

# $V_{ud}$ Outlook : Experiment & Theory

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<http://www.physics.umass.edu/acfi/>

*My pronouns: he/him/his*

***Collaborators: All  
workshop participants***

CKM Workshop  
ACFI May 2019

# ***Workshop Goals***

- ***Review recent theoretical developments in the extraction of  $V_{ud}$  and  $V_{us}$  from experimental measurements***
- ***Seek a consensus on the current values of theoretical inputs and associated theoretical uncertainties***
- ***Develop a roadmap for future, improved theoretical evaluations and experimental tests***

# ***Precision ~ BSM Mass Scale***

*Precision Goal:*

$$\delta \Delta_{CKM} \sim O(10^{-4})$$

*Heavy BSM Physics:*

$$\Delta_{CKM} \sim C (v/\Lambda)^2$$

$$\Lambda \sim 10 \text{ TeV (tree)}$$

$$\Lambda < 1 \text{ TeV (loop)}$$

*Ultralight BSM Physics:*

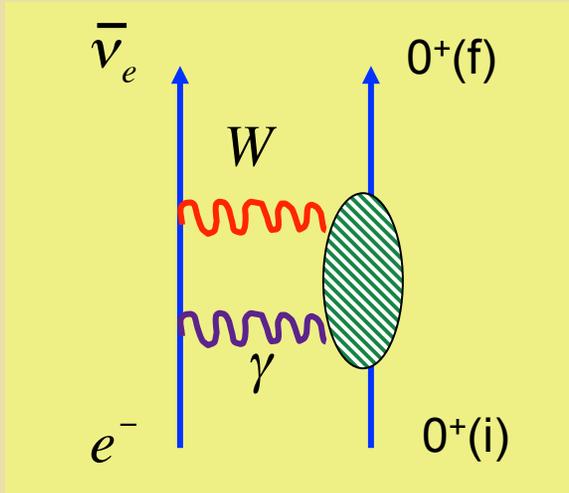
$$\Delta_{CKM} \sim \varepsilon^2 (\alpha/4\pi)$$

$$\varepsilon < 1 \text{ (loop)}$$

## ***Questions for This Discussion***

- ***How robust is the quoted uncertainty on the new value of  $\Delta_R^V$  ?***
- ***What additional tests (theory, experiment) are available ?***
- ***What is the roadmap to refined computation of  $\delta_{NS}$  ?***
- ***How important are contributions from other region of the low-E nuclear response ? How to compute & how to test computations ?***

# Dispersion Relations



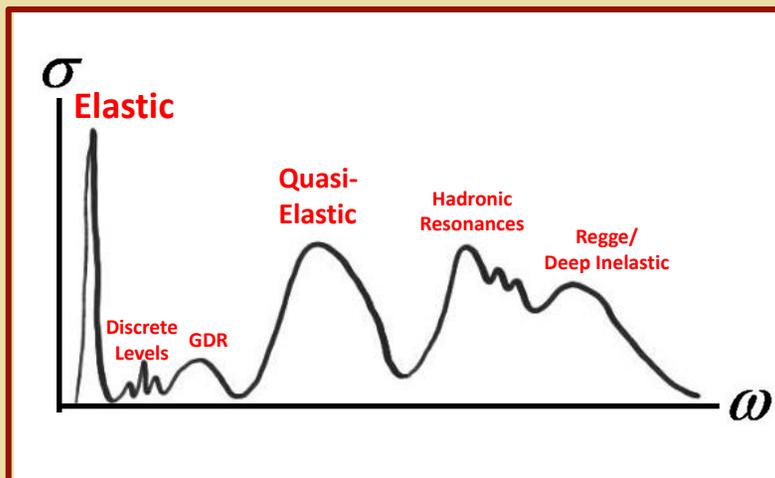
**Radiative Correction:**

$$\begin{aligned} \square_{\gamma W}^{VA(0)} &= \frac{\alpha}{\pi M} \int_0^\infty \frac{dQ^2 M_W^2}{M_W^2 + Q^2} \int_0^\infty d\nu \frac{(\nu + 2q)}{\nu(\nu + q)^2} F_3^{(0)}(\nu, Q^2) \\ &= \frac{3\alpha}{2\pi} \int_0^\infty \frac{dQ^2 M_W^2}{Q^2 [M_W^2 + Q^2]} M_3^{(0)}(1, Q^2) \end{aligned}$$

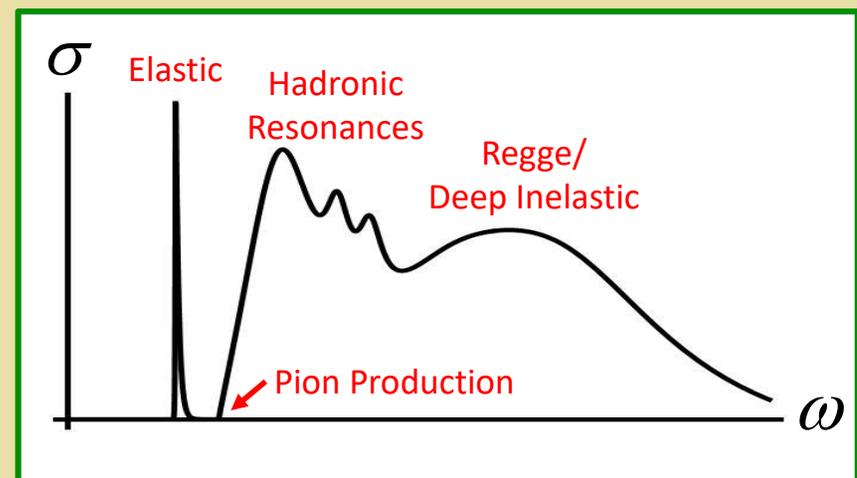
- **Relate  $F_3^{(0)}$  and  $M_3^{(0)}$  to data and/or**
- **Compute  $F_3^{(0)}$  and  $M_3^{(0)}$  using same methods used to describe semi-leptonic scattering processes with nucleon & nuclear targets**

# Leptonproduction: Had & Nuc Response

*Nuclei*



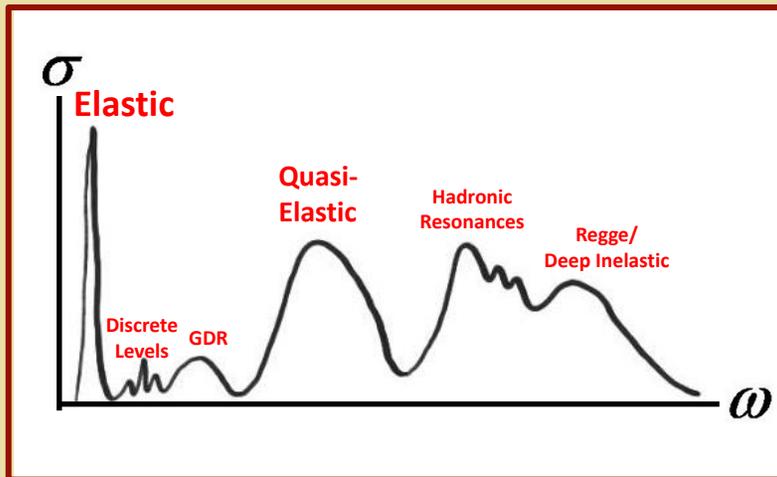
*Free nucleons*



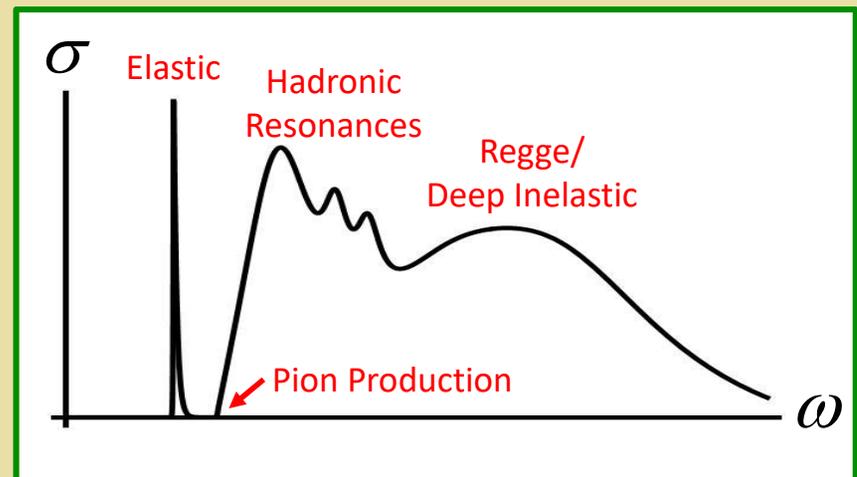
# ***Recent Results: Single Nucleon***

# Leptonproduction: Had & Nuc Response

*Nuclei*



*Free nucleons*



**“CMS” Preliminary**

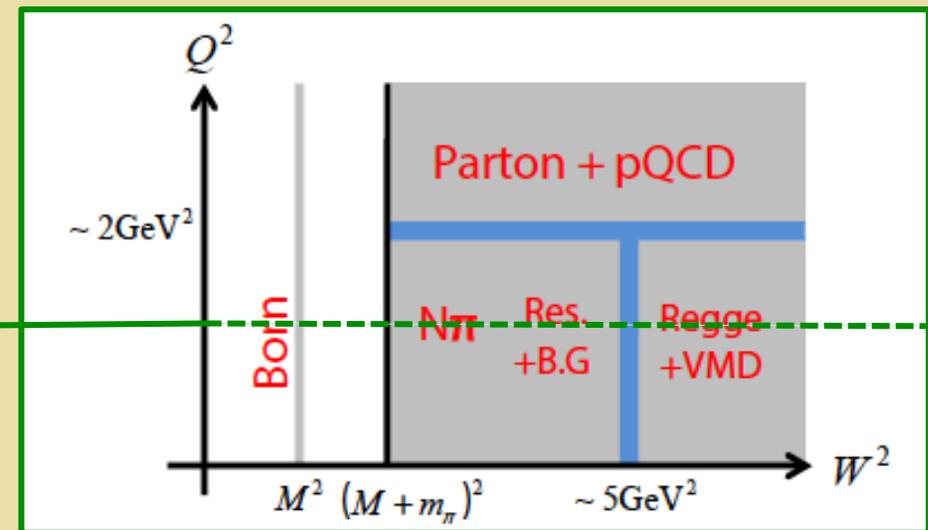
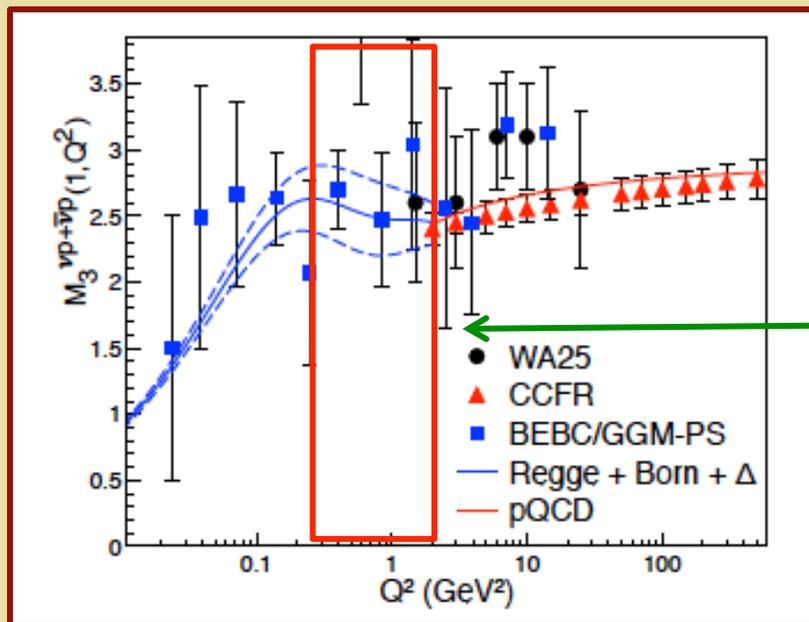
$$\Delta_R^V = 0.02361(38) \rightarrow 0.02433(32)$$

**Single nucleon: PRL 121 (2008) 241804**

$$\Delta_R^V = 0.02361(38) \rightarrow 0.02467(22)$$

*W. Marciano: this workshop*

# Dispersion Relations



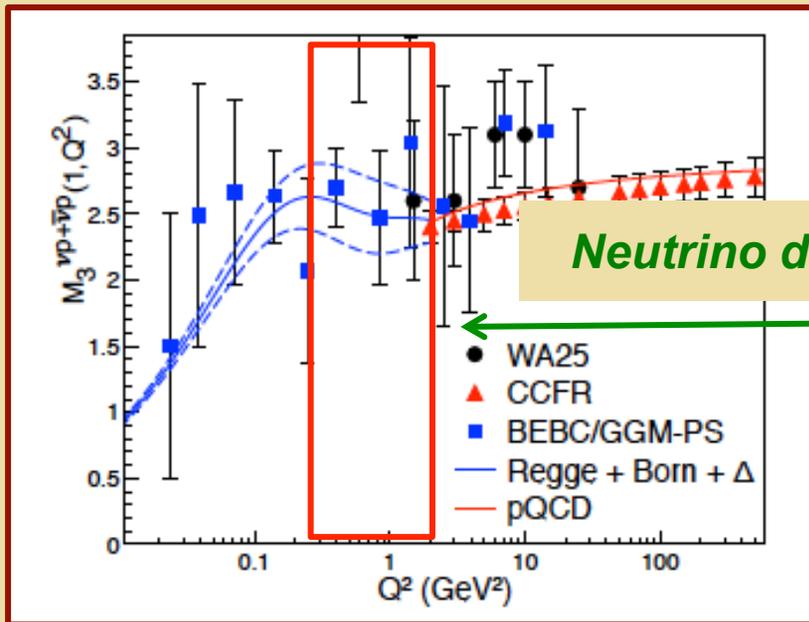
- Compute contributions to  $M_3^{vp+\bar{v}p}$  at each  $Q^2$  from different  $\omega$  regions
- Isospin rotate to  $M_3^{(0)}$

Refinements (WJM):

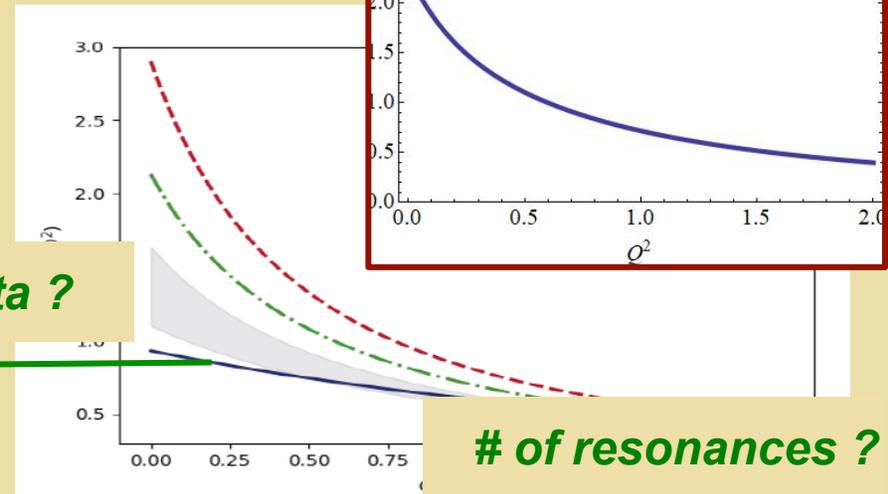
- Implement 4 loop BjSR for pQCD region

# Large $N_C$ Interpolator

Marciano



Neutrino data ?



$$F(Q^2) = \frac{A}{Q^2+m_\rho^2} + \frac{B}{Q^2+m_A^2} + \frac{C}{Q^2+m_{\rho'}^2}$$

$m_\rho = 0.776\text{GeV}$     $m_A = 1.230\text{GeV}$     $m_{\rho'} = 1.465\text{GeV}$

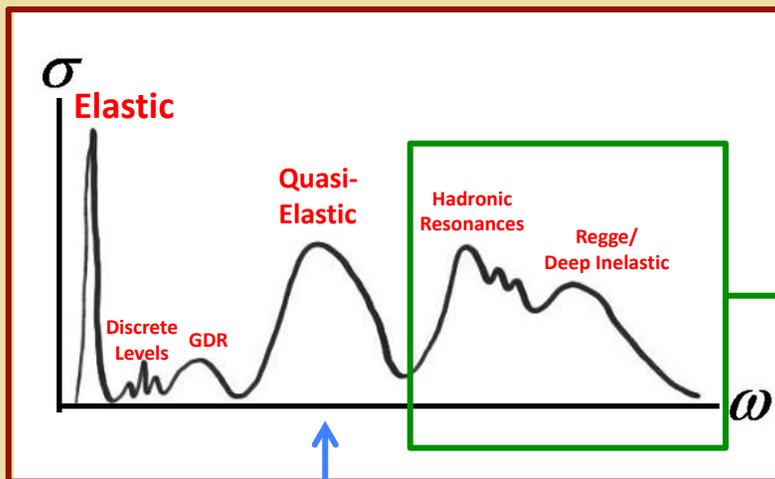
1. Integral  $Q_0^2 - \infty$  equals Bj Function Integral=7.885
2. No  $1/Q^4$  terms in large  $Q^2$  limit
3.  $F(0)$  Arbitrary

$$Q_0^2 = 1.08 \text{ GeV}^2$$

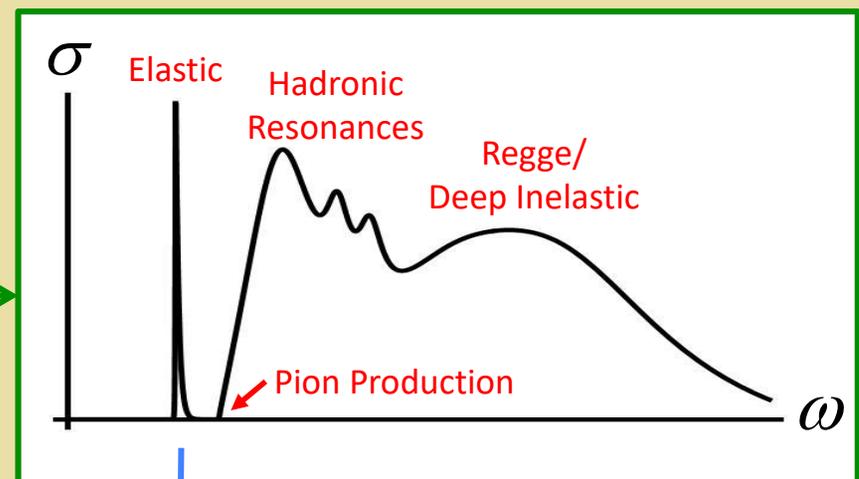
# ***Recent Results: Nuclei***

# Leptonproduction: Had & Nuc Response

*Nuclei*



*Free nucleons*



**Quasielastic response**

Part of  $\delta_{NS}$  : "  $C_B^{Nucl}$  "

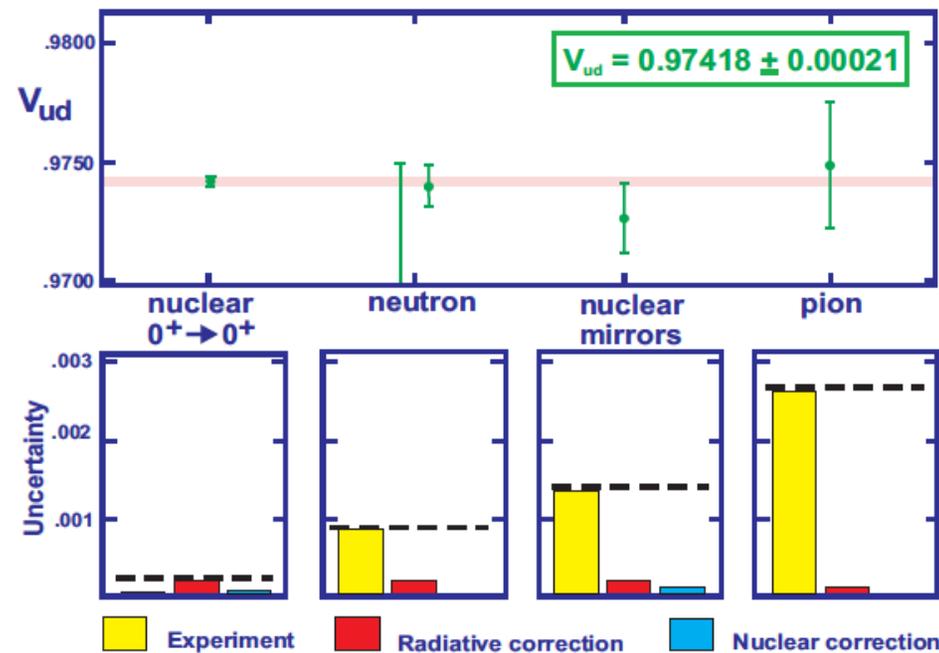
*New work*



# Implications: “Conventional Analysis”

Hardy

## CURRENT STATUS OF $V_{ud}$ AND CKM UNITARITY



$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.99935 \pm 0.00047$$

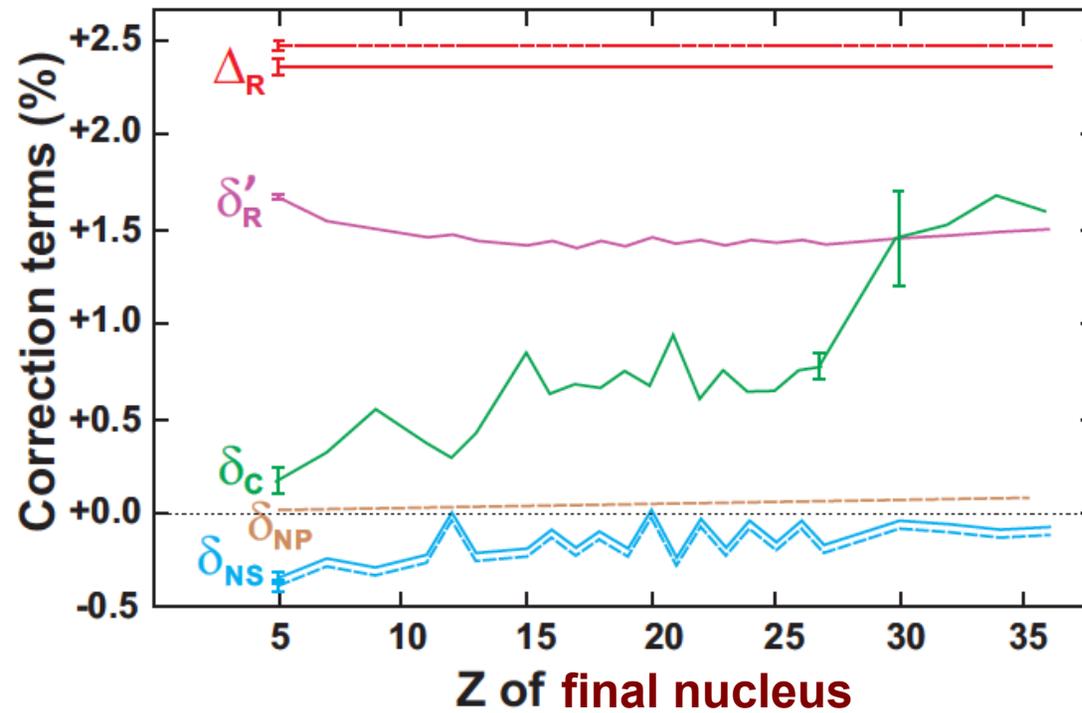
$V_{ud}^2$  nuclear decays muon decay  $0.94902 \pm 0.00041$   
 $V_{us}^2$  PDG kaon decays  $0.05031 \pm 0.00022$   
 $V_{ub}^2$  B decays  $0.00002$

# Implications

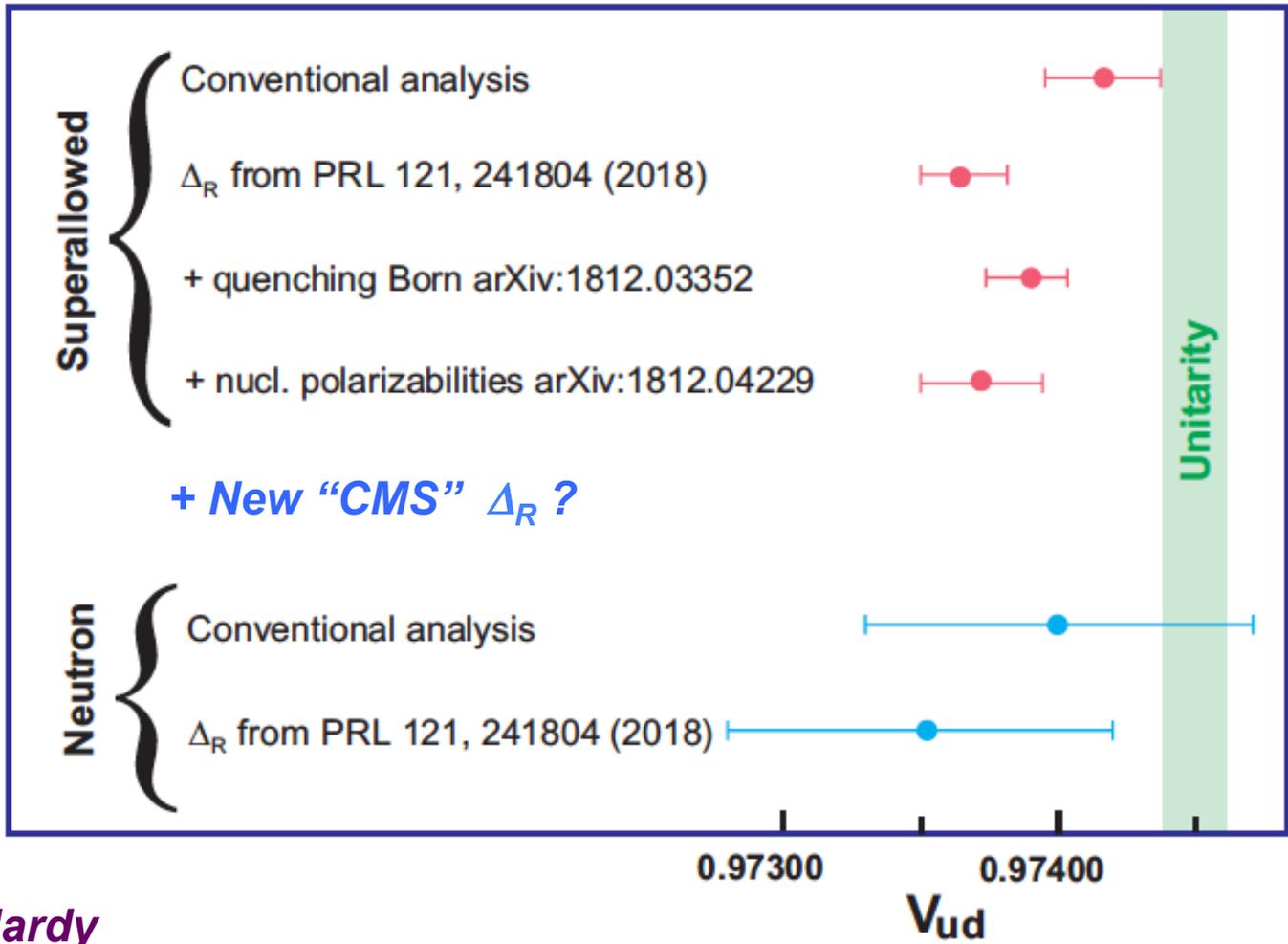
Hardy

## PROMISING FUTURE DIRECTIONS

New Correction terms -----



# Implications



Hardy

# ***Tests & Improvements: Single Nucleon***

# Future Tests

- *Lattice computation of  $M_3^{(0)}(Q^2)$*
- *PV electron scattering*

## *Isospin relation*

$$4F_3^{(0)} = F_{3,\gamma Z}^p - F_{3,\gamma Z}^n$$

- *SoLID ?*
- *EIC ?*
- *More neutrino data for  $M_3^{(0)}(Q^2)$*

# Lattice QCD & $M_3^{(0)}(Q^2)$

Feynman-Hellmann Theorem

Seng

$$\frac{dE_{n,\lambda}}{d\lambda} = \left\langle n_\lambda \left| \frac{\partial H_\lambda}{\partial \lambda} \right| n_\lambda \right\rangle$$

$$H_\lambda = H_0 + 2\lambda_1 \int d^3x \cos(\vec{q} \cdot \vec{x}) J_{em}^2(\vec{x}) - 2\lambda_2 \int d^3x \sin(\vec{q} \cdot \vec{x}) J_A^3(\vec{x})$$

$$\left( \frac{\partial^2 E_{N,\lambda}(\vec{p})}{\partial \lambda_1 \partial \lambda_2} \right)_{\lambda=0} \equiv \frac{iq_x}{Q^2 \omega} T_3^N(\omega, Q^2).$$

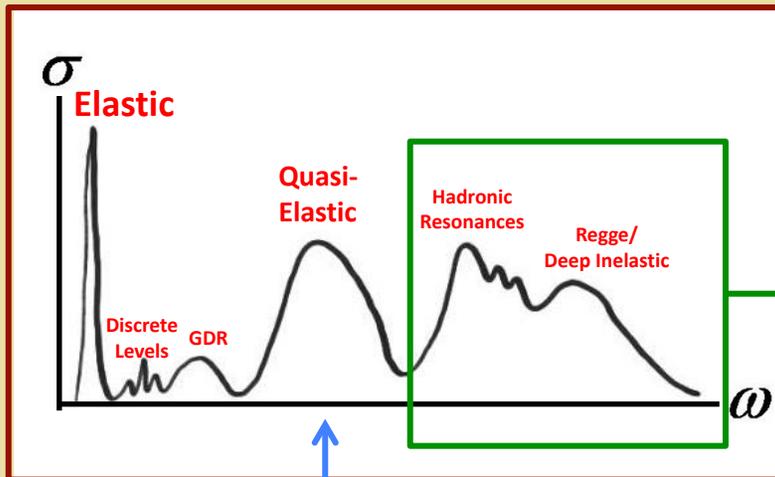
CYS and U.G-Meissner, hep-ph/1903.07969

$$\left( \frac{\partial^2 E_{N,\lambda}(\vec{p})}{\partial \lambda_1 \partial \lambda_2} \right)_{\lambda=0} = \frac{4q_x}{Q^2} \int_0^1 dx \frac{F_3^N(x, Q^2)}{1 - \omega^2 x^2},$$

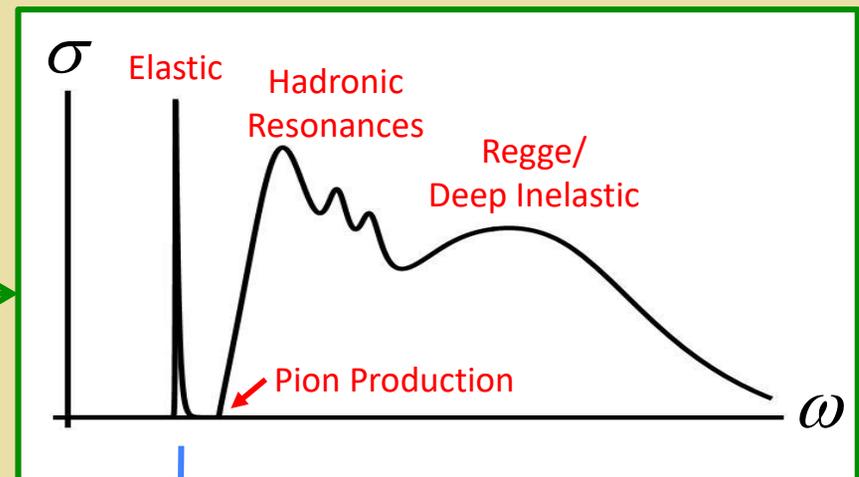
# ***Tests & Improvements: Nuclei***

# Leptonproduction: Had & Nuc Response

*Nuclei*



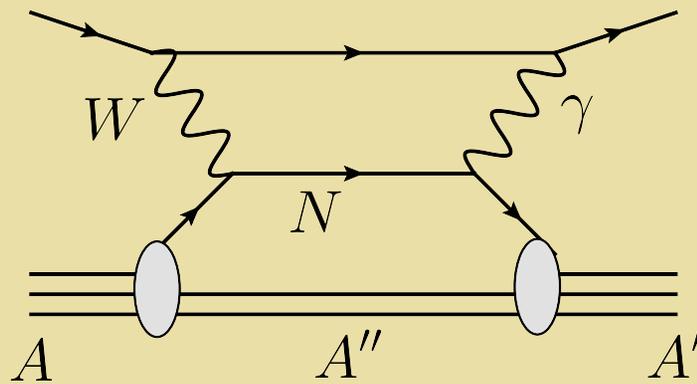
*Free nucleons*



**Quasielastic response**

Part of  $\delta_{NS}$  : "  $C_B^{Nucl}$  "

# Impact on $\delta_{NS}$



$$\Delta \delta_{NS} = \frac{\alpha}{\pi} \left( C_{QE} - q_S^{(0)} q_A C_B \right) = -(4.6 \pm 0.9) \times 10^{-4}$$

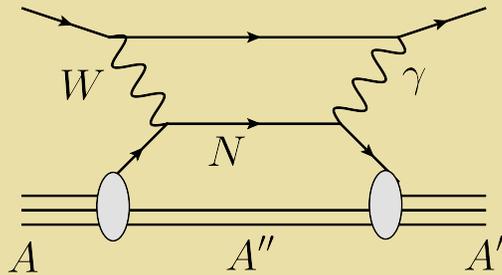
**Neglects A-  
dep variations**

**Ave over 20  
transitions**

**Optimistic:**

- Correlations
- 2-body currents
- Rel corrections

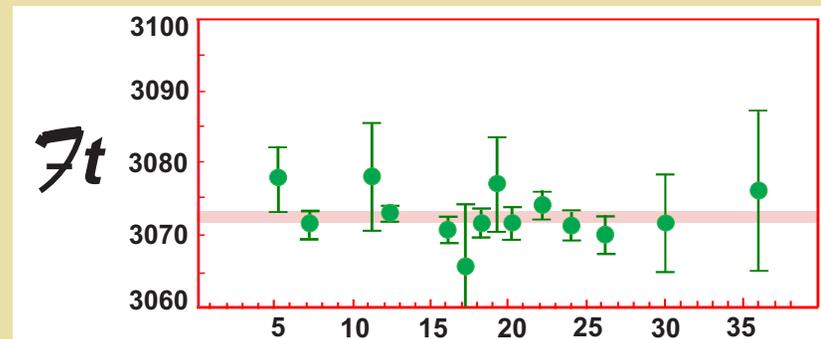
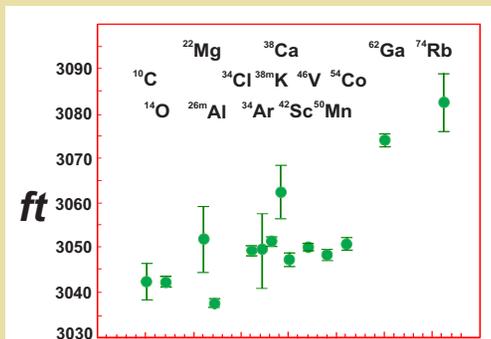
# QE Contribution: Refinements



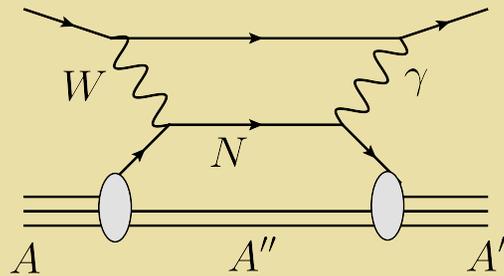
*Apply state-of-art methods*

- *Correlations*
- *2-body currents*
- *Rel corrections*
- *Recent MIT group: 2N knockout*

*Consistency w/ CVC ?*



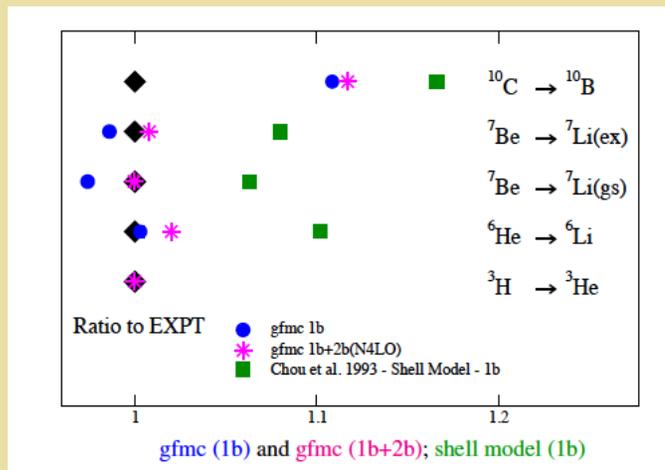
# QE Contribution: Refinements



## Quantum Monte Carlo

- GFMC:  $^{10}\text{B}$
- Auxiliary Field MC: higher  $A$

## LO $\beta$ -Decay



# GFMC & AFMC

Pastore

$$R(q, \omega) = \sum_f \delta(\omega + E_0 - E_f) \langle 0 | O^\dagger(\mathbf{q}) | f \rangle \langle f | O(\mathbf{q}) | 0 \rangle$$

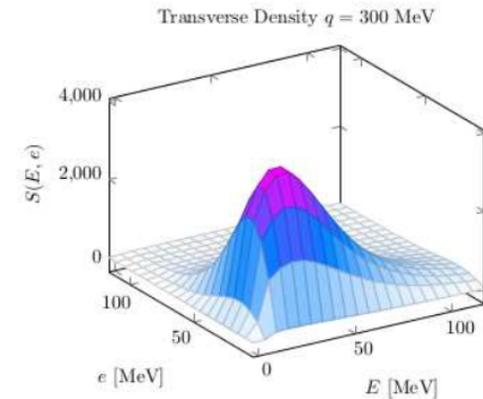
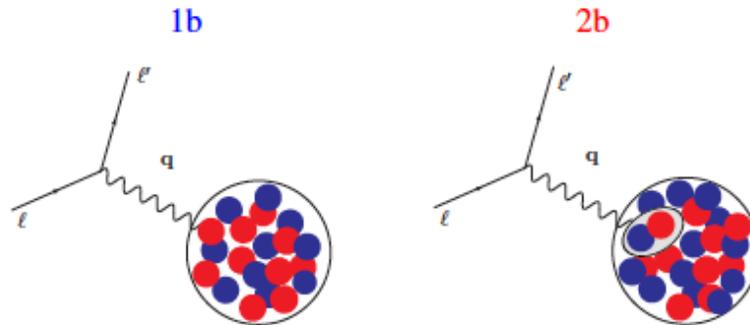
$$R(q, \omega) = \int dt \langle 0 | O^\dagger(\mathbf{q}) e^{i(H-\omega)t} O(\mathbf{q}) | 0 \rangle$$

At short time, expand  $P(t) = e^{i(H-\omega)t}$  and keep up to **2b-terms**

$$H \sim \sum_i t_i + \sum_{i < j} v_{ij}$$

and

$$O_i^\dagger P(t) O_i + O_i^\dagger P(t) O_j + O_i^\dagger P(t) O_{ij} + O_{ij}^\dagger P(t) O$$

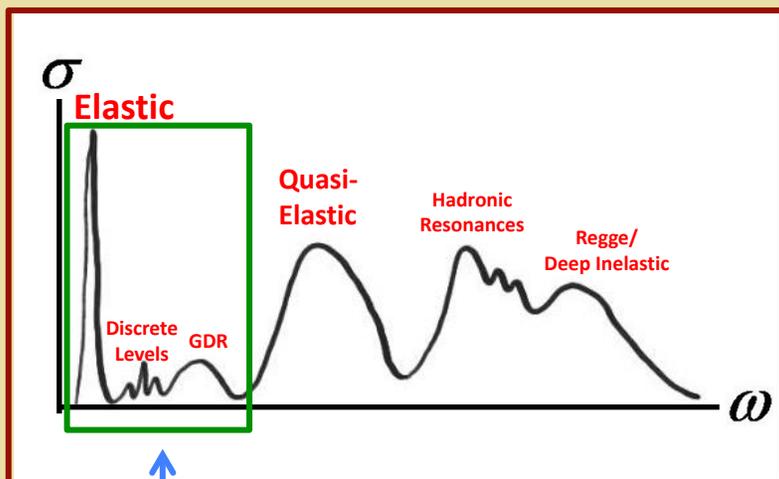


Transverse "response-density"  $1b + 2b$  for  ${}^4\text{He}$   
 $\mathcal{D}(p', P'; q)$

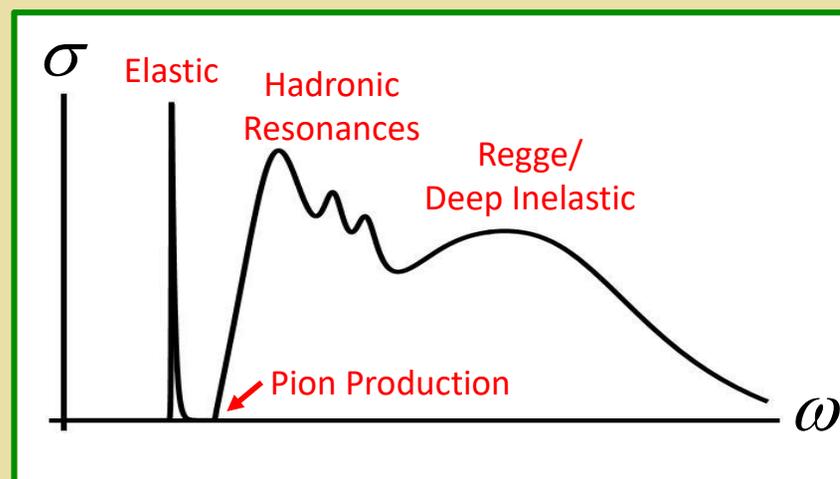
\* Preliminary results \*

# Other Nuclear Corrections

*Nuclei*



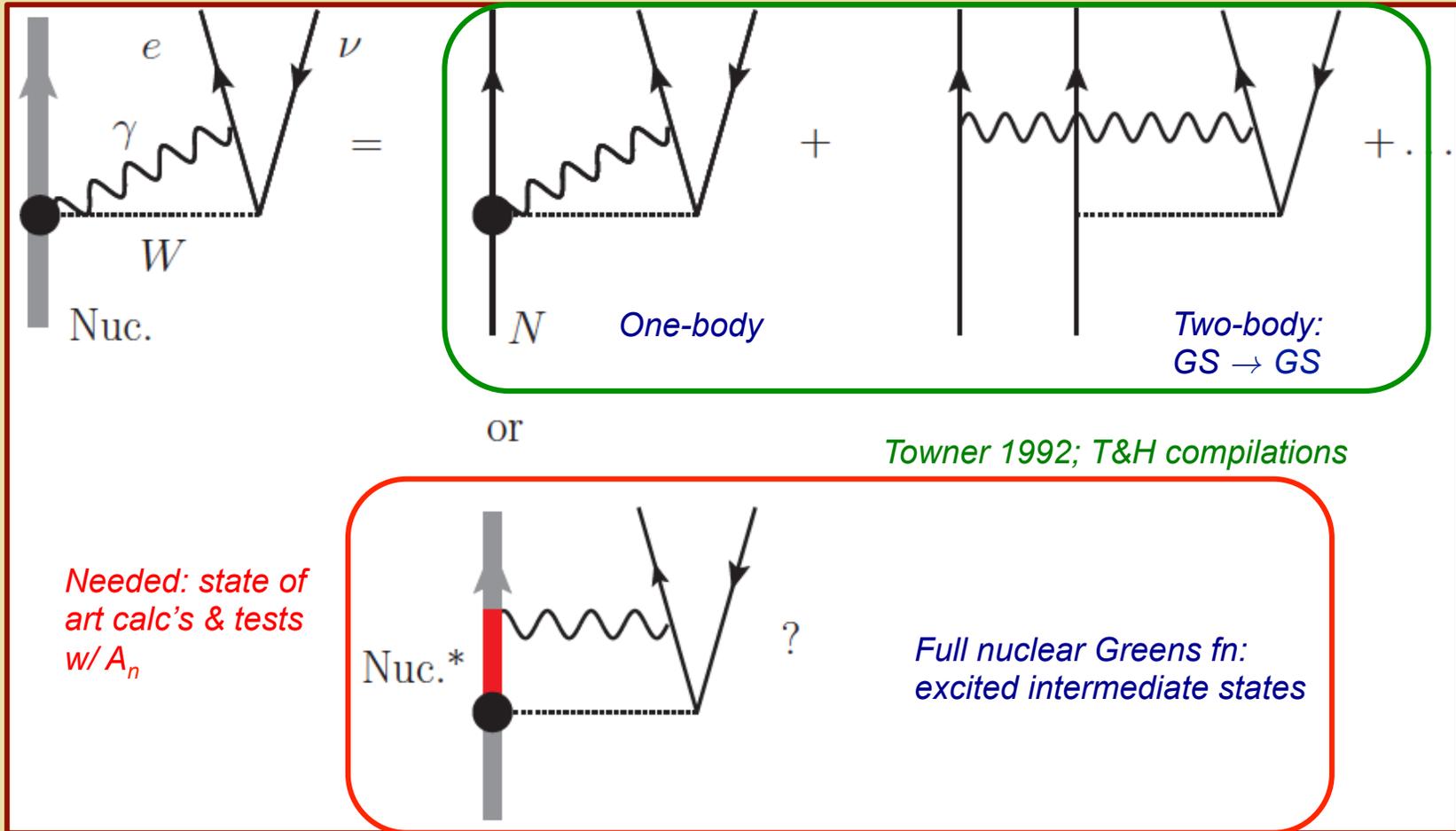
*Free nucleons*



*Low-lying transitions*

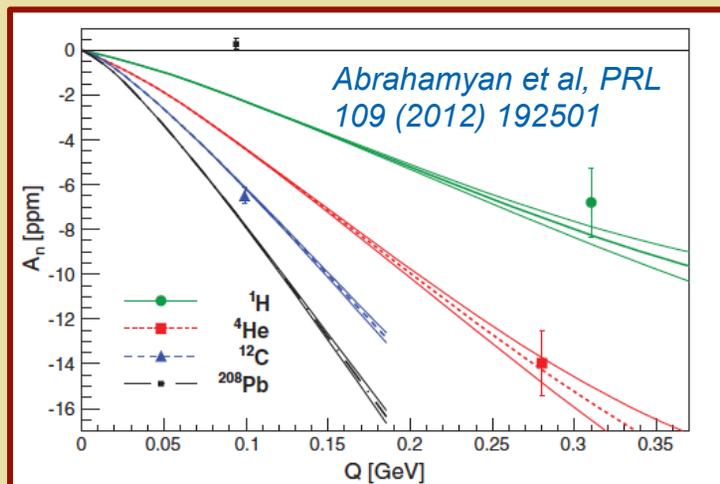
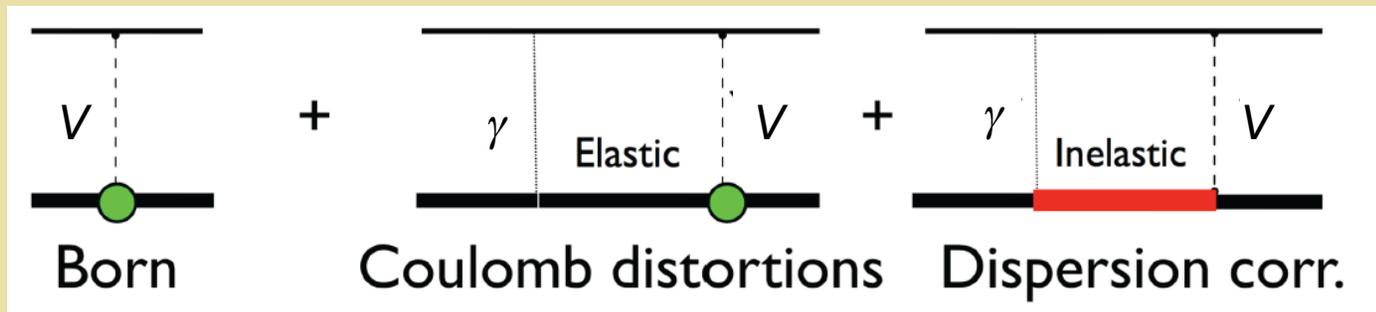
*Part of  $\delta_{NS}$*

# $0^+ \rightarrow 0^+$ Decay: $\delta_{NS}$



# Dispersion Corrections

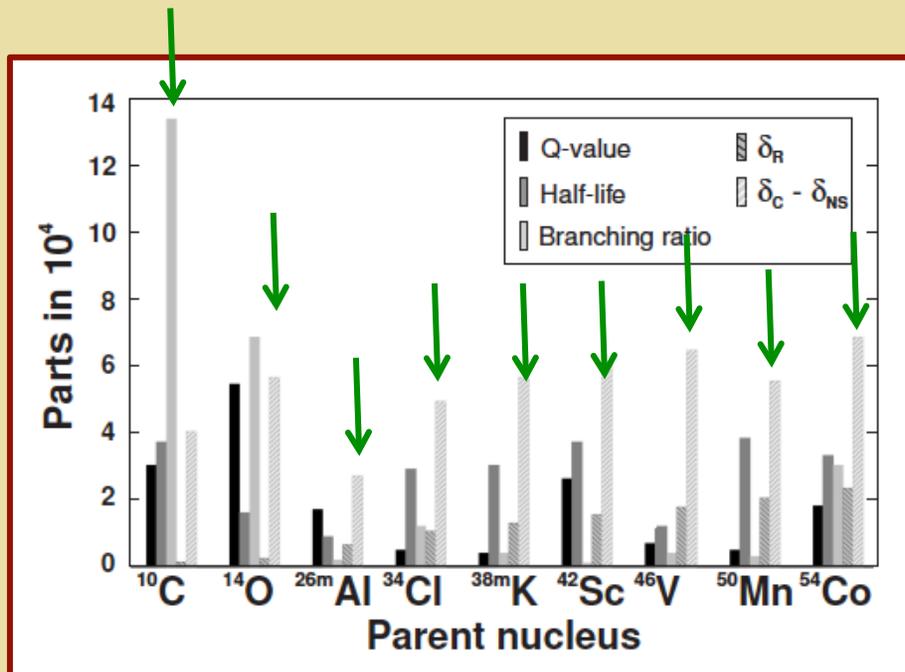
*Two-boson exchange in semileptonic processes: important for elastic PV eN & eA scattering ( $^{12}\text{C}$ ) & nuclear  $\beta$ -decay; beam normal asymmetry, Olympus... provide tests*



$V = \gamma$  *Beam normal asymmetry*

- *J Lab Hall A*
- *Future: Mainz, J Lab*

# $0^+ \rightarrow 0^+$ Dispersion Corrections: $\delta_{NS}$



$b_F$  : scalar currents

Input for  $V_{ud}$  & CKM unitarity test

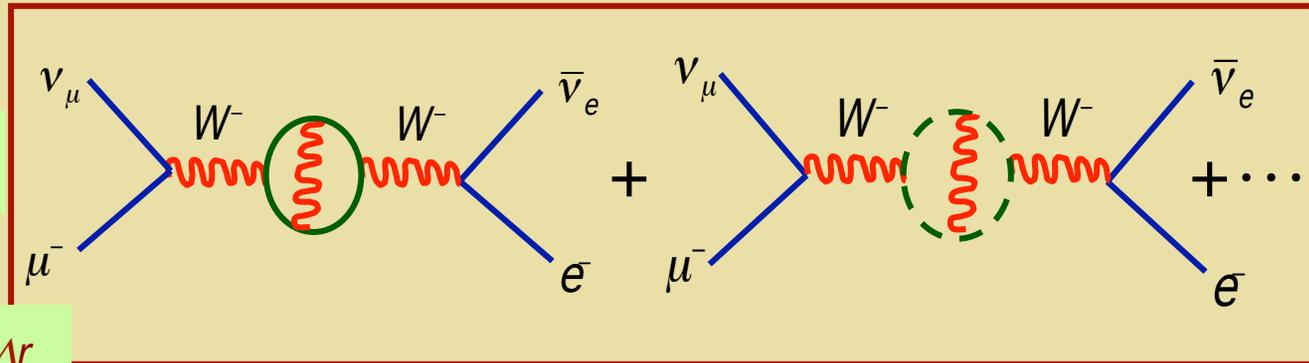
- $e^+$  vs  $e^-$  ?
- Muons ?

Towner & Hardy, PRC 91 (2015) 2, 025501

- Re-compute with state-of-the-art many-body methods
- Test w/  $A_n$  predictions & expt for  $^{10}\text{B}$ ,  $^{14}\text{N}$ ,  $^{26}\text{Mg}$ ,  $^{34}\text{S}$ ,  $^{38}\text{Ar}$ ,  $^{42}\text{Ca}$ ,  $^{46}\text{Ti}$ ,  $^{50}\text{Cr}$ ,  $^{54}\text{Fe}$
- Investigate strategy for obtaining reduced error bars

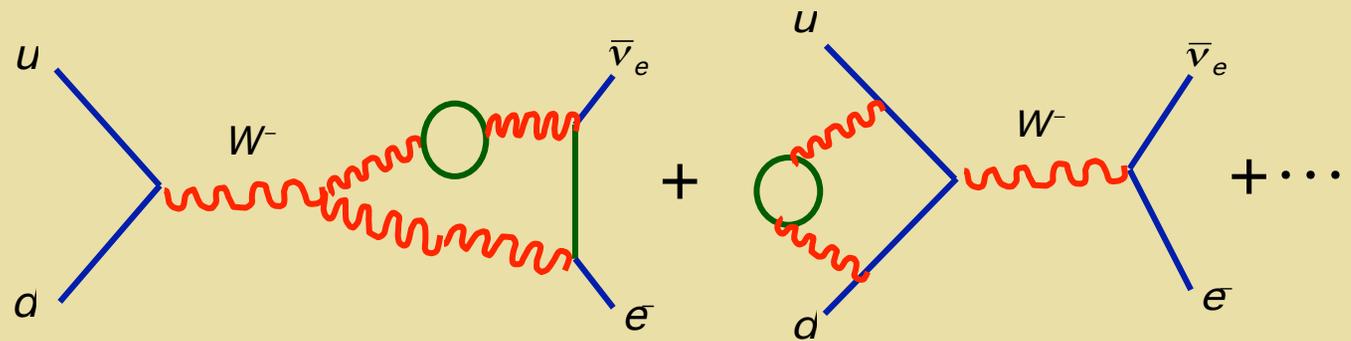
# “Next Frontier” : Higher Order EW RC

Propagator

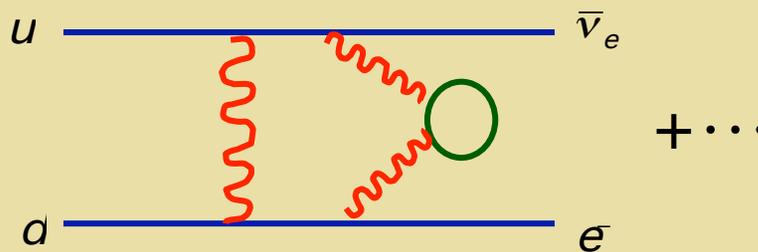


Cancel from  $\Delta r_\beta - \Delta r_\mu$

Vertex & External leg



Box



# Other Issues

## Ivanov: $C_{PS}$ + Wilkinson term

$V_{ud}$  from the neutron beta decay

$$|V_{ud}| = \sqrt{\frac{5099.81}{\tau_n(1 + \Delta_R)(1 + 2.7 \times 10^{-4})_W(1 - 0.33C_{ps})}}$$

$$\Delta_R = \frac{\alpha}{2\pi} \bar{g}(E_0) + \Delta_R^V$$

$$0.33C_{ps} = -3.86 \times 10^{-5} (0.127 + \text{Re}(C_P - \bar{C}_P))$$

$$\text{Neutron beta decay: } V_{ud} = 0.97370(14)$$

## Hayen: RC for GT decays

There is now an additional RC which is *not* included in  $\Delta_V^R$  for GT decays

More generally, based on “old” approach

$$\Delta \frac{dN}{dW} \propto \pm \frac{2}{5} \frac{\alpha Z}{MRc_1} (\pm 2b + d)$$

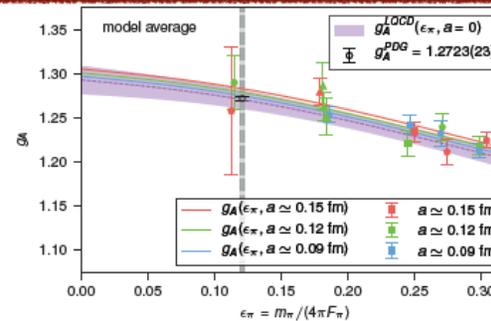
where  $b/A_{c_1}$  is weak magnetism,  $d_{A_{c_1}}$  is induced tensor (0 for isospin multiplet decays)

# Other Issues

## Walker-Loud: $g_A$ from LQCD

### Final result

statistical	0.81%
chiral extrapolation	0.31%
$a \rightarrow 0$	0.12%
$L \rightarrow \infty$	0.15%
isospin	0.03%
model selection	0.43%
<b>total</b>	<b>0.99%</b>



$$g_A^{\text{QCD}} = 1.2711(103)^s(39)^x(15)^a(19)^V(04)^I(55)^M$$

### How precise can we get $g_A$ ?

- Without changing strategy, **we should be able to get 0.5% this year** by improving the three physical pion mass points - with our INCITE allocation on Summit at OLCF
- To achieve 0.2% precision, at least a 4th lattice spacing will be required, ideally at two or more pion masses. This can still be achieved with Summit
- We heard yesterday of a 0.4% radiative correction from QED - we should also compute this and compare
- **How precisely should we push this calculation?**