Renormalization of CP-odd operators of dimension $\leq 5$
Outline

• BSM-induced CP violation at dimension 5
• Operator renormalization in RI-ŠMOM scheme, suitable for lattice implementation
• Matching RI-ŠMOM and \( \overline{\text{MS}} \) at one loop
• Future steps and conclusions

Collaborators:
Tanmoy Bhattacharya, Rajan Gupta, Emanuele Mereghetti, Boram Yoon,
arXiv:1501.xxxx
BSM-induced CP violation at dimension 5
The CP-odd effective Lagrangian

\( \mathcal{L}_{\text{eff}} \) below weak scale, including leading (dim=6) \( \Delta F=0 \) BSM effects:

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}}|_{m_t = 0} - m_i \bar{\psi}_L i \psi_R - m_i^* \bar{\psi}_R i \psi_L - \frac{g^2}{32 \pi^2} \theta G \tilde{G} \\
- \frac{v_{ew}}{2 \Lambda_{\text{BSM}}^2} e \left( d_i^{(\gamma)} \bar{\psi}_L \sigma_{\mu\nu} F^{\mu\nu} \psi_R + d_i^{(\gamma)*} \bar{\psi}_R \sigma_{\mu\nu} F^{\mu\nu} \psi_L \right) \\
- \frac{v_{ew}}{2 \Lambda_{\text{BSM}}^2} g_s \left( d_i^{(g)} \bar{\psi}_L \sigma_{\mu\nu} G^{\mu\nu} \psi_R + d_i^{(g)*} \bar{\psi}_R \sigma_{\mu\nu} G^{\mu\nu} \psi_L \right) \\
+ \frac{d_G}{\Lambda_{\text{BSM}}^2} f^{abc} G_{\mu\nu}^{a} \tilde{G}^{\nu\beta, b} G^{\mu, c} + 4\text{-quark operators}
\]

- Dim 4: CKM + “theta”-term
- Dim 5: quark EDM and CEDM
- Dim 6: gluon CEDM (Weinberg), 4-quark operators
The CP-odd effective Lagrangian

- $\mathcal{L}_{\text{eff}}$ below weak scale, including leading (dim=6) $\Delta F=0$ BSM effects:

$$
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}}|_{m_t=0} - m_i \bar{\psi}_{Li} \psi_{Ri} - m_i^* \bar{\psi}_{Ri} \psi_{Li} - \frac{g^2}{32\pi^2} \theta G \tilde{G} \\
- \frac{v_{ew}}{2\Lambda_{\text{BSM}}^2} e \left( d_{i}^{(\gamma)} \bar{\psi}_{Li} \sigma_{\mu\nu} F^{\mu\nu} \psi_{Ri} + d_{i}^{(\gamma)*} \bar{\psi}_{Ri} \sigma_{\mu\nu} F^{\mu\nu} \psi_{Li} \right) \\
- \frac{v_{ew}}{2\Lambda_{\text{BSM}}^2} g_s \left( d_{i}^{(g)} \bar{\psi}_{Li} \sigma_{\mu\nu} G^{\mu\nu} \psi_{Ri} + d_{i}^{(g)*} \bar{\psi}_{Ri} \sigma_{\mu\nu} G^{\mu\nu} \psi_{Li} \right) \\
+ \frac{d_G}{\Lambda_{\text{BSM}}^2} f^{abc} G_{\mu\nu}^{a} \tilde{G}_{\nu,\beta}^{\beta,b} G_{\mu,c}^{\mu} + 4\text{--quark operators}
$$

- Focus on dim\leq5 operators:
  - Phenomenological relevance of quark EDM & CEDM
  - dim=6 operators not needed to define finite dim=5 operators
The CP-odd effective Lagrangian

- $\mathcal{L}_{\text{eff}}$ below weak scale, including leading (dim=6) $\Delta F=0$ BSM effects:

$$
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}}|_{m_t=0} - m_i \bar{\psi}_{Li} \psi_{Ri} - m_i^* \bar{\psi}_{Ri} \psi_{Li} - \frac{g^2}{32\pi^2} \theta G \tilde{G} \\
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+ \frac{d_G}{\Lambda_{\text{BSM}}^2} f^{abc} G^a_{\mu\nu} \tilde{G}^{\nu\beta,b} G^{\mu,c}_\beta + 4\text{-quark operators}
$$

$$
[d_E] = \frac{v_{ew}}{\Lambda_{\text{BSM}}^2} \begin{pmatrix}
\text{Im} d_u^{(\gamma)} & 0 & 0 \\
0 & \text{Im} d_d^{(\gamma)} & 0 \\
0 & 0 & \text{Im} d_s^{(\gamma)}
\end{pmatrix} \\
[d_M] = \frac{v_{ew}}{\Lambda_{\text{BSM}}^2} \begin{pmatrix}
\text{Re} d_u^{(\gamma)} & 0 & 0 \\
0 & \text{Re} d_d^{(\gamma)} & 0 \\
0 & 0 & \text{Re} d_s^{(\gamma)}
\end{pmatrix}
$$
The CP-odd effective Lagrangian

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}}|_{m_t=0} - m_i \bar{\psi}_L \psi_R - m_i^* \bar{\psi}_R \psi_L - \frac{g^2}{32\pi^2} \theta G \tilde{G} \]
\[ - \frac{v_{ew}}{2\Lambda_{\text{BSM}}^2} \epsilon \left( d_i^{(\gamma)} \bar{\psi}_L \sigma_{\mu\nu} F^{\mu\nu} \psi_R + d_i^{(\gamma)*} \bar{\psi}_R \sigma_{\mu\nu} F^{\mu\nu} \psi_L \right) \]
\[ - \frac{v_{ew}}{2\Lambda_{\text{BSM}}^2} g_s \left( d_i^{(g)} \bar{\psi}_L \sigma_{\mu\nu} G^{\mu\nu} \psi_R + d_i^{(g)*} \bar{\psi}_R \sigma_{\mu\nu} G^{\mu\nu} \psi_L \right) \]
\[ + \frac{d_G}{\Lambda_{\text{BSM}}^2} f^{abc} G^{a}_{\mu\nu} \tilde{G}^{\nu,\beta} G^{\beta,\mu,c} + 4\text{-quark operators} \]

\[ [d_{\text{CE}}] = \frac{v_{ew}}{\Lambda_{\text{BSM}}^2} \begin{pmatrix} \text{Im} \ d_u^{(g)} & 0 & 0 \\ 0 & \text{Im} \ d_d^{(g)} & 0 \\ 0 & 0 & \text{Im} \ d_s^{(g)} \end{pmatrix} \]
\[ [d_{\text{CM}}] = \frac{v_{ew}}{\Lambda_{\text{BSM}}^2} \begin{pmatrix} \text{Re} \ d_u^{(g)} & 0 & 0 \\ 0 & \text{Re} \ d_d^{(g)} & 0 \\ 0 & 0 & \text{Re} \ d_s^{(g)} \end{pmatrix} \]
After vacuum alignment (see Tanmoy Bhattacharya’s talk)

The derivation assumes that quark mass is the dominant source of explicit chiral symmetry breaking.
• After vacuum alignment (see Tanmoy Bhattacharya’s talk)

\[
\delta L_{CPV} = -\bar{\psi} [\delta M] i \gamma_5 \psi - \frac{ie}{2} \bar{\psi} \sigma_{\mu \nu} \gamma_5 F^{\mu \nu} [D_E] Q \psi - \frac{ig_s}{2} \bar{\psi} \sigma_{\mu \nu} \gamma_5 G^{\mu \nu} [D_{CE}] \psi
\]

• No PQ mechanism

\[
\psi = \begin{pmatrix}
u \\ d \\ s
\end{pmatrix}
\]

\[
[\delta M] = m_\star \left( \bar{\theta} - \frac{r}{2} \text{Tr} \left[ M^{-1} \left( [d_{CE}] - m_\star \bar{\theta} M^{-1} [d_{CM}] \right) \right] \right) + \frac{r}{2} \left( [d_{CE}] - m_\star \bar{\theta} M^{-1} [d_{CM}] \right)
\]

\[
M = \begin{pmatrix}
m_u & 0 & 0 \\
0 & m_d & 0 \\
0 & 0 & m_s
\end{pmatrix}
\]

\[
m_\star = \frac{m_u m_d m_s}{m_s (m_u + m_d) + m_u m_d}
\]

\[
r = \frac{\langle \bar{\psi} \sigma^{\mu \nu} g_s G_{\mu \nu} \psi \rangle}{\langle \bar{\psi} \psi \rangle}
\]
• After vacuum alignment (see Tanmoy Bhattacharya’s talk)

\[
\delta L_{\text{CPV}} = -\bar{\psi} [\delta \mathcal{M}] i\gamma_5 \psi - \frac{ie}{2} \bar{\psi} \sigma_{\mu\nu} \gamma_5 F^{\mu\nu} [D_E] Q \psi - \frac{ig_s}{2} \bar{\psi} \sigma_{\mu\nu} \gamma_5 G^{\mu\nu} [D_{CE}] \psi
\]

• No PQ mechanism

Both singlet and non-singlet

Mixture of electric and magnetic s.d. couplings

\[
\mathcal{M} = \begin{pmatrix}
m_u & 0 & 0 \\
0 & m_d & 0 \\
0 & 0 & m_s
\end{pmatrix}
\]

\[
m_* = \frac{m_s m_d m_u}{m_s (m_u + m_d) + m_u m_d}
\]

\[
r = \frac{\langle \bar{\psi} \sigma^{\mu\nu} g_s G_{\mu\nu} \psi \rangle}{\langle \bar{\psi} \psi \rangle}
\]
• After vacuum alignment (see Tanmoy Bhattacharya’s talk)

\[
\delta \mathcal{L}_{CPV} = -\bar{\psi} [\delta \mathcal{M}] \gamma_5 \psi - \frac{ie}{2} \bar{\psi} \sigma_{\mu\nu} \gamma_5 F^{\mu\nu} [D_E] Q \psi - \frac{ig_s}{2} \bar{\psi} \sigma_{\mu\nu} \gamma_5 G^{\mu\nu} [D_{CE}] \psi
\]

• Assume PQ mechanism

\[
\psi = \begin{pmatrix}
  u \\
  d \\
  s
\end{pmatrix}
\]

- Flavor structure controlled by \([d_{CE}]\)
• After vacuum alignment (see Tanmoy Bhattacharya’s talk)

\[ \delta \mathcal{L}_{CPV} = -\bar{\psi} [\delta \mathcal{M}] i\gamma_5 \psi - \frac{ie}{2} \bar{\psi} \sigma_{\mu\nu} \gamma_5 F^{\mu\nu} [D_E] Q \psi - \frac{ig_s}{2} \bar{\psi} \sigma_{\mu\nu} \gamma_5 G^{\mu\nu} [D_{CE}] \psi \]

• To compute \( d_{n,p} (d_E, d_{CE}) \), need nucleon matrix elements of

\[ P = \bar{\psi} i \gamma_5 t^a \psi \]

\[ E = ie \bar{\psi} \sigma_{\mu\nu} \gamma_5 F^{\mu\nu} Q t^a \psi \]

\[ C = ig_s \bar{\psi} \sigma_{\mu\nu} \gamma_5 G^{\mu\nu} t^a \psi \]

t^a represents a flavor diagonal \( n_F \times n_F \) matrix

• Need renormalization of \( P, E, \) and \( C \) in a scheme that can be implemented non-perturbatively, e.g. in lattice QCD
Operator renormalization in RI-ŠMOM scheme
Renormalization: generalities

- **P**: dim=3 quark bilinear, renormalizes multiplicatively
- **E**: tensor quark bilinear \( \times \) EM field strength. Neglecting effects of \( \mathcal{O}(\alpha_{\text{EM}}) \), E renormalizes multiplicatively (as tensor density)

Non-perturbative renormalization well known

Bochicchio et al, 1995
Aoki et al 2009
Renormalization: generalities

- **P**: \( \text{dim}=3 \) quark bilinear, renormalizes multiplicatively

- **E**: tensor quark bilinear \( \times \) EM field strength. Neglecting effects of \( O(\alpha_{\text{EM}}) \), \( E \) renormalizes multiplicatively (as tensor density)

- **C**: self-renormalization + mixing with \( E \) and \( P \)

Even richer mixing structure in subtraction schemes that involve off-shell quarks/gluons and non-zero momentum injection at vertex
Operator basis (I)

- $C = ig_s \overline{\Psi} \sigma_{\mu \nu} \gamma_5 G^{\mu \nu} t^a \Psi$ can mix with two classes of operators:

\[
\begin{pmatrix} O \\ N \end{pmatrix}_{\text{ren}} = \begin{pmatrix} Z_O & Z_{ON} \\ 0 & Z_N \end{pmatrix} \begin{pmatrix} O \\ N \end{pmatrix}_{\text{bare}}
\]

- **O**: gauge-invariant operators with same symmetry properties of $C$, not vanishing by equations of motion (EOM)

- **N**: operators allowed by solution of BRST Ward Identities. Vanish by EOM, need not be gauge invariant. Needed to extract $Z_O$, but do not affect physical matrix elements

Kuger-Stern, Zuber 1975
Joglekar and Lee 1976
Deans-Dixon 1978
Operator basis (II)

- Flavor structure of operators: use "spurion" method

\[ \mathcal{L}_{\text{QED}} + \mathcal{L}_{\text{QCD}} - i g_s / 2 \bar{\Psi} \sigma_{\mu \nu} \gamma_5 G^{\mu \nu} \left[ D_{CE} \right] \Psi \] is invariant under

Quark mass and charge matrices

- Allow only invariant operators, and eventually set

\[ [D_{CE}] \rightarrow t^a \quad (a = 0, 3, 8) \]

\[ t^0 = \frac{1}{\sqrt{6}} I_{3 \times 3} \quad , \quad t^{3,8} = \frac{\lambda^{3,8}}{2} \]
Operator basis (III)

- Dimension-3: 1 operator
  \[ O^{(3)} \equiv P = \bar{\psi} \gamma_5 t^a \psi \]

- Dimension-4: no operators if chiral symmetry is respected

- Dimension-5: 10 + 4 operators
  \[
  O_1^{(5)} \equiv C = i g \bar{\psi} \tilde{\sigma}^{\mu \nu} G_{\mu \nu} t^a \psi
  \]
  \[
  O_2^{(5)} \equiv \partial^2 P = \partial^2 (\bar{\psi} \gamma_5 t^a \psi)
  \]
  \[
  O_3^{(5)} \equiv E = \frac{ie}{2} \bar{\psi} \tilde{\sigma}^{\mu \nu} F_{\mu \nu} \{Q, t^a\} \psi
  \]

\[ \tilde{\sigma}^{\mu \nu} \equiv \frac{1}{2} (\sigma^{\mu \nu} \gamma_5 + \gamma_5 \sigma^{\mu \nu}) \]
Operator basis (III)

- **Dimension-3**: 1 operator
\[ O^{(3)} \equiv P = \bar{\psi} i \gamma_5 t^a \psi \]

- **Dimension-4**: no operators if chiral symmetry is respected

- **Dimension-5**: 10 + 4 operators

\[ O^{(5)}_4 \equiv (m F F) = \text{Tr} \left[ \mathcal{M} Q^2 t^a \right] \frac{1}{2} \epsilon^{\mu \nu \alpha \beta} F_{\mu \nu} F_{\alpha \beta} \]

\[ O^{(5)}_5 \equiv (m G \tilde{G}) = \text{Tr} \left[ \mathcal{M} t^a \right] \frac{1}{2} \epsilon^{\mu \nu \alpha \beta} G_{\mu \nu}^a G_{\alpha \beta}^a \]

\[ O^{(5)}_6 \equiv (m \partial \cdot A)_1 = \text{Tr} \left[ \mathcal{M} t^a \right] \partial_{\mu} (\bar{\psi} \gamma^\mu \gamma_5 \psi) \]

\[ O^{(5)}_7 \equiv (m \partial \cdot A)_2 = \frac{1}{2} \partial_{\mu} (\bar{\psi} \gamma^\mu \gamma_5 \{\mathcal{M}, t^a\} \psi) \quad \text{trace} \]
Operator basis (III)

- Dimension-3: 1 operator

\[ O^{(3)} \equiv P = \bar{\psi} i \gamma_5 t^a \psi \]

- Dimension-4: no operators if chiral symmetry is respected

- Dimension-5: 10 + 4 operators

\[ O_8^{(5)} \equiv (m^2 P)_1 = \frac{1}{2} \bar{\psi} i \gamma_5 \{ \mathcal{M}^2, t^a \} \psi \]

\[ O_9^{(5)} \equiv (m^2 P)_2 = \text{Tr} \left[ \mathcal{M}^2 \right] \bar{\psi} i \gamma_5 t^a \psi \]

\[ O_{10}^{(5)} \equiv (m^2 P)_3 = \text{Tr} \left[ \mathcal{M} t^a \right] \bar{\psi} i \gamma_5 \mathcal{M} \psi \]
Operator basis (III)

- Dimension-3: 1 operator
  \[ O^{(3)} \equiv P = \bar{\psi} i \gamma_5 t^a \psi \]

- Dimension-4: no operators if chiral symmetry is respected

- Dimension-5: 10 + 4 operators
  \[ \psi_E \equiv (iD^\mu \gamma_\mu - \mathcal{M})\psi \]
  \[ O^{(5)}_{11} \equiv P_{EE} = i\bar{\psi}_E \gamma_5 t^a \psi_E \]
  \[ O^{(5)}_{12} \equiv \partial \cdot A_E = \partial_\mu [\bar{\psi}_E \gamma^\mu \gamma_5 t^a \psi + \bar{\psi} \gamma^\mu \gamma_5 t^a \psi_E] \]
  \[ O^{(5)}_{13} \equiv A_\partial = \bar{\psi} \gamma_5 \gamma^a \psi_E - \bar{\psi}_E \gamma_a \gamma_5 \psi \]
  \[ O^{(5)}_{14} \equiv A_{A(\gamma)} = \frac{ie}{2} \left( \bar{\psi} \{Q, t^a\} A^{(\gamma)} \gamma_5 \psi_E - \bar{\psi}_E \{Q, t^a\} A^{(\gamma)} \gamma_5 \psi \right) \]
### Mixing structure

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<thead>
<tr>
<th></th>
<th>$C$</th>
<th>$\delta^2 P$</th>
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Valid in any scheme $\iff$ dimensional analysis, momentum injection, EOM
# Mixing structure

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Physically relevant block \(Z_\circ\)
To identify $[Z_O]_{ij}$, need to study the following Green’s functions:

- $\mathcal{O}_1^{(5)} \equiv C$
- $\mathcal{O}_5^{(5)} \equiv mG\tilde{G}$
- $\mathcal{O}_n^{(5)}$ for $n=2,3,6-10$
Renormalization schemes

- **MS** scheme: use dim-reg and subtract poles in $1/(d-4)$

- Simple, widely used in calculations of Wilson coefficients

- Subtlety: need to specify scheme for $\gamma_5$

  - NDR: $\{\gamma_\mu, \gamma_5\} = 0 \quad \forall \mu$
  - HV: $\{\gamma_\mu, \gamma_5\} = 0$ for $\mu=0-3$, otherwise $[\gamma_\mu, \gamma_5] = 0$
Renormalization schemes

- **\(\overline{\text{MS}}\) scheme**: use dim-reg and subtract poles in \(1/(d-4)\)
- Simple, widely used in calculations of Wilson coefficients
- Subtlety: need to specify scheme for \(\gamma_5\)
  - NDR: \(\{\gamma_\mu, \gamma_5\} = 0 \quad \forall \mu\)
  - HV: \(\{\gamma_\mu, \gamma_5\} = 0\) for \(\mu=0-3\), otherwise \([\gamma_\mu, \gamma_5] = 0\)

- **RI-SMOM** class of schemes: fix finite parts by conditions on quark and gluon amputated Green’s functions in a given gauge, at non-exceptional momentum configurations, such as

\[
p^2 = p'^2 = q^2 = -\Lambda^2
\]

- Regularization independent: can be implemented on the lattice
RI-ŠMOM scheme

- Require conditions on C (14), mG̃ (2), O_{2,3,6-10} (one each)
RI-$\tilde{\text{SMOM}}$ scheme

- Require conditions on $C$ (14), $mG\tilde{G}$ (2), $O_{2,3,6-10}$ (one each)

- On the lattice, first subtract power-divergence ($C \leftrightarrow P$ mixing):

$$C^L \rightarrow C = C^L - Z_{C-P} P$$

\[\gamma_5 \, t^a \text{ projection}\]
\[p^2 = p'^2 = q^2 = -\Lambda_0^2\]

\[= 0 \quad \Rightarrow \quad Z_{C-P} (a, \Lambda_0) \sim 1/a^2\]
RI-$\tilde{\text{S}}$MOM scheme

- Require conditions on $C$ (14), $mG\tilde{G}$ (2), $O_{2,3,6-10}$ (one each)
- Conditions on $C$: amputated 2-pt functions

\[
p^2 = p'^2 = q^2 = -\Lambda^2
\]

Coefficients of 7 spin-flavor structures**

** $\gamma_5 t^a$, $\sigma_{\mu\nu} \gamma_5 p^\mu p'^\nu t^a$, $q^\mu \gamma_\mu \gamma_5 M t^a$, $q^\mu \gamma_\mu \gamma_5 \text{Tr}[M t^a]$, $\gamma_5 M^2 t^a$, $\gamma_5 t^a \text{Tr}[M^2]$, $\gamma_5 M \text{Tr}[M t^a]$
RI-ŠMOM scheme

- Require conditions on $C$ (14), $mG\tilde{G}$ (2), $O_{2,3,6-10}$ (one each)
- Conditions on $C$: amputated 2-pt functions

\[ p^2 = p'^2 = q^2 = -\Lambda^2 \]

Coefficients of 7 spin-flavor structures**

\[ p^2 = p'^2 = q^2 = -\Lambda^2 \]

1 condition for gluons, 1 condition for photons
RI-$\tilde{\text{SMOM}}$ scheme

- Require conditions on $C$ (14), $mG\tilde{G}$ (2), $O_{2,3,6-10}$ (one each)
- Conditions on $C$: amputated 3-pt functions (q-q-gluon)

\[
\begin{align*}
\sigma_{\mu\nu}k^\nu t^a, & \quad \sigma_{\mu\nu} (p-p')^\nu t^a, \quad \gamma^5 (p+p')^\mu t^a
\end{align*}
\]

Kinematics:
\[
\begin{align*}
s &= (p+q)^2 \\
u &= (p-k)^2 \\
t &= (p-p')^2
\end{align*}
\]
RI-$\tilde{\text{S}}$MOM scheme

- Require conditions on $C$ (14), $mG\tilde{G}$ (2), $O_{2,3,6-10}$ (one each)
- Conditions on $C$: amputated 3-pt functions (q-q-gluon)

\[ s = u = t/2 = - \Lambda^2 \]
\[ p^2 = p'^2 = q^2 = k^2 = - \Lambda^2 \]

$S$ point: can't have $s=u=t = - \Lambda^2$ but $s=u = - \Lambda^2$ and conditions on 2pt-function eliminate non-1PI diagrams
RI-$\tilde{\text{S}}$MOM scheme

- Require conditions on $C$ (14), $mG\tilde{G}$ (2), $O_{2,3,6-10}$ (one each)

- Conditions on $C$: amputated 3-pt functions ($q$-$q$-photon)

\[
\begin{align*}
\sigma_{\mu\nu} \gamma_5 k^\nu t^a, & \quad \gamma_5 (p+p')_\mu t^a
\end{align*}
\]

Kinematics:

- $s = (p+q)^2$
- $u = (p-k)^2$
- $t = (p-p')^2$

\[
\begin{align*}
p^2 = p'^2 = q^2 = k^2 &= -\Lambda^2 \\
s = u = t/2 &= -\Lambda^2
\end{align*}
\]

** 2 spin-flavor structures: $\sigma_{\mu\nu} \gamma_5 k^\nu t^a, \quad \gamma_5 (p+p')_\mu t^a$
RI-$\tilde{\text{SMOM}}$ scheme

- Require conditions on $C$ (14), $mG\tilde{G}$ (2), $O_{2,3,6-10}$ (one each)
- Conditions on $(mGG)$: amputated 2-pt functions

\[ p^2 = p'^2 = q^2 = -\Lambda^2 \]

1 spin-flavor structure:
\[ \gamma_{\mu}q^{\mu} \gamma_5 t^a \]

1 condition

\[ p^2 = p'^2 = q^2 = -\Lambda^2 \]

1 tree
RI-ŠMOM scheme

- Require conditions on $C$ (14), $mG\tilde{G}$ (2), $O_{2,3,6-10}$ (one each)
- Conditions on $E$, $(m\partial A)_{1,2}$ and $(m^2P)_{1,2,3}$: amputated 2-pt functions

Conditions are equivalent to RI-SMOM conditions on $A$, $P$, $T$

Aoki et al 2009
Matching RI-ŠMOM and MS at one loop
One-loop calculations

- Insertions of $C$

$Z_{1n}$, $n=2,6-10,11-13$

$Z_{15}$

$Z_{1n}$, $n=1,11-13$

$Z_{1n}$, $n=3,11-14$
One-loop calculations

- Insertions of $mG\tilde{G}$

- Insertions of $E\sim T$, $(m\partial A)_{1,2}$ and $(m^2 P)_{1,2,3}$

$Z_{55}, Z_{56}$

$Z_{nn} \quad n=2,3,6-10$
One-loop calculations

- Schematic form of all 1-loop results

\[ \Gamma = \sum_i (\text{spin } \otimes \text{ flavor})_i \left[ d_i(\xi) \left( \frac{2}{4-d} + \log \frac{\mu^2}{\Lambda^2} \right) + f_i(\xi) \right] \]

- Determine \( Z_\circ \)

\[
Z_{ij}^{\text{MS}} = \delta_{ij} + \frac{1}{\epsilon} \frac{\alpha_s}{4\pi} \, z_{ij}
\]

\[
Z_{ij}^{\text{RI-\tilde{SMOM}}} = \delta_{ij} + \frac{\alpha_s}{4\pi} \left[ \frac{z_{ij}}{\epsilon} + c_{ij} \right]
\]

Depends on scheme adopted for \( \gamma_5 \) (HV, NDR)

Time-consuming part of the calculation

Work in covariant gauge: Landau gauge (\( \xi = 0 \)) can be implemented on the lattice
One-loop calculations

• Schematic form of all 1-loop results

\[ \Gamma = \sum_i (\text{spin } \otimes \text{ flavor})_i \left[ d_i(\xi) \left( \frac{2}{4-d} + \log \frac{\mu^2}{\Lambda^2} \right) + f_i(\xi) \right] \]

 Depends on scheme adopted for \( \gamma_5 \) (HV, NDR)

 Time-consuming part of the calculation

 Work in covariant gauge: Landau gauge (\( \xi=0 \)) can be implemented on the lattice

• Determine \( Z_\Omega \)

\[
Z_{ij}^{\overline{\text{MS}}} = \delta_{ij} + \frac{1}{\epsilon} \frac{\alpha_s}{4\pi} Z_{ij} \\
Z_{ij}^{\text{RI}-\tilde{\text{SMOM}}} = \delta_{ij} + \frac{\alpha_s}{4\pi} \left[ \frac{Z_{ij}}{\epsilon} + \frac{1}{4\pi} c_{ij} \right]
\]

\( \xi \)-independent

\( \xi \)-dependent
One-loop results (I)

- $Z$ in $\overline{\text{MS}}$

\[
\begin{align*}
    z_{11} &= 5C_F - 2C_A \\
    z_{12} &= 0 \\
    z_{13} &= 4C_F \\
    z_{14} &= 0 \\
    z_{15} &= -2 \\
    z_{16} &= C_F - \frac{1}{4} C_A \\
    z_{17} &= 3C_F - \frac{3}{4} C_A \\
    z_{18} &= 6C_F + \frac{3}{2} C_A \\
    z_{19} &= 0 \\
    z_{1,10} &= 0 \\
    z_{1,11} &= 6C_F - \frac{3}{2} C_A \\
    z_{1,12} &= -3C_F + \frac{3}{4} C_A \\
    z_{1,13} &= \frac{3}{4} C_A \\
    z_{1,14} &= \frac{3}{4} C_A . \\
    z_{22} &= -3C_F \\
    z_{33} &= C_F \\
    z_{44} &= 3C_F \\
    z_{55} &= -\frac{11C_A - 4T_F n_F}{3} + 3C_F \\
    z_{56} &= 6C_F \\
    z_{66} &= z_{77} = z_{88} = z_{99} = z_{10,10} = 3C_F \\
    C_F &= \frac{N_C^2 - 1}{2N_C} \\
    C_A &= N_C \\
    T_F &= \frac{1}{2}
\end{align*}
\]
One-loop results (II)

- C-matrix connecting $\overline{\text{MS}}$ and RI-$\tilde{\text{SMOM}}$

\[
O_{i}^{\text{RI-\tilde{SMOM}}} = C_{ij} O_{j}^{\overline{\text{MS}}}
\]

\[
C_{ij} = \left( Z^{\text{RI-\tilde{SMOM}}} \cdot \left( Z^{\overline{\text{MS}}} \right)^{-1} \right)_{ij} \equiv \delta_{ij} + \frac{\alpha_s}{4\pi} c_{ij}
\]
One-loop results (II)

- **C-matrix connecting $\overline{\text{MS}}$ and RI-$\tilde{\text{SMOM}}$**

\[
\begin{align*}
c_{11} &= \frac{C_A(23 + 9\xi)}{12} \psi + \frac{C_A - 2C_F}{2} (1 - \xi) K - \frac{C_A - 2C_F}{2} (1 + \xi) \log 2 \\
&\quad + \frac{10}{9} n_F T_F + C_F \left( \frac{31}{3} - \frac{1}{2} \xi \right) + \frac{C_A}{72} (-646 - 36\xi + 9\xi^2) \\
c_{12} &= \left( 4C_F - \frac{C_A}{6} (3 + \xi) \right) \psi + (C_A - 2C_F)(1 - \xi) K + (C_A - 2C_F)(1 + \xi) \log 2 \\
&\quad + C_F (2 - \xi) + \frac{C_A}{4} (-5 + 2\xi) \\
c_{13} &= \left( -\frac{8}{3} C_F + \frac{C_A}{12} (3 + \xi) \right) \psi - \frac{1}{2} (C_A - 2C_F)(1 - \xi) K - \frac{1}{2} (C_A - 2C_F)(1 + \xi) \log 2 \\
&\quad + C_F \left( \frac{25}{3} + \frac{1}{2} \xi \right) + \frac{C_A}{8} (5 - 2\xi) \\
c_{14} &= 0 \\
c_{15} &= -4 \\
c_{16} &= 3 c_{17} \\
c_{17} &= \frac{C_A}{6} \xi \psi + (C_A - 2C_F)(1 - \xi) K - \frac{1}{2} (C_A - 2C_F)(1 - \xi) \log 2 \\
&\quad + C_F (8 - \xi) + C_A \left( -\frac{13}{4} + \frac{1}{2} \xi \right) \\
c_{18} &= \frac{8C_F - C_A(1 + \xi)}{2} \psi - (C_A - 2C_F)(1 - \xi) K + 2 (C_A - 2C_F) \log 2 \\
&\quad + C_F \left( \frac{10}{3} + \xi \right) + C_A \left( \frac{21}{4} - \frac{1}{2} \xi \right) \\
c_{19} &= 0 \\
c_{11,10} &= 0
\end{align*}
\]

Loop expressed in terms of 1st derivatives of Digamma function:

\[
\begin{align*}
\psi &= \frac{2}{3} \left( \psi^{(1)} \left( \frac{1}{3} \right) - \frac{2}{3} \pi^2 \right) \\
K &= \frac{1}{8} \left( \psi^{(1)} \left( \frac{1}{4} \right) - \pi^2 \right)
\end{align*}
\]

\[
\begin{align*}
c_{22} &= c_{88} = c_{99} = c_{10,10} = \frac{C_F}{2} (3 + \xi) \psi - C_F (12 + \xi) \\
c_{33} &= C_F \left[ (1 - \xi) \left( \frac{4}{3} - \frac{1}{2} \psi \right) + \xi \right] \\
c_{55} &= 2 C_A \left( 1 + \frac{1}{3} \xi \right) \psi + \frac{C_A}{36} (-403 - 18\xi + 9\xi^2) + \frac{20}{9} n_F T_F \\
c_{56} &= -2 C_F (3 + 2\psi) \\
c_{66} &= c_{77} = -4 C_F
\end{align*}
\]
Impact on phenomenology

- Goal: evaluate hadronic CP-odd couplings from

\[
\delta \mathcal{L}_{CPV} \sim c_P^{MS} P^{MS} + c_E^{MS} E^{MS} + c_{CE}^{MS} C^{MS}
\]
Impact on phenomenology

- Goal: evaluate hadronic CP-odd couplings from

\[ \delta \mathcal{L}_{CPV} \sim c_{\overline{MS}}^{P} P^{\overline{MS}} + c_{\overline{MS}}^{E} E^{\overline{MS}} + c_{\overline{MS}}^{CE} C^{\overline{MS}} \]

\[ O_{i}^{\overline{MS}} = [C^{-1}]_{ij} O_{j}^{RI-\tilde{SMOM}} \]

\[ c_{\overline{MS}}^{P} C_{P}^{-1} P^{RI-\tilde{SMOM}} + c_{\overline{MS}}^{E} C_{T}^{-1} E^{RI-\tilde{SMOM}} + c_{\overline{MS}}^{CE} \sum_{n} C_{1n}^{-1} O_{n}^{RI-\tilde{SMOM}} \]

Corrections range from few % to > 30%
Impact on phenomenology

- Goal: evaluate hadronic CP-odd couplings from

\[
\delta \mathcal{L}_{CPV} \sim c_P^{MS} P^{MS} + c_E^{MS} E^{MS} + c_{CE}^{MS} C^{MS}
\]

\[
O_i^{MS} = [C^{-1}]_{ij} O_j^{RI-\tilde{SMOM}}
\]

\[
c_P^{MS} C^{-1}_P P^{RI-\tilde{SMOM}} + c_E^{MS} C^{-1}_T E^{RI-\tilde{SMOM}} + c_{CE}^{MS} \sum_n C^{-1}_{1n} O_n^{RI-\tilde{SMOM}}
\]

Only \( C, E, m\tilde{G}, (m^2P)_{1,2,3} \) contribute to \( \langle n | J^\mu_{EM} \int d^4x O_i(x) | n \rangle \)

Need tensor charge (E) + P, C insertions
Steps towards LQCD implementation

• Neutron EDM from quark EDM (E): tensor charge (see B. Yoon’ talk)

• Neutron EDM from quark CEDM operator (C):

  1. Carry out non-perturbative renormalization: requires qq, gg, qqq correlation functions with insertion of $O_i$, $i=1,14$.

  2. Extract CPV form factor: tensor charge + correlation of P and C with $J_{EM}$ in the nucleon

\[
\langle n | J_{EM}^{\mu} \int d^4x \ O_i(x) | n \rangle
\]
Conclusions

- Defined RI-ŠMOM scheme for CEDM and other CP-odd operators of dim $\leq 5$, suitable for implementation in LQCD
- Computed one-loop matching factors between $\overline{\text{MS}}$ and RI-ŠMOM
- First step towards LQCD calculation of $d_n(d_{CE})$. Future work:
  - Exploratory studies on the lattice, estimate resources
  - CMDM renormalization (vs CEDM), relevant to the extraction $\piNN$ CP-odd couplings
  - Look at dim-6 operators
Backup slides
Axial Ward Identities

- In a given scheme, operators satisfy renormalized PCAC relation

\[ \bar{C}_1(g^2)[\partial \cdot A] = \bar{C}_2(g^2) \ 2i[mP] + \bar{C}_3(g^2) \frac{n_F}{16\pi^2} [g^2 \ G\tilde{G}] + iP_E \]

\[ A_\mu = \bar{\psi}(1/2)[\gamma_\mu, \gamma_5]\psi \quad (mP) \equiv \bar{\psi}M\gamma_5\psi \quad P_E = \bar{\psi}_E\gamma_5\psi + \bar{\psi}\gamma_5\psi_E \]
Axial Ward Identities

- In a given scheme, operators satisfy renormalized PCAC relation

\[
\overline{C}_1(g^2) [\partial \cdot A] = \overline{C}_2(g^2) 2i[mP] + \overline{C}_3(g^2) \frac{n_F}{16\pi^2} [g^2 \tilde{G}\tilde{G}] + iP_E
\]

\[
A_\mu = \bar{\psi}(1/2)[\gamma_\mu, \gamma_5]\psi \quad \quad (mP) \equiv \bar{\psi} \mathcal{M}\gamma_5\psi \quad \quad P_E = \bar{\psi}\gamma_5\psi + \bar{\psi}\gamma_5\psi_E
\]

- $\overline{C}_i(g^2) \neq 1$ are finite coefficients related to $Z_{ij}$ and $\alpha, \beta, \gamma$

\[
\tilde{X} = X + \alpha \partial \cdot A + \beta 2i(mP) + \gamma \tilde{G}\tilde{G}
\]

Evanescent operator: its insertions vanish when removing regulator

Explicit form of $X$ in dim-reg

$X = \frac{1}{2} \bar{\psi} \left\{ \gamma_5, \vec{\gamma} \vec{\nu} - \vec{\nu} \vec{\gamma} \right\} \psi$

$\alpha, \beta, \gamma$ calculable (non)-perturbatively
Axial Ward Identities

- In a given scheme, operators satisfy renormalized PCAC relation

\[
\overline{C}_1(g^2) [\partial \cdot A] = \overline{C}_2(g^2) 2i [mP] + \overline{C}_3(g^2) \frac{n_F}{16\pi^2} [g^2 G\tilde{G}] + iP_E
\]

\[
A_\mu = \overline{\psi}(1/2) [\gamma_\mu, \gamma_5] \psi \quad (mP) \equiv \overline{\psi} M \gamma_5 \psi \quad P_E = \overline{\psi}_E \gamma_5 \psi + \overline{\psi} \gamma_5 \psi_E
\]

Finite rescaling leads to properly normalized WI

\[
[A^\mu]_{WI} = \overline{C}_1(g^2) [A^\mu] \\
[mP]_{WI} = \overline{C}_2(g^2) [mP] \\
[g^2 G\tilde{G}]_{WI} = \overline{C}_3(g^2) [g^2 G\tilde{G}]
\]

\[
\partial \cdot [A]_{WI} = 2i [mP]_{WI} + \frac{n_F}{16\pi^2} [g^2 G\tilde{G}]_{WI} + iP_E
\]

\[
[mP]_{WI} [g^2 G\tilde{G}]_{WI}
\]

have no anomalous dimension, while

\[
\gamma_{AWI} = \frac{\alpha_s}{4\pi} \gamma_{G\tilde{G}, \partial \cdot A}
\]
Axial Ward Identities

- In a given scheme, operators satisfy renormalized PCAC relation

\[
\begin{align*}
\overline{C}_1(g^2)[\partial \cdot A] &= \overline{C}_2(g^2) 2i[mP] + \overline{C}_3(g^2) \frac{n_F}{16\pi^2} [g^2G \tilde{G}] + iP_E \\
A_\mu &= \bar{\psi}(1/2)[\gamma_\mu, \gamma_5]\psi \\
(mP) &\equiv \bar{\psi}M\gamma_5\psi \\
P_E &= \bar{\psi}_E\gamma_5\psi + \bar{\psi}\gamma_5\psi_E
\end{align*}
\]

- Explicit scheme-dependent rescaling:

\[
\begin{align*}
\overline{C}_1 &= 1 - 4C_F \frac{\alpha_s}{4\pi} + O(\alpha_s^2) \\
\overline{C}_2 &= 1 - 8C_F \frac{\alpha_s}{4\pi} + O(\alpha_s^2) \\
\overline{C}_3 &= 1 + O(\alpha_s^2) \\
\overline{C}_1 &= 1 + O(\alpha_s^2) \\
\overline{C}_2 &= 1 + O(\alpha_s^2) \\
\overline{C}_3 &= 1 + \frac{Z_g^2}{Z_{G\tilde{G}}} \bigg|_{\text{RI-SMOM}}
\end{align*}
\]

\[\text{MS-HV} \quad \text{RI-SMOM}\]
Extraction of nEDM from qCEDM

- Extraction of the CPV form factor

\[
\langle n | J_{\mu}^{EM} | n \rangle \sim d_n \bar{\psi}_n \sigma_{\mu\nu} \gamma_5 q^\nu \psi_n
\]

\[
J_{\mu}^{EM} = \frac{2}{3} \bar{u} \gamma_\mu u - \frac{1}{3} \bar{d} \gamma_\mu d - \frac{1}{3} \bar{s} \gamma_\mu s
\]

- Requires 4-point function
Extraction of nEDM from qCEDM

- Extraction of the CPV form factor

\[
\langle n | J_{\mu}^{\text{EM}} | n \rangle \sim d_n \bar{\psi}_n \sigma_{\mu \nu} \gamma_5 q^\nu \psi_n
\]

\[
J_{\mu}^{\text{EM}} = \frac{2}{3} \bar{u} \gamma_{\mu} u - \frac{1}{3} \bar{d} \gamma_{\mu} d - \frac{1}{3} \bar{s} \gamma_{\mu} s
\]

- Or 3-point function in external background E field

\[
\langle n | J_{\mu}^{\text{EM}} | \int d^4 x \ O_i(x) \ | n \rangle = \frac{\partial}{\partial A_\mu} \langle n | \int d^4 x \ O_i(x) \ | n \rangle_E
\]