Lattice QCD Calculation of Nucleon Tensor Charge

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Neutron EDM, Quark EDM and Tensor Charge

- Quark EDMs at dim=5

$$\mathcal{L} = -\frac{i}{2} \sum_{q=u,d,s} d_q \; \bar{q}\sigma_{\mu\nu}\gamma_5 q F^{\mu\nu}$$

- Neutron EDM from qEDMs

$$d_N = d_u \; g_T^{u,N} + d_d \; g_T^{d,N} + d_s \; g_T^{s,N}$$

- Hadronic part: nucleon tensor charge

$$\langle N| \; \bar{q}\sigma_{\mu\nu} q \; |N\rangle = g_T^{q,N} \; \bar{\psi}_N \sigma_{\mu\nu} \psi_N$$

Neutron EDM, Quark EDM and Tensor Charge

- $d_q \propto m_q$ in many models

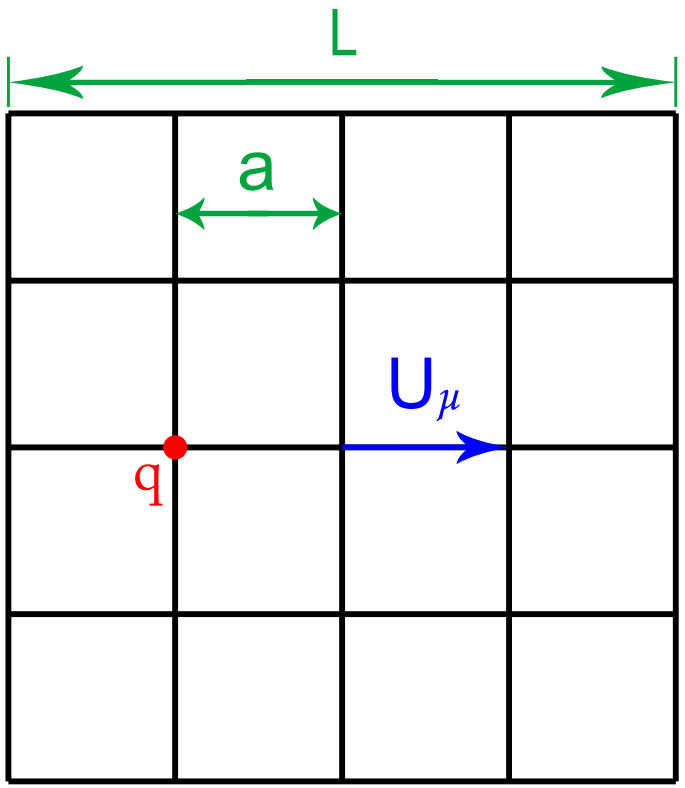
$$d_q = y_q \delta_q; \qquad \frac{y_u}{y_d} \approx \frac{1}{2}, \qquad \frac{y_s}{y_d} \approx 20$$

$$\begin{aligned} d_N &= d_u \, g_T^{u,N} + d_d \, g_T^{d,N} + d_s \, g_T^{s,N} \\ &= d_d \left[g_T^{d,N} + \frac{1}{2} \frac{\delta_u}{\delta_d} \, g_T^{u,N} + 20 \frac{\delta_s}{\delta_d} \, g_T^{s,N} \right] \end{aligned}$$

$\Rightarrow$ Precision determination of $g_T^{s,N}$ is important

Lattice QCD

- **Non-perturbative** approach to understand QCD
- Formulated on **discretized Euclidean space-time**
  - Hypercubic lattice
  - Lattice spacing "a"
  - Quark fields placed on sites
  - Gauge fields on the links between sites; U_μ



Physical Results from Unphysical Simulations

- **Finite Lattice Spacing**
  - Simulations at finite lattice spacings $a \approx 0.06, 0.09$ & 0.12 fm

  $\Rightarrow$ Extrapolate to continuum limit, $a = 0$

- **Heavy Pion Mass**
  - Lattice simulation:

    Smaller quark mass $\longrightarrow$ Larger computational cost
  - Simulations at (heavy) pion masses $M_\pi \approx 130, 210$ & 310 MeV

  $\Rightarrow$ Extrapolate to physical pion mass, $M_\pi = M_\pi^{\text{phys}}$

- **Finite Volume**
  - Simulations at finite lattice volume

    $M_\pi L = 3.2 \sim 5.4$ ($L = 2.9 \sim 5.8\,\text{fm}$)

  $\Rightarrow$ Extrapolate to infinite volume, $M_\pi L = \infty$

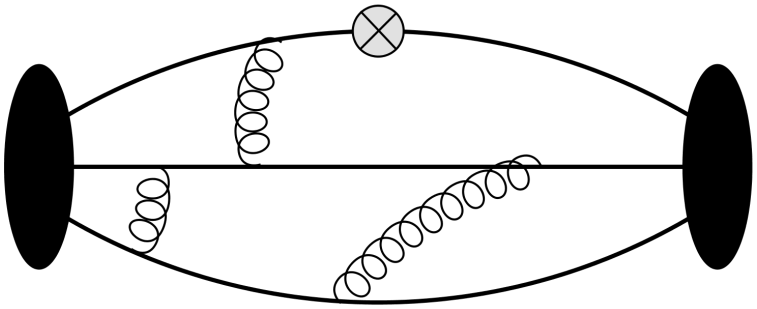
MILC HISQ Lattices, $n_f = 2 + 1 + 1$

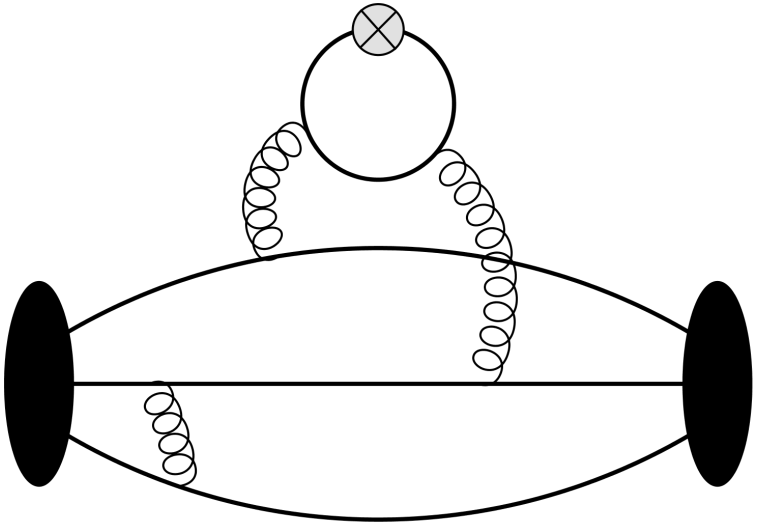
ID	a (fm)	M_π (MeV)	$L^3 \times T$	$M_\pi L$
a12m310	0.1207(11)	305.3(4)	$24^3 \times 64$	4.54
a12m220S	0.1202(12)	218.1(4)	$24^3 \times 64$	3.22
a12m220	0.1184(10)	216.9(2)	$32^3 \times 64$	4.29
a12m220L	0.1189(09)	217.0(2)	$40^3 \times 64$	5.36
a09m310	0.0888(08)	312.7(6)	$32^3 \times 96$	4.50
a09m220	0.0872(07)	220.3(2)	$48^3 \times 96$	4.71
a09m130	0.0871(06)	128.2(1)	$64^3 \times 96$	3.66
a06m310	0.0582(04)	319.3(5)	$48^3 \times 144$	4.51
a06m220	0.0578(04)	229.2(4)	$64^3 \times 144$	4.25

- Fermion discretization : Clover (valence) on HISQ (sea)
- HYP smearing – reduce discretization artifact
- $m_u = m_d$

Three-point Function Diagrams

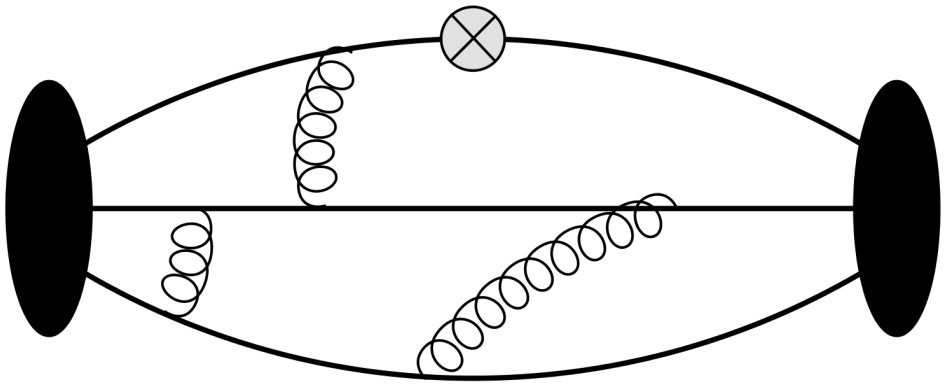
$$\text{ME} \sim \langle N| \, \overline{q}_i \sigma_{\mu\nu} q_j \, |N\rangle$$





- Quark-line connected / disconnected diagrams
- Disconnected diagrams : complicated and expensive on lattice

Connected Quark Loop Contribution



Nucleon Charge on Lattice

- Nucleon tensor charge g_T^q is defined by

$$\langle N| \, \bar{q}\sigma_{\mu\nu}q \, |N\rangle = g_T^q \, \bar{\psi}_N \sigma_{\mu\nu}\psi_N$$

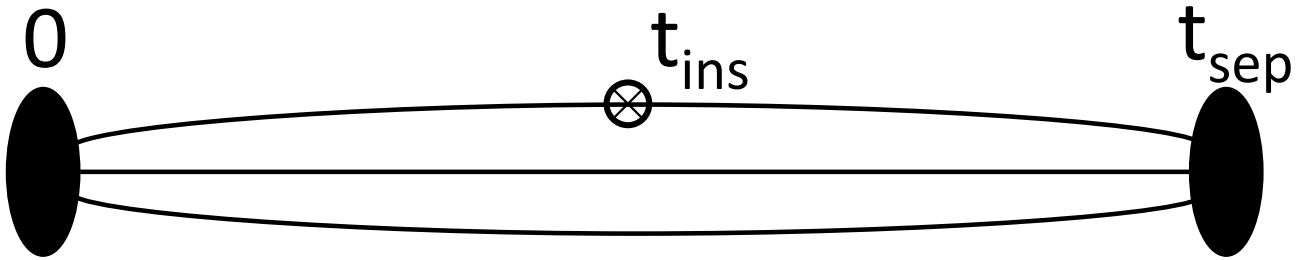
- On lattice, g_T^q is extracted from ratio of 3-pt and 2-pt function

$$C^{\text{3pt}}/C^{\text{2pt}} \longrightarrow g_\Gamma^q$$

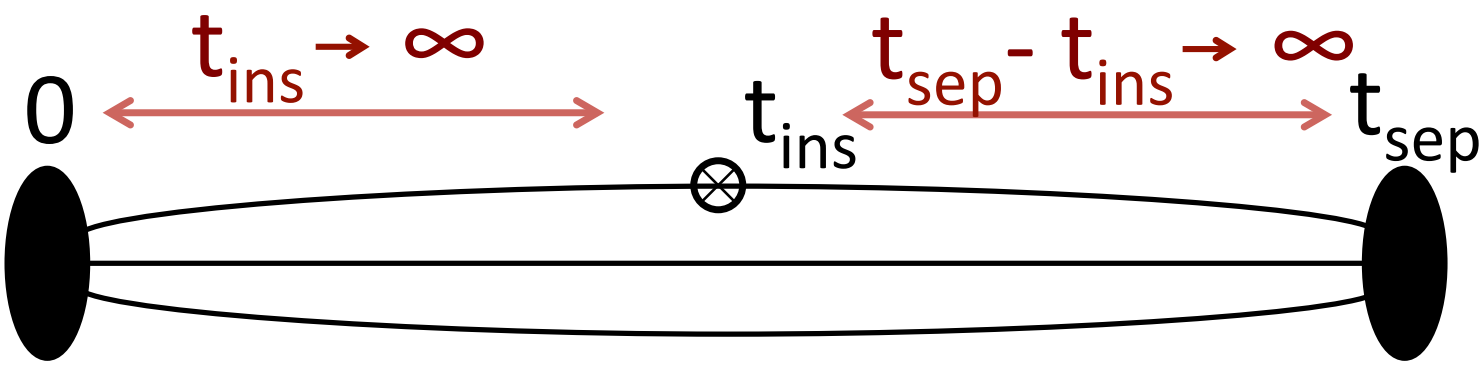
  – $C^{\text{2pt}} = \langle 0| \, \chi(t_{\text{s}}) \, \overline{\chi}(0) \, |0\rangle, \quad C^{\text{3pt}} = \langle 0| \, \chi(t_{\text{s}}) \, \mathcal{O}(t_{\text{i}}) \, \overline{\chi}(0) \, |0\rangle$

  – χ : **interpolating operator** of proton

- χ introduces **excited states** of proton



Removing Excited States Contamination

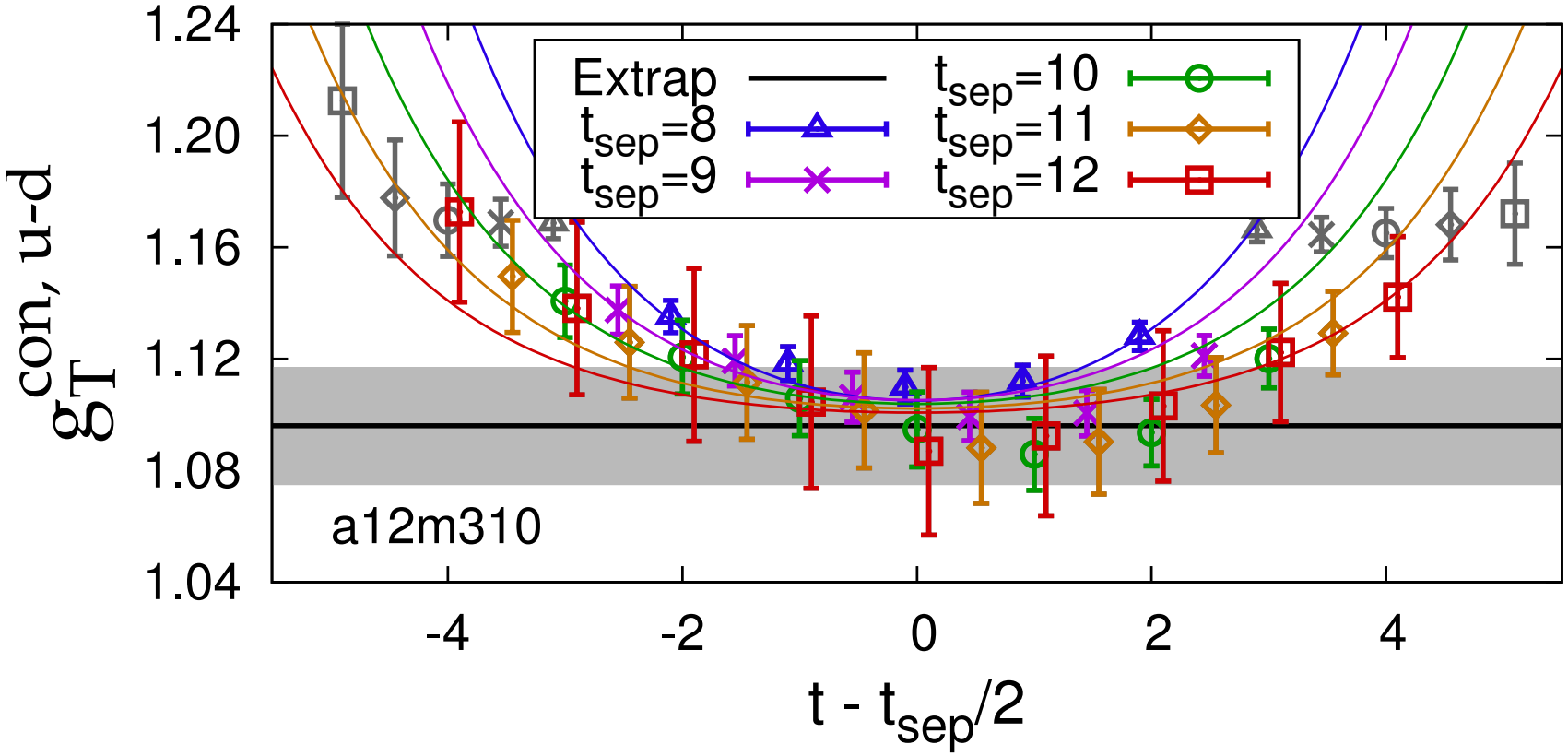


- **Separating** proton sources **far from each other** $\longrightarrow$ small excited state effect, but **weak signal**
- Put operator reasonable range, remove excited state by fitting to

$$C^{\text{2pt}}(t_{\text{sep}}) = A_1 e^{-M_0 t_{\text{sep}}} + A_2 e^{-M_1 t_{\text{sep}}}$$

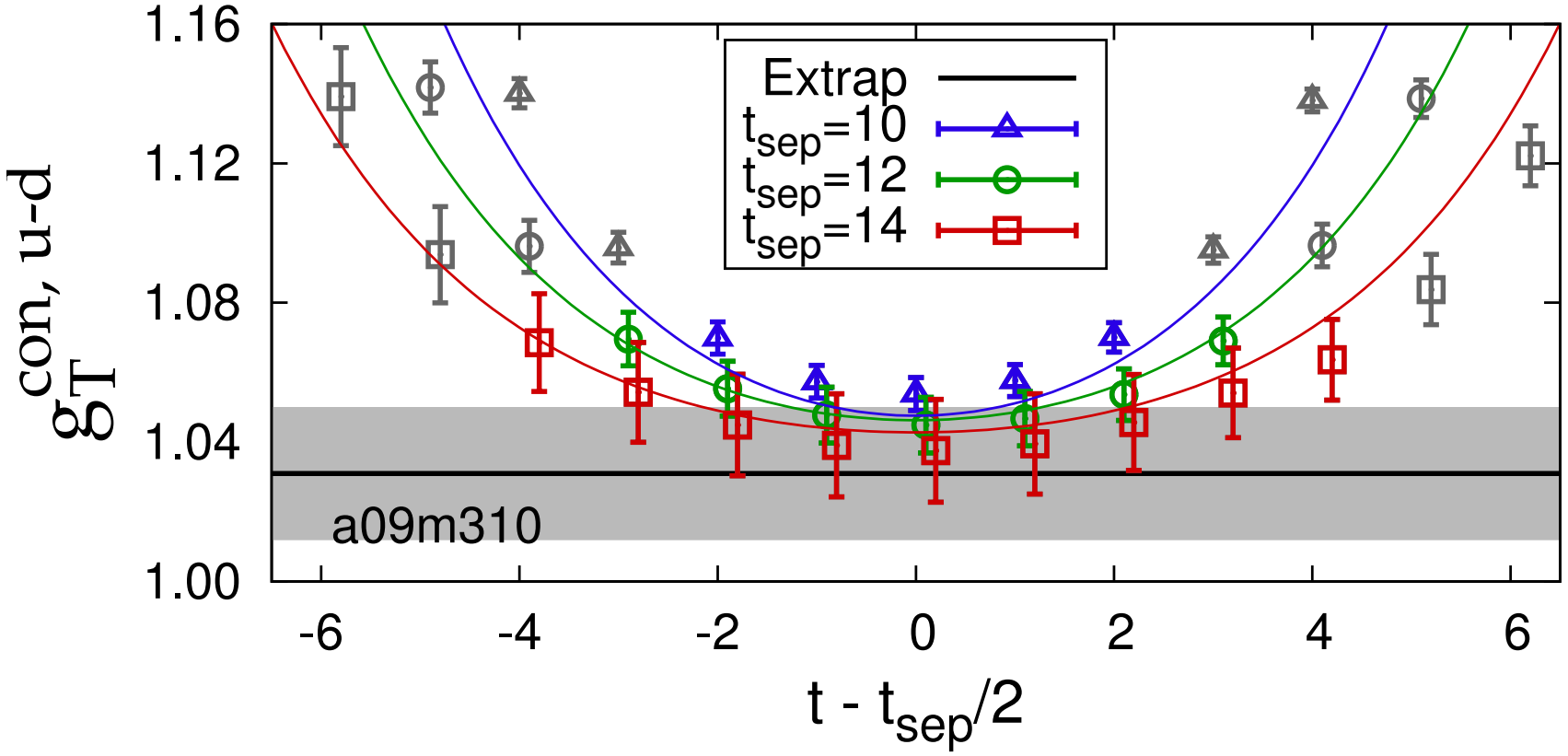
$$C^{\text{3pt}}(t_{\text{sep}}, t_{\text{ins}}) = B_1 e^{-M_0 t_{\text{sep}}} + B_2 e^{-M_1 t_{\text{sep}}} + B_{12}\left[e^{-M_0 t_{\text{ins}}} e^{-M_1 (t_{\text{sep}} - t_{\text{ins}})} + e^{-M_1 t_{\text{ins}}} e^{-M_0 (t_{\text{sep}} - t_{\text{ins}})}\right]$$

Removing Excited States Contamination (a12m310)



- Small excited state contamination (compared to g_A, g_S)

Removing Excited States Contamination (a09m310)



- Small excited state contamination (compared to g_A, g_S)

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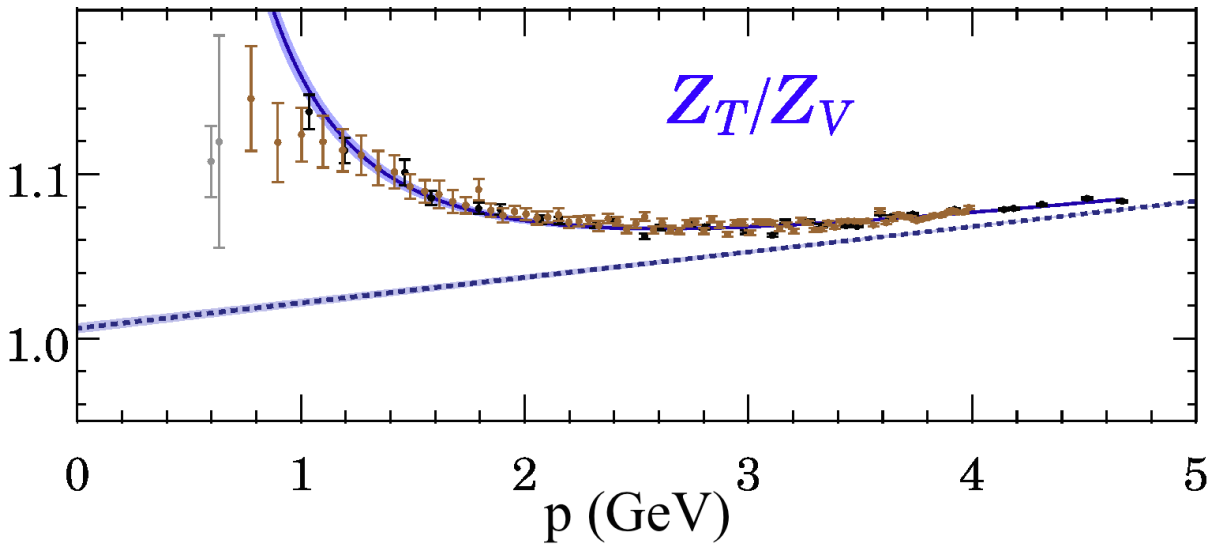
Renormalization of Bilinear Operators $\overline{q}\sigma_{\mu\nu}q$

- Lattice results $\Longrightarrow$ $\overline{\text{MS}}$ scheme at 2GeV
- **Non-perturbative** renormalization using RI-sMOM scheme
- Calculate ratio Z_T/Z_V : reduce lattice artifact
- Renormalized Tensor Charge :

$$g_T^{\text{renorm}} = \frac{Z_T}{Z_V} \times \frac{g_T^{\text{bare}}}{g_V^{\text{bare}}} \qquad \text{(Use } Z_V g_V^{u-d} = 1\text{)}$$

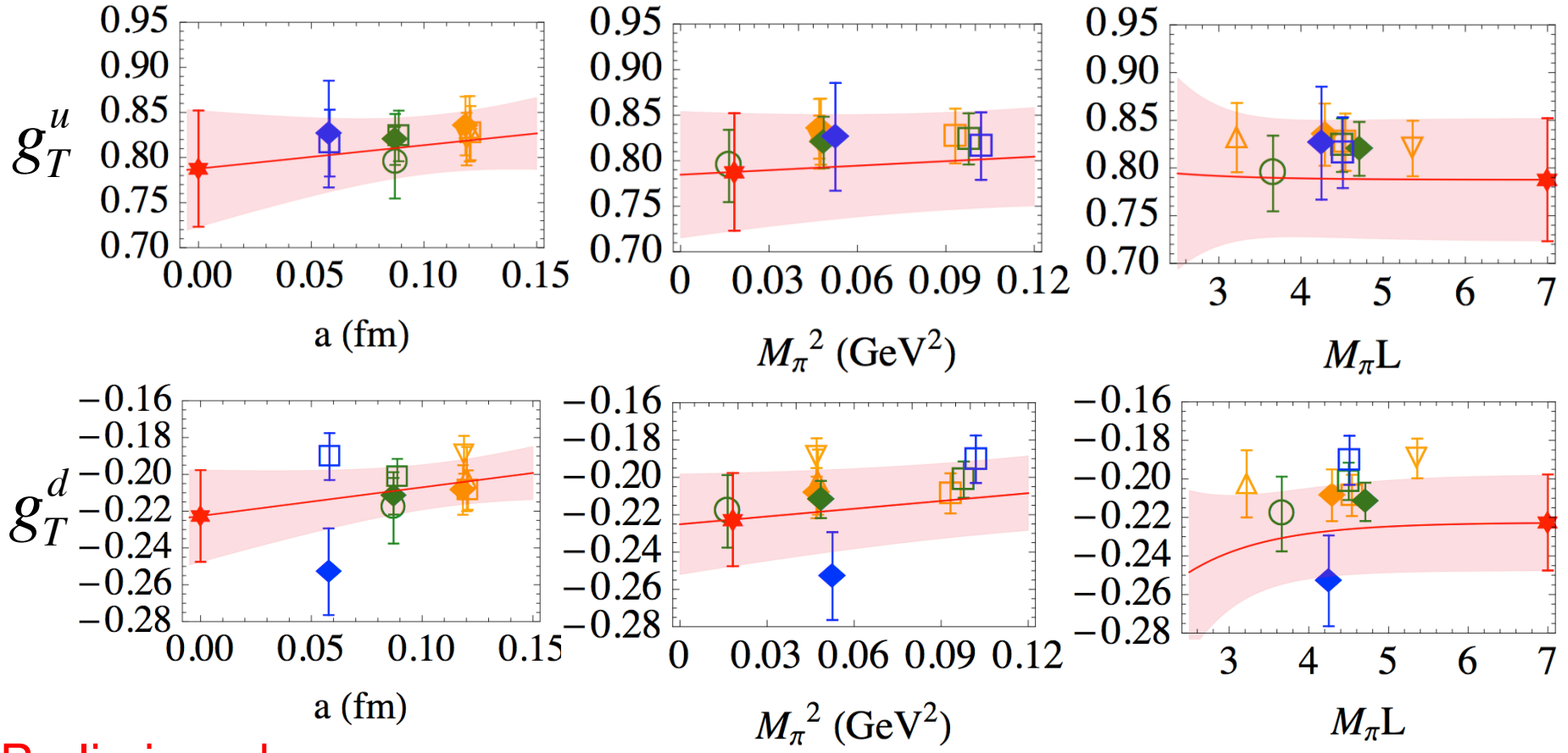


a (fm)	Z_T/Z_V
0.12	1.01(3)
0.09	1.05(3)
0.06	1.07(4)

Simultaneous extrapolation of $(a, M_\pi, M_\pi L)$

$$g_T(a, M_\pi, L) = c_1 + c_2 a + c_3 M_\pi^2 + c_4 e^{-M_\pi L}$$

Lattice Spacing $a \to 0$ **Pion Mass** $M_\pi \to M_\pi^{phys}$ **Lattice Volume** $M_\pi L \to \infty$



Preliminary!

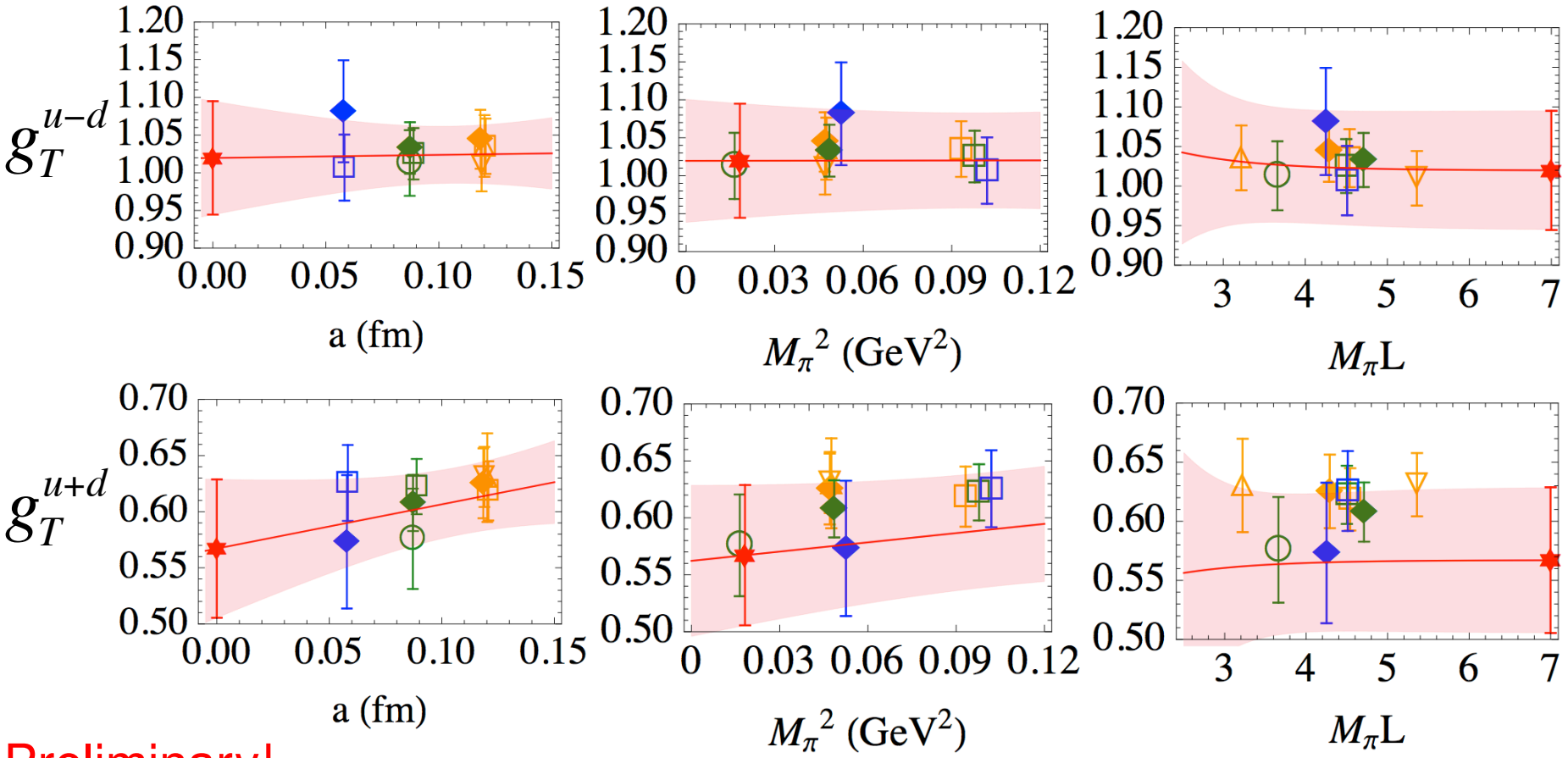
Orange: a=0.12; Green: a=0.09; Blue: a=0.06fm

□=310; ◆=220; ○=130 MeV

Simultaneous extrapolation of $(a, M_\pi, M_\pi L)$

$$g_T(a, M_\pi, L) = c_1 + c_2 a + c_3 M_\pi^2 + c_4 e^{-M_\pi L}$$

Lattice Spacing $a \to 0$ **Pion Mass** $M_\pi \to M_\pi^{phys}$ **Lattice Volume** $M_\pi L \to \infty$



Preliminary!

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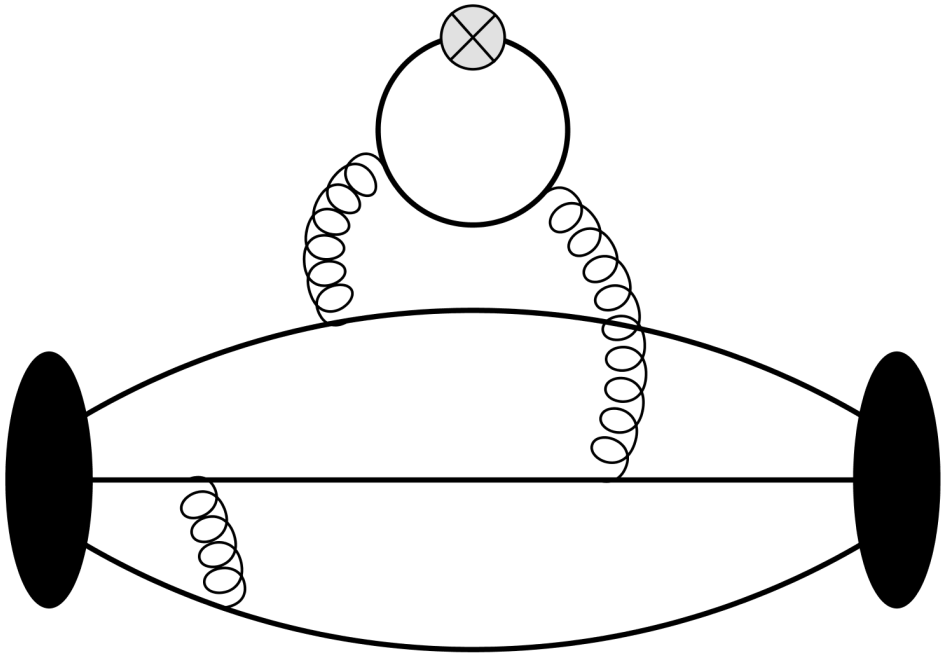
Disconnected Quark Loop Contribution

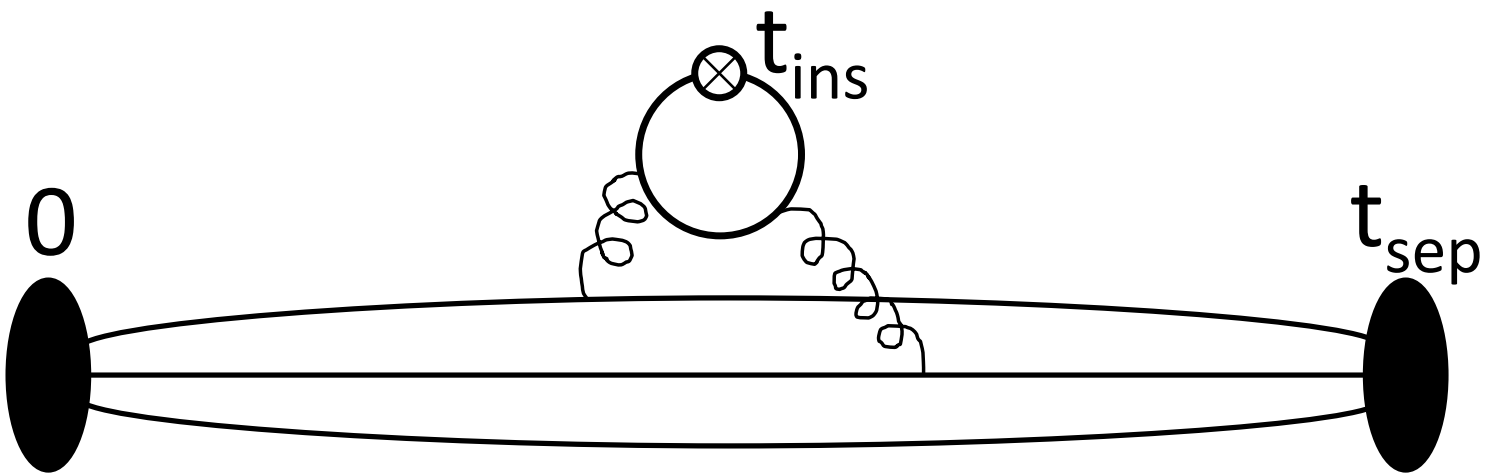


Disconnected Contribution to the Nucleon Charges

Disconnected part of the ratio of 3pt func to 2pt func

$$\left[\frac{C^{\text{3pt}}}{C^{\text{2pt}}}\right]^{\text{disc}} = -\frac{\langle C^{\text{2pt}}(t_{\text{s}}) \sum_{\mathbf{x}} \mathrm{Tr}[M^{-1}(t_{\text{i}}, \mathbf{x}; t_{\text{i}}, \mathbf{x})\sigma_{\mu\nu}]\rangle}{\langle C^{\text{2pt}}(t_{\text{s}})\rangle}$$

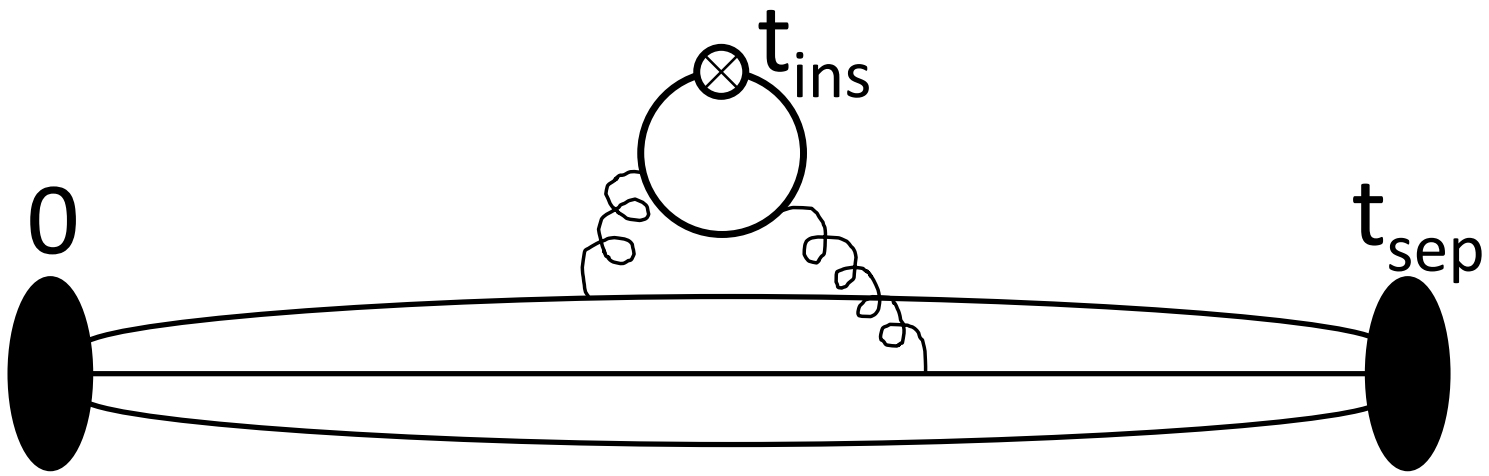
- M : Dirac operator
- $\mathrm{Tr}[M^{-1}(t_{\text{i}}, \mathbf{x}; t_{\text{i}}, \mathbf{x})\sigma_{\mu\nu}]$: disconnected quark loop



Difficulties in Disconnected Diagram Calculation

$$\left[\frac{C^{\text{3pt}}}{C^{\text{2pt}}}\right]^{\text{disc}} = -\frac{\langle C^{\text{2pt}}(t_{\text{s}}) \sum_{\mathbf{x}} \mathrm{Tr}[M^{-1}(t_{\text{i}}, \mathbf{x}; t_{\text{i}}, \mathbf{x})\sigma_{\mu\nu}]\rangle}{\langle C^{\text{2pt}}(t_{\text{s}})\rangle}$$

- Connected calculation needs only point–to–all propagators
  Disconnected quark loop needs all–$\mathbf{x}$–to–all propagators
  $\Rightarrow$ Computationally L^3 times more expensive; need new technique
- Noisy signal $\Rightarrow$ Need more statistics



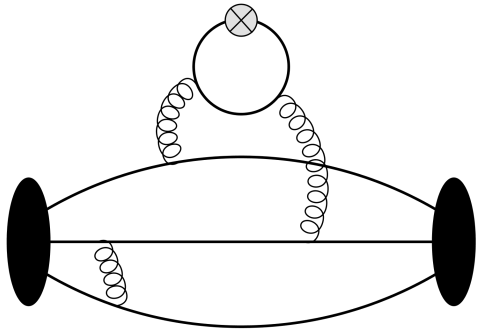
Improvement & Error Reduction Techniques

- Multigrid Solver [Osborn, *et al.*, 2010; Babich, *et al.*, 2010]
- All-Mode Averaging (AMA) for Two-point Correlators [Blum, Izubuchi and Shintani, 2013]
- Hopping Parameter Expansion (HPE) [Thron, *et al.*, 1998; McNeile and Michael , 2001]
- Truncated Solver Method (TSM) [Bali, Collins and Schäfer, 2007]
- Dilution [Bernardson, *et al.*, 1994; Viehoff, *et al.*, 1998]

Improved Estimator of Two-point Function

$$C^{\text{2pt, imp}} = \underbrace{\frac{1}{N_{\text{LP}}} \sum_{i=1}^{N_{\text{LP}}} C^{\text{2pt}}_{\text{LP}}(\mathbf{x}_i)}_{\text{LP estimate}} + \underbrace{\frac{1}{N_{\text{HP}}} \sum_{j=1}^{N_{\text{HP}}} \left[C^{\text{2pt}}_{\text{HP}}(\mathbf{x}_j) - C^{\text{2pt}}_{\text{LP}}(\mathbf{x}_j) \right]}_{\text{Crxn term}}$$

- All-mode averaging (AMA) [Blum, Izubuchi and Shintani, 2013] with Multigrid solver for Clover in Chroma [Osborn, *et al.*, 2010]
- Exploiting translation symmetry & small fluctuation of low-modes
- "LP" term is cheap low-precision estimate
- "HP" (high-precision) correction term
  Systematic error $\Rightarrow$ Statistical error

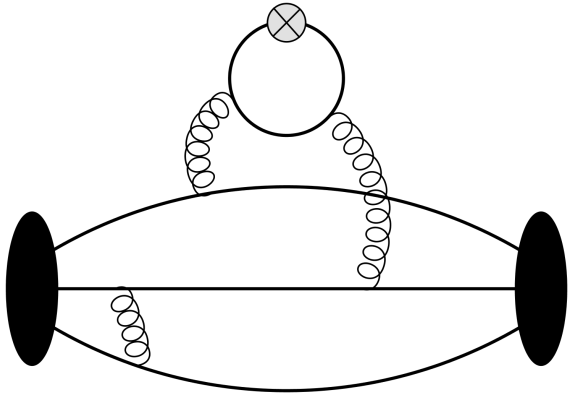


- $N_{\text{LP}} \gg N_{\text{HP}}$ brings computational gain (e.g., N_{LP} = 60, N_{HP} = 4)

Truncated Solver Method (TSM)

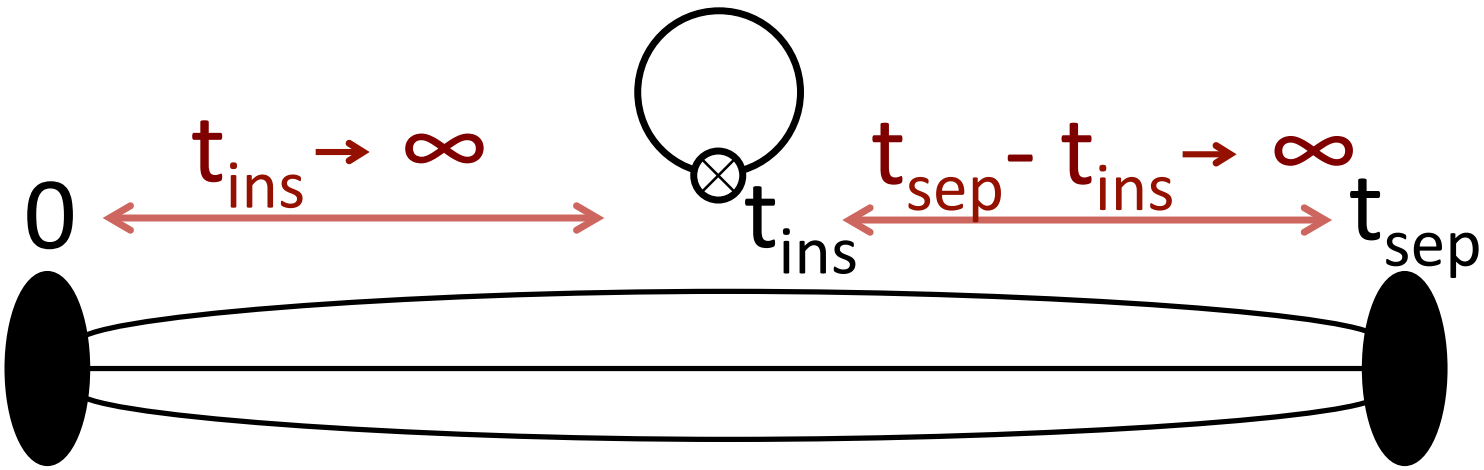
$$M_E^{-1} = \underbrace{\frac{1}{N_{\text{LP}}} \sum_{i=1}^{N_{\text{LP}}} |s_i\rangle_{\text{LP}} \langle \eta_i|}_{\text{LP estimate}} + \underbrace{\frac{1}{N_{\text{HP}}} \sum_{i=N_{\text{LP}}+1}^{N_{\text{LP}}+N_{\text{HP}}} \Big(|s_i\rangle_{\text{HP}} - |s_i\rangle_{\text{LP}} \Big) \langle \eta_i|}_{\text{Crxn term}}$$

- Stochastic estimate of M^{-1} [Bali, Collins and Schäfer, 2007]
  – Do calculate exact M^{-1}, but estimate with reasonable error
  – Computational cost : $\frac{1}{100} \sim \frac{1}{10000}$ of exact calculation
- Same form as AMA
  - $C^{\text{2pt}} \longrightarrow M^{-1}$
  - Sum over source positions
    $\longrightarrow$ Sum over random noise sources



- $|\eta_i\rangle$: complex random noise vector
- $|s_i\rangle$: solution vector; $M|s_i\rangle = |\eta_i\rangle$

Removing Excited States Contamination

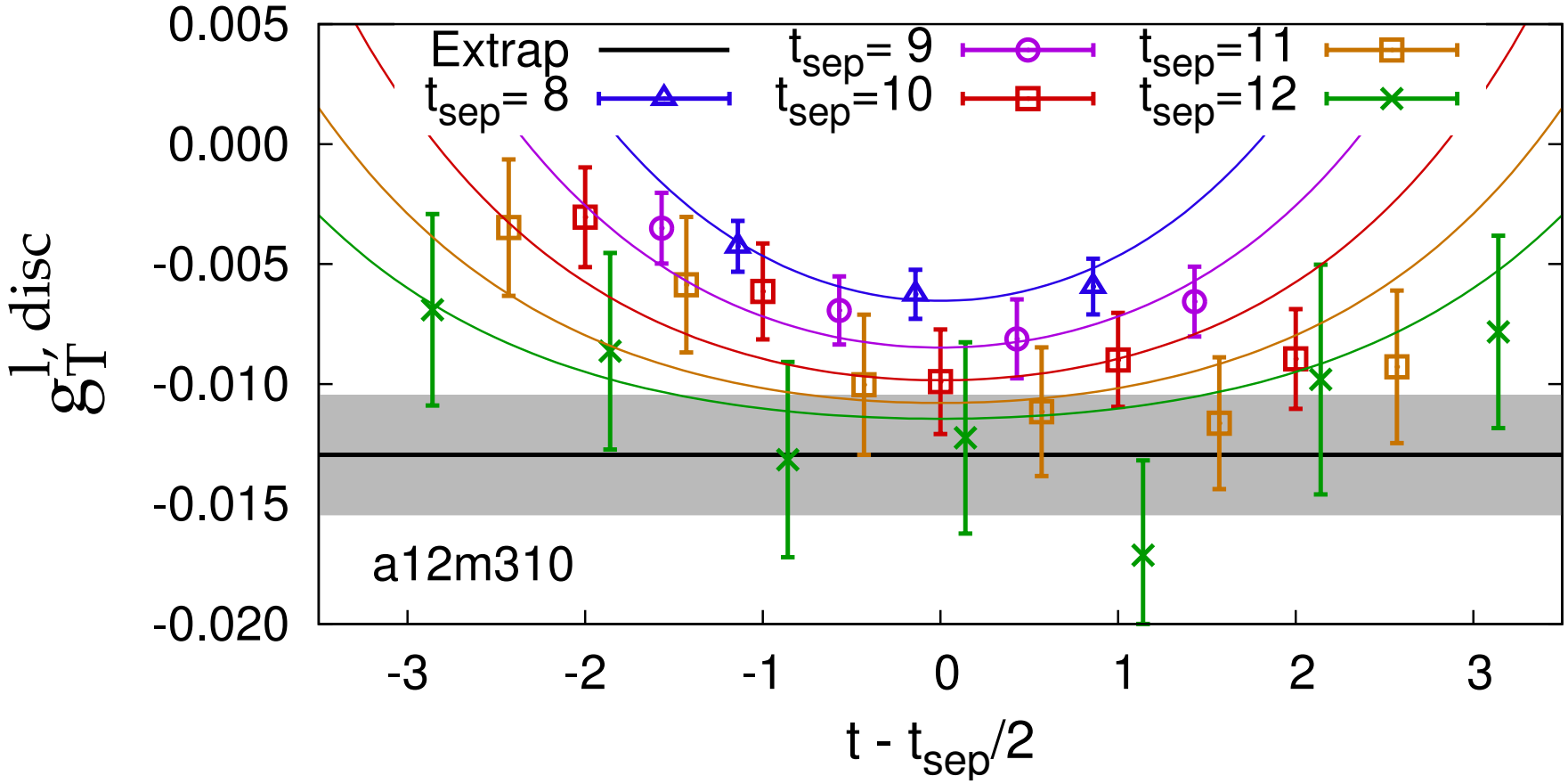


- Interpolating operator introduces excited state contamination
- Remove excited state by fitting to

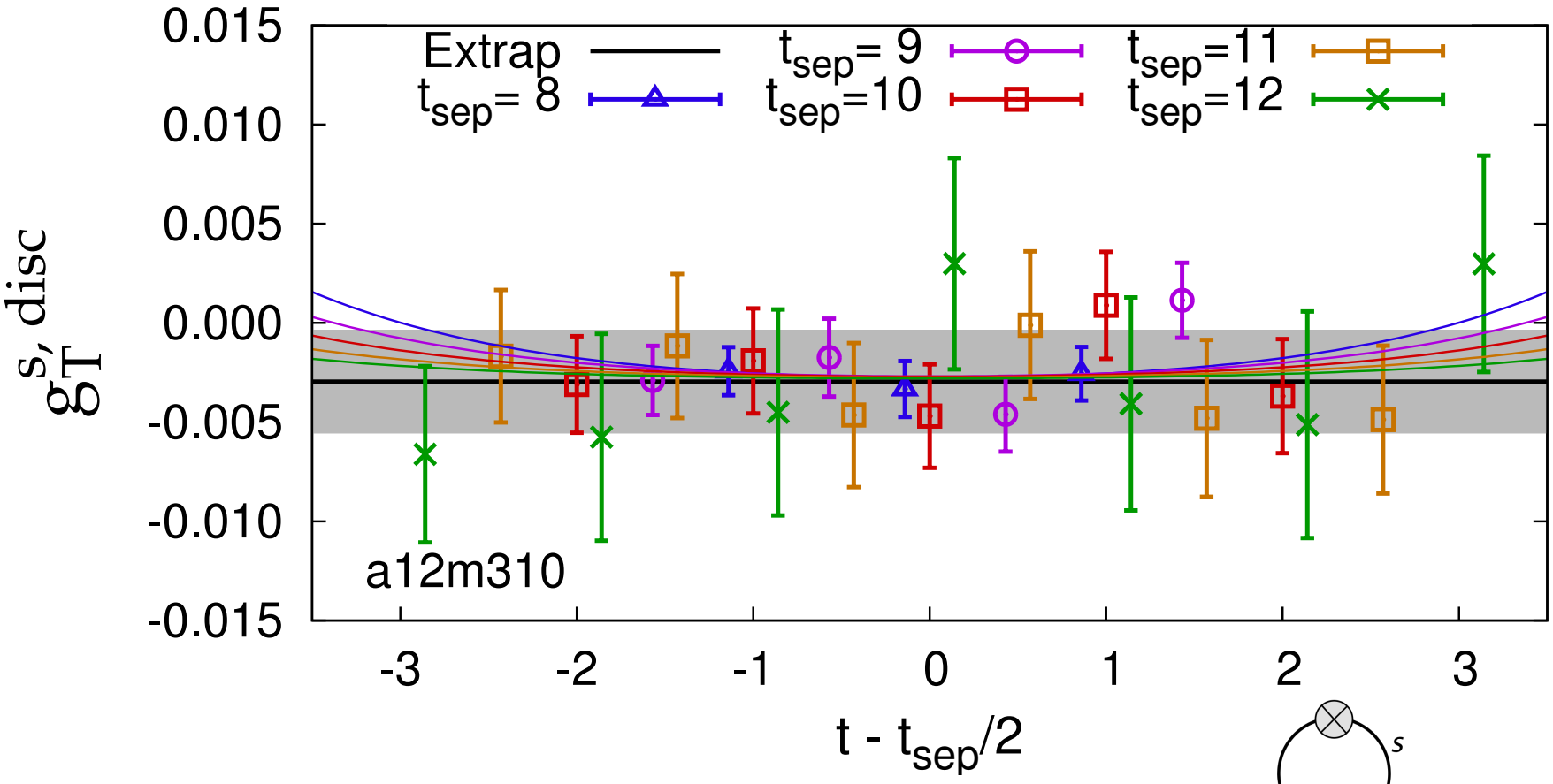
$$C^{\text{2pt}}(t_{\text{sep}}) = A_1 e^{-M_0 t_{\text{sep}}} + A_2 e^{-M_1 t_{\text{sep}}}$$

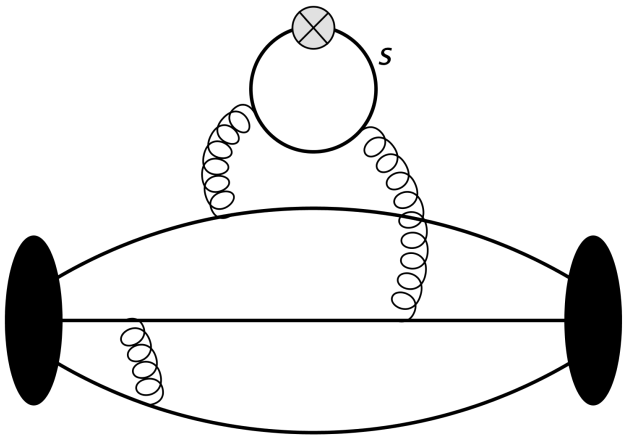
$$C^{\text{3pt}}(t_{\text{sep}}, t_{\text{ins}}) = B_1 e^{-M_0 t_{\text{sep}}} + B_2 e^{-M_1 t_{\text{sep}}} + B_{12}\left[e^{-M_0 t_{\text{ins}}} e^{-M_1 (t_{\text{sep}} - t_{\text{ins}})} + e^{-M_1 t_{\text{ins}}} e^{-M_0 (t_{\text{sep}} - t_{\text{ins}})}\right]$$

Removing Excited States Contamination (a12m310, l)



Removing Excited States Contamination (a12m310, s)





Proton Tensor Charge : Connected / Disconnected

- **Connected Contribution**

g_T^u	g_T^d	g_T^{u-d}	g_T^{u+d}
0.788(64)	$-0.223(25)$	1.020(75)	0.567(62)

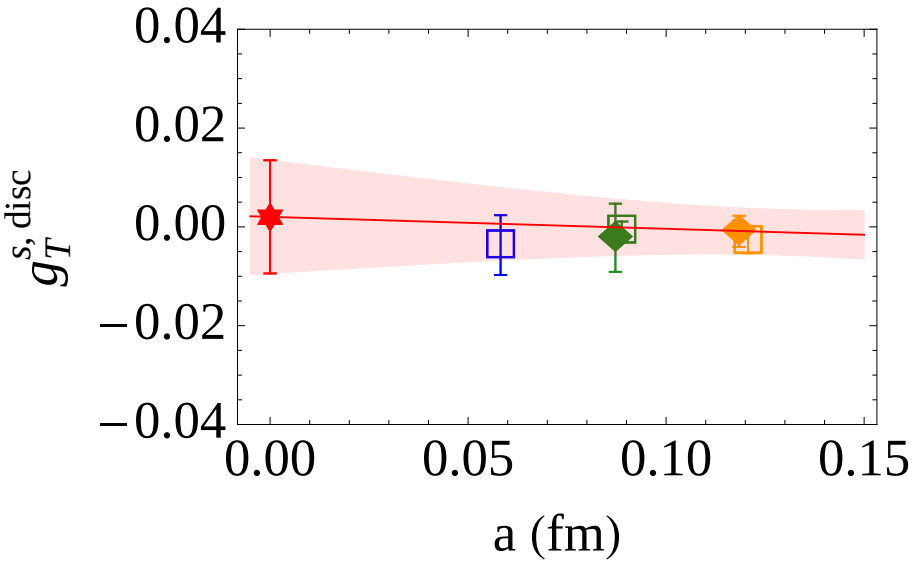
- **Disconnected Contribution**

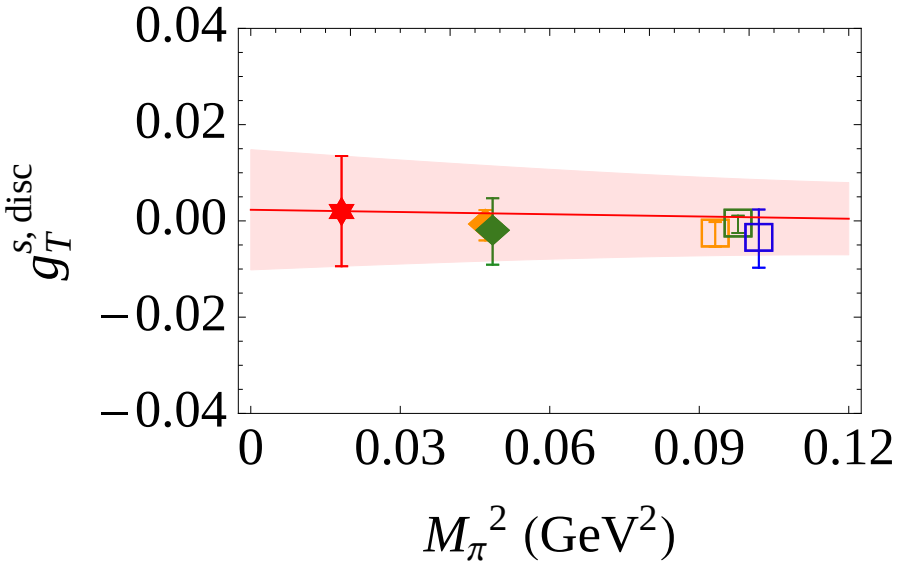
Ens	g_T^l	g_T^s
a12m310	$-0.0122(24)$	$-0.0027(24)$
a12m220	$-0.0030(46)$	$-0.0009(32)$
a09m310	$-0.0052(19)$	$-0.0001(25)$
a09m220	—	$-0.0022(69)$
a06m310	$-0.0051(94)$	$-0.0037(60)$

  – $g_T^{l,\text{disc}}$ is tiny compared to the connected contributions
  $\Rightarrow$ Take maximum value as systematic error

  – No connected diagrams for g_T^s $\Rightarrow$ Extrapolate to physical point

Simultaneous extrapolation of g_T^s in (a, M_π)





$$g_T^s = 0.002(11)$$

Results

Lattice Results of Nucleon Tensor Charge

Preliminary!

- **Proton Tensor Charge** ($\mu^{\overline{\text{MS}}} = 2\,\text{GeV}$)

$$
\begin{aligned}
g_T^u &= 0.79(7) \\
g_T^d &= -0.22(3) \\
g_T^{u-d} &= 1.02(8) \\
g_T^{u+d} &= 0.57(6) \\
g_T^s &= -0.002(11)
\end{aligned}
$$

- **Neutron Tensor Charge**

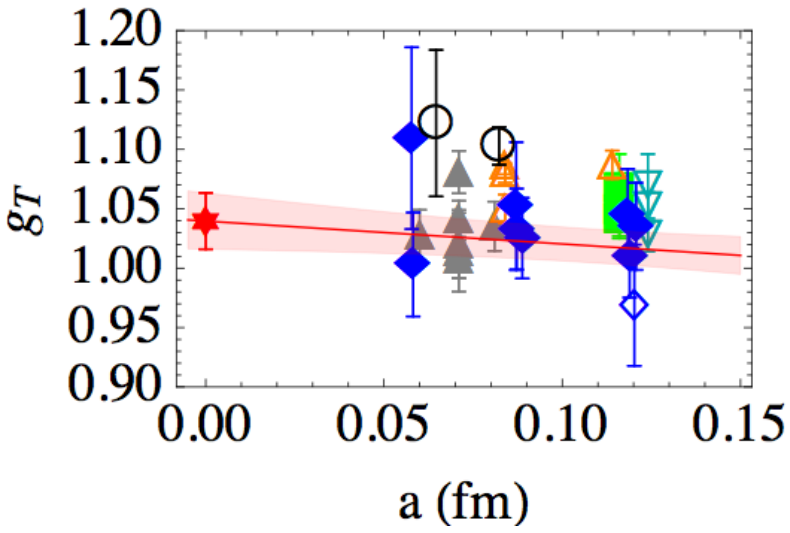
  In isospin limit ($m_u = m_d$), $u \leftrightarrow d$ from proton g_T

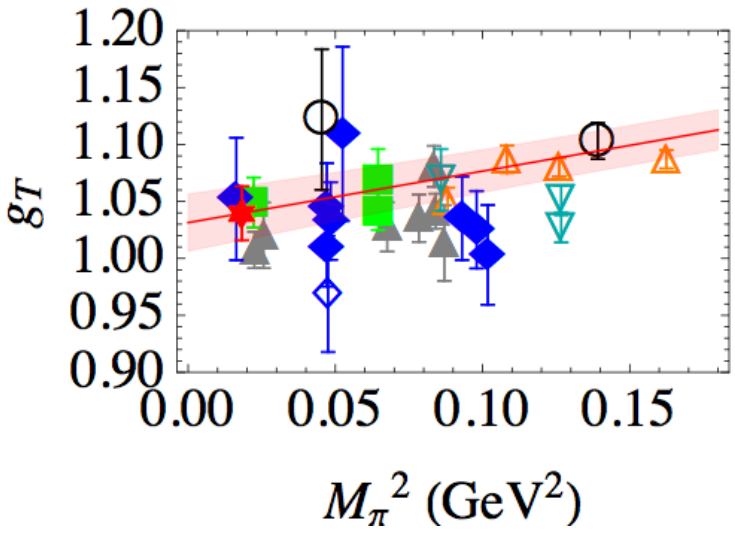
Proton Tensor Charge

- This study $(\mu^{\overline{\text{MS}}} = 2\,\text{GeV})$

$$g_T^u = 0.79(7), \qquad g_T^d = -0.22(3)$$
$$g_T^{u-d} = 1.02(8) \qquad g_T^{u+d} = 0.57(6)$$

- Lattice QCD estimates for g_T^{u-d}





[LHPC, ETMC, RQCD, PNDME]

Proton Tensor Charge

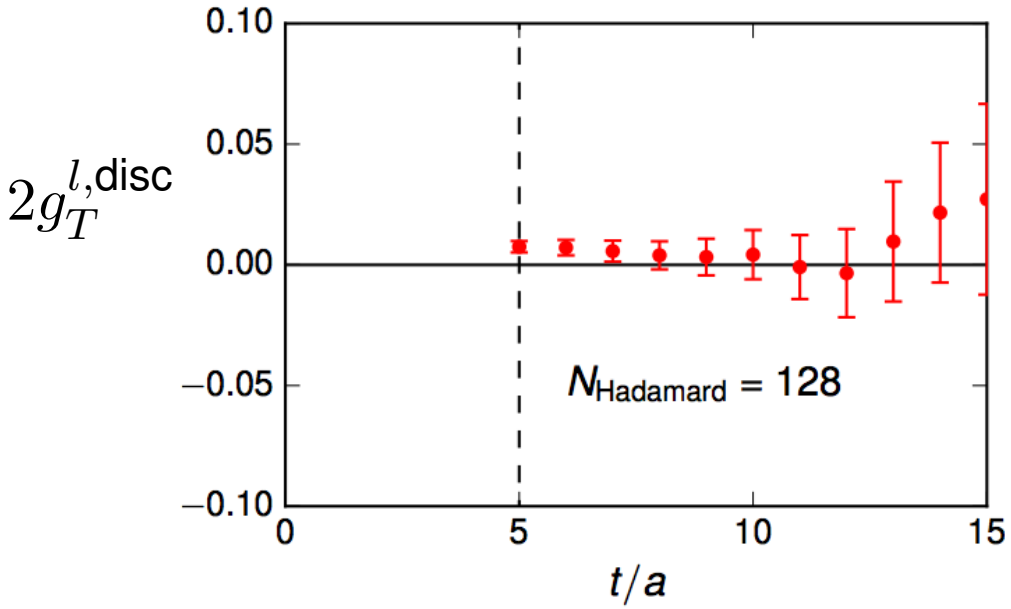
- This study

$$|g_T^{l,\text{disc}}| \leq 0.0122, \qquad g_T^{s,\text{disc}} = 0.002(11)$$

- Lattice, Abdel-Rehim, *et al.*, 2014,
  $a = 0.082\,\text{fm}$, $M_\pi = 370\,\text{MeV}$, Twisted mass

$$g_T^{l,\text{disc}} = 0.0008(7)$$

- Lattice, S. Meinel, *et al.*, 2014,
  $a = 0.11\,\text{fm}$, $M_\pi = 317\,\text{MeV}$, Clover



Proton Tensor Charge

- This study

$$g_T^u = 0.79(7), \qquad g_T^d = -0.22(3) \qquad (\mu^{\overline{\text{MS}}} = 2\,\text{GeV})$$

- Quark model

$$g_T^u = \frac{4}{3}, \qquad g_T^d = -\frac{1}{3}$$

- Dyson-Schwinger [Pitschmann, *et al.*, 2014]

$$g_T^u = 0.55(8), \qquad g_T^d = -0.11(2) \qquad (\zeta_2 = 2\,\text{GeV})$$

- Experiments (HERMES and COMPASS)

$$g_T^u = 0.57(21), \qquad g_T^d = -0.18(33) \qquad (Q^2 = 1.0\,\text{GeV}^2)$$

[Bacchetta, *et al.*, JHEP 2013]

$$g_T^u = 0.39^{+0.18}_{-0.12}, \qquad g_T^d = -0.25^{+0.30}_{-0.10} \qquad (Q^2 = 0.8\,\text{GeV}^2)$$

[Anselmino, *et al.*, PRD 2013]

qEDM and Tensor Charge

$$d_N = d_u\, g_T^{u,N} + d_d\, g_T^{d,N} + d_s\, g_T^{s,N}$$

- Known parameters

  $|d_N| < 2.9 \times 10^{-26} e$ cm (90% C.L.) [Baker, *et al.*, PRL 2006]

  $g_T^{u,N}$ = $-$0.22(3)

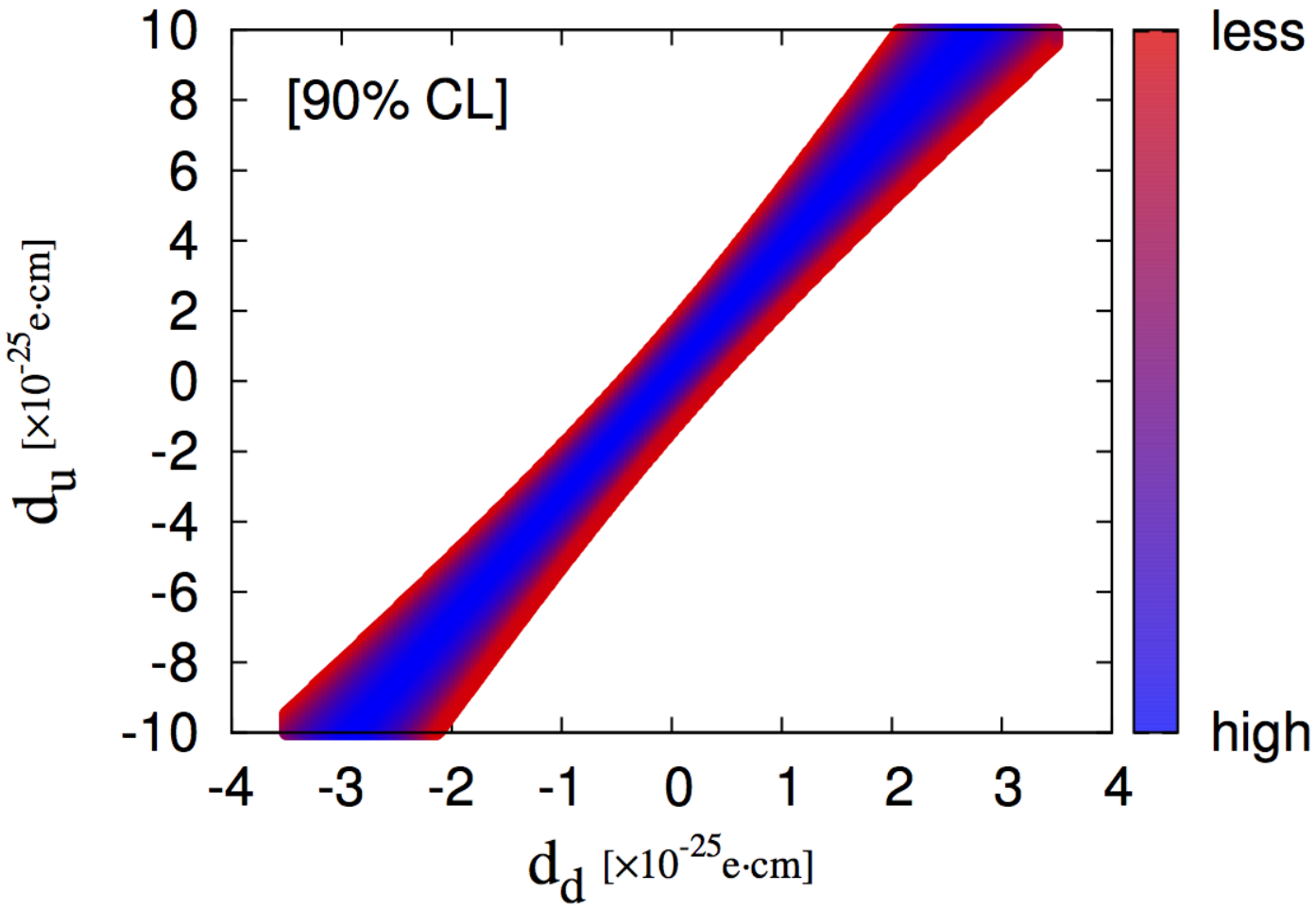
  $g_T^{d,N}$ = 0.79(7)

  $g_T^{s,N}$ = $-$0.002(11)

$\Rightarrow$ Place constraints on d_q

qEDM Constraints

- 90% C.L. parameter space of d_u and d_d, assuming $g_T^s = 0$

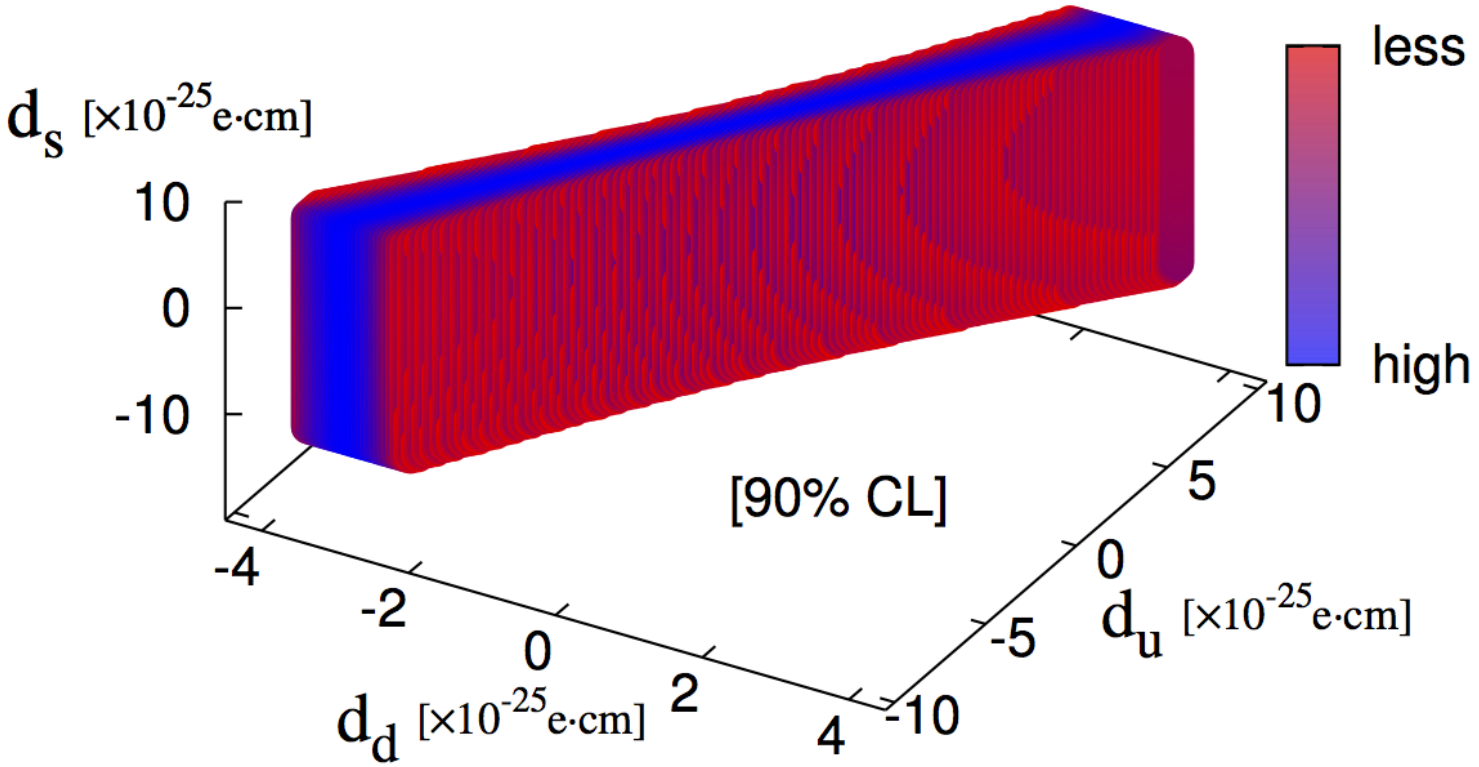


qEDM Constraints

$$d_N = d_u\, g_T^{u,N} + d_d\, g_T^{d,N} + d_s\, g_T^{s,N}$$

$$g_T^{u,N} = -0.223(28), \quad g_T^{d,N} = 0.788(68), \quad g_T^{s,N} = -0.002(11)$$

- Since $g_T^s = 0$ within error, cannot give constraints on d_s



Conclusion

- Presented first lattice QCD calculation of nucleon tensor charge including all systematics (a, M_π, $M_\pi L$, disconnected diagrams)
- Constrained qEDMs by the results combined with experiment
- Need more study on g_T^s to constrain d_s