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Jefferson Lab

$K\pi$ form factors from a dispersive approach

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« Current and Future Status of the First-Row CKM Unitarity » Workshop

UMass Amherst, Amherst, USA., May 17 2019

*Supported by NSF

Outline :

1. Introduction and Motivation
2. Dispersive representation of the $K\pi$ form factors
3. Improvement: Combination of $\tau \to K\pi\nu_\tau$ and K_{l3} decays
4. Applications
5. Conclusion and outlook

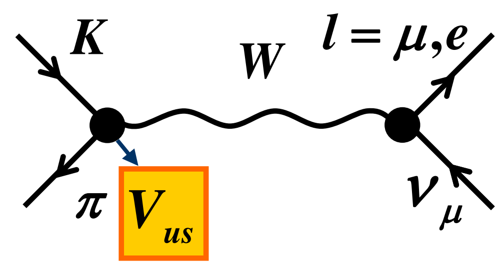
1. Introduction and Motivation

1.1 Precise test of the Standard Model

- Studying τ and K_{l3} decays $\Rightarrow$ indirect searches of new physics, several possible high-precision tests:
  - Extraction of V_{us}

$$\left(K \to \pi l \nu_l\right)$$

$$\left(l = e, \mu\right)$$



$$\boxed{\Gamma_{K^{+/0}l3} = N\left|f_+(0)\mathrm{V}_{us}\right|^2 I^l_{K^{+/0}}}$$

with

$$\boxed{I^l_{K^{+/0}} = \int dt \frac{1}{m^8_{K^{+/0}}}\lambda^{3/2} F\left(t, \overline{f}_+(t), \overline{f}_0(t)\right)}$$

$f_+(0)$ ← From Lattice QCD

$$\boxed{f_+(0)\left|\mathrm{V}_{us}\right|} \longrightarrow \boxed{\left|\mathrm{V}_{us}\right|} \Rightarrow \text{Test of unitarity} \quad \boxed{\left|V_{ud}\right|^2 + \left|V_{us}\right|^2 + \left|V_{ub}\right|^2 \stackrel{?}{=} 1}$$

$0^+ \rightarrow 0^+$ β decays

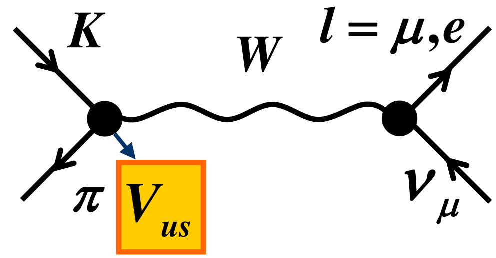
K_{l3} decays

Negligible (B decays)

1.1 Precise test of the Standard Model

- Studying τ and K_{l3} decays ⇨ indirect searches of new physics, several possible high-precision tests:
  - ➢ Extraction of V_{us}

$$\left(K \to \pi l \nu_l\right) \qquad \left(l = e, \mu\right)$$



$$\boxed{\Gamma_{K^{+/0} l3} = N \left| f_+(0) \mathbf{V}_{us} \right|^2 I^l_{K^{+/0}}}$$

with

$$\boxed{I^l_{K^{+/0}} = \int dt \frac{1}{m^8_{K^{+/0}}} \lambda^{3/2} F\left(t, \bar{f}_+(t), \bar{f}_0(t)\right)}$$

⇨ Knowledge of the two form factors:

$$\boxed{\left\langle \pi(p_\pi) \right| \bar{s}\gamma_\mu u \left| K(p_K) \right\rangle = \left[\left(p_K + p_\pi\right)_\mu - \frac{\Delta_{K\pi}}{t}\left(p_K - p_\pi\right)_\mu \right] \underset{\text{vector}}{\underset{\uparrow}{f_+(t)}} + \frac{\Delta_{K\pi}}{t}\left(p_K - p_\pi\right)_\mu \underset{\text{scalar}}{\underset{\uparrow}{f_0(t)}}}$$

$$t = q^2 = (p_\mu + p_{\nu_\mu})^2 = (p_K - p_\pi)^2, \quad \bar{f}_{0,+}(t) = \frac{f_{0,+}(t)}{f_+(0)}$$

1.2 Callan-Treiman Low Energy Theorem

Bernard, Oertel, E.P., Stern'06, '08

- Callan-Treiman theorem:

$$C = \bar{f}_0(\Delta_{K\pi}) = \frac{F_K}{F_\pi f_+(0)} + \Delta_{CT} = \underbrace{\frac{F_K |V^{us}|}{F_\pi |V^{ud}|} \frac{1}{f_+(0)|V^{us}|} |V^{ud}|}_{\text{Very precisely known from Br(Kl2/}\pi\text{l2), }\Gamma\text{(Ke3) and }|V_{ud}|} r + \Delta_{CT}$$

$$\Delta_{K\pi} = m_K^2 - m_\pi^2$$

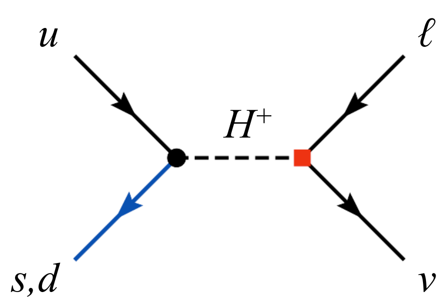
$$\Rightarrow \boxed{B_{\exp} = 1.2446(41)}$$

- In the Standard Model : $\boxed{r = 1}$ $\left(\ln C_{SM} = 0.2141(73)\right)$ $\boxed{\Delta_{CT} = (-3.5 \pm 8).10^{-3}}$

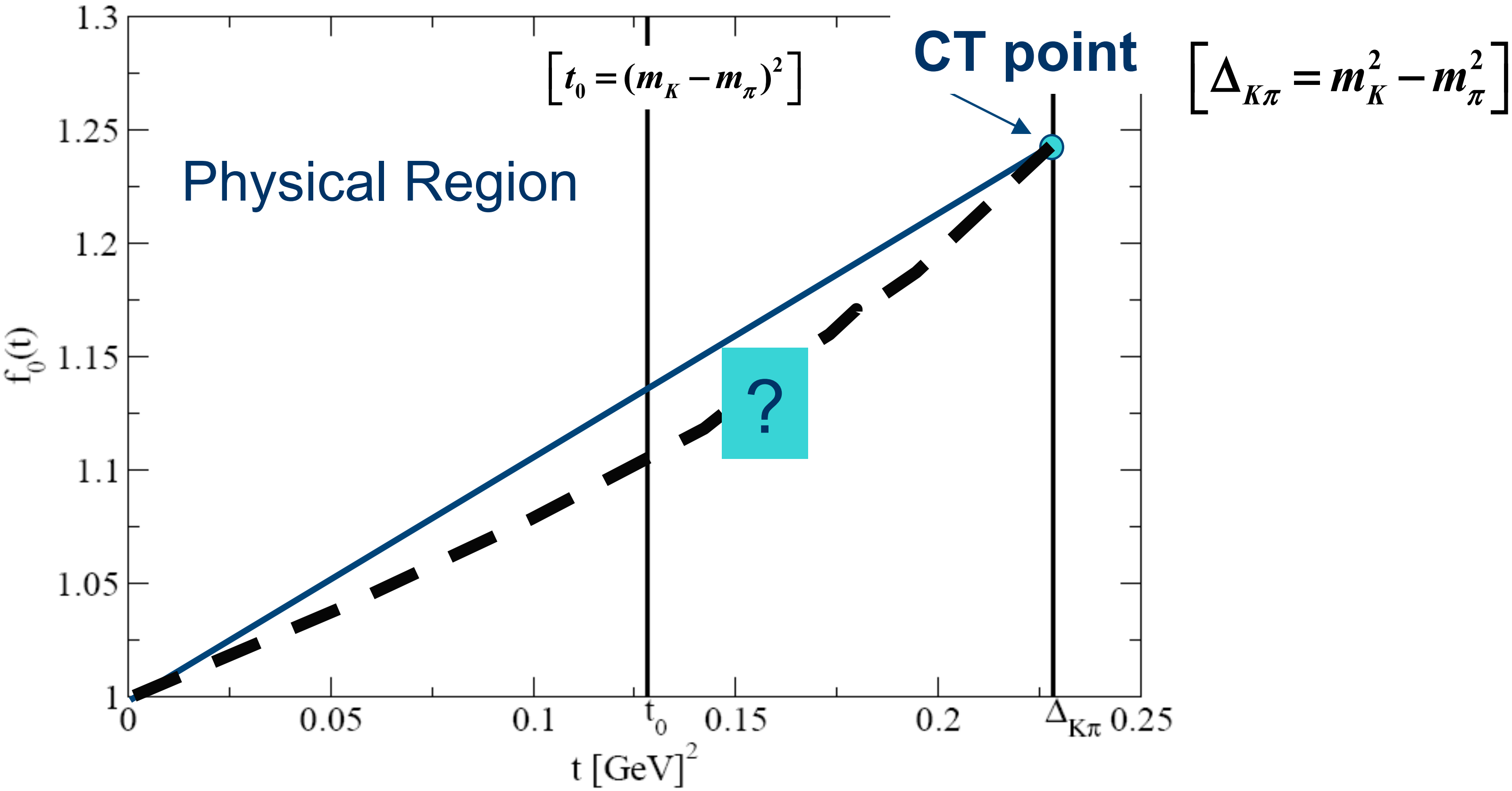
NLO value + large error bars in agreement with *Bijnens&Ghorbani'07* *Kastner & Neufeld'08*

- In presence of new physics, new couplings : $\boxed{r \neq 1}$

- Ex:

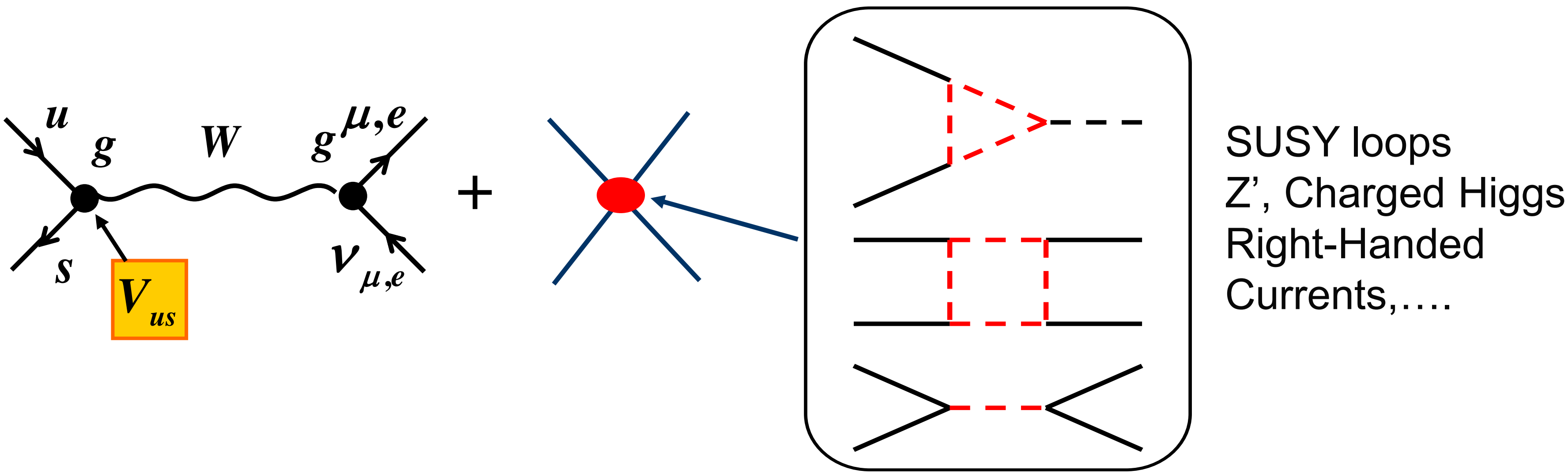


1.2 Callan-Treiman Low Energy Theorem



1.3 Test of New Physics

➡ Test of New Physics :



[E.g. Bernard et al'06,'07, Deschamps et al'09, Cirigliano et al'10, Jung et al'10, Buras et al'10…]

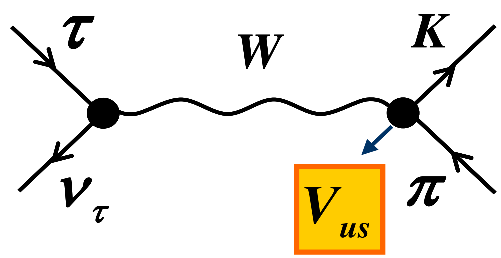
1.4 Kπ form factors

- $\bar{f}_+(t)$ accessible in K_{e3} and $K_{\mu 3}$ decays

- $\bar{f}_0(t)$ only accessible in $K_{\mu 3}$ (suppressed by m_l^2/M_K^2) + correlations
  ⇨ difficult to measure

- Data from *Belle* and *BaBar* on $\tau \to K\pi\nu_\tau$ decays
  ⇨ Use them to constrain the form factors and especially $\bar{f}_0$

- $\tau \to K\pi\nu_\tau$ decays



Hadronic matrix element: Crossed channel

$$\boxed{\langle \mathbf{K}\pi |\ \bar{\mathrm{s}}\gamma_\mu \mathbf{u}\ | 0 \rangle = \left[\left(p_K - p_\pi \right)_\mu + \frac{\Delta_{K\pi}}{s} \left(p_K + p_\pi \right)_\mu \right] f_+(s) - \frac{\Delta_{K\pi}}{s} \left(p_K + p_\pi \right)_\mu f_0(s)}$$

vector　　　scalar

with $s = q^2 = (p_K + p_\pi)^2$

1.5 Parametrization of the Kπ form factors

- $$\langle \pi(p_\pi) | \, \bar{s}\gamma_\mu u \, | K(p_K) \rangle = f_+(t)(p_K + p_\pi)_\mu + f_-(t)(p_K - p_\pi)_\mu$$

$$f_s(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

Normalisation: $\bar{f}_+(t) = \frac{f_+(t)}{f_+(0)}$ and $\bar{f}_0(t) = \frac{f_s(t)}{f_+(0)}$

- Taylor Expansion: $$\bar{f}_{+,0}(t) = 1 + \lambda'_{+,0} \frac{t}{m_\pi^2} + \frac{1}{2}\lambda''_{+,0} \frac{t}{m_\pi^2} + \ldots$$

Ok for K_{l3} but can not combine with tau data and large correlations

λ'_0	1	-0.9996	-0.97	0.91
λ''_0		1	0.98	-0.92
λ'_+			1	-0.98
λ''_+				1

[Franzini, Kaon'08]

⇒ Only slope accessible for the scalar FF

1.5 Parametrization of the Kπ form factors

- $$\langle \pi(p_\pi) | \, \bar{s}\gamma_\mu u \, | K(p_K) \rangle = f_+(t)(p_K + p_\pi)_\mu + f_-(t)(p_K - p_\pi)_\mu$$

$$f_s(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

Normalisation: $\bar{f}_+(t) = \dfrac{f_+(t)}{f_+(0)}$ and $\bar{f}_0(t) = \dfrac{f_s(t)}{f_+(0)}$,

- Taylor Expansion: $$\bar{f}_{+,0}(t) = 1 + \lambda'_{+,0} \frac{t}{m_\pi^2} + \frac{1}{2} \lambda''_{+,0} \frac{t}{m_\pi^2} + \ldots$$

  Ok for K_{l3} but can not combine with tau data and large correlations

- Pole parametrization:

$$\bar{f}_{+,0}(t) = \frac{m_{V,S}^2}{m_{V,S}^2 - t}$$

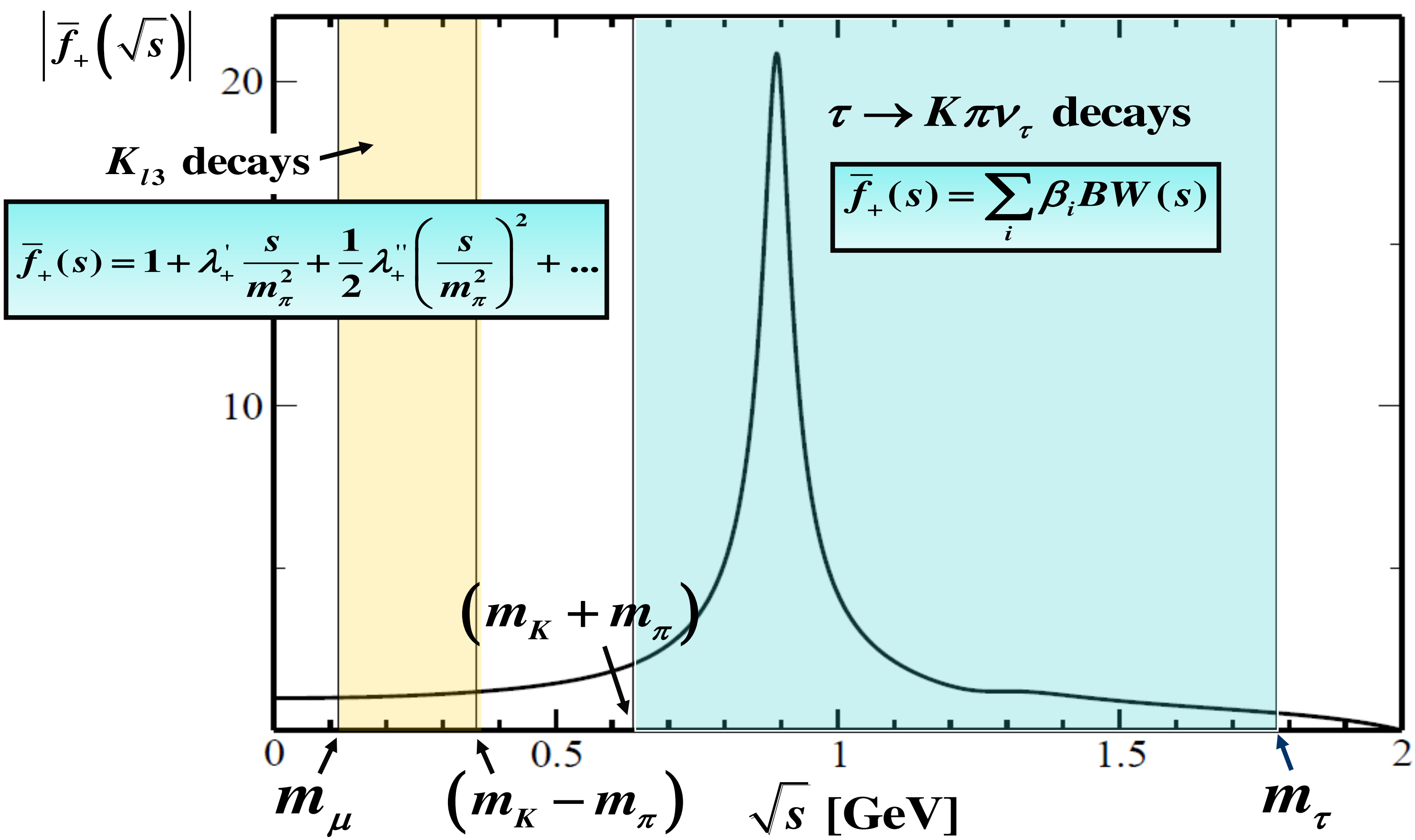
with $m_{V,S}$ to be fitted from data

Ok for K_{l3} but can not combine with tau data: will explode at the resonance mass! ⇨ Ok for vector but not so obvious for scalar

2. Dispersive representation of the Kπ form factors

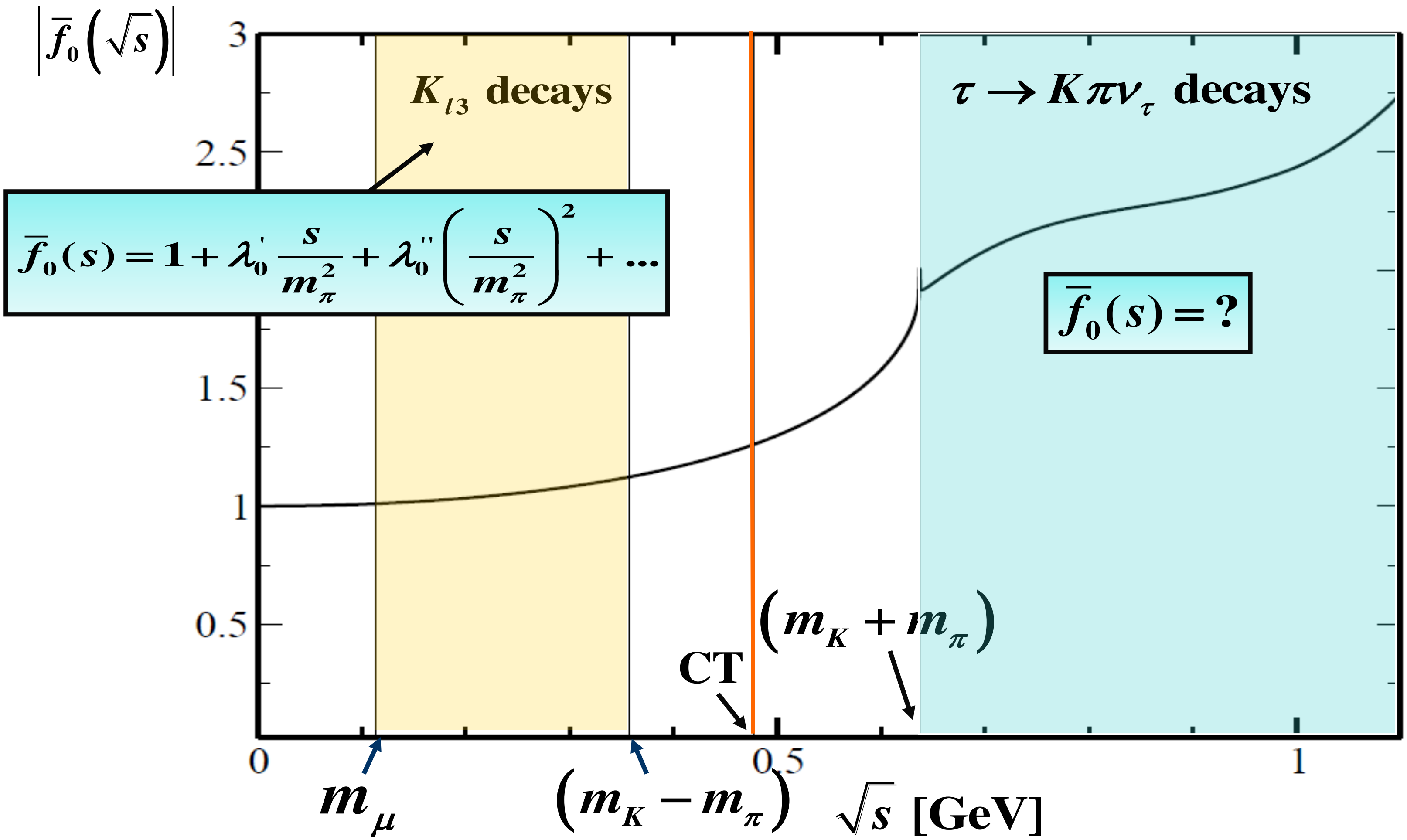
2.1 Introduction

- Parametrization to analyse both K_{l3} and τ
  - Vector form factor: ⇒ Dominance of K*(892) resonance



2.1 Introduction

- Parametrization to analyse both K_{l3} and τ
  - Scalar form factor: ⇨ No obvious dominance of a resonance



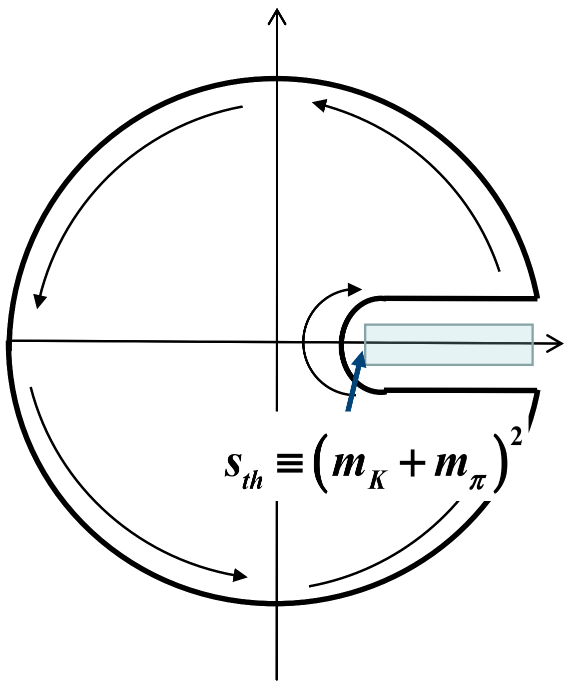
2.2 Dispersive representation for the form factors

- Parametrization to analyse both K_{l3} and $\tau \to K\pi\nu_\tau$ ⇨ Use dispersion relations

Unitarity:

$$disc\left[\bar{f}_{0,+}(s)\right] \propto t_\ell^{I*}(s)\,\bar{f}_{0,+}(s)$$

- Omnès representation: ⇨ $$\boxed{\bar{f}_{+,0}(s) = \exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\phi_{+,0}(s')}{s'-s-i\varepsilon}\right]}$$



$\phi_{+,0}(s)$: phase of the form factor

- $s < s_{in}$: $\phi_{+,0}(s) = \delta_{K\pi}(s)$

Kπ scattering phase

- $s \geq s_{in}$: $\phi_{+,0}(s)$ unknown

⇨ $\phi_{+,0}(s) = \phi_{+,0as}(s) = \pi \pm \pi \quad \left(\bar{f}_{+,0}(s) \to 1/s\right)$

Brodsky & Lepage

- Subtract dispersion relation to weaken the high energy contribution of the phase. Improve the convergence but sum rules to be satisfied!

2.2 Dispersive representation

Bernard, Oertel, E.P., Stern'06,'09

- Dispersion relation with n subtractions in $\bar{s}$:

$$\boxed{\bar{f}_{+,0}(s) = \exp\left[P_{n-1}(s) + \frac{(s-\bar{s})^n}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{(s'-\bar{s})^n}\frac{\phi_{+,0}(s')}{s'-s-i\varepsilon}\right]}$$

➢ $\bar{f}_0(s)$ ⇨ dispersion relation with 2 subtractions: 1 in s=0 and 1 in s=$\Delta_{K\pi}$

[Callan-Treiman]

$$\boxed{\bar{f}_0(s) = \exp\left[\frac{s}{\Delta_{K\pi}}\left(\ln C + \frac{(s-\Delta_{K\pi})}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{ds'}{s'}\frac{\phi_0(s')}{(s'-\Delta_{K\pi})(s'-s-i\varepsilon)}\right)\right]}$$

For s < s_{in}: Kπ scattering phase extracted from the data

Buettiker, Descotes-Genon, & Moussallam'02

1 parameter to fit to the data: $\boxed{\ln C = \ln \bar{f}(\Delta_{K\pi})}$

2.2 Dispersive representation

Bernard, Oertel, E.P., Stern'06,'09

- Dispersion relation with n subtractions in $\bar{s}$:

$$\boxed{\bar{f}_{+,0}(s) = \exp\left[P_{n-1}(s) + \frac{(s-\bar{s})^n}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{(s'-\bar{s})^n} \frac{\phi_{+,0}(s')}{s'-s-i\varepsilon} \right]}$$

➢ $\bar{f}_+(s)$ ⇨ dispersion relation with 2 subtractions in s=0

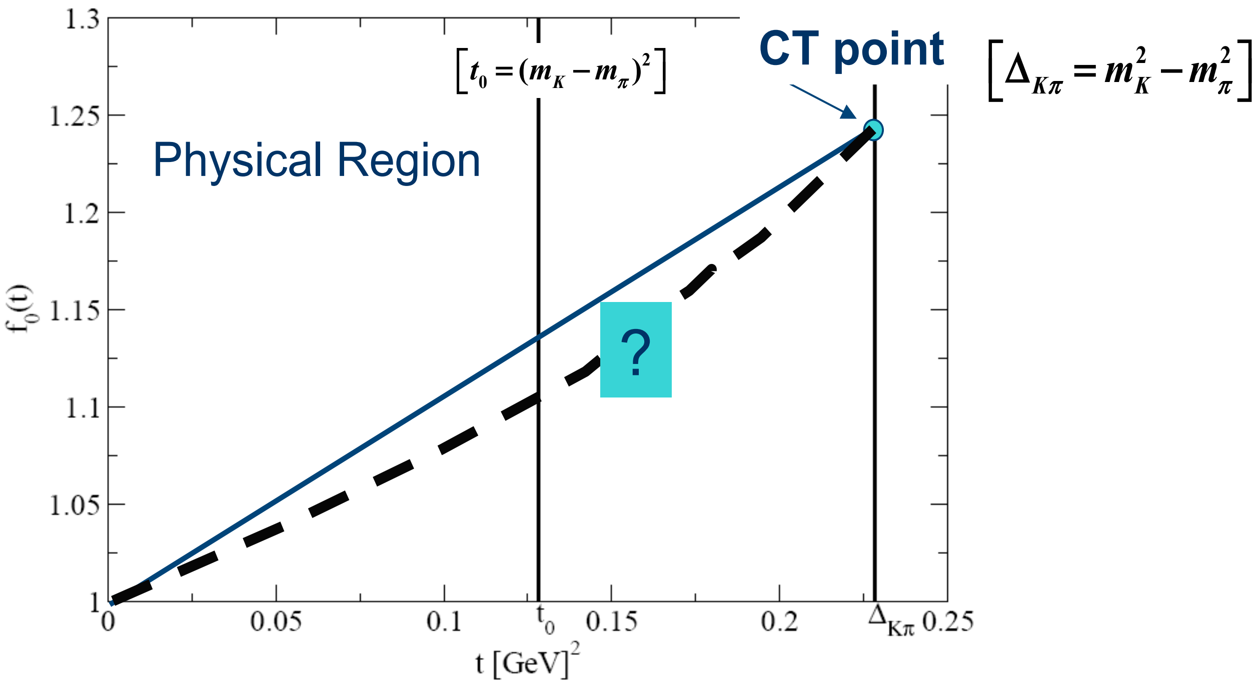
$$\boxed{\bar{f}_+(s) = \exp\left[\lambda_+^{'} \frac{s}{m_\pi^2} + \frac{s^2}{\pi} \int_{(m_K+m_\pi)^2}^{\infty} \frac{ds'}{s'^2} \frac{\phi_+(s')}{(s'-s-i\varepsilon)} \right]}$$

Extracted from a model including
1 resonances K*(892)
a la Gounaris-Sakourai

1 parameter to fit to the data: $\boxed{\lambda_+^{'}}$

2.2 Dispersive Representation

- Take the Kπ rescattering into account *Bernard, Oertel, E.P., Stern'06, '09*
- Allow to determine the slope and *curvature* of the form factors: only 2 param.



- Use the CT theorem for the scalar FF ⇨ Write a twice substracted dispersion relation for ln f(t) at t=0 and at the CT point for the scalar FF

2.3 Scalar form factor

Bernard, Oertel, E.P., Stern'06,'09

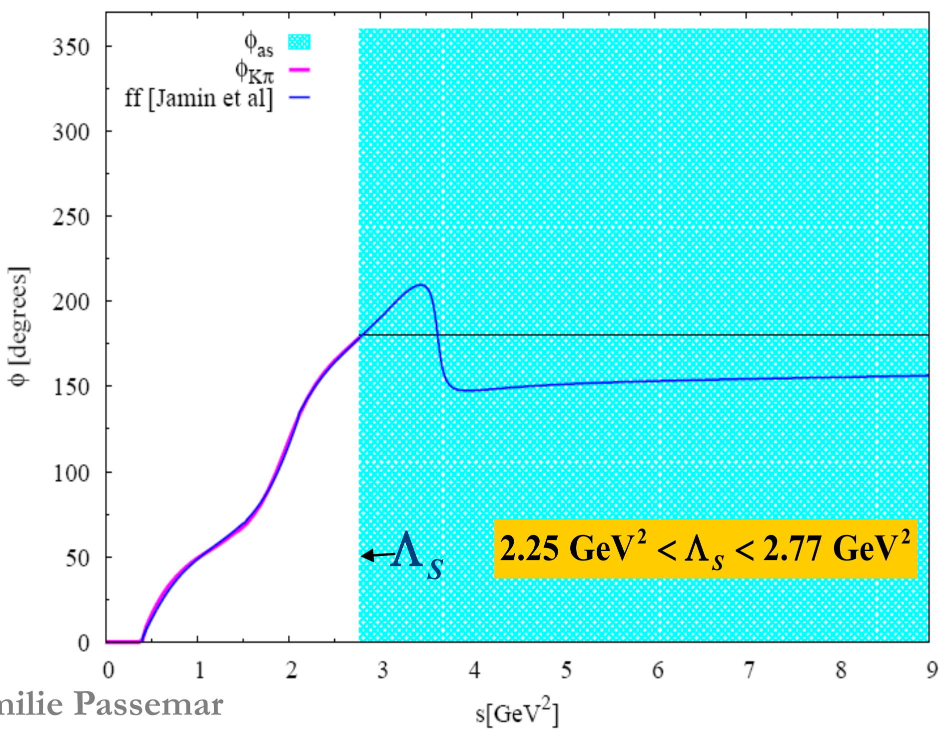
- Scalar form factor:

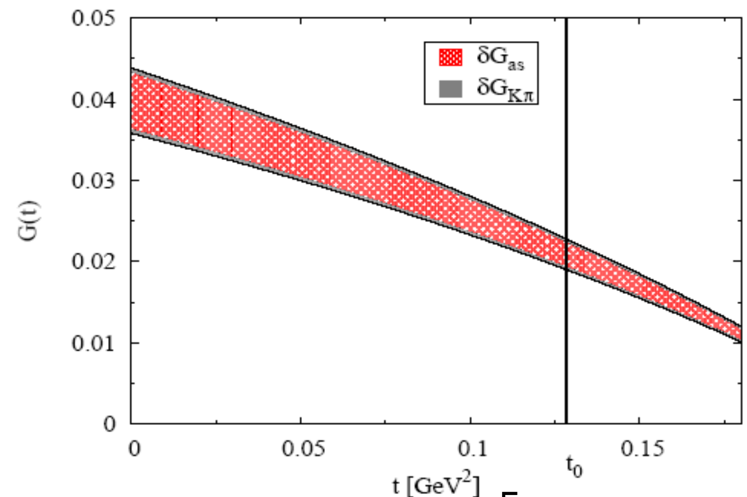
$$\bar{f}_0(t) = \exp\left[\frac{t}{\Delta_{K\pi}}\left(\ln C - G(t)\right)\right]$$

with

$$G(t) = \frac{\Delta_{K\pi}(\Delta_{K\pi} - t)}{\pi} \int_{t_{K\pi}}^{\infty} \frac{ds}{s} \frac{\phi(s)}{(s-\Delta_{K\pi})(s-t)}$$

- Phase used:





$\left[t_0 = (m_K - m_\pi)^2\right]$

Maximal value for G(t) :

$$G(0) = 0.0398 \pm 0.0040$$

does not exceed 20% of lnC ~ 0.20

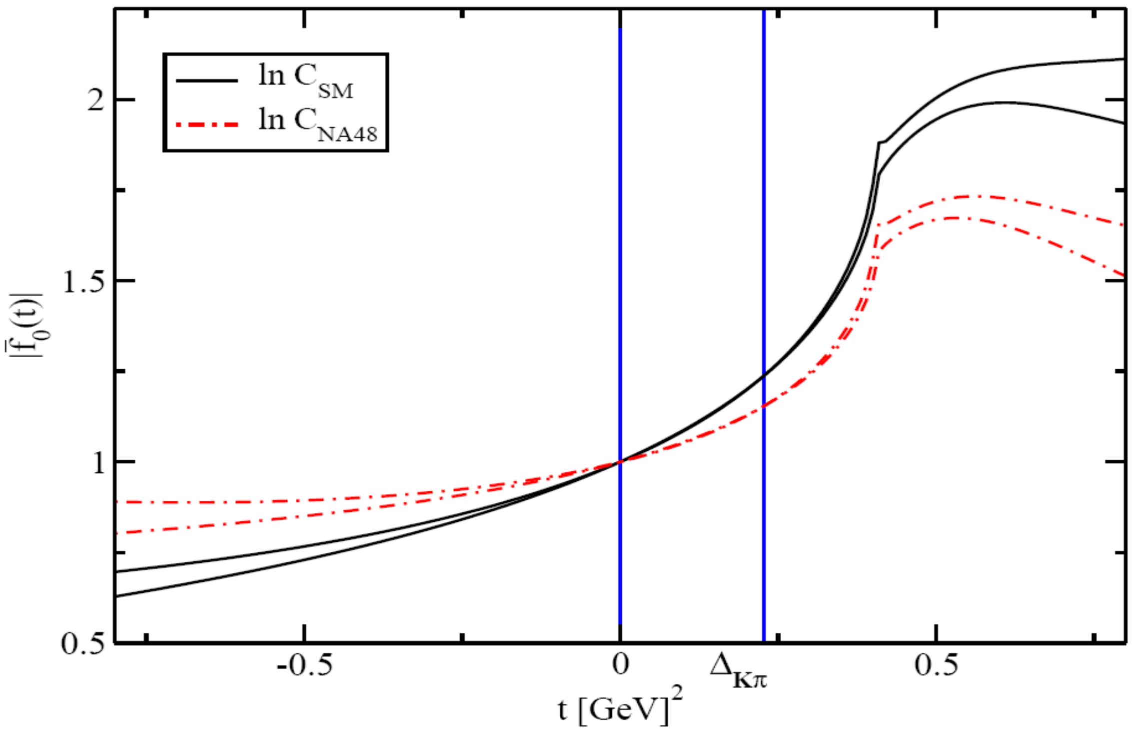
2.3 Scalar form factor

Bernard, Oertel, E.P., Stern'06,'09

- Scalar form factor:

$$\bar{f}_0(t) = \exp\left[\frac{t}{\Delta_{K\pi}}\left(\ln C - G(t)\right)\right] \quad \text{with} \quad G(t) = \frac{\Delta_{K\pi}(\Delta_{K\pi}-t)}{\pi}\int_{t_{K\pi}}^{\infty}\frac{ds}{s}\frac{\phi(s)}{(s-\Delta_{K\pi})(s-t)}$$

- Form factor:



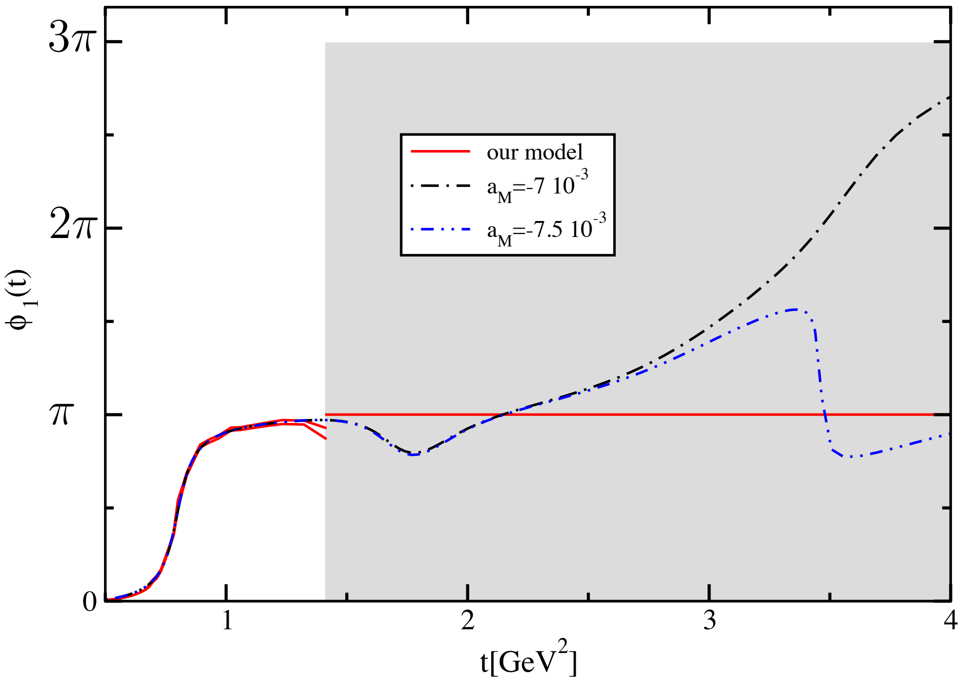
2.4 Vector form factor

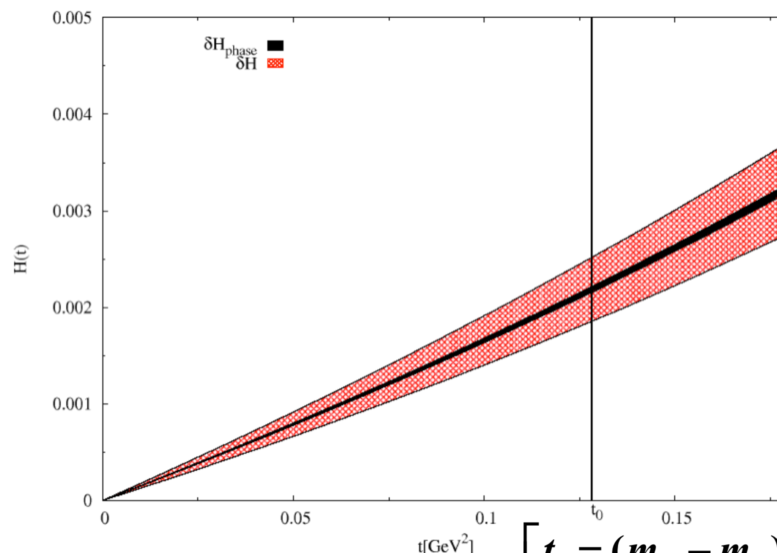
Bernard, Oertel, E.P., Stern'06,'09

- Vector form factor:

$$\bar{f}_+(t) = \exp\left[\frac{t}{m_\pi^2}\,(\Lambda_+ + H(t))\right] \quad \text{with} \quad H(t) = \frac{m_\pi^2 t}{\pi}\int_{t_{\pi K}}^{\infty}\frac{ds}{s^2}\frac{\varphi(s)}{(s-t)}$$

- Phase used:





$$\left[t_0 = (m_K - m_\pi)^2\right]$$

Maximal value for H(t) :

$H(t_0) = (2.16 +/- 0.04 +/- 0.33) \times 10^{-3}$

does not exceed 10% of $\Lambda_+ \sim 24 \times 10^{-3}$

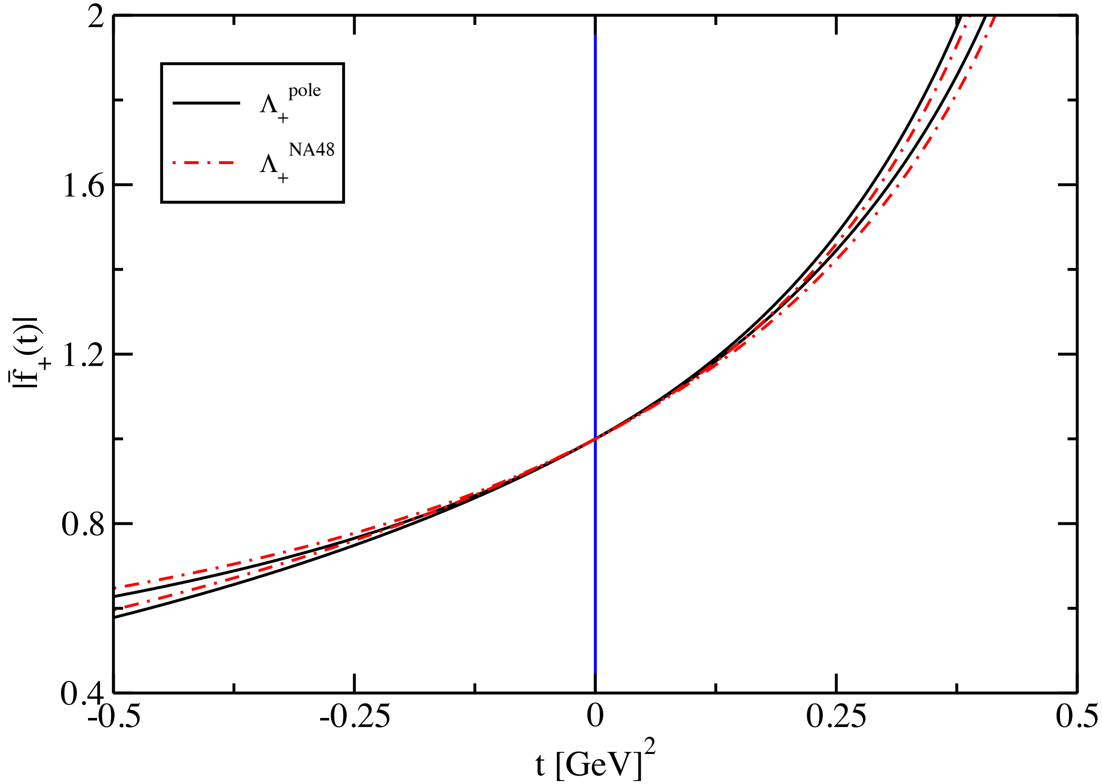
2.4 Vector form factor

Bernard, Oertel, E.P., Stern'06,'09

- Vector form factor:

$$\bar{f}_+(t) = \exp\left[\frac{t}{m_\pi^2}\left(\Lambda_+ + H(t)\right)\right]$$ with $$H(t) = \frac{m_\pi^2 t}{\pi}\int_{t_{\pi K}}^{\infty}\frac{ds}{s^2}\frac{\varphi(s)}{(s-t)}$$

- Form factor:



3. Combining K_{l3} and $\tau \rightarrow K\pi\nu_\tau$ to improve the $K\pi$ form factors determination

3.1 Kπ form factors from $\tau \to K\pi\nu_\tau$ and K_{l3} decays

- Fit to the $\tau \to K\pi\nu_\tau$ decay data
  - from *Belle [Epifanov et al'08]* (*BaBar?*)

$$N_{events} \propto N_{tot}\; b_w\; \frac{1}{\Gamma_{K\pi}} \frac{d\Gamma_{K\pi}}{d\sqrt{s}}$$

Number of events/bin

bin width

$$\Rightarrow \quad \chi^2_\tau = \sum_{bins} \left(\frac{N_{events} - N_\tau}{\sigma_{N_\tau}} \right)^2$$

with

$$\frac{d\Gamma_{K\pi}}{d\sqrt{s}} = \frac{G_F^2 m_\tau^3}{32\pi^3 s} C_K^2 S_{EW} \left| f_+(0) V_{us} \right|^2 \left(1 - \frac{s}{m_\tau^2} \right)^2 \left[\left(1 + \frac{2s}{m_\tau^2} \right) q^3_{K\pi}(s) \left| \bar{f}_+(s) \right|^2 + \frac{3\Delta^2_{K\pi}}{4s} q_{K\pi}(s) \left| \bar{f}_0(s) \right|^2 \right]$$

$$\Rightarrow \quad \frac{1}{\Gamma_{K\pi}} \frac{d\Gamma_{K\pi}}{d\sqrt{s}}$$ fit independent of V_{us}

3.2 Dispersive representation for the FFs

Bernard, Boito, E.P.'11

Bernard'14

- Dispersion relation with n subtractions in $\bar{s}$:

$$\bar{f}_{+,0}(s) = \exp\left[P_{n-1}(s) + \frac{(s-\bar{s})^n}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{(s'-\bar{s})^n}\frac{\phi_{+,0}(s')}{s'-s-i\varepsilon}\right]$$

➢ $\bar{f}_0(s)$ ⇨ dispersion relation with 3 subtractions: 2 in s=0 and 1 in s=$\Delta_{K\pi}$

Callan-Treiman

$$\bar{f}_0(s) = \exp\left[\frac{s}{\Delta_{K\pi}}\left(\ln C + (s-\Delta_{K\pi})\left(\frac{\ln C}{\Delta_{K\pi}} - \frac{\lambda_0'}{m_\pi^2}\right)\right) + \frac{\Delta_{K\pi}s(s-\Delta_{K\pi})}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{ds'}{s'^2}\frac{\phi_0(s')}{(s'-\Delta_{K\pi})(s'-s-i\varepsilon)}\right]$$

➢ $\bar{f}_+(s)$ ⇨ dispersion relation with 3 subtractions in s=0

Boito, Escribano, Jamin'09,'10

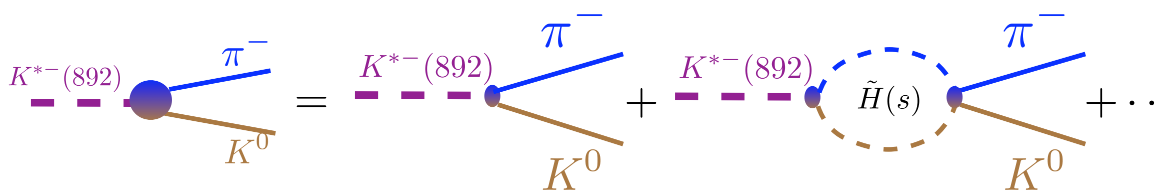
$$\bar{f}_+(s) = \exp\left[\lambda_+'\frac{s}{m_\pi^2} + \frac{1}{2}(\lambda_+'' - \lambda_+'^2)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{ds'}{s'^3}\frac{\phi_+(s')}{(s'-s-i\varepsilon)}\right]$$

Jamin, Pich, Portolés'08

Extracted from a model including
2 resonances K*(892) and K*(1414)

Modeling of the phase

- Model for $\phi_+(s)$:



$$\boxed{\tilde{f}_+(s) = \left[\frac{m_{K^*}^2 - \kappa_{K^*}\left(\mathrm{Re}\,\tilde{H}_{K\pi}(0) + \mathrm{Re}\,\tilde{H}_{K\eta}(0)\right) + \beta s}{D\left(m_{K^*}, \Gamma_{K^*}\right)} - \frac{\beta s}{D\left(m_{K^{*\prime}}, \Gamma_{K^{*\prime}}\right)}\right]}$$

$K^*(892)$ $\qquad$ $K^*(1410)$

with $\boxed{D\left(m_n, \Gamma_n\right) = m_n^2 - s - \kappa_n \sum \mathrm{Re}\,\tilde{H} - i m_n \Gamma_n(s)}$

Boito, Escribano, Jamin'09,'10

Jamin, Pich, Portolés'08

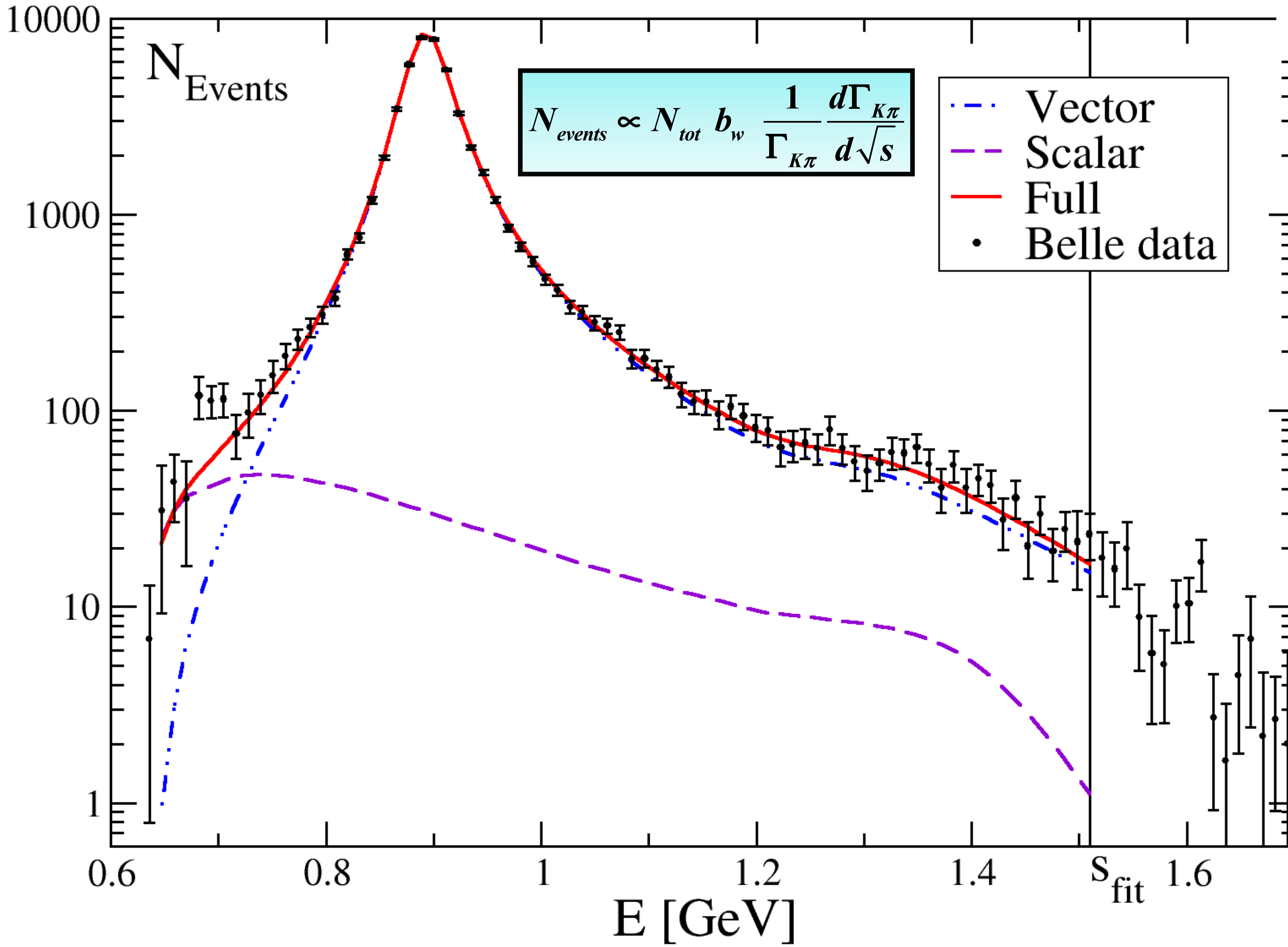
Bernard'14

$\Rightarrow$ $\boxed{\tan\delta_{K\pi}^{P,1/2} = \frac{\mathrm{Im}\,\tilde{f}_+(s)}{\mathrm{Re}\,\tilde{f}_+(s)}}$

Fit to the $\tau \to K\pi\nu_\tau$ decay data + K_{l3} constraints

Bernard, Boito, E.P.'11

Bernard'14



3.3 Determination of the form factors

Bernard, Boito, E.P.'11
Antonelli, Cirigliano Lusiani, E.P.'13

- Results of the fits:

	$\tau \to K\pi\nu_\tau$ & $K_{\ell 3}$ Belle	$\tau \to K\pi\nu_\tau$ & $K_{\ell 3}$ SuperB
$\ln C$	0.20193 ± 0.00892	0.20034 ± 0.00557
$\lambda'_0 \times 10^3$	13.139 ± 0.965	13.851 ± 0.592
m_{K^*} [MeV]	892.09 ± 0.22	892.01 ± 0.21
Γ_{K^*} [MeV]	46.287 ± 0.417	46.494 ± 0.436
$m_{K^{*\prime}}$ [MeV]	1292.5 ± 47.2	1259.8 ± 27.2
$\Gamma_{K^{*\prime}}$ [MeV]	171.64 ± 234.65	205.41 ± 10.27
β	-0.0204 ± 0.0289	-0.0350 ± 0.0229
$\lambda'_+ \times 10^3$	25.714 ± 0.332	25.655 ± 0.268
$\lambda''_+ \times 10^3$	1.1988 ± 0.0313	1.2176 ± 0.0089
$\chi^2/d.o.f$	$59.7/67$	$56.5/67$
I_K^τ	0.7655 ± 0.0416	0.7857 ± 0.0105
$f_+(0)V_{us}$	0.2134 ± 0.0061	0.21103 ± 0.0037

Very accurate determination of K*(892)!

3.4 $K\pi$ form factors from $\tau \to K\pi\nu_\tau$ and K_{l3} decays

- Precise extraction of $K\pi$ scattering phase and good determination of K*

  $\boxed{m_{K^*} = 892.02 \pm 0.21 \text{ MeV}}$ and $\boxed{\Gamma_{K^*} = 46.300 \pm 0.426 \text{ MeV}}$

  PDG : $m_{K^*} = 891.66 \pm 0.26 \text{ MeV}$ and $\Gamma_{K^*} = 50.8 \pm 0.9 \text{ MeV}$

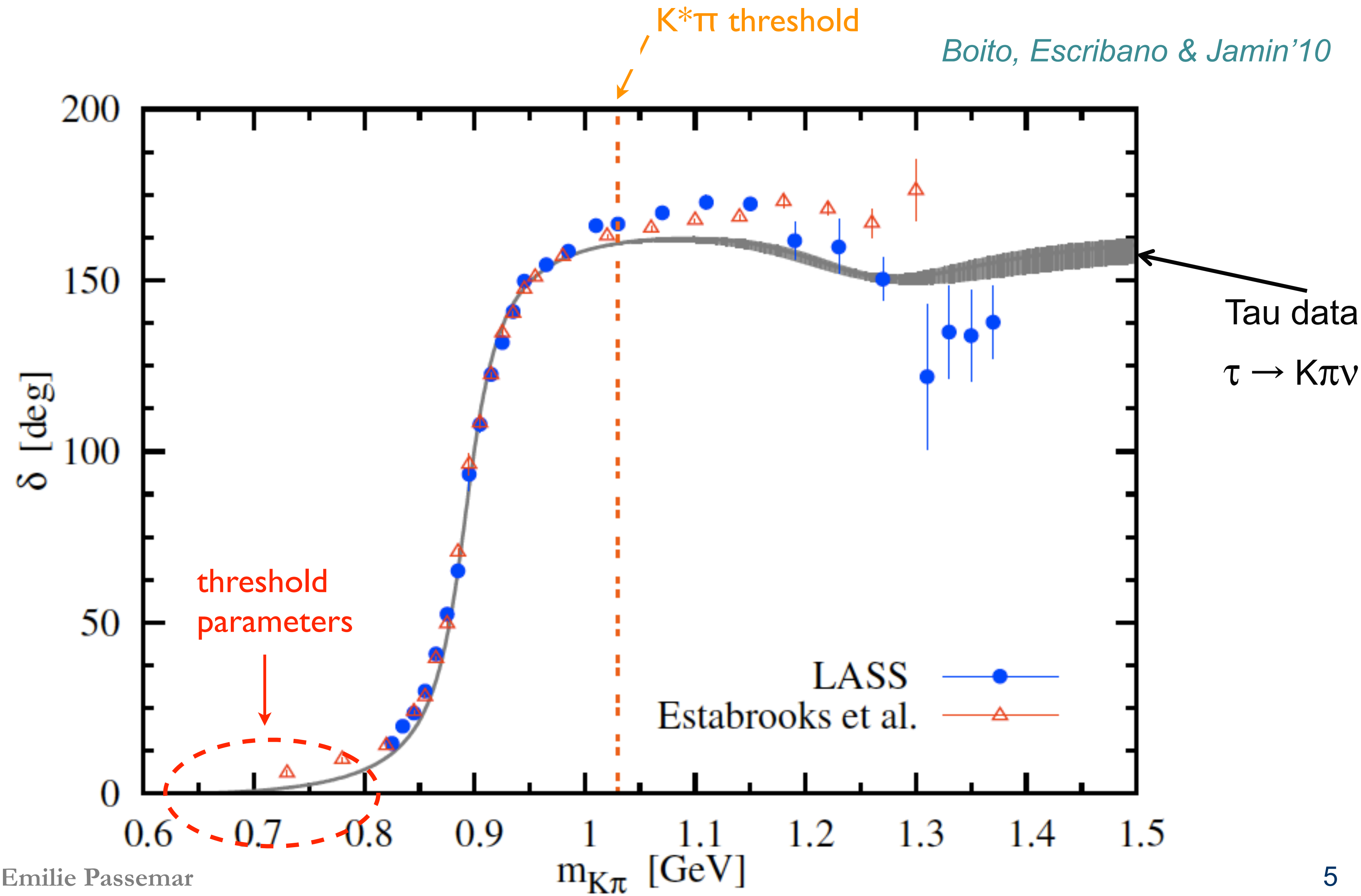
- Callan-Treiman test or lattice QCD test (F_K/F_π and $f_+(0)$)

- V_{us} from $\tau \to K\pi\nu_\tau$: $\boxed{\Gamma_{\tau\to K\pi\nu_\tau} = N \left| f_+(0) V_{us} \right|^2 I_K^\tau}$ with $\boxed{I_K^\tau = \int ds\, F\left(s, \bar{f}_+(s), \bar{f}_0(s)\right)}$

- Prediction of the strange Brs and V_{us}

- Use of the form factors for CPV tests, etc.

3.5 Kπ phase shift



4. Applications

4.1 Extraction of V_{us} from $\tau \to K\pi\nu_\tau$

- Decay rate master formula

Antonelli, Cirigliano, Lusiani, E.P.'13

$$\boxed{\Gamma\left(\tau \to \overline{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \delta_{EM}^{K\tau} + \cancel{\tilde{\delta}_{SU(2)}^{K\pi}}\right)^2}$$

$$BR\left(\tau \to \overline{K^0}\pi^-\nu_\tau\right) = (0.416 \pm 0.008)\%$$

Belle'14

$$S_{ew} = 1.0201$$

Marciano & Sirlin'88, Braaten & Li'90, Erler'04

$$\delta_{EM}^{\bar{K}^0\tau} = (-0.15 \pm 0.2)\%$$

$$I_{K^0}^\tau = 0.50432 \pm 0.01721$$

$$f_+(0) = 0.9677(27)$$

FLAG'19 $N_f = 2+1$

$$\Rightarrow \quad f_+(0)\left|V_{us}\right| = 0.2141 \pm 0.0014_{I_K} \pm 0.0021_{\rm exp} \quad \Rightarrow \quad \left|V_{us}\right| = 0.2212 \pm 0.0027$$

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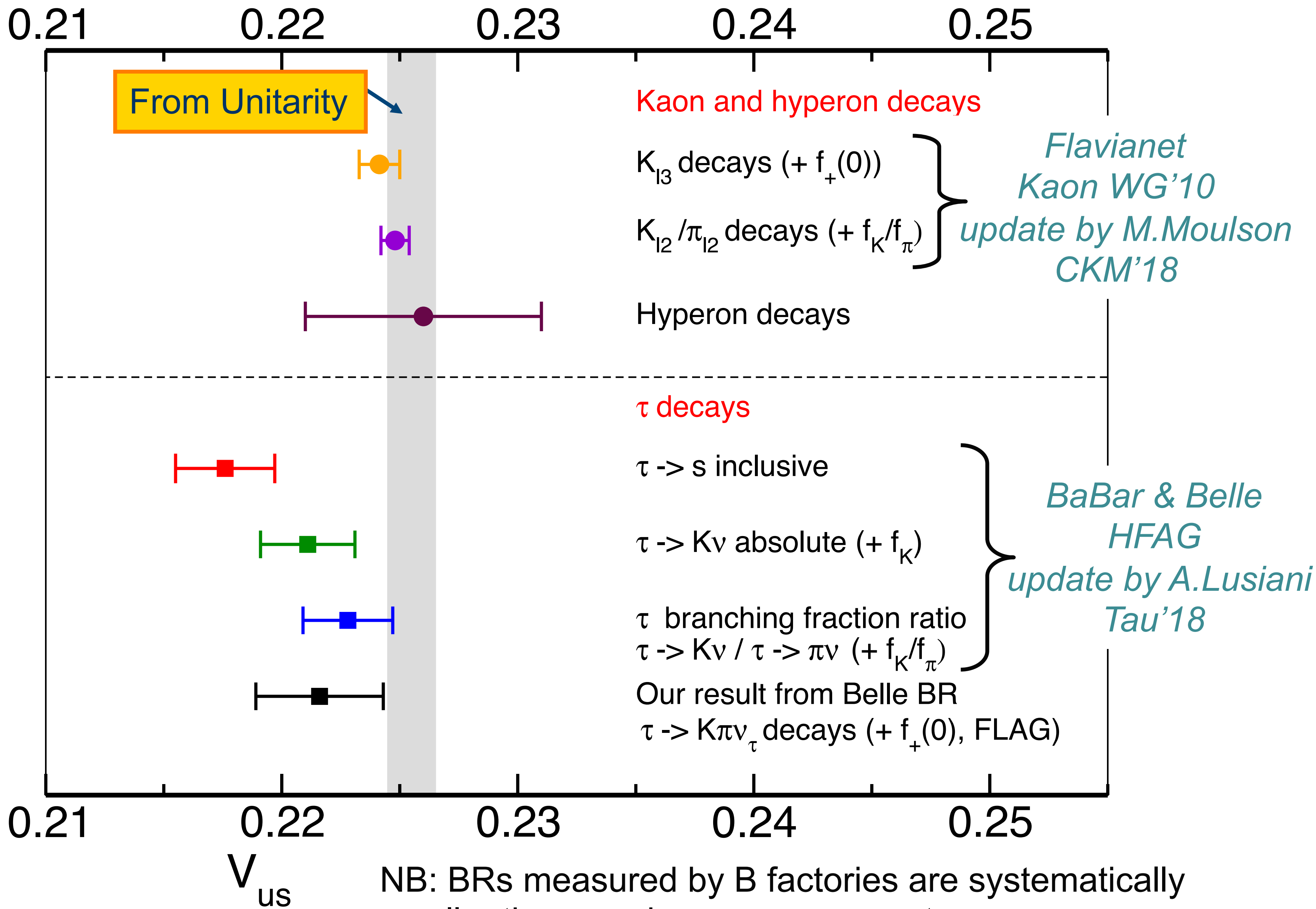
$N_f = 2+1$

$\Rightarrow$ $$\boxed{f_+(0)\left|V_{us}\right| = 0.2141 \pm 0.0014_{I_K} \pm 0.0021_{\text{exp}}}$$ $\Rightarrow$ $$\boxed{\left|V_{us}\right| = 0.2212 \pm 0.0027}$$

- Result of fit to K_{l3} + $\tau \to K\pi\nu_\tau$ and $K\pi$ scattering data including inelasticities in the dispersive FFs

$\Rightarrow$ $$\boxed{f_+(0)\left|V_{us}\right| = 0.2163 \pm 0.0014}$$

Bernard'14



NB: BRs measured by B factories are systematically smaller than previous measurements

4.2 V_{us} using info on Kaon decays and $\tau \to K\pi\nu_\tau$

- Modes measured in the strange channel for $\tau \to s$:

HFAG'12

Branching fraction	HFAG Winter 2012 fit
$\Gamma_{10} = K^- \nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$
$\Gamma_{16} = K^- \pi^0 \nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$
$\Gamma_{23} = K^- 2\pi^0 \nu_\tau$ (ex. K^0)	$(0.0630 \pm 0.0222) \cdot 10^{-2}$
$\Gamma_{28} = K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	$(0.0419 \pm 0.0218) \cdot 10^{-2}$
$\Gamma_{35} = \pi^- \overline{K}^0 \nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$
$\Gamma_{40} = \pi^- \overline{K}^0 \pi^0 \nu_\tau$	$(0.3649 \pm 0.0108) \cdot 10^{-2}$
$\Gamma_{44} = \pi^- \overline{K}^0 \pi^0 \pi^0 \nu_\tau$	$(0.0269 \pm 0.0230) \cdot 10^{-2}$
$\Gamma_{53} = \overline{K}^0 h^- h^- h^+ \nu_\tau$	$(0.0222 \pm 0.0202) \cdot 10^{-2}$
$\Gamma_{128} = K^- \eta \nu_\tau$	$(0.0153 \pm 0.0008) \cdot 10^{-2}$
$\Gamma_{130} = K^- \pi^0 \eta \nu_\tau$	$(0.0048 \pm 0.0012) \cdot 10^{-2}$
$\Gamma_{132} = \pi^- \overline{K}^0 \eta \nu_\tau$	$(0.0094 \pm 0.0015) \cdot 10^{-2}$
$\Gamma_{151} = K^- \omega \nu_\tau$	$(0.0410 \pm 0.0092) \cdot 10^{-2}$
$\Gamma_{801} = K^- \phi \nu_\tau (\phi \to KK)$	$(0.0037 \pm 0.0014) \cdot 10^{-2}$
$\Gamma_{802} = K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	$(0.2923 \pm 0.0068) \cdot 10^{-2}$
$\Gamma_{803} = K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	$(0.0411 \pm 0.0143) \cdot 10^{-2}$
$\Gamma_{110} = X_s^- \nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$

~70% of the decay modes crossed channels from Kaons!

4.2 V_{us} using info on Kaon decays and $\tau \rightarrow K\pi\nu_\tau$

Antonelli, Cirigliano, Lusiani, E.P. '13

Branching fraction	HFAG Winter 2012 fit	
$\Gamma_{10} = K^-\nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$	➡ **(0.713 ± 0.003)%**
$\Gamma_{16} = K^-\pi^0\nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$	➡ **(0.471 ± 0.018)%**
$\Gamma_{23} = K^-2\pi^0\nu_\tau$ (ex. K^0)	$(0.0630 \pm 0.0222) \cdot 10^{-2}$	
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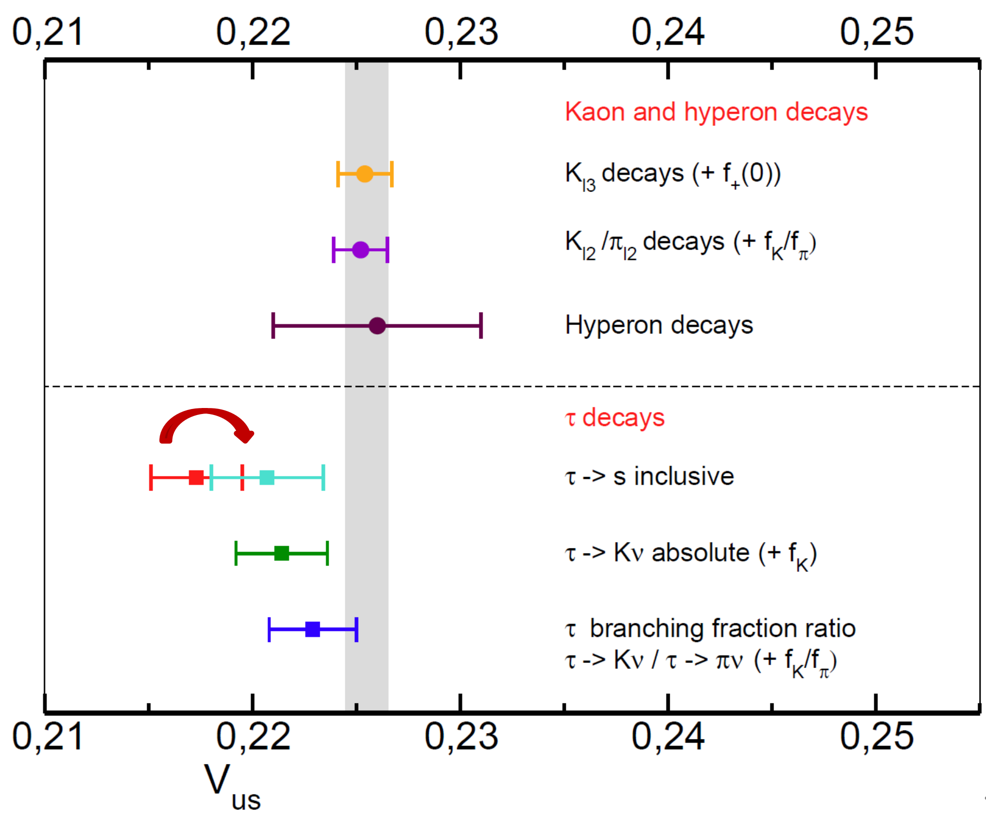
$\Gamma_{35} = \pi^-\bar{K}^0\nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$	➡ **(0.857 ± 0.030)%**
$\Gamma_{110} = X_s^-\nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$	➡ **(2.967 ± 0.060)%**

- Longstanding inconsistencies between τ and kaon decays in extraction of V_{us} seem to have been resolved !
  R. Hudspith, R. Lewis, K. Maltman, J. Zanotti'17

- Crucial input:
  $\tau \rightarrow K\pi\nu_\tau$ Br + spectrum

$$|V_{us}| = 0.2229 \pm 0.0022_{exp} \pm 0.0004_{theo}$$

➡ need new data



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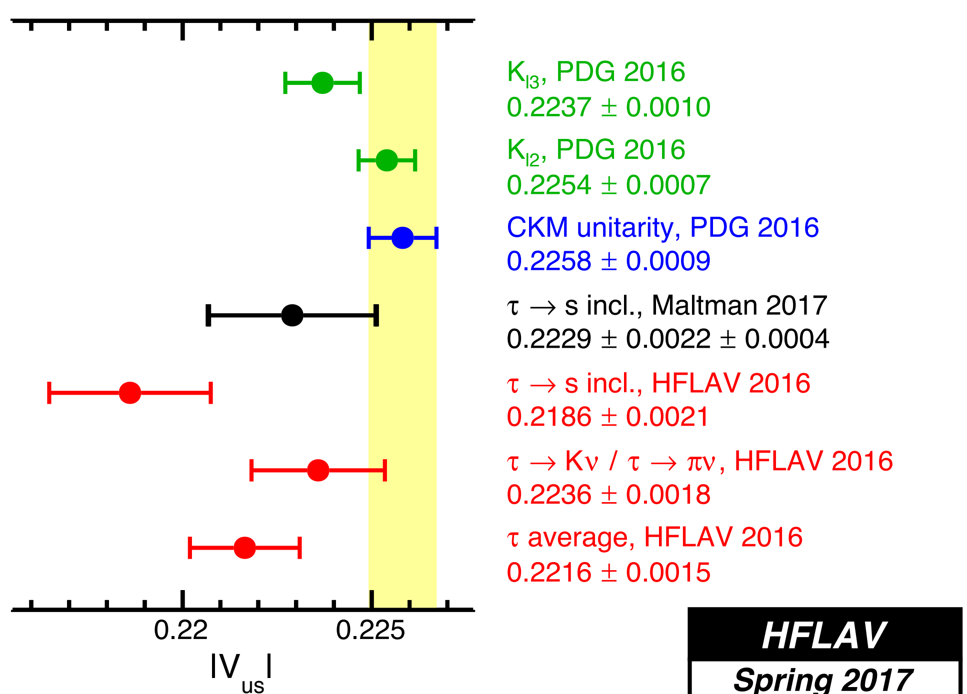

4.3 Callan-Treiman theorem and test of new physics

Bernard, Oertel, E.P., Stern'06, '08

- Callan-Treiman theorem:

$$C = \bar{f}_0(\Delta_{K\pi}) = \frac{F_K}{F_\pi f_+(0)} + \Delta_{CT} = \underbrace{\frac{F_K |V^{us}|}{F_\pi |V^{ud}|} \frac{1}{f_+(0)|V^{us}|} |V^{ud}|}_{\text{Very precisely known from Br(Kl2/}\pi\text{l2), }\Gamma\text{(Ke3) and } |V_{ud}|} r + \Delta_{CT}$$

$\Delta_{K\pi} = m_K^2 - m_\pi^2$

$\boxed{B_{\exp} = 1.2446(41)}$

- In the Standard Model : $\boxed{r = 1}$ $\quad \left(\ln C_{SM} = 0.2141(73)\right)$ $\quad \Delta_{CT} = (-3.5 \pm 8).10^{-3}$

- In presence of new physics, new couplings : $\boxed{r \neq 1}$

Experiment $K_{e3}+K_{\mu 3}$	ln C
NA48'07 **($K_{\mu 3}$ alone)**	**0.144(14)**
KLOE'08	**0.204(25)**
KTeV'10	**0.192(12)**
NA48/NA62'18	**0.184(15)**

NLO value + large error bars in agreement with

Bijnens&Ghorbani'07
Kastner & Neufeld'08

5. Conclusion and outlook

Conclusion and outlook

- $K\pi$ form factors (shape and normalization) are an important input in the determination of V_{us}

- In this talk we discussed the determination of the shape of the vector and scalar form factors using a dispersive approach
  Main input: $K\pi$ scattering phase-shifts.
  Unknown: $K\pi$ phase in the inelastic region ⇨ source of systematic uncertainty

- Possible improvement comes from combining $\tau \to K\pi\nu_\tau$ and K_{l3} decays
  ⇨ model of the phase at higher energies
  Will allow to reduce the large 2π band in the inelastic region

- It would be great to have more precise data from Tau sector

- Many possible applications:
  - V_{us} extraction from K_{l3} and $\tau \to K\pi\nu_\tau$ data
  - Callan-Treiman test of the Standard Model and New Physics

- It would be great to have lattice information on the shape of these FFs

6. Back-up