Dispersion Theory of the γW-Box For free and bound neutron β-decay

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Based on 3 papers:

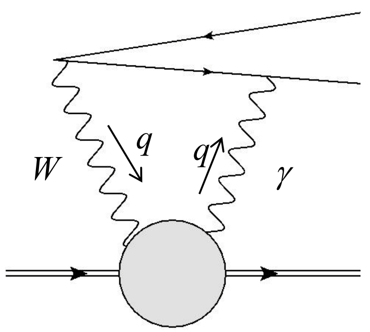
arXiv: 1807.10197
arXiv: 1812.03352
arXiv: 1812.04229

Outline

Superallowed nuclear decays:

$$|V_{ud}|^2 = \frac{2984.43s}{\mathcal{F}t(1+\Delta_R^V)}$$

$$\mathcal{F}t = ft(1+\delta'_R)[1-(\delta_C-\delta_{NS})]$$



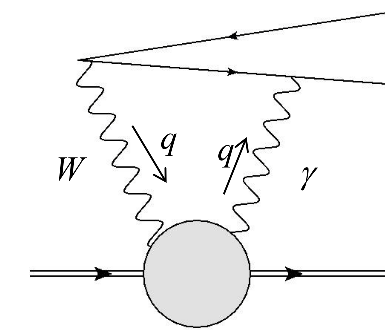
Free neutron decay:

$$|V_{ud}|^2 = \frac{5099.34s}{\tau_n(1+3\lambda^2)(1+\Delta_R)}$$

1. Dispersion formalism for the γW-box
2. Calculation of the universal free-neutron RC Δ_R^V
3. Splitting the full nuclear RC into free-neutron Δ_R^V and nuclear modification δ_{NS}
4. Splitting the full RC into “outer” and “inner”

1. γW-box from dispersion relations

γW-box



Box at zero momentum transfer* (but with energy dependence)

$$T_{\gamma W} \;=\; \sqrt{2}e^2 G_F V_{ud}\int \frac{d^4q}{(2\pi)^4}\frac{\bar{u}_e\gamma^\mu(\not{k}-\not{q}+m_e)\gamma^\nu(1-\gamma_5)v_\nu}{q^2[(k-q)^2-m_e^2]}\frac{M_W^2}{q^2-M_W^2}T^{\gamma W}_{\mu\nu}$$

***Precision goal: 10^{-4}; RC ~ $\alpha/2\pi$ ~ 10^{-3}; recoil on top - negligible**

Hadronic tensor: two-current correlator

$$T^{\mu\nu}_{\gamma W} = \int dx e^{iqx}\langle f\,|\,T[J^\mu_{em}(x)J^{\nu,\pm}_W(0)]\,|\,i\rangle$$

General gauge-invariant decomposition of a spin-independent tensor

$$T^{\mu\nu}_{\gamma W} \;=\; \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)T_1+\frac{1}{(p\cdot q)}\left(p-\frac{(p\cdot q)}{q^2}q\right)^\mu\left(p-\frac{(p\cdot q)}{q^2}q\right)^\nu T_2+\frac{i\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta}{2(p\cdot q)}T_3$$

Loop integral with generally unknown forward amplitudes

$$T_{\gamma W} = -\frac{\alpha}{2\pi}G_F V_{ud}\int\frac{d^4qM_W^2}{q^2(M_W^2-q^2)}\bar{u}_e\gamma_\beta(1-\gamma_5)u_\nu\sum_i C_i^\beta(E,\nu,q^2)T_i^{\gamma W}(\nu,q^2)$$

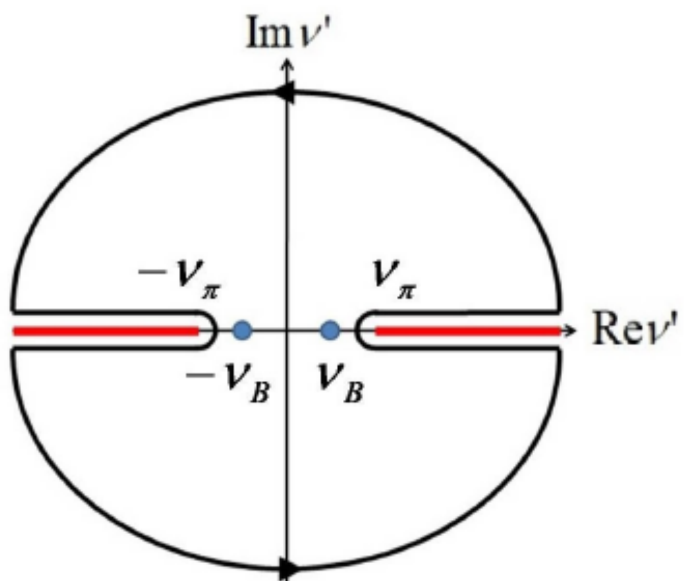
$$p^\mu = (M,\vec{0})$$

$$E = (pk)/M$$

$$\nu = (pq)/M$$

Known algebraic functions of external energy E and loop variables ν, q^2

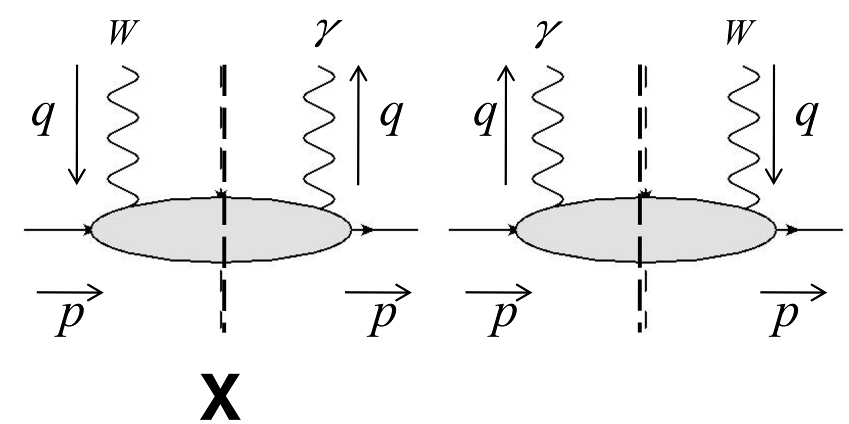
γW-box from Dispersion Relations



$T_{1,2,3}$ - analytic functions inside the contour C in the complex ν-plane determined by their singularities on the real axis - poles + cuts

$$T_i^{\gamma W}(\nu, Q^2) = \frac{1}{2\pi i} \oint dz \frac{T_i^{\gamma W}(z, Q^2)}{z - \nu}, \ \ \nu \in C$$

Forward amplitudes T_i - unknown;
Their absorptive parts can be related to production of on-shell intermediate states
—> a γW-analog of structure functions $F_{1,2,3}$

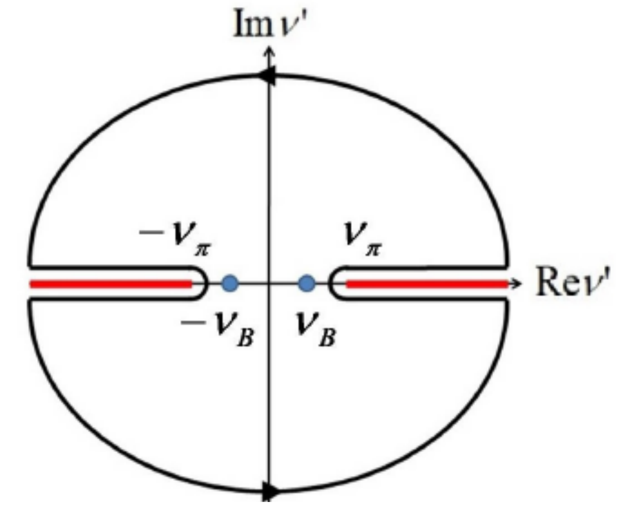


X = inclusive strongly-interacting on-shell physical states

Structure functions $F_i^{\gamma W}$ are NOT data
But they can be related to data

$$\mathrm{Im}\, T_i^{\gamma W}(\nu, Q^2) = 2\pi F_i^{\gamma W}(\nu, Q^2)$$

γW-box from Dispersion Relations



Crossing behavior: relate the left and right hand cut
Mismatch between the initial and final states - asymmetric;
Symmetrize - γ is a mix of I=0 and I=1

$$T_i^{\gamma W,a} = T_i^{(0)}\tau^a + T_i^{(-)}\frac{1}{2}[\tau^3,\tau^a]$$

$$T_i^{(I)}(-\nu,Q^2) = \xi_i^{(I)} T_i^{(I)}(\nu,Q^2)$$

$$\xi_1^{(0)} = +1,\ \ \xi_{2,3}^{(0)} = -1; \quad \xi_i^{(-)} = -\xi_i^{(0)}$$

Two types of dispersion relations for scalar amplitudes

$$T_i^{(I)}(\nu,Q^2) = 2\int_0^\infty d\nu'\left[\frac{1}{\nu'-\nu-i\epsilon} + \frac{\xi_i^{(I)}}{\nu'-\nu-i\epsilon}\right] F_i^{(I)}(\nu',Q^2)$$

Substitute into the loop and calculate leading energy dependence

$$\mathrm{Re}\,\Box_{\gamma W}^{even} = \frac{\alpha}{\pi M N}\int_0^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2}\int_0^\infty \frac{d\nu}{\nu}\frac{\nu+2q}{(\nu+q)^2}F_3^{(0)}(\nu,Q^2) + O(E^2)$$

$$\mathrm{Re}\,\Box_{\gamma W}^{odd}(E) = \frac{8\alpha E}{3\pi N M}\int\limits_0^\infty dQ^2 \int\limits_{\nu_{thr}}^\infty \frac{d\nu}{(\nu+q)^3}\left[\mp F_1^{(0)} \mp \left(\frac{3\nu(\nu+q)}{2Q^2}+1\right)\frac{M}{\nu}F_2^{(0)} + \frac{\nu+3q}{4\nu}F_3^{(-)}\right] + O(E^3)$$

$$q = \sqrt{\nu^2+Q^2}$$

2. Universal inner RC Δ_R^V

Inner universal RC from DR

γW-box at zero energy
$$\mathrm{Re}\,\Box_{\gamma W}^{even} = \frac{\alpha}{\pi M}\int_0^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2}\int_0^\infty \frac{d\nu}{\nu}\frac{\nu+2q}{(\nu+q)^2}F_3^{(0)}(\nu,Q^2)$$

$$\mathrm{Re}\,\Box_{\gamma W}^{odd}(E=0)=0$$

Connection to MS: rewrite in terms of the first Nachtmann moment of F_3

$$M_3^{(0)}(1,Q^2) = \frac{4}{3}\int_0^1 dx \frac{1+2\sqrt{1+4M^2x^2/Q^2}}{(1+\sqrt{1+4M^2x^2/Q^2})^2}F_3^{(0)}(x,Q^2) \qquad x=\frac{Q^2}{2M\nu}$$

$$\mathrm{Re}\,\Box_{\gamma W}^{even} = \frac{3\alpha}{2\pi}\int_0^\infty \frac{dQ^2 M_W^2}{Q^2(M_W^2+Q^2)}M_3^{(0)}(1,Q^2) = \frac{\alpha}{8\pi}\int_0^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2}F_{MS}(Q^2)$$

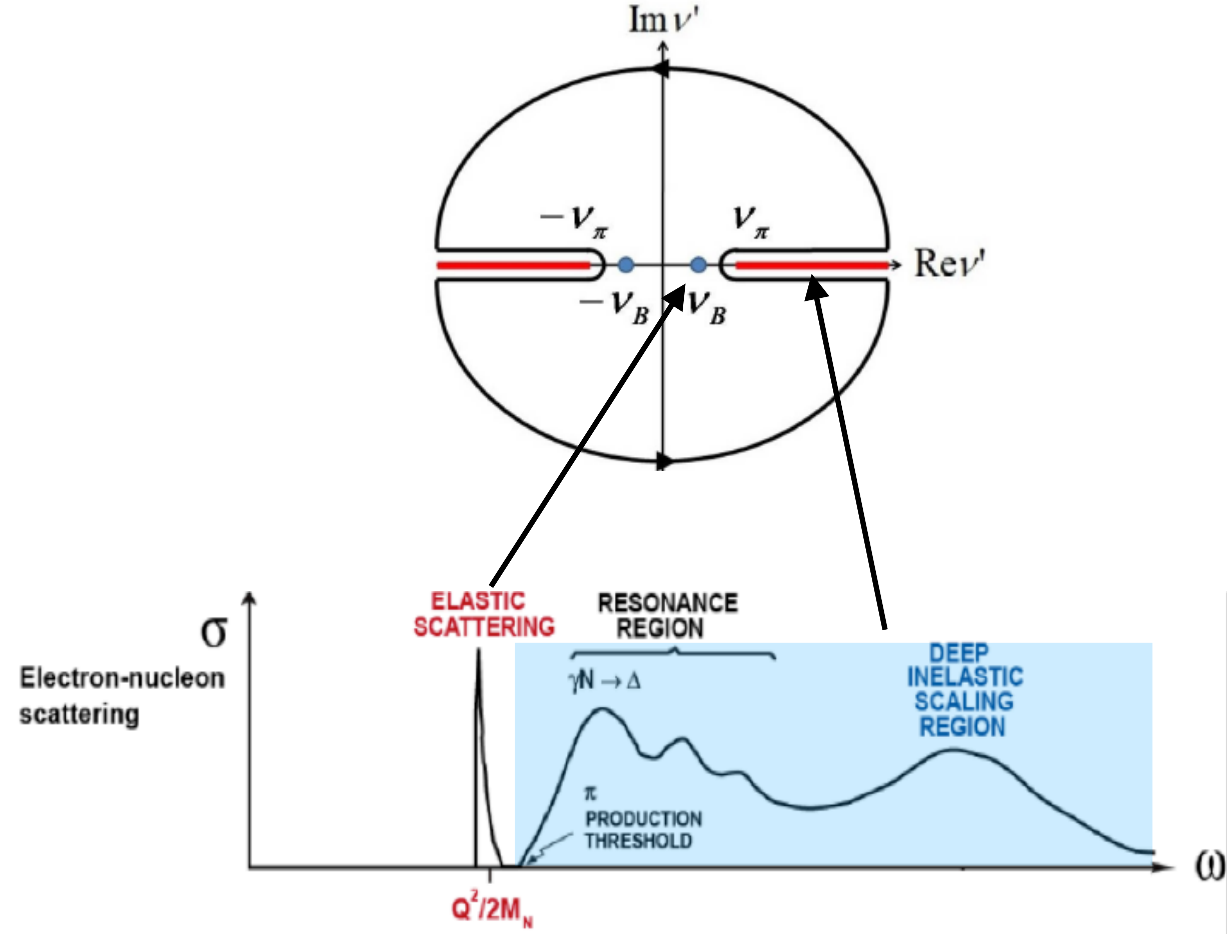
MS loop fn. $F(Q^2)$ directly related to $M_3^{(0)}$
$$F_{MS}(Q^2)=\frac{12}{Q^2}M_3^{(0)}(1,Q^2)$$

SF F_3 - commutator of em and weak currents - insert complete set of on-shell hadronic states

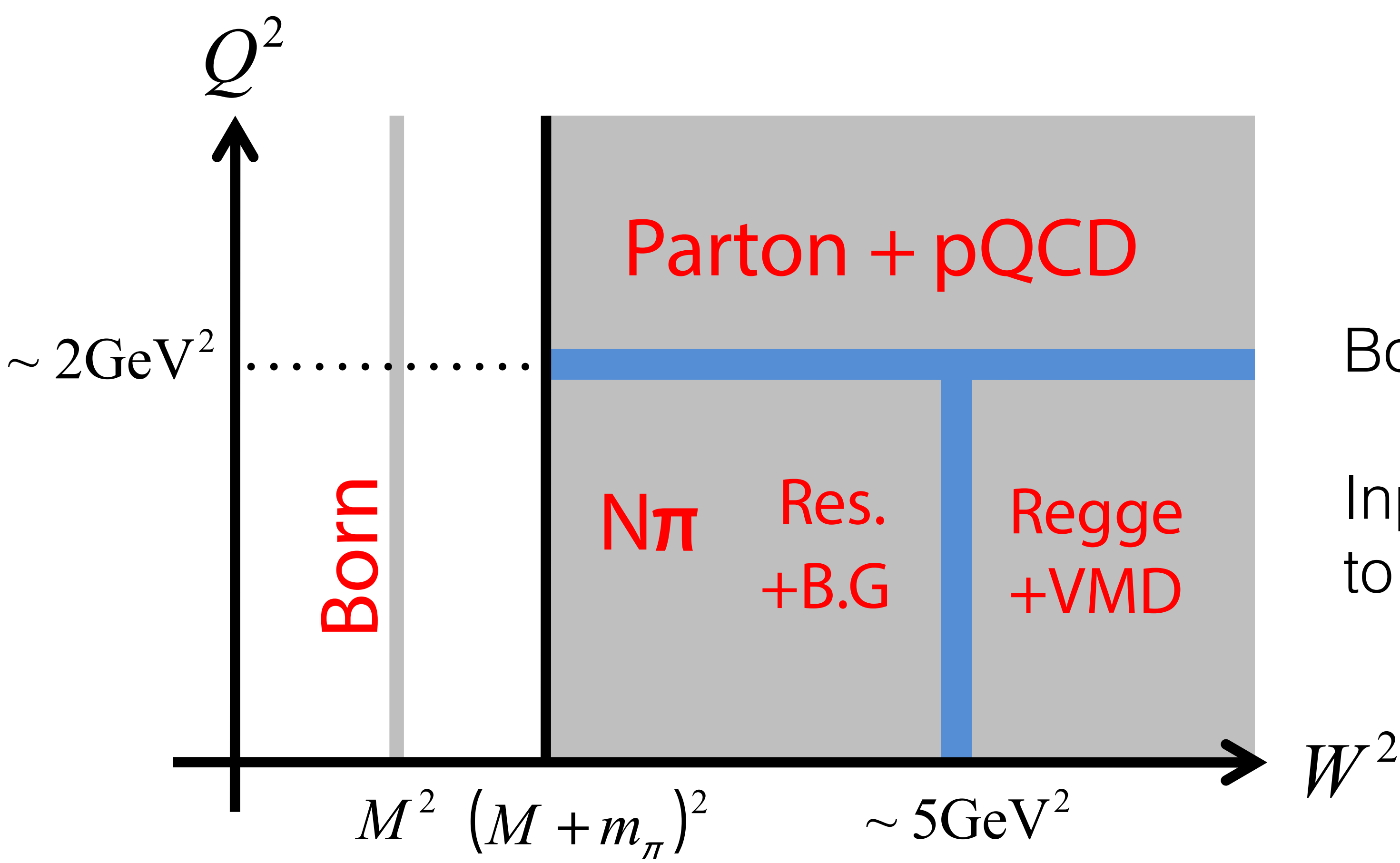
$$F_3^{(0)} \propto \int dx e^{iqx}\langle p|[J_{em}^{\mu,(0)}(x),J_W^{\nu,+}(0)]|n\rangle \sim \int dx e^{iqx}\sum_X \langle p|J_{em}^{\mu,(0)}(x)|X\rangle\langle X|J_W^{\nu,+}(0)|n\rangle$$

Input into dispersion integral

Dispersion in energy: $W^2 = M^2 + 2M\nu - Q^2$
scanning hadronic intermediate states



Dispersion in Q²:
scanning dominant physics pictures

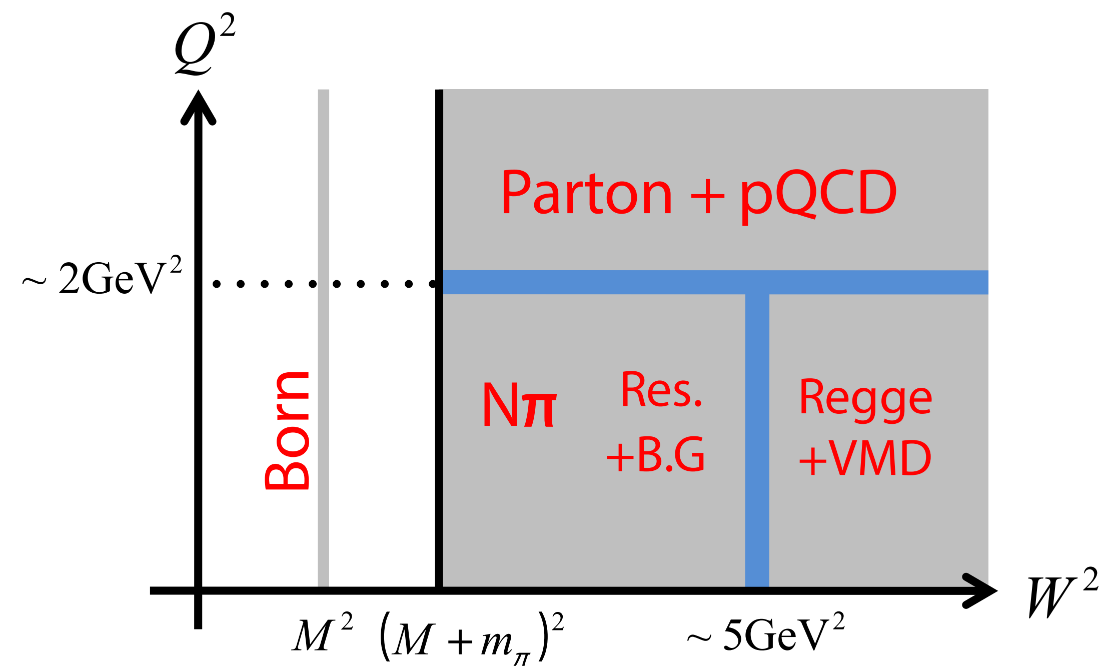


Boundaries between regions - approximate

Input to DR related (directly or indirectly) to experimentally accessible data

Input into dispersion integral

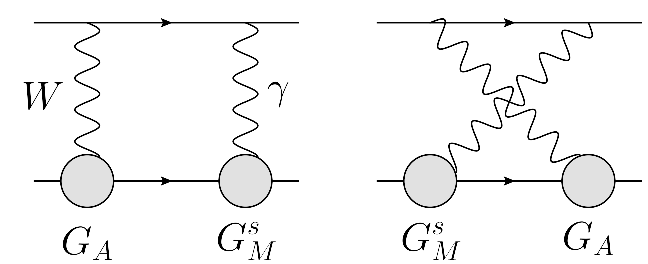
Our parametrization of the needed SF follows from this diagram



$$F_3^{(0)} = F_{\mathrm{Born}} + \begin{cases} F_{\mathrm{pQCD}}, & Q^2 \gtrsim 2\ \mathrm{GeV}^2 \\ F_{\pi N} + F_{\mathrm{res}} + F_{\mathbb{R}}, & Q^2 \lesssim 2\ \mathrm{GeV}^2 \end{cases}$$

Born: elastic FF from e$^-$, ν scattering data

$$\Box_{\gamma W}^{VA,\mathrm{Born}} = -\frac{\alpha}{\pi}\int_0^\infty dQ \frac{2\sqrt{4M^2+Q^2}+Q}{\left(\sqrt{4M^2+Q^2}+Q\right)^2} G_A(Q^2) G_M^S(Q^2)$$



πN:

relativistic ChPT calculation plus nucleon FF

Resonances:

axial excitation from PCAC (Lalakulich et al 2006) - neutrino scattering

isoscalar photo-excitation from MAID and PDG - electron and γ inelastic scattering

Above resonance region:

multiparticle continuum economically described by Regge exchanges

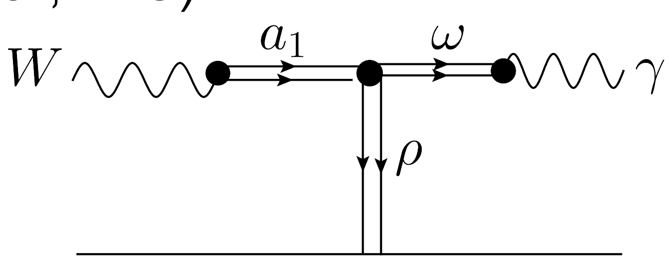
Inelastic states - low Q^2, high W

Scattering at high energy can be very effectively described by Regge exchanges

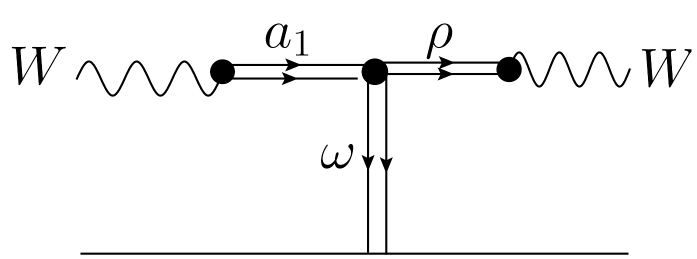
$$F_3^{(0),\mathrm{Regge}}(\nu, Q^2) = C_R(Q^2)\left(\frac{\nu}{\nu_0}\right)^{\alpha_\rho}$$

Regge behavior in EW processes: hadron-like behavior of HE electroweak probes - Vector/Axial Vector Dominance is the proper language

γW-box: conversion of $W^\pm$ (charged, I=1, axial) to γ (neutral, vector, I=0)
requires charged vector exchange w. I=1 - $\rho^\pm$
effective a_1 - ρ - ω vertex



Inclusive ν scattering: conversion of $W^\pm$ (charged, I=1, axial) to $W^\pm$ (charged, I=1, axial)
requires neutral vector exchange w. I=0 - ω
effective a_1 - ω - ρ vertex



Minimal model for both reactions - check with data.

VM propagators $1/(M_a^2+Q^2)/(M_\rho^2+Q^2) \sim 1/Q^4$, more natural for hadronic amplitudes

Compare to Bill's $F(Q^2) \sim 1/Q^2$ at high-Q^2

Input into dispersion integral

Unfortunately, no data can be obtained for $F_3^{\gamma W\,(0)}$

Data exist for the pure CC processes

$$\frac{d^2\sigma^{\nu(\bar\nu)}}{dxdy} = \frac{G_F^2 ME}{\pi}\left[xy^2F_1 + \left(1 - y - \frac{Mxy}{2E}\right)F_2 \pm x\left(y - \frac{y^2}{2}\right)F_3\right]$$

$$\sigma^{\nu p} - \sigma^{\bar\nu p} \sim F_3^{\nu p} + F_3^{\bar\nu p} = u_v^p(x) + d_v^p(x)$$

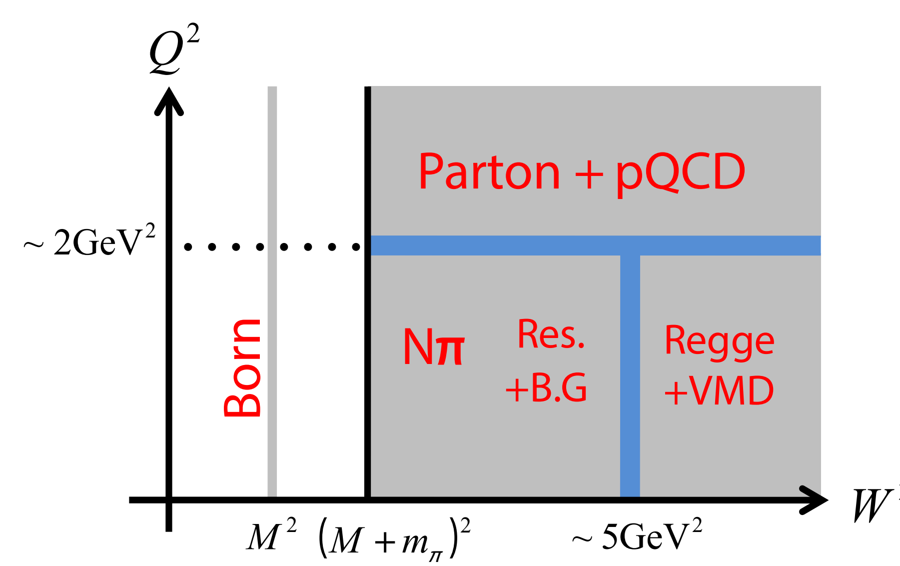
Gross-Llewellyn-Smith sum rule $\quad \int_0^1 dx(u_v^p(x) + d_v^p(x)) = 3$

Validate the model for CC process; apply an isospin rotation to obtain γW

$$F^{\nu p+\bar\nu p}_{3,\,\mathrm{low}-Q^2} = F^{\nu p+\bar\nu p}_{3,\,el.} + F^{\nu p+\bar\nu p}_{3,\,\pi N} + F^{\nu p+\bar\nu p}_{3,\,R} + F^{\nu p+\bar\nu p}_{3,\,\mathrm{Regge}}$$

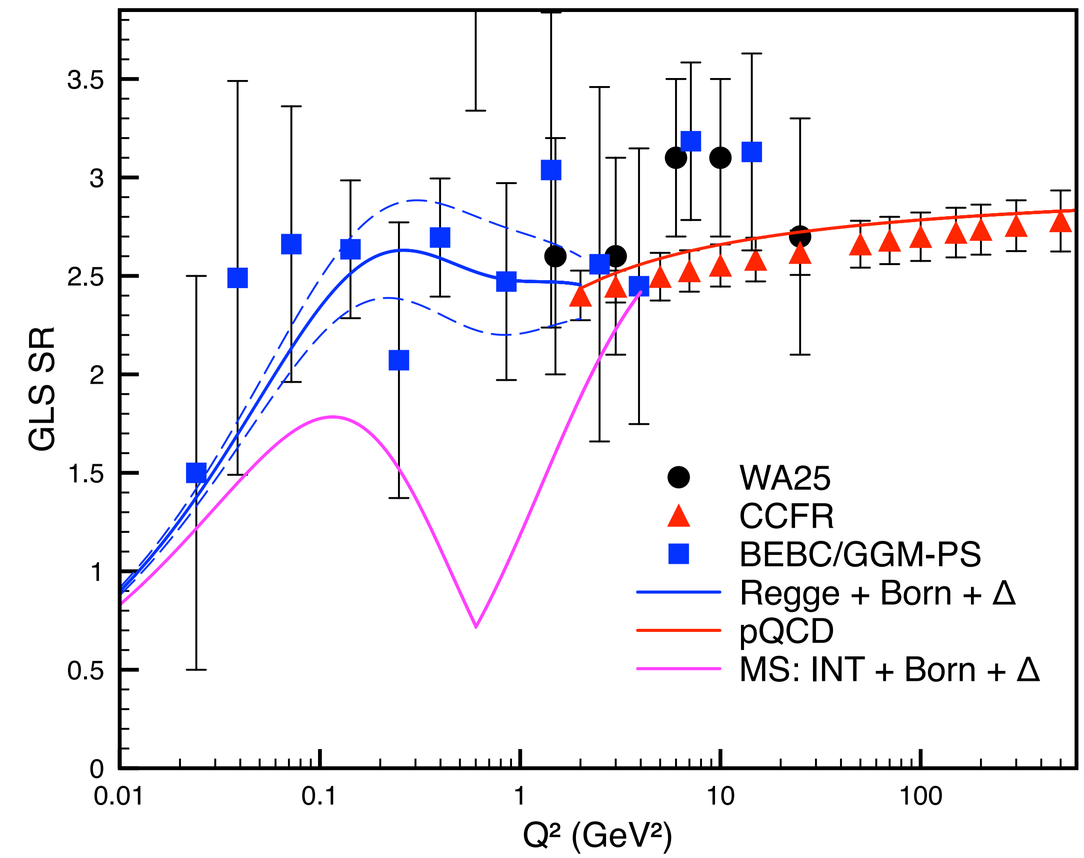
Low-W part of spectrum:
neutrino data from MiniBooNE, Minerva, …
- axial FF, resonance contributions, pi-N continuum

High-W: Regge behavior $F_3 \sim q^{\alpha} \sim x^{-\alpha}$, $\alpha \sim 0.5\text{-}0.7$



Parameters of the Regge model from neutrino scattering

Low $Q^2 < 0.1$ GeV^2: Born + Δ(1232) dominate
Not fitted: modern data more precise but cover only limited energy range
Fit driven by 4 data points between 0.2 and 2 GeV^2

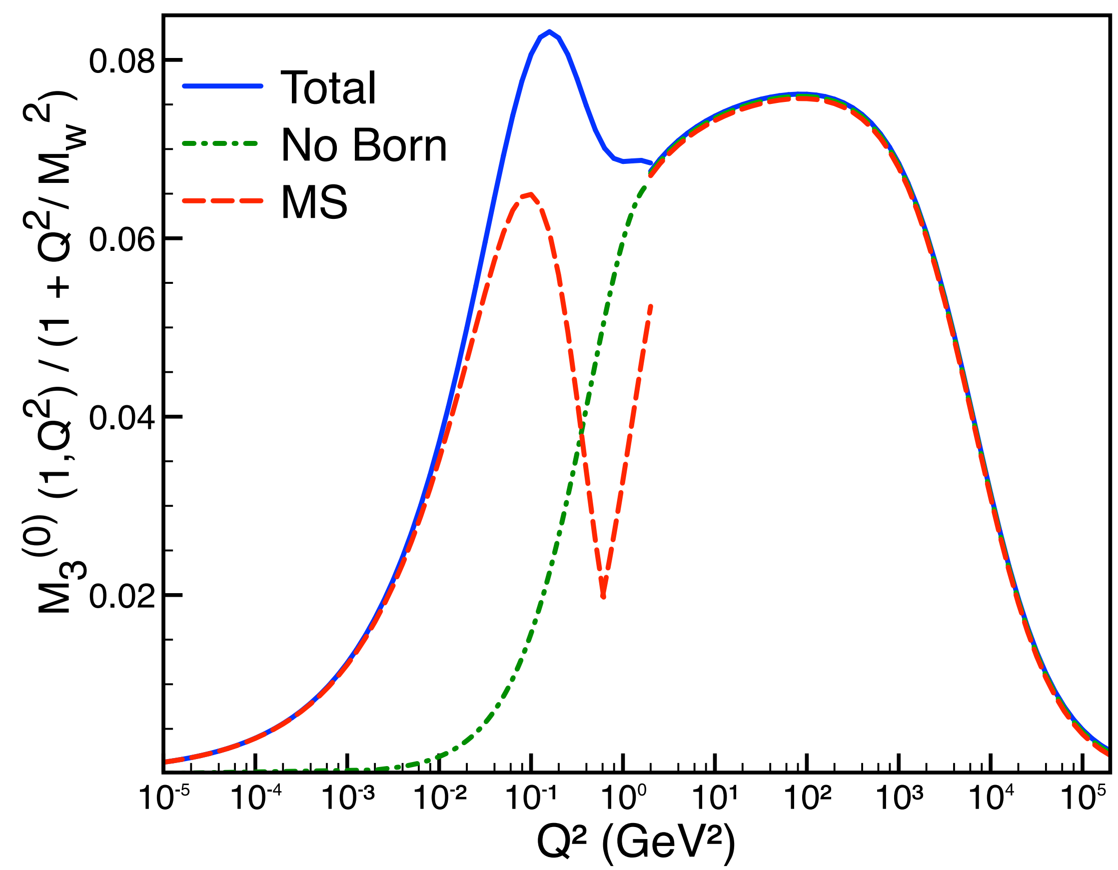


Model & Uncertainty fully specified
- compare M&S vs This work

$M_3^{WW}(1,Q^2)$

Isospin symmetry

$M_3^{\gamma W}(1,Q^2)$



M&S: integrand discontinuous at $Q^2 = 2.25$ GeV^2

Log scale for x-axis: integral = surface under the curve

$$\text{MS Total}: \quad \Box^{(0)}_{\gamma W} = 0.00324 \pm 0.00018$$

$$\text{New Total}: \quad \Box^{(0)}_{\gamma W} = 0.00379 \pm 0.00010$$

Uncertainty reduced by almost factor 2;
~ 3-5 sigma shift from the old value

2. Nuclear structure modification of Δ_R^V

C-Y Seng, MG, M J Ramsey-Musolf, arXiv: 1812.03352

Splitting the γW-box into Universal and Nuclear Parts

General structure of RC for nuclear decay

$$ft(1+RC) = Ft(1+\delta'_R)(1-\delta_C+\delta_{NS})(1+\Delta_R^V)$$

RC on a free neutron

$$\Delta_R^V \propto F_3^{\text{free n}} \propto \int dx e^{iqx} \sum_X \langle p | J_{em}^{\mu,(0)}(x) | X \rangle \langle X | J_W^{\nu,+}(0) | n \rangle$$

RC on a nucleus

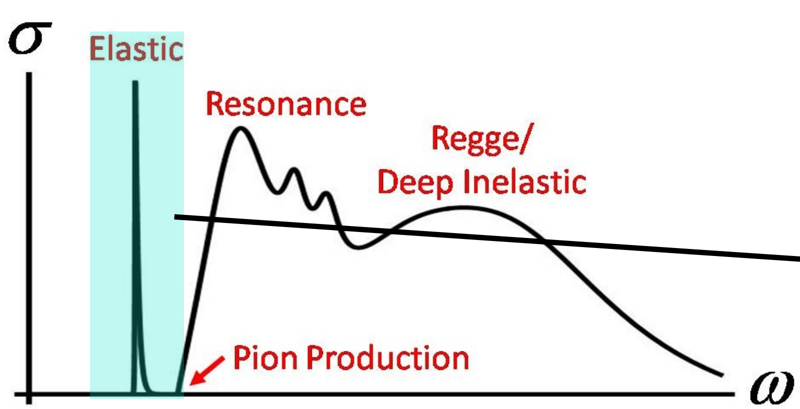
$$\Delta_R^V + \delta_{NS} \propto F_3^{\text{Nucl.}} \propto \int dx e^{iqx} \sum_{X'} \langle A' | J_{em}^{\mu,(0)}(x) | X' \rangle \langle X' | J_W^{\nu,+}(0) | A \rangle$$

NS correction reflects this extraction of the free box

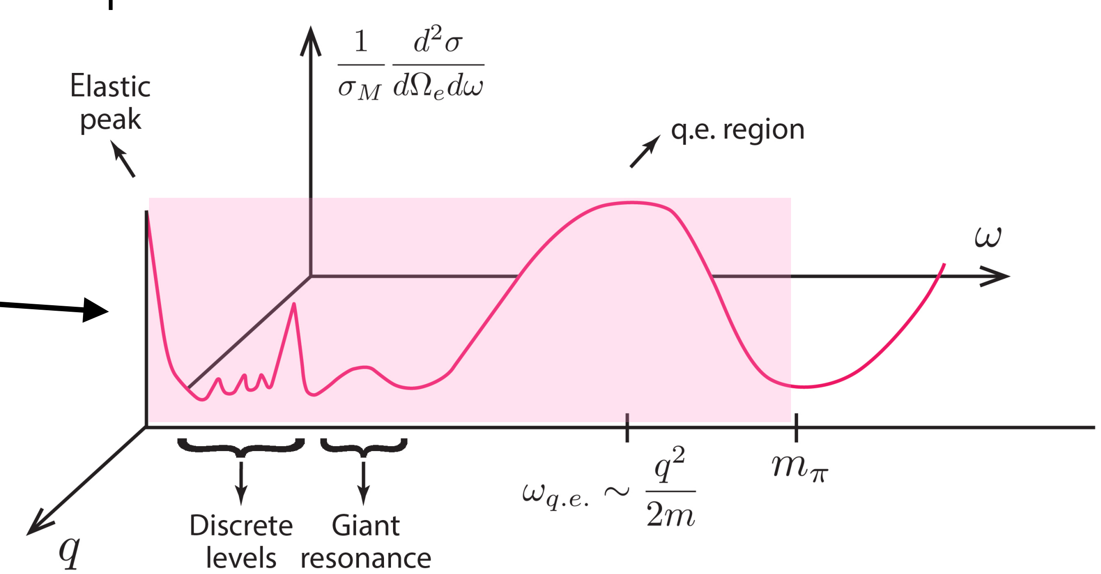
$$\Box_{\gamma W}^{\text{VA, Nucl.}} = \Box_{\gamma W}^{\text{VA, free n}} + \left[\Box_{\gamma W}^{\text{VA, Nucl.}} - \Box_{\gamma W}^{\text{VA, free n}}\right]$$

Nuclear modification in the lower part of the spectrum

Input in the DR for the universal RC



Input in the DR for the RC on a nucleus



Nuclear γW-box

$$\Box_{\gamma W}^{VA,\ Nucl.} = \frac{\alpha}{N\pi M}\int\limits_0^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2}\int\limits_0^\infty d\nu \frac{(\nu+2q)}{\nu(\nu+q)^2}\; F_{3,\,\gamma W}^{(0),\,\mathrm{Nucl.}}(\nu,Q^2)$$

Need to know the full nuclear Green's function
indices k, l count the nucleon d.o.f. in a nucleus

$$T_{\mu\nu}^{\gamma W\ \mathrm{nuc}} \sim \sum_{k,\ell}\langle f|J_\mu^W(k)\,G_{\mathrm{nuc}}\,J_\nu^{\mathrm{EM}}(\ell)|i\rangle$$

Two cases: (A) same active nucleon
(B) two nucleons correlated by G

$$T_{\mu\nu}^{A} = \sum_{k}\langle f|J_\mu^W(k)\,G_{\mathrm{nuc}}\,J_\nu^{\mathrm{EM}}(k)|i\rangle$$

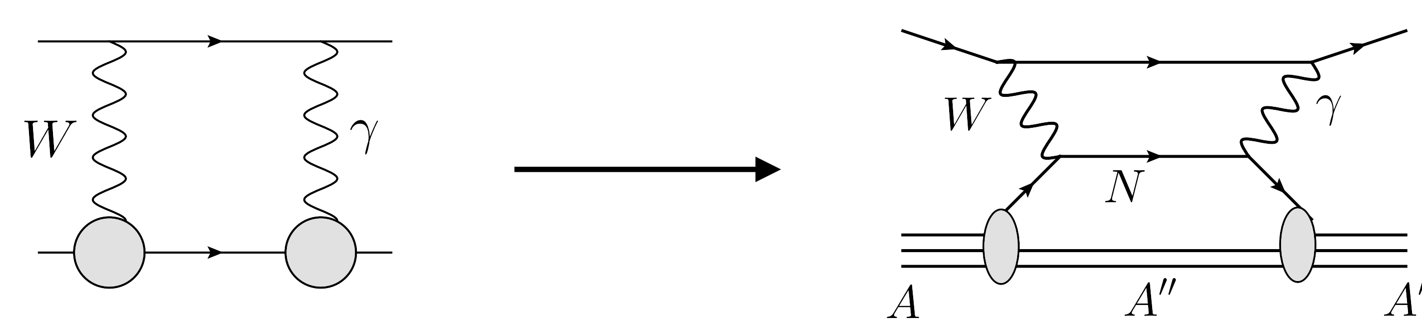
$$T_{\mu\nu}^{B} = \sum_{k\neq\ell}\langle f|_\mu^W(k)\,G_{\mathrm{nuc}}\,J_\nu^{\mathrm{EM}}(\ell)|i\rangle$$

Case (A): on-shell neutron propagating between interaction vertices
Case (B): all two-nucleon contributions (QE 2p2h and nuclear excitations)

Insert on-shell intermediate states:

$$T_{\mu\nu}^{A} \to \sum_{k}\langle f\,|\,J_\mu^W(k)[S_F^N \otimes G_{nuc}^{A''}]J_\nu^{EM}(k)\,|\,i\rangle$$

The elastic nucleon box
is replaced by a single N QE knockout



Universal vs. Nuclear Corrections

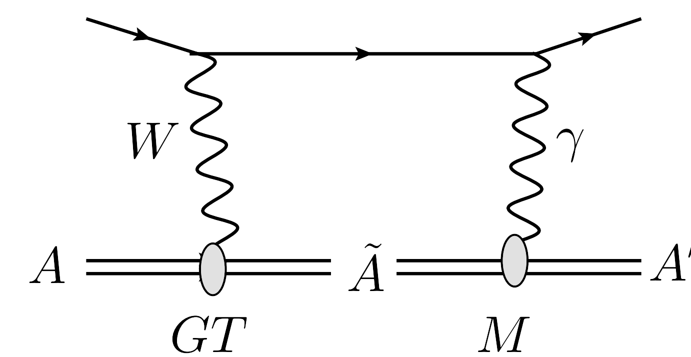
universal

Towner 1994 and ever since:
$$\Box^{\rm Nucl}_{\gamma W} - \Box^{\rm free\,n}_{\gamma W} = [q_S^{(0)} q_A - 1]\,\Box^{\rm free\,n}_{\gamma W}$$

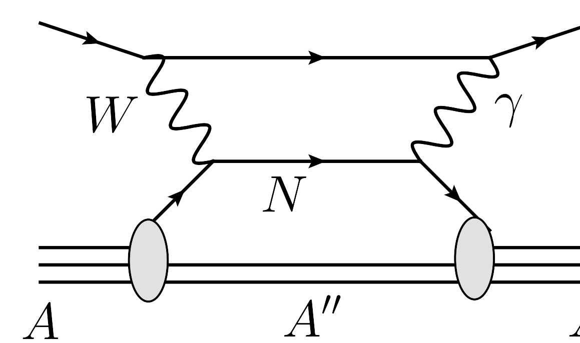
nuclear δ_{NS}

Idea: calculate Gamow-Teller and magnetic nuclear transitions in the shell model;
Insert the single nucleon spin operators —> predict the strength of nuclear transitions
"Quenching of spin operators in nuclei": shell model overestimates those strengths!
Each vertex is suppressed by 10-15%

Numerically: on average between the 14 superallowed decays
$$\delta_{NS}^{quenched\,Born} = [q_S^{(0)} q_A - 1]2\,\Box^{\rm free\,n,\,Born}_{\gamma W} \approx -0.058(14)\,\%$$



But from dispersion relation perspective it corresponds
to a contribution of an excited nuclear state,
not to the modified box on a free nucleon!
The correct estimate should base on quasielastic knockout
with an on-shell N + spectator in the intermediate state



Modification of C_B in a nucleus - QE

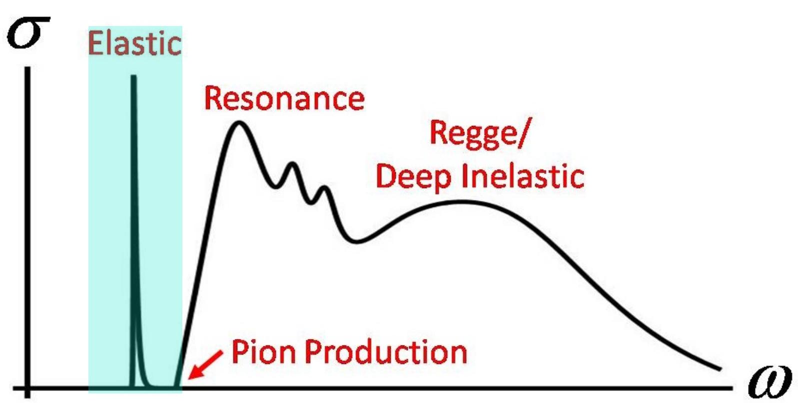
Integral is peaked at low ν, Q^2

$$\Box^{VA,\,Nucl.}_{\gamma W} = \frac{\alpha}{N\pi M}\int_0^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2}\int_0^\infty d\nu\,\frac{(\nu+2q)}{\nu(\nu+q)^2}\; F^{(0),\,\mathrm{Nucl.}}_{3,\,\gamma W}(\nu,Q^2)$$

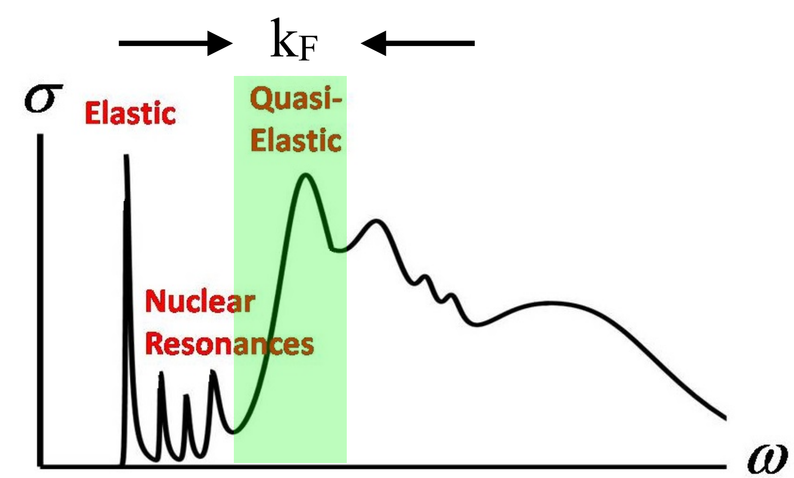
Born on free n:

$$F_3^{(0),\,B} = -\frac{Q^2}{4}G_A G_M^S\,\delta(2M\nu - Q^2)$$

Reduction for QE:
finite threshold ϵ (binding energy) + Fermi momentum k_F



$\nu = Q^2/2M$



$\nu \geq Q^2/2M + \epsilon$

QE calculation in free Fermi gas model with Pauli blocking
assign a generous 30% model uncertainty

C-Y Seng, MG, M J Ramsey-Musolf, arXiv: 1812.03352

New δ^{QE}_{NS} ~ - 0.11(3)% instead of the previous estimate δ^{q}_{NS} ~-0.058(14)%

3.Splitting of the RC into inner and outer

MG, arXiv: 1812.04229

Splitting the RC into "inner" and "outer"

Radiative corrections ~ α/π ~ 10^{-3}

Precision goal: ~ 10^{-4}

When does energy dependence matter?
Correction ~ E_e/Λ, with Λ ~ relevant mass (m_e; M_p; M_A)
Maximal E_e ranges from 1 MeV to 10.5 MeV

Electron mass regularizes the IR divergent parts - (E_e/m_e important) - "outer" correction

If Λ of hadronic origin (at least m_π) —> E_e/Λ small, correction ~ 10^{-5} —> negligible

- certainly true for the neutron decay
- hadronic contributions do not distort the spectrum, may only shift it as a whole

However, in nuclei binding energies ~ few MeV — similar to Q-values

A scenario is possible when RC ~ $(\alpha/\pi) \times (E_e/\Lambda^{Nucl})$ ~ 10^{-3}

Nuclear structure may distort the electron spectrum

With dispersion relations can be checked straightforwardly!

Nuclear structure distorts the β-spectrum!

Evaluate the E-dependent contribution

$$\mathrm{Re}\,\Box_{\gamma W}^{odd}(E) = \frac{8\alpha E}{3\pi N M}\int_0^\infty dQ^2 \int_{\nu_{thr}}^\infty \frac{d\nu}{(\nu+q)^3}\left[\mp F_1^{(0)} \mp \left(\frac{3\nu(\nu+q)}{2Q^2}+1\right)\frac{M}{\nu}F_2^{(0)} + \frac{\nu+3q}{4\nu}F_3^{(-)}\right] + O(E^3)$$

Estimate with nuclear polarizabilities and size

Photonuclear sum rule: $$\alpha_E = \frac{2\alpha}{M}\int_\epsilon^\infty \frac{d\nu}{\nu^3}F_1(\nu,0) = 2\alpha\int_\epsilon^\infty \frac{d\nu}{\nu^2}\frac{\partial}{\partial Q^2}F_2(\nu,0)$$

Supplement with the nuclear form factor: $$\alpha_E(Q^2) \sim \alpha_E(0) \times e^{-R_{Ch}^2 Q^2/6}$$

Radius and polarizability scale with A: $$R_{Ch} \sim 1.2\,\mathrm{fm}\,A^{1/3},\ \ \alpha_E \sim 2.25\times 10^{-3}\,\mathrm{fm}^3\,A^{5/3}$$

Dimensional analysis estimate: $$\delta_{NS}(E) = 2\times 10^{-5}\left(\frac{E}{MeV}\right)\frac{A}{N}$$

Estimate in Fermi gas model (same as for E-independent)

$$\delta_{NS}(E) = (2.8 \pm 0.4 \pm 0.8)\times 10^{-4}\left(\frac{E}{MeV}\right)$$

Uncertainty: spread in **ϵ** and k_F, plus 30% on model

Nuclear structure and E-dependent RC

Use the two estimates as upper and lower bound of the effect

$$\delta_{NS}(E) = (1.6 \pm 1.6) \times 10^{-4} \left(\frac{E}{MeV} \right)$$

Spectrum distortion due to nuclear polarizabilities ~ 0.016 % per MeV

Roughly independent of the nucleus;

The total rate will depend on nucleus: different Q-values!

Correction to Ft values: integrate over spectrum (only total rate measured)

$$\tilde{\mathcal{F}}t = ft(1 + \delta_R')(1 - \delta_C + \delta_{NS} + \delta_{NS}^E)$$

$$\delta_{NS}^E = \frac{\int_{m_e}^{Q} dE E p (Q-E)^2 \delta_{NS}(E)}{\int_{m_e}^{Q} dE E p (Q-E)^2} \longrightarrow \delta_{NS}^E \approx (8 \pm 8) \times 10^{-5} \frac{Q}{\text{MeV}}$$

Average Q ~ 6 MeV —> expect an effect $\langle \delta_{NS}^E \rangle \approx (1.5 \pm 1.5) \times 10^{-4}$

4.Putting numbers together

Universal correction

$$|V_{ud}|^2 = \frac{2984.432(3)\,\mathrm{s}}{\mathcal{F}t(1+\Delta_R^V)}$$

Marciano & Sirlin 2006

$$\Delta_R^V = 0.02361(38)$$

Dispersion relations

$$\Delta_R^V = 0.02467(22)$$

DR allowed to ~ halve the uncertainty in Δ_R^V due to the use of neutrino data

Nuclear corrections - E-dependent and independent

$$\mathscr{F}t = ft(1+\delta'_R)(1-\delta_C+\delta_{NS}) \longrightarrow \tilde{\mathscr{F}}t = ft(1+\delta'_R)(1-\delta_C+\tilde{\delta}_{NS}+\delta^E_{NS})$$

New δ_{NS} uses QE estimate instead of the quenched Born estimate

$$\delta_{NS}^{\text{quenched Born}} = -0.058(14)\,\% \longrightarrow \delta_{NS}^{QE} = -0.11(3)\,\%$$

$$\delta_{NS}^{E} = 0 \qquad \delta_{NS}^{QE} = +0.05(5)\,\%$$

No net shift to the Ft central value

$$\overline{\mathscr{F}t} = (3072.1 \pm 0.7)s \;\rightarrow\; \overline{\mathscr{F}t} = (3072 \pm 2)s$$

$$|V_{ud}|^2 = \frac{2984.432(3)\,\mathrm{s}}{\mathcal{F}t(1+\Delta_R^V)}$$

MS 2006 + HT 2014-2018 $\longrightarrow$ Dispersion relations

$$\Delta_R^V = 0.02361(38) \longrightarrow \Delta_R^V = 0.02467(22)$$

$$\overline{\mathscr{F}t} = (3072.1 \pm 0.7)s \longrightarrow \overline{\mathscr{F}t} = (3072 \pm 2)s$$

$$|V_{ud}| = 0.97420(10)_{Ft}(18)_{\Delta_R^V} \longrightarrow |V_{ud}| = 0.97370(30)_{Ft}(10)_{\Delta_R^V}$$

Unitarity in the top row

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994 \pm 0.0005 \longrightarrow |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9984 \pm 0.0006$$

Substantial shift in the central value by two old sigmas (due to Δ_R^V)

Discrepancy with the unitarity mainly depends now on nuclear uncertainties

Conclusions & Outlook

- The γW-box in the forward dispersion relation framework
- Hadronic and nuclear corrections in a unified framework
- Nuclear structure leaks in the outer correction, distorts the beta decay spectrum
- Better calculations than free Fermi gas should be done
- Nuclear uncertainties shift the emphasis on free neutron decay
- Tensions with CKM unitarity: $\Sigma_{i=d,s,b} |V_{ui}|^2 - 1 = -0.0016(6)$

Nuclear correction δ_{NS}
DR allow to address hadronic and nuclear parts of the calculation on the same footing
The full nuclear correction should be calculated (not just QE) - further test of H&T δ_{NS}

Decay spectra and nuclear polarizabilities
Can contaminate the extraction of Fierz interference from precise spectra!

Conclusions & Outlook

However… the largest correction to Ft is ISB δ_C non-dispersive

Range from 0.15% to 1.5%

Can its calculation be related to neutron skin calculations for PVES?

Which ingredients are common and which are not?

MESA@Mainz will measure the weak radius of C-12 to <1% (2023 on)

Other nuclei (including symmetric ones) possible in the future

Potentially a strong statement between two fields

Backup slide

Nuclear structure distorts the β-spectrum!

$$\tilde{\mathscr{F}}t = ft(1+\delta'_R)(1-\delta_C+\delta_{NS}+\delta^E_{NS})$$

Absolute shift in Ft values

$$\delta\mathscr{F}t = \mathscr{F}t \times \delta^E_{NS}$$

Decay	Q (MeV)	$\Delta_E^{NS}(10^{-4})$	$\delta\mathcal{F}t(s)$	$\mathcal{F}t(s)$ [3]
^{10}C	1.91	1.5	0.5	3078.0(4.5)
^{14}O	2.83	2.3	0.7	3071.4(3.2)
^{22}Mg	4.12	3.3	1.0	3077.9(7.3)
^{34}Ar	6.06	4.8	1.5	3065.6(8.4)
^{38}Ca	6.61	5.3	1.6	3076.4(7.2)
^{26m}Al	4.23	3.4	1.0	3072.9(1.0)
^{34}Cl	5.49	4.4	1.4	$3070.7^{+1.7}_{-1.8}$
^{38m}K	6.04	4.8	1.5	3071.6(2.0)
^{42}Sc	6.43	5.1	1.6	3072.4(2.3)
^{46}V	7.05	5.6	1.7	3074.1(2.0)
^{50}Mn	7.63	6.1	1.9	3071.2(2.1)
^{54}Co	8.24	6.6	2.0	$3069.8^{+2.4}_{-2.6}$
^{62}Ga	9.18	7.3	2.2	3071.5(6.7)
^{74}Rb	10.42	8.3	2.6	3076(11)

Shift due to δ^E_{NS}: comparable to precision of 7 best-known decays

$$\overline{\mathcal{F}t} = 3072.07(63)\text{s} \to \overline{\mathcal{F}t} = 3073.6(0.6)(1.5)\text{s}$$

Decay electron polarizes the daughter nucleus

As a result the spectrum is slightly distorted towards the upper end

Positive-definite correction to Ft ~ 0.05%

Previously found: E-independent piece lowers the Ft value by about the same amount

$$\overline{\mathcal{F}t} = 3072.07(63)s \to [\overline{\mathcal{F}t}]^{\text{new}} = 3070.50(63)(98)s$$

Nuclear structure uncertainties might be underestimated

QE calculation - effect on Ft values and V_{ud}

Adopting a new estimate of the in-nucleus modification of the free-nucleon Born

Shifts the Ft value according to $\overline{\mathcal{F}t} \to \overline{\mathcal{F}t}(1 + \delta_{NS}^{new} - \delta_{NS}^{old})$

Numerically: $\overline{\mathcal{F}t} = 3072.07(63)s \to [\overline{\mathcal{F}t}]^{\text{new}} = 3070.50(63)(98)s$

Will affect the extracted V_{ud} $|V_{ud}|^2 = \dfrac{2984.432(3)\,\text{s}}{\mathcal{F}t(1+\Delta_R^V)}$

Compensates for a part of the shift due to a new evaluation of Δ^V_R

$$V_{ud}^{\text{old}} = 0.97420(21) \ \to \ |V_{ud}^{\text{new}}| = 0.97370(14) \to |V_{ud}^{\text{new, QE}}| = 0.97395(14)(16)$$

Brings the first row closer to the unitarity (4σ ➞ 2.2σ)

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9984 \pm 0.0004 \ \to |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9989 \pm 0.0005$$

and 1 sigma away from the PDG: 0.9994 ± 0.0005

Important messages:
a nuclear contribution may shift by 2 sigma if evaluated with a different method
dispersion relations as a unified tool for treating hadronic and nuclear parts of RC