Hadronic matrix elements for probes of CP violation
ACFI, January 22-24 2015

# Renormalization of CP-odd operators of dimension $\leq 5$

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# Outline

- BSM-induced CP violation at dimension 5
- Operator renormalization in RI-$\tilde{\text{S}}$MOM scheme, suitable for lattice implementation
- Matching RI-$\tilde{\text{S}}$MOM and $\overline{\text{MS}}$ at one loop
- Future steps and conclusions

Collaborators:
Tanmoy Bhattacharya, Rajan Gupta, Emanuele Mereghetti, Boram Yoon,
arXiv:1501.xxxx

# BSM-induced CP violation at dimension 5

# The CP-odd effective Lagrangian

- $\mathcal{L}_{\text{eff}}$ below weak scale, including leading (dim=6) ΔF=0 BSM effects:

$$
\begin{aligned}
\mathcal{L}_{\text{eff}} \;=\; & \mathcal{L}_{\text{SM}}|_{m_i=0} - m_i \bar{\psi}_{Li}\,\psi_{Ri} - m_i^* \bar{\psi}_{Ri}\,\psi_{Li} - \frac{g^2}{32\pi^2}\theta G\tilde{G} \\
- \; & \frac{v_{ew}}{2\Lambda_{\text{BSM}}^2} e \left( d_i^{(\gamma)}\,\bar{\psi}_{Li}\sigma_{\mu\nu}F^{\mu\nu}\,\psi_{Ri} \;+\; d_i^{(\gamma)*}\,\bar{\psi}_{Ri}\sigma_{\mu\nu}F^{\mu\nu}\,\psi_{Li} \right) \\
- \; & \frac{v_{ew}}{2\Lambda_{\text{BSM}}^2} g_s \left( d_i^{(g)}\,\bar{\psi}_{Li}\sigma_{\mu\nu}G^{\mu\nu}\,\psi_{Ri} \;+\; d_i^{(g)*}\,\bar{\psi}_{Ri}\sigma_{\mu\nu}G^{\mu\nu}\,\psi_{Li} \right) \\
+ \; & \frac{d_G}{\Lambda_{\text{BSM}}^2}\, f^{abc} G^a_{\mu\nu} \tilde{G}^{\nu\beta,b} G^{\mu,c}_{\beta} \;+\; 4\text{−quark operators}
\end{aligned}
$$

- Dim 4: CKM + “theta”-term
- Dim 5: quark EDM and CEDM
- Dim 6: gluon CEDM (Weinberg), 4-quark operators

# The CP-odd effective Lagrangian

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-\;& \frac{v_{ew}}{2\Lambda_{\text{BSM}}^2} e \left( d_i^{(\gamma)}\,\bar{\psi}_{Li}\sigma_{\mu\nu}F^{\mu\nu}\,\psi_{Ri} \;+\; d_i^{(\gamma)*}\,\bar{\psi}_{Ri}\sigma_{\mu\nu}F^{\mu\nu}\,\psi_{Li} \right) \\
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\end{aligned}
$$

- Focus on dim≤5 operators:
  - Phenomenological relevance of quark EDM & CEDM
  - dim=6 operators not needed to define finite dim=5 operators

# The CP-odd effective Lagrangian

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&- \frac{v_{ew}}{2\Lambda_{\text{BSM}}^2} e \left( d_i^{(\gamma)}\, \bar{\psi}_{Li}\sigma_{\mu\nu}F^{\mu\nu}\,\psi_{Ri} \;+\; d_i^{(\gamma)*}\, \bar{\psi}_{Ri}\sigma_{\mu\nu}F^{\mu\nu}\,\psi_{Li} \right) \\
&- \frac{v_{ew}}{2\Lambda_{\text{BSM}}^2} g_s \left( d_i^{(g)}\, \bar{\psi}_{Li}\sigma_{\mu\nu}G^{\mu\nu}\,\psi_{Ri} \;+\; d_i^{(g)*}\, \bar{\psi}_{Ri}\sigma_{\mu\nu}G^{\mu\nu}\,\psi_{Li} \right) \\
&+ \frac{d_G}{\Lambda_{\text{BSM}}^2}\, f^{abc} G^a_{\mu\nu} \tilde{G}^{\nu\beta,b} G^{\mu,c}_{\beta} \;+\; 4\text{−quark operators}
\end{aligned}
$$

$$
[d_E] = \frac{v_{\text{ew}}}{\Lambda_{\text{BSM}}^2} \begin{pmatrix} \text{Im}\, d_u^{(\gamma)} & 0 & 0 \\ 0 & \text{Im}\, d_d^{(\gamma)} & 0 \\ 0 & 0 & \text{Im}\, d_s^{(\gamma)} \end{pmatrix}
\qquad
[d_M] = \frac{v_{\text{ew}}}{\Lambda_{\text{BSM}}^2} \begin{pmatrix} \text{Re}\, d_u^{(\gamma)} & 0 & 0 \\ 0 & \text{Re}\, d_d^{(\gamma)} & 0 \\ 0 & 0 & \text{Re}\, d_s^{(\gamma)} \end{pmatrix}
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# The CP-odd effective Lagrangian

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&- \frac{v_{ew}}{2\Lambda_{\text{BSM}}^2} g_s \left( d_i^{(g)}\,\bar{\psi}_{Li}\sigma_{\mu\nu}G^{\mu\nu}\,\psi_{Ri} \;+\; d_i^{(g)*}\,\bar{\psi}_{Ri}\sigma_{\mu\nu}G^{\mu\nu}\,\psi_{Li} \right) \\
&+ \frac{d_G}{\Lambda_{\text{BSM}}^2}\, f^{abc} G^a_{\mu\nu} \tilde{G}^{\nu\beta,b} G^{\mu,c}_{\beta} \;+\; 4\text{−quark operators}
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$$
[d_{CE}] = \frac{v_{\text{ew}}}{\Lambda_{\text{BSM}}^2} \begin{pmatrix} \text{Im}\, d_u^{(g)} & 0 & 0 \\ 0 & \text{Im}\, d_d^{(g)} & 0 \\ 0 & 0 & \text{Im}\, d_s^{(g)} \end{pmatrix}
\qquad
[d_{CM}] = \frac{v_{\text{ew}}}{\Lambda_{\text{BSM}}^2} \begin{pmatrix} \text{Re}\, d_u^{(g)} & 0 & 0 \\ 0 & \text{Re}\, d_d^{(g)} & 0 \\ 0 & 0 & \text{Re}\, d_s^{(g)} \end{pmatrix}
$$

- After vacuum alignment (see Tanmoy Bhattacharya's talk)

$$\delta\mathcal{L}_{CPV} = -\bar{\psi}\,[\delta\mathcal{M}]\,i\gamma_5\psi - \frac{ie}{2}\,\bar{\psi}\sigma_{\mu\nu}\gamma_5 F^{\mu\nu}\,[D_E]\,Q\,\psi - \frac{ig_s}{2}\,\bar{\psi}\sigma_{\mu\nu}\gamma_5\,G^{\mu\nu}\,[D_{CE}]\,\psi$$

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

The derivation assumes that quark mass is the dominant source of explicit chiral symmetry breaking

- After vacuum alignment (see Tanmoy Bhattacharya's talk)

$$\delta\mathcal{L}_{CPV} = -\bar{\psi}\,[\delta\mathcal{M}]\,i\gamma_5\psi - \frac{ie}{2}\,\bar{\psi}\sigma_{\mu\nu}\gamma_5 F^{\mu\nu}\,[D_E]\,Q\,\psi - \frac{ig_s}{2}\,\bar{\psi}\sigma_{\mu\nu}\gamma_5\,G^{\mu\nu}\,[D_{CE}]\,\psi$$

- No PQ mechanism

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

Both singlet and non-singlet

$$\begin{aligned}[\delta\mathcal{M}] &= m_*\left(\bar{\theta} - \frac{r}{2}\mathrm{Tr}\Big[\mathcal{M}^{-1}\left([d_{CE}] - m_*\bar{\theta}\mathcal{M}^{-1}\,[d_{CM}]\right)\Big]\right) \\ &+ \frac{r}{2}\Big(\,[d_{CE}] - m_*\bar{\theta}\mathcal{M}^{-1}\,[d_{CM}]\,\Big)\end{aligned}$$

$$\mathcal{M} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \qquad m_* = \frac{m_s m_d m_u}{m_s(m_u + m_d) + m_u m_d} \qquad r = \frac{\langle\bar{\psi}\sigma^{\mu\nu} g_s G_{\mu\nu}\psi\rangle}{\langle\bar{\psi}\psi\rangle}$$

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$$\begin{aligned}
[\delta\mathcal{M}] &= m_*\left(\bar{\theta} - \frac{r}{2}\mathrm{Tr}\Big[\mathcal{M}^{-1}\left([d_{CE}] - m_*\bar{\theta}\mathcal{M}^{-1}\,[d_{CM}]\right)\Big]\right) \\
&+ \frac{r}{2}\Big(\,[d_{CE}] - m_*\bar{\theta}\mathcal{M}^{-1}\,[d_{CM}]\,\Big) \\
[D_{CE}] &= [d_{CE}] - m_*\bar{\theta}\mathcal{M}^{-1}\,[d_{CM}] \\
[D_E] &= [d_E] - m_*\bar{\theta}\mathcal{M}^{-1}\,[d_M]
\end{aligned}$$

Mixture of electric and magnetic s.d. couplings

$$\mathcal{M} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \qquad m_* = \frac{m_s m_d m_u}{m_s(m_u + m_d) + m_u m_d} \qquad r = \frac{\langle\bar{\psi}\sigma^{\mu\nu} g_s G_{\mu\nu}\psi\rangle}{\langle\bar{\psi}\psi\rangle}$$

- After vacuum alignment (see Tanmoy Bhattacharya's talk)

$$\delta\mathcal{L}_{CPV} = -\bar{\psi}\,[\delta\mathcal{M}]\,i\gamma_5\psi - \frac{ie}{2}\,\bar{\psi}\sigma_{\mu\nu}\gamma_5 F^{\mu\nu}\,[D_E]\,Q\,\psi - \frac{ig_s}{2}\,\bar{\psi}\sigma_{\mu\nu}\gamma_5\,G^{\mu\nu}\,[D_{CE}]\,\psi$$

- Assume PQ mechanism

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

Flavor structure controlled by $[d_{CE}]$

$$\begin{aligned} [\delta\mathcal{M}] &= \frac{r}{2}\,[d_{CE}] \\ [D_{CE}] &= [d_{CE}] \\ [D_E] &= [d_E] \end{aligned}$$

- After vacuum alignment (see Tanmoy Bhattacharya's talk)

$$\delta\mathcal{L}_{CPV} = -\bar{\psi}\,[\delta\mathcal{M}]\,i\gamma_5\psi - \frac{ie}{2}\,\bar{\psi}\sigma_{\mu\nu}\gamma_5 F^{\mu\nu}\,[D_E]\,Q\,\psi - \frac{ig_s}{2}\,\bar{\psi}\sigma_{\mu\nu}\gamma_5\,G^{\mu\nu}\,[D_{CE}]\,\psi$$

- To compute $d_{n,p}$ ($d_E$, $d_{CE}$), need nucleon matrix elements of

$$P = \bar{\psi} i\gamma_5 t^a \psi$$

$$E = ie\,\bar{\psi}\sigma_{\mu\nu}\gamma_5 F^{\mu\nu}\,Q t^a\,\psi$$

$$C = ig_s\,\bar{\psi}\sigma_{\mu\nu}\gamma_5\,G^{\mu\nu}\,t^a\,\psi$$

$t^a$ represents a flavor diagonal $n_F \times n_F$ matrix

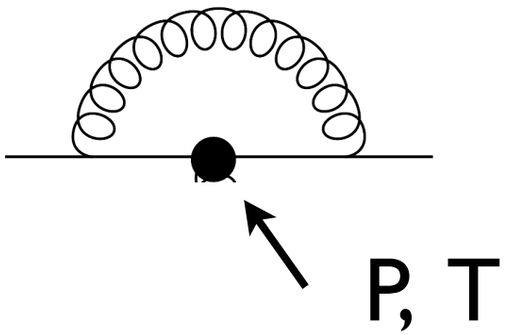
- Need renormalization of P, E, and C in a scheme that can be implemented non-perturbatively, e.g. in lattice QCD

# Operator renormalization in $\widetilde{\text{RI-SMOM}}$ scheme

# Renormalization: generalities

- P: dim=3 quark bilinear, renormalizes multiplicatively

- E: tensor quark bilinear x EM field strength. Neglecting effects of $O(\alpha_{EM})$, E renormalizes multiplicatively (as tensor density)




Non-perturbative renormalization well known

Bochicchio et al,1995
...
Aoki et al 2009

# Renormalization: generalities

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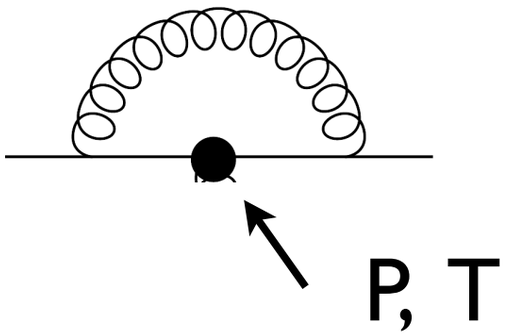
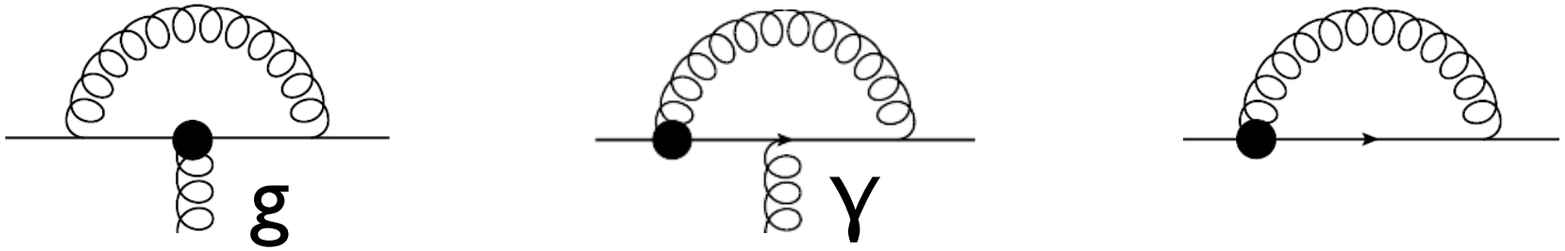



- C: self-renormalization + mixing with E and P




Even richer mixing structure in subtraction schemes that involve off-shell quarks/gluons and non-zero momentum injection at vertex

# Operator basis (I)

- $C = ig_s \bar{\Psi}\sigma_{\mu\nu}\gamma_5 G^{\mu\nu}t^a\Psi$ can mix with two classes of operators:

Kuger-Stern Zuber 1975
Joglekar and Lee 1976
Deans-Dixon 1978

$$\begin{pmatrix} O \\ N \end{pmatrix}_{\text{ren}} = \begin{pmatrix} Z_O & Z_{ON} \\ 0 & Z_N \end{pmatrix} \begin{pmatrix} O \\ N \end{pmatrix}_{\text{bare}}$$

- O: gauge-invariant operators with same symmetry properties of C, not vanishing by equations of motion (EOM)

- N: operators allowed by solution of BRST Ward Identities.
  Vanish by EOM, need not be gauge invariant.
  Needed to extract $Z_O$, but do not affect physical matrix elements

# Operator basis (II)

- Flavor structure of operators: use "spurion" method
- $\mathcal{L}_{QED} + \mathcal{L}_{QCD} - i(g_s/2)\,\overline{\Psi}\sigma_{\mu\nu}\gamma_5 G^{\mu\nu}\,[D_{CE}]\Psi$ invariant under

$$\boxed{\begin{aligned} \psi_{L,R} &\to U_{L,R}\,\psi_{L,R} \\ \mathcal{M} &\to U_L \mathcal{M} U_R^\dagger \\ Q &\to U_{L,R} Q U_{L,R}^\dagger \\ [D_{CE}] &\to U_L [D_{CE}] U_R^\dagger \end{aligned}} \qquad U_{L,R} \in SU(3)_{L,R}$$

Quark mass and charge matrices

- Allow only invariant operators, and eventually set

$$\boxed{[D_{CE}] \to t^a \qquad (a = 0, 3, 8)} \qquad t^0 = \frac{1}{\sqrt{6}} I_{3\times 3}\ , \quad t^{3,8} = \frac{\lambda^{3,8}}{2}$$

# Operator basis (III)

- Dimension-3: 1 operator $O^{(3)} \equiv P = \bar{\psi} i \gamma_5 t^a \psi$

- Dimension-4: no operators if chiral symmetry is respected

- Dimension-5: 10 + 4 operators

$O\left(\mathcal{M}^0\right)$

$$O_1^{(5)} \equiv C = ig\,\bar{\psi}\tilde{\sigma}^{\mu\nu} G_{\mu\nu} t^a \psi$$

$$O_2^{(5)} \equiv \partial^2 P = \partial^2 \left(\bar{\psi} i \gamma_5 t^a \psi\right)$$

$$O_3^{(5)} \equiv E = \frac{ie}{2}\,\bar{\psi}\tilde{\sigma}^{\mu\nu} F_{\mu\nu} \{Q, t^a\} \psi$$

$$\tilde{\sigma}^{\mu\nu} \equiv \frac{1}{2}\left(\sigma^{\mu\nu}\gamma_5 + \gamma_5 \sigma^{\mu\nu}\right)$$

# Operator basis (III)

- Dimension-3: 1 operator $O^{(3)} \equiv P = \bar{\psi} i \gamma_5 t^a \psi$

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$O\left(\mathcal{M}^1\right)$

$$O_4^{(5)} \equiv (m\, F\tilde{F}) = \mathrm{Tr}\left[\mathcal{M} Q^2 t^a\right] \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

$$O_5^{(5)} \equiv (m\, G\tilde{G}) = \mathrm{Tr}\left[\mathcal{M} t^a\right] \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} G^b_{\mu\nu} G^b_{\alpha\beta}$$

$$O_6^{(5)} \equiv (m\, \partial \cdot A)_1 = \mathrm{Tr}\left[\mathcal{M} t^a\right] \partial_\mu \left(\bar{\psi}\gamma^\mu\gamma_5\psi\right)$$

$$O_7^{(5)} \equiv (m\, \partial \cdot A)_2 = \frac{1}{2}\partial_\mu \left(\bar{\psi}\gamma^\mu\gamma_5 \left\{\mathcal{M}, t^a\right\}\psi\right) - \text{trace}$$

# Operator basis (III)

- Dimension-3: 1 operator $O^{(3)} \equiv P = \bar{\psi} i \gamma_5 t^a \psi$

- Dimension-4: no operators if chiral symmetry is respected

- Dimension-5: 10 + 4 operators

$O\left(\mathcal{M}^2\right)$

$$O_8^{(5)} \equiv (m^2 P)_1 = \frac{1}{2} \bar{\psi} i \gamma_5 \left\{\mathcal{M}^2, t^a\right\} \psi$$

$$O_9^{(5)} \equiv (m^2 P)_2 = \mathrm{Tr}\left[\mathcal{M}^2\right] \; \bar{\psi} i \gamma_5 t^a \psi$$

$$O_{10}^{(5)} \equiv (m^2 P)_3 = \mathrm{Tr}\left[\mathcal{M} t^a\right] \; \bar{\psi} i \gamma_5 \mathcal{M} \psi$$

# Operator basis (III)

- Dimension-3: 1 operator $O^{(3)} \equiv P = \bar{\psi} i \gamma_5 t^a \psi$

- Dimension-4: no operators if chiral symmetry is respected

- Dimension-5: 10 + 4 operators

$$\psi_E \equiv (i D^\mu \gamma_\mu - \mathcal{M})\psi$$

$$O_{11}^{(5)} \equiv P_{EE} = i\bar{\psi}_E \gamma_5 t^a \psi_E$$

$$O_{12}^{(5)} \equiv \partial \cdot A_E = \partial_\mu [\bar{\psi}_E \gamma^\mu \gamma_5 t^a \psi + \bar{\psi} \gamma^\mu \gamma_5 t^a \psi_E]$$

$$O_{13}^{(5)} \equiv A_\partial = \bar{\psi} \gamma_5 \not{\partial} t^a \psi_E - \bar{\psi}_E \overleftarrow{\not{\partial}} \gamma_5 t^a \psi$$

$$O_{14}^{(5)} \equiv A_{A^{(\gamma)}} = \frac{ie}{2} \left( \bar{\psi} \{Q, t^a\} \not{A}^{(\gamma)} \gamma_5 \psi_E - \bar{\psi}_E \{Q, t^a\} \not{A}^{(\gamma)} \gamma_5 \psi \right)$$

EOM

# Mixing structure

| | $C$ | $\partial^2 P$ | $E$ | $mF\tilde{F}$ | $mG\tilde{G}$ | $(m\partial\cdot A)_1$ | $(m\partial\cdot A)_2$ | $(m^2P)_1$ | $(m^2P)_2$ | $(m^2P)_3$ | $P_{EE}$ | $\partial\cdot A_E$ | $A_\partial$ | $A_{A(\gamma)}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $C$ | X | X | X | X | X | X | X | X | X | X | X | X | X | X |
| $\partial^2 P$ | | X | | | | | | | | | | | | |
| $E$ | | | X | | | | | | | | | | | |
| $mF\tilde{F}$ | | | | X | | | | | | | | | | |
| $mG\tilde{G}$ | | | | | X | X | | | | | | | | |
| $(m\partial\cdot A)_1$ | | | | | | X | | | | | | | | |
| $(m\partial\cdot A)_2$ | | | | | | | X | | | | | | | |
| $(m^2P)_1$ | | | | | | | | X | | | | | | |
| $(m^2\hat{P})_2$ | | | | | | | | | X | | | | | |
| $(m^2\hat{P})_3$ | | | | | | | | | | X | | | | |
| $P_{EE}$ | | | | | | | | | | | X | X | X | X |
| $\partial\cdot A_E$ | | | | | | | | | | | | X | | |
| $A_\partial$ | | | | | | | | | | | X | X | X | X |
| $A_{A(\gamma)}$ | | | | | | | | | | | | | | X |

Valid in any scheme $\Leftarrow$ dimensional analysis, momentum injection, EOM

# Mixing structure

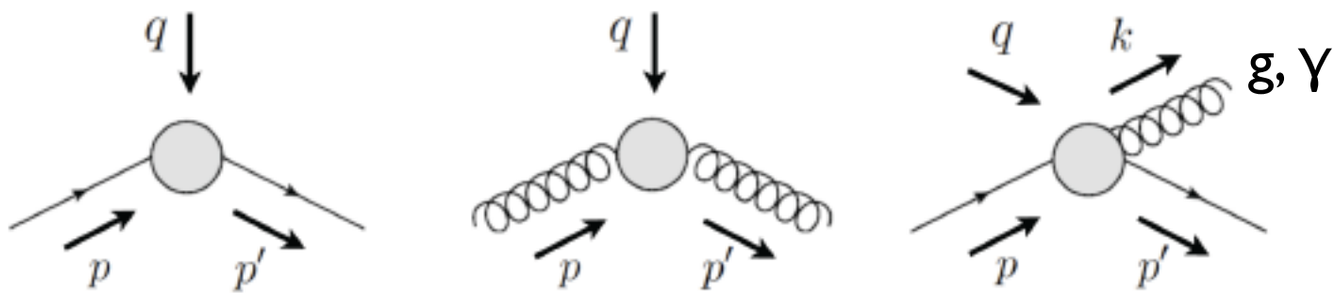
| | $C$ | $\partial^2 P$ | $E$ | $mF\tilde{F}$ | $mG\tilde{G}$ | $(m\partial\cdot A)_1$ | $(m\partial\cdot A)_2$ | $(m^2P)_1$ | $(m^2P)_2$ | $(m^2P)_3$ | $P_{EE}$ | $\partial\cdot A_E$ | $A_\partial$ | $A_{A(\gamma)}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $C$ | X | X | X | X | X | X | X | X | X | X | X | X | X | X |
| $\partial^2 P$ | | X | | | | | | | | | | | | |
| $E$ | | | X | | | | | | | | | | | |
| $mF\tilde{F}$ | | | | X | | | | | | | | | | |
| $mG\tilde{G}$ | | | | | X | X | | | | | | | | |
| $(m\partial\cdot A)_1$ | | | | | | X | | | | | | | | |
| $(m\partial\cdot A)_2$ | | | | | | | X | | | | | | | |
| $(m^2P)_1$ | | | | | | | | X | | | | | | |
| $(m^2\hat{P})_2$ | | | | | | | | | X | | | | | |
| $(m^2\hat{P})_3$ | | | | | | | | | | X | | | | |
| $P_{EE}$ | | | | | | | | | | | X | X | X | X |
| $\partial\cdot A_E$ | | | | | | | | | | | | X | | |
| $A_\partial$ | | | | | | | | | | | X | X | X | X |
| $A_{A(\gamma)}$ | | | | | | | | | | | | | | X |

Physically relevant block $Z_0$

# Mixing structure

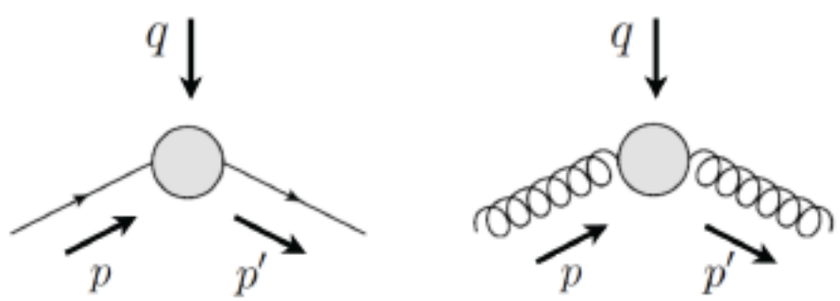
- To identify $[Z_O]_{ij}$ , need to study the following Green's functions:

$O_1^{(5)} \equiv C$




| | $C$ | $\partial^2 P$ | $E$ | $mF\tilde{F}$ | $mG\tilde{G}$ | $(m\partial\cdot A)_1$ | $(m\partial\cdot A)_2$ | $(m^2P)_1$ | $(m^2P)_2$ | $(m^2P)_3$ | $P_{EE}$ | $\partial\cdot A_E$ | $A_\partial$ | $A_{A^{(\gamma)}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $C$ | x | x | x | x | x | x | x | x | x | x | x | x | x | x |
| $\partial^2 P$ | | x | | | | | | | | | | | | |
| $E$ | | | x | | | | | | | | | | | |
| $mF\tilde{F}$ | | | | x | | | | | | | | | | |
| $mG\tilde{G}$ | | | | | x | x | | | | | | | | |
| $(m\partial\cdot A)_1$ | | | | | | x | | | | | | | | |
| $(m\partial\cdot A)_2$ | | | | | | | x | | | | | | | |
| $(m^2P)_1$ | | | | | | | | x | | | | | | |
| $(m^2\tilde{P})_2$ | | | | | | | | | x | | | | | |
| $(m^2\tilde{P})_3$ | | | | | | | | | | x | | | | |
| $P_{EE}$ | | | | | | | | | | | x | x | x | |
| $\partial\cdot A_E$ | | | | | | | | | | | | x | | |
| $A_\partial$ | | | | | | | | | | | x | x | x | |
| $A_{A^{(\gamma)}}$ | | | | | | | | | | | | | | x |

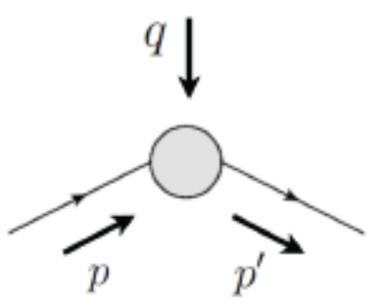
$O_5^{(5)} \equiv mG\tilde{G}$




| | $C$ | $\partial^2 P$ | $E$ | $mF\tilde{F}$ | $mG\tilde{G}$ | $(m\partial\cdot A)_1$ | $(m\partial\cdot A)_2$ | $(m^2P)_1$ | $(m^2P)_2$ | $(m^2P)_3$ | $P_{EE}$ | $\partial\cdot A_E$ | $A_\partial$ | $A_{A^{(\gamma)}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $C$ | x | x | x | x | x | x | x | x | x | x | x | x | x | x |
| $\partial^2 P$ | | x | | | | | | | | | | | | |
| $E$ | | | x | | | | | | | | | | | |
| $mF\tilde{F}$ | | | | x | | | | | | | | | | |
| $mG\tilde{G}$ | | | | | x | x | | | | | | | | |
| $(m\partial\cdot A)_1$ | | | | | | x | | | | | | | | |
| $(m\partial\cdot A)_2$ | | | | | | | x | | | | | | | |
| $(m^2P)_1$ | | | | | | | | x | | | | | | |
| $(m^2\tilde{P})_2$ | | | | | | | | | x | | | | | |
| $(m^2\tilde{P})_3$ | | | | | | | | | | x | | | | |
| $P_{EE}$ | | | | | | | | | | | x | x | x | |
| $\partial\cdot A_E$ | | | | | | | | | | | | x | | |
| $A_\partial$ | | | | | | | | | | | x | x | x | |
| $A_{A^{(\gamma)}}$ | | | | | | | | | | | | | | x |

$O_n^{(5)}$

n=2,3,6-10




| | $C$ | $\partial^2 P$ | $E$ | $mF\tilde{F}$ | $mG\tilde{G}$ | $(m\partial\cdot A)_1$ | $(m\partial\cdot A)_2$ | $(m^2P)_1$ | $(m^2P)_2$ | $(m^2P)_3$ | $P_{EE}$ | $\partial\cdot A_E$ | $A_\partial$ | $A_{A^{(\gamma)}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $C$ | x | x | x | x | x | x | x | x | x | x | x | x | x | x |
| $\partial^2 P$ | | x | | | | | | | | | | | | |
| $E$ | | | x | | | | | | | | | | | |
| $mF\tilde{F}$ | | | | x | | | | | | | | | | |
| $mG\tilde{G}$ | | | | | x | x | | | | | | | | |
| $(m\partial\cdot A)_1$ | | | | | | x | | | | | | | | |
| $(m\partial\cdot A)_2$ | | | | | | | x | | | | | | | |
| $(m^2P)_1$ | | | | | | | | x | | | | | | |
| $(m^2\tilde{P})_2$ | | | | | | | | | x | | | | | |
| $(m^2\tilde{P})_3$ | | | | | | | | | | x | | | | |
| $P_{EE}$ | | | | | | | | | | | x | x | x | |
| $\partial\cdot A_E$ | | | | | | | | | | | | x | | |
| $A_\partial$ | | | | | | | | | | | x | x | x | |
| $A_{A^{(\gamma)}}$ | | | | | | | | | | | | | | x |

# Renormalization schemes

- $\overline{\text{MS}}$ scheme: use dim-reg and subtract poles in $1/(d-4)$
  - Simple, widely used in calculations of Wilson coefficients
  - Subtlety: need to specify scheme for $\gamma_5$

    - NDR: $\{\gamma_\mu, \gamma_5\} = 0 \quad \forall\ \mu$
    - HV: $\{\gamma_\mu, \gamma_5\} = 0$ for $\mu$=0-3, otherwise $[\gamma_\mu, \gamma_5] = 0$

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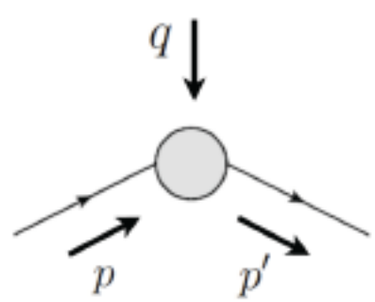
- RI-SMOM class of schemes: fix finite parts by conditions on quark and gluon amputated Green's functions in a given gauge, at non-exceptional momentum configurations, such as




= tree level

$p^2 = p'^2 = q^2 = -\Lambda^2$

  - Regularization independent: can be implemented on the lattice

# RI-$\widetilde{\text{S}}$MOM scheme

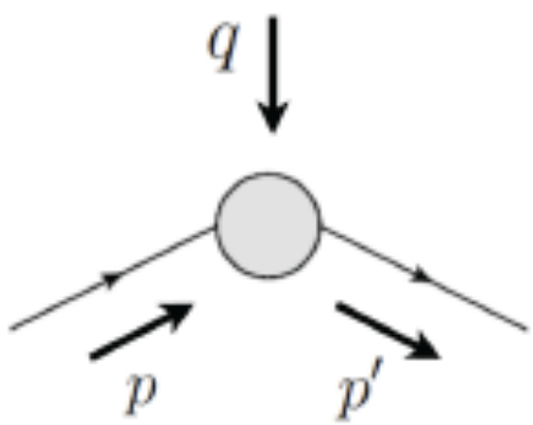
- Require conditions on C (14), $m_G\widetilde{G}$ (2), $O_{2,3,6\text{-}10}$ (one each)

# RI-$\tilde{S}$MOM scheme

- Require conditions on C (14), $m G\tilde{G}$ (2), $O_{2,3,6\text{-}10}$ (one each)

- On the lattice, first subtract power-divergence (C ↔ P mixing) :

$$C^L \rightarrow C = C^L - Z_{C\text{-}P}\, P$$



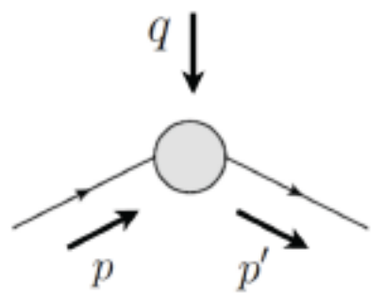

$= 0 \quad \Rightarrow \quad Z_{C\text{-}P}(a, \Lambda_0) \sim 1/a^2$

$\gamma_5\, t^a$ projection

$p^2 = p'^2 = q^2 = -\Lambda_0^2$

# RI-$\tilde{S}$MOM scheme

- Require conditions on C (14), $m G\tilde{G}$ (2), $O_{2,3,6\text{-}10}$ (one each)
- Conditions on C: amputated 2-pt functions




$= 0$

$p^2 = p'^2 = q^2 = -\Lambda^2$

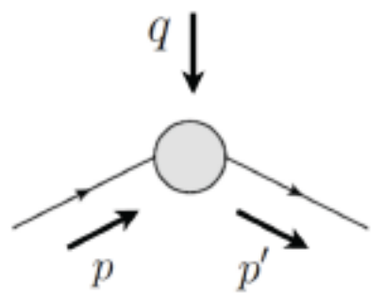
Coefficients of
7 spin-flavor structures**

$\Lambda \neq \Lambda_0$

** $\gamma_5 t^a$, $\sigma_{\mu\nu}\gamma_5 p^\mu p'^\nu t^a$, $q^\mu\gamma_\mu \gamma_5 M t^a$, $q^\mu\gamma_\mu \gamma_5 \mathrm{Tr}[M t^a]$, $\gamma_5 M^2 t^a$, $\gamma_5 t^a \mathrm{Tr}[M^2]$, $\gamma_5 M \mathrm{Tr}[M t^a]$

# RI-$\widetilde{\text{S}}$MOM scheme

- Require conditions on C (14), $m G\widetilde{G}$ (2), $O_{2,3,6\text{-}10}$ (one each)
- Conditions on C: amputated 2-pt functions




$= 0$

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Coefficients of
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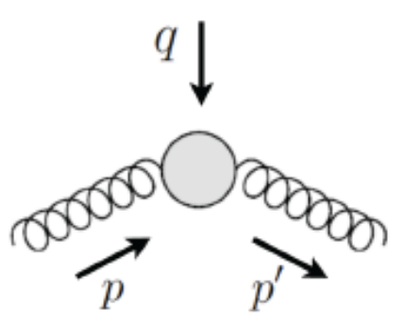


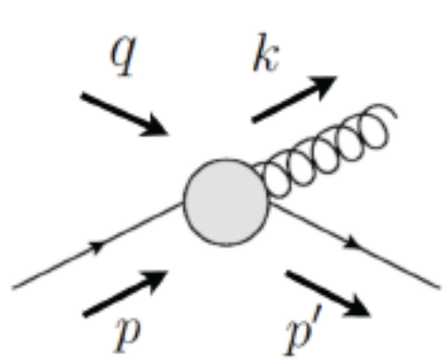

$= 0$

$p^2 = p'^2 = q^2 = -\Lambda^2$

1 condition for gluons,
1 condition for photons

# RI-$\widetilde{\text{S}}$MOM scheme

- Require conditions on C (14), $mG\widetilde{G}$ (2), $O_{2,3,6\text{-}10}$ (one each)
- Conditions on C: amputated 3-pt functions (q-q-gluon)




= tree-level**

$p^2 = p'^2 = q^2 = k^2 = -\Lambda^2$

$s = u = t/2 = -\Lambda^2$

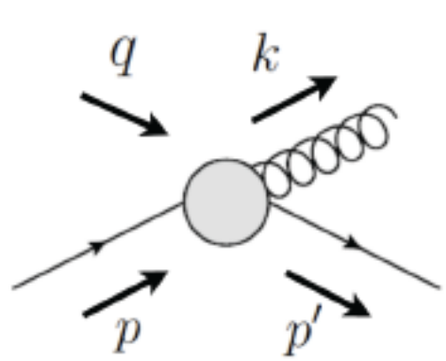
Kinematics:

$s = (p+q)^2$
$u = (p-k)^2$
$t = (p-p')^2$

** 3 spin-flavor structures: $\sigma_{\mu\nu}\gamma_5 k^\nu t^a$, $\sigma_{\mu\nu}\gamma_5 (p-p')^\nu t^a$, $\gamma_5 (p+p')_\mu t^a$

# RI-$\tilde{S}$MOM scheme

- Require conditions on C (14), $mG\tilde{G}$ (2), $O_{2,3,6\text{-}10}$ (one each)
- Conditions on C: amputated 3-pt functions (q-q-gluon)




= tree-level**

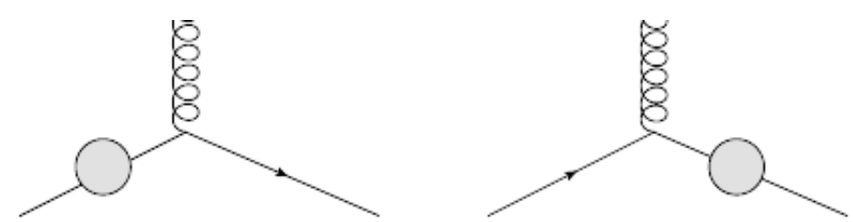
$p^2 = p'^2 = q^2 = k^2 = -\Lambda^2$

$s = u = t/2 = -\Lambda^2$

Kinematics:

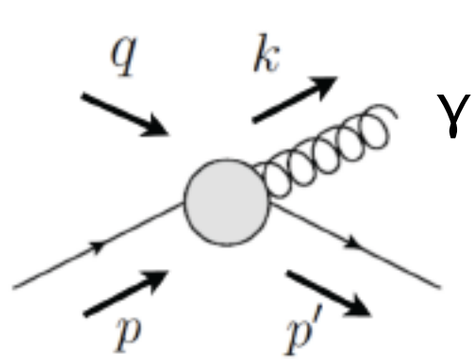
$s = (p+q)^2$
$u = (p-k)^2$
$t = (p-p')^2$

$\tilde{S}$ point: can't have $s=u=t = -\Lambda^2$ but $s=u = -\Lambda^2$ and conditions on 2pt-function eliminate non-1PI diagrams



# RI-$\tilde{S}$MOM scheme

- Require conditions on C (14), $mG\tilde{G}$ (2), $O_{2,3,6\text{-}10}$ (one each)
- Conditions on C: amputated 3-pt functions (q-q-photon)




$\gamma$

$= 0$ **

$p^2 = p'^2 = q^2 = k^2 = -\Lambda^2$

$s = u = t/2 = -\Lambda^2$

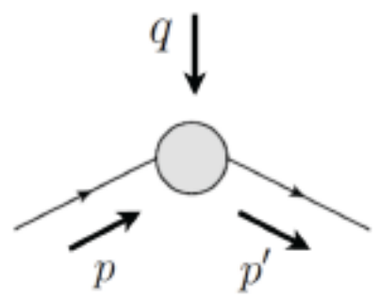
Kinematics:

$s = (p+q)^2$
$u = (p-k)^2$
$t = (p-p')^2$

** 2 spin-flavor structures: $\sigma_{\mu\nu}\gamma_5 k^\nu t^a$, $\gamma_5 (p+p')_\mu t^a$

# RI-$\widetilde{\text{S}}$MOM scheme

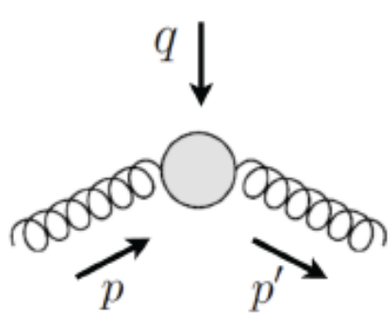
- Require conditions on C (14), mG$\widetilde{\text{G}}$ (2), $O_{2,3,6\text{-}10}$ (one each)
- Conditions on (mGG): amputated 2-pt functions




= 0

$p^2 = p'^2 = q^2 = -\Lambda^2$

1 spin-flavor structure:
$\gamma_\mu q^\mu \gamma_5 t^a$



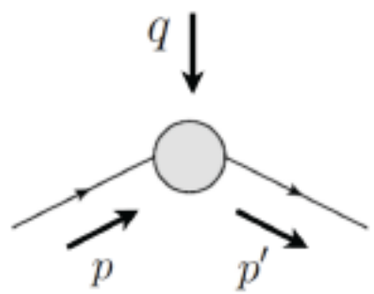

= tree

$p^2 = p'^2 = q^2 = -\Lambda^2$

1 condition

# RI-$\widetilde{\text{S}}$MOM scheme

- Require conditions on C (14), $m G\widetilde{G}$ (2), $O_{2,3,6\text{-}10}$ (one each)
- Conditions on E, $(m\partial A)_{1,2}$ and $(m^2P)_{1,2,3}$: amputated 2-pt functions




= tree

1 spin-flavor structure each

$p^2 = p'^2 = q^2 = -\Lambda^2$

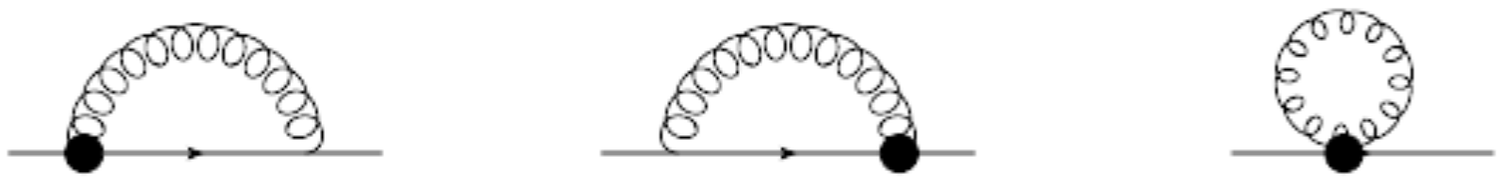
Conditions are equivalent to RI-SMOM conditions on A, P, T

Aoki et al 2009

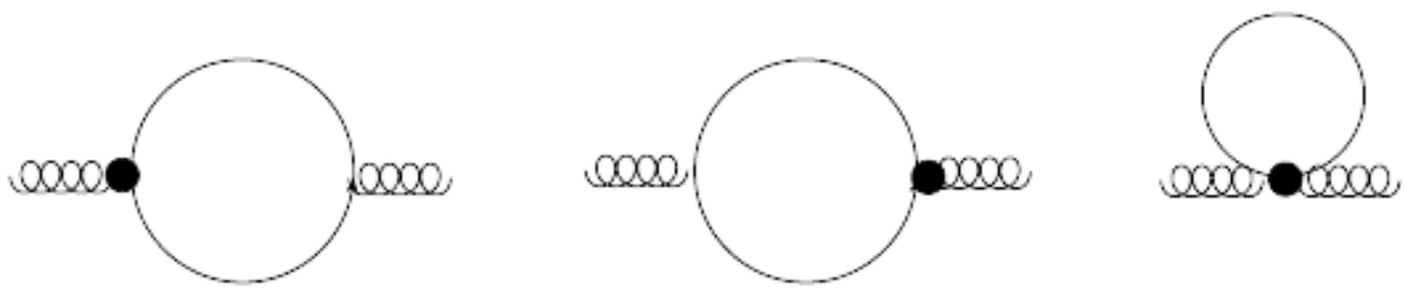
# Matching RI-$\tilde{\text{S}}$MOM and $\overline{\text{MS}}$ at one loop

# One-loop calculations

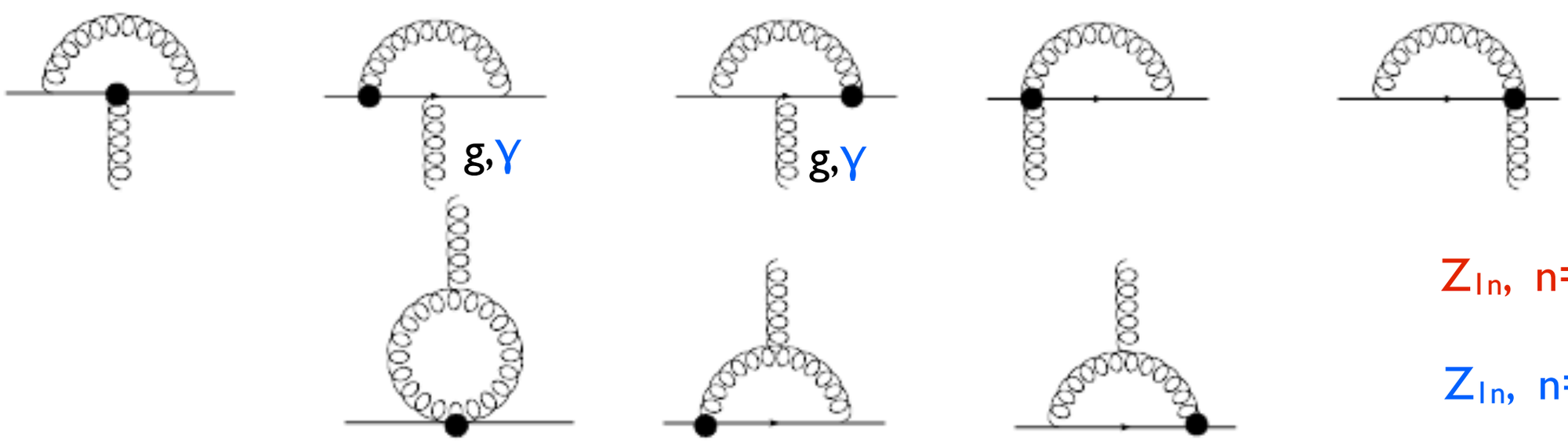
- Insertions of C



$Z_{1n}$
n= 2,6-10, 11-13



$Z_{15}$

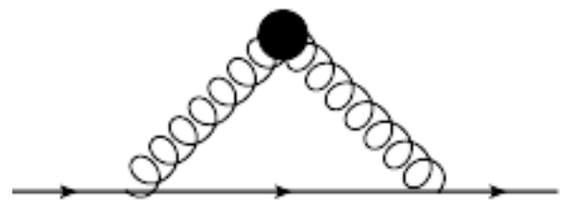



$Z_{1n}$, n=1, 11-13

$Z_{1n}$, n=3,11-14

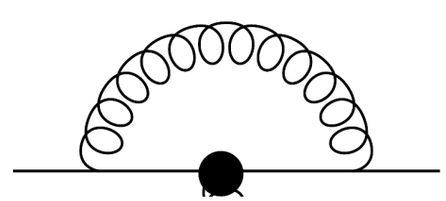
# One-loop calculations

- Insertions of $m G\tilde{G}$



$Z_{55}$, $Z_{56}$

- Insertions of $E \sim T$, $(m\partial A)_{1,2}$ and $(m^2 P)_{1,2,3}$



A, P, T

$Z_{nn}$ n=2,3, 6-10

# One-loop calculations

- Schematic form of all 1-loop results

$$\Gamma = \sum_i (\text{spin} \otimes \text{flavor})_i \left[ d_i(\xi) \left( \frac{2}{4-d} + \log \frac{\mu^2}{\Lambda^2} \right) + f_i(\xi) \right]$$

Depends on scheme adopted for $\gamma_5$ (HV, NDR)
Time-consuming part of the calculation
Work in covariant gauge: Landau gauge ($\xi$=0) can be implemented on the lattice

- Determine $Z_O$

$$Z_{ij}^{\overline{\text{MS}}} = \delta_{ij} + \frac{1}{\epsilon} \frac{\alpha_s}{4\pi} z_{ij}$$

$$Z_{ij}^{\text{RI}-\tilde{\text{S}}\text{MOM}} = \delta_{ij} + \frac{\alpha_s}{4\pi} \left[ \frac{z_{ij}}{\epsilon} + c_{ij} \right]$$

# One-loop calculations

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- Determine $Z_O$

$\xi$-independent

$$Z_{ij}^{\overline{\text{MS}}} = \delta_{ij} + \frac{1}{\epsilon} \frac{\alpha_s}{4\pi} z_{ij}$$

$\xi$-dependent

$$Z_{ij}^{\text{RI}-\tilde{\text{S}}\text{MOM}} = \delta_{ij} + \frac{\alpha_s}{4\pi} \left[ \frac{z_{ij}}{\epsilon} + c_{ij} \right]$$

# One-loop results (I)

- Z in $\overline{\text{MS}}$

$$z_{11} = 5C_F - 2C_A$$
$$z_{12} = 0$$
$$z_{13} = 4C_F$$
$$z_{14} = 0$$
$$z_{15} = -2$$
$$z_{16} = C_F - \frac{1}{4}C_A$$
$$z_{17} = 3C_F - \frac{3}{4}C_A$$
$$z_{18} = 6C_F + \frac{3}{2}C_A$$
$$z_{19} = 0$$
$$z_{1,10} = 0$$

$$z_{1,11} = 6C_F - \frac{3}{2}C_A$$
$$z_{1,12} = -3C_F + \frac{3}{4}C_A$$
$$z_{1,13} = \frac{3}{4}C_A$$
$$z_{1,14} = \frac{3}{4}C_A \ .$$

$$z_{22} = -3C_F$$
$$z_{33} = C_F$$
$$z_{44} = 3C_F$$
$$z_{55} = -\frac{11C_A - 4T_F n_F}{3} + 3C_F$$
$$z_{56} = 6C_F$$
$$z_{66} = z_{77} = z_{88} = z_{99} = z_{10,10} = 3C_F$$

$$C_F = \frac{N_C^2 - 1}{2N_C} \qquad C_A = N_C \qquad T_F = \frac{1}{2}$$

# One-loop results (II)

- C-matrix connecting $\overline{\text{MS}}$ and RI-$\tilde{\text{S}}$MOM

$$O_i^{\text{RI}-\tilde{\text{S}}\text{MOM}} = C_{ij}\, O_j^{\overline{\text{MS}}}$$

$$C_{ij} = \left( Z^{\text{RI}-\tilde{\text{S}}\text{MOM}} \cdot \left( Z^{\overline{\text{MS}}} \right)^{-1} \right)_{ij} \equiv \delta_{ij} \; + \; \frac{\alpha_s}{4\pi} c_{ij}$$

# One-loop results (II)

- C-matrix connecting $\overline{\text{MS}}$ and RI-$\widetilde{\text{S}}$MOM

$$c_{11} = \frac{C_A(23+9\xi) - 32C_F}{12}\,\psi + \frac{C_A - 2C_F}{2}(1-\xi)\,K - \frac{C_A - 2C_F}{2}(1+\xi)\,\log 2$$
$$+ \frac{10}{9}n_F T_F + C_F\left(\frac{31}{3} - \frac{1}{2}\xi\right) + \frac{C_A}{72}\left(-646 - 36\xi + 9\,\xi^2\right)$$
$$c_{12} = \left(4C_F - \frac{C_A}{6}(3+\xi)\right)\,\psi + (C_A - 2C_F)(1-\xi)\,K + (C_A - 2C_F)(1+\xi)\,\log 2$$
$$+ C_F(2-\xi) + \frac{C_A}{4}(-5+2\xi)$$
$$c_{13} = \left(-\frac{8}{3}C_F + \frac{C_A}{12}(3+\xi)\right)\,\psi - \frac{1}{2}(C_A - 2C_F)(1-\xi)\,K - \frac{1}{2}(C_A - 2C_F)(1+\xi)\,\log 2$$
$$+ C_F\left(\frac{25}{3} + \frac{1}{2}\xi\right) + \frac{C_A}{8}(5-2\xi)$$
$$c_{14} = 0$$
$$c_{15} = -4$$
$$c_{16} = 3\,c_{17}$$
$$c_{17} = \frac{C_A\,\xi}{6}\,\psi + (C_A - 2C_F)(1-\xi)\,K - \frac{1}{2}(C_A - 2C_F)(1-\xi)\,\log 2$$
$$+ C_F(8-\xi) + C_A\left(-\frac{13}{4} + \frac{1}{2}\xi\right)$$
$$c_{18} = \frac{8C_F - C_A(1+\xi)}{2}\,\psi - (C_A - 2C_F)(1-\xi)\,K + 2\,(C_A - 2C_F)\,\log 2$$
$$+ C_F\left(\frac{10}{3} + \xi\right) + C_A\left(\frac{21}{4} - \frac{1}{2}\xi\right)$$
$$c_{19} = 0$$
$$c_{1,10} = 0$$

Loop expressed in terms of 1st derivatives of Digamma function:

$$\psi = \frac{2}{3}\left(\psi^{(1)}\left(\frac{1}{3}\right) - \frac{2}{3}\pi^2\right)$$

$$K = \frac{1}{8}\left(\psi^{(1)}\left(\frac{1}{4}\right) - \pi^2\right)$$

$$c_{22} = c_{88} = c_{99} = c_{10,10} = \frac{C_F}{2}(3+\xi)\,\psi - C_F(12+\xi)$$
$$c_{33} = C_F\left[(1-\xi)\left(\frac{4}{3} - \frac{1}{2}\psi\right) + \xi\right]$$
$$c_{55} = 2\,C_A\left(1 + \frac{1}{3}\xi\right)\,\psi + \frac{C_A}{36}(-403 - 18\xi + 9\xi^2) + \frac{20}{9}n_F T_F$$
$$c_{56} = -2C_F\,(3 + 2\psi)$$
$$c_{66} = c_{77} = -4\,C_F$$

# Impact on phenomenology

- Goal: evaluate hadronic CP-odd couplings from

$$\delta\mathcal{L}_{CPV} \sim c_P^{\overline{\rm MS}}\, P^{\overline{\rm MS}} + c_E^{\overline{\rm MS}}\, E^{\overline{\rm MS}} + c_{CE}^{\overline{\rm MS}}\, C^{\overline{\rm MS}}$$

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$$O_i^{\overline{\rm MS}} = \left[C^{-1}\right]_{ij}\, O_j^{\rm RI-\tilde{S}MOM}$$

$$c_P^{\overline{\rm MS}}\, C_P^{-1}\, P^{\rm RI-\tilde{S}MOM} + c_E^{\overline{\rm MS}}\, C_T^{-1}\, E^{\rm RI-\tilde{S}MOM} + c_{CE}^{\overline{\rm MS}} \sum_n C_{1n}^{-1}\, O_n^{\rm RI-\tilde{S}MOM}$$

Corrections range from few % to > 30%

# Impact on phenomenology

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$$\delta\mathcal{L}_{CPV} \sim c_P^{\overline{\rm MS}}\, P^{\overline{\rm MS}} + c_E^{\overline{\rm MS}}\, E^{\overline{\rm MS}} + c_{CE}^{\overline{\rm MS}}\, C^{\overline{\rm MS}}$$

$$O_i^{\overline{\rm MS}} = \left[C^{-1}\right]_{ij}\, O_j^{\rm RI-\tilde{S}MOM}$$

$$c_P^{\overline{\rm MS}}\, C_P^{-1}\, P^{\rm RI-\tilde{S}MOM} + c_E^{\overline{\rm MS}}\, C_T^{-1}\, E^{\rm RI-\tilde{S}MOM} + c_{CE}^{\overline{\rm MS}} \sum_n C_{1n}^{-1}\, O_n^{\rm RI-\tilde{S}MOM}$$

n=1,3,5,8-10

Only C, E, $m\tilde{G}G$, $(m^2P)_{1,2,3}$ contribute to $\langle n|J^\mu_{\rm EM} \int d^4x\, O_i(x)|n\rangle$

Need tensor charge (E) + P, C insertions

# Steps towards LQCD implementation

- Neutron EDM from quark EDM (E): tensor charge (see B. Yoon' talk)

- Neutron EDM from quark CEDM operator (C):

  1. Carry out non-perturbative renormalization: requires qq, gg, qqg correlation functions with insertion of $O_i$, i=1,14.

  2. Extract CPV form factor: tensor charge + correlation of P and C with $J_{EM}$ in the nucleon

$$\langle n | J^{\mu}_{\rm EM} \int d^4x \, O_i(x) | n \rangle$$

# Conclusions

- Defined RI-$\widetilde{\text{S}}$MOM scheme for CEDM and other CP-odd operators of dim $\leq$ 5, suitable for implementation in LQCD
- Computed one-loop matching factors between $\overline{\text{MS}}$ and RI-$\widetilde{\text{S}}$MOM
- First step towards LQCD calculation of $d_n(d_{CE})$. Future work:
  - Exploratory studies on the lattice, estimate resources
  - CMDM renormalization (vs CEDM), relevant to the extraction $\pi$NN CP-odd couplings
  - Look at dim-6 operators

# Backup slides

# Axial Ward Identities

- In a given scheme, operators satisfy renormalized PCAC relation

$$\overline{C}_1(g^2)[\partial \cdot A] = \overline{C}_2(g^2)\, 2i[mP] + \overline{C}_3(g^2)\frac{n_F}{16\pi^2}\,[g^2\, G\tilde{G}] + iP_E$$

$$A_\mu = \bar{\psi}(1/2)[\gamma_\mu, \gamma_5]\psi \qquad (mP) \equiv \bar{\psi}\mathcal{M}\gamma_5\psi \qquad P_E = \bar{\psi}_E\gamma_5\psi + \bar{\psi}\gamma_5\psi_E$$

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- $\overline{C}_i\,(g^2) \neq 1$ are finite coefficients related to $Z_{ij}$ and $\alpha, \beta, \gamma$

$$\bar{X} = X \;+\; \alpha\, \partial\cdot A \;+\; \beta\; 2i(mP) \;+\; \gamma\; G\tilde{G}$$

Evanescent operator: its insertions vanish when removing regulator

$$X = \frac{1}{2}\bar{\psi}\left\{\gamma_5, \overrightarrow{\not{D}} - \overleftarrow{\not{D}}\right\}\psi$$

Explicit form of X in dim-reg

$\alpha, \beta, \gamma$ calculable (non)-perturbatively

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Finite rescaling leads to properly normalized WI

$$\begin{aligned}
[A^\mu]_{\rm WI} &= \overline{C}_1(g^2)\,[A^\mu] \\
[mP]_{\rm WI} &= \overline{C}_2(g^2)\,[mP] \\
\left[g^2 G\tilde{G}\right]_{\rm WI} &= \overline{C}_3(g^2)\,[g^2 G\tilde{G}]
\end{aligned}$$

$$\partial\cdot[A]_{\rm WI} = 2i[mP]_{\rm WI} + \frac{n_F}{16\pi^2}[g^2\, G\tilde{G}]_{\rm WI} + iP_E$$

$[mP]_{\rm WI}$ $\left[g^2 G\tilde{G}\right]_{\rm WI}$ have no anomalous dimension, while $\gamma_{A_{WI}} = \frac{\alpha_s}{4\pi}\gamma_{G\tilde{G},\partial\cdot A}$

# Axial Ward Identities

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- Explicit scheme-dependent rescaling:

$$\begin{aligned}
\overline{C}_1 &= 1 - 4C_F\,\frac{\alpha_s}{4\pi} + O(\alpha_s^2)\\
\overline{C}_2 &= 1 - 8C_F\,\frac{\alpha_s}{4\pi} + O(\alpha_s^2)\\
\overline{C}_3 &= 1 + O(\alpha_s^2)
\end{aligned}$$

$\overline{\text{MS}}$-HV

$$\begin{aligned}
\overline{C}_1 &= 1 + O(\alpha_s^2)\\
\overline{C}_2 &= 1 + O(\alpha_s^2)\\
\overline{C}_3 &= 1 + \frac{Z_g^2}{Z_{\text{G}\tilde{\text{G}}}}\Big|_{\text{RI-SMOM}}
\end{aligned}$$

RI-SMOM

# Extraction of nEDM from qCEDM

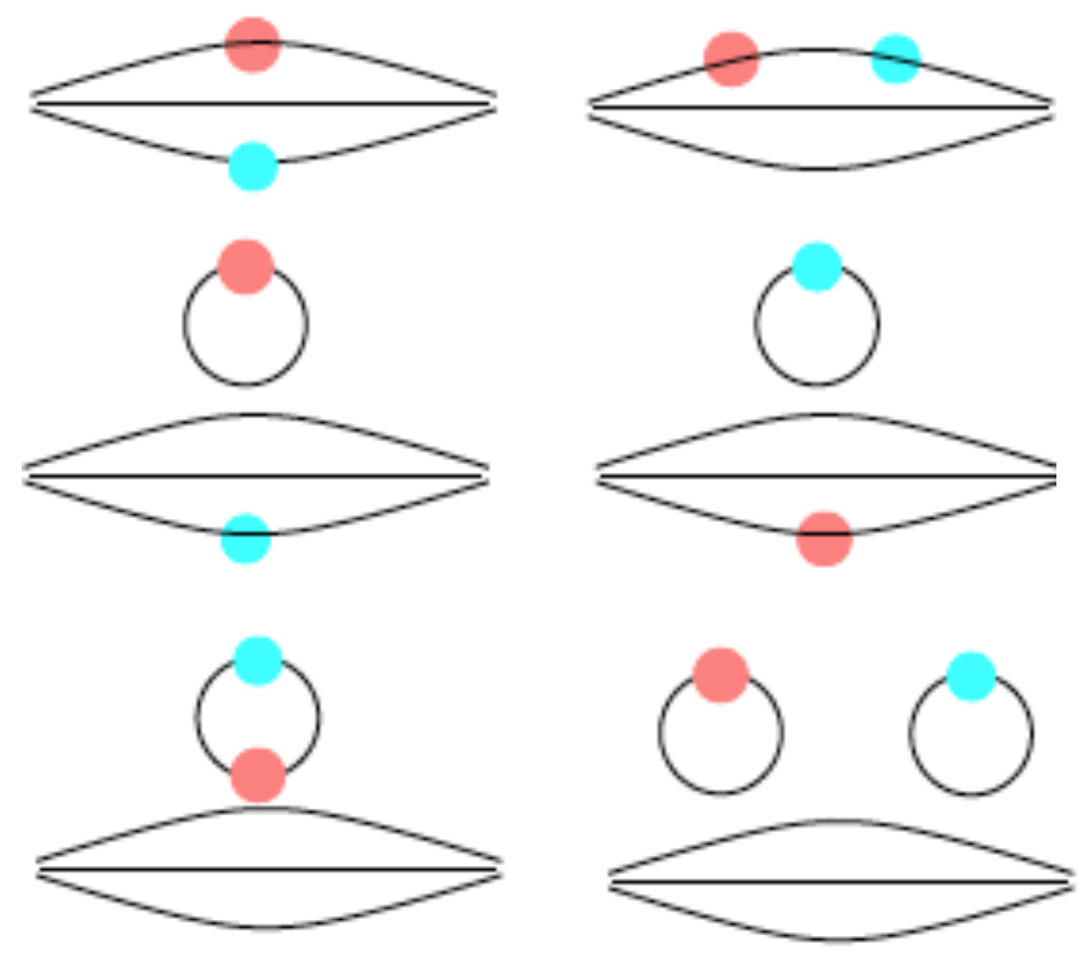
- Extraction of the CPV form factor

$$\langle n | J_\mu^{\rm EM} | n \rangle \sim d_n \, \bar{\psi}_n \, \sigma_{\mu\nu}\gamma_5 q^\nu \, \psi_n$$

$$J_\mu^{\rm EM} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s$$

- Requires 4-point function

$$\langle n | J_\mu^{\rm EM} \int d^4x \, O_i(x) \, | n \rangle$$



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- Or 3-point function in external background E field

$$\langle n | J_\mu^{\mathrm{EM}} \int d^4x \, O_i(x) \, | n \rangle = \frac{\partial}{\partial A_\mu} \langle n | \int d^4x \, O_i(x) \, | n \rangle_E$$