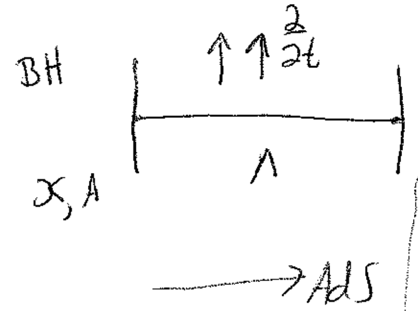
Cosmic Hair (and precluded (?) info)

[J. Traschen 4-16]

I

AdS



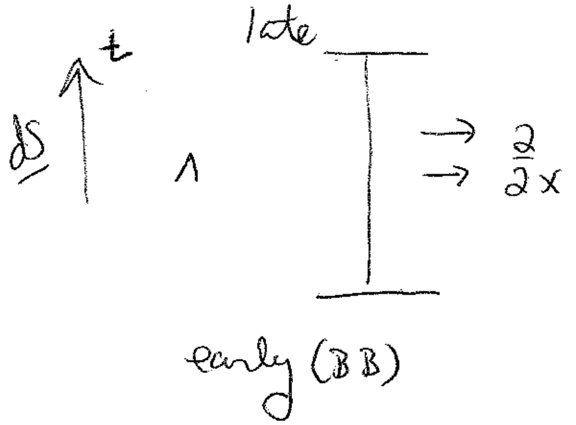
∞ ADM charge | Symm

$M, \mathcal{T}_j$

- $\frac{\partial}{\partial t} \Rightarrow (D-3)M = (D-2)\kappa A + 2\Lambda V_{ther}$

- scaling $\Rightarrow M + \sum_{j=1}^{D-2} \mathcal{T}_j^{AdS} L_j = 0$

dS



$\mathcal{T}_j$

- $\frac{\partial}{\partial x} \Rightarrow$?

(late BT) = (early BT) + ΛV_{cos}

- scaling $\sum_{j=1}^{D-1} L_j \mathcal{T}_j^{cos} = 0$

questions

1. What are the late time $\mathcal{T}_j$, and why are they not inflated away?
2. Cosmolog Smarr rel is ?
3. V_{cos} is ?

Fiducial metrics $ds^2 = -dt^2 + \sum a_k^2(t)(dx^k)^2$, $D=4$

where $a_k^2(t) \rightarrow e^{2H_0 t}\left[1 + c_k e^{-3Ht}\right]$

II. Working example (like AdS BH) $\qquad z \longleftrightarrow T$

$(D=4)$

$$ds^2 = -\frac{\ell^2}{T^2}\frac{dT^2}{F(T)} + \frac{T^2}{\ell^2}\sum_k F(T)^{p_k}(dx_k)^2, \qquad F(T) = 1 - \left(\frac{T_0}{T}\right)^3$$

dS-Kasner

$$\Sigma p_k = \Sigma p_k^2 = 1$$

(1a) Why are $\mathcal{T}_i$ nonzero ?

- As in AdS : fall off is exp in t_{prop}

  But da grows exp.

(1b) What ?

- Noether charge (Hamilt)

  Asympt $\frac{\partial}{\partial x}$ sym $\cdot \longrightarrow$ charge $Q(\frac{\partial}{\partial x})$

  covariant th. $\qquad$ (see App. 1)

- Find $\quad \mathcal{T}_x = \int_{t_{late}} (K^x{}_x - (K^x{}_x)_{dS})$

$$= \int \left(\frac{\dot{a}_x}{a_x} - H_{dS}\right)$$

Ex $\quad$ Kas-dS $\qquad\qquad (D=4)$

$$\mathcal{T}_k \propto T_0^3 \cdot \Lambda(3p_k - 1)$$

$$\Sigma L_k \mathcal{T}_k = 0$$

[I factors : $16\pi \mathcal{T}_k = \dfrac{v\, T_0^3}{L_x \ell^4}(3p_k - 1)$, $\quad \Lambda = 3/\ell^2$]

III. Smarr Rel : Stokes Thm construct
based on a KV ξ in S.T.

late BT

$\Rightarrow \frac{\partial}{\partial X}$ $\qquad \nabla_a \nabla^a \xi^c = -R^c_d \xi^d$

$\to X_c$ (See App. 2)

early BT

$$\Downarrow$$

a) $$\underset{\text{late}}{\int da}\, K^x{}_x \equiv \underset{\text{early}}{\int da}\, K^x{}_x + \Lambda \int_{t_i}^{t_f} (\xi^a X_a)\, dv$$

$$\infty \qquad\qquad ? \qquad\qquad \infty$$

OK... • props of KV $\Rightarrow$ can rewrite $\int dV$ as a BT,

b) $$\int dv\, (\xi^a X_a) \equiv \underbrace{\begin{pmatrix}\text{balances}\\ \text{div of}\\ \text{late time BT}\end{pmatrix}}_{\underset{\text{late}}{\int} (K^x{}_x)_{ds}} + \text{finite} \qquad + \text{finite}$$

(See App 3)

c) $$T_x = \int_{T_f} \left[K^x{}_x - (K^x{}_x)_{ds}\right] = \Big(\underset{\text{Big Bang}\ T=T_0}{\int} K^x{}_x \Big) + \Lambda V_{car}$$

where BB contrib $$\int_{T_0} K^x{}_x \propto T_0^3 \Lambda^3 p_x \qquad \text{Kas-ds}$$

d) $$\begin{cases} da \to 0 \\ K^x{}_x \to \infty \end{cases}$$ but prod. finite

<u>momentum is finite</u>

$T_x \propto T_0^3 \Lambda (3 p_x - 1)$ Kas-dS

$\Lambda V_{cos} \propto \Lambda T_0^3$

Fiddling

$$\Lambda V_{cos} = \int_{late} KP - \int_{early\ T_0} KP$$

$$= \int_{T=0}^{T_0} \sqrt{-g}\Big|_{dS} = \left(\begin{array}{c} \text{vol from } T=0 \\ \text{until BB} \end{array} \right).$$

e)

<u>Summarize</u>

$$\underset{\substack{\text{late time Tension} \\ (\text{renorm. expansion in } \hat{x}) \\ \Updownarrow \\ \text{observable}}}{T_x} \equiv \underbrace{\left(\begin{array}{c}\text{grav mom.} \\ \text{at BB}\end{array}\right) + \Lambda V_{\text{before BB}}}_{\substack{\Updownarrow \\ \text{early univ info}}}$$

~~Summarize~~

$$\cancel{\left(\begin{array}{c}\text{late time} \\ \text{Tension}_x \\ \propto H_x\end{array}\right) = \int_{\text{Big Bang}} \Pi + \Lambda V}$$

IV.

(End of Inflation info) $\leftarrow$ early univ phys

<u>ideas for connecting to observables</u>: 1. Study $\langle H_x^2 - H_y^2 \rangle$

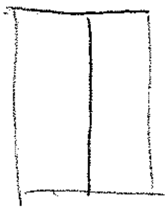
2. generalize to $\Lambda_{early} \rightarrow \Lambda_{late}$

[need to include $\int T_{ab}$]

3. Do 1^{st} law analysis, so can include perturb. anisotropies.

V. ? Connection to Action & prob./complexity

late



foliation of M, $\hat{n}$ = normal to Σ

cosmo tension $\hat{n} = \hat{x}$. $\quad x \cdot x = 1$

Hawking - Horowitz $\quad \hat{n} = \hat{t}$, $t \cdot t = -$

with KV $\vec{\xi} = F\hat{n} + \cancel{\vec{\beta}}\, 0$

$$16\pi(S_V + S_{BT}) = \int_M \sqrt{-g}\, d^D x\, (R - 2\Lambda) + \int_{\partial M} (da)_\partial (K - K_0)_\partial$$

$$= \int_M [\text{constraint eqs} + \pi \mathcal{L}_{\vec{\xi}}\, g_{ab}]$$

(taking $\vec{\beta} = 0$)

$$+ (-n \cdot n) \int_{\partial M} da_b V^b + 2\int_{\partial M} (da)_\partial (K - K$$

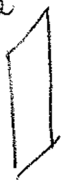
where $V^b = 2(n^b \nabla_c n^c - n^c \nabla_c n^b)$ $\quad$ from changing to Hamilt vars

and $(da)_\partial = da$

$\hat{n} =$	$\hat{n}$		bdry normal	
$+1$	$\hat{t}$	∞	$\hat{r}$	
-1	$\hat{t}$	Horizon	$-\hat{r}$	(no $\tilde{K}_0$)
-1	$\hat{x}$	late	$-\hat{t}$	
$+1$	$\hat{x}$	early	$\hat{t}$	(no $\tilde{K}_0$)

Find: total late time B.T (using $\frac{\partial}{\partial x}$)

late



$$\doteq -2\int_{T_f} da \left[(K^y{}_y + K^z{}_z) - (K_{(0)}{}^y{}_y + K_{(0)}{}^z{}_z)\right]$$

$$= -2\int_{T_f} da\,(K^x{}_x - K_{(0)}{}^x{}_x)$$

using sumrule

$$= -2\,T_x$$

then use Smarr ⇒

$$S = \Big(\int_{T_{early}} K^x{}_x\Big)(1 \pm 1) \pm \frac{2\Lambda V}{D-2}$$

Work in progress!

Need a better understanding of (Action) ↔ probs, entropy in cosmology

It's all in the B.T.s (and the ~~[illegible]~~ signs ☺)

$D = n+1$

Appendix 1

def of ADM tension at future ∞

asympt dS in "Poincare style coords"

$$ds^2 \to -\frac{\ell^2}{T^2}\left(1 + \frac{c_T}{T^n}\right)dt^2 + \frac{T^2}{\ell^2}\sum_k \left(1 + \frac{c_k}{T^n}\right)(dx^k)^2 + \ldots$$

$$\equiv g^{dS}_{ab} + \delta g_{ab} \qquad \text{eg} \quad h'_{11} = \frac{c_k}{T^n}$$

Hamilt framework

$$g_{ab} = \hat{x}_a \hat{x}_b + s_{ab}, \qquad x \cdot x = +1$$

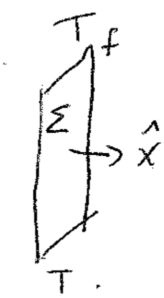
$$s_{ab} = s^{dS}_{ab} + h_{ab}, \qquad \pi^{ab} = \cancel{\pi^{ab}_{dS}} + p^{ab}$$

($\pi^{ab}_{dS} \to 0$)

KV $\quad \vec{\xi} = \frac{\partial}{\partial x} = F\hat{x}^a$

then linear

$$\delta(F G_{ab} x^a x^b) = 8\pi\, \delta(F T_{ab} x^a x^b)$$



on sols find $\to$

$$\int_{\partial\Sigma} da_c B^c = 16\pi \int_\Sigma \delta p_x \sqrt{-s}$$

Note analyzing this rel. $\to$ a "first law"

define asympt charge by late time B.T.

$$Q(\vec{\xi}) = -\frac{1}{16\pi}\int_{T_{late}} da_c B^c,$$

here $\quad B^c = F(D^c h - D_b h^{bc}) - h D^c F + h^{cb} D_b F$

For $\frac{\partial}{\partial x}$, $\quad F = \frac{T}{\ell} \quad \Rightarrow$ expression in $\{c_\alpha\}$

App 2.

$$\int_{\partial\Sigma} da_b\, x_c \nabla^b \xi^c = -\int_{\Sigma} \sqrt{-g}\, x_b \xi^c \left(\frac{2\Lambda}{D-2} g^b_c + 2\pi \left(T^b_c - \frac{T}{D-2} g^b_c \right.\right.$$

x^j periodic, $0 \le x^j \le L^j$

top $T = T_f$: $da_b = da\,(-t_b)$, $t_b = -|g_{tt}|^{1/2} \nabla_b t$

bott $T = T_i$: $da_b = da\, t_b$

process BT

let $K_{(x)}(t) = \int_t da_b\, t_c \nabla^b \xi^c = \int da\, K^x_x(t)$

$\sim \int \pi^x_x$

Komar charges are the grav momenta at t

App 3 set $T_{ab} = 0$

$\int_{T_i}^{T_f} \Lambda \to \infty$ as $T_f \to \infty$

and $K_x(T_f) \to \infty$ — " —

in Both cases the divergence from dS, and ness. cancel taking T_i = finite

$T_i = 0$

[1 why: $\int_{T=T_i} BT = 0$

Smarr: $\int_{T_f} K^y_y = \int_{\Sigma} \Lambda$ in dS in a more organized (general) way,

define Killing Pot. ω^{bc} $\quad \nabla_c \omega^{bc} = \xi^b$

$$\Rightarrow \quad \int_\Sigma \sqrt{-g}\, \Lambda\, x_b \xi^b = \Lambda \int_{\partial\Sigma} da_{\varepsilon}\, x_b\, \omega^{bc}$$

(D−1) dim Vol integral $\qquad$ (D−2) Boundary

define $\quad K_{(x)}^{ren}(T_f) = K_x(T_f) - \dfrac{2\Lambda}{D-2} \int_{T_f} da\, t_c\, x_b\, \omega_{ds}^{cd}$

K.P. in ds

"gauge" choice $\quad \omega_{ds}\big|_{T=0} = 0$

Smarr $\longrightarrow$

$$\underbrace{K_x^{ren}(T_f)}_{T_x} = K_x(T_0) + \frac{2\Lambda}{D-2} V_{cos}$$

where $\quad V_{cos} = \int_{T_f} (\omega - \omega_{ds}) - \int_{T_0} \omega$

process : $\quad V_{cos} = \int_{T=0}^{T_0} \sqrt{-g}\Big|_{ds} = \left(\begin{array}{c}\text{vol between} \\ T=0 \text{ and BB}\end{array}\right)$