# Theory overview on rare eta decays

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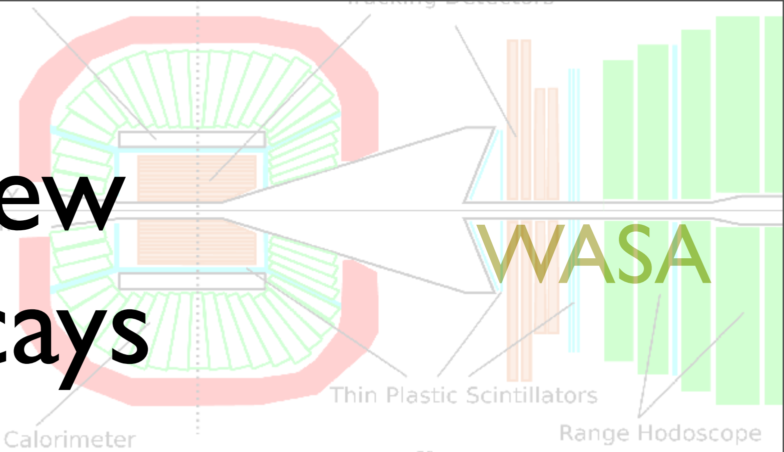
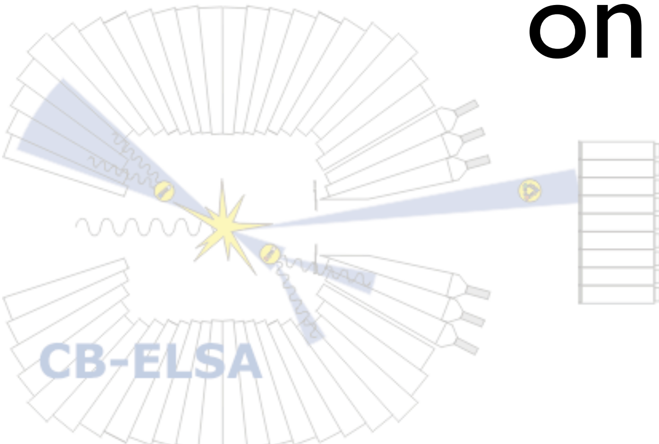
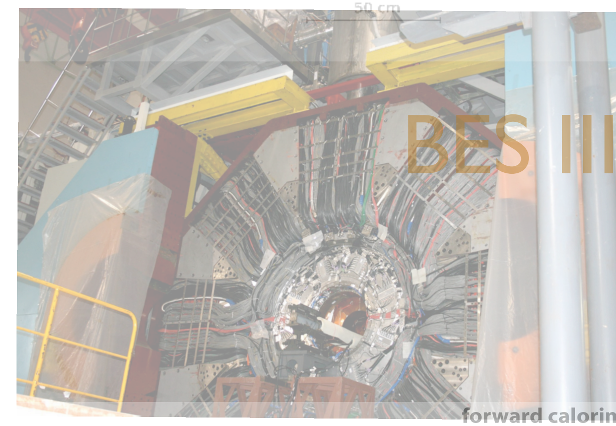
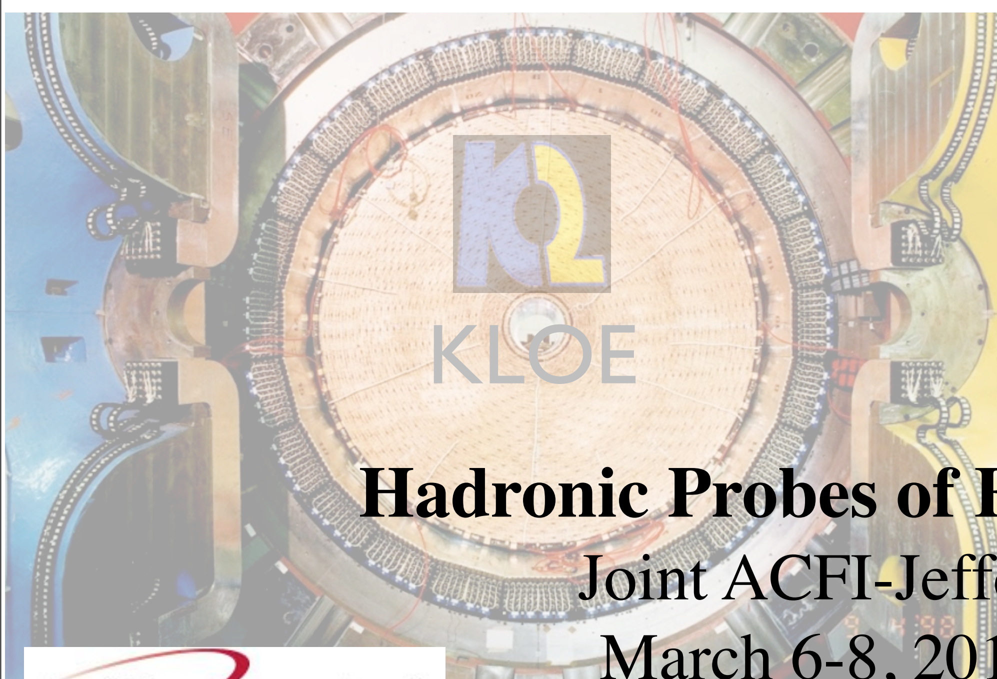
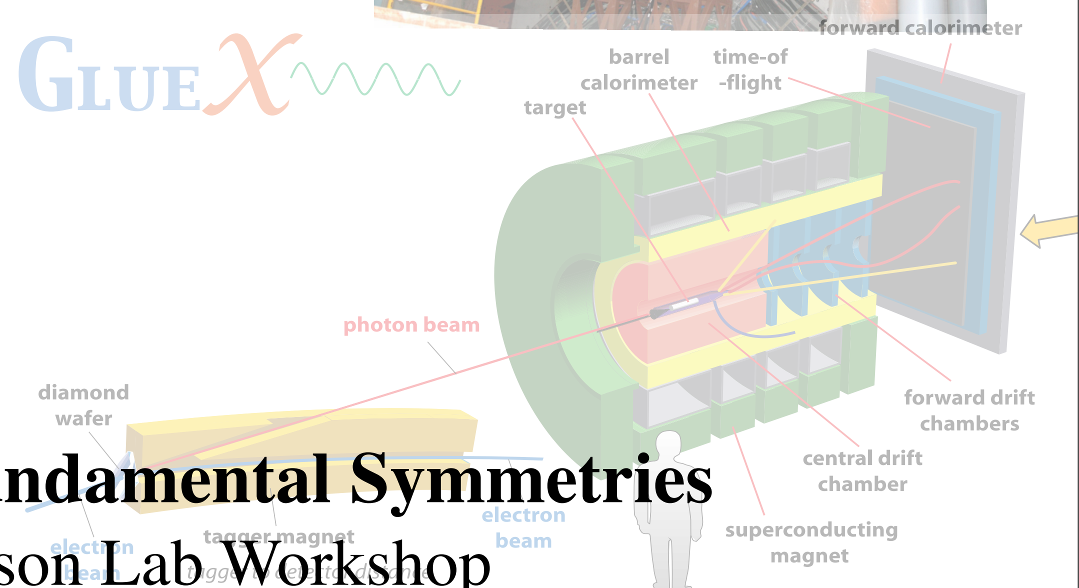



**Hadronic Probes of Fundamental Symmetries**

Joint ACFI-Jefferson Lab Workshop

March 6-8, 2014 UMass Amherst

Jefferson Lab

HAMPTON UNIVERSITY

# Motivation

Main motivation for the workshop
The JEF proposal submitted to JLab Hall D
(talk by Liping Gan)

Physics of the proposal
- accurate measurement of $\eta \to \pi^0\gamma\gamma$
- improve bounds on $\eta \to \pi^0\pi^0$ and $\eta \to \gamma\gamma\gamma$

One objective of the workshop is to learn more about the possibilities of searching for BSM effects in the $\eta$ decays

$\eta$ has been and is investigated in precision experiments at several facilities

# The special role of the $\eta$

- Self conjugate $0^{-+}$ meson
- Is a Goldstone Boson despite being a little overweight
- Mostly an octet, but mixes with the $\eta'$
- The decays $\eta \to 3\pi$ give access to $\frac{m_u - m_d}{m_s}$
- Chiral anomaly in $\eta \to \gamma\gamma$, ...

Can we ask more from $\eta$ ?

# Outline

- $\eta \to \pi^0\gamma\gamma$ : generalities
- other QED mediated decays $\eta \to \pi^0\mu^+\mu^-, \ \eta \to \mu^+\mu^-$
- BSM with $\eta$
- Discussion of the two pertinent decays $\eta \to \pi\pi \quad \eta \to 3\gamma$
- Possible points of discussion

# The decay $\eta \to \pi^0\gamma\gamma$

$$\Gamma^{\rm Exp} = 0.35 \pm 0.07 \text{ eV}$$

Has changed significantly over time

## Challenge for ChPT

Ametller et al ('91)

- No tree level contributions up to $\mathcal{O}(p^4)$
- Finite contributions to 1-loop $\mathcal{O}(p^4)$
- 1-loop contributions are very small
- Dominant contributions are NLO in ChPT.
- Must be estimated using models
- Window into $\mathcal{O}(p^6)$ ChPT

$$\gamma\gamma \to \eta\pi^0 \quad \neq \quad \gamma\gamma \to \pi^0\pi^0$$

$$M = \alpha\, \omega_1 \omega_2\, \vec{\epsilon}_1 \cdot \vec{\epsilon}_2 + \beta\, (\vec{\epsilon}_1 \times \vec{k}_1) \cdot (\vec{\epsilon}_2 \times \vec{k}_2)$$

$\alpha,\ \beta$ functions of meson masses, $M_{\gamma\gamma},\ \omega_1 + \omega_2$

Only known meson decay that proceeds through polarizability pathway

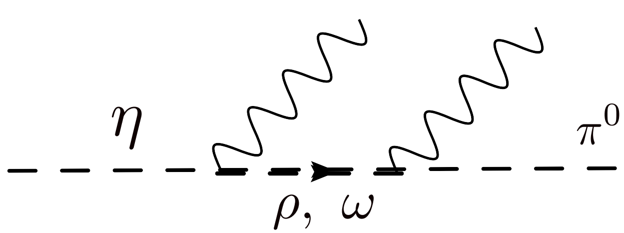
one-loop contributions

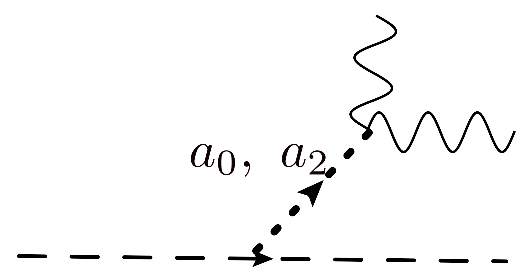
$$\Gamma^{1-loop} = 3.9 \times 10^{-3}\ \mathrm{eV}$$

Estimation of dominant contributions

Ametller et al; Ko; Oset et al; Piccioto; Ng & Peters

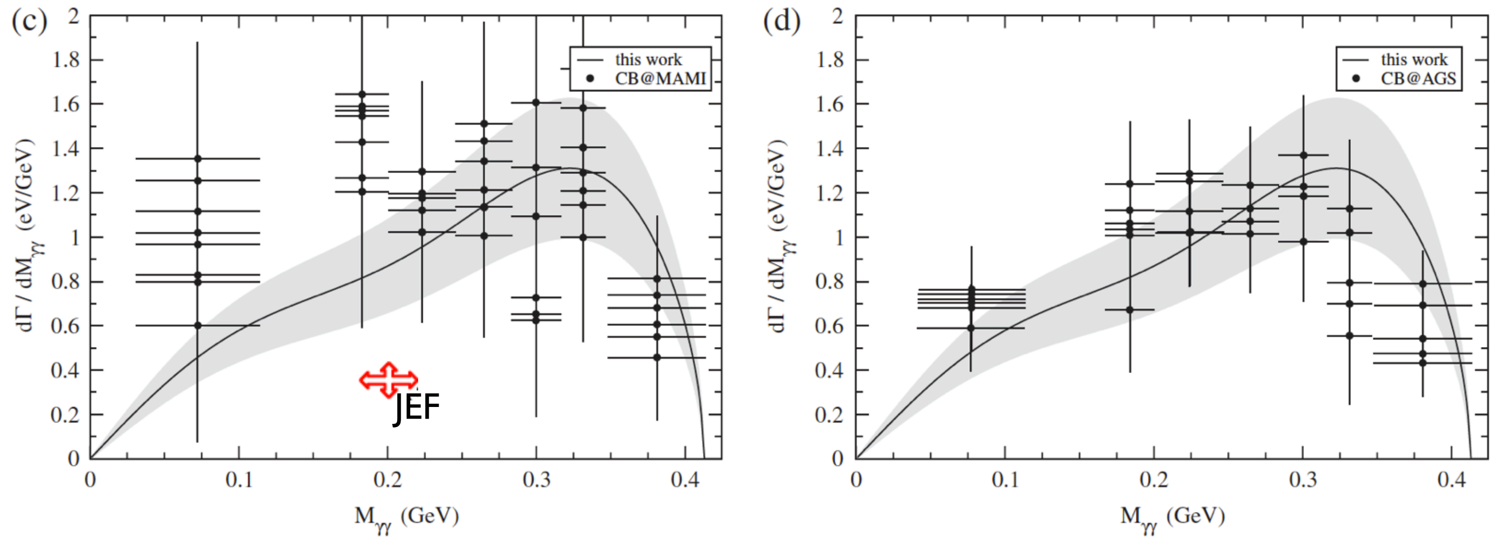
Resonance dominance models $\omega,\ \rho,\ a_0,\ a_2$



# Important constraints from spectrum

from Oset et al

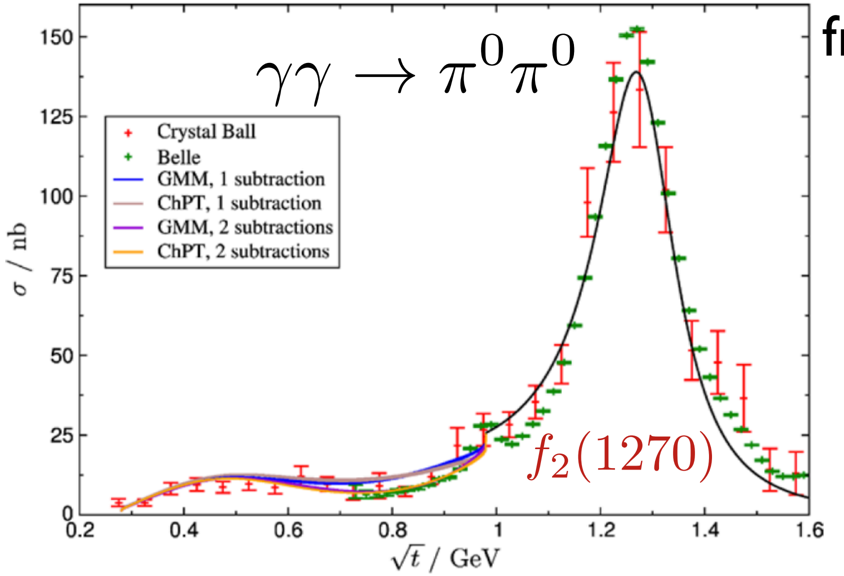



Left to do

- More accurate measurement of the spectrum JEF proposal (Liping's talk)
- Possible improvement of predictions?
- Possible future access from Lattice QCD?

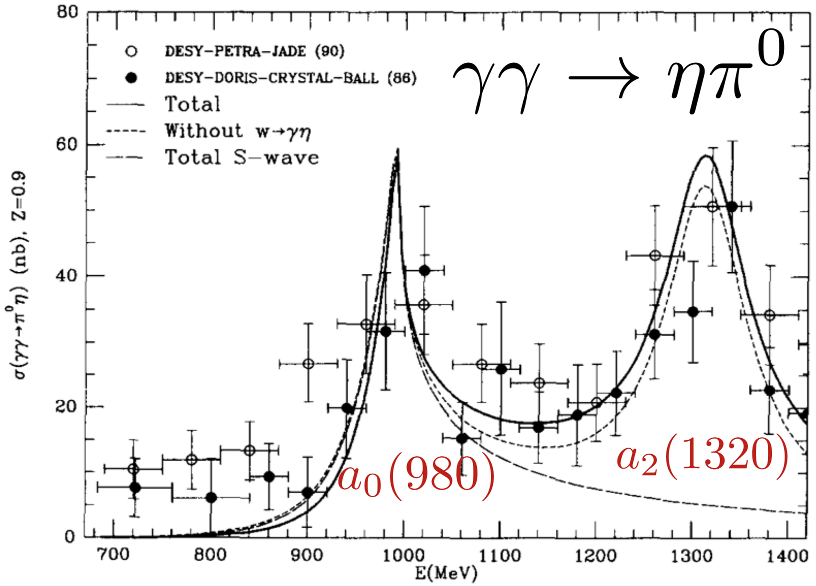
Pion E polarizabilities studied in LQCD: Detmold et al: still large errors

Possible interesting point of discussion in workshop....




from Hoferichter et al

from Oset et al




Could it be improved?

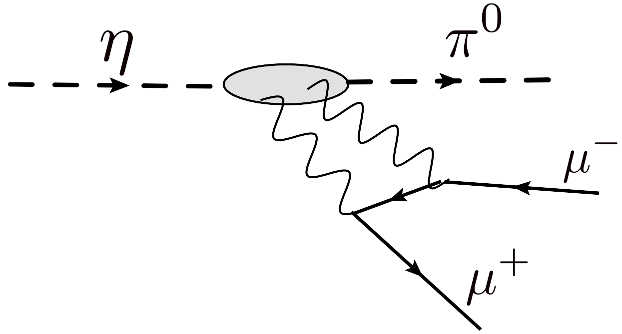
The expert's review: Hans Bijnens' talk

# $\eta \to \pi^0 \mu^+ \mu^-$

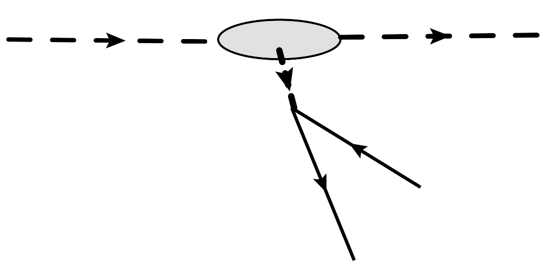
Current Exp bound $\Gamma_{PDG}(\eta \to \pi^0 \mu^+ \mu^-) < 6.5\ meV$

Calculation by Ng & Peters

$$\Gamma(\eta \to \pi^0 \mu^+ \mu^-) \sim .6 \pm .3 \mu eV$$




Any useful bound on new physics? e.g: heavy scalar



$$\frac{d\Gamma}{dE_\pi} \sim \frac{1}{12\pi^3} \left( \frac{\epsilon g_{qS} g_{\ell S}}{M_S^2} \right)^2 M_\eta \mid p_\pi \mid^3 \qquad \epsilon \sim 10^{-2}$$

$$M_S < \epsilon\ g_{qS}\ g_{\ell S} \times 13 \text{ GeV}$$

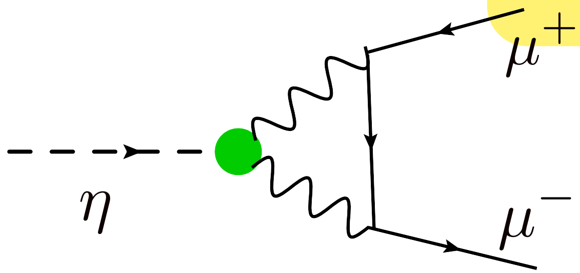
Bound @ EM contribution $\quad M_S > \sqrt{\epsilon\, g_{qS}\, g_{\ell S}} \times 130 \text{ GeV}$

Bound from $\mu\ (g-2)$ $\quad M_S > g_{\ell S} \times 240 \text{ GeV}$

$g_{qS},\ g_{\ell S}$ are likely to be very small

# A more sensitive case

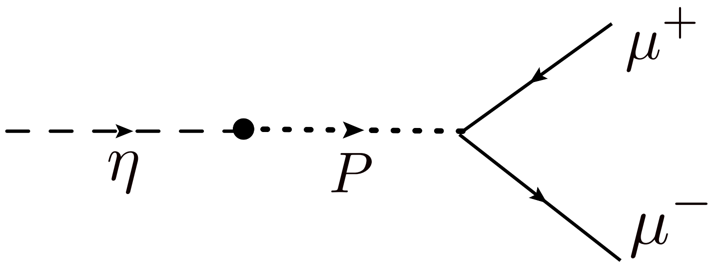
## $\eta \to \mu^+ \mu^-$




$$BR^{Exp} = 5.8 \pm 0.8 \times 10^{-6}$$

Theoretically understood at the level of the experimental error: e.g. Luke et al.

## Bound for a heavy pseudoscalar mass?




$$\delta A_P = g_{qP}\, g_{\mu P}\, \frac{M_\eta F_\eta}{M_P^2} \bar{u}_\mu \gamma_5 v_\mu$$

Sensitivity @ exp error $\quad M_P > \sqrt{g_{gP}\, g_{\mu P}} \times 240\ \mathrm{GeV}$

Bound from $\mu\ (g-2) \quad M_P > g_{\mu P} \times 240\ \mathrm{GeV}$

Contribution from Z0 exchange $\quad \delta BR_{Z^0} \sim 5 \times 10^{-8}$

Q: any model BSM that can receive a meaningful bound from $\eta \to X\ \mu^+ \mu^-$

# BSM Physics

The case for it

- Hierarchy problem
- quark and lepton masses
- Family problem, or who ordered them?
- Mechanism to explain $\bar{\theta}_{QCD} \sim 0$ : axion?
- BAU
- Dark matter is out there

Many models motivated by these facts: SUSY, SUGRA, string theory phenomenology, extra dimensions, technicolor, dark matter models, ...

# Effective Theory for BSM Physics

$$\mathcal{L}(M_W) = \mathcal{L}_{SM} + \mathcal{L}_5 + \mathcal{L}_6 + \cdots$$

$$\mathcal{L}_n = \frac{1}{\Lambda_{NP}^{n-4}} \sum_i C_i^{(n)} \; O_i^{(n)}$$

Composite operators respect SM gauge symmetries

$$\mathcal{L}(\Lambda_{QCD}) = \mathcal{L}_{QCD} + \mathcal{L}_{QED} + \mathcal{L}_{Weak} + \mathcal{L}_{BSM}$$

Bases of composite operators up to dim 6

Grzadkowski et al

For sensitivity to BSM physics the structure of operators is key; some hadronic matrix elements need to be estimated and at some point evaluated using LQCD... already possible for known nonleptonic decays

# Dim 6 Operators

## Operators not giving FCNC (MFV models)

Array of fields in family multiplets $Q_L,\ U_R,\ D_R,\ L_L,\ \ell_R,\ \ \nu_R$

$U(N_f)^6$ symmetry group

$U(N_f)^6 \times SU_c(3) \times SU_L(2) \times U_Y(1)$ singlet operators will not give FCNC

Operators built with products of bilinears of the most general forms:

$$\bar{Q}_L \gamma_\mu \lambda_c T_f Q_L\ ,\quad \bar{U}_R \gamma_\mu \lambda_c U_R\ ,\quad \bar{D}_R \gamma_\mu \lambda_c U_R\ , \cdots$$

$\lambda_c$ color matrices; $T_f$ flavor matrices

Only permitted the combinations with quarks and leptons:

$\bar{Q}_L \gamma_\mu Q_L\ \bar{L}_L \gamma_\mu L_L$ and other flavor and chirality combinations

Effects of these operators are non observable in FCNC decays; "observable" in the purely leptonic decays of the $\eta$, ... but $Z^0$ also contributes !...

I think this is important point of discussion

Search strategy for NP; look at quantities which

- Vanish at tree level in the SM
- Are calculable (finite) at 1-loop level in SM

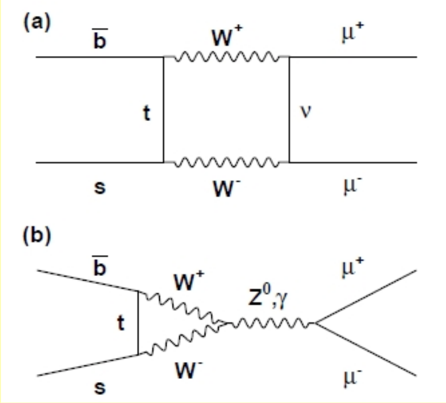
  - EW precision observables fixed by EW symmetry
  - FCNC (GIM)
  - CPV
  - B, L violation
  - EDMs
  - (g-2)s
  - etc

# Low energy SM tests and searches BSM

$$K^0 \to \mu^+\mu^- : \; GIM$$

$$B - \bar{B} \text{ mixing} : \; \text{t quark}$$

The newest test of FCNC $\quad B_s \to \mu^+\mu^-$




Very suppressed and reliably calculable in SM  Buras et al

$$\mathcal{B}(B_s \to \mu^+\mu^-)^{SM} = \left(3.23 \pm 0.15 \pm 0.23_{F_{B_s}}\right) \times 10^{-9}$$

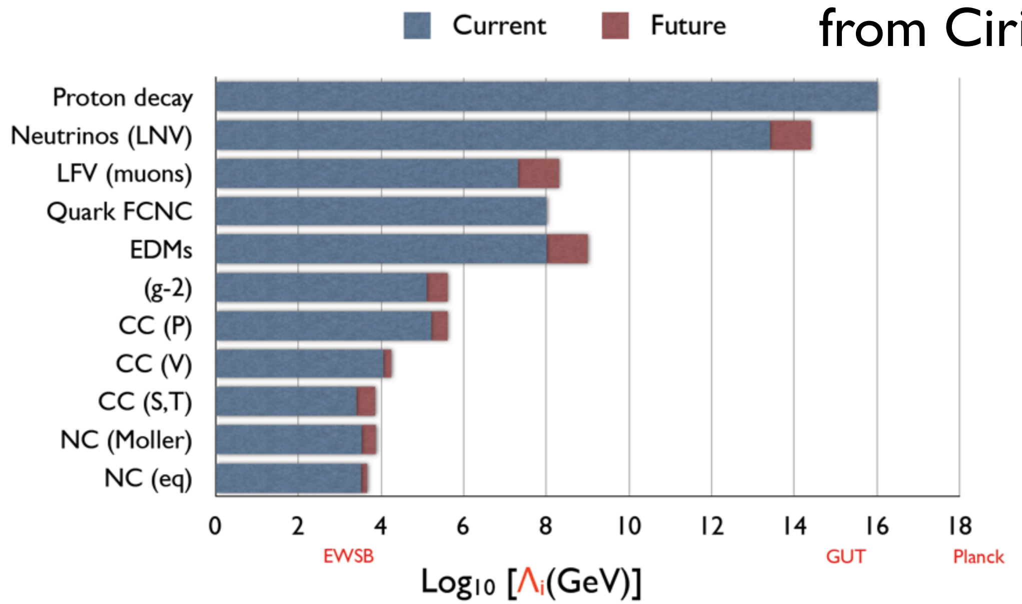
Measured by LHCb $\quad \overline{\mathcal{B}}^{exp} = \left(3.2^{+1.5}_{-1.2}\right) \times 10^{-9}$

Constrains MSSM
Isidori et al.

Tight constraints on extra scalar particles

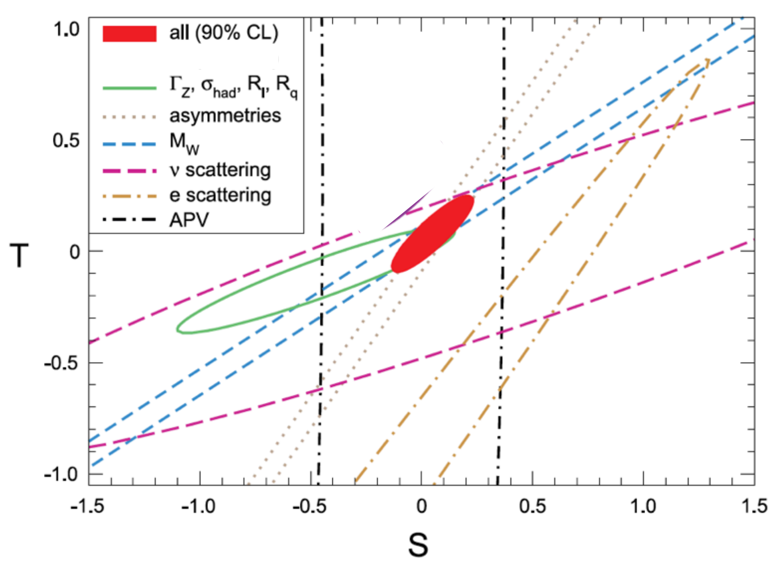
Future $\quad \dfrac{\mathcal{B}(B_s \to \tau^+\tau^-)^{SM}}{\mathcal{B}(B_s \to \mu^+\mu^-)^{SM}} = 215$

from Cirigliano & Ramsey-Musolf

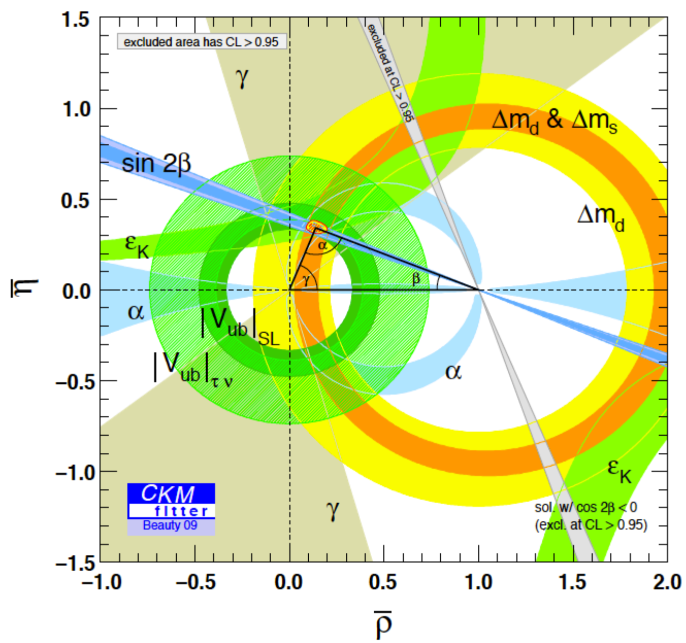



...if only suppression is by the scale of new physics

# Example with precision EW physics




PDG



# Symmetry tests with $\eta$ decays

| Mode | Branching Ratio | Symmetry Highlight |
|---|---|---|
| $\pi^0 2\gamma$ | $(2.7 \pm 0.5) \times 10^{-4}$ | $\chi$PTh, $\mathcal{O}(p^6)$ |
| $\pi^0\pi^0$ | $< 3.5 \times 10^{-4}$ | CP, P |
| $4\pi^0$ | $< 6.9 \times 10^{-7}$ | CP, P |
| $\pi^+\pi^-$ | $< 1.3 \times 10^{-5}$ | CP, P |
| $4\gamma$ | $< 2.8 \times 10^{-4}$ | suppressed ($< 10^{-11}$) |
| $\pi^0\gamma$ | $< 9 \times 10^{-5}$ | C, L |
| $3\gamma$ | $< 1.6 \times 10^{-5}$ | C |
| $2\pi^0\gamma$ | $< 5 \times 10^{-4}$ | C |
| $3\pi^0\gamma$ | $< 6 \times 10^{-5}$ | C |
| $\pi^0 e^+ e^-$ | $< 4 \times 10^{-5}$ | C |
| $e^+ e^-$ | $< 2.7 \times 10^{-5}$ | helicity suppressed |

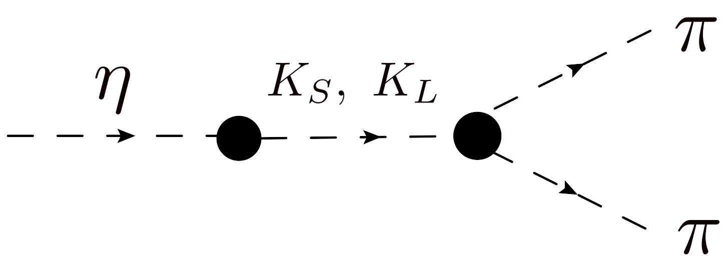
- $\pi^0, \eta, \eta'$ are the only self conjugated light mesons
- Test sources for FC non violating physics
- Most sensitive to SM C violation
- SM CPV is very suppressed
- Main disadvantage: short lifetimes
- Bounds on BSM physics of various kinds

# $\eta \to \pi\pi$

CP violating; current bound $BR_{Exp}(\eta \to \pi\pi) < 10^{-5}$

EW contribution

Flavor conserving CP violation: requires second order weak interaction




$\propto G_F^2$

$$O_1 = (\bar{s_L}\gamma_\mu d_L)(\bar{u_L}\gamma_\mu u_L) - (\bar{s_L}\gamma_\mu u_L)(\bar{u_L}\gamma_\mu d_L) \quad (\{8_f\}, \Delta I = 1/2),$$

$$O_2 = 2(\bar{s_L}\gamma_\mu d_L)\sum_{q=u,d,s}(\bar{q_L}\gamma_\mu q_L) - O_1 \quad (\{8_f\}, \Delta I = 1/2),$$

$$O_3 = O_2 - 5(\bar{s_L}\gamma_\mu d_L)(\bar{s_L}\gamma_\mu s_L) \quad (\{27\}, \Delta I = 1/2),$$

$$O_4 = (\bar{s_L}\gamma_\mu d_L)(\bar{u_L}\gamma_\mu u_L) + (\bar{s_L}\gamma_\mu u_L)(\bar{u_L}\gamma_\mu d_L) - (\bar{s_L}\gamma_\mu d_L)(\bar{d_L}\gamma_\mu d_L) \quad (\{27\}, \Delta I = 3/2),$$

$$O_5 = (\bar{s_L}\gamma_\mu \lambda^a d_L)\left(\sum_{q=u,d,s}\bar{q_R}\gamma_\mu \lambda^a q_R\right) \quad (\{8\}, \Delta I = 1/2),$$

$$O_6 = (\bar{s_L}\gamma_\mu d_L)\left(\sum_{q=u,d,s}\bar{q_R}\gamma_\mu q_R\right) \quad (\{8\}, \Delta I = 1/2).$$

calculation: Jarlskog & Shabalin $BR_{EW}(\eta \to \pi\pi) \sim 10^{-27}$

## $\theta_{QCD}$ contribution

$$\Gamma_\theta(\eta \to \pi\pi) = \frac{\mid g_\theta \mid^2}{16\pi M_\eta}\sqrt{1 - 4\frac{M_\pi^2}{M_\eta^2}}$$

$$g_\theta = -\theta \frac{M_\pi^2}{\sqrt{8}F_\pi}(\cos\theta_{\eta-\eta'} - \sqrt{2}\sin\theta_{\eta-\eta'})$$

$$\Gamma_\theta(\eta \to \pi\pi) \sim 100\,\theta^2\,\mathrm{keV}$$

n EDM $\theta < 1.5 \times 10^{-10}$ Guo & Meissner

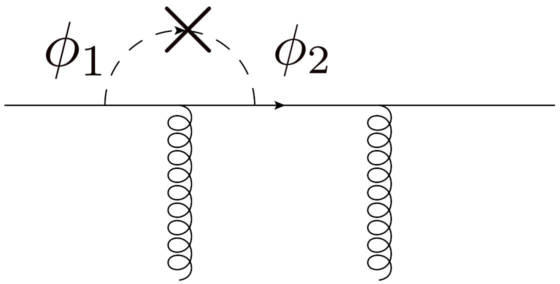
$$BR_\theta < 1.6 \times 10^{-18}$$

## BSM physics:

What is needed for a possibly observable effect?

A 12 orders of magnitude enhancement: Spontaneous CP violation

Estimate in Weinberg's model




Order of magnitude estimate by Jarlskog & Shabalin

$$BR_{SCPV}(\eta \to \pi\pi) \sim 10^{-15}$$

A question for the workshop:
Is there any other model BSM that could give even larger contributions?

# $\eta \to \gamma\gamma\gamma$

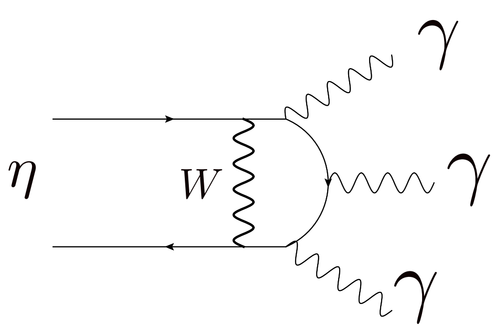
CP conserving mode $\quad H_{\rm eff} = g_{\eta 3\gamma}\, \eta\, F_{\mu\nu} F^{\nu\rho} F_{\rho}{}^{\mu}$

In SM need for one W in Feynman diagram

Back of the envelope order of magnitude estimate

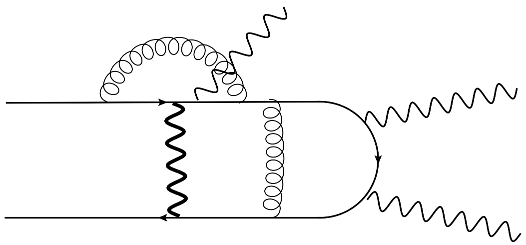
Loop contribution estimate: only short distances $\qquad$ Dicus

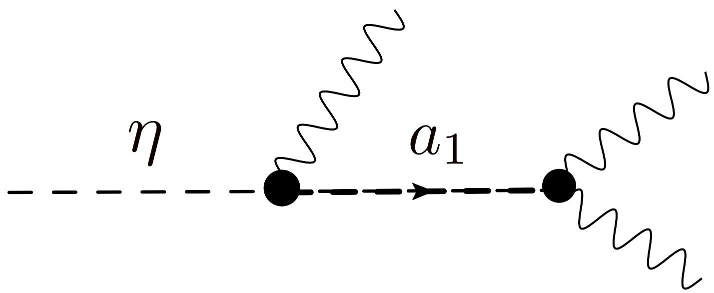
$$A \sim \frac{e^3\, G_F\, F_\eta\, k^3}{M_W^2}$$




$$\Gamma(\eta \to 3\gamma) \sim \frac{M_\eta \mid A \mid^2}{(8\pi)^3} \sim 10^{-23}\ \mathrm{eV} \qquad BR \sim 10^{-26}$$

# Enhancement via $a_1(1260)$






$$A \sim \frac{e^3\, G_F\, F_\eta\, g_{a_1\gamma\gamma}\, k^3}{M_{a_1}^2}$$

$$\Gamma_{a_1} \sim g_{a_1\gamma\gamma}^2 \times 10^{-15}\ \mathrm{eV} \qquad BR \sim g_{a_1\gamma\gamma}^2 \times 10^{-18}$$

Expect CPV $\eta \to 3\gamma$ to be significantly smaller

Q: is $\eta \to 3\gamma$ a good place to look for new physics?
what new physics could a $BR > 10^{-6}$ constrain?
A point of discussion which seems interesting.

# Possible points for discussion

- What possible non FC NP could be constrained with eta decays?. Any specific models?
- Update on experimental BRs achievable in present and proposed experiments.
- QCD physics: impact the JEF experiment would have for understanding $\eta \to \pi^0\gamma\gamma$
- Other decays not addressed: anomalous $\eta \to \pi^+\pi^-\gamma$ CV in $\eta \to \pi^0\mu^+\mu^-$ @ $\mathcal{O}(\alpha)$
- lepton family violation $\eta \to \mu^+ e^-$ & $\mu^- e^+$