Electroweak Phase Transition and Sphaleron

Eibun Senaha (Natl Taiwan U)

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ACFI Workshop:
Making the Electroweak Phase Transition (Theoretically) Strong

@Umass-Amherst

Outline

- part1: EWPT and sphaleron in a complex-extended SM (cxSM)

  CW Chiang, M. Ramsey-Musolf, E.S., in progress

- part2: Band structure effect on sphaleron rate at high-T.

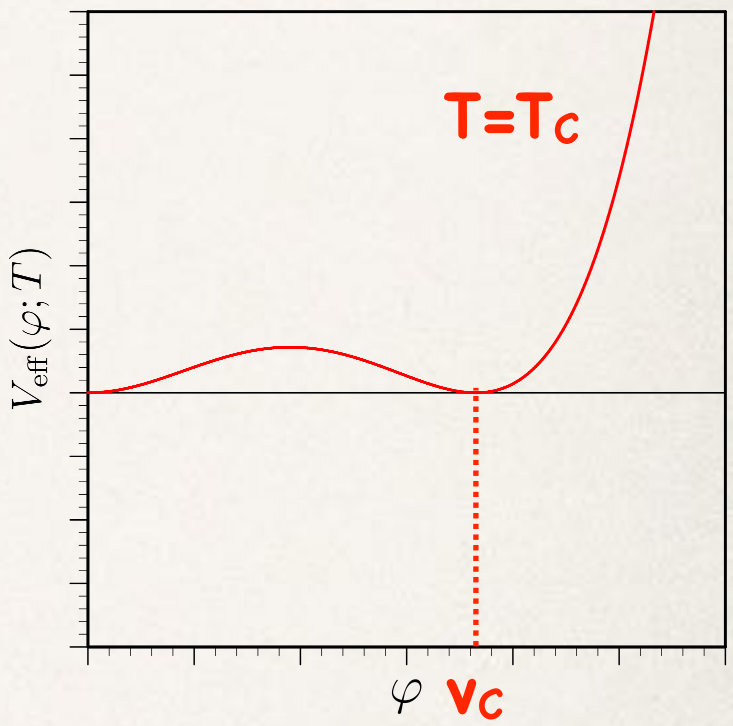
  K. Funakubo, K. Fuyuto, E.S., arXiv:1612.05431

Introduction

- Standard perturbative treatment of EWPT is gauge-dependent.

T_C: T at which V_{eff} has degenerate minima

v_C: minimum of V_{eff} at T_C



- Gauge-independent methods:

(1) v_C and T_C are determined by $V^{\text{high-}T}(\varphi_i;T)=V_0(\varphi_i)+\frac{1}{2}\Sigma_i(T)\varphi_i^2$

(2) Patel-Ramsey-Musolf (PRM) scheme [JHEP07(2011)029]

v_C and T_C are determined separately.

We analyze EWPT and sphaleron in the cxSM using 2 methods.

SM with a complex scalar (cxSM)

H: SU(2)-doublet Higgs, S: SU(2)-singlet Higgs

$$V_0(H,\mathbb{S}) = \frac{m^2}{2}H^\dagger H + \frac{\lambda}{4}(H^\dagger H)^2 + \frac{\delta_2}{2}H^\dagger H|\mathbb{S}|^2 + \frac{b_2}{2}|\mathbb{S}|^2 + \frac{d_2}{4}|\mathbb{S}|^4 + \left[a_1\mathbb{S} + \frac{b_1}{4}\mathbb{S}^2 + \text{h.c.}\right].$$

We assume all parameters are real. $m^2,\ \lambda,\ \delta_2,\ b_2,\ d_2,\ a_1,\ b_1$

$$H(x) = \begin{pmatrix} G^+(x) \\ \frac{1}{\sqrt{2}}\left(v_0 + h(x) + iG^0(x)\right) \end{pmatrix},$$

$$\mathbb{S}(x) = \frac{1}{\sqrt{2}}\left(v_{S0} + S(x) + iA(x)\right),$$

$$\downarrow$$

$v_0,\ v_{S0},\ m_{H_1},\ m_{H_2},\ \alpha,\ m_A,\ a_1$

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$\boxed{v_0,}\ v_{S0},\ m_{H_1},\ m_{H_2},\ \alpha,\ m_A,\ a_1$

246GeV

SM with a complex scalar (cxSM)

H: SU(2)-doublet Higgs, S: SU(2)-singlet Higgs

$$
\begin{aligned}
&V_0(H, \mathbb{S}) \\
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&\quad + \left[a_1 \mathbb{S} + \frac{b_1}{4}\mathbb{S}^2 + \text{h.c.} \right] .
\end{aligned}
$$

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$$
\downarrow
$$

$$
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$$

$$
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$$

$\underbrace{\boxed{v_0}}_{246\text{GeV}},\ v_{S0},\ \underbrace{\boxed{m_{H_1}}}_{125\text{GeV}},\ m_{H_2},\ \alpha,\ m_A,\ a_1$

SM with a complex scalar (cxSM)

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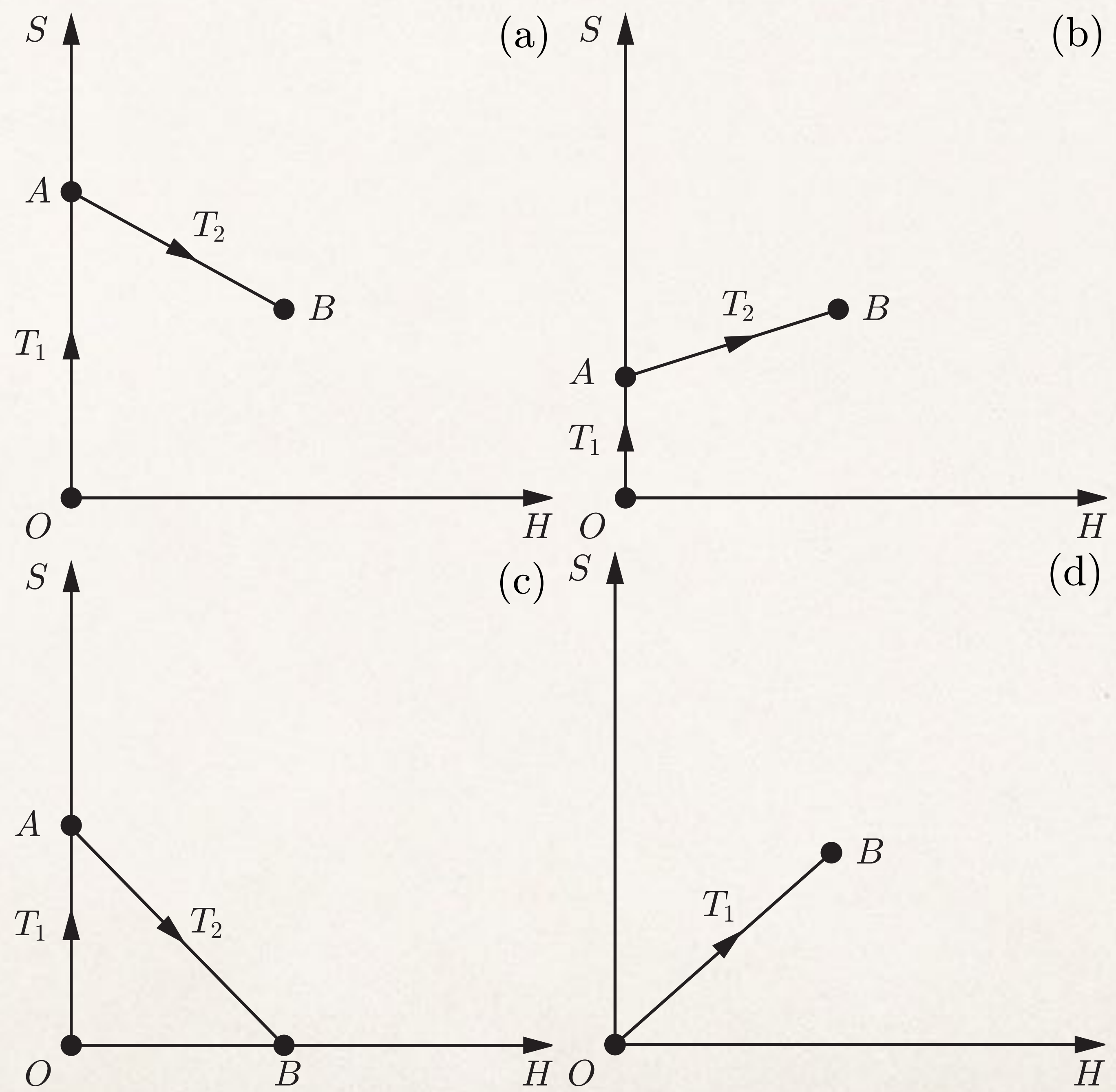
$\downarrow$

$\boxed{v_0},\ v_{S0},\ \boxed{m_{H_1}},\ m_{H_2},\ \alpha,\ m_A,\ a_1$

246GeV 125GeV

Patterns of PT

Because of 2 fields, there are many patterns of phase transitions.



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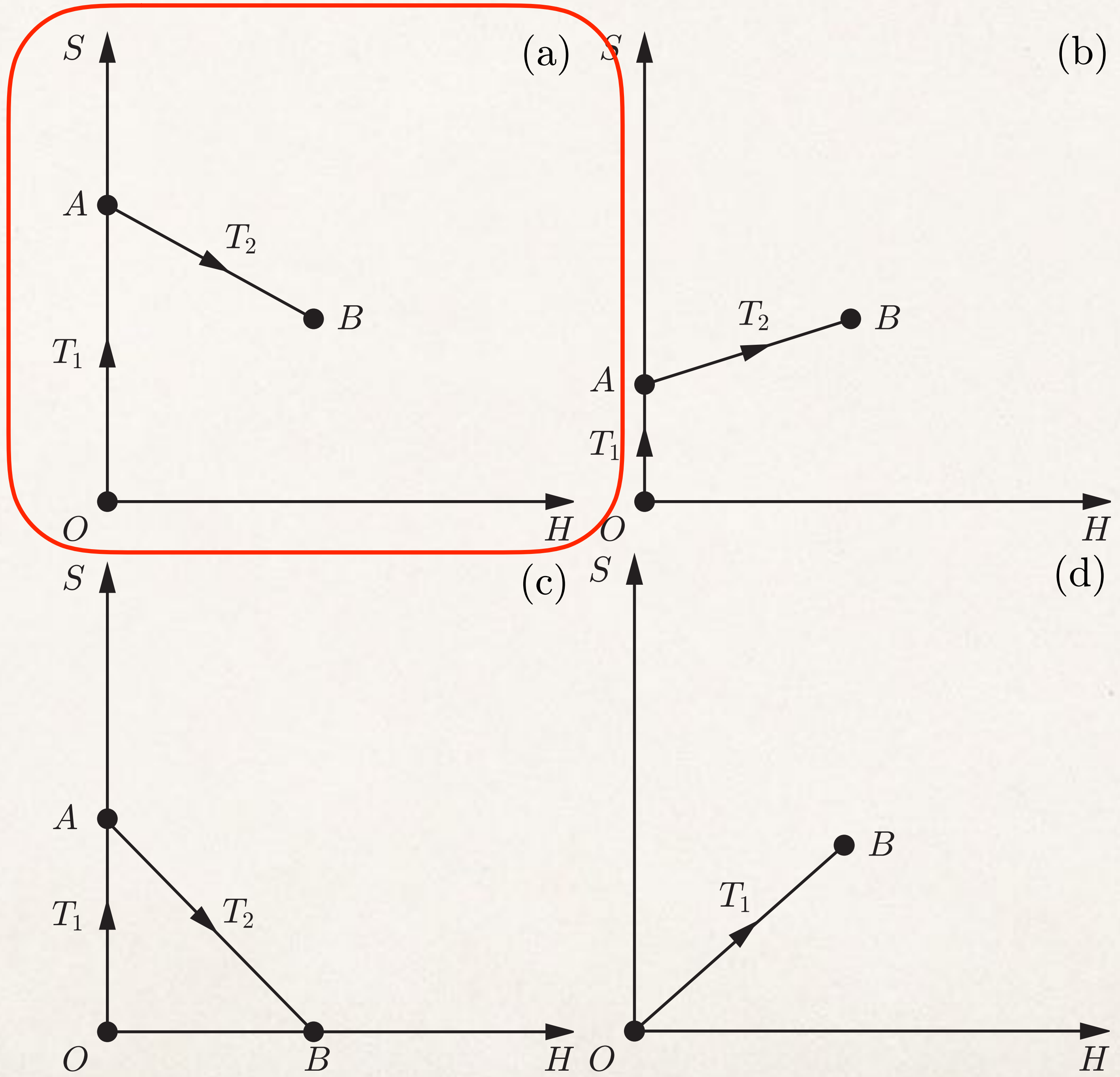


We will focus on type (a) PT.

Leading order analysis

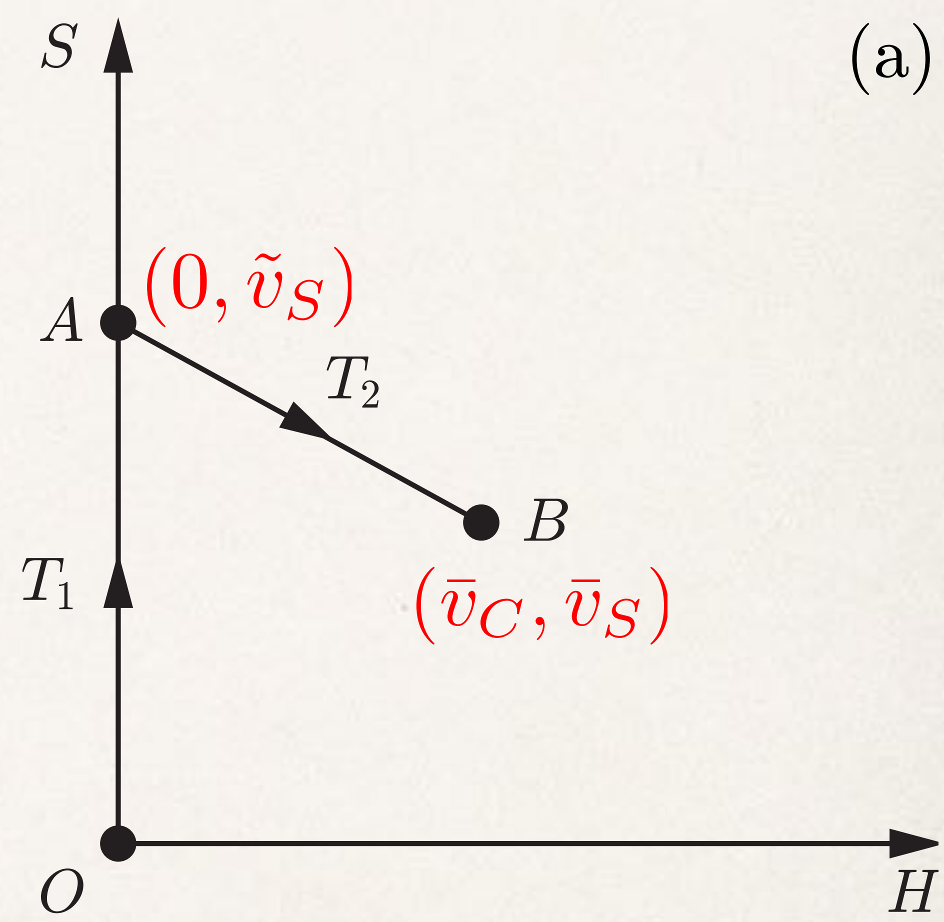
$$V^{\text{high-}T}(\varphi, \varphi_S; T) = V_0(\varphi, \varphi_S) + \frac{1}{2}\Sigma_H T^2 \varphi^2 + \frac{1}{2}\Sigma_S T^2 \varphi_S^2 \ ,$$

where
$$\Sigma_H = \frac{\lambda}{8} + \frac{\delta_2}{24} + \frac{3g_2^2 + g_1^2}{16} + \frac{y_t^2}{4} \ ,$$
$$\Sigma_S = \frac{\delta_2 + d_2}{12} \ .$$

Approximate formulas:

$$\bar{v}_C \simeq \sqrt{\frac{2\delta_2 \tilde{v}_S(T_C)}{\lambda}\left(\tilde{v}_S(T_C) - \bar{v}_S(T_C)\right)} \ ,$$

$$T_C \simeq \sqrt{\frac{1}{2\Sigma_H}\left(-m^2 - \frac{\tilde{v}_S^2(T_C)}{2}\delta_2\right)} \ .$$



- large positive δ_2 (negative α) gives larger v_C/T_C.
- However, too large positive δ_2 (negative α) leads to unstable vacuum.

Leading order analysis

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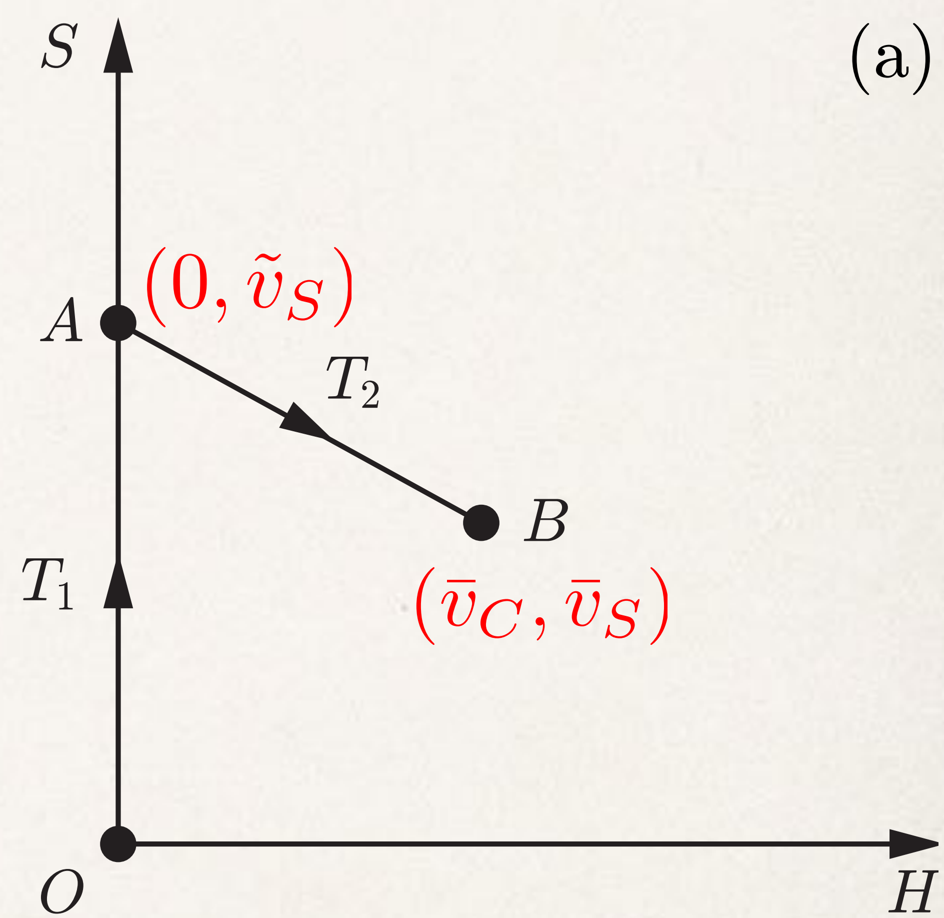


- large positive δ_2 (negative α) gives larger v_C/T_C.
- However, too large positive δ_2 (negative α) leads to unstable vacuum.

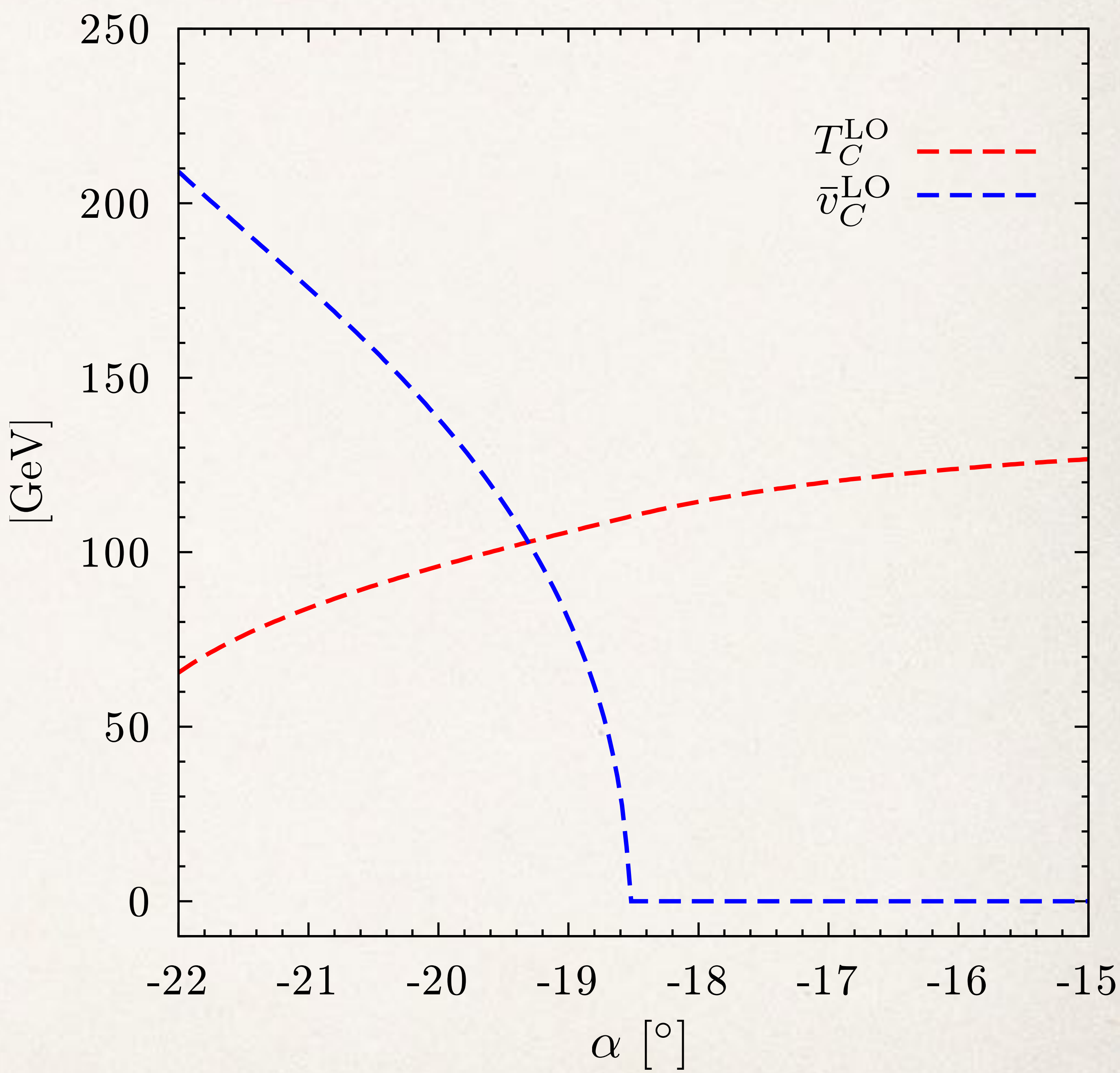
Leading order analysis

An example:

$$m_{H_2} = 230 \text{ GeV}, v_{S0} = 40 \text{ GeV},$$
$$a_1 = -(110 \text{ GeV})^3$$

- v_C and T_C are determined numerically.
- smaller α (large δ_2)gives larger v_C/T_C.
- EW vacuum becomes metastable for a small alpha.

-> upper bound on v_C/T_C



- Stronger upper bound on v_C/T_C comes from bubble nucleation (see later.)

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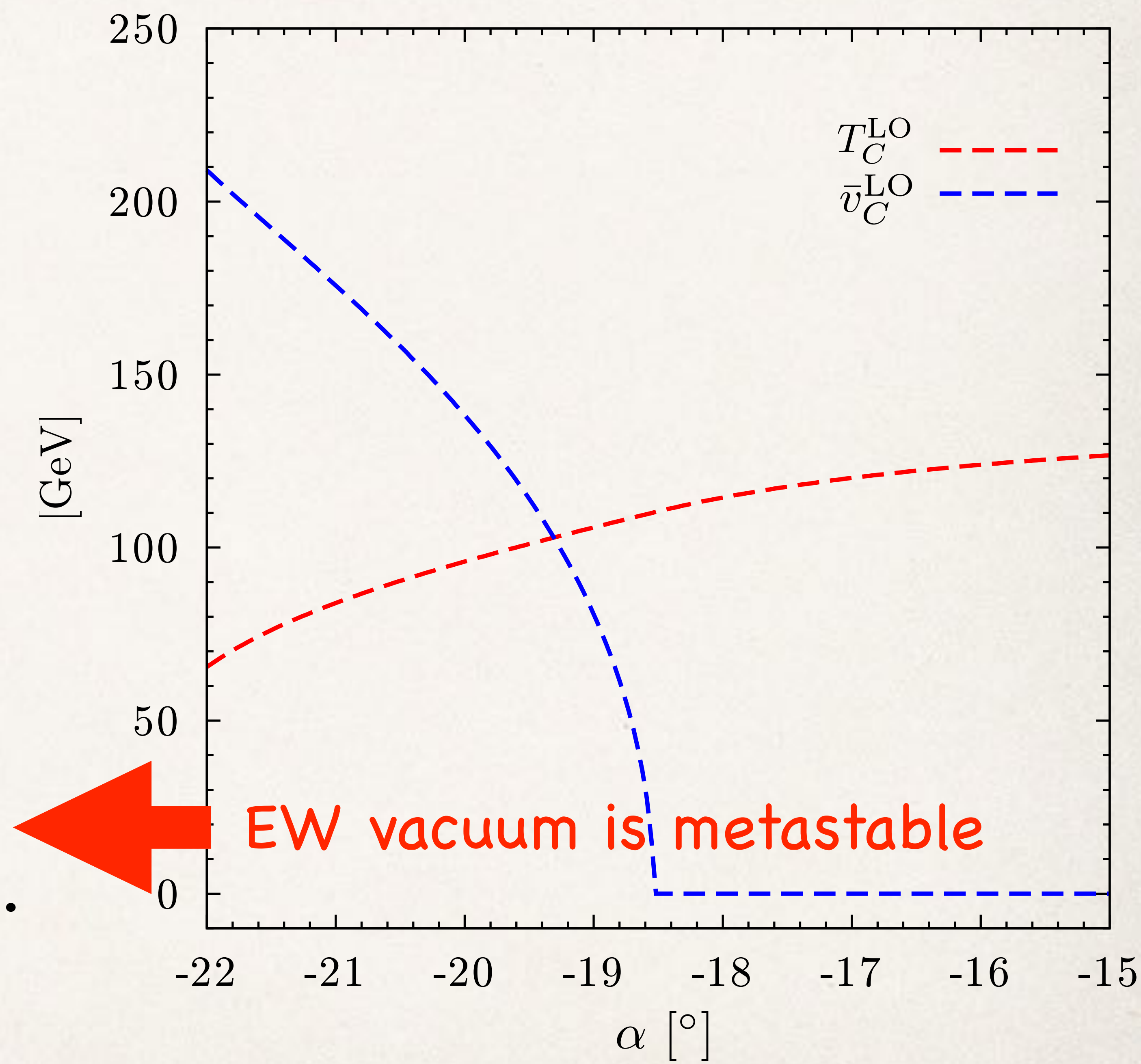


- Stronger upper bound on v_C/T_C comes from bubble nucleation (see later.)

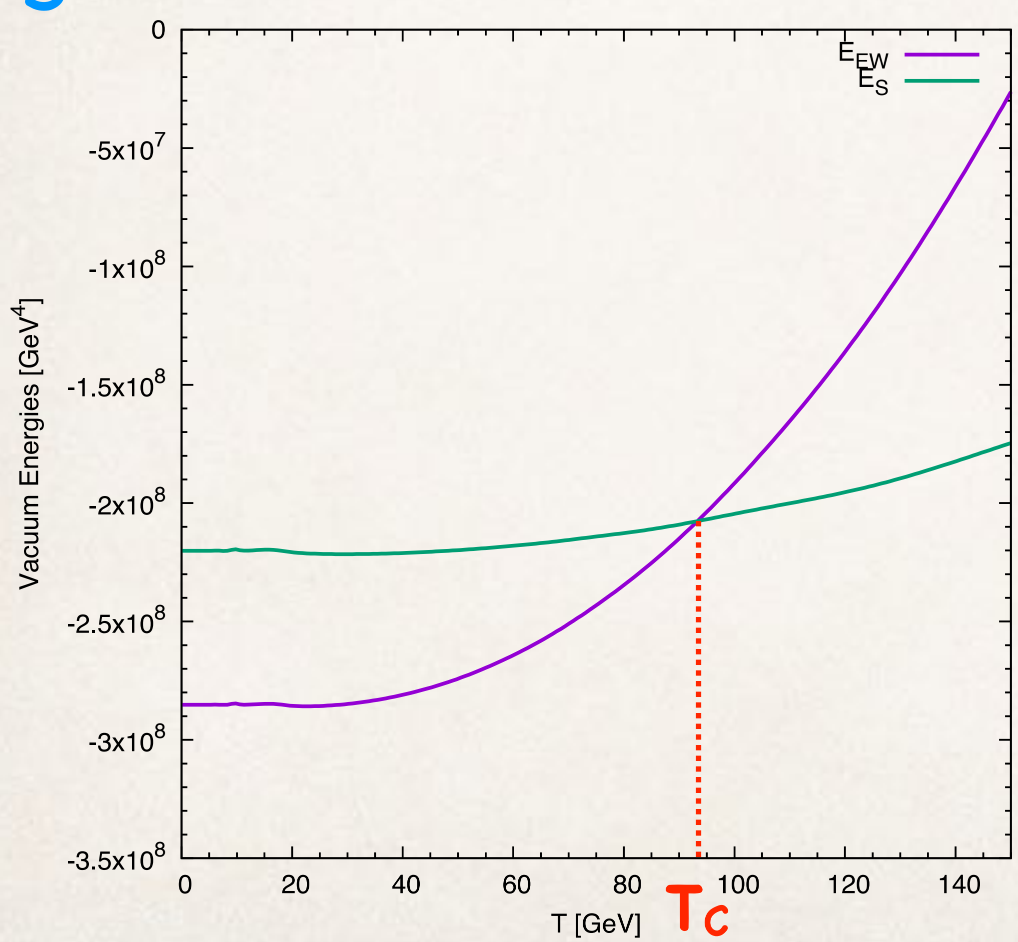
NLO analysis

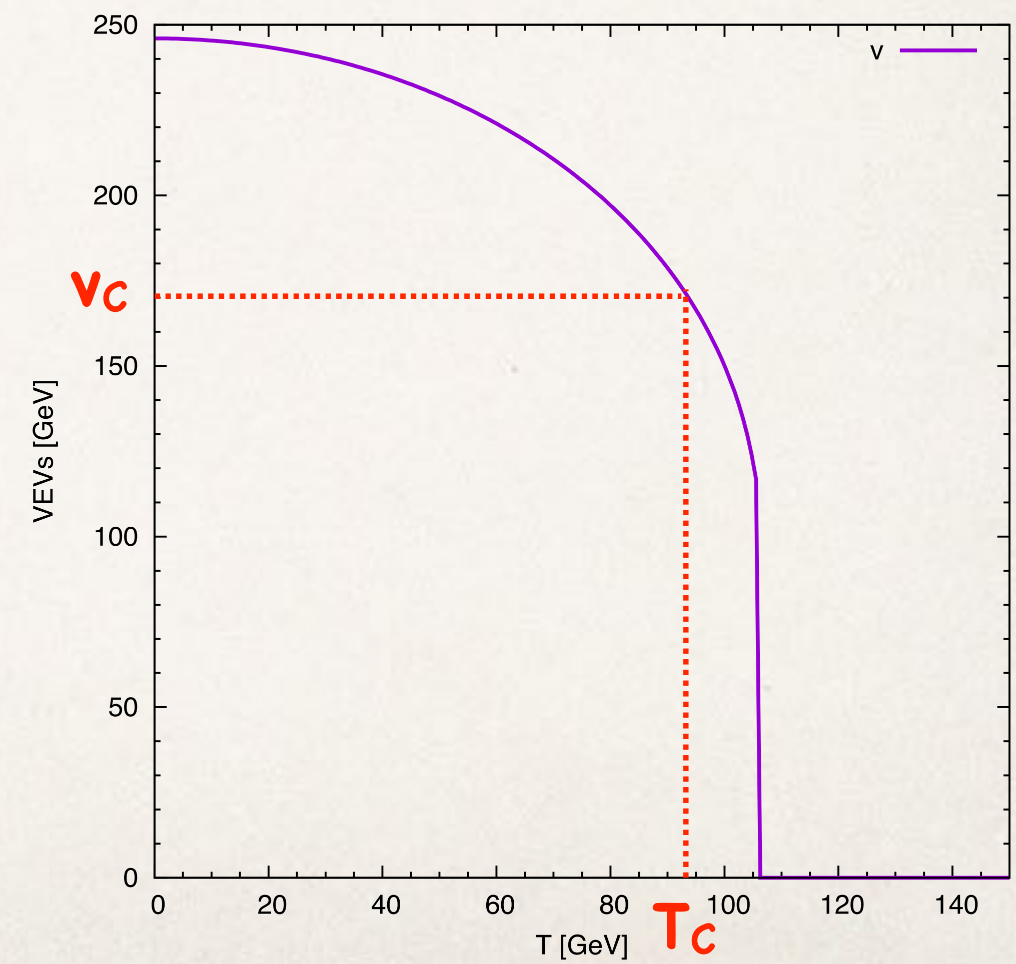
– PRM scheme –

O(hbar)

T_C	v_C
$$\boxed{V_{\text{eff}}(\boldsymbol{v}_0^{(1)};T_C) - V_{\text{eff}}(\boldsymbol{v}_0^{(2)};T_C) = 0}$$	$$V^{\text{high-}T}(\varphi_i;T) = V_0(\varphi_i) + \frac{1}{2}\Sigma_i(T)\varphi_i^2$$
$v_0^{(i=1,2)}$ are stationary points of "V_0"	v_C = minimum of high-T potential at T_C

e.g.





μ dependence

PRM scheme is gauge independent but scale dependent.

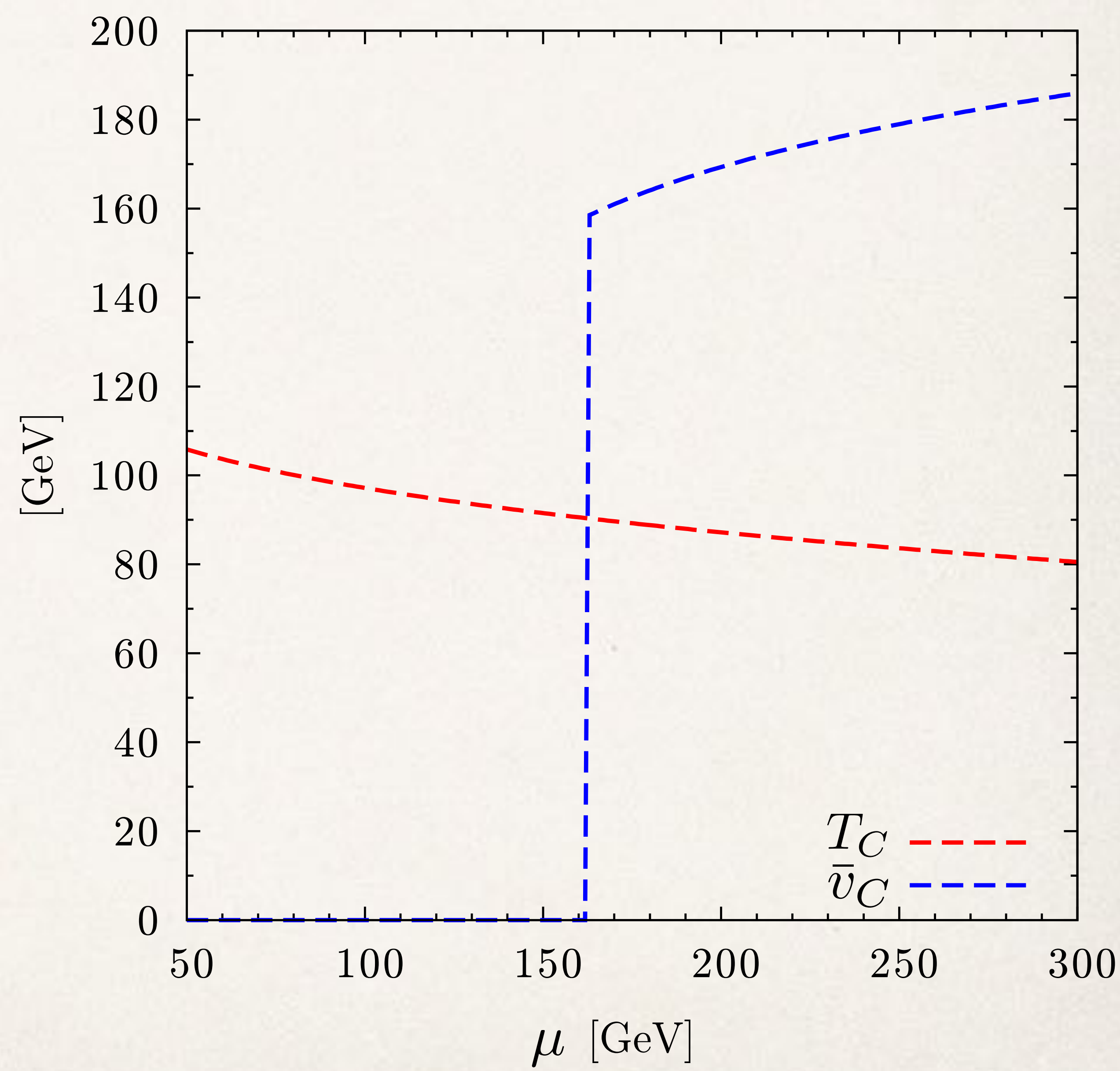
origin:

$$V_{\text{eff}} \ni V_{\text{CW}}(m^2) = \frac{m^4}{64\pi^2}\left(\ln\frac{m^2}{\mu^2} - c\right)$$

Different scales give different orders of phase transition:

2^{nd} order for μ ≲ 160 GeV

1^{st} order for μ ≳ 160 GeV



Improved-RPM scheme

idea: μ dependence is reduced by renormalization group eq.

$$V_{\rm eff}(\varphi_i;T) = V_0(\varphi_i) + V_1(\varphi;T)$$

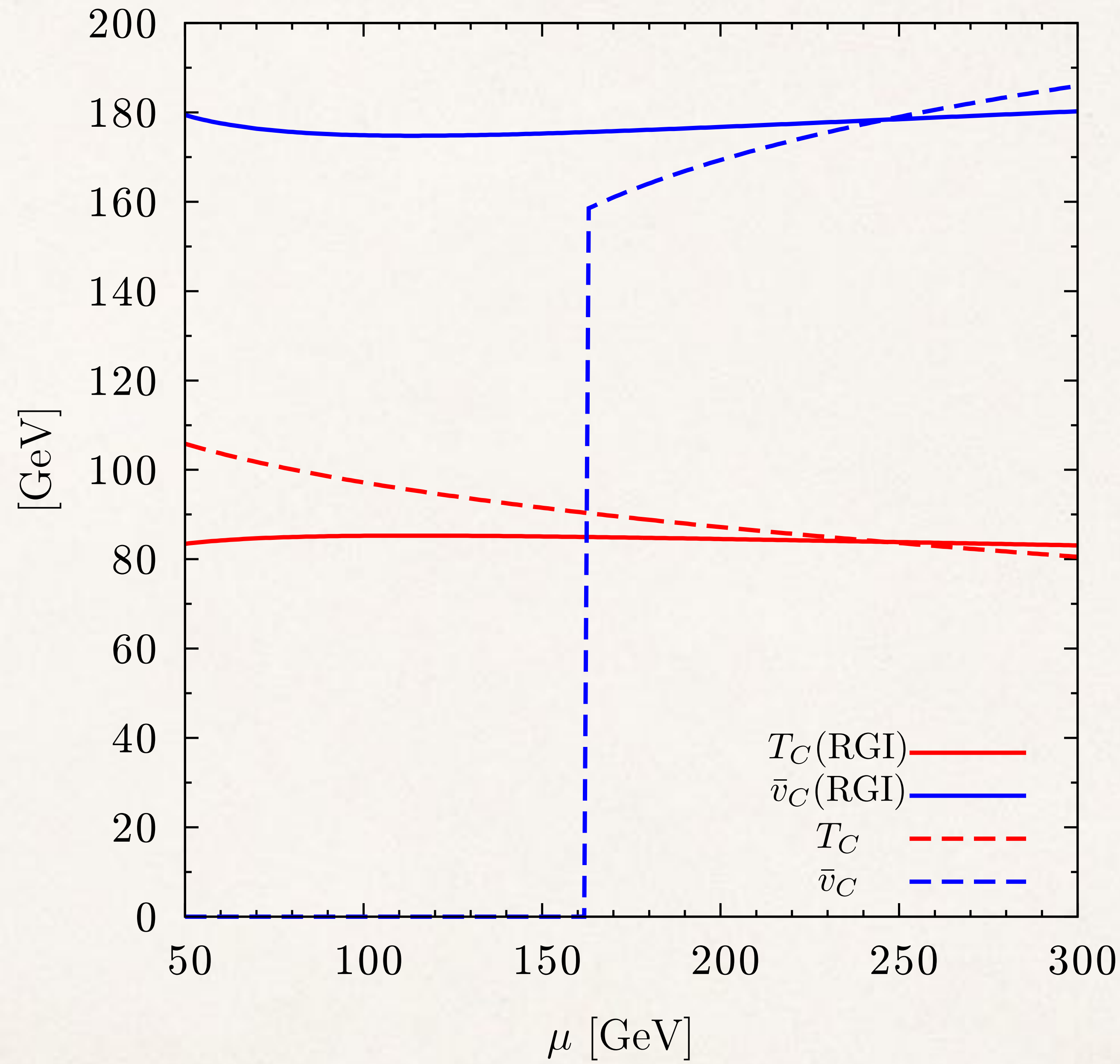
our scheme:

All parameters in $V_0(\varphi_i)$ are replaced by the running parameters.

$$V_0(\varphi_i) \quad \Rightarrow \quad \tilde{V}_0(\varphi_i)$$

$$m_j^2 \text{ and } \lambda_j \quad \Rightarrow \quad m_j^2(\mu) \text{ and } \lambda_j(\mu)$$

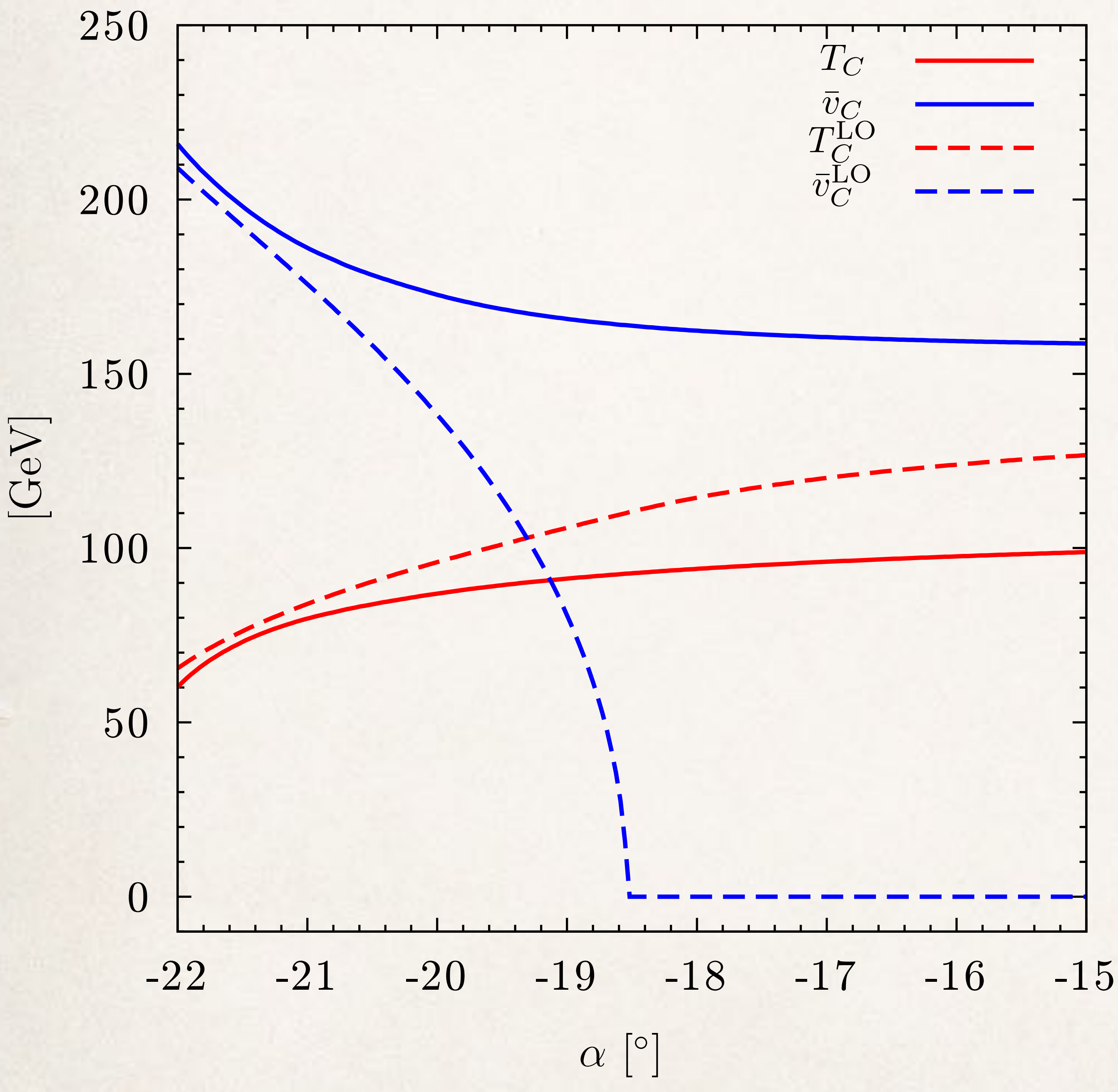
Improved-RPM scheme



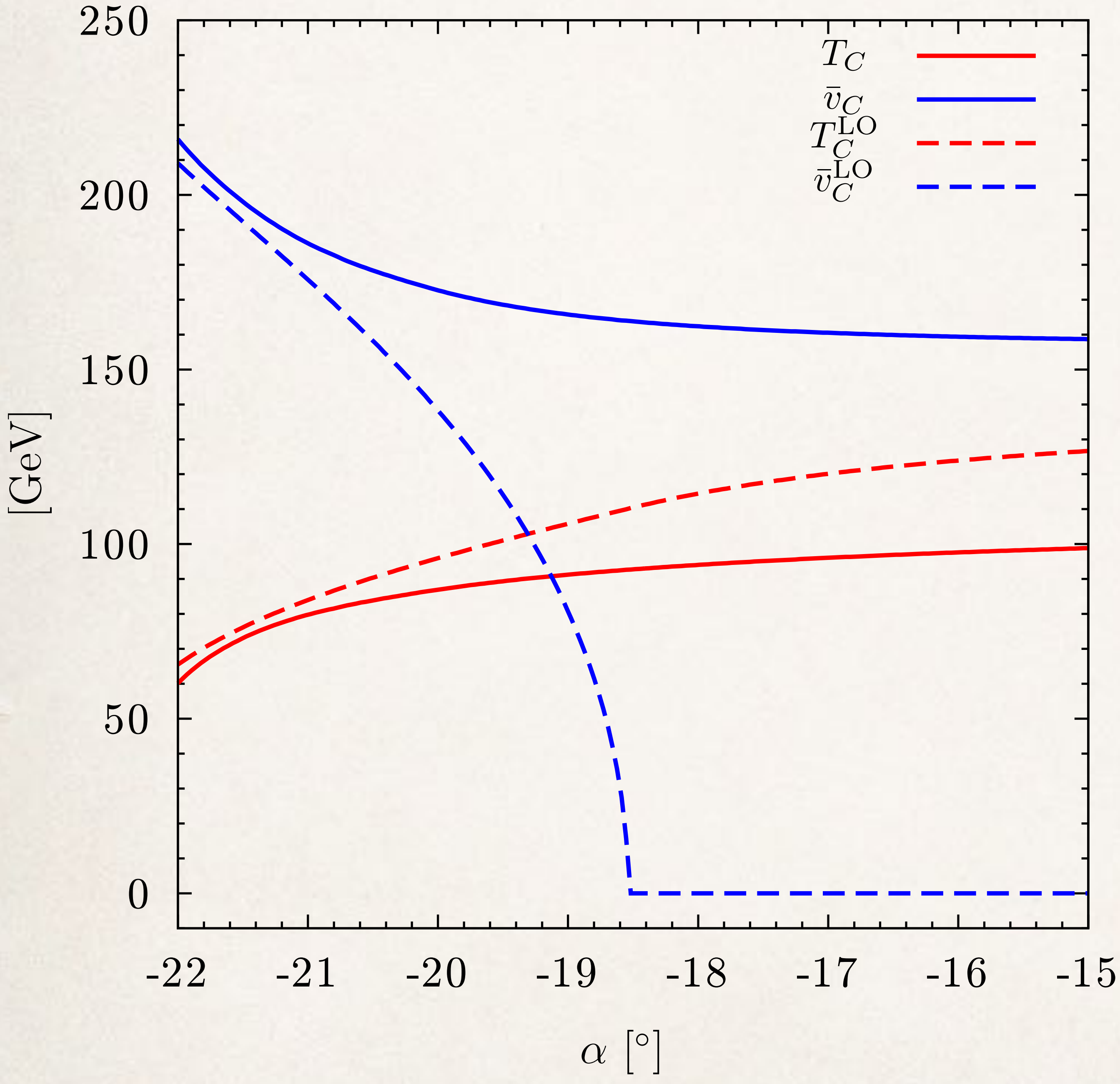
μ dependence is significantly reduced by the RG improvement.

In this example, phase transition is 1st order.

LO vs. NLO

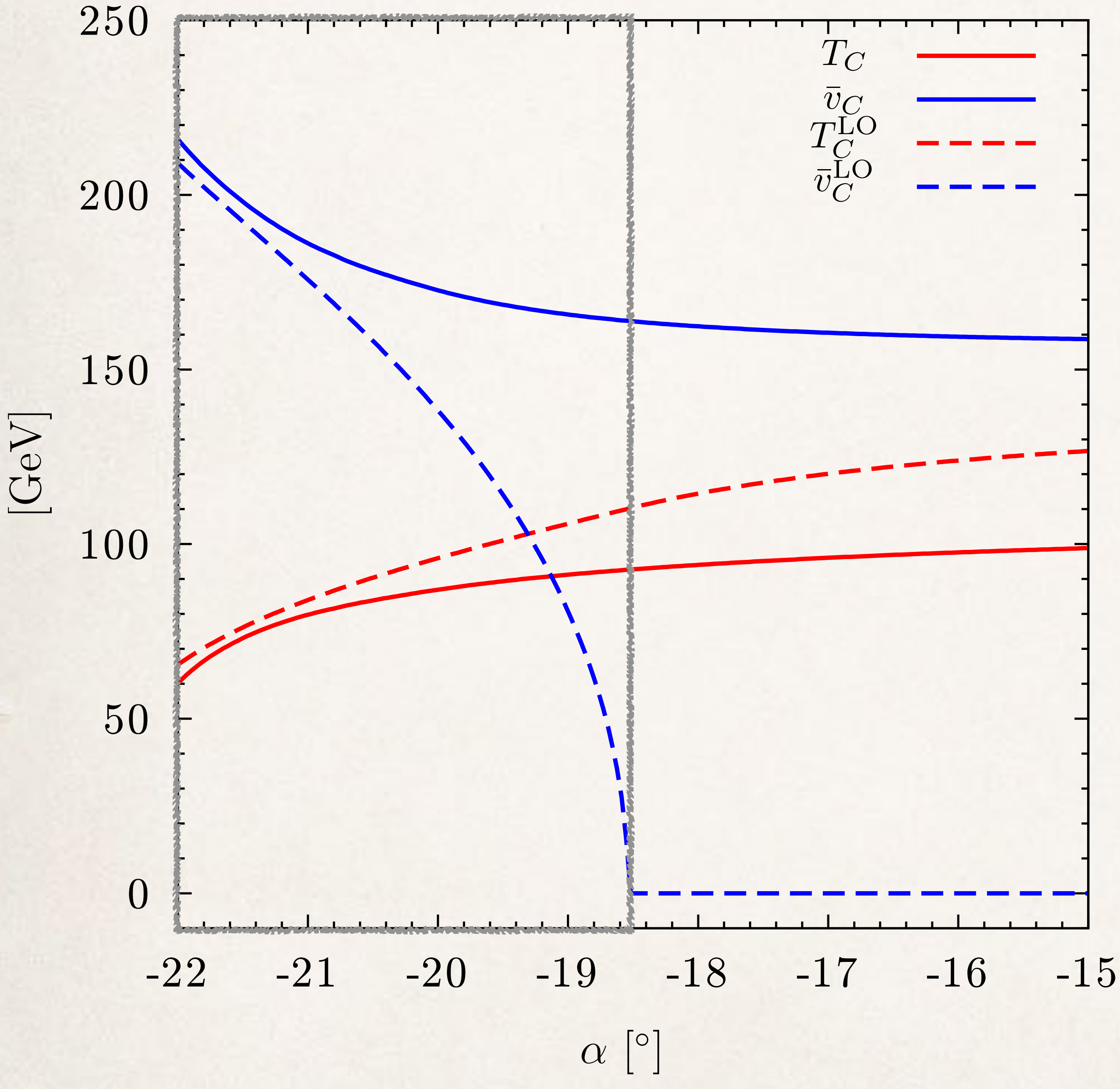

T_C
$\bar{v}_C$
T_C^{LO}
$\bar{v}_C^{\mathrm{LO}}$
250
200
150
100
50
0
[GeV]
-22
-21
-20
-19
-18
-17
-16
-15
α [°]

LO vs. NLO



$$T_C < T_C^{\mathrm{LO}}$$

LO vs. NLO



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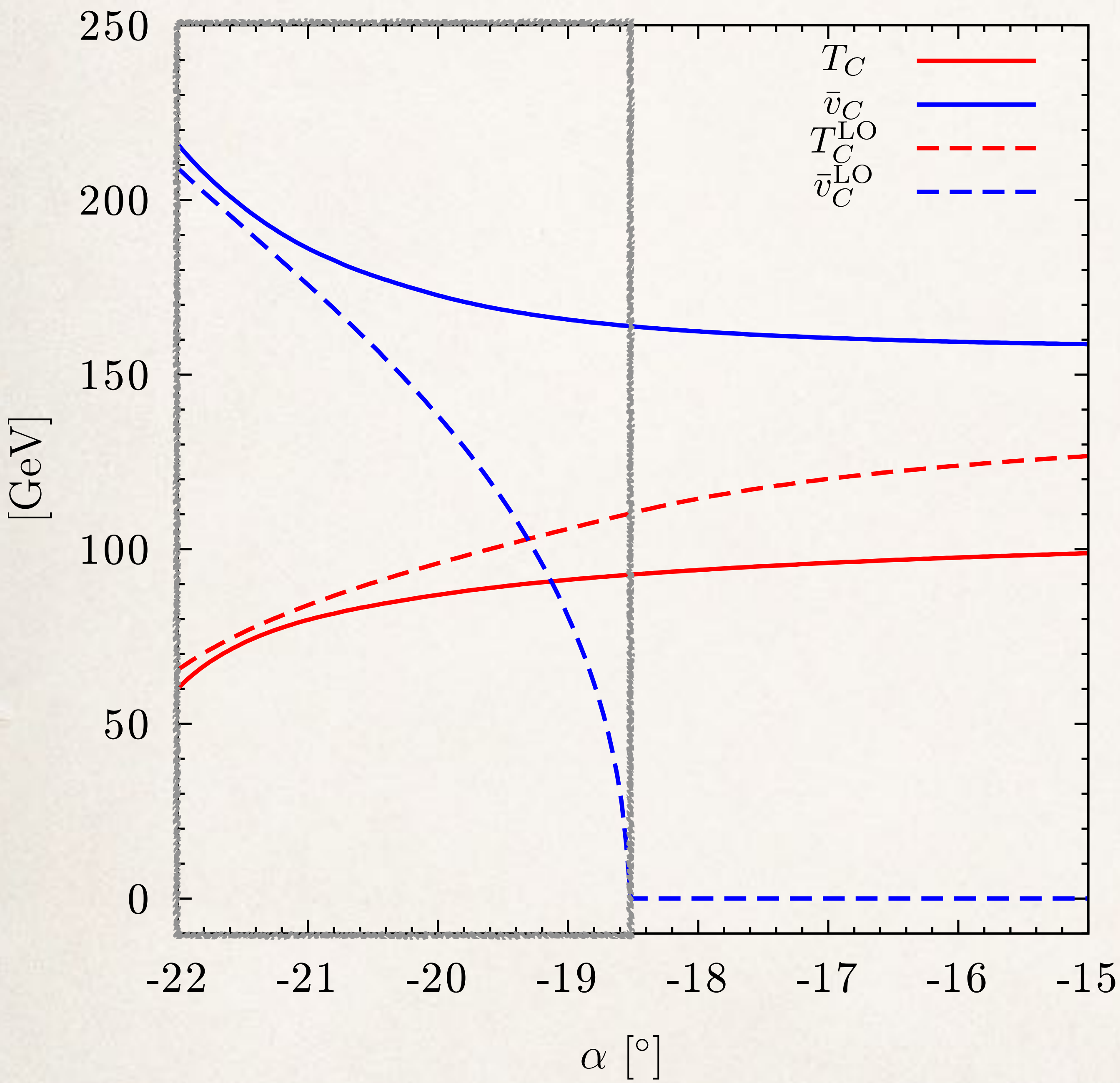
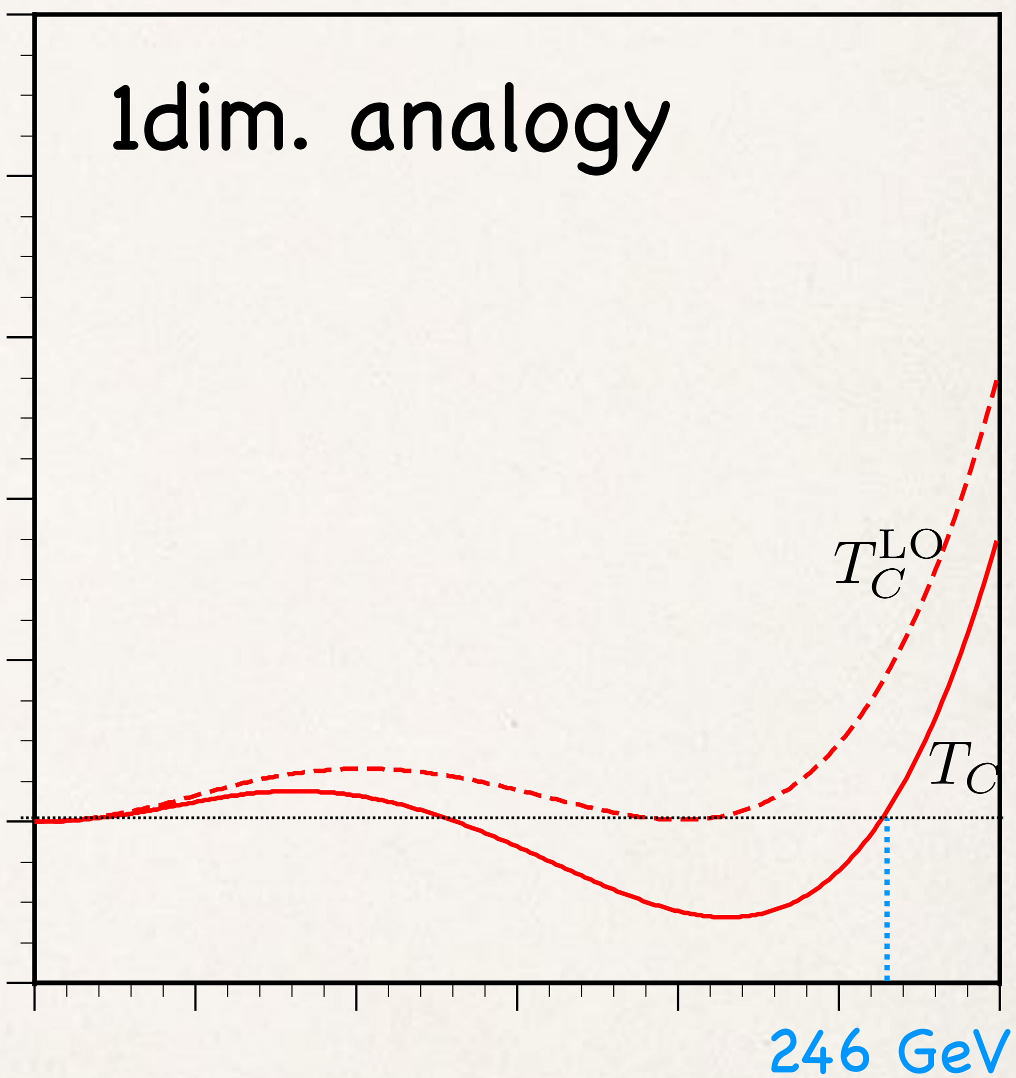


$$T_C < T_C^{\rm LO}$$



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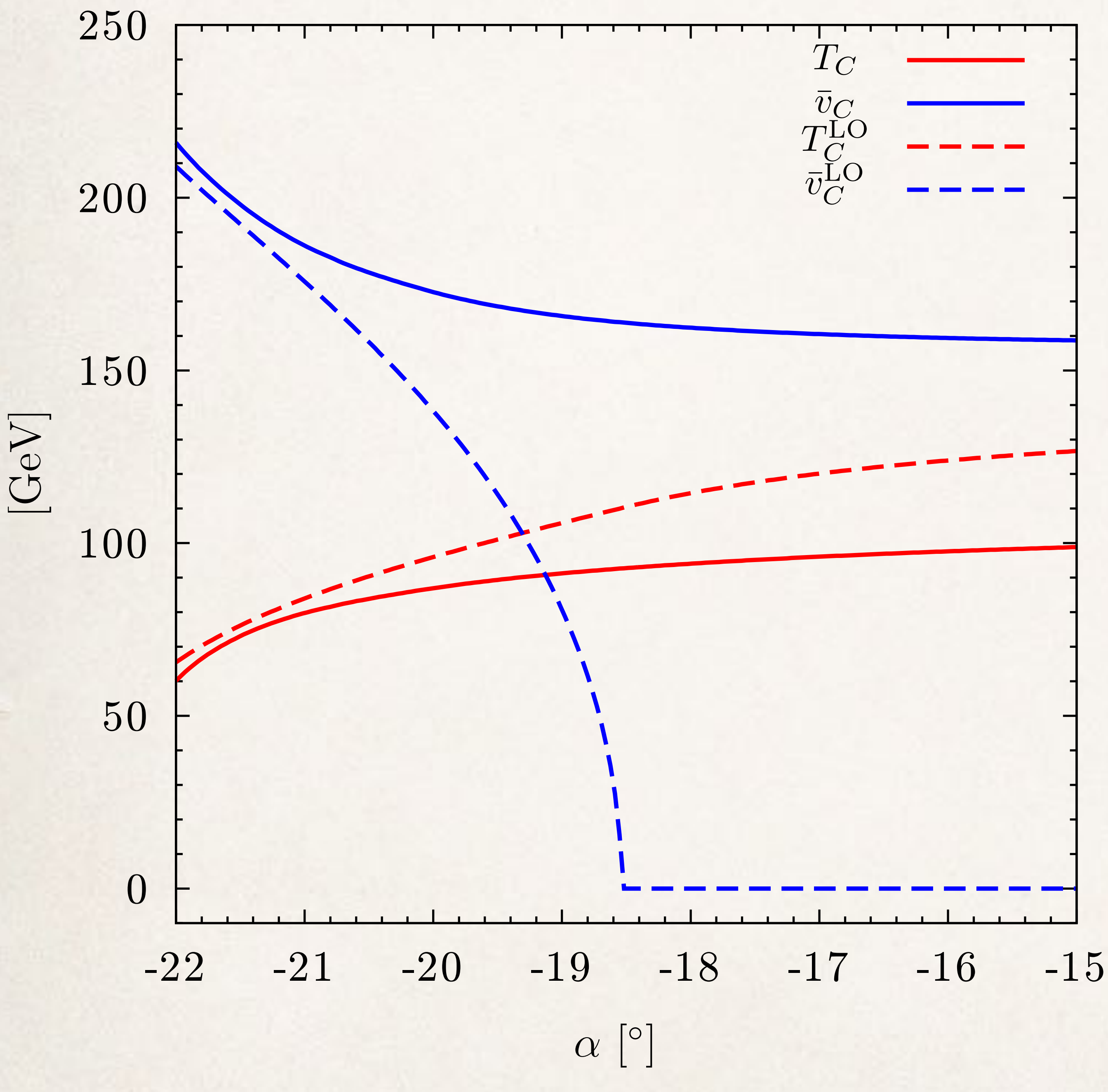

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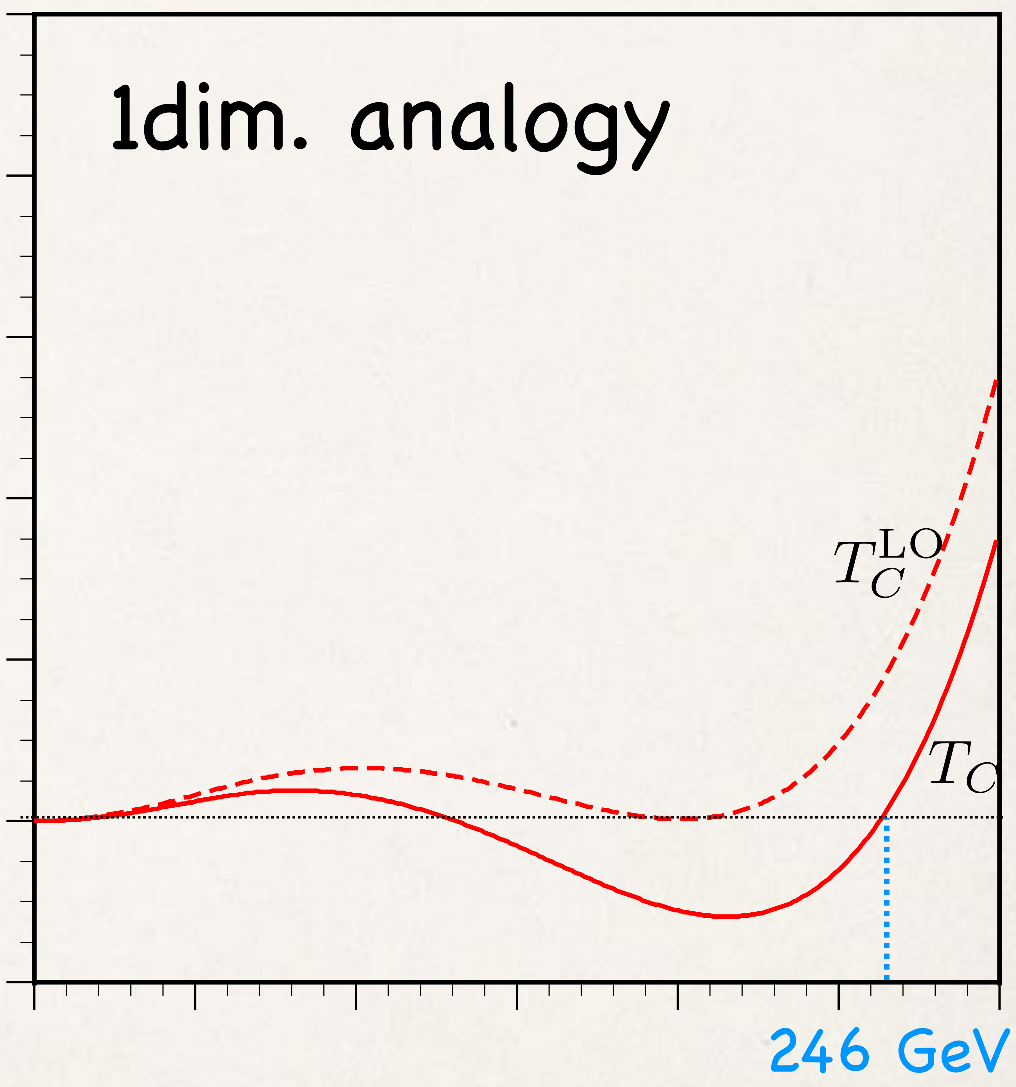

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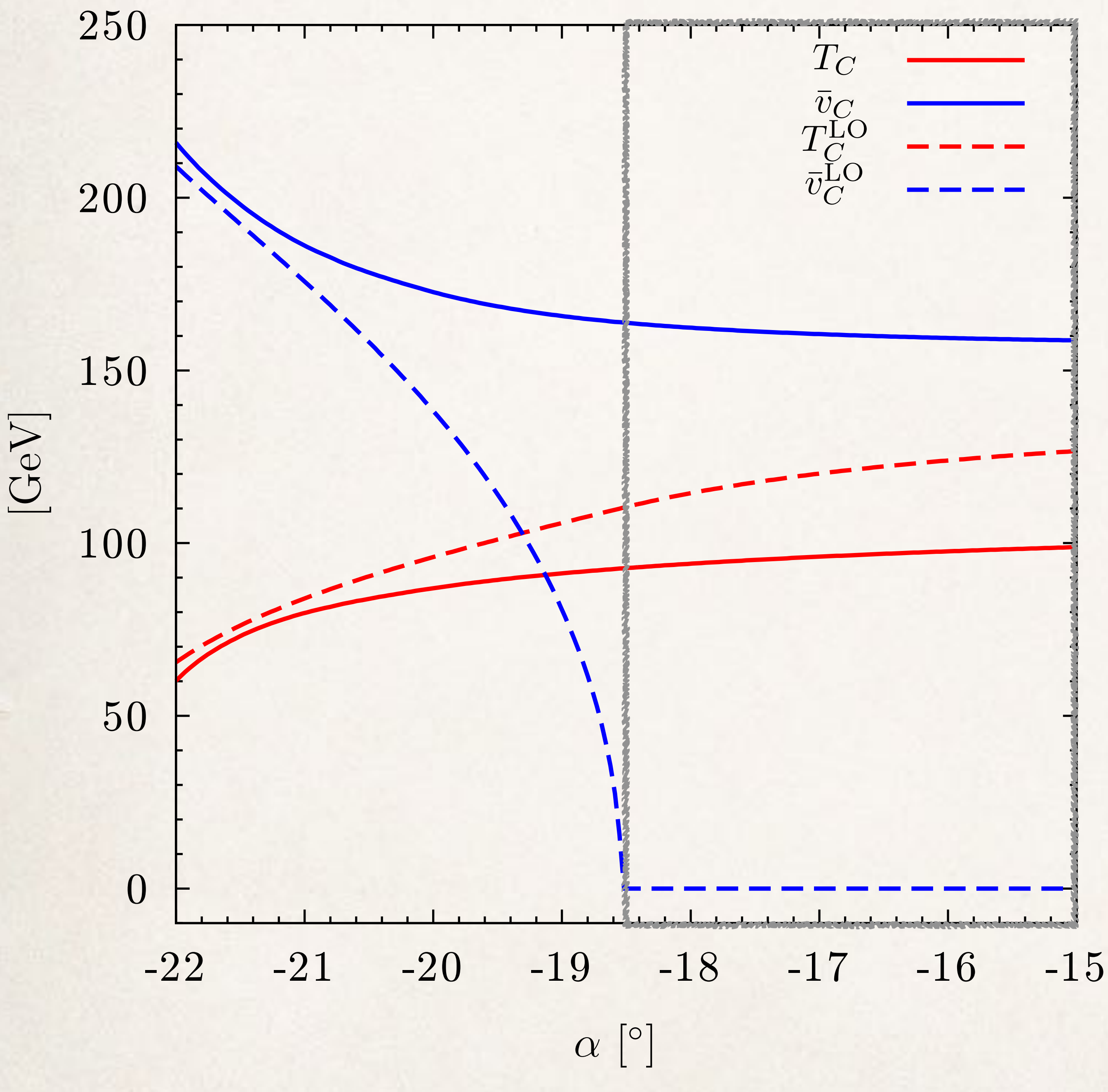

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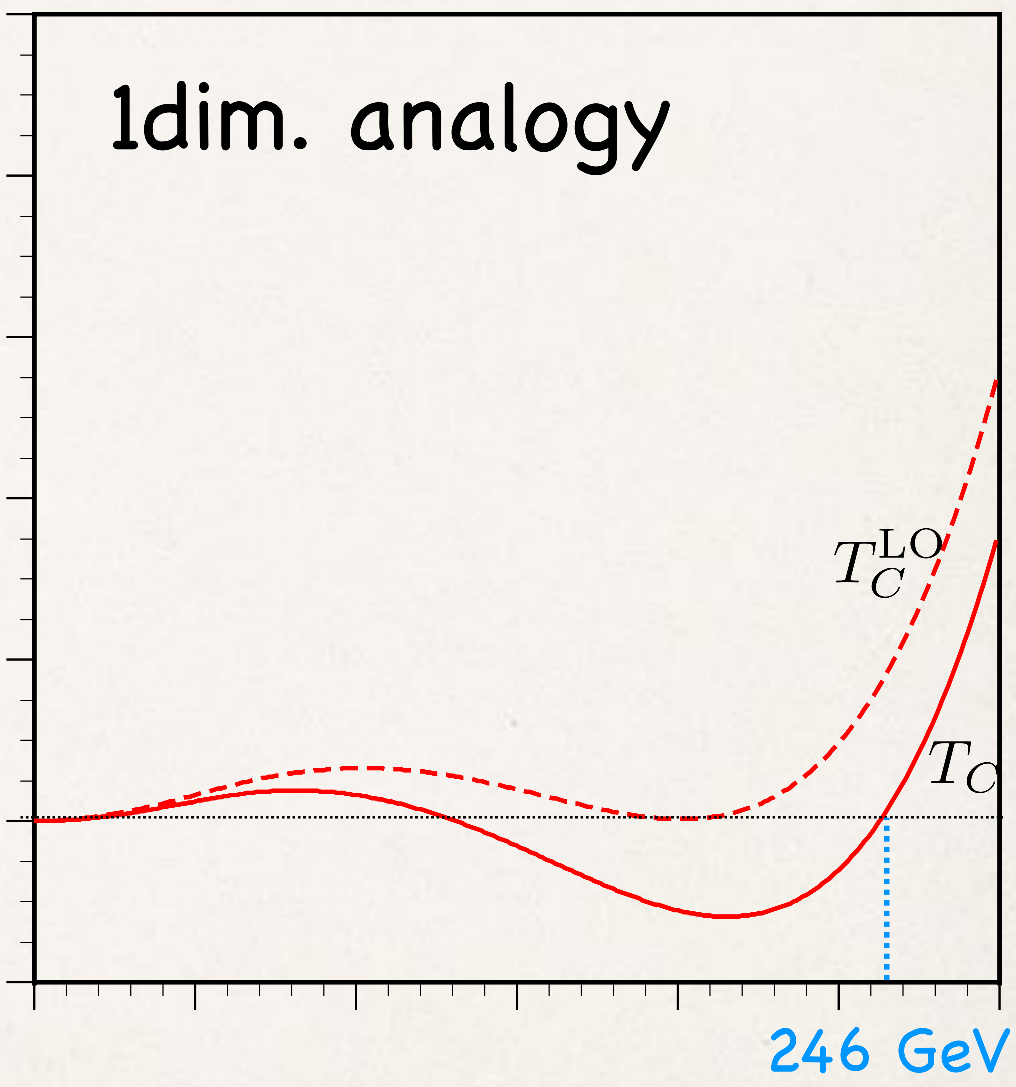
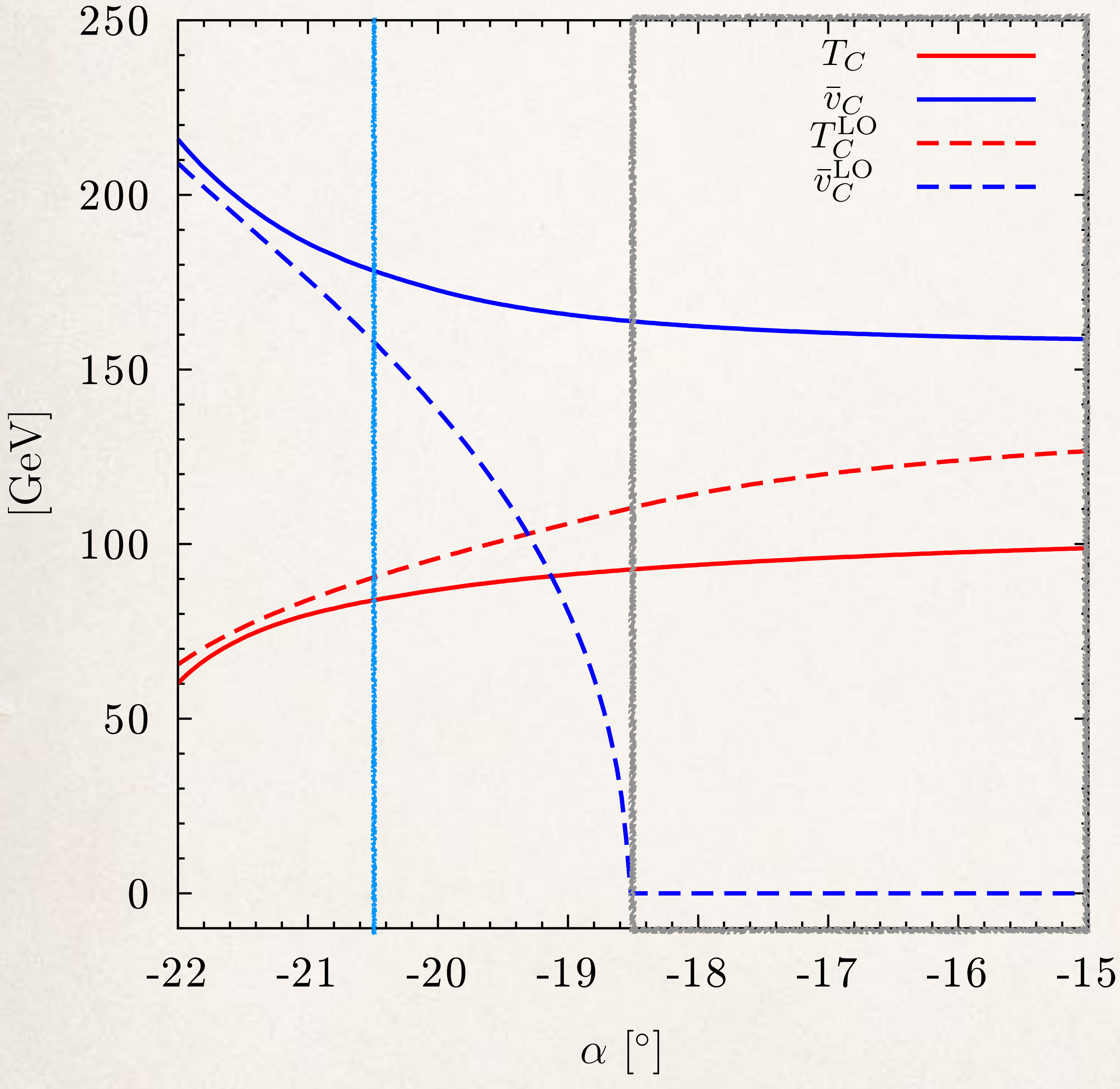


LO vs. NLO

α=-20.5°



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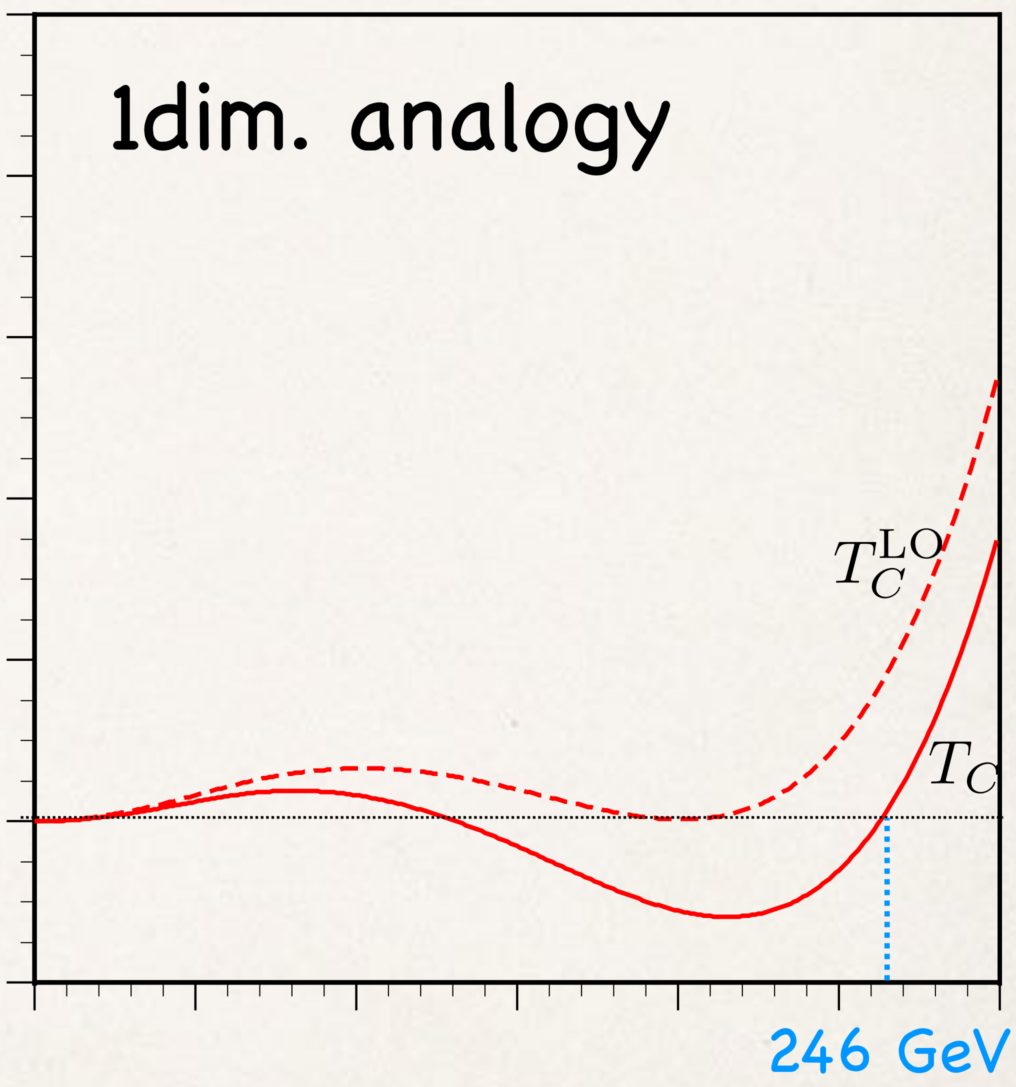


In the following, α=-20.5° is taken.

LO vs. NLO

benchmark point:

$$m_{H_2} = 230 \text{ GeV},\ v_{S0} = 40 \text{ GeV},$$
$$\alpha = -20.5^\circ,\ a_1 = -(110 \text{ GeV})^3$$

Leading Order:

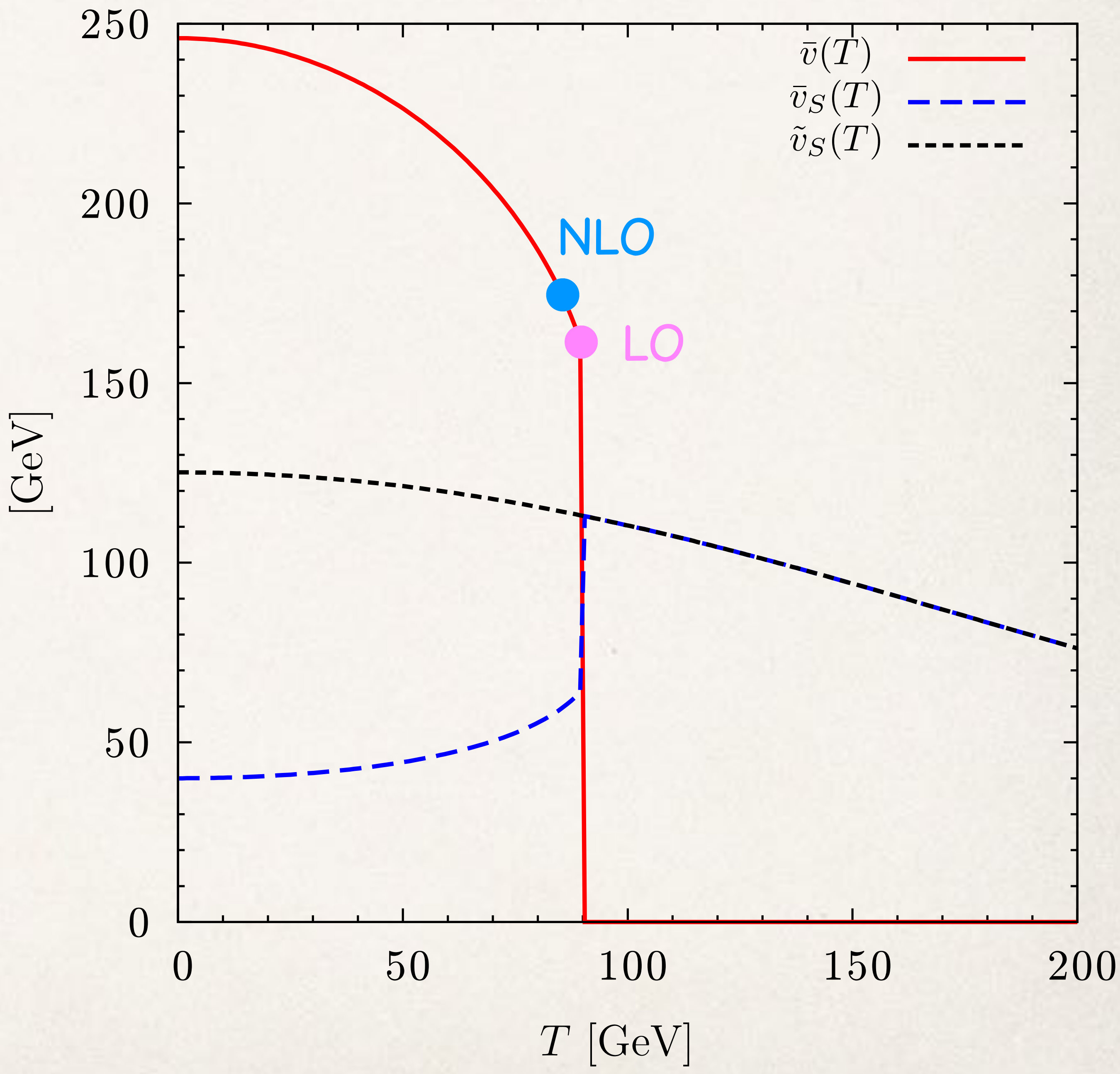
$$T_C^{\rm LO} = 90.4 \text{ GeV},\ \bar{v}_C^{\rm LO} = 158.2 \text{ GeV}$$

Next-to-Leading Order:

$$T_C = 83.1 \text{ GeV},\ \bar{v}_C = 180.2 \text{ GeV}$$

$$\frac{\bar{v}_C}{T_C} > \frac{\bar{v}_C^{\rm LO}}{T_C^{\rm LO}}$$

minima of $V^{\text{high}-T}(\varphi_i; T)$



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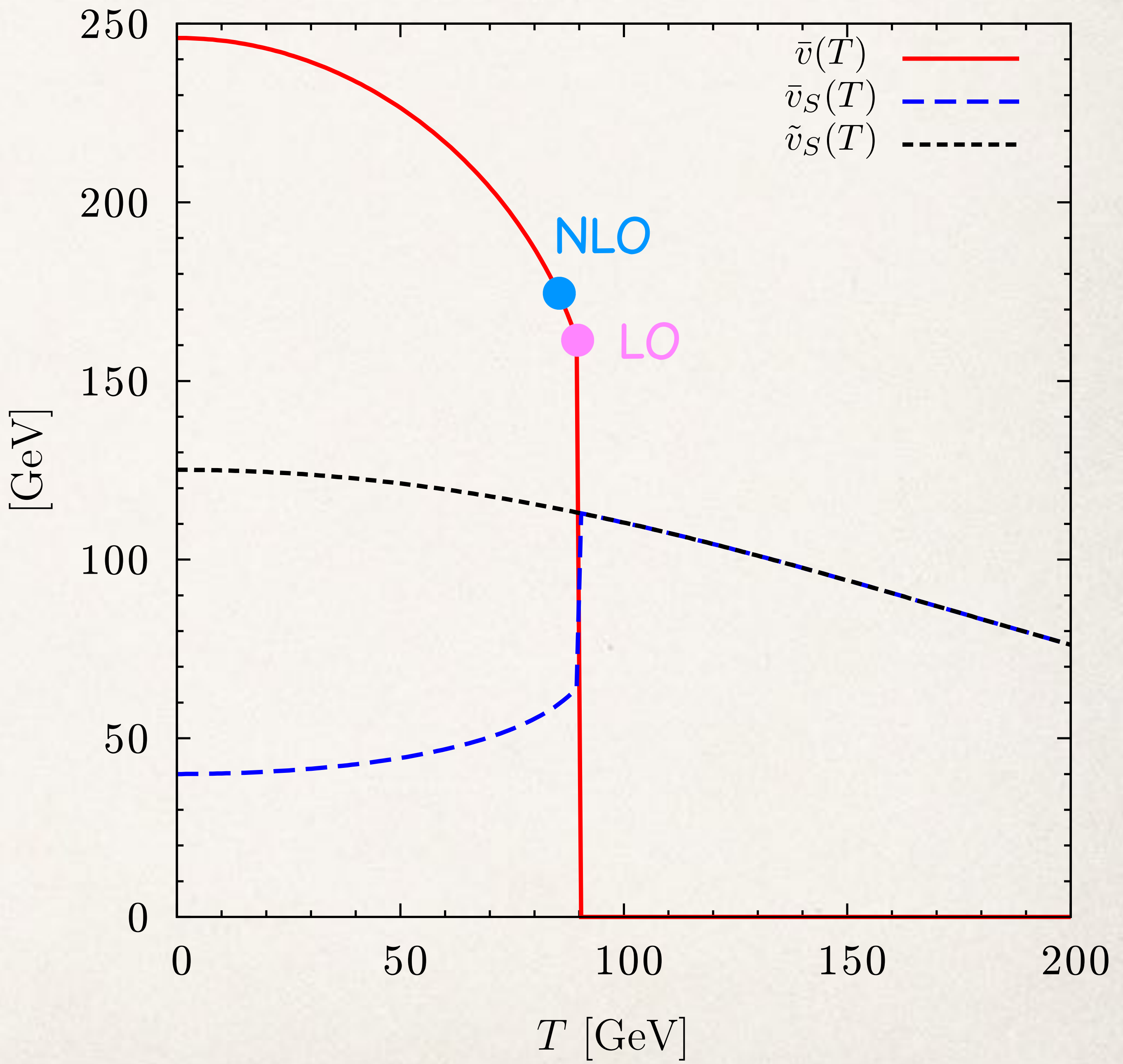
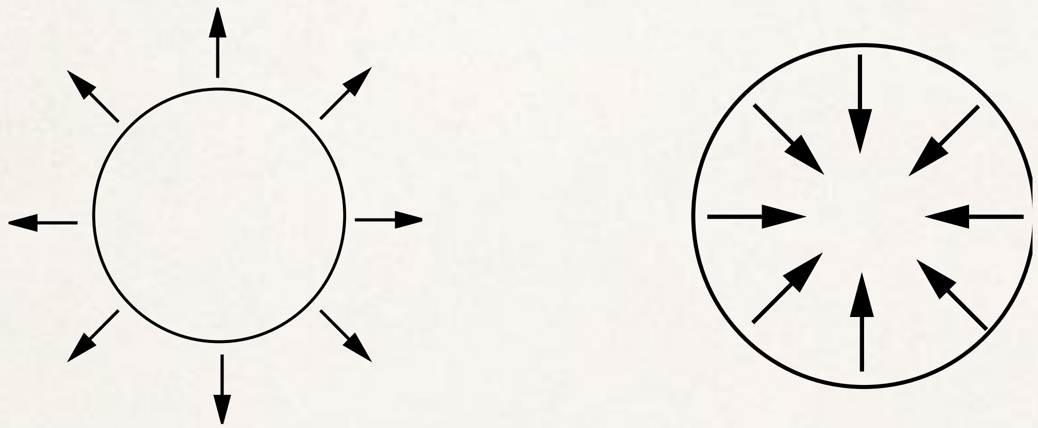


How about nucleation temperature?

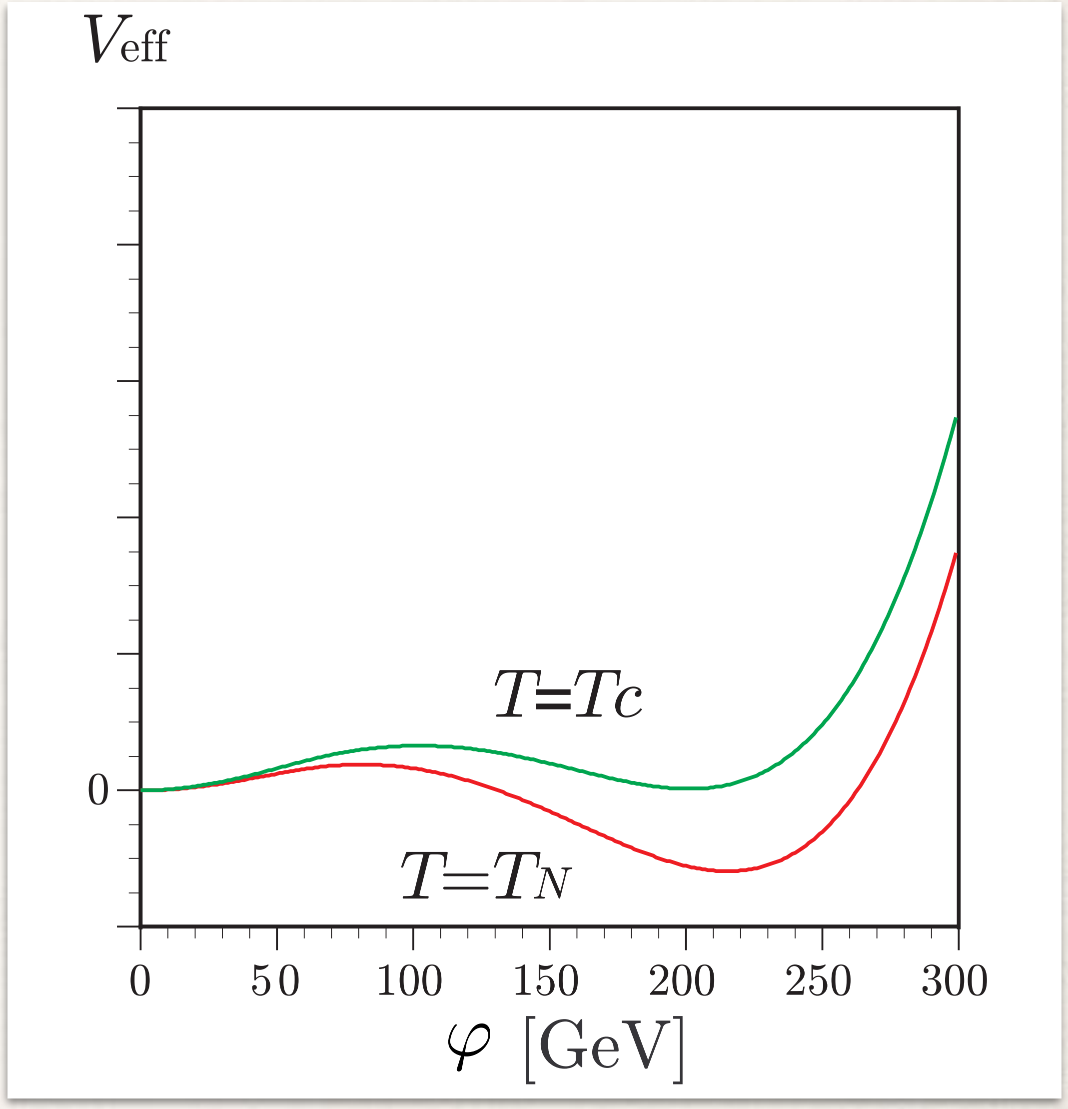
Onset of PT

- T_C is not onset of the PT.
- Nucleation starts somewhat below T_C.

"Not all bubbles can grow"



expand? or shrink?



volume energy vs. surface energy

$\propto$ (radius)^3 $\propto$ (radius)^2

There is a critical value of radius -> critical bubble

Nucleation temperature

- Nucleation rate per unit time per unit volume

$$\Gamma_N(T) \simeq T^4 \left(\frac{S_3(T)}{2\pi T}\right)^{3/2} e^{-S_3(T)/T}$$

[A.D. Linde, NPB216 ('82) 421]

$S_3(T)$: energy of the critical bubble at T

- Definition of nucleation temperature (T_N)

horizon scale $\simeq H(T)^{-1}$

$$\boxed{\Gamma_N(T_N) H(T_N)^{-3} = H(T_N)}$$

$$\frac{S_3(T_N)}{T_N} - \frac{3}{2}\ln\left(\frac{S_3(T_N)}{T_N}\right) = 152.59 - 2\ln g_*(T_N) - 4\ln\left(\frac{T_N}{100\ \mathrm{GeV}}\right)$$

Roughly, $S_3(T)/T \lesssim 150$ is needed for the development of the EWPT.

$S_3(T)/T$

- LO case -

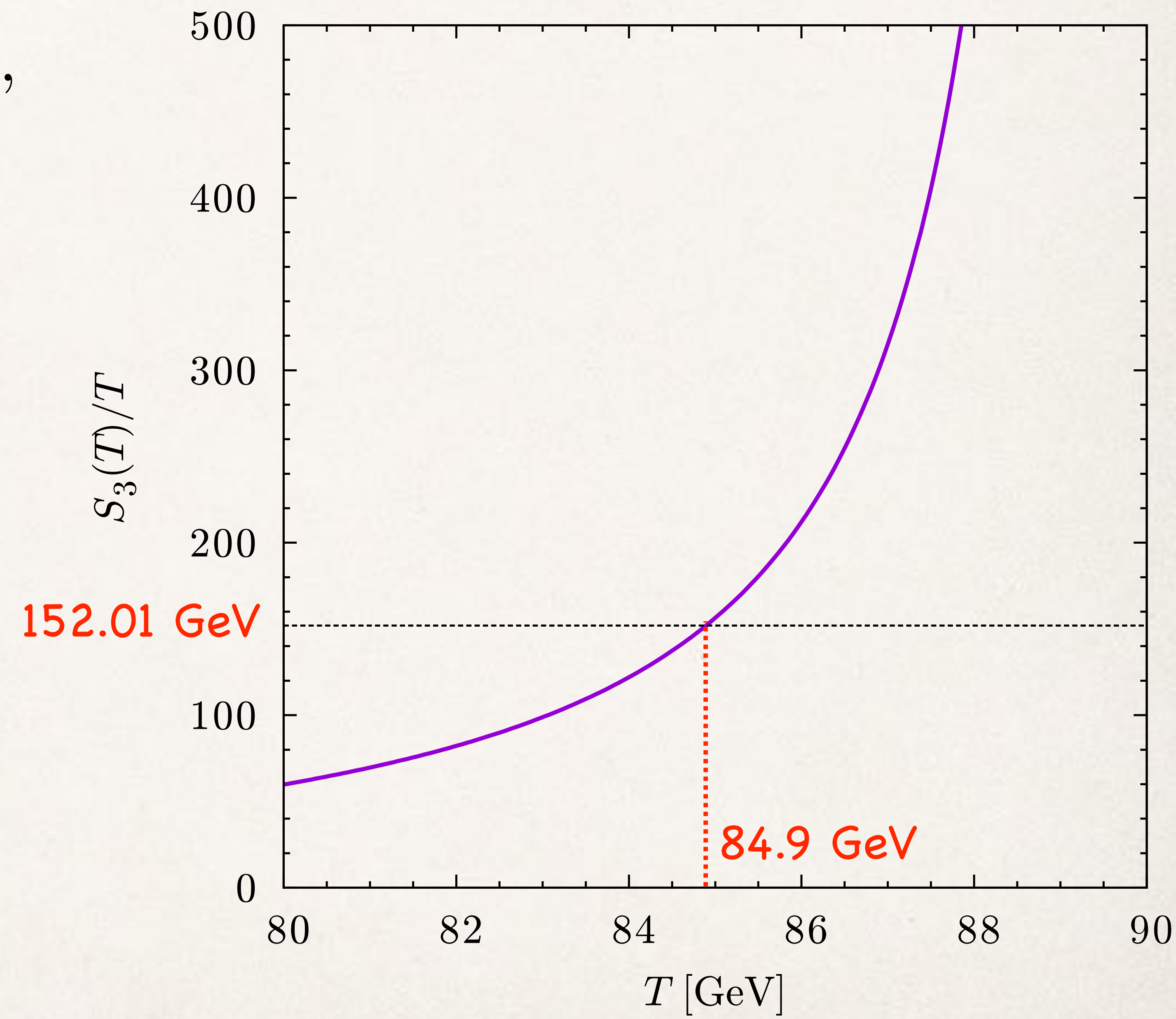
benchmark point:

$$m_{H_2} = 230 \text{ GeV},\ v_{S0} = 40 \text{ GeV},$$
$$\alpha = -20.5^\circ,\ a_1 = -(110 \text{ GeV})^3$$

$$T_N = 84.9 \text{ GeV}$$
$$\frac{S_3(T_N)}{T_N} = 152.01 \text{ GeV}$$

$$\frac{T_C^{\rm LO} - T_N}{T_C^{\rm LO}} \simeq 6.1\%$$

cf., MSSM: O(0.1)%



T_C = 83.1 GeV; T_N (LO) = 84.9 GeV, T_C (LO) = 90.4 GeV

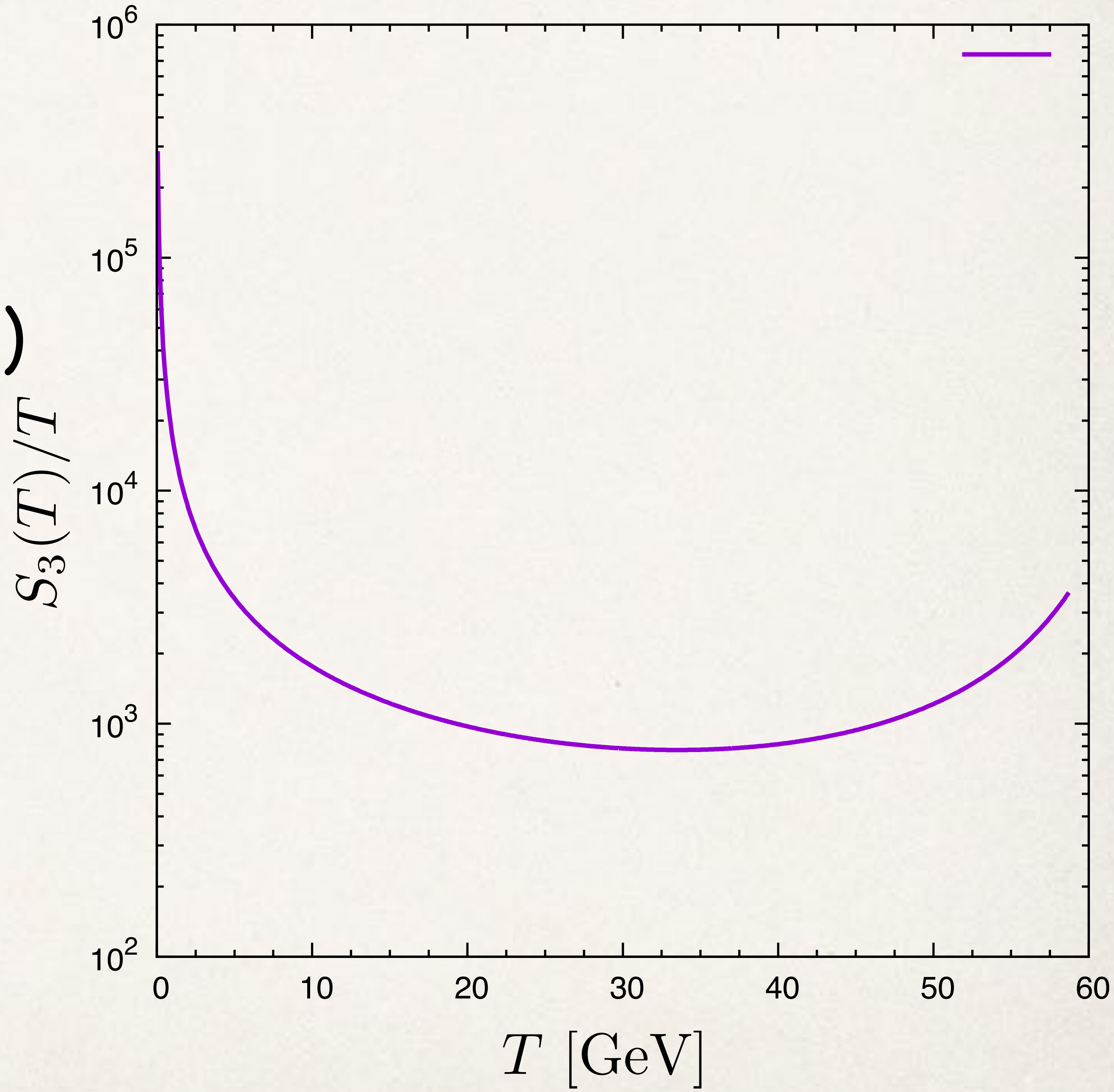
No nucleation case

- α=-22.0°

- V_C/T_C = (209.1GeV)/(65.52GeV) = 3.2

- Too strong 1st-order EWPT may not be consistent!!

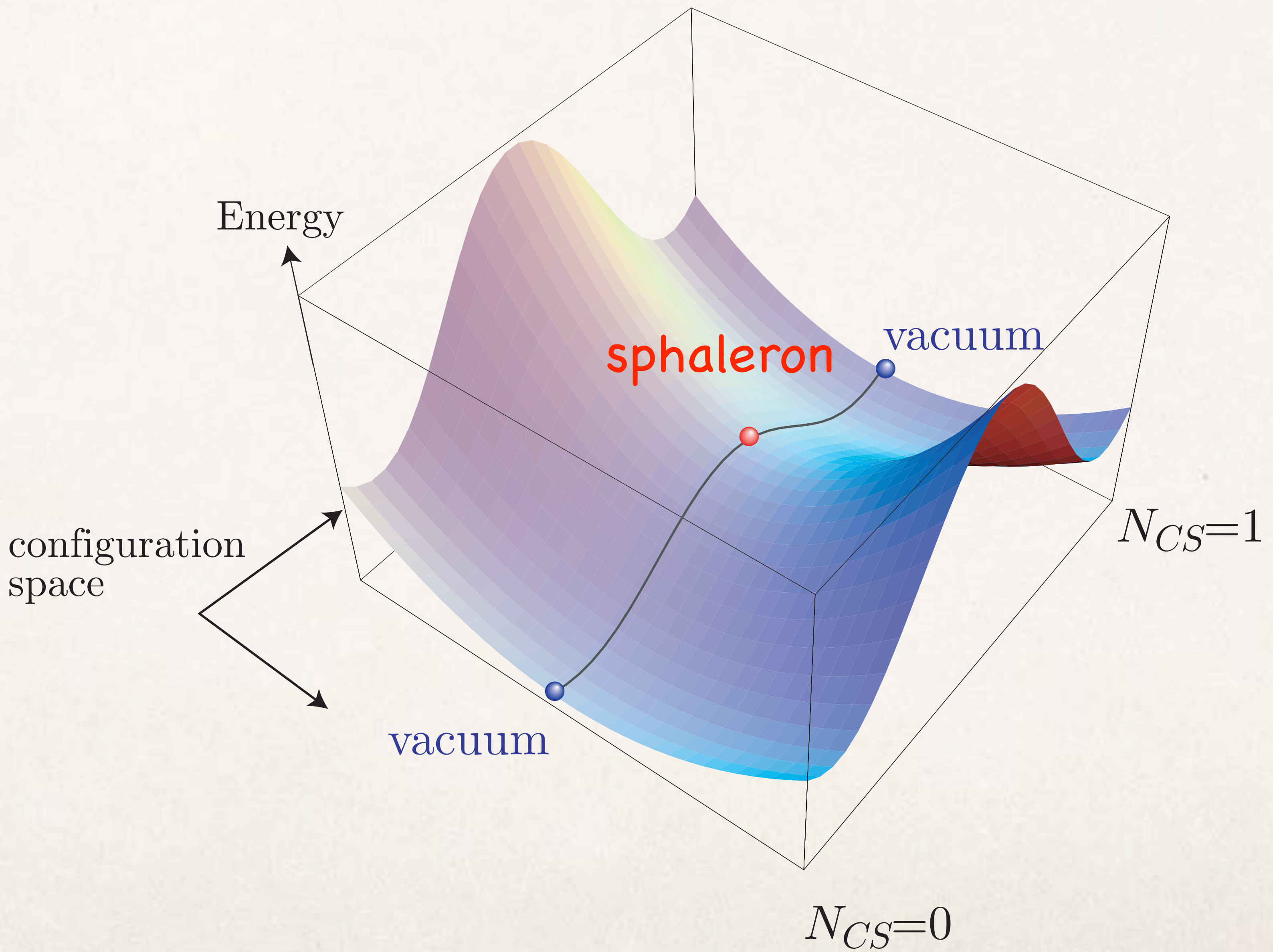
- No nucleation for α <-21.4°



Sphaleron

σ φ α λ ε ρ ο s (sphaleros) "ready to fall"

[F.R.Klinkhamer and N.S.Manton, PRD30, 2212 (1984)]



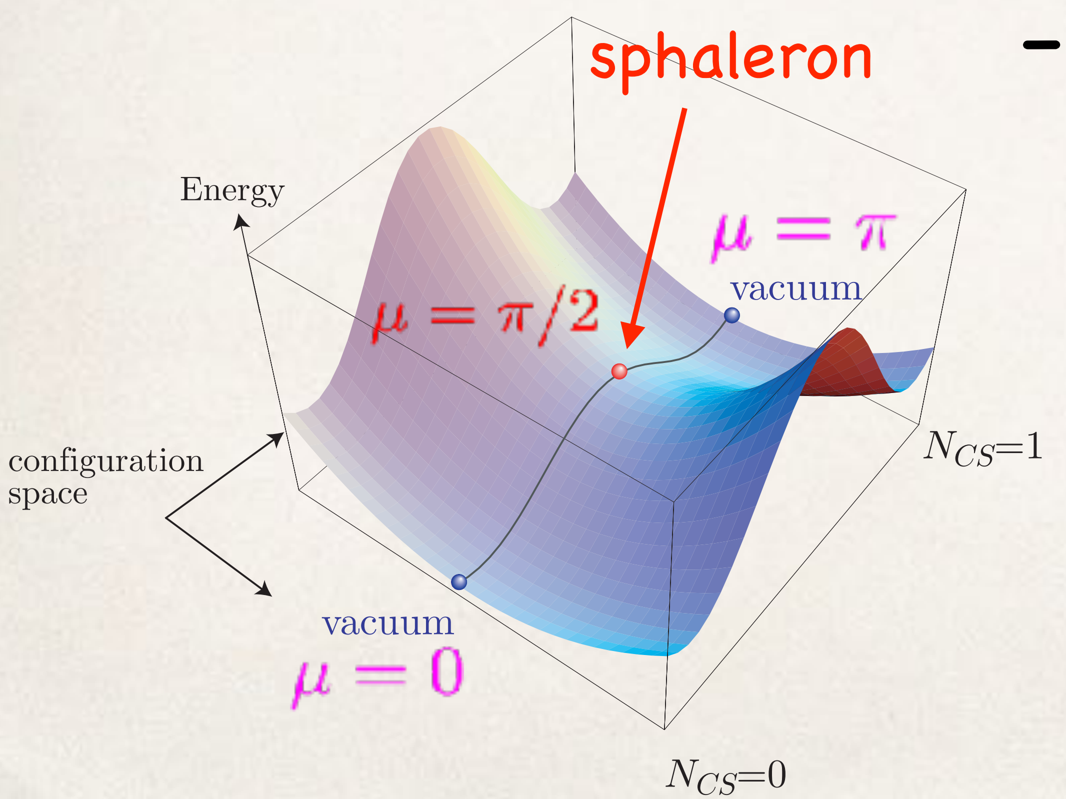
Sphaleron in SU(2) gauge-Higgs system

$$\mathcal{L}_{\text{gauge+Higgs}} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + (D_\mu\Phi)^\dagger D^\mu\Phi - V(\Phi),$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_2\epsilon^{abc}A^b_\mu A^c_\nu,$$

$$D_\mu\Phi = \left(\partial_\mu + ig_2 A^a_\mu \frac{\tau^a}{2}\right)\Phi, \quad V(\Phi) = \lambda\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2.$$

How do we find a saddle point configuration?



-> use of a noncontractible loop.

$$\mu \in [0, \pi]$$

In the limit of $r = |\boldsymbol{x}| = \infty$,

$$A_i^\infty(\mu, \boldsymbol{x}) = \frac{i}{g_2}\partial_i U(\mu,\theta,\phi)U^{-1}(\mu,\theta,\phi),$$

$$\Phi^\infty(\mu, \boldsymbol{x}) = U(\mu,\theta,\phi)\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

Manton's ansatz

[N.S. Manton, PRD28 ('83) 2019]

$$A_i(\mu, r, \theta, \phi) = \frac{i}{g_2} f(r) \partial_i U(\mu, \theta, \phi) U^{-1}(\mu, \theta, \phi),$$

$$\Phi(\mu, r, \theta, \phi) = \frac{v}{\sqrt{2}} \left[(1 - h(r)) \begin{pmatrix} 0 \\ e^{-i\mu} \cos \mu \end{pmatrix} + h(r) U(\mu, \theta, \phi) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right].$$

$\mu = \pi/2 \quad \Rightarrow \quad$ saddle point configuration (sphaleron)

$\mu = 0, \pi \quad \Rightarrow \quad$ vacuum configuration

Changing the variable $r = \sqrt{\boldsymbol{x}^2}$ to ξ, one gets

Energy functional $\left(\mu = \frac{\pi}{2}\right)$

$$E_{\rm sph} = \frac{4\pi v}{g_2} \int_0^\infty d\xi \left[4 \left(\frac{df}{d\xi} \right)^2 + \frac{8}{\xi^2} (f - f^2)^2 + \frac{\xi^2}{2} \left(\frac{dh}{d\xi} \right)^2 + h^2 (1 - f)^2 + \frac{\lambda}{4 g_2^2} \xi^2 (h^2 - 1)^2 \right]$$

$$= \frac{4\pi v}{g_2} \mathcal{E}_{\rm sph}, \quad \text{where} \quad \xi = g_2 v r.$$

Manton's ansatz

[N.S. Manton, PRD28 ('83) 2019]

$$A_i(\mu, r, \theta, \phi) = \frac{i}{g_2} \boxed{f(r)} \partial_i U(\mu, \theta, \phi) U^{-1}(\mu, \theta, \phi),$$

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input: $\frac{\lambda}{g_2^2} \simeq 0.3$ (SM)

Sphaleron energy

Equations of motion for the sphaleron

$$\frac{d^2}{d\xi^2} f(\xi) = \frac{2}{\xi^2} f(\xi)(1 - f(\xi))(1 - 2f(\xi)) - \frac{1}{4} h^2(\xi)(1 - f(\xi)),$$

$$\frac{d}{d\xi}\left(\xi^2 \frac{dh(\xi)}{d\xi}\right) = 2h(\xi)(1 - f(\xi))^2 + \frac{\lambda}{g_2^2}(h^2(\xi) - 1)h(\xi)$$

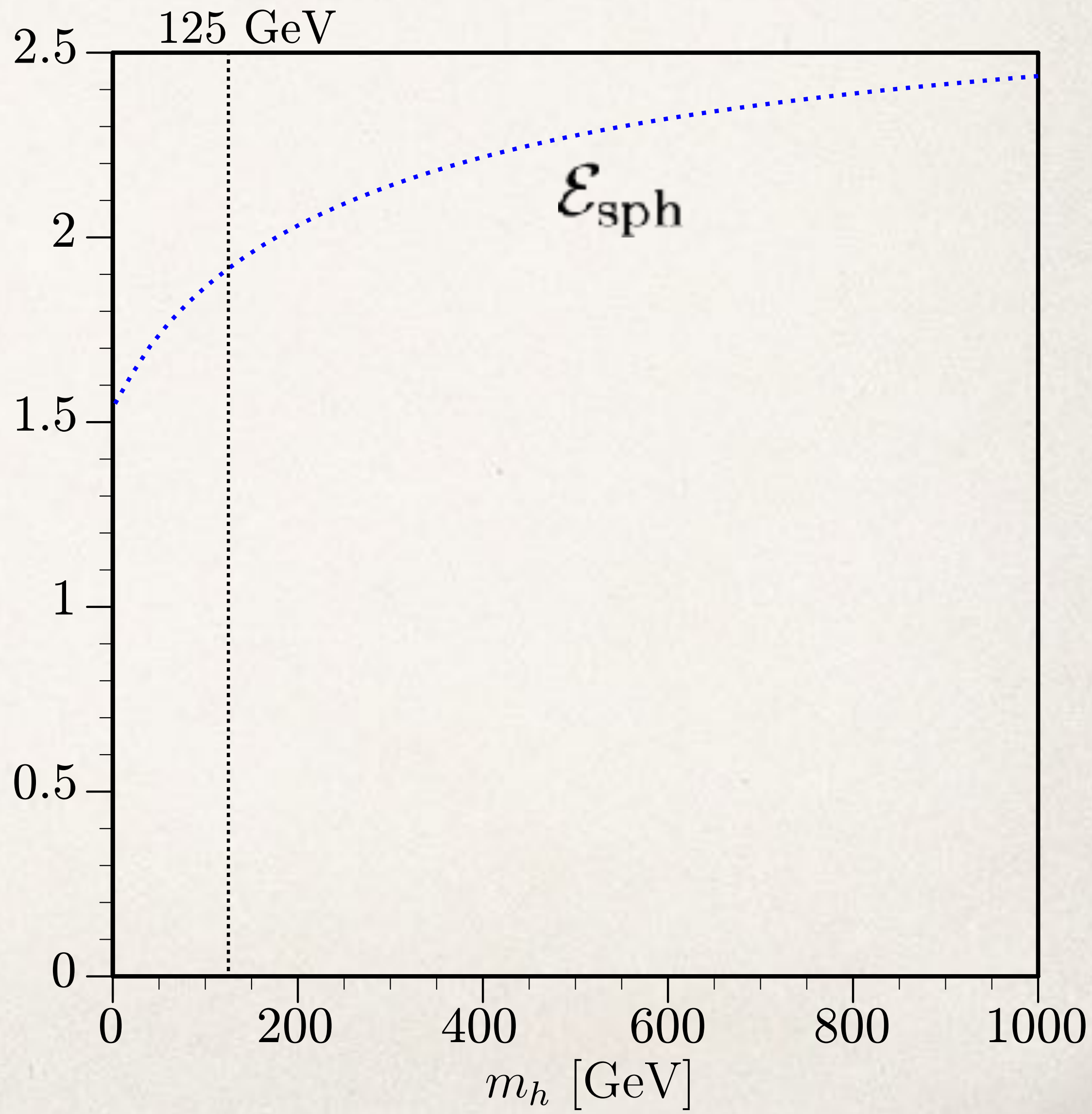
with the boundary conditions:

$$\lim_{\xi \to 0} f(\xi) = 0, \quad \lim_{\xi \to 0} h(\xi) = 0,$$

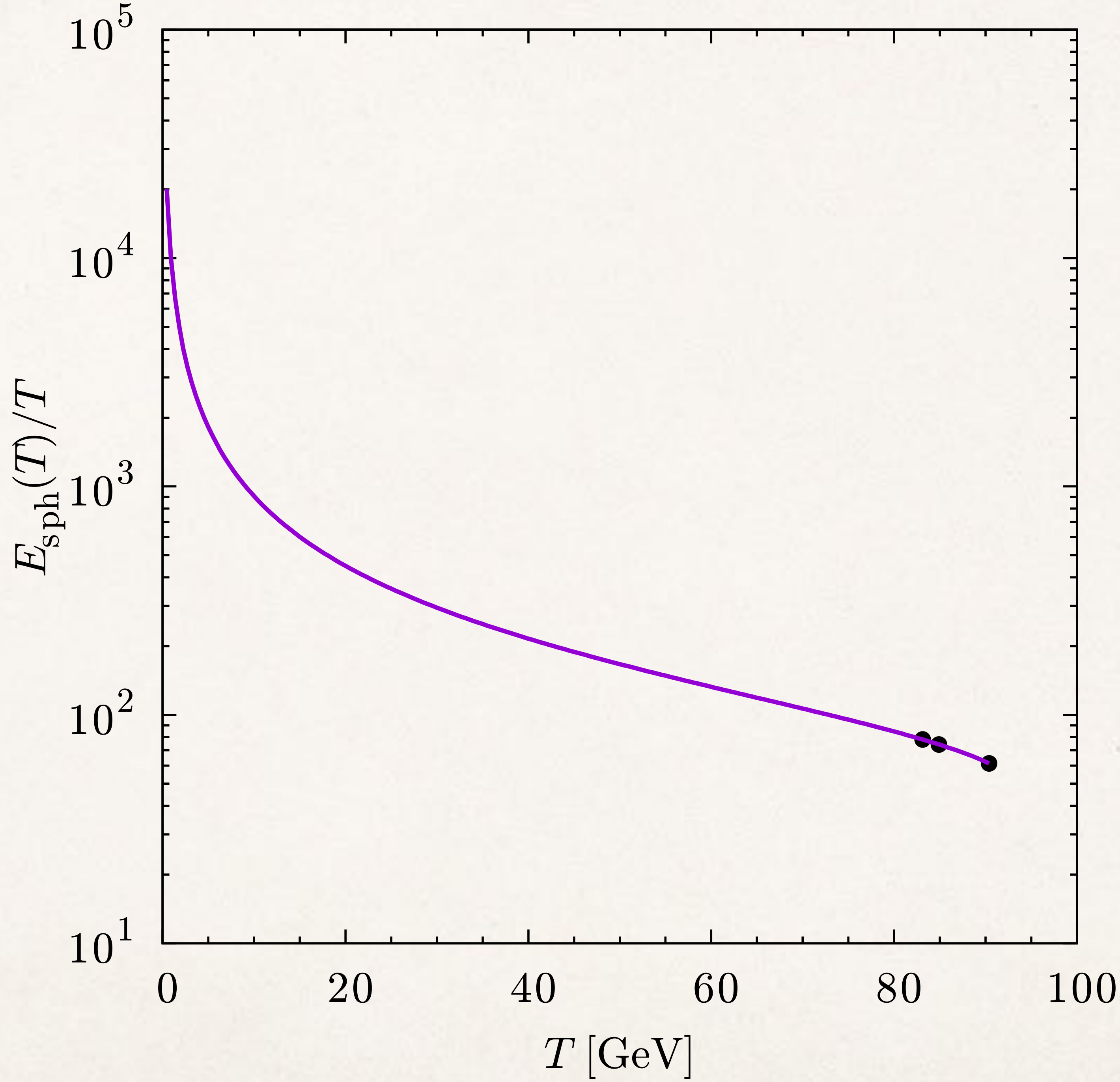
$$\lim_{\xi \to \infty} f(\xi) = 1, \quad \lim_{\xi \to \infty} h(\xi) = 1.$$

For $m_h = 125$ GeV,

$$\boxed{\begin{array}{l} \mathcal{E}_{\rm sph} = 1.92, \\ (E_{\rm sph} = 9.08\ \text{TeV}) \end{array}}$$



$E_{sph}(T)/T$ in cxSM



$$\frac{E_{\rm sph}(T_C)}{T_C} = 78.00, \quad \frac{E_{\rm sph}(T_N)}{T_N} = 74.23, \quad \frac{E_{\rm sph}(T_C^{\rm LO})}{T_C^{\rm LO}} = 61.31,$$

T-dependence of $E_{sph}(T)$

$$E_{\rm sph}(T) = \frac{4\pi\bar{v}(T)}{g_2}\mathcal{E}(T)$$

If T-dependence comes from v(T) only, one has

$$E_{\rm sph}(T) = E_{\rm sph}(0)\frac{\bar{v}(T)}{v_0}$$

Is this scaling law valid?

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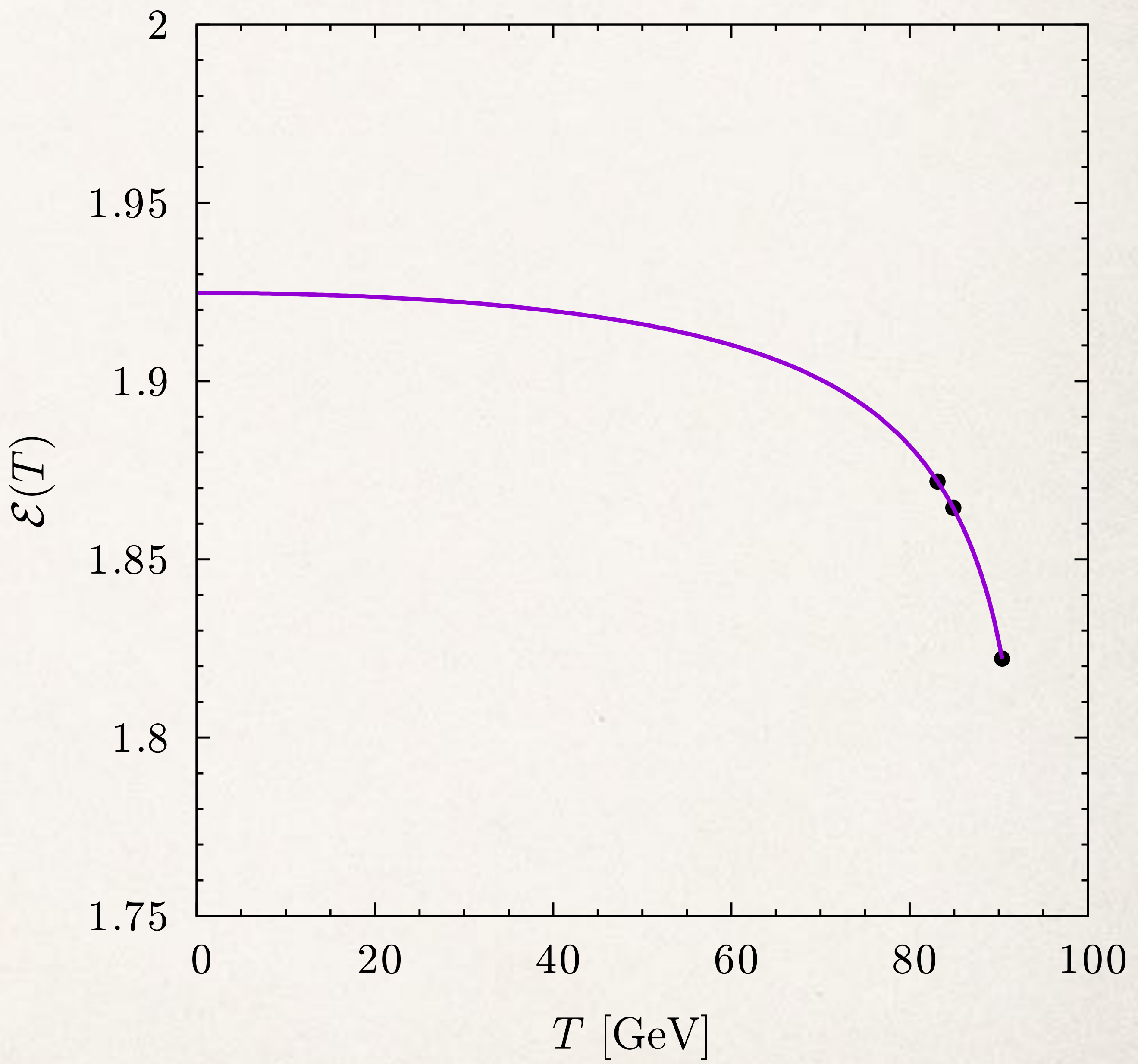

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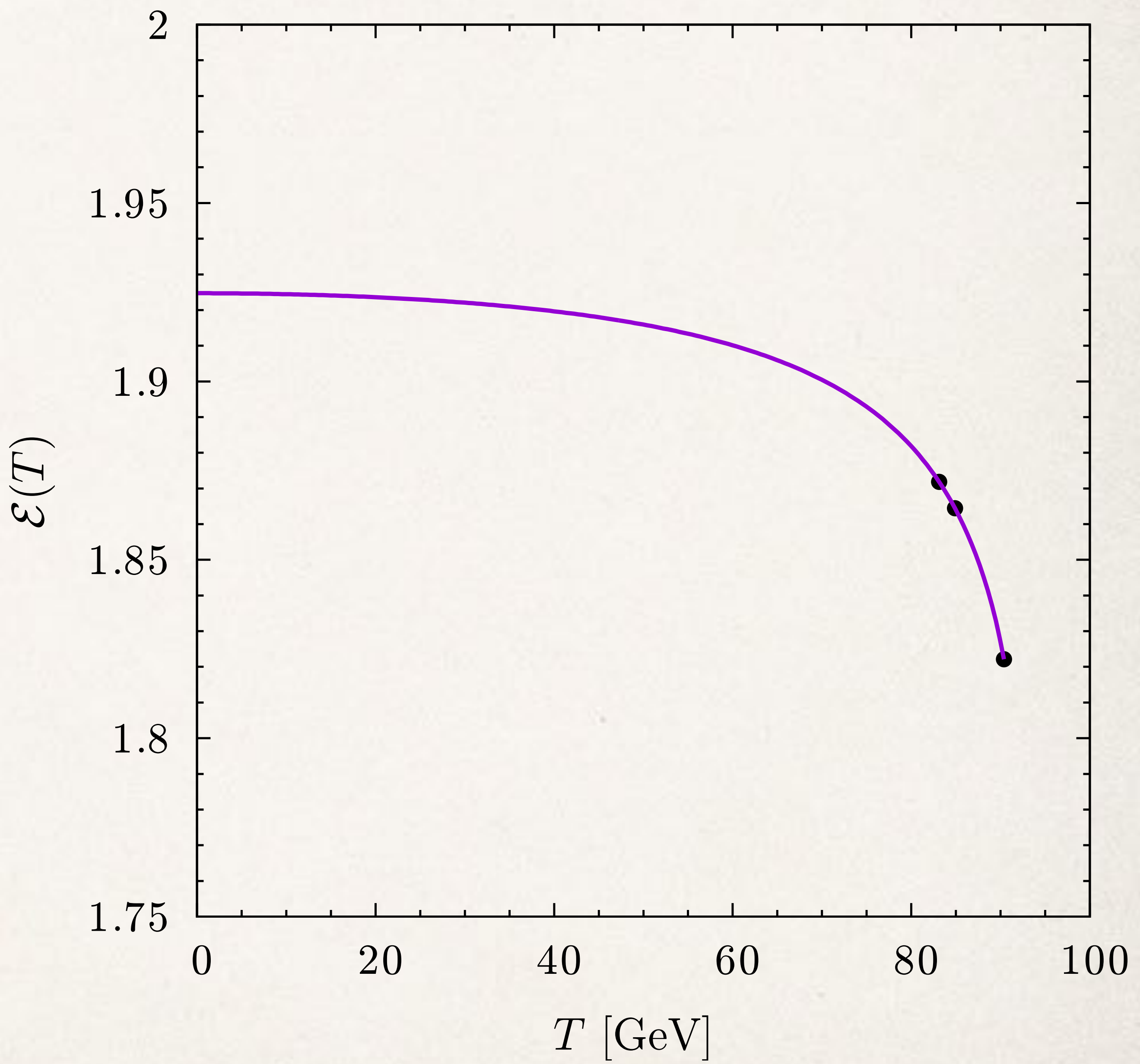

No, it breaks down especially when T approaches T_C.

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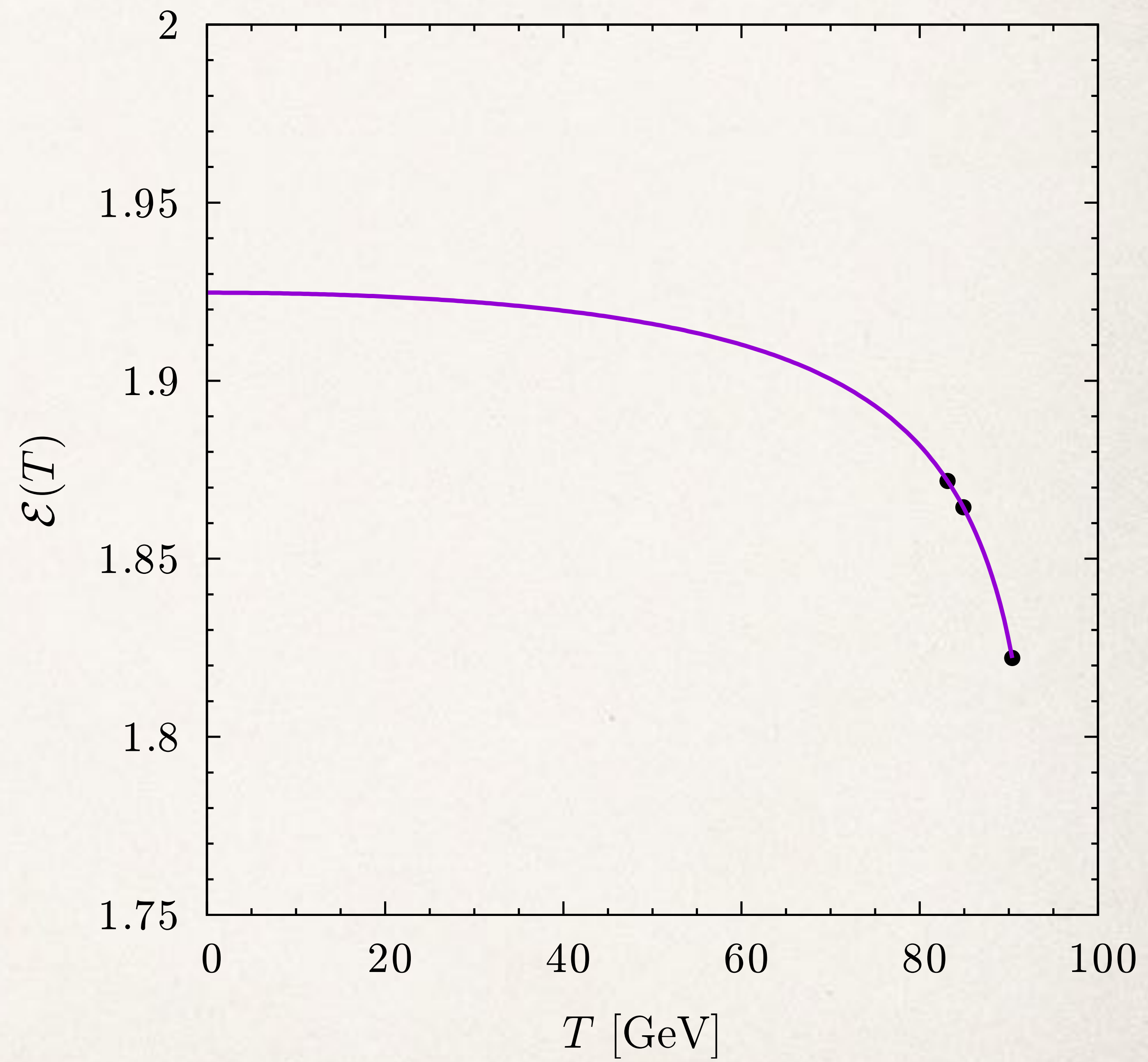

No, it breaks down especially when T approaches T_C.

$\because$ presence of $v_S(T)$.

Summary of the 1st part

- We have evaluated v_C and T_C using GI methods in the cxSM.

-μ dependence can be alleviated by the RG improvement.

- v_C/T_C is greater than the LO result.

$$\rightarrow \quad \frac{E_{\rm sph}(T_C)}{T_C} > \frac{E_{\rm sph}(T_C^{\rm LO})}{T_C^{\rm LO}}$$

- Around phase transition point, T_C is subject to the large theoretical errors. -> higher-order corrections are needed.

Band structure effect on B preservation criteria

based on the collaborators with

Koichi Funakubo (Saga U), Kaori Fuyuto (UMass-Amherst)

Ref. arXiv:1612.05431

B preservation criteria

$$\Gamma_B^{(b)}(T_C) < H(T_C)$$

modified by band effect?

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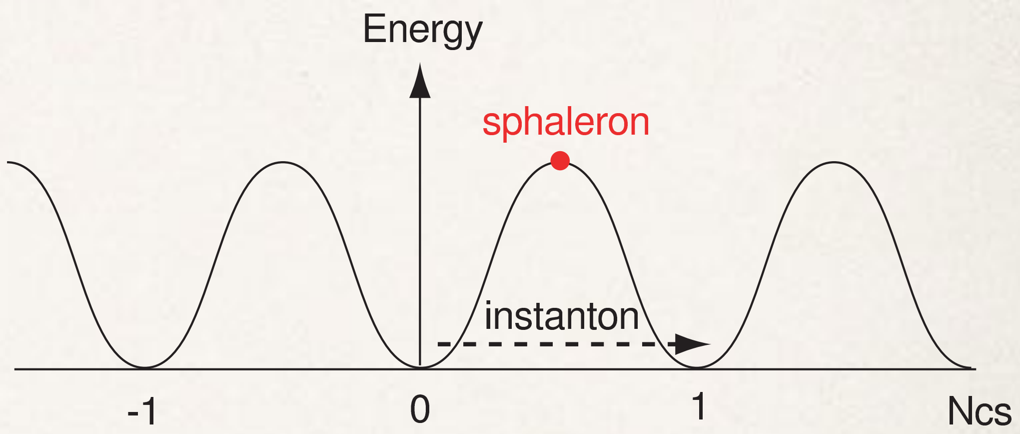
$\longrightarrow$ EWBG-viable region must be re-analyzed!!

B+L violation

- (B+L) is violated by a chiral anomaly in EW theories.



Vacuum transition (instanton)

['t Hooft, PRL37,8 (1976), PRD14,3432 (1976)]

$$\sigma_{\text{instanton}} \simeq e^{-2S_{\text{instanton}}} = e^{-4\pi/\alpha_W} \simeq 10^{-162}$$

Transition rate at finite-E

instanton-based [Ringwald, NPB330,(1990)1, Espinosa, NPB343 (1990)310]

$$\sigma(E) \sim \exp\left(\frac{4\pi}{\alpha_W} F(E)\right) \qquad E \nearrow \implies \sigma(E) \nearrow$$

- But, instanton-based calculation is not valid at $E > E_{sph}$

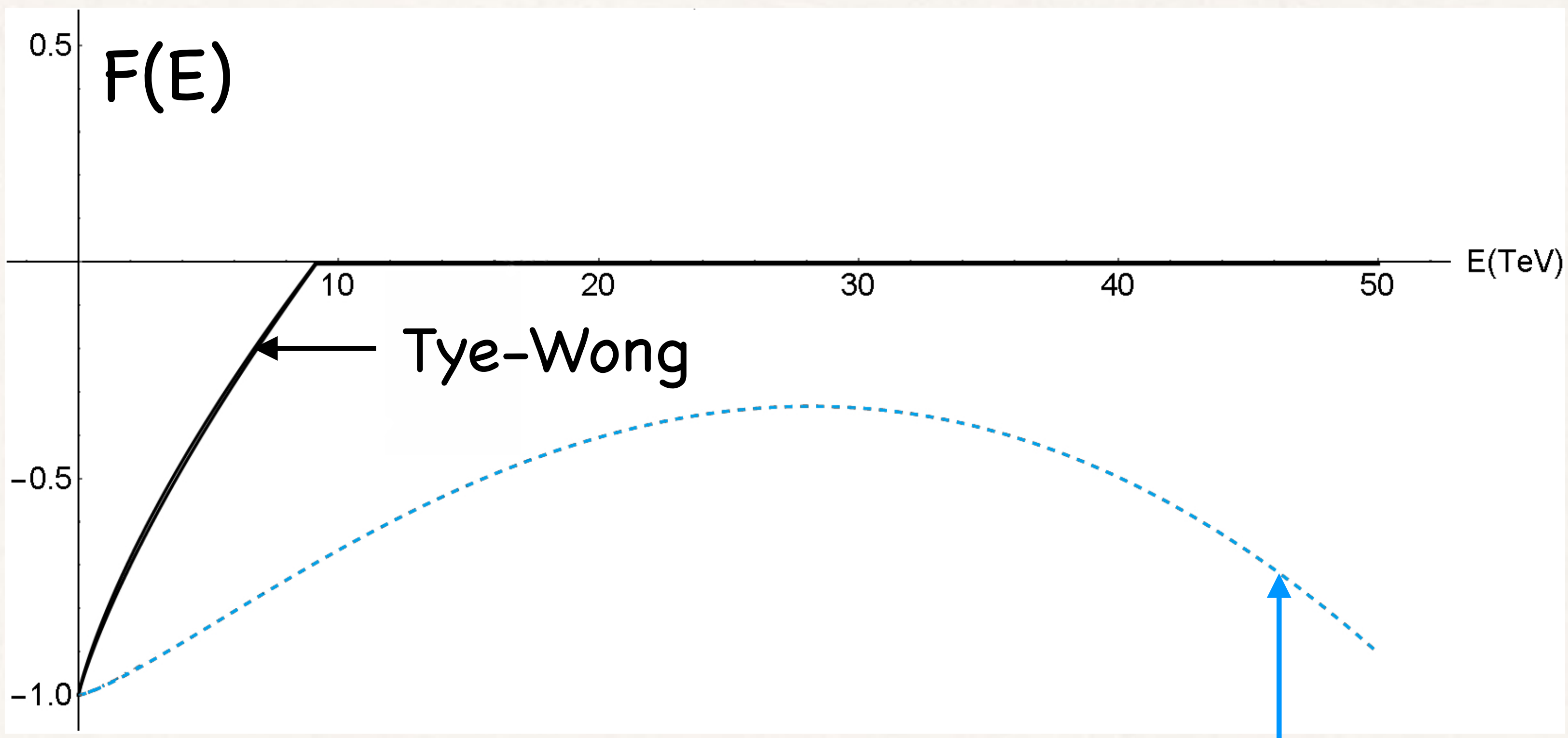
Bounce is more appropriate (transition between the finite-E states)

-> Reduced model.

[Aoyama, Goldberg, Ryzak, PRL60, 1902 ('88)]
[Funakubo, Otsuki, Takenaga, Toyoda, PTP87,663('92), PTP89,881('93)]
[H. Tye, S. Wong, PRD92,045005 ('15)]

Tye-Wong's work

[H. Tye, S. Wong, PRD92,045005 (2015)]



$$F(E) = -1 + \frac{9}{8}\left(\frac{E}{E_0}\right)^{4/3} - \frac{9}{16}\left(\frac{E}{E_0}\right)^{2} + \cdots$$ (instanton calculus) $E_0 \simeq 15$ TeV

F(E) = 0 for $E > E_{sph}$ (Tye-Wong) ∵ a band structure

Q: Does the band affect sphaleron process at finite-T?

Reduced model

[Aoyama, Goldberg, Ryzak, PRL60, 1902 (1988)]
[Funakubo, Otsuki, Takenaga, Toyoda, PTP87, 663 (1992), PTP89, 881 (1993)]
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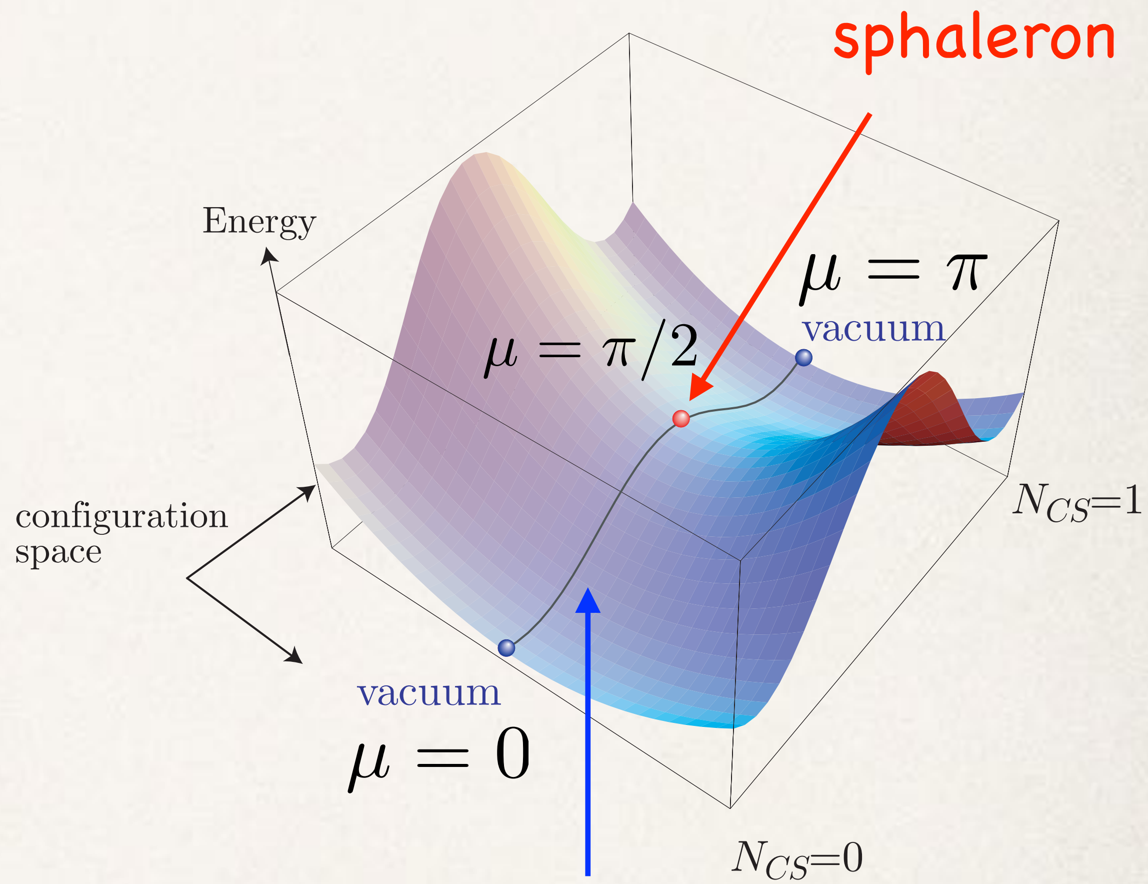
Let us promote μ to a dynamical variable:

$$\boxed{\mu \Rightarrow \mu(t)}$$

$\mu(-\infty)=0$, $\mu(+\infty)=\pi$: vacuum,
$\mu(t_{sph})=\pi/2$: sphaleron



- We construct a reduced model by adopting a Manton's ansatz.

Comparison with Tye-Wong's work

Some differences between our work and Tye-Wong's (TW's).

	A_0	Sphaleron mass	Method for band structure
this work	$A_0 \neq 0$	μ-dependent	WKB w/ 3 connection formulas
Tye-Wong	$A_0 = 0$	μ-independent	Schroedinger eq. numerically

We use the Manton's ansatz with $A_0 = \frac{i}{g_2} f(r) \partial_0 U U^{-1}$.

Unlike the previous studies, our method is fully gauge invariant.

N.B.

If $A_0=0$ is naively adopted with the Manton's ansatz, an unwanted divergence would appear in DΦ at the region r=∞. -> some prescription is needed!!

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Classical action

$$S[\mu] = g_2 v \int dt \left[\frac{M(\mu)}{2}\left(\frac{d}{dt}\frac{\mu(t)}{g_2 v}\right)^2 - V(\mu)\right],$$

$$\begin{aligned}M(\mu) = \frac{4\pi}{g_2^2}\int_0^\infty d\xi\ \xi^2 \Bigg[&4\left\{f'^2\frac{4+2c_\mu^2}{3} + \frac{4}{\xi^2}(f-f^2)^2\frac{8+2c_\mu^2}{3}s_\mu^2\right\} \\ &+(1-h)^2 + 2h(1-h)(1-f) + 2(1-h)^2 f c_\mu^2 \\ &+\frac{4+2c_\mu^2}{3}\left\{h^2(1-f)^2 + \Big((1-h)^2(f^2-2f) - 2h(1-h)f(1-f)\Big)c_\mu^2\right\}\Bigg]\end{aligned}$$

$$\begin{aligned}V(\mu) = \frac{4\pi}{g_2^2}\int_0^\infty d\xi\ \xi^2 \Bigg[&\frac{4}{\xi^2}\left\{f'^2 + \frac{2}{\xi^2}(f-f^2)^2 s_\mu^2\right\}s_\mu^2 \\ &+\frac{s_\mu^2}{2}\left\{h'^2 + \frac{2}{\xi^2}\Big(h^2(1-f)^2 - 2h(1-h)f(1-f)c_\mu^2 + f^2(1-h)^2c_\mu^2\Big)\right\} \\ &+\frac{\lambda}{4g_2^2}(1-h^2)^2 s_\mu^4\Bigg].\end{aligned}$$

f, h are determined by the EOM for the sphaleron.

Classical action

$$S[\mu] = g_2 v \int dt \left[\frac{M(\mu)}{2} \left(\frac{d}{dt} \frac{\mu(t)}{g_2 v} \right)^2 - V(\mu) \right],$$

where $M(\mu) = \frac{4\pi}{g_2^2} \left(\alpha_0 + \alpha_1 \cos^2 \mu + \alpha_2 \cos^4 \mu \right), \quad V(\mu) = \frac{4\pi}{g_2^2} \sin^2 \mu \left(\beta_1 + \beta_2 \sin^2 \mu \right).$

$$\alpha_0 = 19.42, \quad \alpha_1 = -1.937, \quad \alpha_2 = -2.656,$$
$$\beta_1 = 1.313, \quad \beta_2 = 0.603,$$

$$M_{\rm sph} = g_2 v M \left(\frac{\pi}{2} \right) \simeq 92.01 \text{ TeV}, \quad E_{\rm sph} = g_2 v V \left(\frac{\pi}{2} \right) \simeq 9.08 \text{ TeV}.$$

c.f., TW's: M_{sph} = 17.1 TeV. With same normalization, M_{sph}(ours) -> 23.0 TeV.

Number of band edges are affected by the size of M_{sph} (see later).

Band structure

E_{sph}=9.08 TeV $\qquad$ E_{sph}=9.11 TeV

this work	Units:	TeV	Tye-Wong
Band Centre E	Band Width	Band Centre E	Band Width
14.054	0.0744	?	?
13.980	0.0741	?	?
⋮	⋮	⋮	⋮
9.072	0.0104	9.113	0.0156
9.044	4.85×10^{-3}	9.081	7.19×10^{-3}
9.012	1.61×10^{-3}	9.047	2.62×10^{-3}
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0.1015	1.88×10^{-199}	0.1027	$\sim 10^{-177}$
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Band gaps still exist $E>E_{sph}$ due to nonzero reflection rate.

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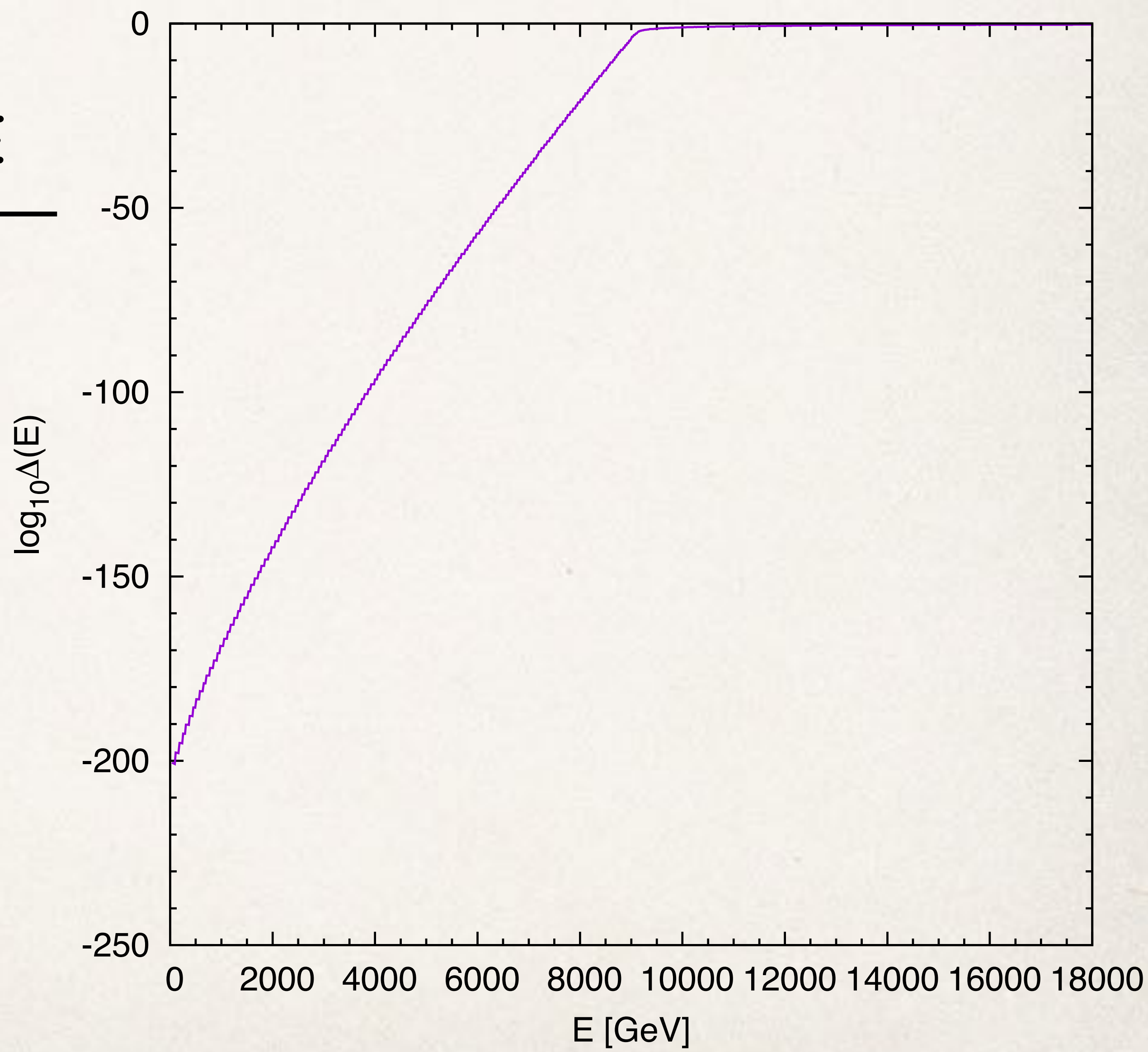
Transition factor

tunneling factor

$$\sigma_{\Delta(B+L)=\pm 1} \propto \begin{cases} 1 \times \exp\left(\frac{4\pi}{\alpha_W} F(E)\right) & \text{instanton calculus} \\ \Delta(E) \times 1 & \text{band picture} \end{cases}$$

$$\Delta(E) \simeq \frac{\text{sum of band widths up to E}}{\text{energy (E)}}$$



Band picture:

- State of density is restricted.
- Exponential suppression at $E \ll E_{sph}$ is due to the tiny band width.

Vacuum decay rate at finite-T

Ordinary case: [Affleck, PRL46,388 (1981)]

$$\boxed{\begin{aligned}\Gamma_A(T) &= \frac{1}{Z_0(T)}\int_0^\infty dE\ J(E)e^{-E/T}\\ &\simeq \frac{1}{Z_0}\frac{\omega_-}{4\pi\sin\left(\frac{\omega_-}{2T}\right)}e^{-E_{\rm sph}/T} \quad \text{for } T > \underbrace{\frac{\omega_-}{2\pi}}_{\simeq 14\ \text{GeV}},\end{aligned}}$$

$$J(E) = \frac{T(E)}{2\pi},\ Z_0(T) = \left[2\sinh\left(\frac{\omega_0}{2T}\right)\right]^{-1},\ \frac{\omega_0}{g_2 v} = \underbrace{\sqrt{\frac{V''(0)}{M(0)}}}_{\simeq 0.42},\ \frac{\omega_-}{g_2 v} = \underbrace{\sqrt{\frac{V''(\pi/2)}{M(\pi/2)}}}_{\simeq 0.51}$$

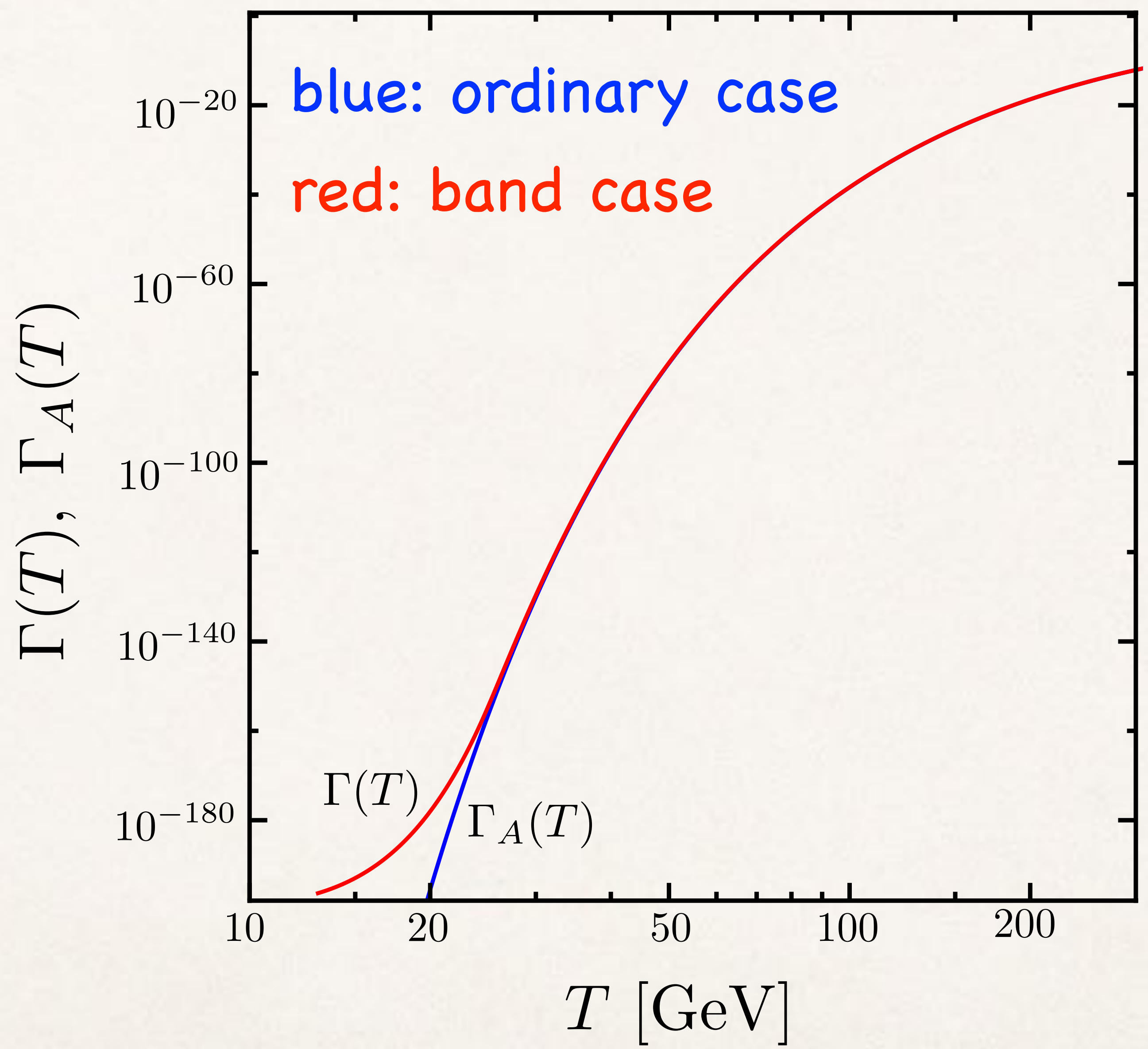
Band case: $J(E) \to \eta(E)/2\pi$

$$\boxed{\Gamma(T) = \frac{1}{Z_0(T)}\int_0^\infty dE\ \frac{\eta(E)}{2\pi}e^{-E/T}}$$

$\eta(E) = 1$ for the conducting band, $\eta(E) = 0$ for the band gap

Impact of band

For simplicity, we use the band structure obtained before.



For T=100 GeV, Γ / Γ_A = 1.06.

How about B-number preservation criteria?

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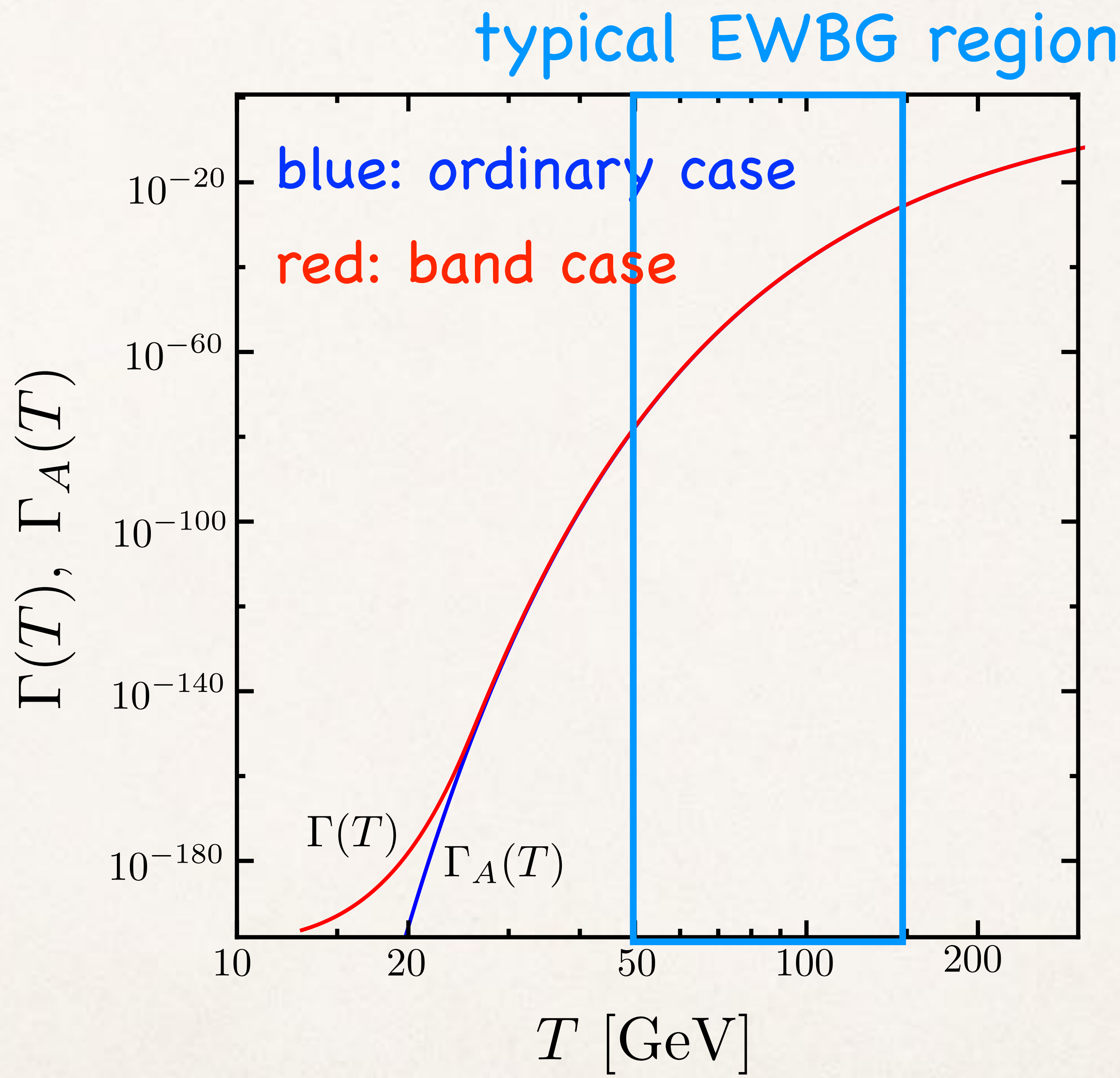

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How about B-number preservation criteria?

Baryon number preservation criteria

$$\Gamma(T) < H(T)$$

Including the band effect, $\Gamma(T) = R(T)\Gamma_A(A)$

$$\boxed{\frac{v(T)}{T} > \frac{g_2}{4\pi\mathcal{E}_{\rm sph}}\left[42.97 + \log\mathcal{N} + \log R(T) + \cdots\right]}$$

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$$\log R(T = 100\ \mathrm{GeV}) \simeq 0.05$$

Band effect has little effect on the B preservation criteria.

Summary of the 2^{nd} part

- We have discussed the band effect on the sphaleron processes at $T \neq 0$.
- At $T \simeq 100$ GeV, sphaleron process is virtually unaffected.
  -> no impact on EWBG.

Backup

Eigenvalue problem

Hamiltonian:

$$\hat{H}(\mu, p) = g_2 v \left[\hat{p} \frac{1}{2M(\hat{\mu})} \hat{p} + V(\hat{\mu}) \right], \qquad [\hat{\mu}, \hat{p}] = i$$

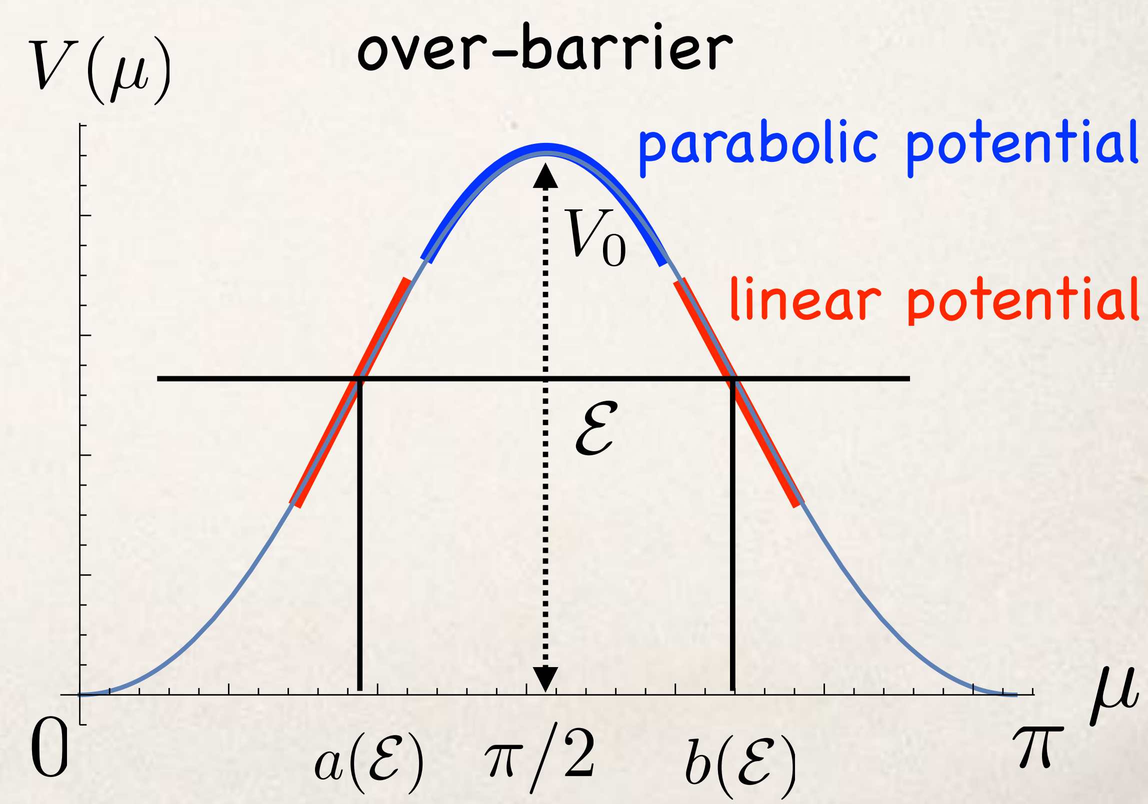
Band energy is determined by solving [N.L.Balazs, Ann.Phys.53,421 (1969)]

$$\boxed{\cos(\Phi(\mathcal{E})) = \pm\sqrt{T(\mathcal{E})}}$$

with 3 connection formulas depending on energy.

$$\Phi(\mathcal{E}) = \begin{cases} \frac{1}{\hbar} \int_{b(\mathcal{E})}^{a(\mathcal{E})} d\mu \; p(\mu) & \text{for } \mathcal{E} < V_0, \\ \frac{1}{\hbar} \int_{-\pi/2}^{\pi/2} d\mu \; p(\mu) & \text{for } \mathcal{E} \geq V_0, \end{cases}$$

$$p(\mu) = \sqrt{M(\mu)(\mathcal{E} - V(\mu))}$$



Δ(B+L)≠0 process

[Funakubo, Otsuki, Takenaga, Toyoda, PTP87, 663 (1992), PTP89, 881 (1993)]

transition amplitude:

$$S_{fi} = \langle f|\hat{S}|i\rangle \sim \int\int \langle f|\phi(y),\pi(y)\rangle\langle\phi(y),\pi(y)|\hat{S}|\phi(x),\pi(x)\rangle\langle\phi(x),\pi(x)|i\rangle$$

path integral using coherent state $|\phi,\pi\rangle$

$\because$ appropriate for describing classical configuration

- tunneling suppression appears in the intermediate process.
- overlap issue: suppressions from $\langle f|\phi,\pi\rangle$ and $\langle\phi,\pi|i\rangle$.

This point is not properly discussed in the work of Tye and Wong.

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overlap factor

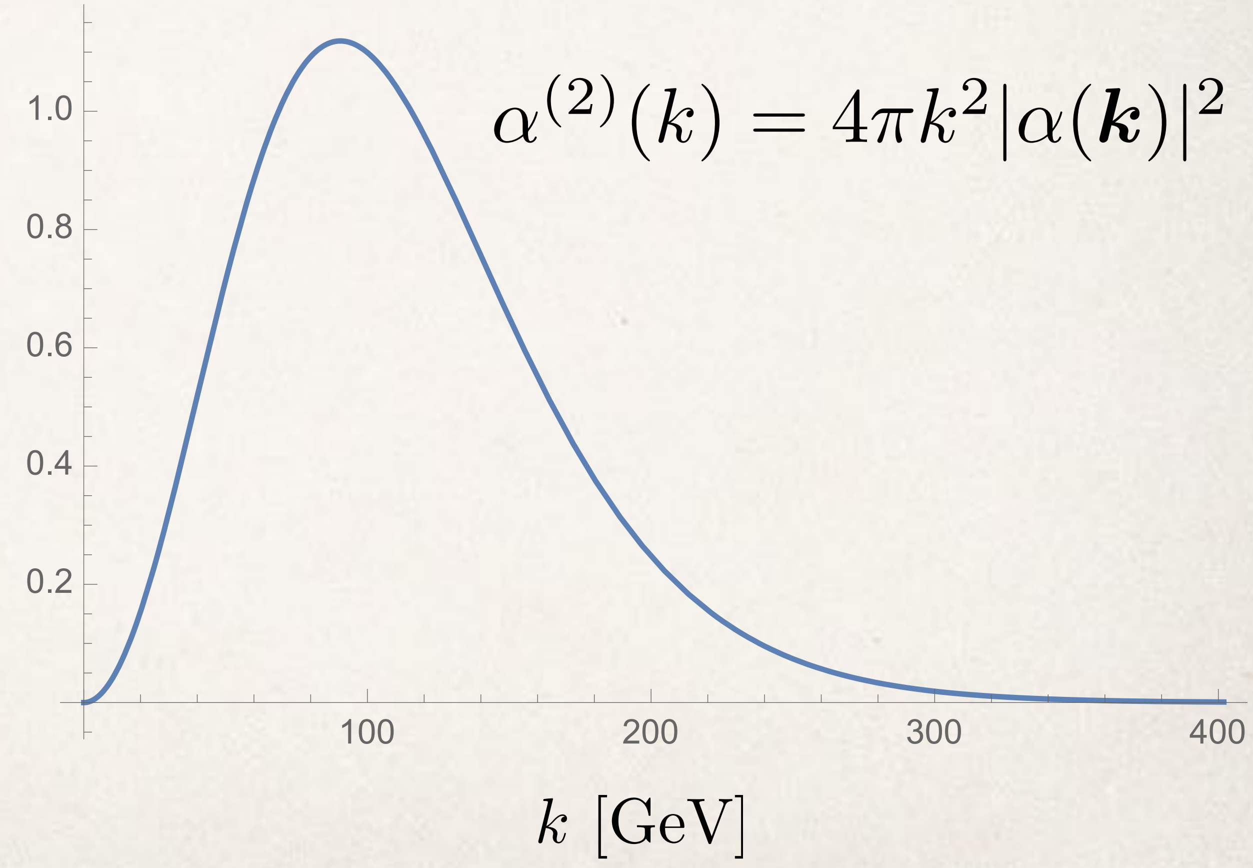
inner product between n particle state and coherent state:

$$\langle 0|\hat{a}(\boldsymbol{k}_1)\hat{a}(\boldsymbol{k}_2)\cdots\hat{a}(\boldsymbol{k}_n)|\phi(x),\pi(x)\rangle = \exp\left[-\frac{1}{2}\int d\boldsymbol{k}|\alpha(\boldsymbol{k})|^2\right]\alpha(\boldsymbol{k}_1)\alpha(\boldsymbol{k}_2)\cdots\alpha(\boldsymbol{k}_n)$$

$$\alpha(k) = \int \frac{d^{d-1}\boldsymbol{x}}{(2\pi)^{d-1}}\frac{1}{\sqrt{2\omega_{\boldsymbol{k}}}}\Big[\omega_{\boldsymbol{k}}\phi(x) + i\pi(x)\Big]e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}$$

- cross section $\propto |\alpha_1|^2 \ldots |\alpha_n|^2$

- $|\alpha|^2$ has a peak at $k=m_W$.



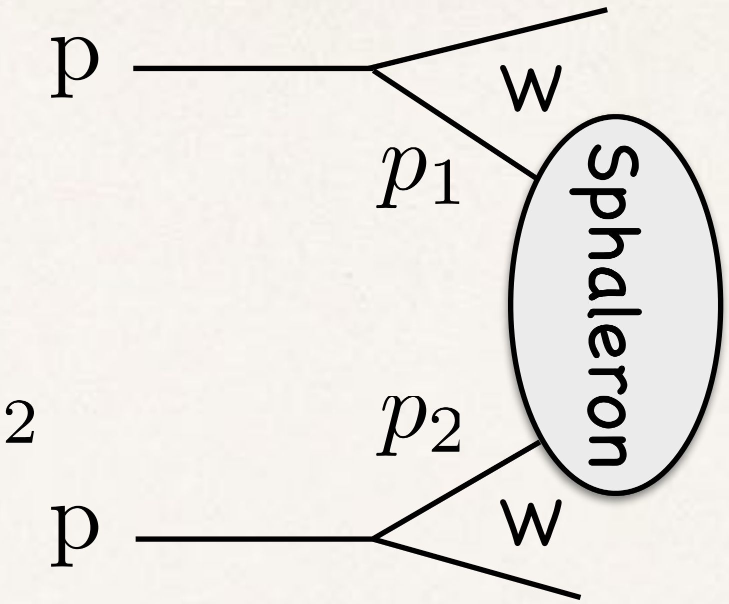
Sphaleron at colliders

<u>Case1:</u> 2 -> sphaleron

For $|p_1|=|p_2|\simeq E_{sph}/2$

$$|\langle\phi(x),\pi(x)|\boldsymbol{p}_1\boldsymbol{p}_2\rangle|^2 \ni |\alpha(\boldsymbol{p}_1)|^2|\alpha(\boldsymbol{p}_2)|^2$$

$$\sim e^{-\pi E_{\rm sph}/m_W} \sim \boxed{10^{-155}}$$



Creation of sphaleron from the 2 energetic particles is difficult.

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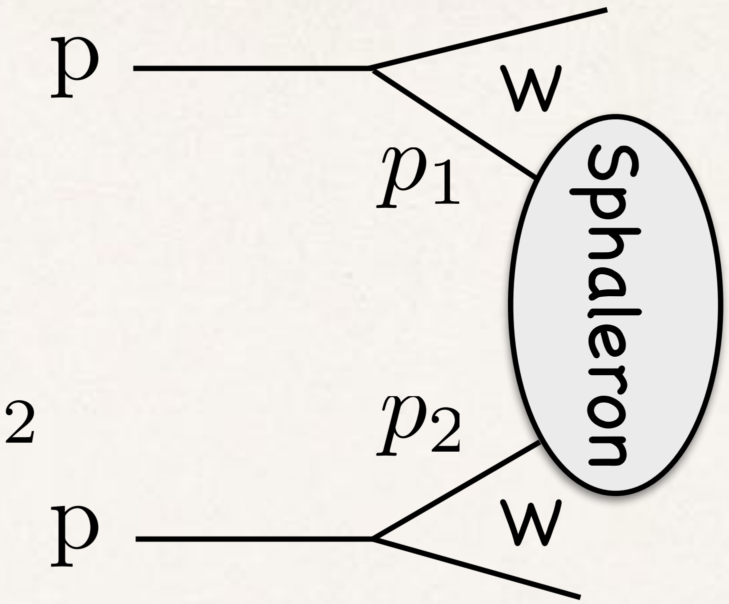
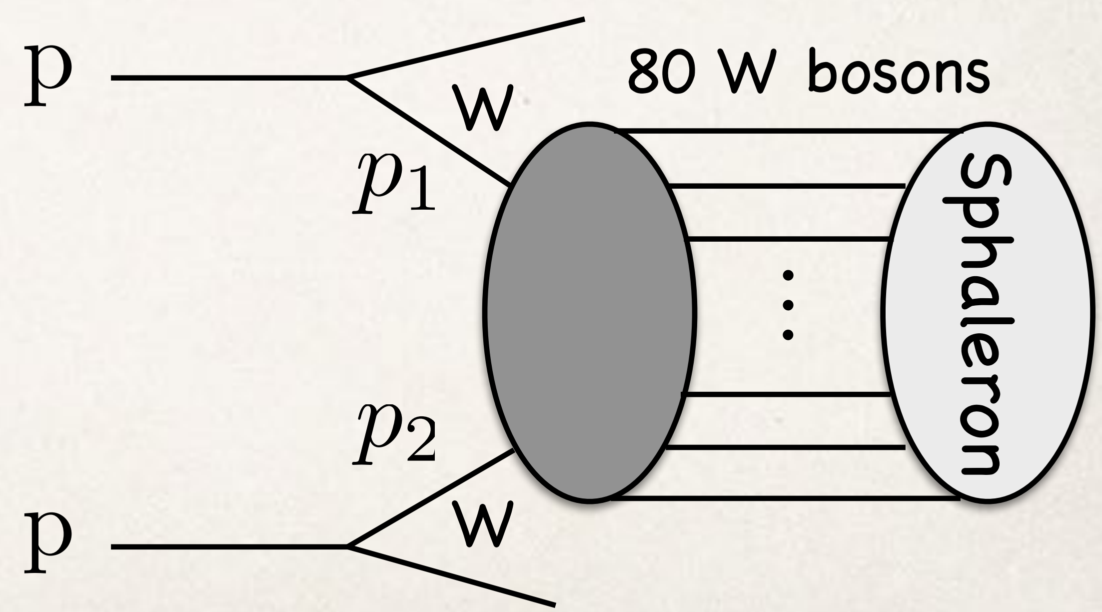


Creation of sphaleron from the 2 energetic particles is difficult.

Case2: 2 -> n W -> sphaleron

$n\simeq 80$ since $E_{sph}/\surd 2m_W$

phase space factor: $\sim \left(\frac{1}{(4\pi)^2}\right)^{80} \sim \boxed{10^{-176}}$



difficult to produce about 80 W bosons.