SU(2)_L X U(1)_Y$:

$$\hat{O} \sim \bar{p}\gamma_\mu n \bar{e}_L \gamma_\nu \upsilon_L \tilde{F}^{\mu\nu}$$

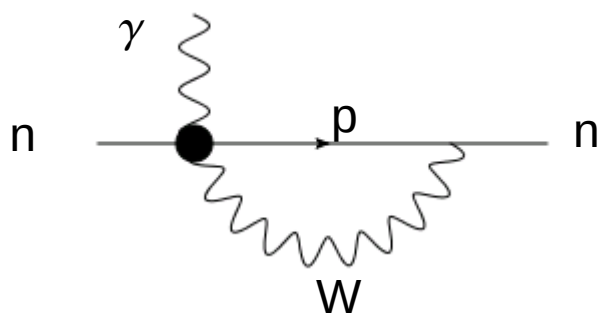
(S. Gardner, *Hadronic Probes of Fundamental Symmetries* workshop, 2014); Harvey, Hill and Hill, **hep-ph/0708.1281v2**

Before integrating out the W-boson, it looks like:

$$\hat{O} \sim \bar{p}\gamma_\mu n W_\nu \tilde{F}^{\mu\nu}$$

(gauge invariance is ensured by some complicated anomaly analysis)

- We think it is subject to EDM constraints as well via loop corrections like:



Unless there’s some peculiar cancelation between diagrams.

Is there any interesting operator in lower dimension?

- We can write down a dim-5, flavor non-conserving PCTV operator:

$$\boxed{\mathcal{L}_{\mathrm{PCTV}} = iv\frac{c}{\Lambda_{TV}^2}(\bar{u}\sigma^{\mu\nu}dW_{\mu\nu}^{+} - \bar{d}\sigma^{\mu\nu}uW_{\mu\nu}^{-})}$$

which could arise from dim-6 T-violating operators before EWSB:

$$\hat{\vartheta}_1 = i(\bar{d}_R\sigma^{\mu\nu}H^{+}Q_L - \bar{Q}_L H\sigma^{\mu\nu}d_R)B_{\mu\nu}$$

$$\hat{\vartheta}_2 = i(\bar{d}_R\sigma^{\mu\nu}H^{+}\frac{\tau^i}{2}Q_L - \bar{Q}_L\frac{\tau^i}{2}H\sigma^{\mu\nu}d_R)W_{\mu\nu}^{i}$$

$$\hat{\vartheta}_3 = i(\bar{u}_R\sigma^{\mu\nu}\tilde{H}^{+}Q_L - \bar{Q}_L\tilde{H}\sigma^{\mu\nu}u_R)B_{\mu\nu}$$

$$\hat{\vartheta}_4 = i(\bar{u}_R\sigma^{\mu\nu}\tilde{H}^{+}\frac{\tau^i}{2}Q_L - \bar{Q}_L\frac{\tau^i}{2}\tilde{H}\sigma^{\mu\nu}u_R)W_{\mu\nu}^{i}$$

Perform a set of linear combinations :

$$\hat{\vartheta}_1' = -\frac{\sqrt{2}}{c_w}\hat{\vartheta}_1$$

$$\hat{\vartheta}_2' = \frac{\sqrt{2}}{c_w}\hat{\vartheta}_3$$

$$\hat{\vartheta}_3' = t_w(\hat{\vartheta}_1 - \hat{\vartheta}_3) + 2(\hat{\vartheta}_2 + \hat{\vartheta}_4)$$

$$\hat{\vartheta}_4' = -t_w(\hat{\vartheta}_1 + \hat{\vartheta}_3) + 2(-\hat{\vartheta}_2 + \hat{\vartheta}_4)$$

After EWSB and neglecting Z and W^+W^- terms :

$$\hat{\vartheta}_1' \to vi\bar{d}\sigma^{\mu\nu}\gamma_5 dF_{\mu\nu} \qquad \text{(tree - level d - EDM)}$$

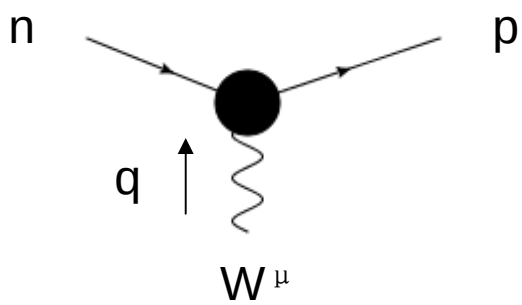
$$\hat{\vartheta}_2' \to vi\bar{u}\sigma^{\mu\nu}\gamma_5 uF_{\mu\nu} \qquad \text{(tree - level u - EDM)}$$

$$\hat{\vartheta}_3' \to vi(\bar{d}\sigma^{\mu\nu}uW^-_{\mu\nu} - \bar{u}\sigma^{\mu\nu}dW^+_{\mu\nu}) \qquad \text{(PCTV operator)}$$

$$\hat{\vartheta}_4' \to vi(\bar{d}\sigma^{\mu\nu}\gamma_5 uW^-_{\mu\nu} + \bar{u}\sigma^{\mu\nu}\gamma_5 dW^+_{\mu\nu}) \qquad \text{("weak transition EDM")}$$

3. Numerical estimations of the dim-5 contribution to T-violating experimental observables

(a). D-coefficient induced by the dim-5 operator



$$\langle p|\bar{u}\sigma^{\mu\nu}d|n\rangle = g_T \bar{u}_p \sigma^{\mu\nu} u_n$$

Nucleon tensor charge

$$iM^{\mu} = \frac{ig}{2\sqrt{2}} V_{ud} \bar{u}_p (g_V \gamma^{\mu} - g_A \gamma^{\mu}\gamma_5 + iF\sigma^{\mu\nu} q_\nu + \ldots) u_n$$

The imaginary part of F induces a D-coefficient

With our dim-5 operator we obtain (set $g_V \approx 1$):

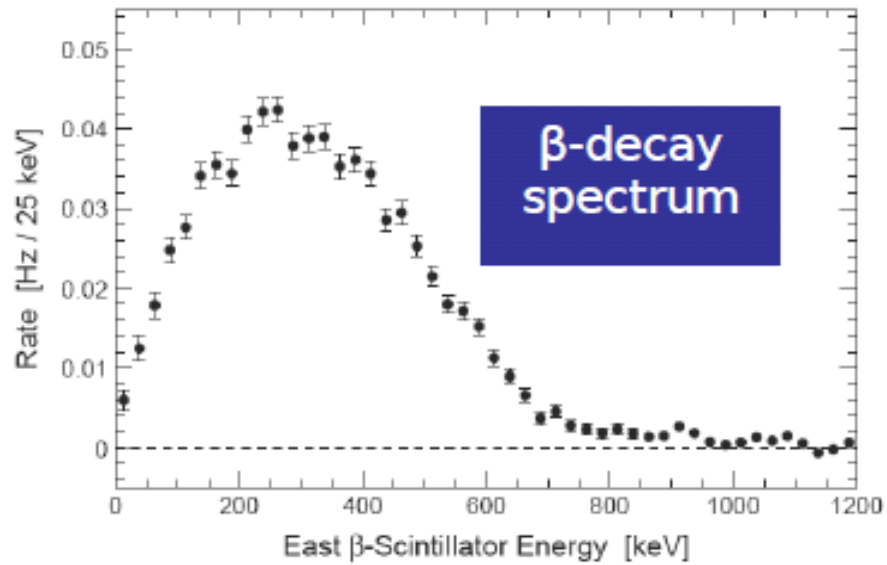
$$\boxed{D(E_e) = -\frac{4\sqrt{2} g_T}{g V_{ud}(1+3g_A^2)} c \frac{v}{\Lambda_{TV}^2} \{-2(1+g_A)(m_n - m_p) + 4E_e\}}$$

using:

$$g_T \approx 1.038$$ (lattice calculation, hep-ph1303.6953)

with a typical energy of the outgoing electron:

$$(K.E)_e \approx 0.25\,\mathrm{MeV}$$
$$\Rightarrow E_e \approx 0.76\,\mathrm{MeV}$$



J. Martin, JLab Hall C Summer Meeting, 2007

and the current bound on the D-coefficient:

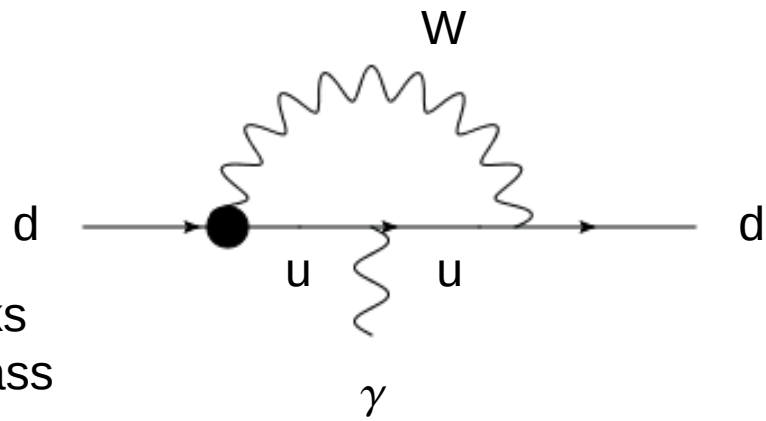
$$|D| < 4\times10^{-4}$$

we obtain:

$$\boxed{\left|\frac{c}{\Lambda_{TV}^2}\right| < 4\times10^{-4}\ \mathrm{GeV}^{-2}}$$

(b). Nucleon EDM induced by the dim-5 operator

Quark EDMs could be generated via diagrams like:



Since the PCTV operator already breaks chiral symmetry so there's no quark-mass suppression to the EDM.

Naïve dimensional analysis (NDA) estimation:

$$d_q \sim \frac{eV_{ud}g}{16\pi^2} c \frac{v}{\Lambda_{TV}^2} \ln \frac{m_W}{\mu}$$

For order of magnitude estimation, assume:

$$d_n \sim d_q$$

Current neutron EDM bound:

$$d_n < 2.9\times10^{-26}\, e\,\mathrm{cm} \approx 1.5\times10^{-12}\, e\,\mathrm{GeV}$$

$$\text{By naively taking } \ln\frac{m_W}{\mu} \sim 1 \text{ we get :}$$

$$\boxed{\left|\frac{c}{\Lambda_{TV}^2}\right| < 10^{-12}\,\mathrm{GeV}^{-2}}$$

- This is a much more stringent bound than that set by direct PCTV observables!
- Question: how do we distinguish effects between tree-level PVTV operators and loop-level PCTV operators?

- More rigorous analysis: compute the mixing of various dim-6 T-violating operators, and run it down from Λ_{TV} to EW scale.

$$\hat{\vartheta}_1 = i(\bar{d}_R \sigma^{\mu\nu} H^+ Q_L - \bar{Q}_L H \sigma^{\mu\nu} d_R) B_{\mu\nu}$$

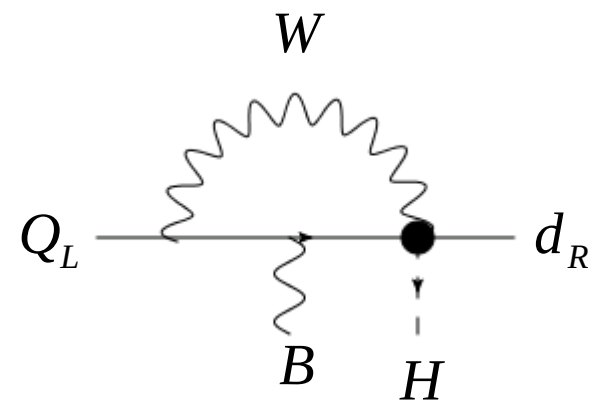
$$\hat{\vartheta}_2 = i(\bar{d}_R \sigma^{\mu\nu} H^+ \frac{\tau^i}{2} Q_L - \bar{Q}_L \frac{\tau^i}{2} H \sigma^{\mu\nu} d_R) W^i_{\mu\nu}$$

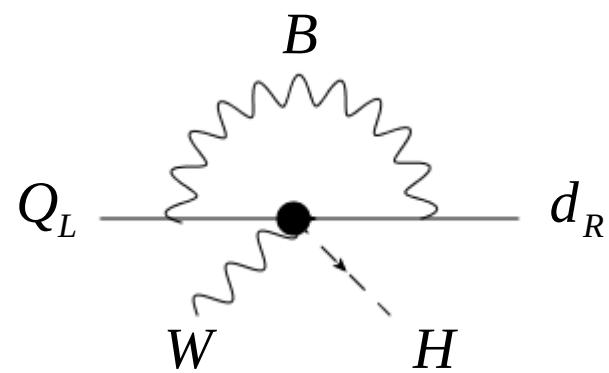
$$\hat{\vartheta}_3 = i(\bar{u}_R \sigma^{\mu\nu} \tilde{H}^+ Q_L - \bar{Q}_L \tilde{H} \sigma^{\mu\nu} u_R) B_{\mu\nu}$$

$$\hat{\vartheta}_4 = i(\bar{u}_R \sigma^{\mu\nu} \tilde{H}^+ \frac{\tau^i}{2} Q_L - \bar{Q}_L \frac{\tau^i}{2} \tilde{H} \sigma^{\mu\nu} u_R) W^i_{\mu\nu}$$

$$\hat{\vartheta}_5 = i\bar{d}_R H^+ Q_L H^+ H + h.c$$

$$\hat{\vartheta}_6 = i\bar{u}_R \tilde{H}^+ Q_L \tilde{H}^+ \tilde{H} + h.c$$

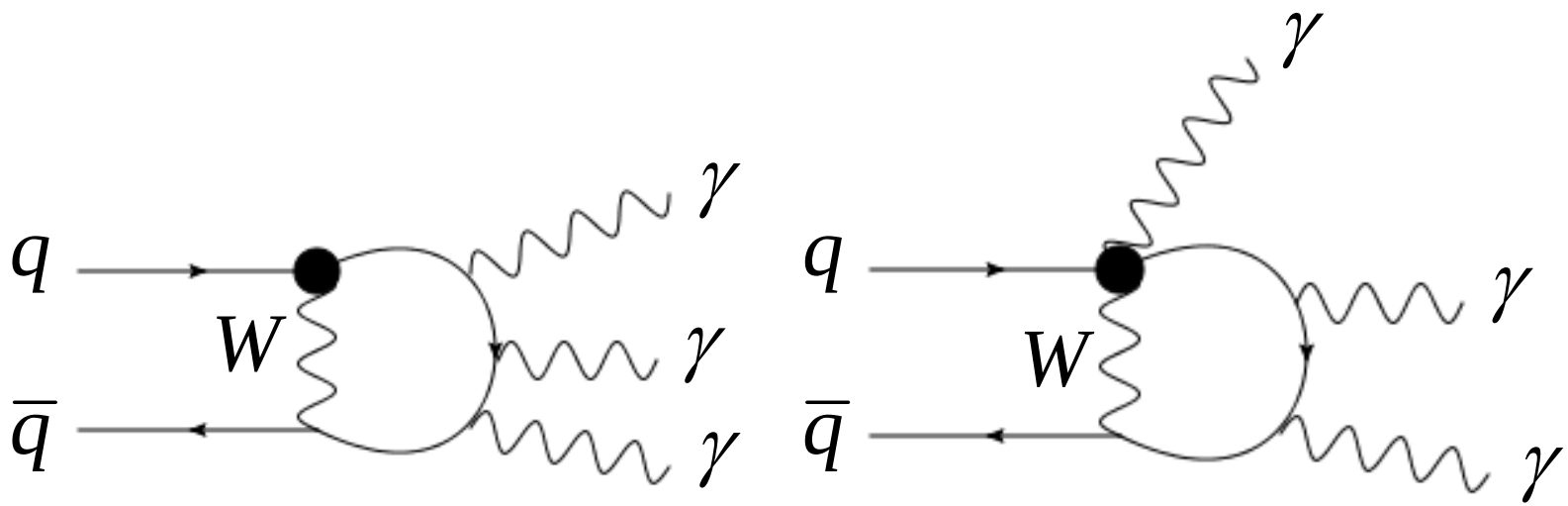




CALCULATION IN PROGRESS!

(c). P-even $\phi \to 3\gamma$ induced by the dim-5 operator

- The dim-5 PCTV operator could induce a $\phi \to 3\gamma$ decay which is C-odd and P-even via diagrams like:



- Effective Operator Analysis: write down lowest order $\phi \to 3\gamma$ operators in terms of ϕ, $F_{\mu\nu}$ (and its dual) and derivatives.
- Useful rules:

$$F_{\mu\alpha}F^{\alpha}{}_{\nu}F^{\mu\nu} = 0$$

$$\widetilde{F}_{\mu\alpha}F^{\alpha}{}_{\nu}F^{\mu\nu} = 0$$

$$\partial_{\mu}\widetilde{F}^{\mu\nu} = 0$$

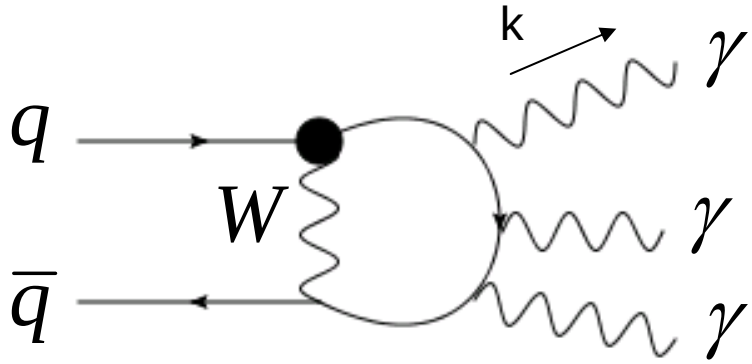
$$\partial^{\lambda}F^{\mu\nu} + \partial^{\mu}F^{\nu\lambda} + \partial^{\nu}F^{\lambda\mu} = 0$$

$$\partial_{\mu}F^{\mu\nu} \to 0 \quad \text{(EOM)}$$

- The effective C-violating, P-conserving $\phi \to 3\gamma$ operator begins at dim-9! Only ONE independent operator:

$$\boxed{\hat{O}^{CVPC}_{\phi\to 3\gamma} = (\partial^{\alpha}\phi)(\partial^{\beta}\widetilde{F}_{\alpha\mu})F_{\beta\nu}F^{\mu\nu}}$$

- An "overestimated" NDA analysis: P-even $\eta \to 3\gamma$ LEC: $\mathcal{L} = g_{\eta\gamma\gamma\gamma}\hat{O}^{CVPC}_{\eta\to 3\gamma}$



$$g_{\eta\gamma\gamma\gamma} \sim \frac{1}{16\pi^2}\frac{c}{\Lambda_{TV}^2}\frac{ge^3v}{m_W^2}\frac{1}{F_\eta^2}$$

(SM flavor-diagonal T-violations are many-loop suppressed)

That gives: $$\Gamma(\eta \to 3\gamma) \sim \frac{1}{2m_\eta}\frac{m_\eta^2}{32(2\pi)^3} g_{\eta\gamma\gamma\gamma}^2 k^{10} \sim \frac{c^2}{\Lambda_{TV}^4}\times 10^{-19}\,\mathrm{GeV}^5$$

3-particle phase space

(assuming $k \sim m_\eta/3$)

With current data: $\Gamma_{\eta,\mathrm{tot}} = 1.30\times 10^{-6}\,\mathrm{GeV}, \quad \mathrm{BR}(\eta \to 3\gamma) < 1.6\times 10^{-5}$

This leads to: $$\left|\frac{c}{\Lambda_{TV}^2}\right| < 10^4\,\mathrm{GeV}^{-2}$$

Not very useful!

Replacing η by π does not help because $m_\pi < m_\eta$ causes a large suppression.

Summary

- Current direct bounds on PCTV observables are far less stringent than their PVTV counterparts.
- Lowest-order PCTV operators with SM fields have dimension 5, and could arise from dim-6 T-violating operators before EWSB
- Induced PVTV observables such as EDMs place the most stringent bound on PCTV interactions.
- Current $\eta \to 3\gamma$ data does not seem to set any significant constraint on the dim-5 PCTV interactions.