Overview of two-photon and two-boson exchange

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Outline

- Recent advances in TPE theory (2008-present)

  Review: Afanasev, PGB, Hassell, Raue, Prog. Nucl. Part. Phys. (2017)

  - improved hadronic model parameters (fit to data)
  - use of dispersion relations and connection to data
  - new experimental results

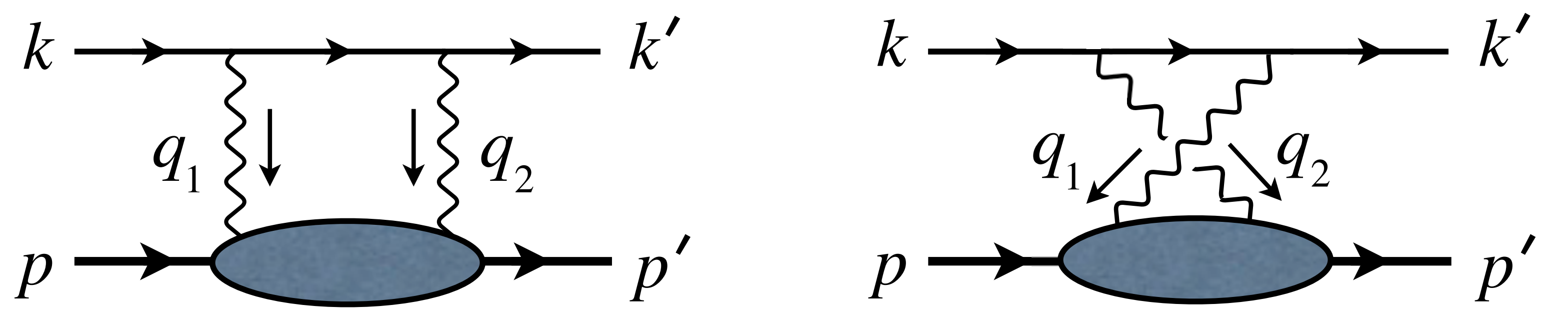
- γZ box contributions to PV electron scattering
  - amenable to dispersion analysis in forward limit ($Q^2 \to 0$)
  - distinction between axial and vector hadron coupling
  - use of inelastic PV data in resonance and DIS regions

Hadronic Approach

Low to moderate Q^2:

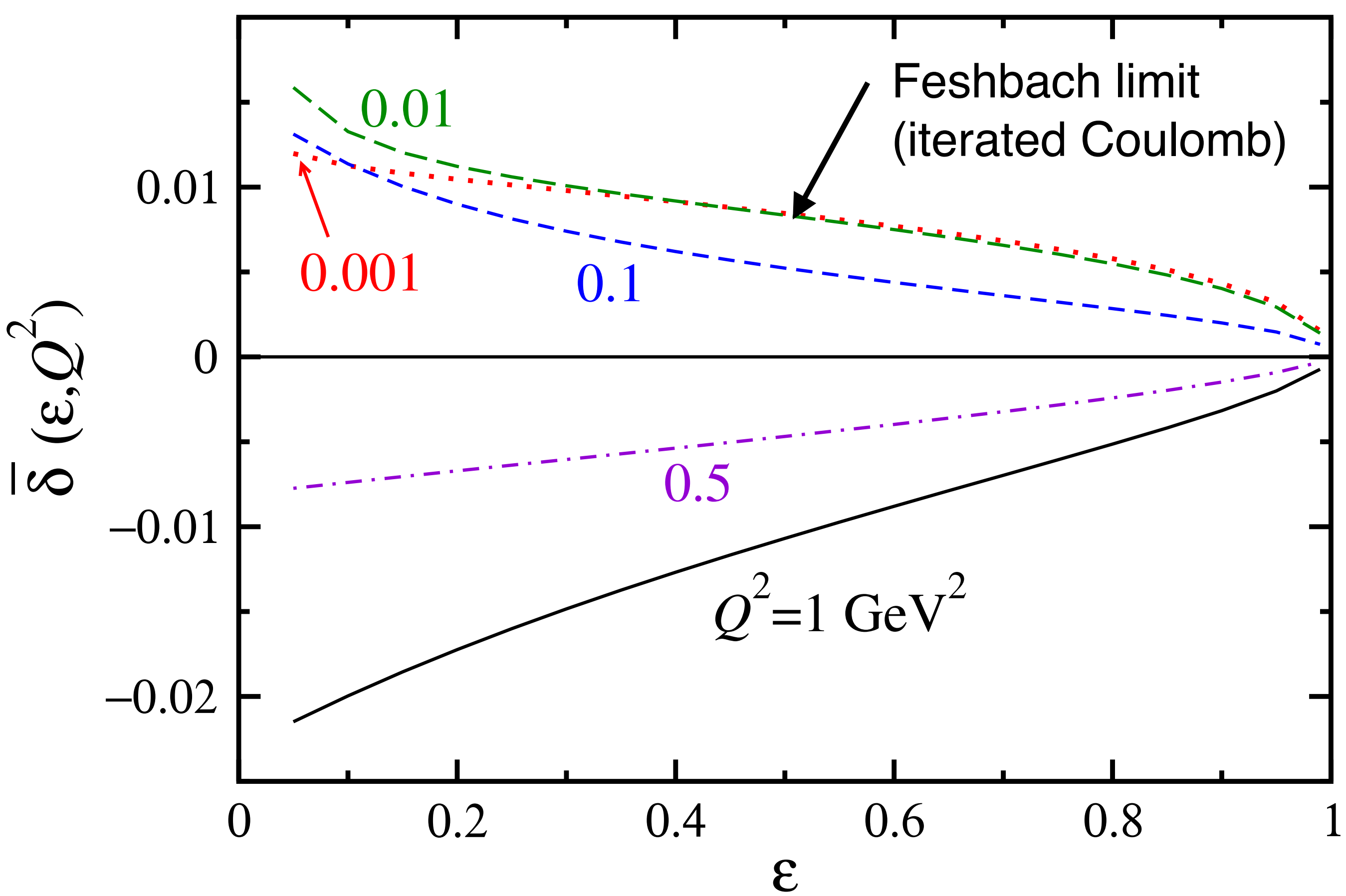
hadronic: $N + \Delta + N^*$ etc.

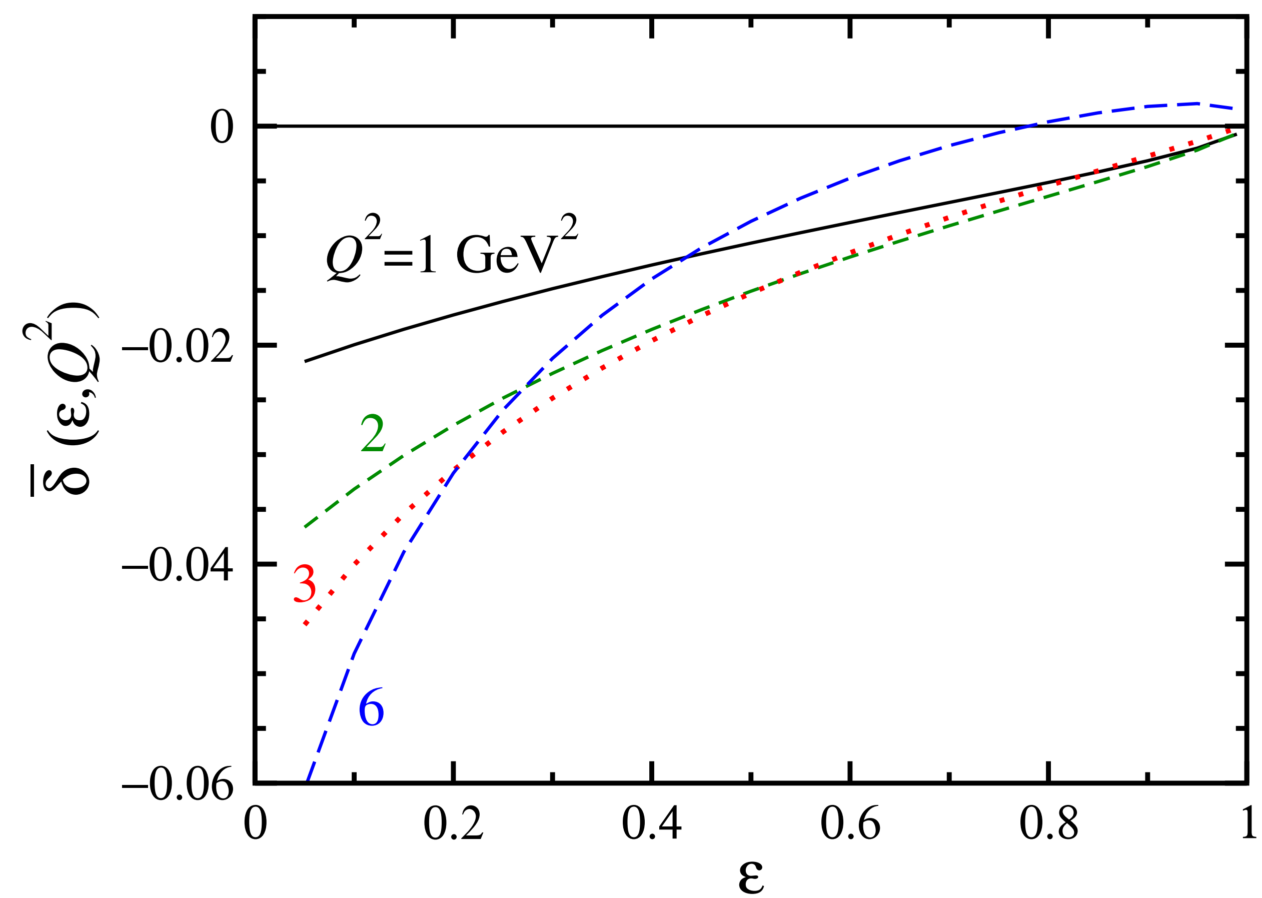
- as Q^2 increases more and more parameters
- Loop integration using sum of monopole transition form factors fit to spacelike Q^2



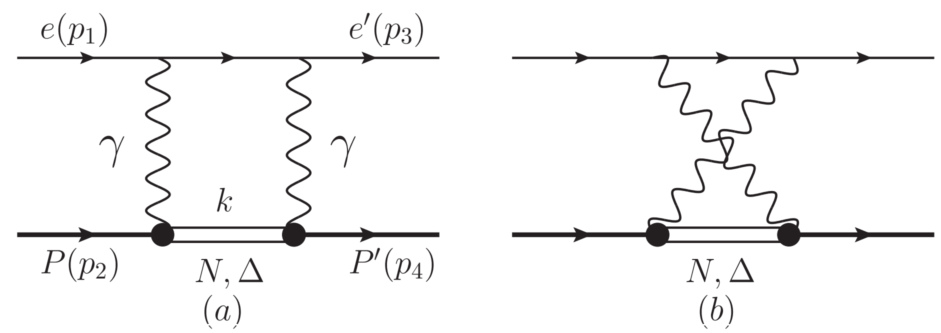
PGB, Melnitchouk, & Tjon, PRL **91**, 142304 (2003)

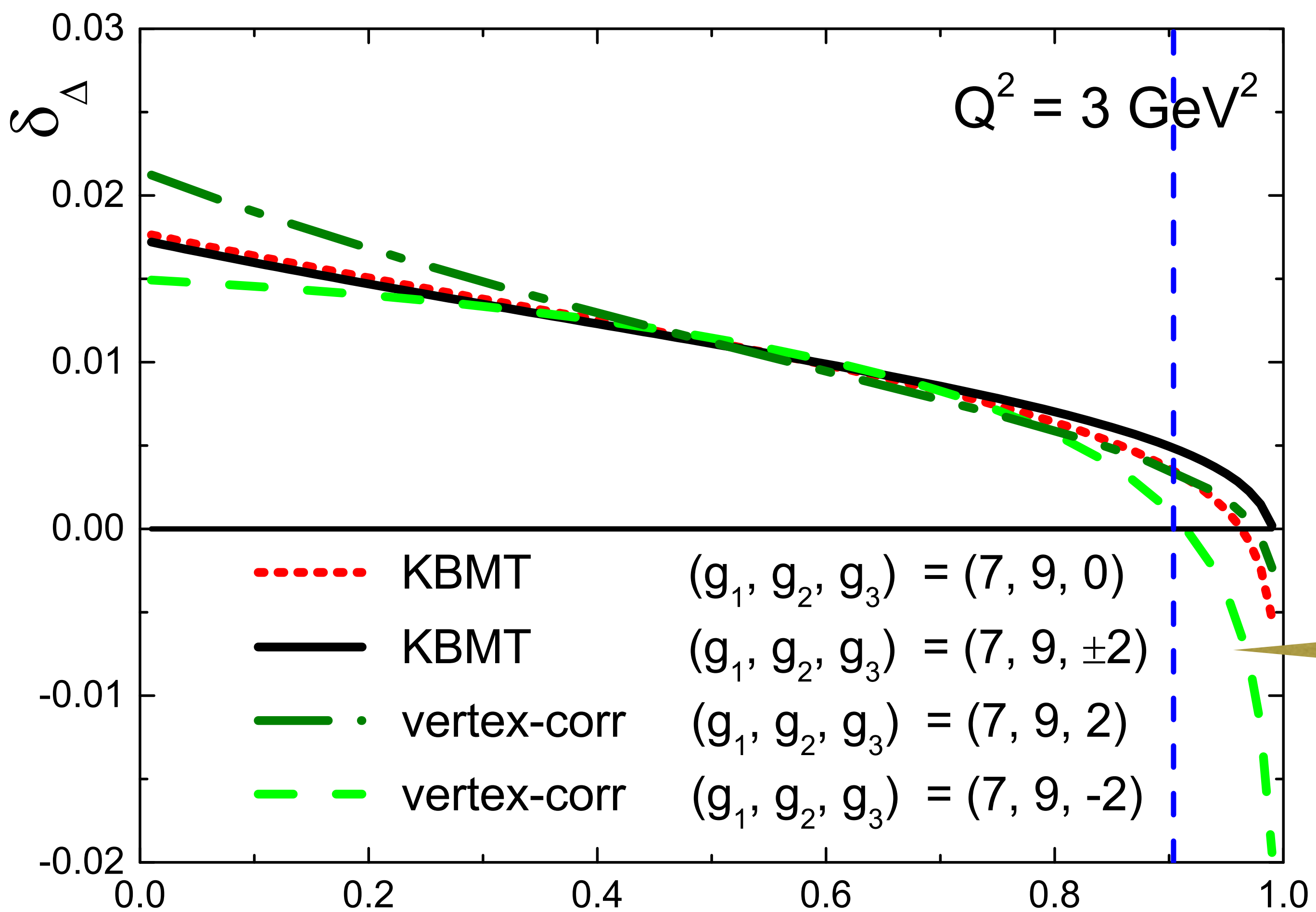
Nucleon (elastic) intermediate state





Δ and N* intermediate states





Direct loop integration method

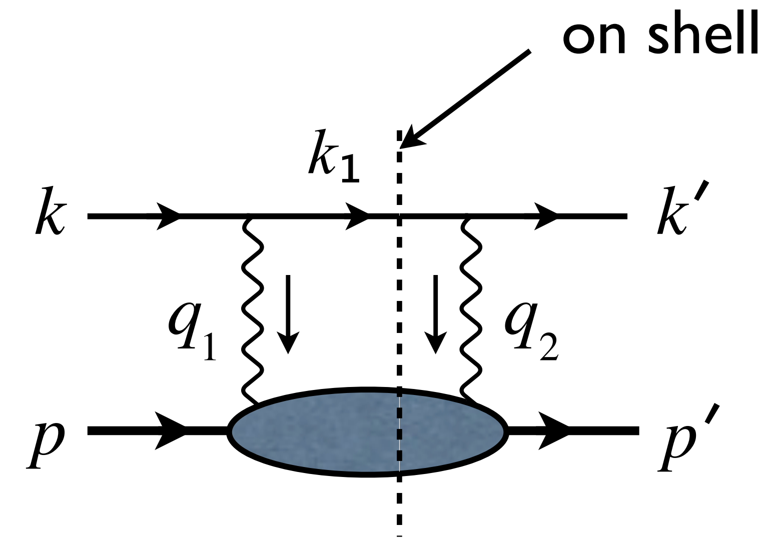
Kondratyuk *et al.*, PRL **95**, 172503 (2005)

Zhou & Yang, Eur. Phys. J. A. **51**, 105 (2015)

Unphysical divergence

- Include all 3 $N \rightarrow \Delta$ multipoles, with form factors fit to CLAS data
- Opposite sign to nucleon contribution
- Qualitatively correct, BUT diverges as $\varepsilon \rightarrow 1$, implying a violation of unitarity (Froissart bound)

Dispersive method



$$S = 1 + i\mathcal{M}$$

$$S^\dagger = 1 - i\mathcal{M}^\dagger$$

$$SS^\dagger = 1$$

Unitarity → $-i\left(\mathcal{M} - \mathcal{M}^\dagger\right) = 2\Im m\,\mathcal{M} = \mathcal{M}^\dagger\mathcal{M}$

$$\boxed{\Im m\,\langle f|\mathcal{M}|i\rangle = \frac{1}{2}\int d\rho\,\sum_n \langle f|\mathcal{M}^*|n\rangle\langle n|\mathcal{M}|i\rangle}$$

$$d\rho = \frac{d^3k_1}{(2\pi)^3 2E_{k_1}} \sim dW_n\,dQ_1^2\,dQ_2^2$$

- Imaginary part determined by unitarity
- Uses only on-shell form factors
  - Use form factors directly fit to data, not reparametrized by sum of monopoles
- Real part determined from dispersion relations

TPE using dispersion relations

Generalized form factors

$$\mathcal{M}_{\gamma\gamma} \to (\gamma_\mu)^{(e)} \otimes \left(F_1'(Q^2,\nu)\gamma^\mu + F_2'(Q^2,\nu)\frac{i\sigma^{\mu\nu}q_\nu}{2M} \right)^{(p)}$$
$$+ (\gamma_\mu\gamma_5)^{(e)} \otimes \left(G_a'(Q^2,\nu)\gamma^\mu\gamma_5 \right)^{(p)}$$

$$\delta_{\gamma\gamma} = 2\mathrm{Re}\,\frac{\varepsilon G_E(F_1' - \tau F_2') + \tau G_M(F_1' + F_2') + \nu(1-\varepsilon)G_M G_a'}{\varepsilon G_E^2 + \tau G_M^2}$$

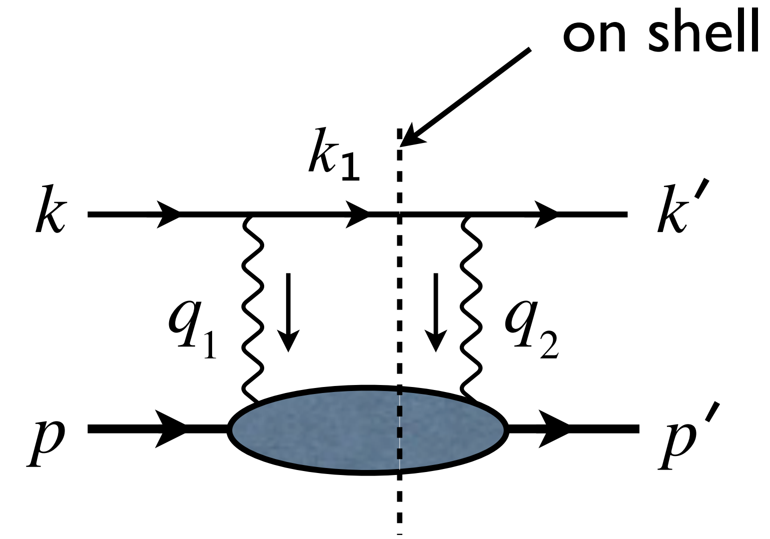
Dispersion relations

$$\mathrm{Re}\ F_1'(Q^2,\nu) = \frac{2}{\pi}\mathcal{P}\int_{-\tau}^{\infty} d\nu'\ \frac{\nu}{\nu'^2 - \nu^2}\ \mathrm{Im}\ F_1'(Q^2,\nu')\,,$$
$$\mathrm{Re}\ F_2'(Q^2,\nu) = \frac{2}{\pi}\mathcal{P}\int_{-\tau}^{\infty} d\nu'\ \frac{\nu}{\nu'^2 - \nu^2}\ \mathrm{Im}\ F_2'(Q^2,\nu')\,,$$
$$\mathrm{Re}\ G_a'(Q^2,\nu) = \frac{2}{\pi}\mathcal{P}\int_{-\tau}^{\infty} d\nu'\ \frac{\nu'}{\nu'^2 - \nu^2}\ \mathrm{Im}\ G_a'(Q^2,\nu')\,.$$

Integral extends into ``unphysical region'' down to zero energy ($\cos\theta <$ -1)

A few technical details

$$\frac{\alpha}{4\pi}Q^2\frac{1}{i\pi^2}\int d^4q_1\ \frac{\mathrm{Im}\left\{L_{\alpha\mu\nu}H^{\alpha\mu\nu}\right\}}{(q_1^2-\lambda^2)(q_2^2-\lambda^2)}$$



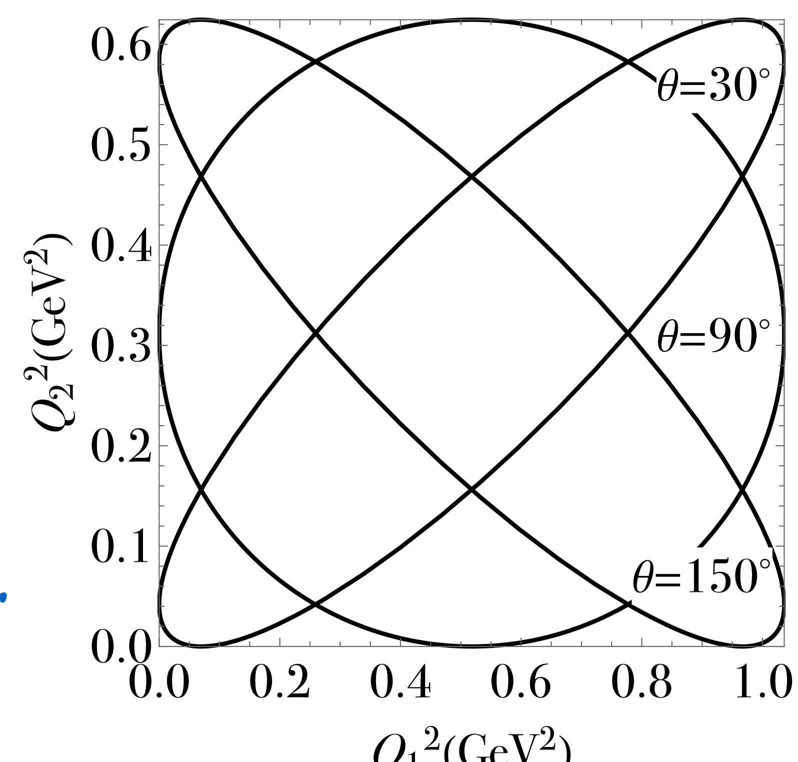
$$\longrightarrow\quad \frac{s-W^2}{4s}\int d\Omega_{k_1}\ \frac{f\left(Q_1^2,Q_2^2\right)G_1(Q_1^2)\,G_2(Q_2^2)}{\left(Q_1^2+\lambda^2\right)\left(Q_2^2+\lambda^2\right)}$$

- L and H are leptonic and hadronic tensors
- f is a polynomial in photon virtualities $Q_1{}^2$ and $Q_2{}^2$
- $G_i(Q_i{}^2)$ is a transition form factor with poles in the complex $Q_i{}^2$ plane

Use numerical contour integration

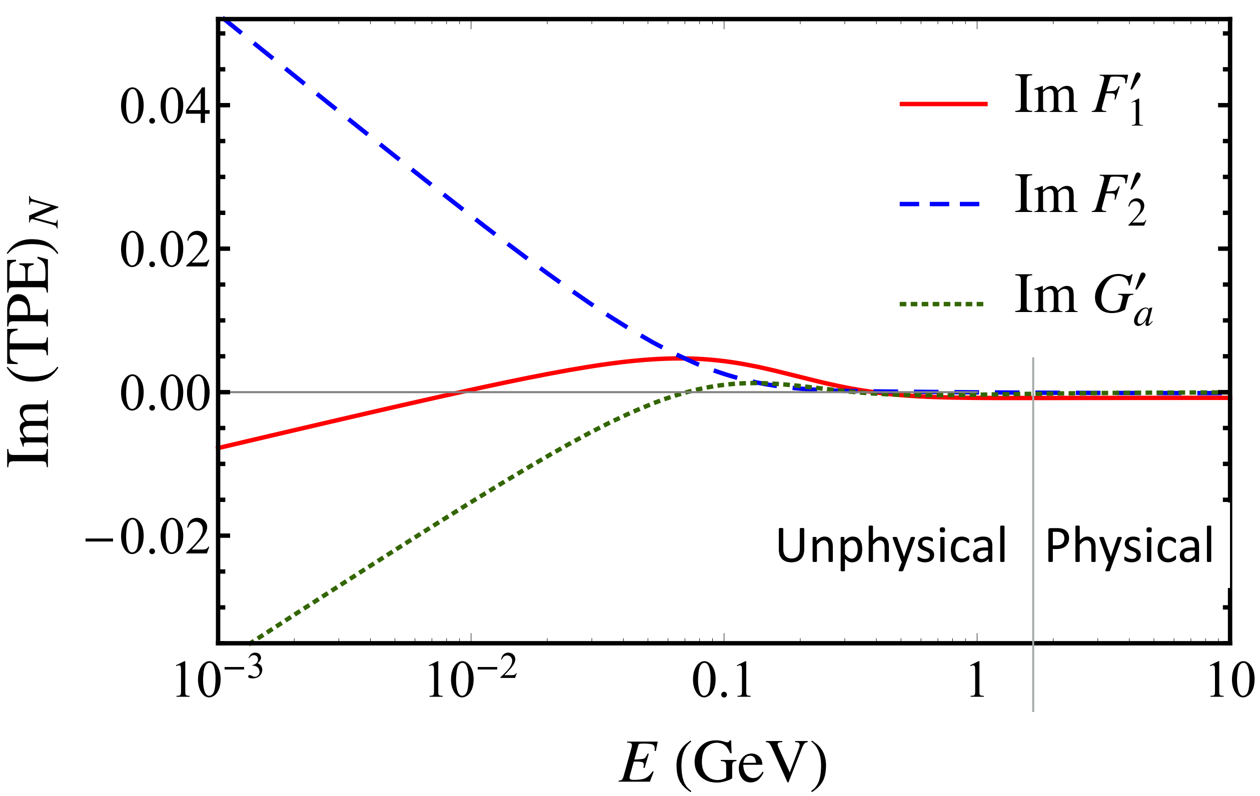
→ Allows for use of arbitrary functional forms for transition form factors $G_i(Q_i{}^2)$

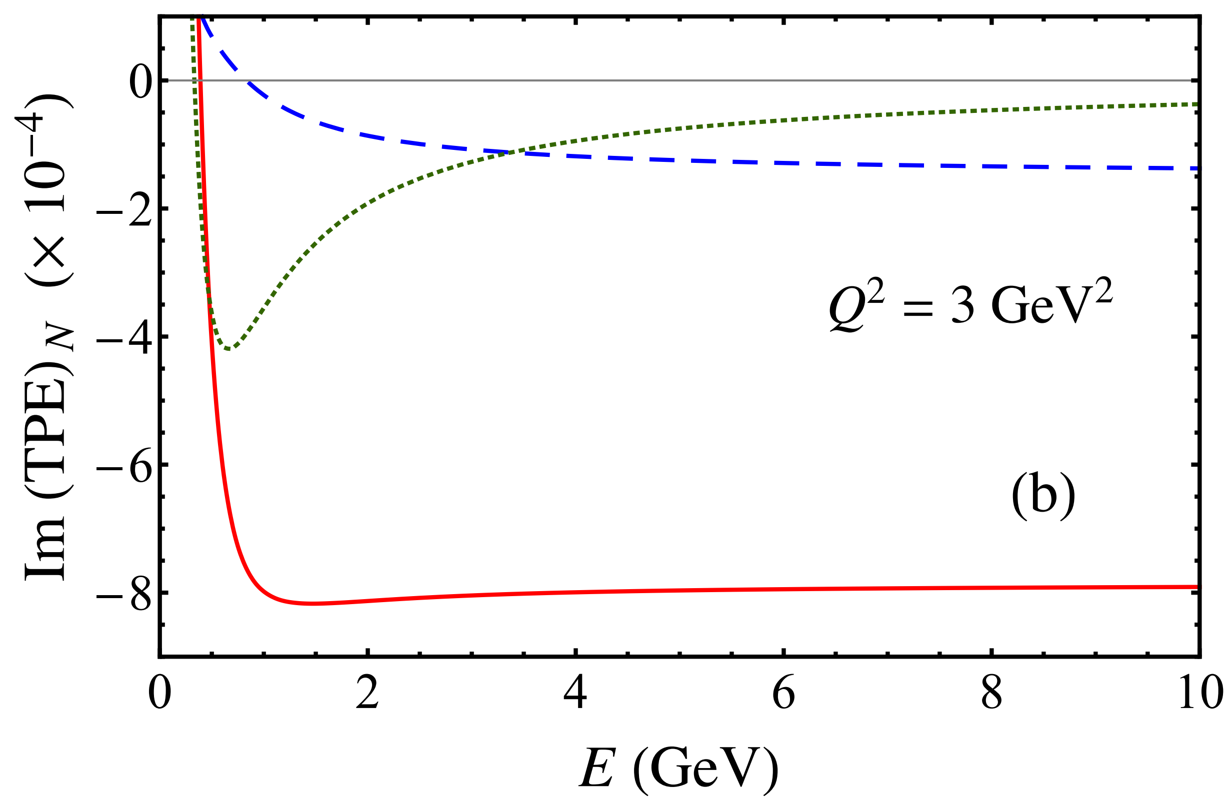
Contours are concentric ellipses of radial parameter r



Nucleon (elastic) intermediate state

$Q^2 = 3\ \text{GeV}^2$

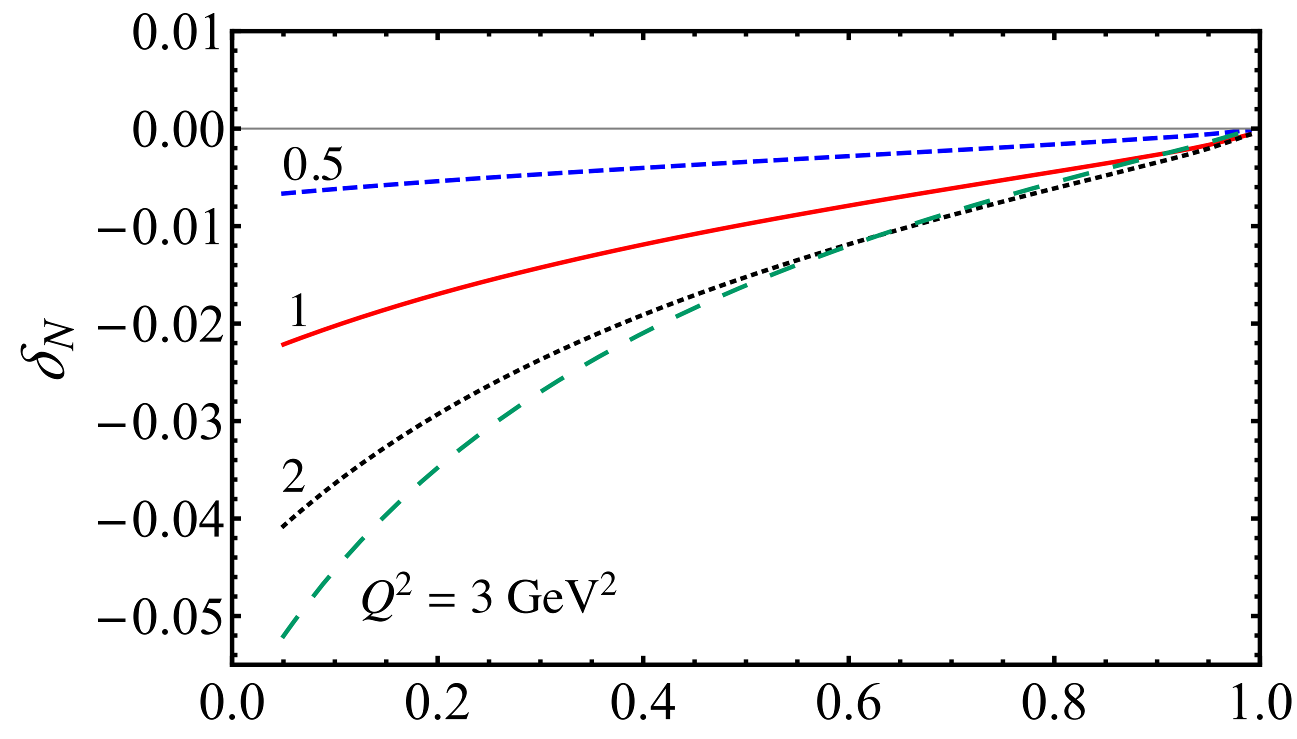




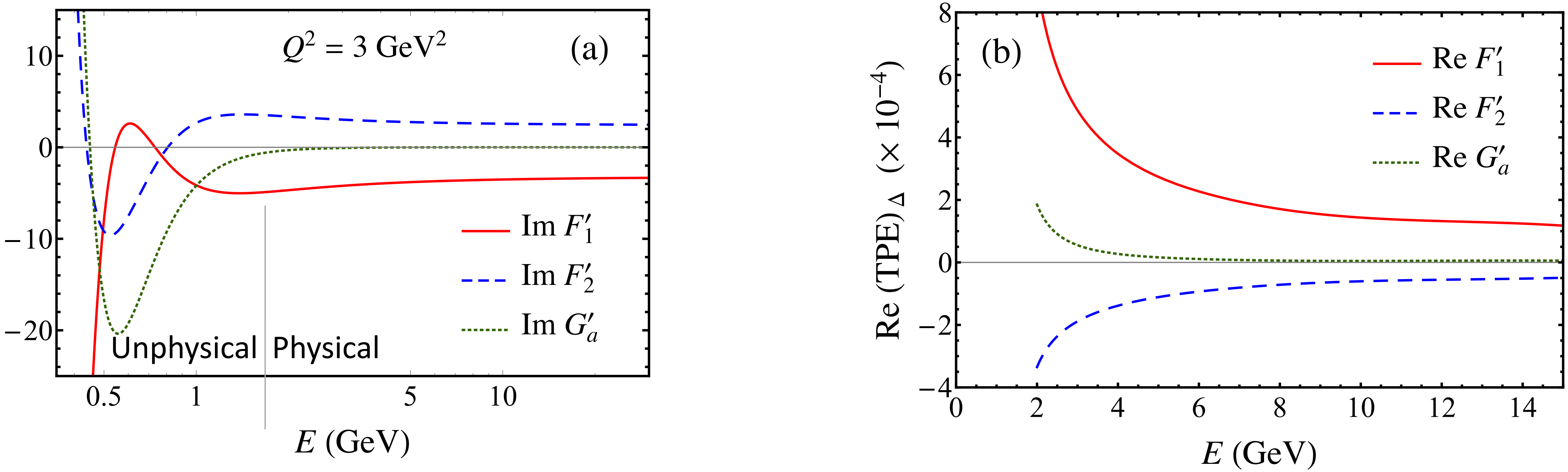
Logarithmic divergence at low energies

No subtractions needed

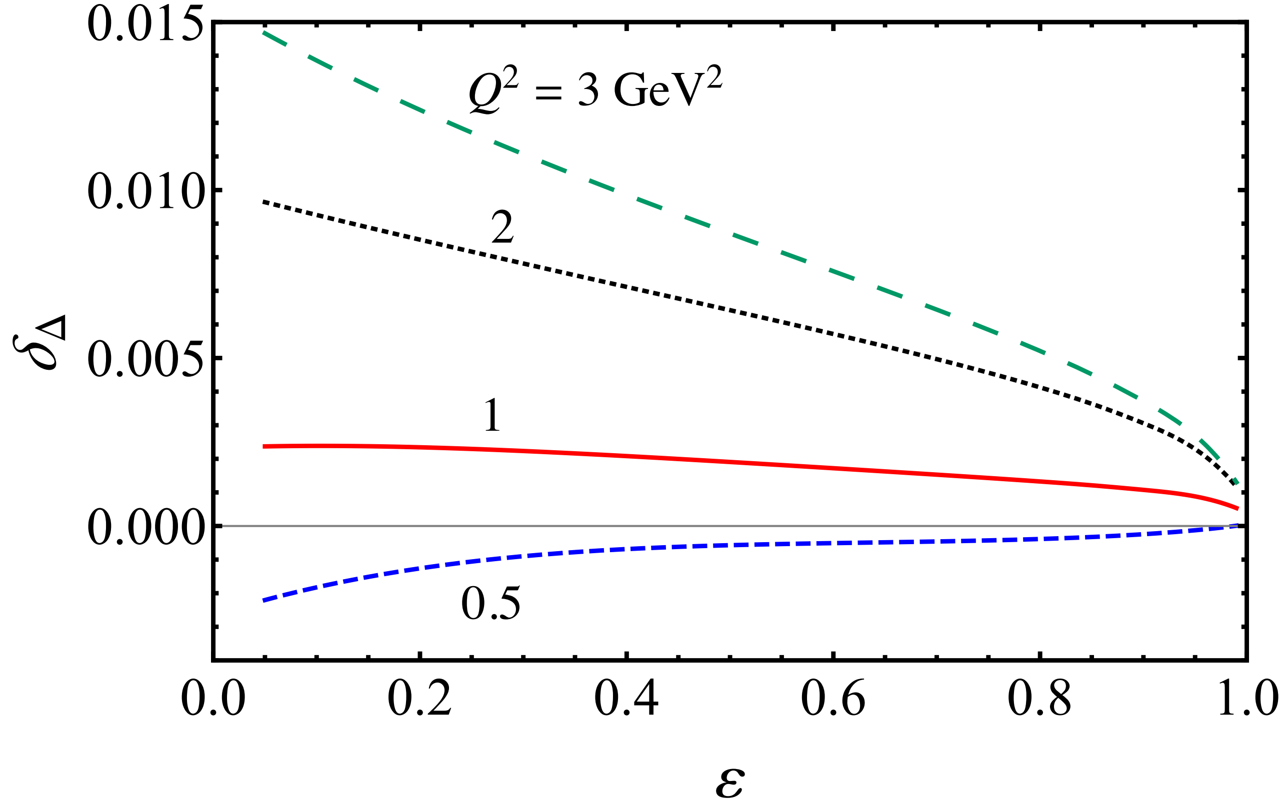
Agrees with old loop integration method



Δ intermediate state (zero width approximation)



- Include all 3 multipoles, with form factors fit to recent CLAS data
- $G_M{}^*$ x $G_M{}^*$ dominates, but $G_M{}^*$ x $G_E{}^*$ interference is significant



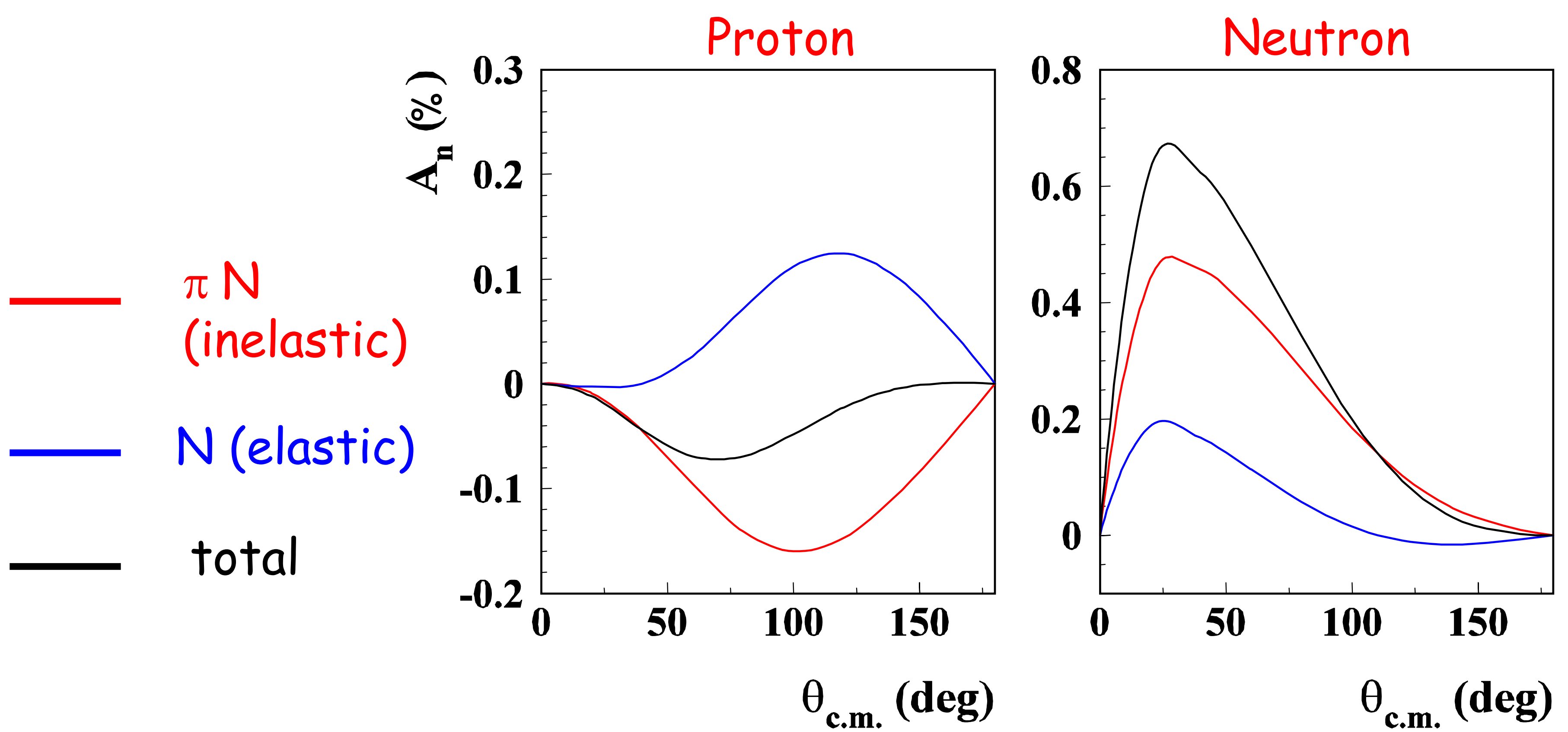
No unphysical divergence at $\varepsilon \to 1$

changes sign at $Q^2 \approx 0.6$ GeV^2

Direct measurements of Im part

Target normal spin asymmetry

Ee = 0.570 GeV



(taken from Pasquini & Vanderhaeghen)

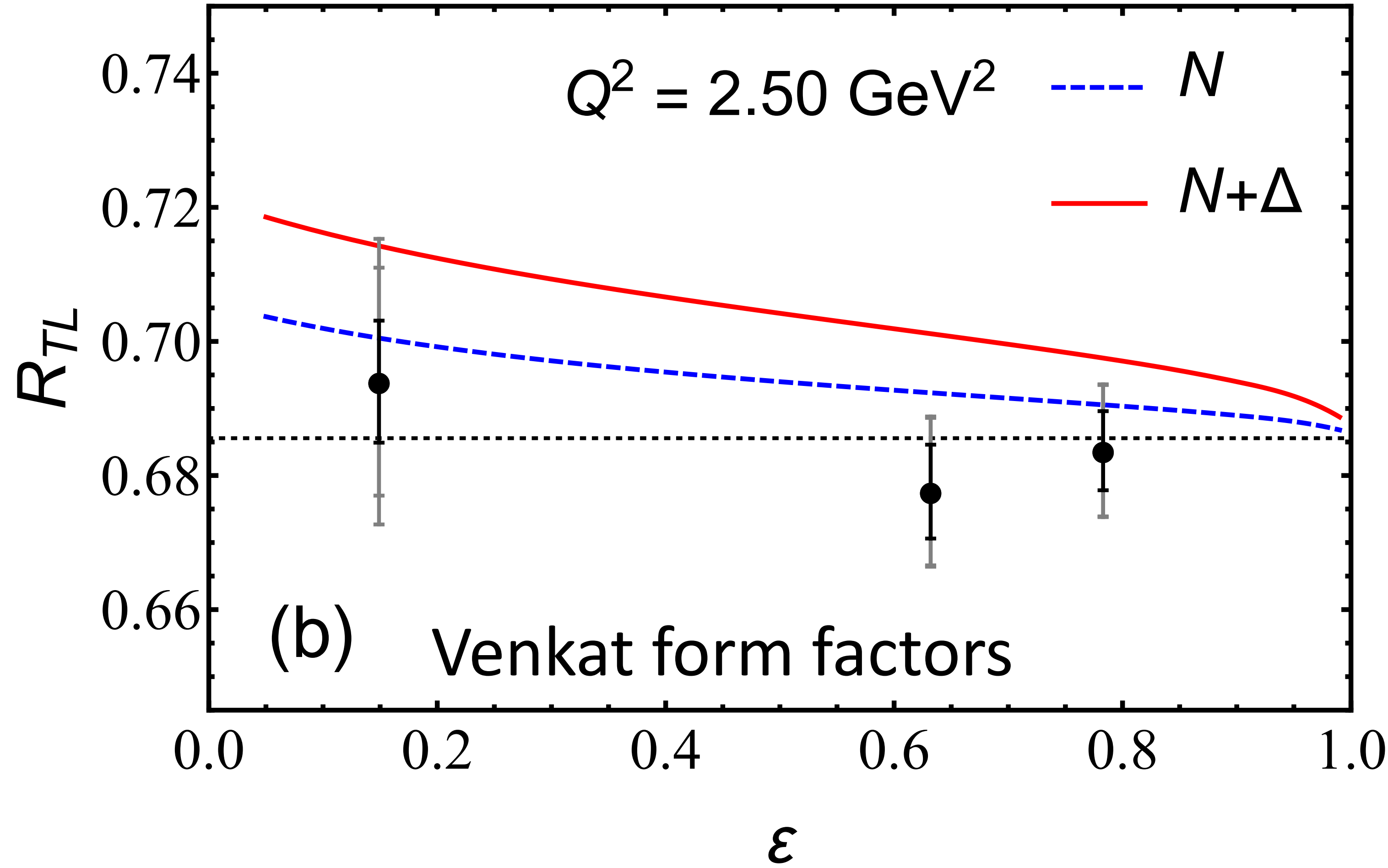
$$A_n = \sqrt{\frac{2\,\varepsilon\,(1+\varepsilon)}{\tau}}\;\left(G_M^2 + \frac{\varepsilon}{\tau}\,G_E^2\right)^{-1}$$

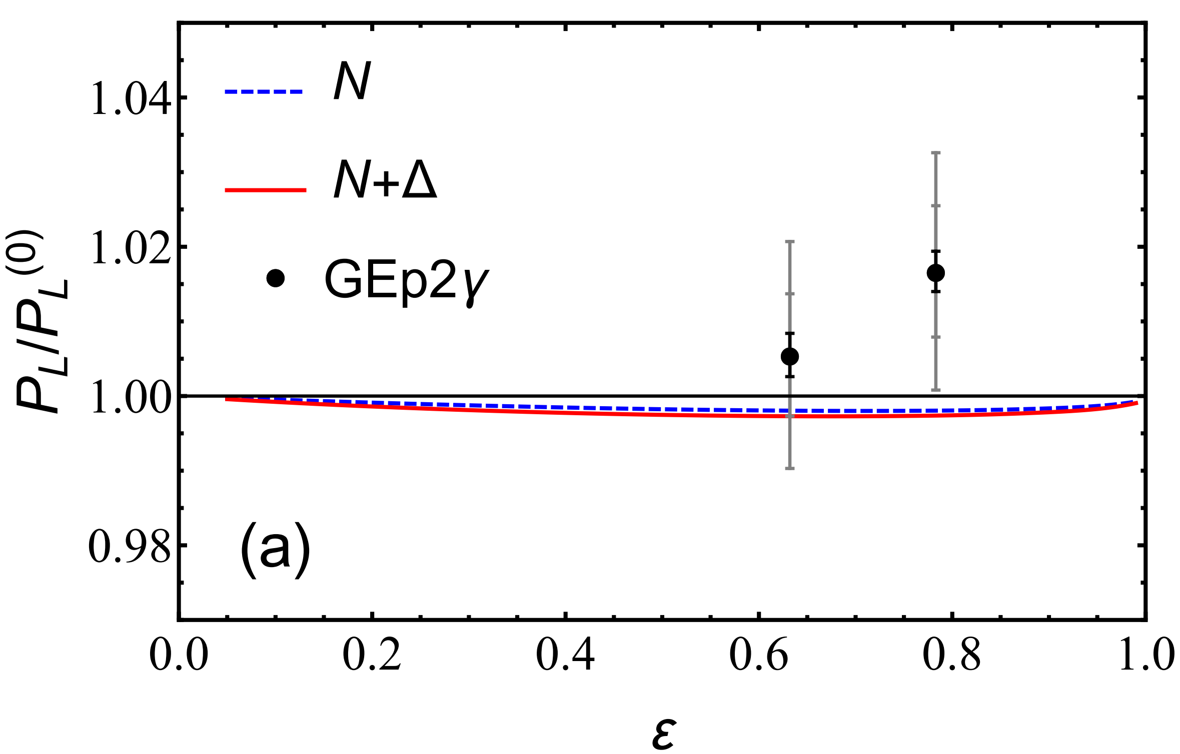
$$\times \quad \left\{ -\,G_M\,\mathcal{I}m\left(\delta\tilde{G}_E + \frac{\nu}{M^2}\tilde{F}_3\right) + G_E\,\mathcal{I}m\left(\delta\tilde{G}_M + \left(\frac{2\varepsilon}{1+\varepsilon}\right)\frac{\nu}{M^2}\tilde{F}_3\right)\right\}$$

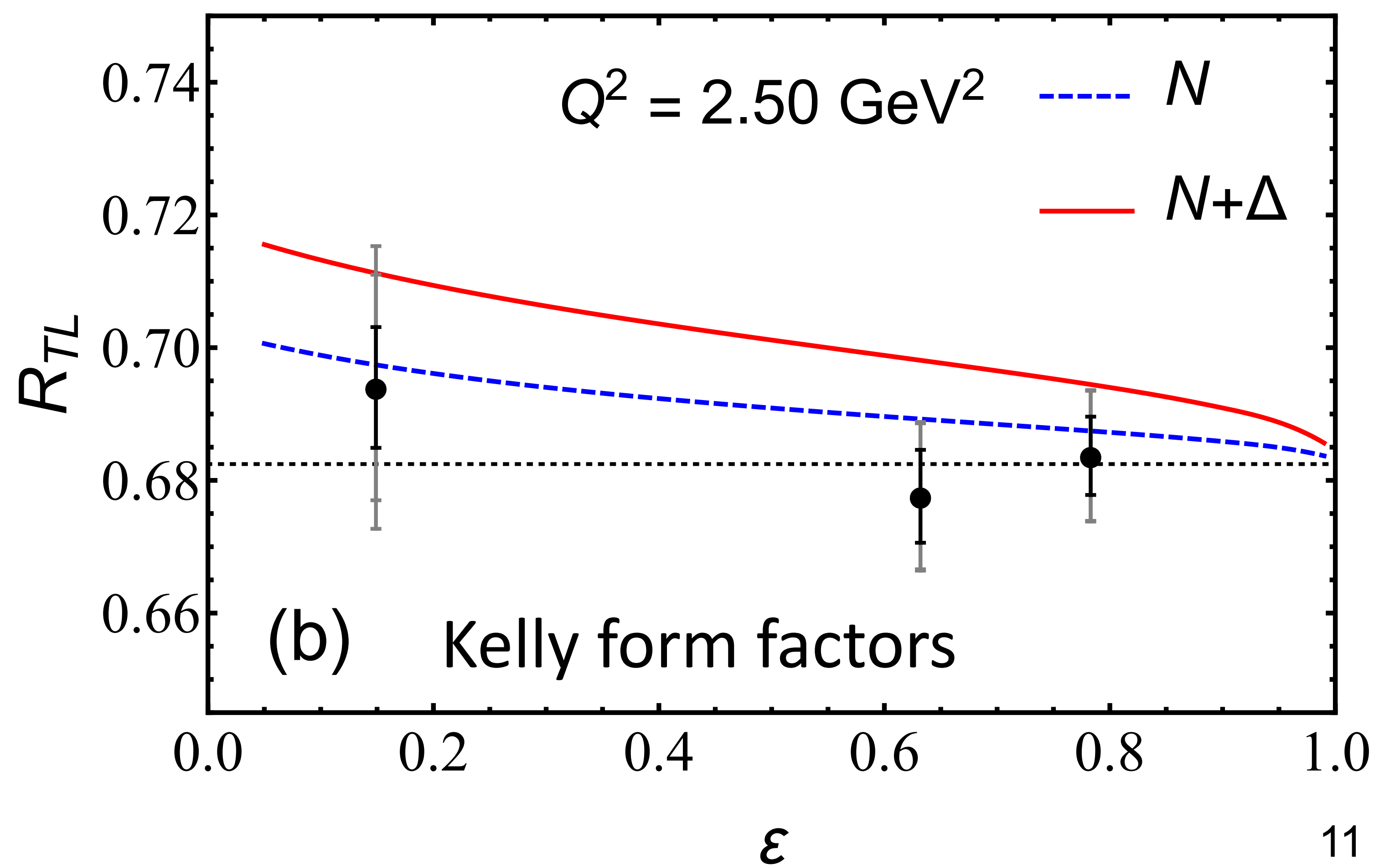
This is all in the physical region.

Polarization data

R_{TL} indicates mild sensitivity to G_E form factor at low ε





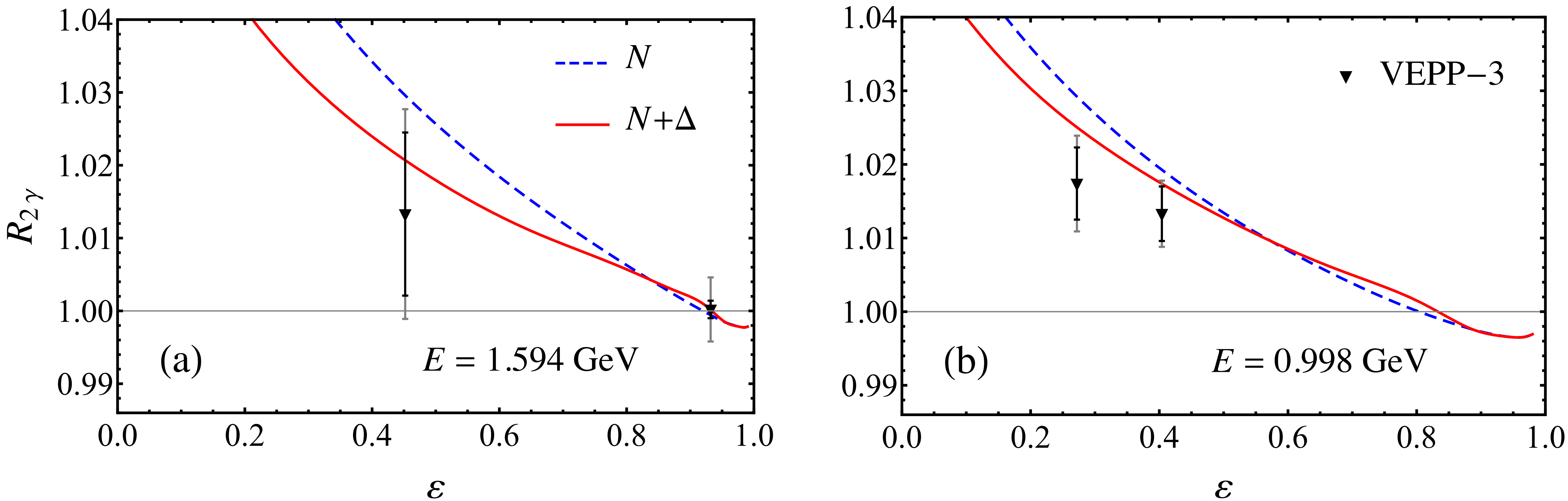


TPE effect on ratio of e^+p to e^-p cross sections

TPE interference changes sign for positrons vs electrons

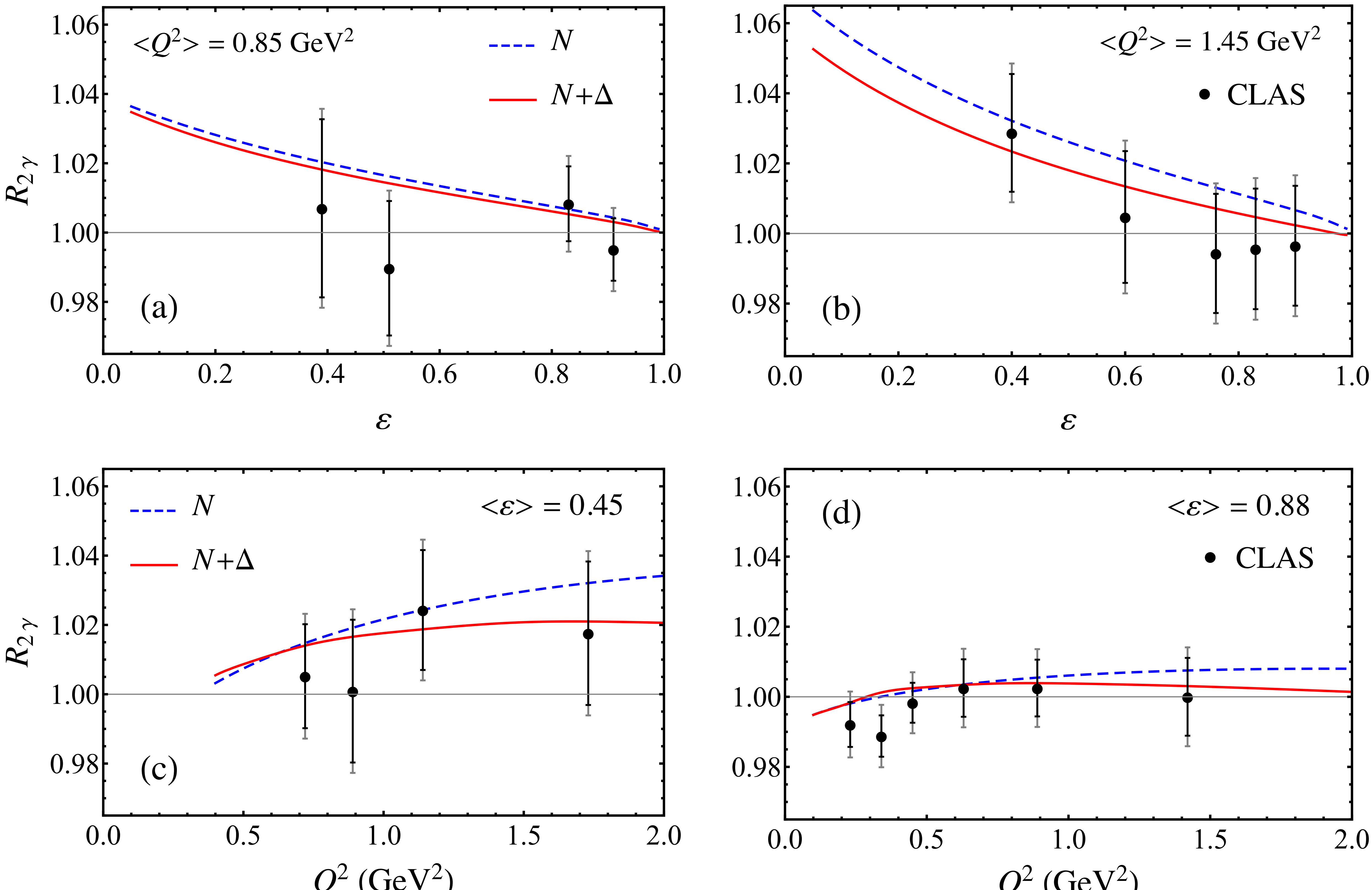
$$R_{2\gamma} = \frac{\sigma^{e^+}}{\sigma^{e^-}} \approx 1 - 2\delta_{\gamma\gamma}$$

VEPP-3 (Novosibirsk)



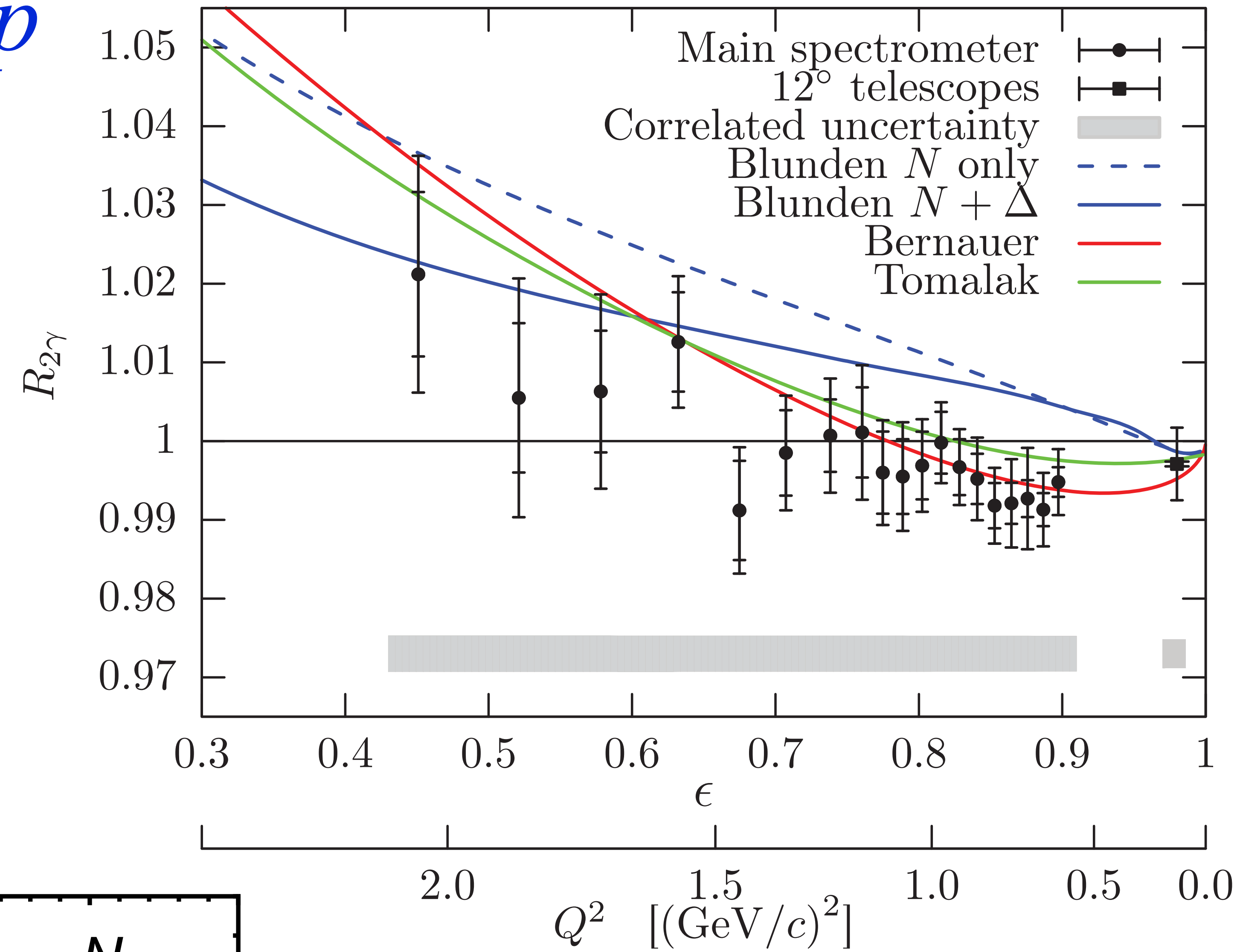
TPE effect on ratio of e^+p to e^-p cross sections

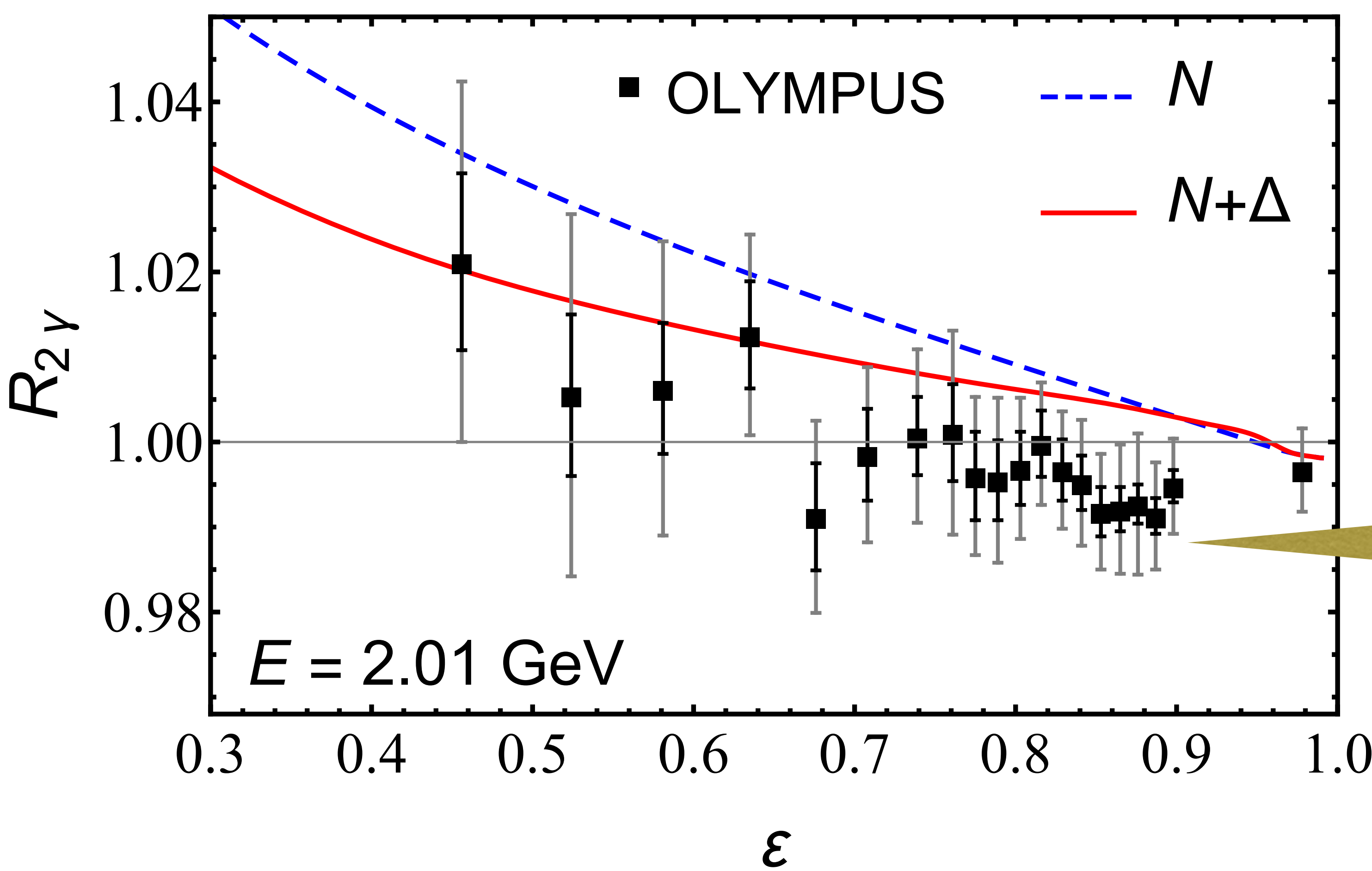
CLAS (Jefferson Lab)



TPE effect on ratio of e^+p to e^-p cross sections

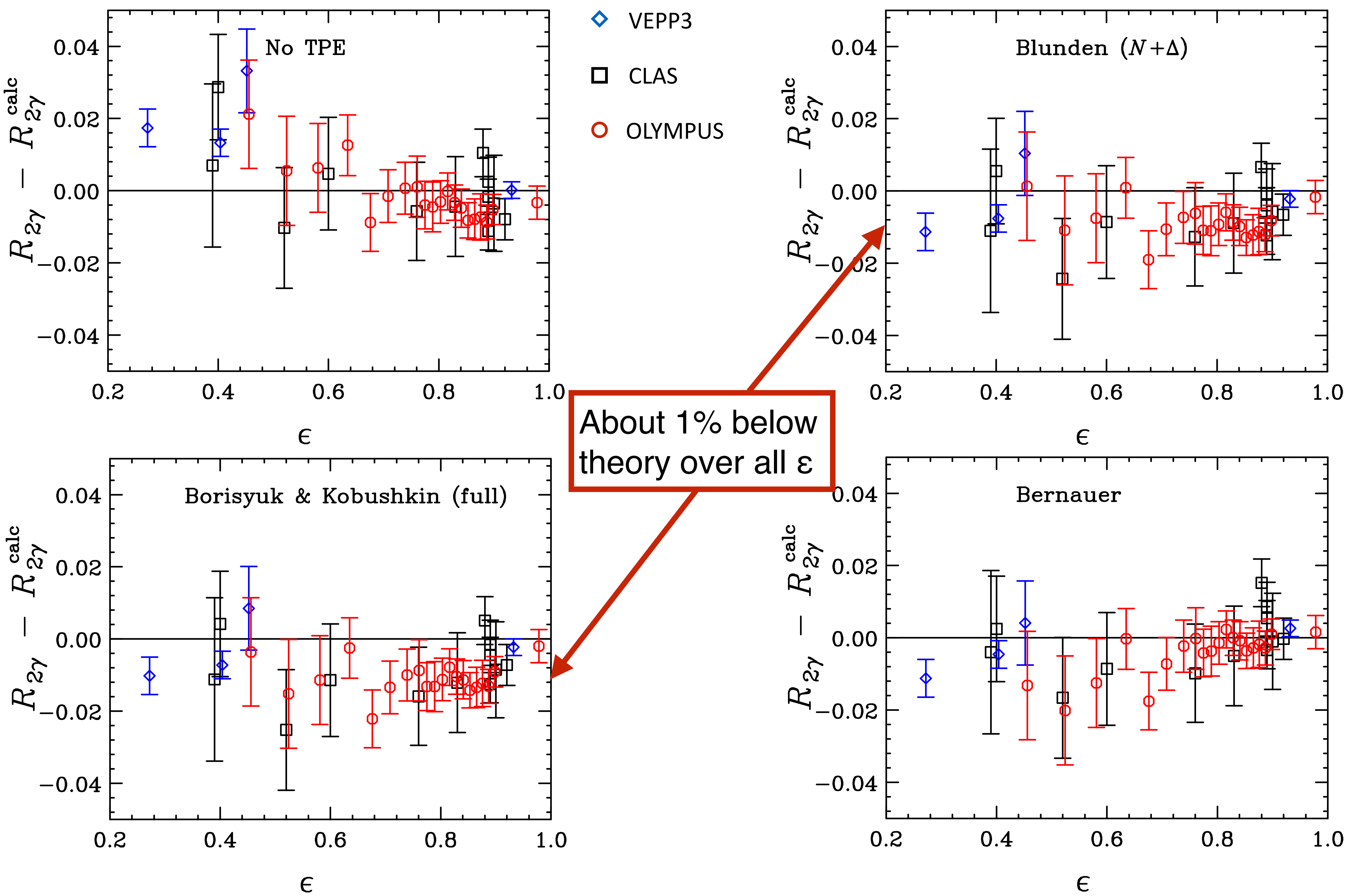
OLYMPUS
(Doris ring @ DESY)



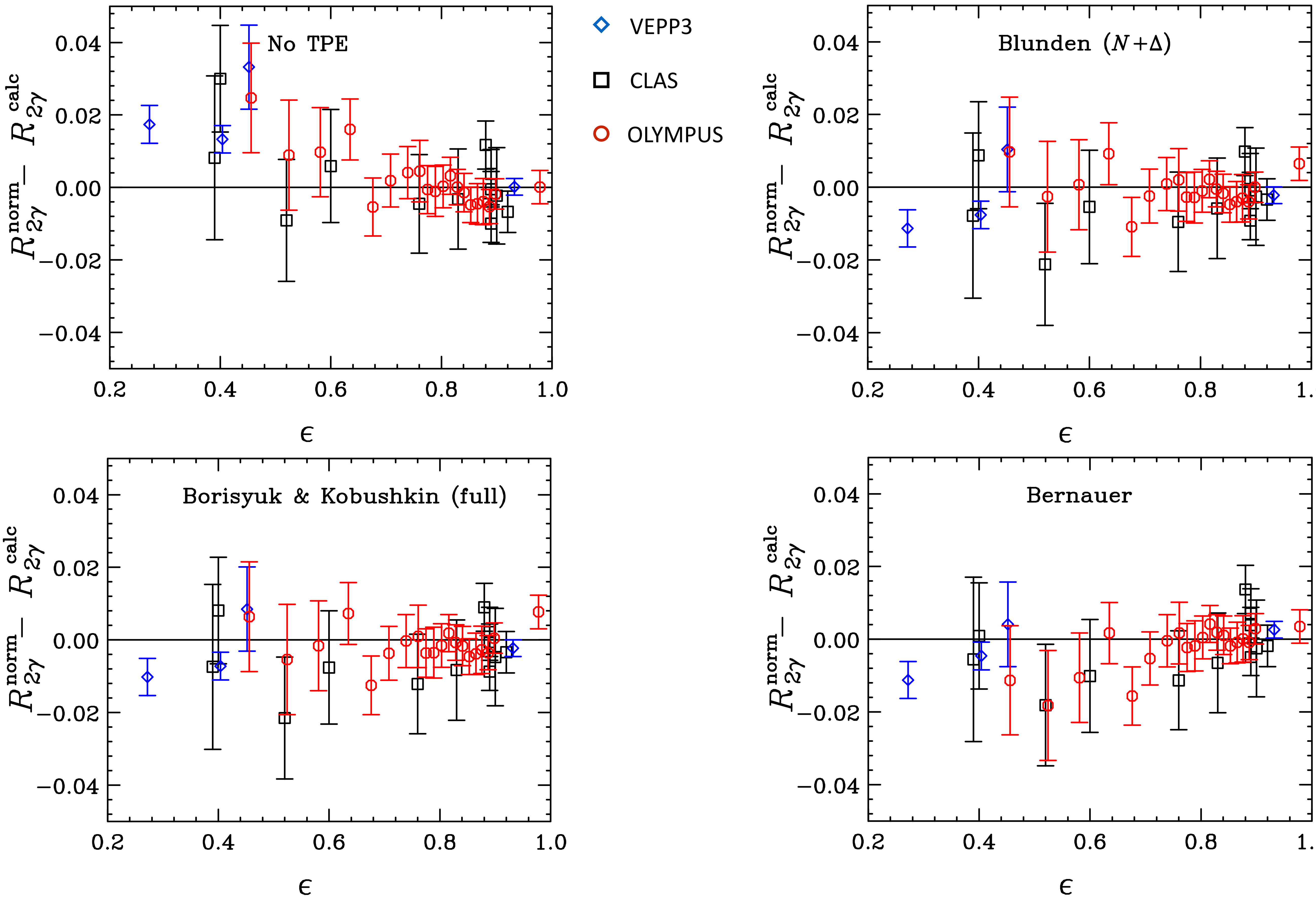


What is going on at low Q^2?

Comparing theory and experiment



Allowing normalization to float



Correction to proton weak charge

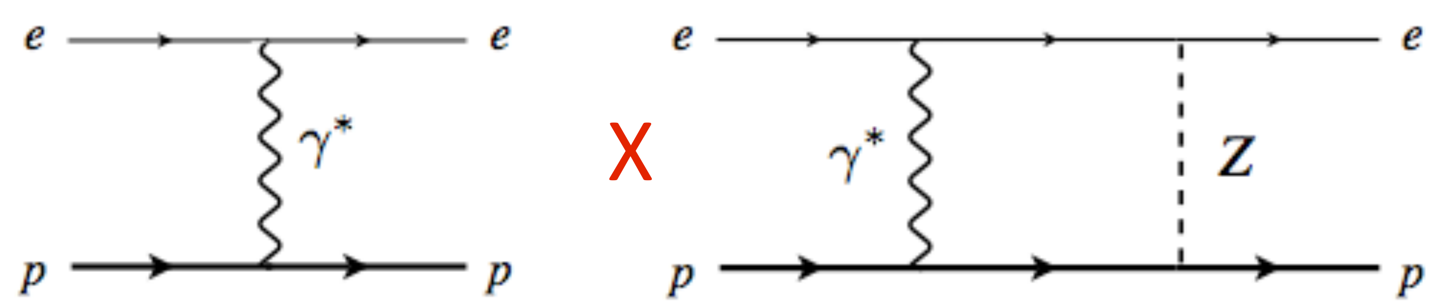
- including one-loop radiative corrections

e.w. vertex corrections

$$Q_W^p = \rho\left(1 - 4\kappa_{\mathrm{PT}}(0)\hat{s}^2 + \Delta'_e + \Delta_W\right)$$
$$+ \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}$$

box diagrams

Box corrections



→ *WW* and *ZZ* box diagrams large but dominated by short distances; can be evaluated perturbatively

→ γZ box diagram sensitive to long distance physics, has two contributions: $\Box_{\gamma Z} = \Box_{\gamma Z}^{A} + \Box_{\gamma Z}^{V}$

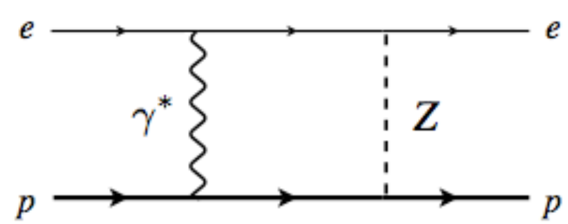
V(e) x A(h)
(finite at E=0)

A(e) x V(h)
(inelastic vanishes at E=0)

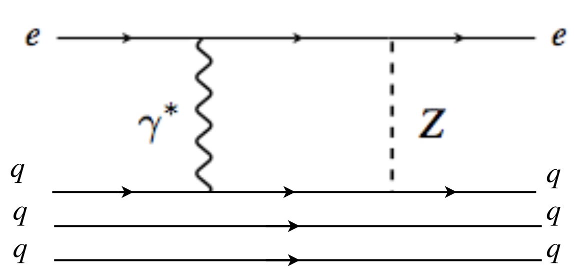
Axial h correction

- <u>axial</u> h correction $\Box^{A}_{\gamma Z}$ dominant γZ correction in atomic parity violation at very low (zero) energy

$\longrightarrow$ computed by Marciano & Sirlin in 1983 as sum of two parts:



★ low-energy part approximated by *Born* contribution (elastic intermediate state)



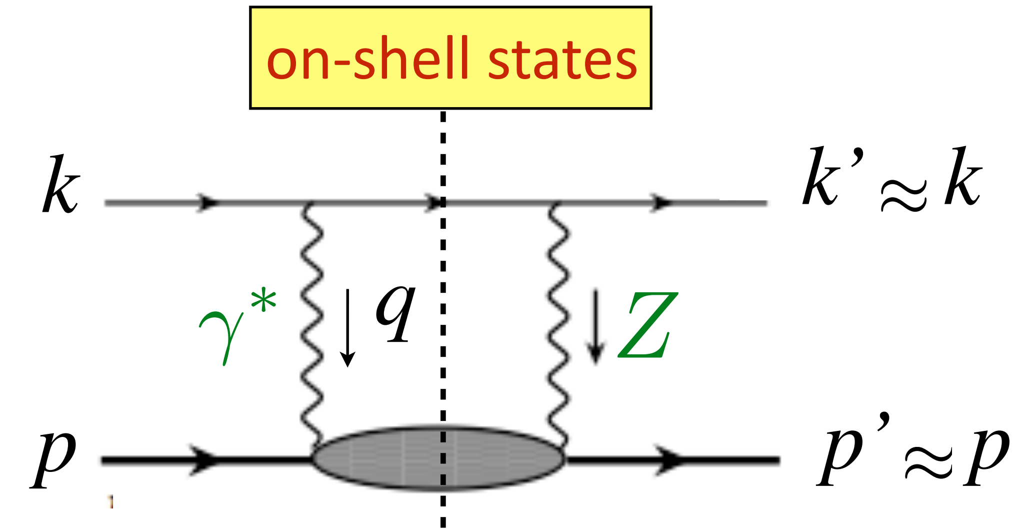
★ high-energy part (above scale $\Lambda \sim 1$ GeV) computed perturbatively in terms of scattering from *free quarks*

$$\Box^{A}_{\gamma Z} = (1 - 4\hat{s}^2)\,\frac{5\alpha}{2\pi}\underbrace{\int_{\Lambda^2}^{\infty}\frac{dQ^2}{Q^2(1+Q^2/M_Z^2)}\left(1-\frac{\alpha_s(Q^2)}{\pi}\right)}_{\sim\log\frac{M_Z^2}{\Lambda^2}+c}$$

Marciano, Sirlin, PRD ***29*** *(1984) 75; Erler et al., PRD* ***68*** *(2003) 016006*

Forward angle dispersion method

Gorchtein, Horowitz, PRL **102** (2009) 091806



$$S = 1 + i\mathcal{M}$$

$$S^\dagger = 1 - i\mathcal{M}^\dagger$$

$$SS^\dagger = 1$$

Unitarity $\rightarrow \quad -i\left(\mathcal{M} - \mathcal{M}^\dagger\right) = 2\Im m\,\mathcal{M} = \mathcal{M}^\dagger\mathcal{M}$

$$\boxed{\Im m\,\langle f|\mathcal{M}|i\rangle = \frac{1}{2}\int d\rho\,\sum_n \langle f|\mathcal{M}^*|n\rangle\langle n|\mathcal{M}|i\rangle}$$

Forward scattering amplitude: $|\,f\,\rangle \approx |\,i\,\rangle$

$$\Im m\langle i|\mathcal{M}|i\rangle = \frac{1}{2}\int d\rho \sum_n |\langle n|\mathcal{M}|i\rangle|^2 \sim \int d^3k_1\;\frac{L_{\mu\nu}W^{\mu\nu}}{q^2(q^2 - M_Z^2)}$$

vector h

axial h

hadronic tensor: $MW^{\mu\nu}_{\gamma Z} = -g^{\mu\nu}F_1^{\gamma Z} + \frac{p^\mu p^\nu}{p\cdot q}F_2^{\gamma Z} - i\varepsilon^{\mu\nu\lambda\rho}\frac{p_\lambda q_\rho}{2p\cdot q}F_3^{\gamma Z}$

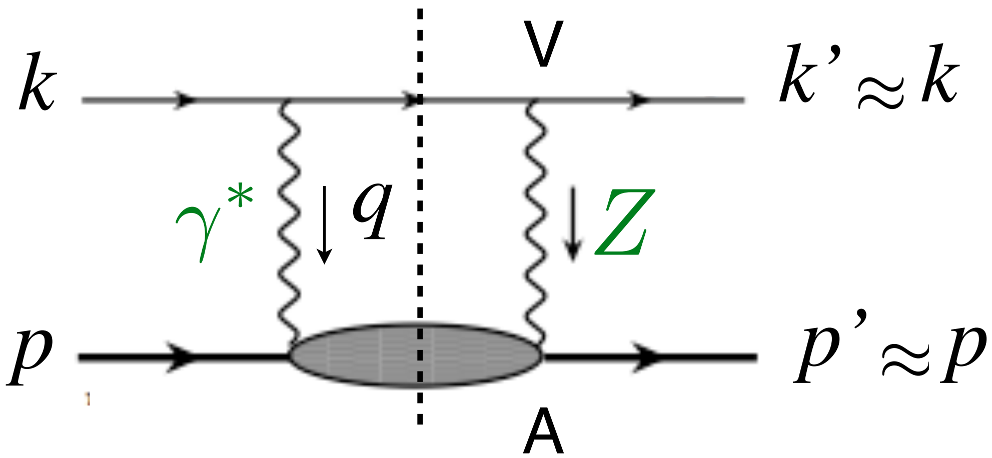
Axial h correction

- At low energy, dominant $V_e \times A_h$ correction evaluated using forward dispersion relations

$$\Re e\,\Box^A_{\gamma Z}(E) = \frac{2}{\pi}\int_0^\infty dE'\,\frac{E'}{E'^2 - E^2}\,\Im m\,\Box^A_{\gamma Z}(E')$$

$\longrightarrow$ imaginary part given by $F_3^{\gamma Z}$ structure function

$$\Im m\,\Box^A_{\gamma Z}(E) = \frac{1}{(2ME)^2}\int_{M^2}^{s} dW^2 \int_0^{Q^2_{\max}} dQ^2\,\frac{v_e(Q^2)\,\alpha(Q^2)}{1+Q^2/M_Z^2}$$
$$\times\left(\frac{2ME}{W^2 - M^2 + Q^2} - \frac{1}{2}\right) F_3^{\gamma Z}$$



with $v_e(Q^2) = 1 - 4\kappa(Q^2)\hat{s}^2$

Axial h correction DIS part (dominant contribution)

- DIS part dominated by leading twist PDFs at small x

(MSTW, CTEQ, Alekhin parametrizations)

$$F_3^{\gamma Z(\mathrm{DIS})}(x,Q^2) = \sum_q 2\, e_q\, g_A^q \,\left(q(x,Q^2) - \bar{q}(x,Q^2)\right)$$

$\longrightarrow$ in DIS region ($Q^2 \gtrsim 1\ \mathrm{GeV}^2$), expand integrand in powers of x

$$\Re e\,\Box_{\gamma Z}^{\mathrm{A(DIS)}}(E) = \frac{3}{2\pi}\int_{Q_0^2}^{\infty} dQ^2\, \frac{v_e(Q^2)\alpha(Q^2)}{Q^2(1+Q^2/M_Z^2)} \times \left[M_3^{(1)}(Q^2) + \frac{2M^2}{9Q^4}(5E^2 - 3Q^2) M_3^{(3)}(Q^2) + \ldots \right]$$

with moments $\quad M_3^{\gamma Z(n)} = \int_0^1 dx\, x^{n-1} F_3^{\gamma Z}(x,Q^2)$

Axial h correction

- structure function moments

$n = 1$ $$M_3^{\gamma Z(1)}(Q^2) = \frac{5}{3}\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)$$

$\longrightarrow$ γZ analog of Gross-Llewellyn Smith sum rule

$$\mathcal{R}e\,\Box_{\gamma Z}^{A(\mathrm{DIS})} \approx (1-4\hat{s}^2)\,\frac{5\alpha}{2\pi}\int_{Q_0^2}^{\infty}\frac{dQ^2}{Q^2(1+Q^2/M_Z^2)}\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)$$

$\longrightarrow$ precisely result from Marciano & Sirlin! $\sim \log\frac{M_Z^2}{Q_0^2}$

(works because result depends on lowest moment of *valence* PDF, with <u>model-independent normalization</u>!)

$n = 3$ $$M_3^{\gamma Z(3)}(Q^2) = \frac{1}{3}\left(2\langle x^2\rangle_u + \langle x^2\rangle_d\right)\left(1 + \frac{5\alpha_s(Q^2)}{12\pi}\right)$$

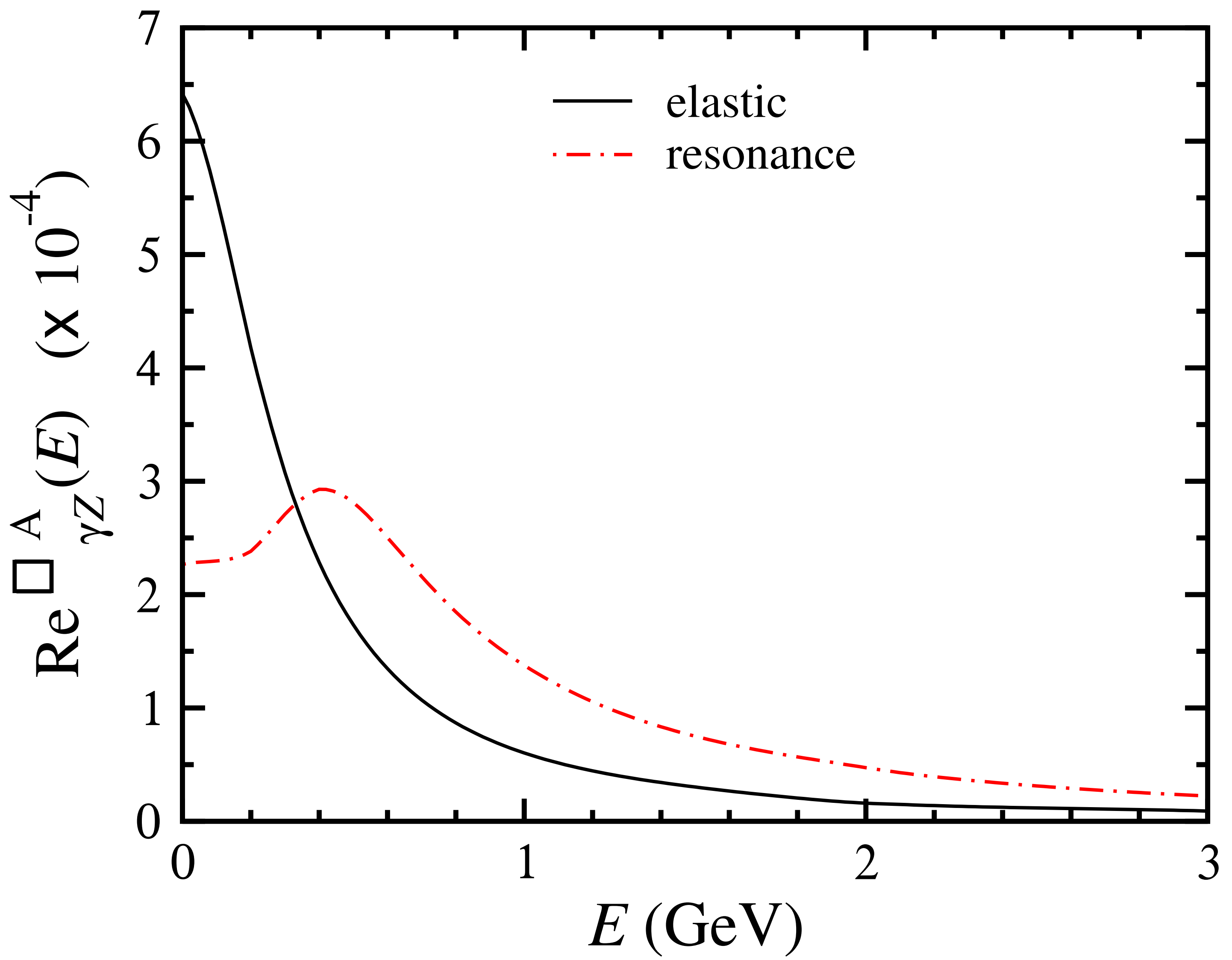
$\longrightarrow$ related to x^2-weighted moment of valence quarks

Axial h elastic + resonance correction

- elastic part: $F_3^{\gamma Z(\mathrm{el})}(Q^2) = -Q^2 G_M^p(Q^2) G_A^Z(Q^2) \delta(W^2 - M^2)$

- resonance part from parametrization of ν scattering data; includes lowest four spin 1/2 and 3/2 states (Lalakulich-Paschos)

→ Least well-constrained part of the calculation



Vector h correction

- vector h correction $\Box^V_{\gamma Z}$ vanishes at $E = 0$, but experiment has $E \sim 1$ GeV - what is energy dependence?

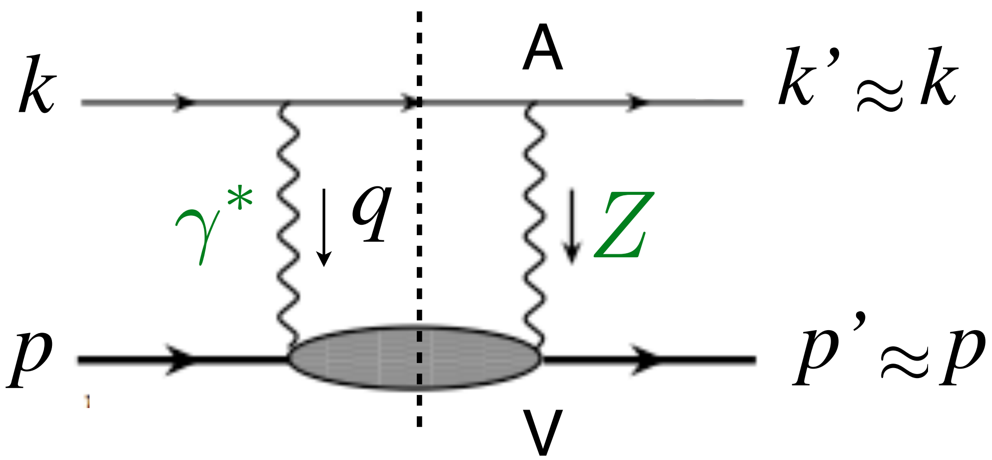
$\longrightarrow$ forward dispersion relation

$$\Re e\, \Box^V_{\gamma Z}(E) = \frac{2E}{\pi} \int_0^\infty dE' \frac{1}{E'^2 - E^2}\, \Im m\, \Box^V_{\gamma Z}(E')$$

$\longrightarrow$ imaginary part given by

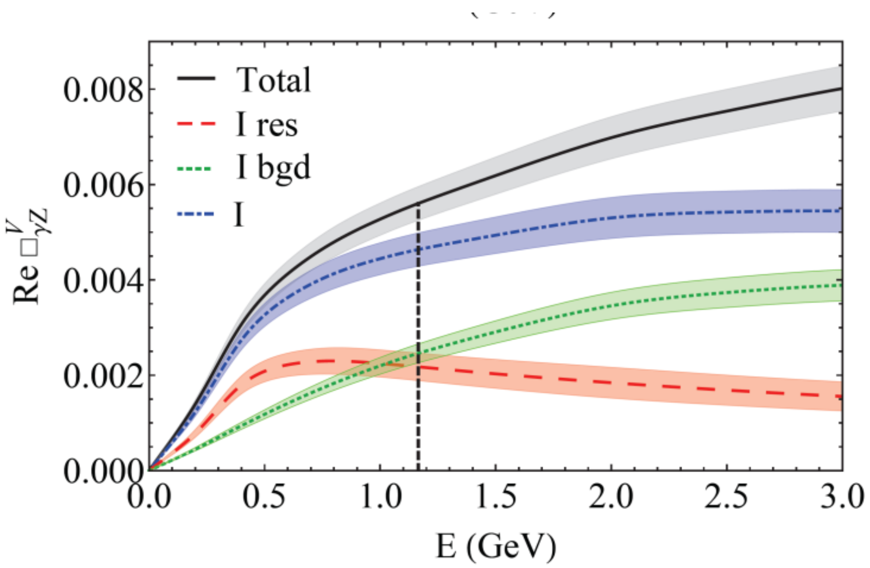
$$\Im m\, \Box^V_{\gamma Z}(E) = \frac{\alpha}{(s - M^2)^2} \int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{\max}} \frac{dQ^2}{1 + Q^2/M_Z^2}$$

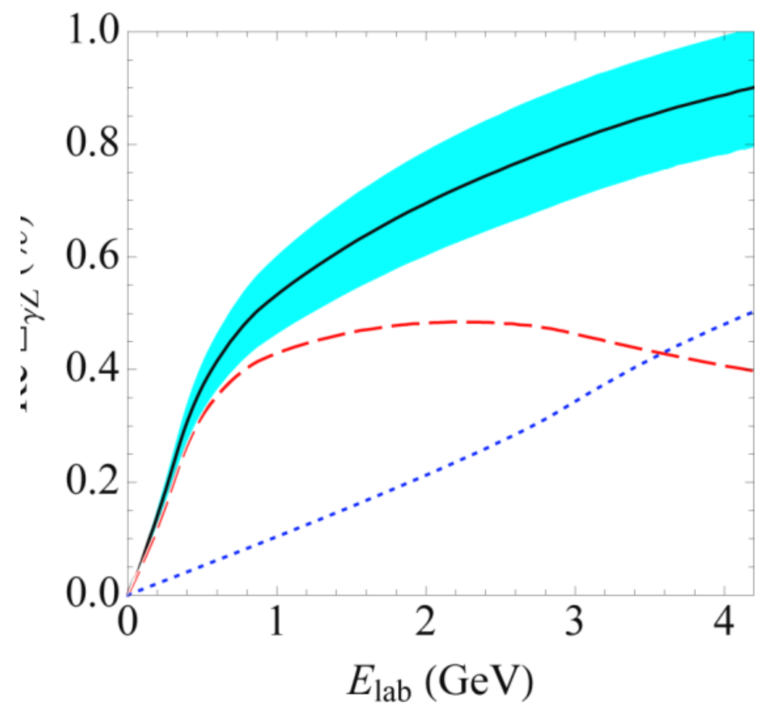
$$\times \left(F_1^{\gamma Z} + F_2^{\gamma Z} \frac{s\,(Q^2_{\max} - Q^2)}{Q^2(W^2 - M^2 + Q^2)} \right)$$

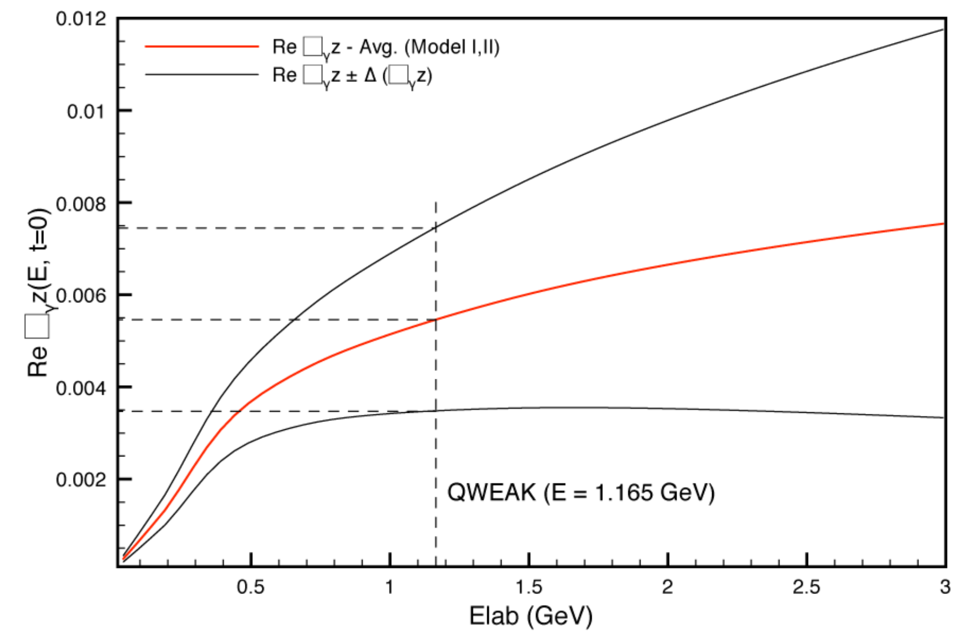


3 groups doing independent analyses

Hall *et al.*	Carlson and Rislow	Gorchtein *et al.*
PRD 88, 013011 (2013)	PRD 83, 113007 (2011)	PRC 84, 015502 (2011)







Qweak energy: $\mathrm{Re}\Box^V_{\gamma Z}(E = 1.165\ \mathrm{GeV})$ **8% correction!**

$(5.6 \pm 0.36) \times 10^{-3}$ $(5.7 \pm 0.9) \times 10^{-3}$ $(5.4 \pm 2.0) \times 10^{-3}$

- Mainly different treatments of low Q^2, low W region background contributions
- Agree on overall magnitude of 8% correction, but disagree on errors and details

AJM structure function model

- Accurate knowledge of $\gamma\gamma$ and γZ structure functions (at all kinematics) vital for determination of radiative corrections

- Wealth of data on $F_i^{\gamma\gamma}$ structure functions over large range of kinematics in Q^2 and W (or x) – with some gaps

- Relatively little known about $F_i^{\gamma Z}$ interference structure functions below HERA measurements, with $Q^2 \geq 1500\ \mathrm{GeV}^2$

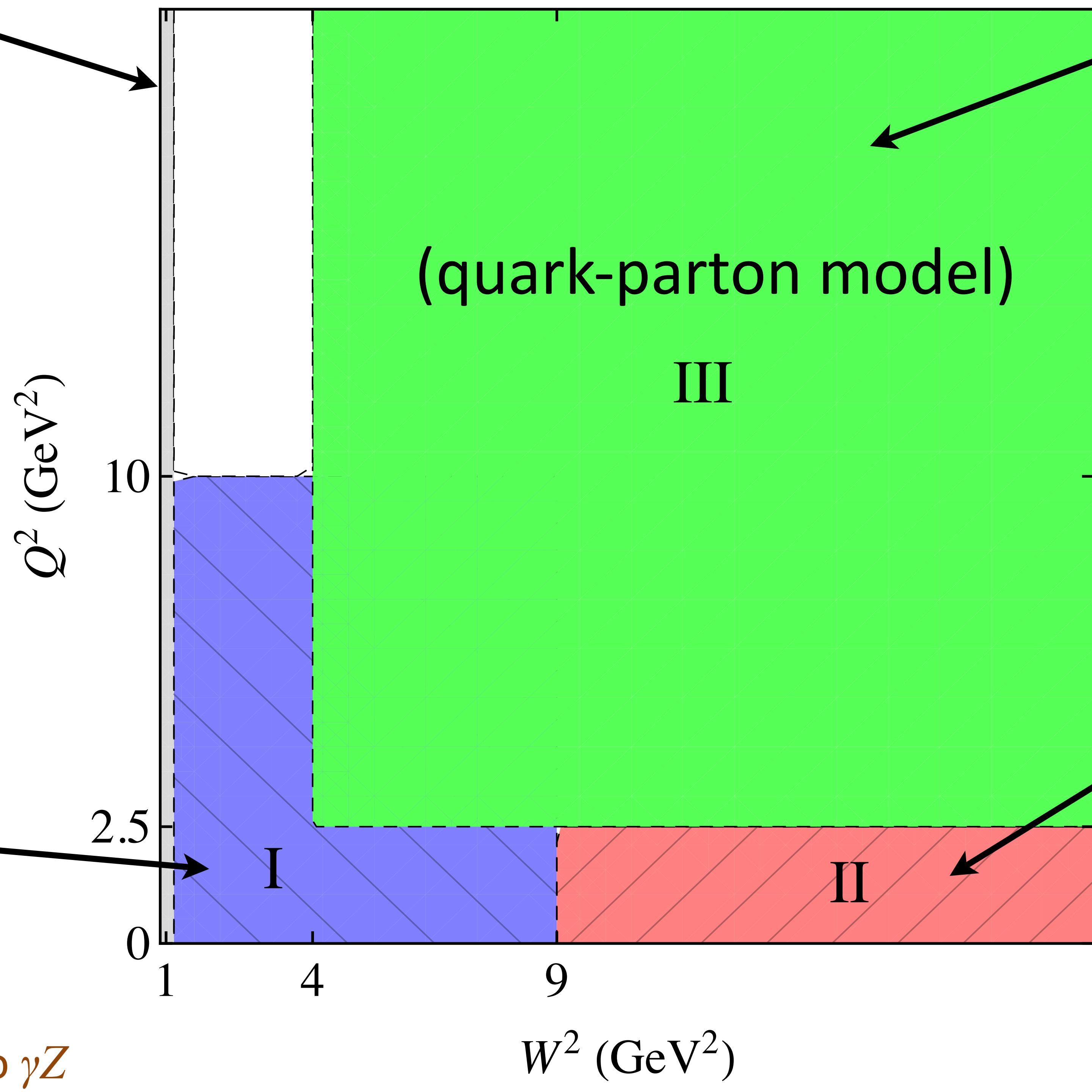
- Fit $F_i^{\gamma\gamma}$ over all kinematics in Q^2 and W, then "rotate" to $F_i^{\gamma Z}$ using available theoretical/phenomenological constraints

⟶ *e.g.* isospin symmetry

$$\langle N^*|J_Z^\mu|p\rangle = (1 - 4\sin^2\theta_W)\langle N^*|J_\gamma^\mu|p\rangle - \langle N^*|J_\gamma^\mu|n\rangle$$

Integration region (structure function map)

“elastic”



“DIS”

Leading twist PDFs

“Regge”

Pomeron and Reggeon exchange

“resonance”

Bosted-Christy $\gamma\gamma$ fit

+ isospin rotation to γZ

Basic issue: how to relate $F_{1,2}^{\gamma Z}$ to $F_{1,2}^{\gamma}$?

Scaling region III

$$F_2^{\gamma} = \sum_q e_q^2 x(q+\bar{q})$$

$$F_2^{\gamma Z} = \sum_q 2 e_q g_V^q x(q+\bar{q})$$

$$x = \frac{Q^2}{W^2 - M^2 + Q^2}$$

Resonance region I largest contribution (unlike $F_3^{\gamma Z}$)

For $\gamma\gamma$ use Christy-Bosted (CB) fit to e-p cross sections

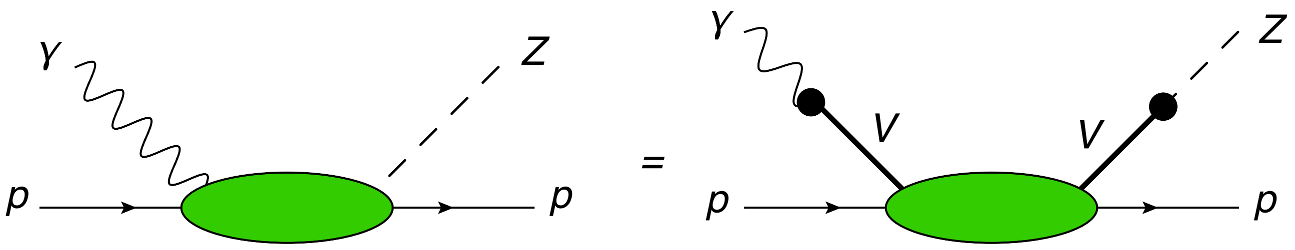
$$\sigma_{T,L} = \sigma_{T,L}(\mathrm{res}) + \sigma_{T,L}(\mathrm{bg})$$

$\sigma_{T,L}(\mathrm{res})$

- Includes 7 most prominent N* resonances below 2 GeV.
- Generally agrees with data to $\sim 5\%$
- For γZ modify fit by ratio of weak to e.m. transition amplitudes.

Background $\sigma_{T,L}(\mathrm{bg})$

- Use Vector Meson Dominance (VMD) models fit to high energy data, plus isospin rotations



V = ρ, ω, φ + continuum

$$\sigma_V^{\gamma Z} = \kappa_V\, \sigma_V^{\gamma\gamma}$$

$$\frac{\sigma^{\gamma Z}}{\sigma^{\gamma\gamma}} = \frac{\kappa_\rho + \kappa_\omega R_\omega + \kappa_\phi R_\phi + \kappa_C R_C}{1 + R_\omega + R_\phi + R_C}$$

Isospin rotation: $\kappa_\rho = 2 - 4\sin^2\theta_W\,,\;\; \kappa_\omega = -4\sin^2\theta_W\,,\;\; \kappa_\phi = 3 - 4\sin^2\theta_W$

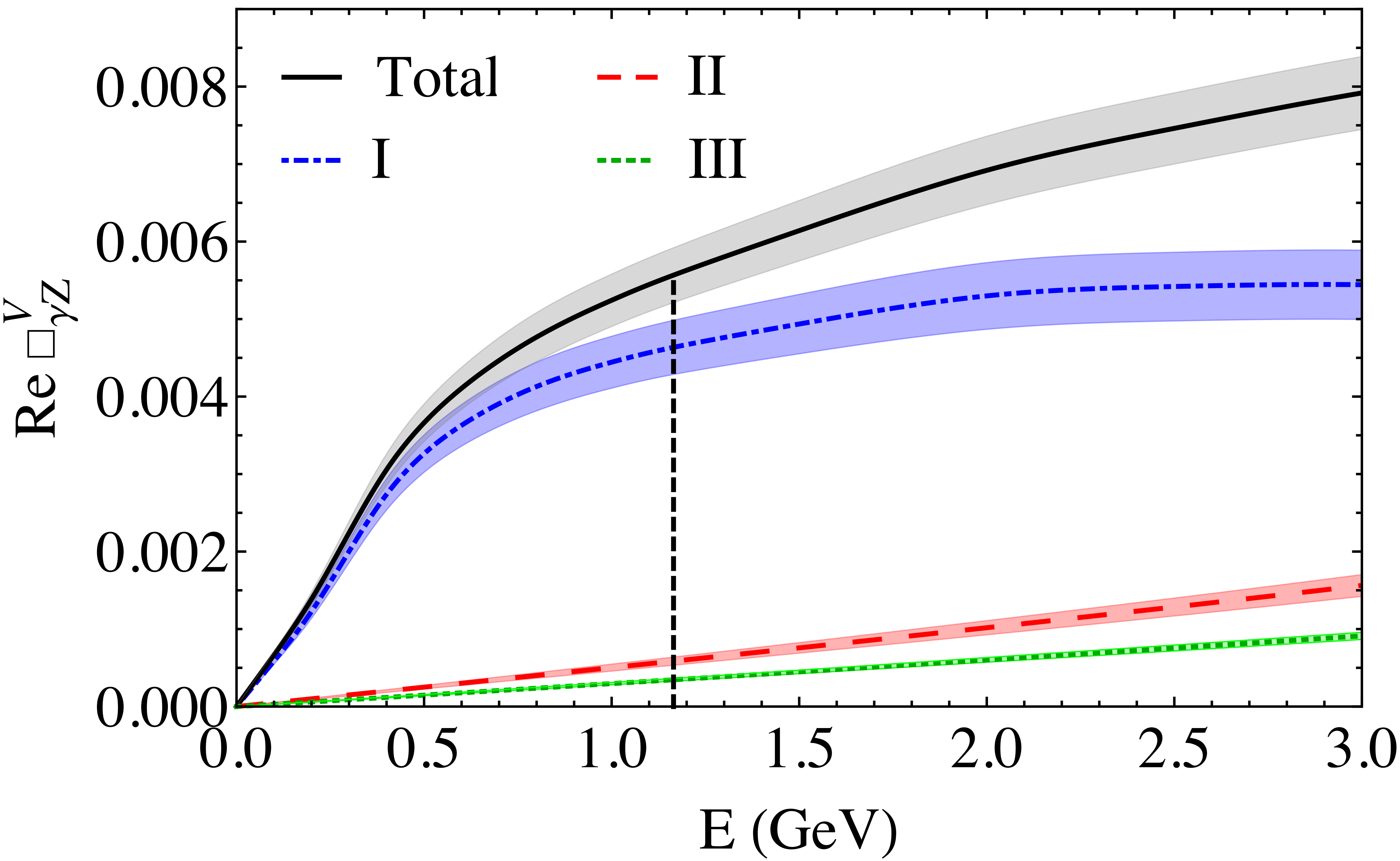
$\longrightarrow$ continuum parameter κ_C not constrained in VMD

- GHRM: assign 100% uncertainty on continuum contribution (dominates errors)

- AJM model: constrain continuum (higher Q^2) contribution by matching with PDF ratios (γZ to $\gamma\gamma$) across boundaries of Regions I, II and III.

Contribution from different regions to $\Box^V_{\gamma Z}$

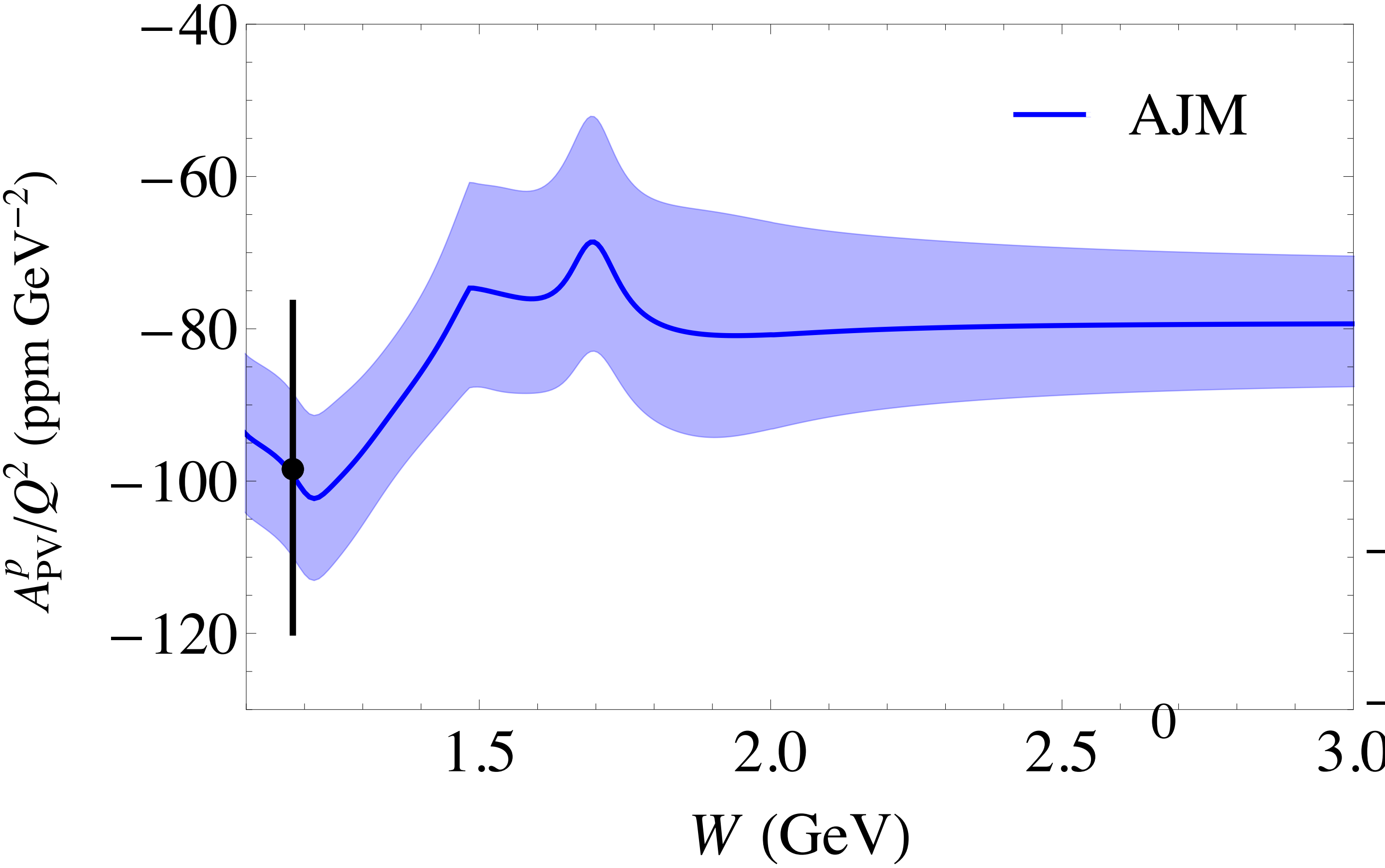
(relative to weak charge of 0.0713)

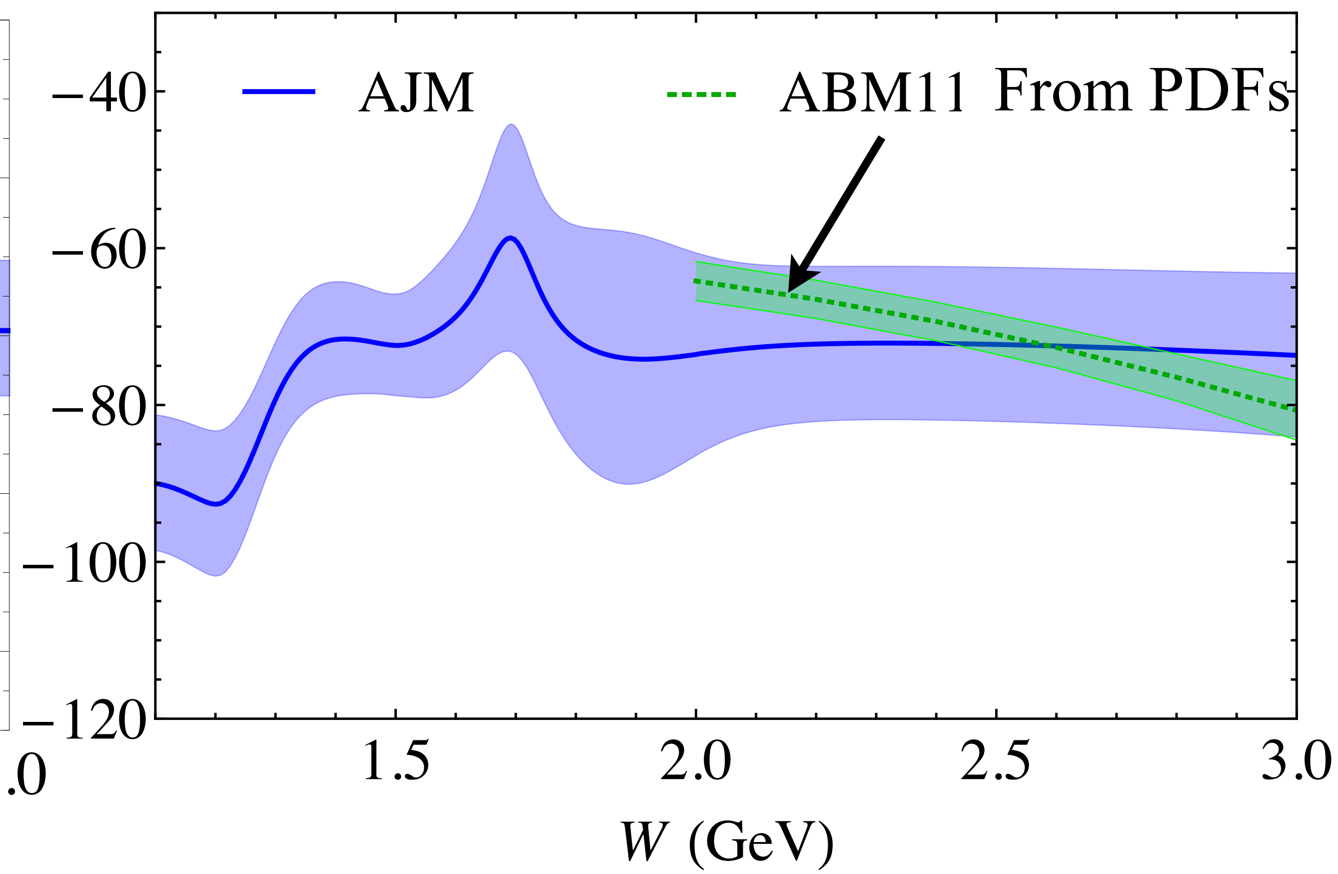


AJM γZ model direct test

- Parity-violating Deep Inelastic Scattering (PVDIS) asymmetry allows a <u>direct</u> measurement of the γZ structure functions

$$A_{\rm PV} = g_A^e \left(\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\right) \frac{xy^2 F_1^{\gamma Z} + (1-y)F_2^{\gamma Z} + \frac{g_V^e}{g_A^e}(y - y^2/2)xF_3^{\gamma Z}}{xy^2 F_1^{\gamma\gamma} + (1-y)F_2^{\gamma\gamma}}$$

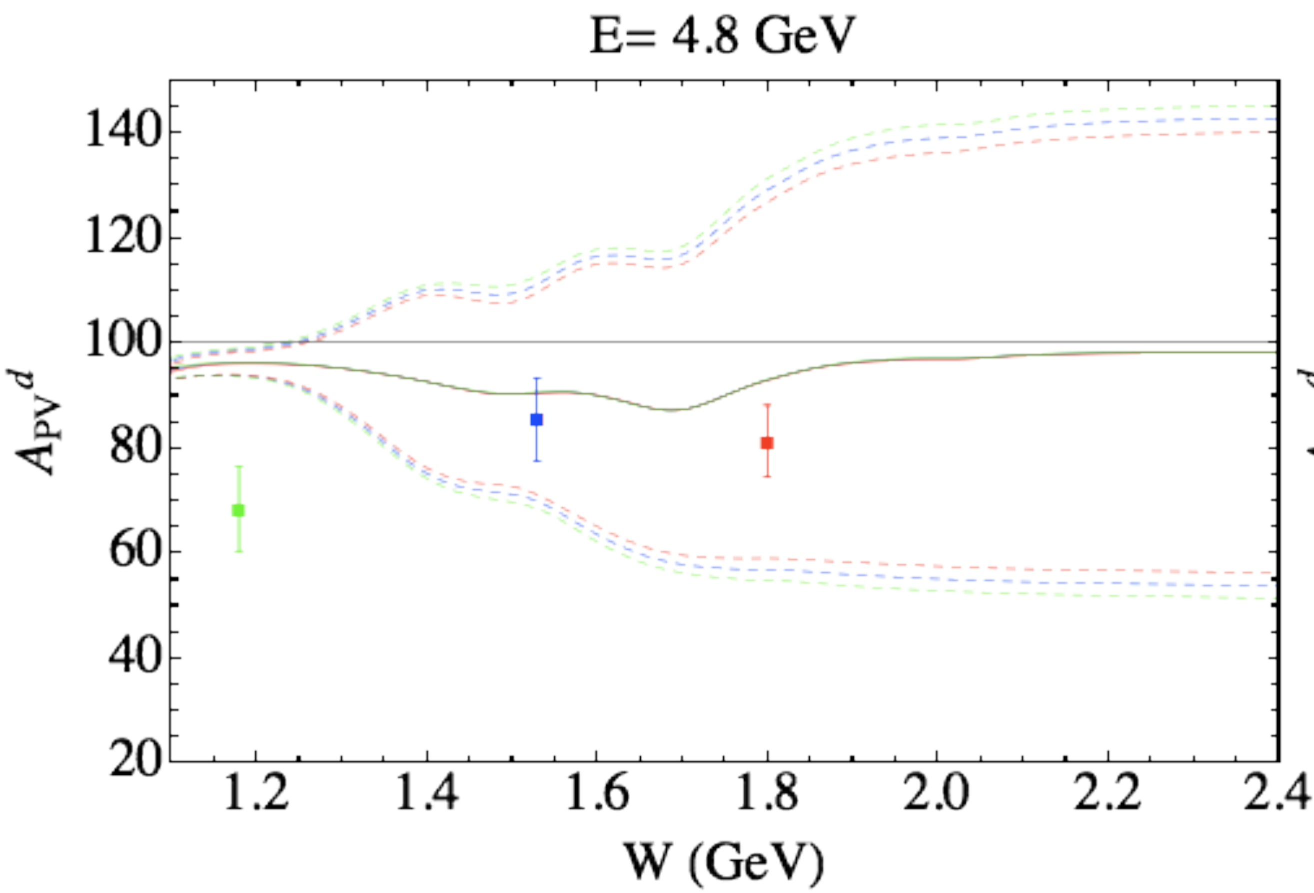


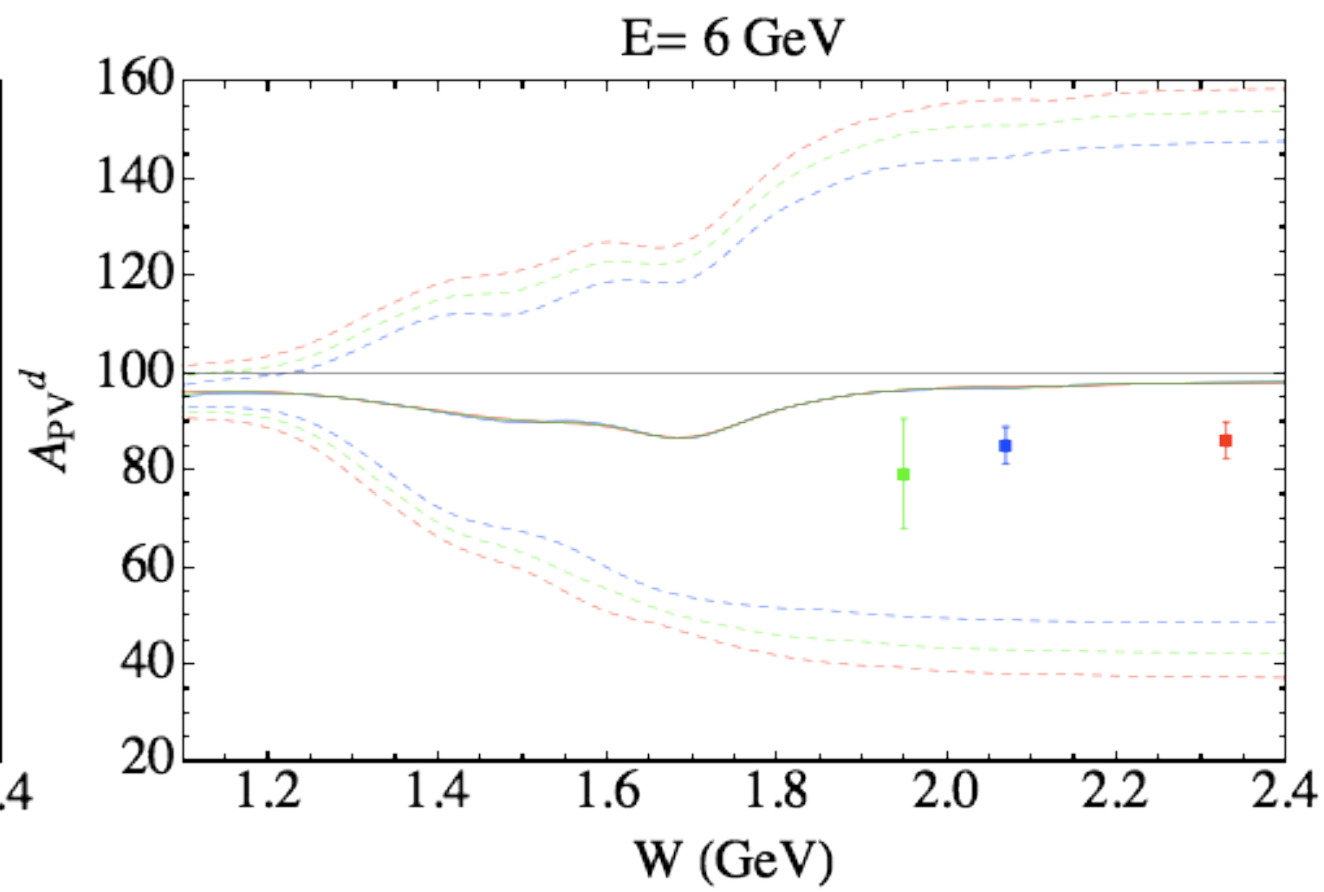


Q^2=0.34 GeV², E=0.69 GeV

Q^2 = 2.5 GeV², E = 6 GeV

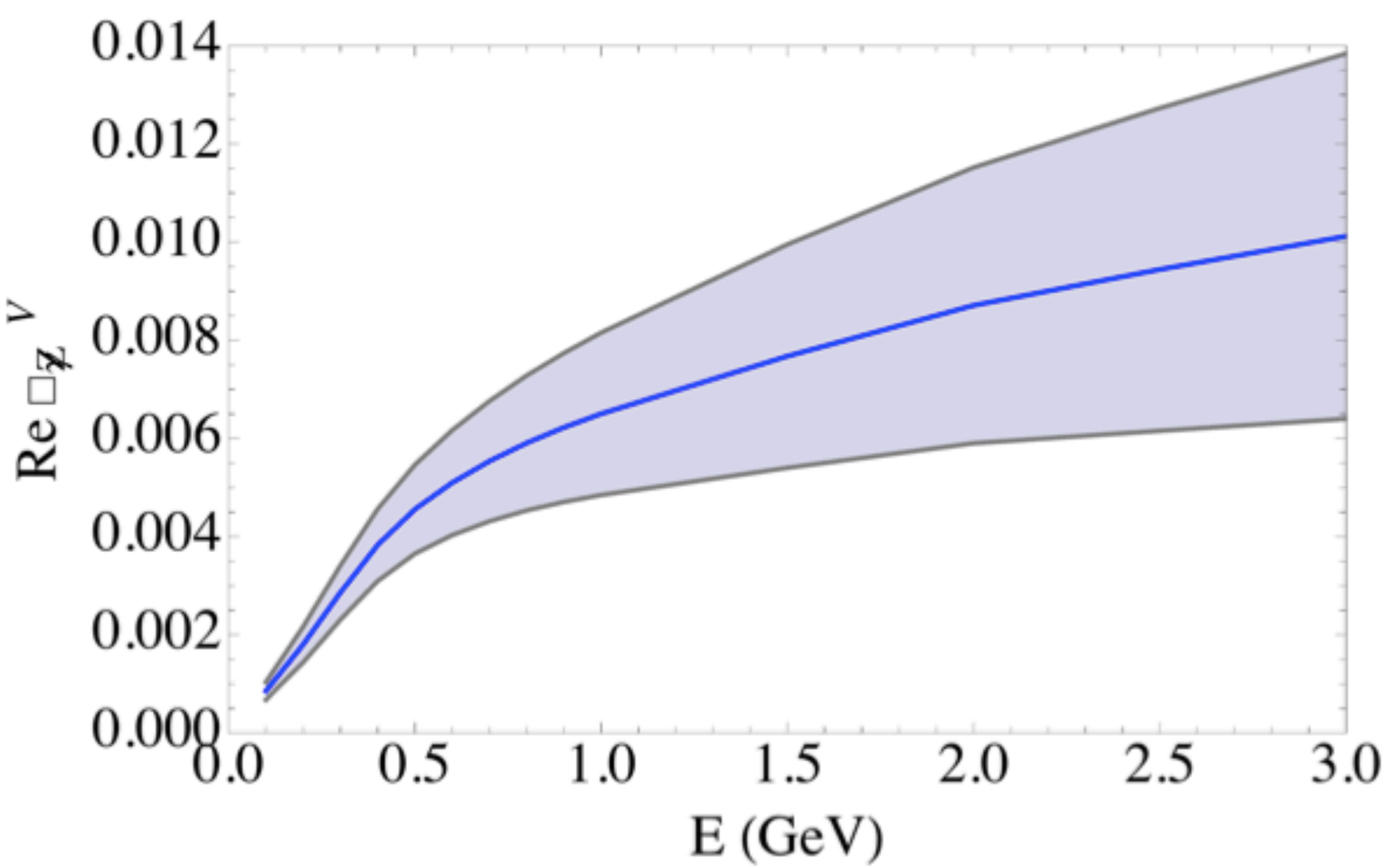
Androic et al. (G0 collaboration), arXiv:1212.1637

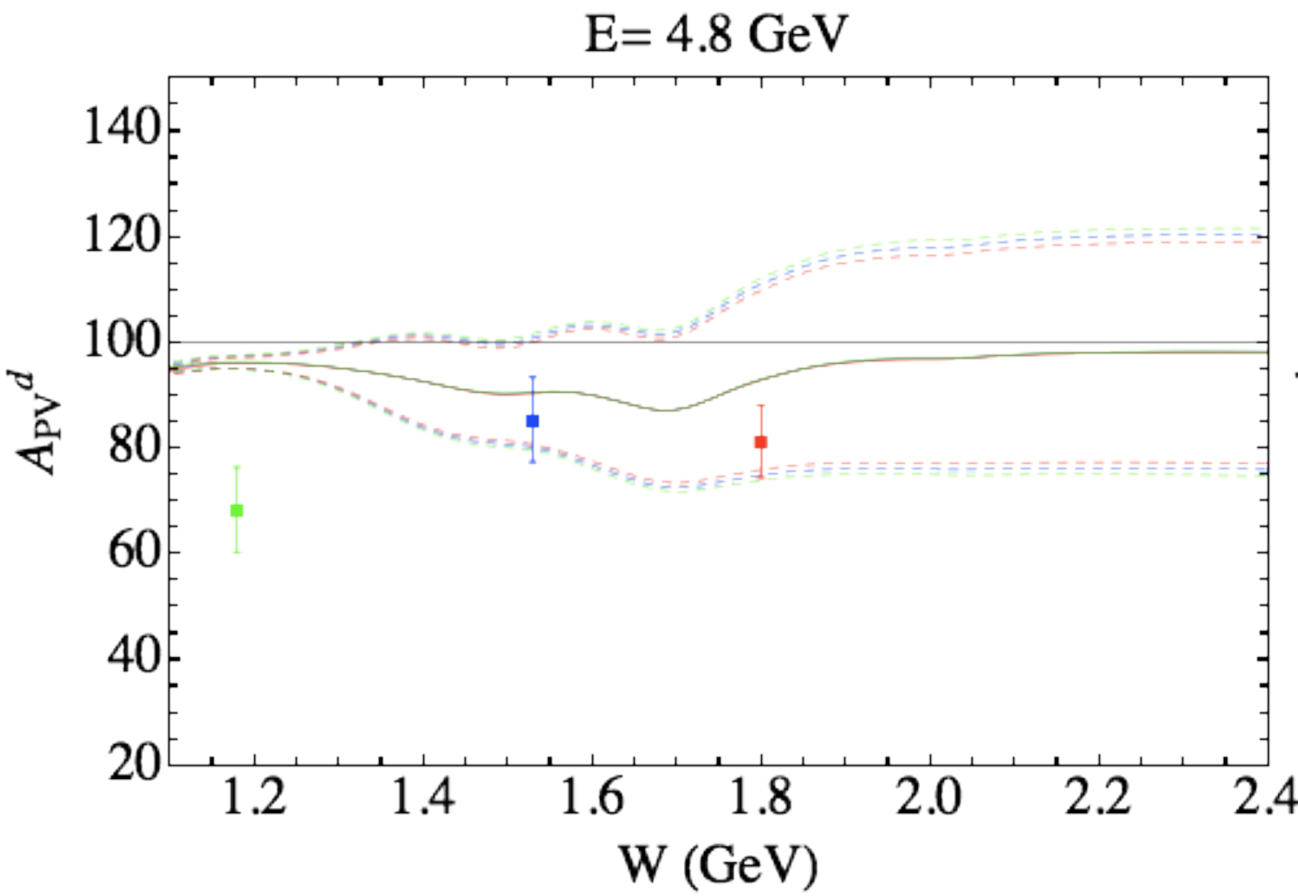


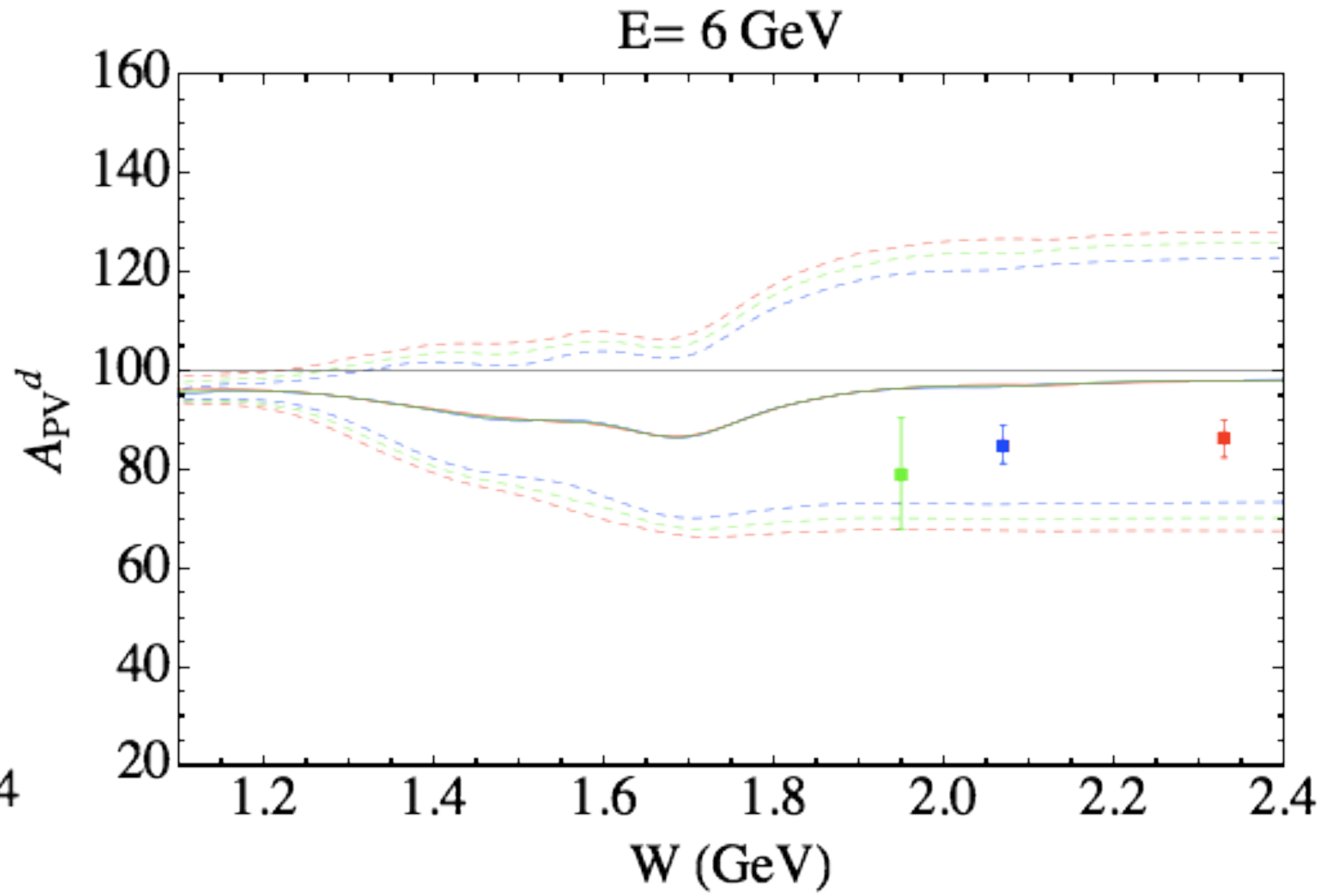


Potential impact of constraints from <u>deuteron</u> PV inelastic asymmetries

100% uncertainty on continuum background

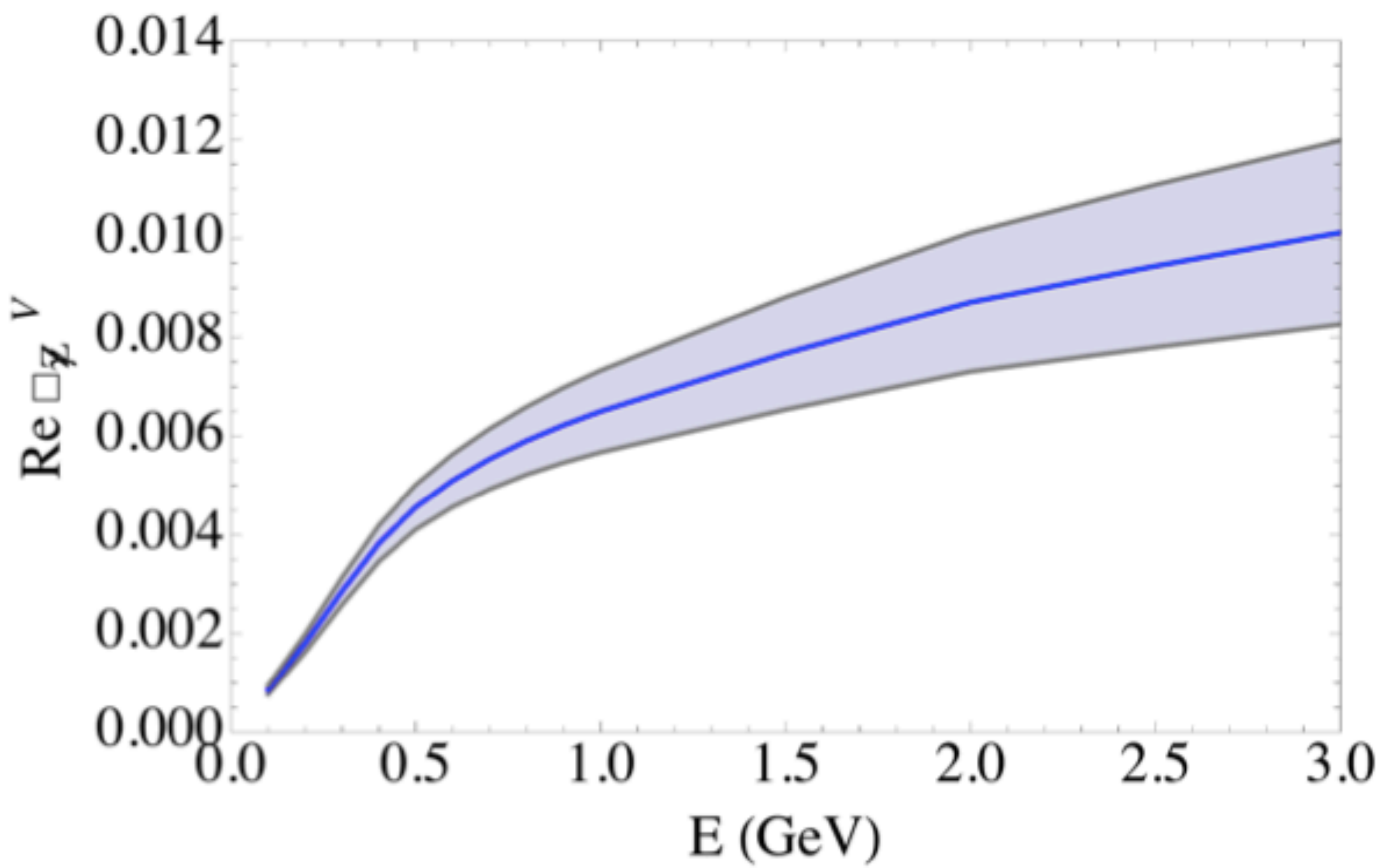


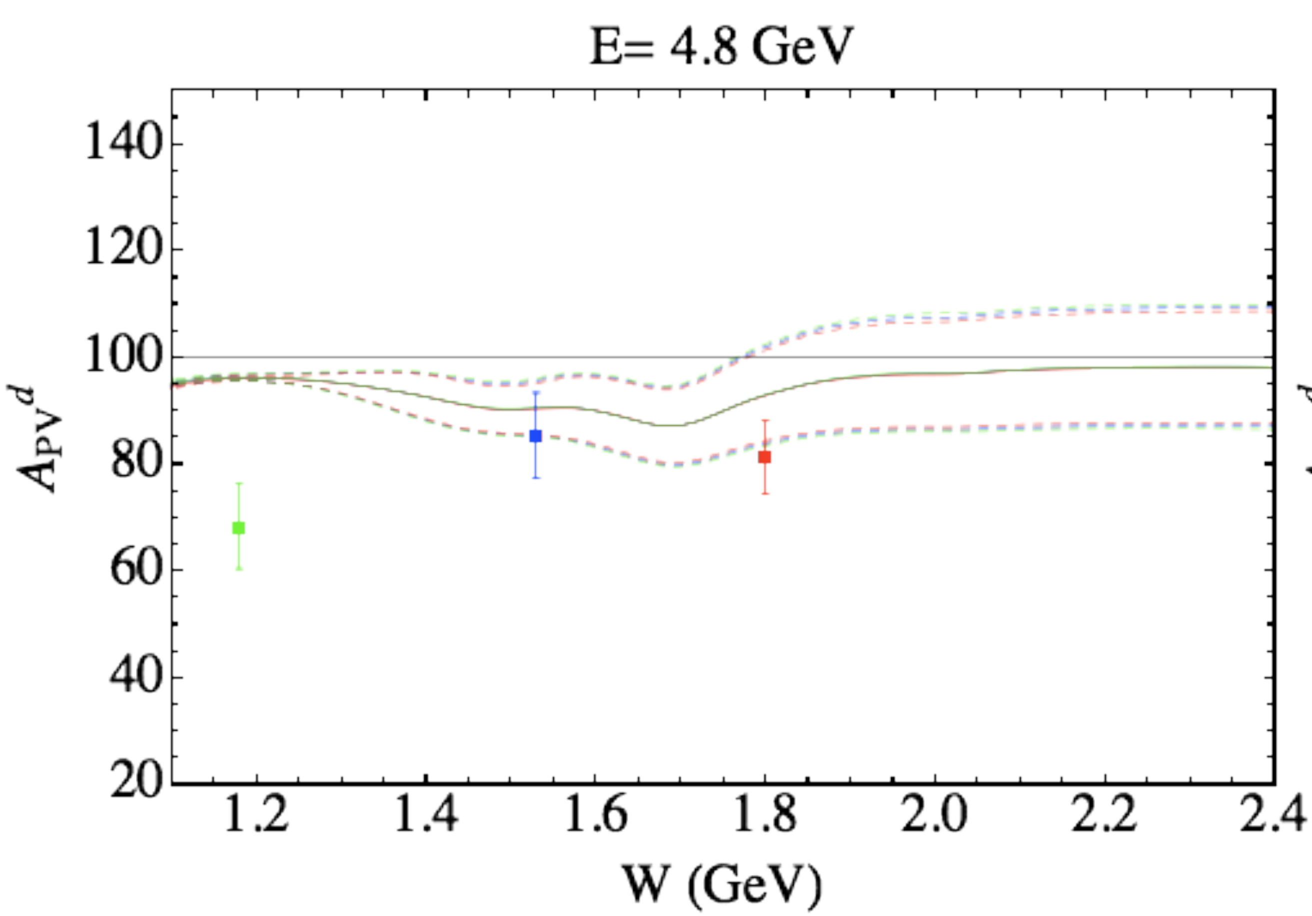


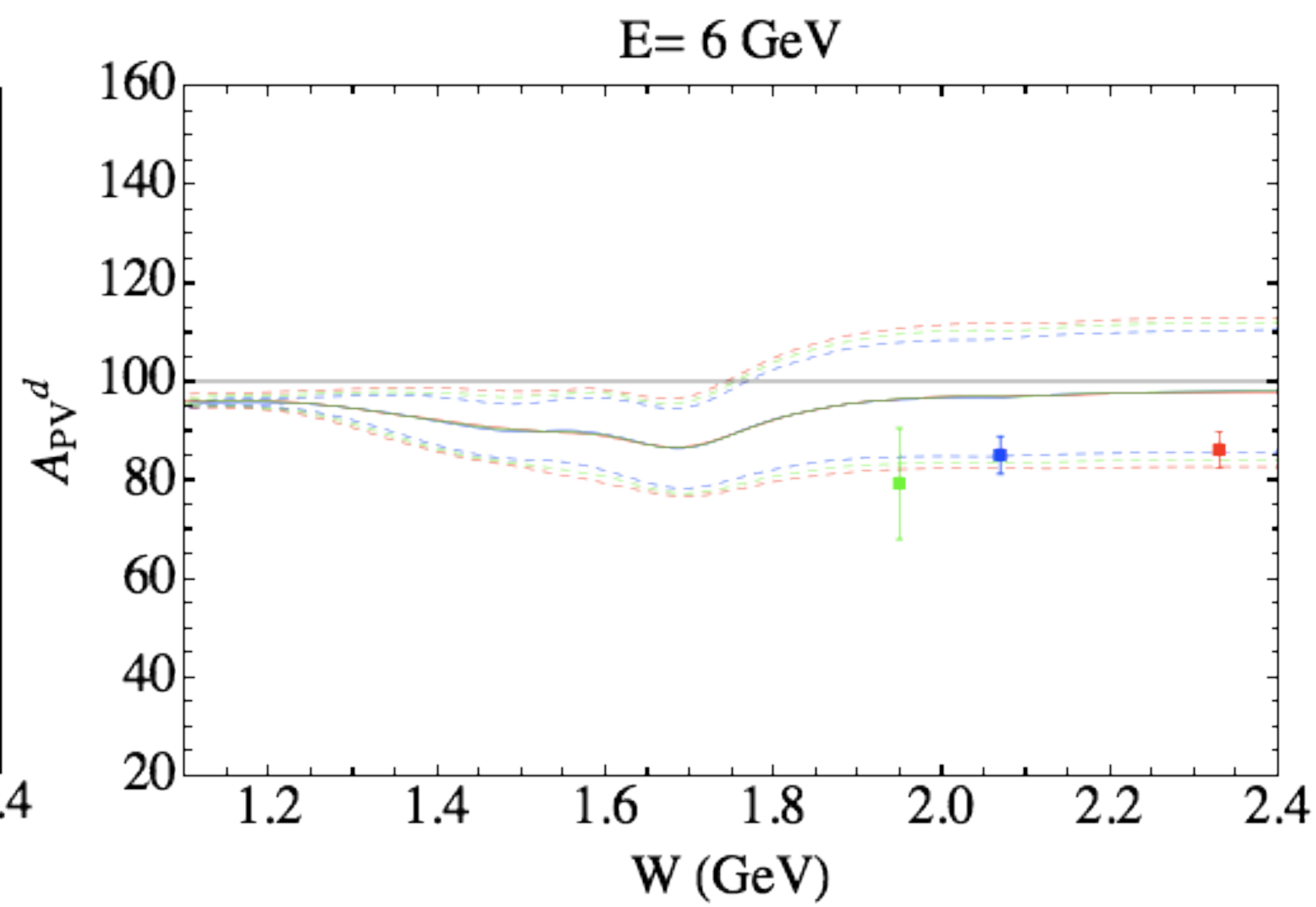


Potential impact of constraints from <u>deuteron</u> PV inelastic asymmetries

50% uncertainty on continuum background

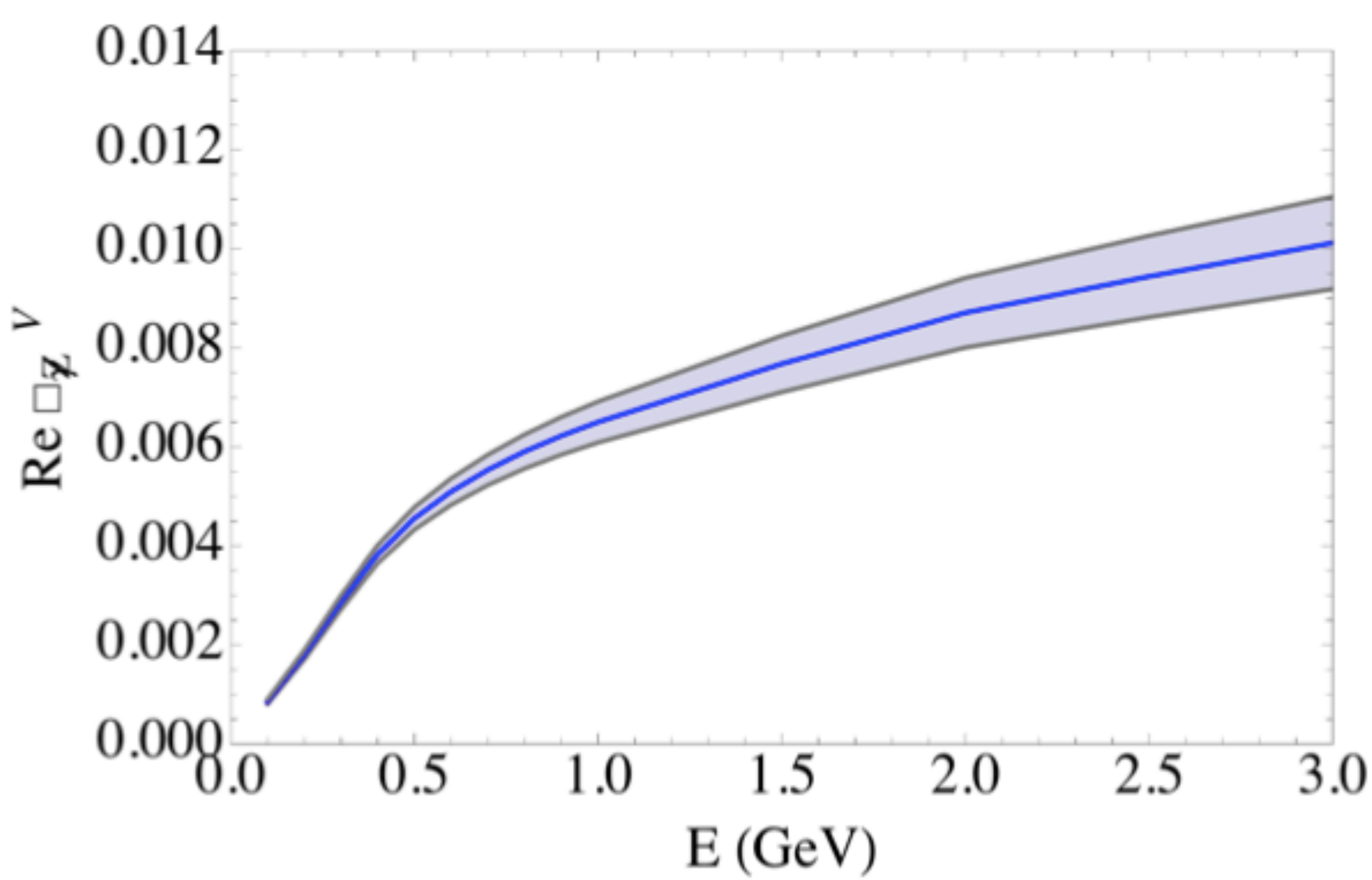




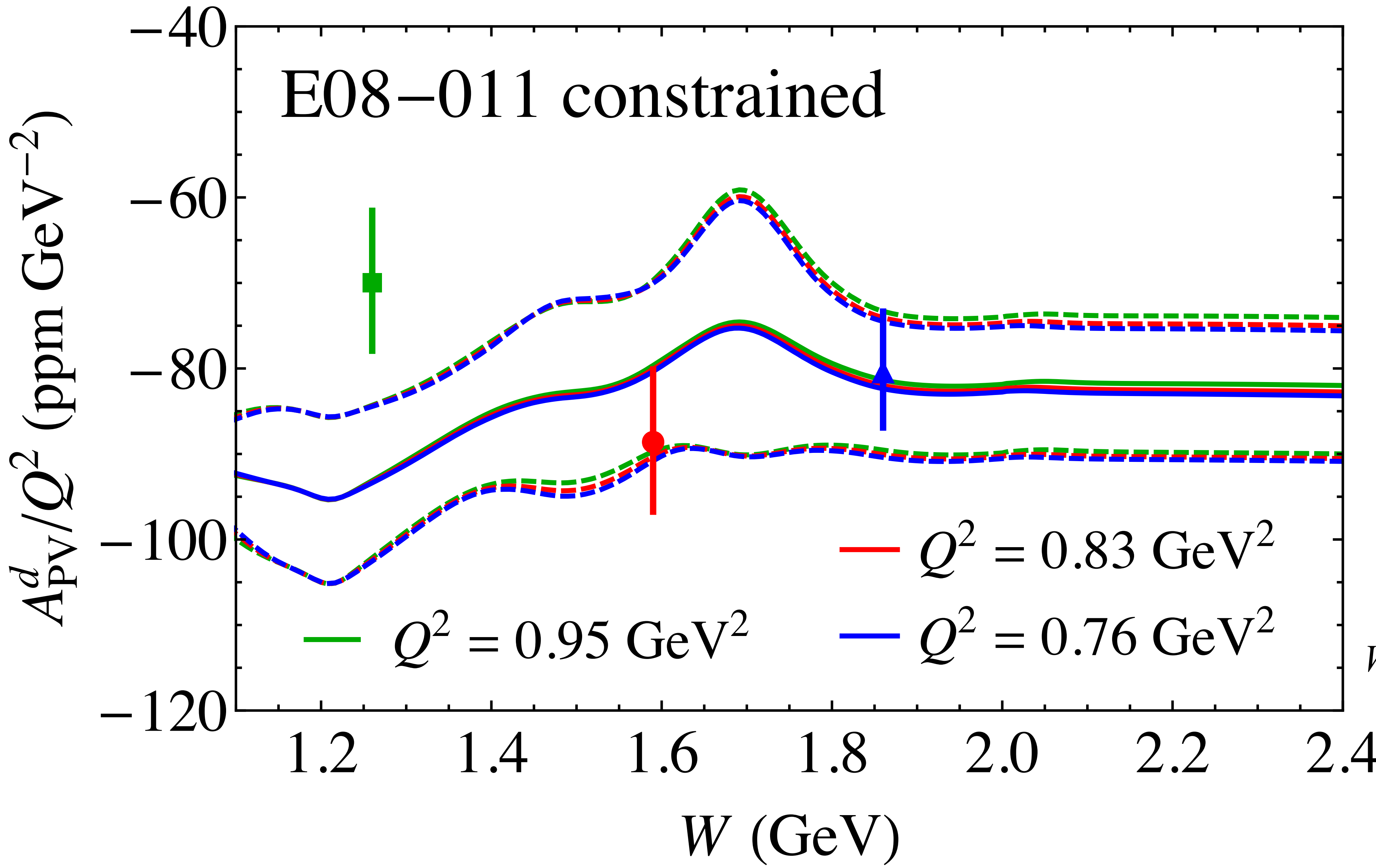


Potential impact of constraints from <u>deuteron</u> PV inelastic asymmetries

25% uncertainty on continuum background



Constraints from PV inelastic asymmetries

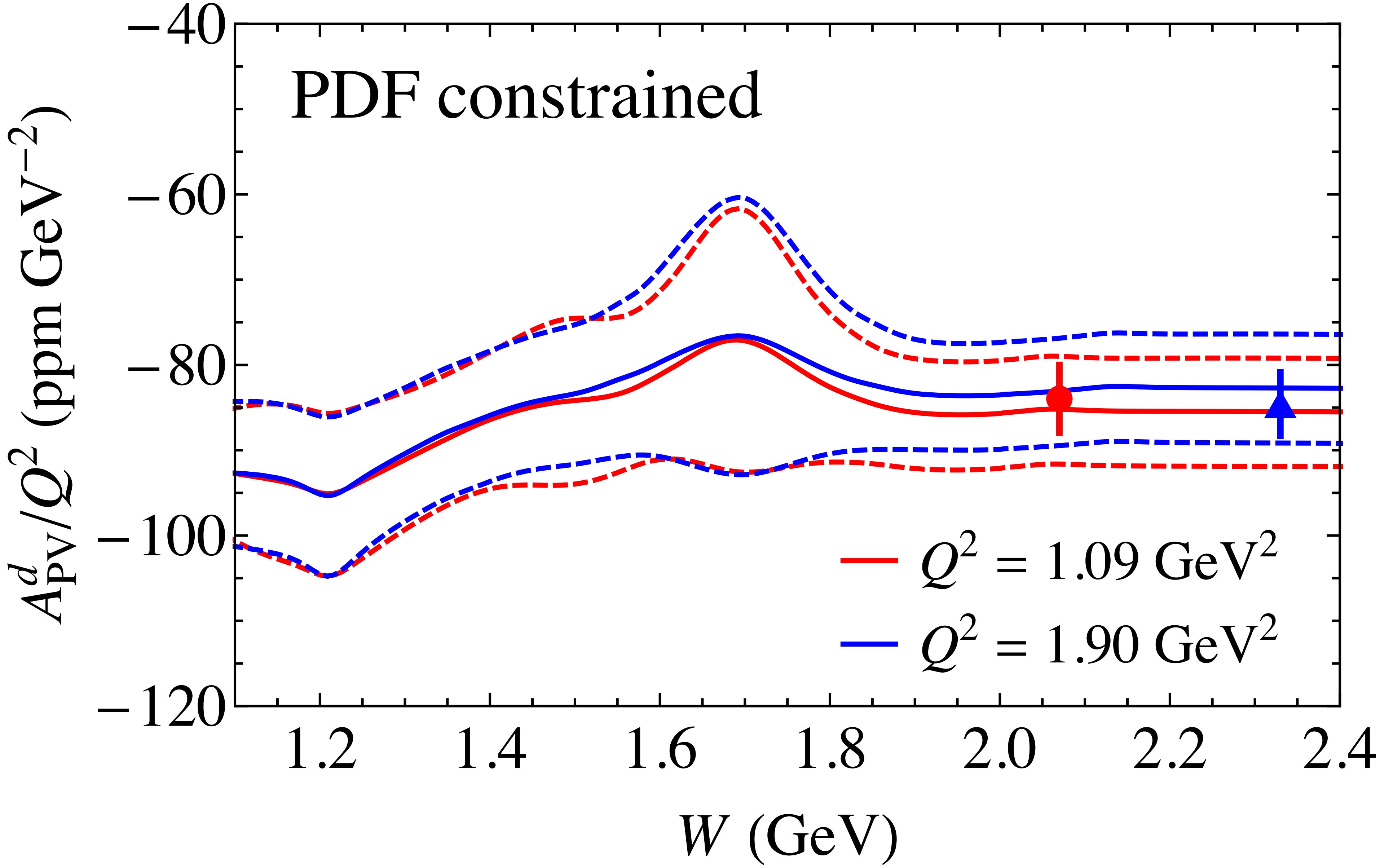


*Wang et al. PRL **111**, 082501 (2013)*

AJM model asymmetries and uncertainties for PV deuteron asymmetry constrained by fit to E08-011 data

Hall et al. (2013)

Predictions for PV deuteron asymmetry in DIS kinematics



Prediction: *Hall et al. (2013)*

$A_{\mathrm{PV}} = -92.4 \pm 6.8$ ppm

$A_{\mathrm{PV}} = -157.2 \pm 12.2$ ppm

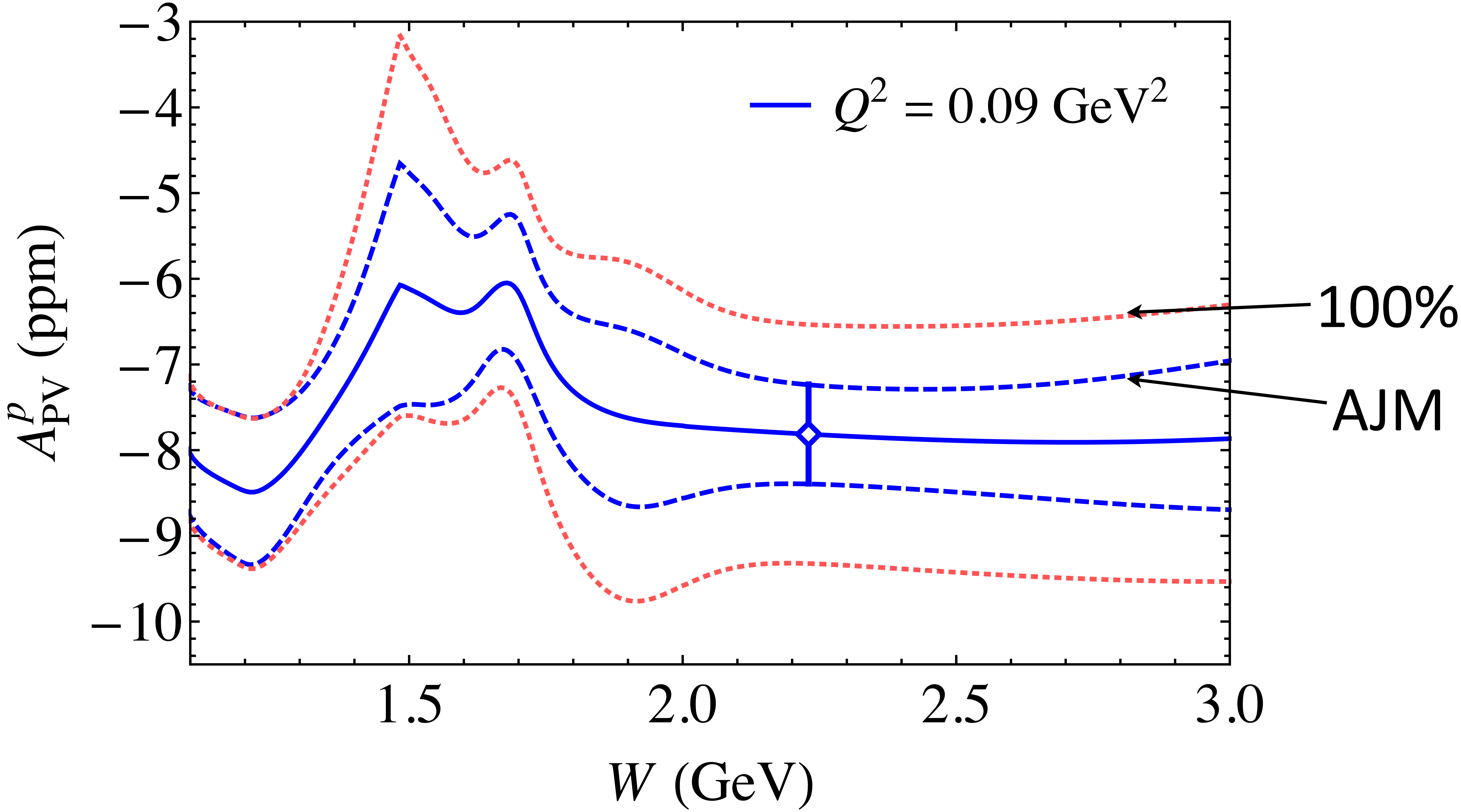
E08-011: *Wang et al. Nature* ***506****, 67 (2014)*

$A_{\mathrm{PV}} = -91.1 \pm 4.3$ ppm

$A_{\mathrm{PV}} = -160.8 \pm 7.1$ ppm

Parity-violating inelastic asymmetries

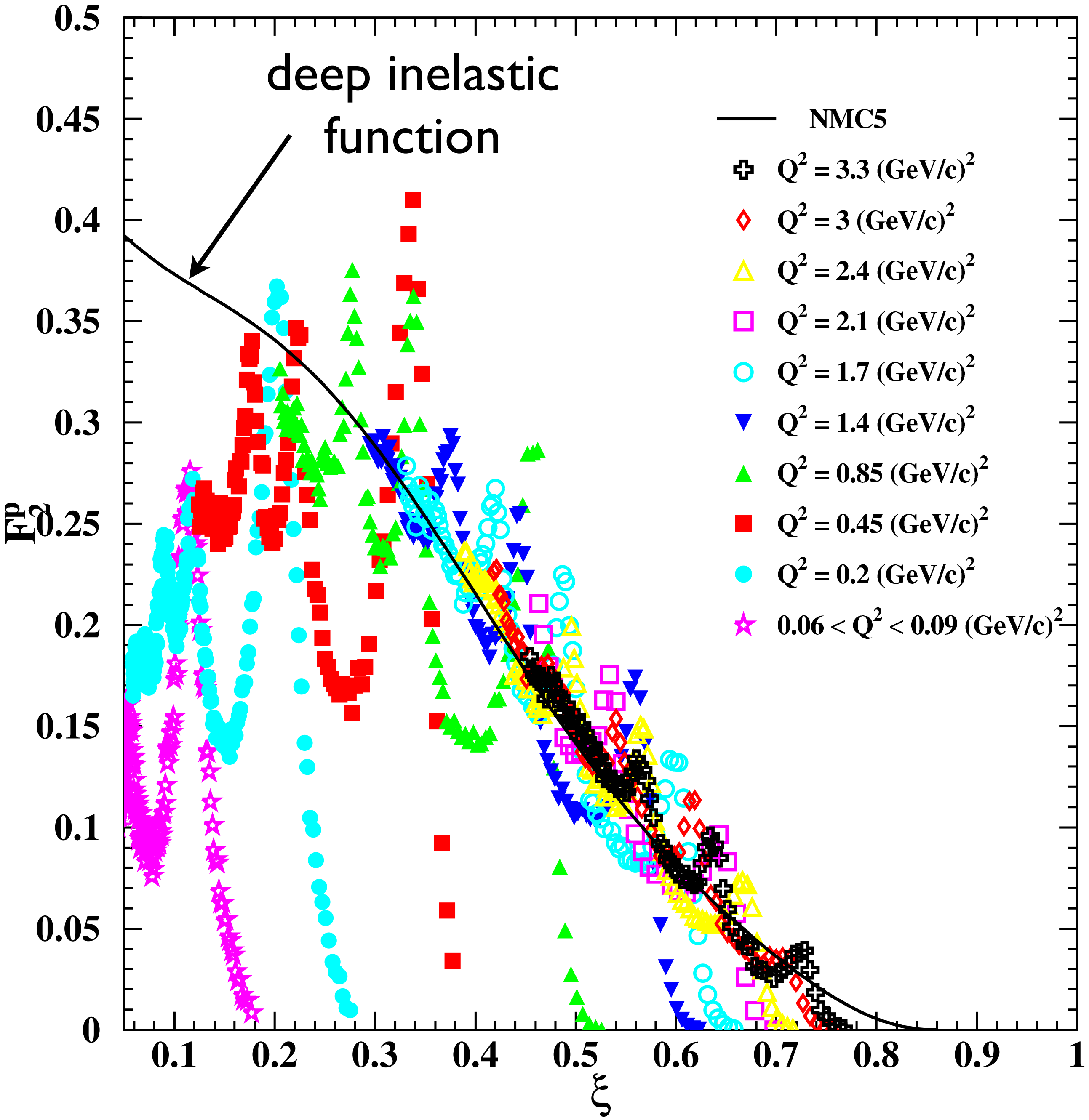
- Expected inelastic asymmetry data from Qweak



→ AJM model uncertainties compared with 100% on continuum contribution

Hall et al. (2013)

Duality in electron-nucleon scattering



Niculescu et al., PRL ***85****, 1182 (2000)*
WM, Ent, Keppel, PRep. ***406****, 127 (2005)*

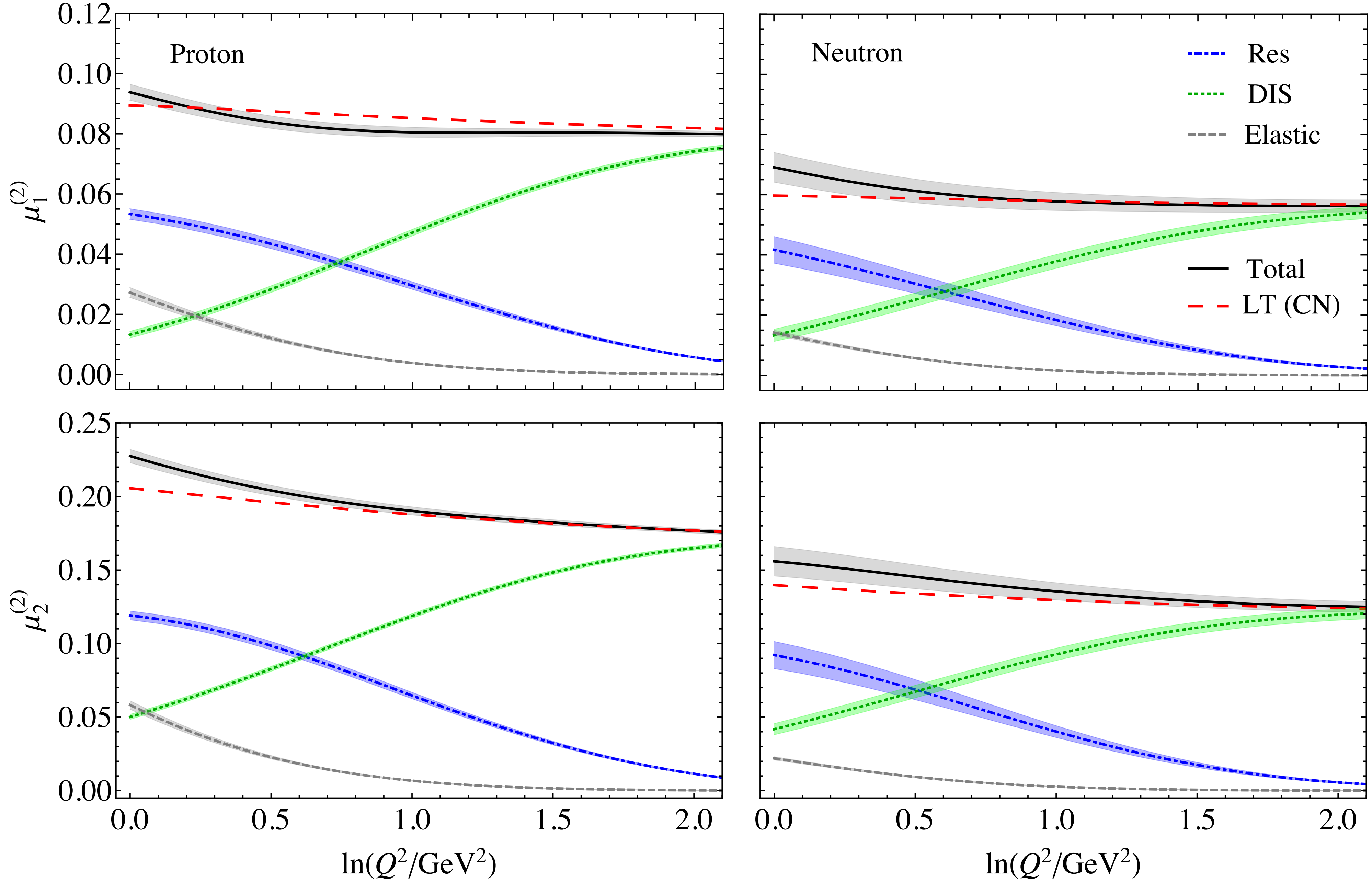
average over
(strongly Q^2 dependent)
resonances
$\approx Q^2$ independent
scaling function

"Nachtmann" scaling variable

$$\xi = \frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}}$$

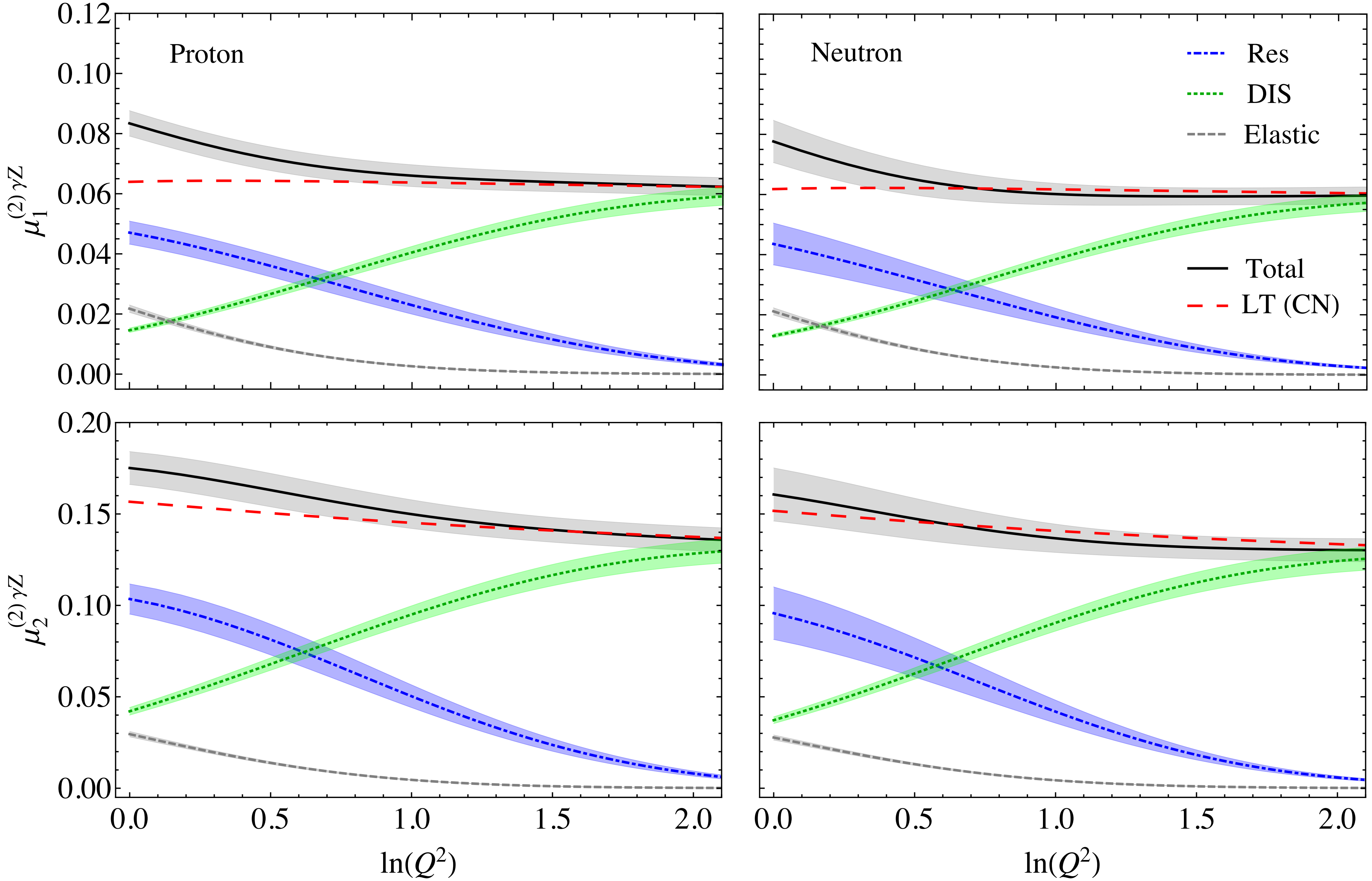
Separates higher twist (HT) effects from target mass corrections to leading twist (LT)

$\gamma\gamma$ Leading Twist (LT) $F_{1,2}$ moments vs. Nachtmann moments



- Compare total <u>empirical</u> moments of structure functions to leading twist (LT) contributions down to low Q^2
- Difference indicative of highter twist (HT) contributions
- Sum is approximately independent of Q^2
- Note isospin independence $\longrightarrow$ Apply to γZ structure functions?

γZ Leading Twist (LT) moments vs. Nachmann



Allows us to extend PDF region down to Q^2=1 GeV² (from Q^2=2.5)

$$\Box^V_{\gamma Z}@1.165\ \text{GeV}: \quad (5.6 \pm 0.4) \times 10^{-3}, \quad 2013$$
$$(5.4 \pm 0.4) \times 10^{-3}, \quad 2015$$

Summary

- Lots of interesting new theoretical work motivated by new experimental results
- Dispersive method only feasible approach for TPE, with connection to data in forward angle limit
- Efforts underway to incorporate electroproduction data throughout the resonance region, including background
- Clear need for definitive e^+p measurements at high Q^2, low ε

- Dispersion approach significant improvement over old methods
- PDF region provides constraints on model-dependence of isospin rotation
- Direct comparison with PV inelastic data in resonance and DIS regions
- e-d PVDIS asymmetry strongly constrains the uncertainty
- checking Δ region at Mainz or JLab would be useful
- quark-hadron duality approach allows further constraints on uncertainties