Lattice QCD Calculation of Nucleon Tensor Charge

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PNDME Collaboration

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Neutron EDM, Quark EDM and Tensor Charge

- Quark EDMs at dim=5

\[ \mathcal{L} = -\frac{i}{2} \sum_{q=u,d,s} d_q \bar{q} \sigma_{\mu\nu} \gamma_5 q F^{\mu\nu} \]

- Neutron EDM from qEDMs

\[ d_N = d_u g^{u,N}_T + d_d g^{d,N}_T + d_s g^{s,N}_T \]

- Hadronic part: nucleon tensor charge

\[ \langle N| \bar{q} \sigma_{\mu\nu} q |N \rangle = g^{q,N}_T \bar{\psi}_N \sigma_{\mu\nu} \psi_N \]
Neutron EDM, Quark EDM and Tensor Charge

- $d_q \propto m_q$ in many models

\[ d_q = y_q \delta_q; \quad \frac{y_u}{y_d} \approx \frac{1}{2}, \quad \frac{y_s}{y_d} \approx 20 \]

\[ d_N = d_u \ g_{T}^{u,N} + d_d \ g_{T}^{d,N} + d_s \ g_{T}^{s,N} \]
\[ = d_d \left[ g_{T}^{d,N} + \frac{1}{2} \frac{\delta_u}{\delta_d} \ g_{T}^{u,N} + 20 \frac{\delta_s}{\delta_d} \ g_{T}^{s,N} \right] \]

$\Rightarrow$ Precision determination of $g_{T}^{s,N}$ is important
Lattice QCD

- **Non-perturbative** approach to understand QCD
- Formulated on **discretized** Euclidean space-time
  - Hypercubic lattice
  - Lattice spacing “$a$”
  - Quark fields placed on sites
  - Gauge fields on the links between sites; $U_\mu$
Physical Results from Unphysical Simulations

- **Finite Lattice Spacing**
  - Simulations at finite lattice spacings $a \approx 0.06, 0.09 \& 0.12$ fm
  $\Rightarrow$ Extrapolate to continuum limit, $a = 0$

- **Heavy Pion Mass**
  - Lattice simulation:
    - Smaller quark mass $\rightarrow$ Larger computational cost
  - Simulations at (heavy) pion masses $M_\pi \approx 130, 210 \& 310$ MeV
  $\Rightarrow$ Extrapolate to physical pion mass, $M_\pi = M_\pi^{\text{phys}}$

- **Finite Volume**
  - Simulations at finite lattice volume
    $M_\pi L = 3.2 \sim 5.4$ ($L = 2.9 \sim 5.8$ fm)
  $\Rightarrow$ Extrapolate to infinite volume, $M_\pi L = \infty$
MILC HISQ Lattices, $n_f = 2 + 1 + 1$

<table>
<thead>
<tr>
<th>ID</th>
<th>$a$ (fm)</th>
<th>$M_\pi$ (MeV)</th>
<th>$L^3 \times T$</th>
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<tbody>
<tr>
<td>a12m310</td>
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- Fermion discretization: Clover (valence) on HISQ (sea)
- HYP smearing – reduce discretization artifact
- $m_u = m_d$
Three-point Function Diagrams

\[ \text{ME} \sim \langle N | \bar{q}_i \sigma_{\mu\nu} q_j | N \rangle \]

- Quark-line connected / disconnected diagrams
- Disconnected diagrams: complicated and expensive on lattice
Connected Quark Loop Contribution
Nucleon Charge on Lattice

- Nucleon tensor charge $g_T^q$ is defined by
  \[ \langle N | \bar{q} \sigma_{\mu\nu} q | N \rangle = g_T^q \bar{\psi}_N \sigma_{\mu\nu} \psi_N \]

- On lattice, $g_T^q$ is extracted from ratio of 3-pt and 2-pt function
  \[ \frac{C^{3\text{pt}}}{C^{2\text{pt}}} \rightarrow g_T^q \]
  - $C^{2\text{pt}} = \langle 0 | \chi(t_s) \bar{\chi}(0) | 0 \rangle$
  - $C^{3\text{pt}} = \langle 0 | \chi(t_s) \mathcal{O}(t_i) \bar{\chi}(0) | 0 \rangle$
  - $\chi$: interpolating operator of proton

- $\chi$ introduces excited states of proton
Removing Excited States Contamination

- **Separating** proton sources far from each other
  $\rightarrow$ small excited state effect, but **weak signal**

- Put operator reasonable range, remove excited state by fitting to

\[
C^{2pt}(t_{sep}) = A_1 e^{-M_0 t_{sep}} + A_2 e^{-M_1 t_{sep}}
\]
\[
C^{3pt}(t_{sep}, t_{ins}) = B_1 e^{-M_0 t_{sep}} + B_2 e^{-M_1 t_{sep}}
+ B_{12} \left[ e^{-M_0 t_{ins}} e^{-M_1 (t_{sep} - t_{ins})} + e^{-M_1 t_{ins}} e^{-M_0 (t_{sep} - t_{ins})} \right]
\]
Removing Excited States Contamination (a12m310)

- Small excited state contamination (compared to $g_A$, $g_S$)
Removing Excited States Contamination (a09m310)

- Small excited state contamination (compared to $g_A$, $g_S$)
MILC HISQ Lattices, $n_f = 2 + 1 + 1$

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Renormalization of Bilinear Operators $\bar{q}\sigma_{\mu\nu}q$

- Lattice results $\Rightarrow$ $\overline{\text{MS}}$ scheme at 2GeV

- Non-perturbative renormalization using RI-sMOM scheme

- Calculate ratio $Z_T/Z_V$ : reduce lattice artifact

- Renormalized Tensor Charge :

\[
g_{T}^{\text{renorm}} = \frac{Z_T}{Z_V} \times \frac{g_{T}^{\text{bare}}}{g_{V}^{\text{bare}}} \tag{Use \(Z_V g_{V}^{u-d} = 1\)}
\]

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<tr>
<td>0.12</td>
<td>1.01(3)</td>
</tr>
<tr>
<td>0.09</td>
<td>1.05(3)</td>
</tr>
<tr>
<td>0.06</td>
<td>1.07(4)</td>
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</tbody>
</table>
Simultaneous extrapolation of \((a, M_\pi, M_\pi L)\)

\[
g_T(a, M_\pi, L) = c_1 + c_2 a + c_3 M_\pi^2 + c_4 e^{-M_\pi L}
\]

**Lattice Spacing** \(a \rightarrow 0\)

**Pion Mass** \(M_\pi \rightarrow M_\pi^{\text{phys}}\)

**Lattice Volume** \(M_\pi L \rightarrow \infty\)

---

**Preliminary!**

- **Orange:** \(a=0.12\);
- **Green:** \(a=0.09\);
- **Blue:** \(a=0.06\) fm

- Square: 310 MeV
- Diamond: 220 MeV
- Circle: 130 MeV
Simultaneous extrapolation of \((a, M_\pi, M_\pi L)\)

\[ g_T(a, M_\pi, L) = c_1 + c_2 a + c_3 M_\pi^2 + c_4 e^{-M_\pi L} \]

**Lattice Spacing** \(a \to 0\)

**Pion Mass** \(M_\pi \to M_\pi^{phys}\)

**Lattice Volume** \(M_\pi L \to \infty\)

---

**Preliminary!**

Orange: \(a=0.12\); Green: \(a=0.09\); Blue: \(a=0.06\) fm

\(\square=310; \quad \blacklozenge=220; \quad \bigcirc=130\) MeV
Disconnected Quark Loop Contribution
Disconnected Contribution to the Nucleon Charges

Disconnected part of the ratio of 3pt func to 2pt func

\[
\left[ \frac{C^{3\text{pt}}}{C^{2\text{pt}}} \right]^{\text{disc}} = -\frac{\langle C^{2\text{pt}}(t_s) \sum_x \text{Tr}[M^{-1}(t_i, x; t_i, x)\sigma_{\mu\nu}] \rangle}{\langle C^{2\text{pt}}(t_s) \rangle}
\]

- \( M \) : Dirac operator
- \( \text{Tr}[M^{-1}(t_i, x; t_i, x)\sigma_{\mu\nu}] \) : disconnected quark loop
Difficulties in Disconnected Diagram Calculation

\[
\left[ \frac{C^{3\text{pt}}}{C^{2\text{pt}}} \right]^{\text{disc}} = - \frac{\langle C^{2\text{pt}}(t_s) \sum_x \text{Tr}[M^{-1}(t_i, x; t_i, x) \sigma_{\mu\nu}] \rangle}{\langle C^{2\text{pt}}(t_s) \rangle}
\]

- Connected calculation needs only point–to–all propagators
  Disconnected quark loop needs all–x–to–all propagators
  \( \Rightarrow \) Computationally \( L^3 \) times more expensive; need new technique
- Noisy signal \( \Rightarrow \) Need more statistics
Improvement & Error Reduction Techniques

- Multigrid Solver [Osborn, et al., 2010; Babich, et al., 2010]

- All-Mode Averaging (AMA) for Two-point Correlators [Blum, Izubuchi and Shintani, 2013]

- Hopping Parameter Expansion (HPE) [Thron, et al., 1998; McNeile and Michael, 2001]

- Truncated Solver Method (TSM) [Bali, Collins and Schäfer, 2007]

- Dilution [Bernardson, et al., 1994; Viehoff, et al., 1998]
Improved Estimator of Two-point Function

\[
C_{2pt, \text{imp}} = \frac{1}{N_{\text{LP}}} \sum_{i=1}^{N_{\text{LP}}} C_{\text{LP}}^{2pt}(x_i) + \frac{1}{N_{\text{HP}}} \sum_{j=1}^{N_{\text{HP}}} \left[ C_{\text{HP}}^{2pt}(x_j) - C_{\text{LP}}^{2pt}(x_j) \right]
\]

- All-mode averaging (AMA) [Blum, Izubuchi and Shintani, 2013] with Multigrid solver for Clover in Chroma [Osborn, et al., 2010]
- Exploiting translation symmetry & small fluctuation of low-modes
- “LP” term is cheap low-precision estimate
- “HP” (high-precision) correction term
- Systematic error ⇒ Statistical error
- \(N_{\text{LP}} \gg N_{\text{HP}}\) brings computational gain (e.g., \(N_{\text{LP}} = 60, N_{\text{HP}} = 4\))
Truncated Solver Method (TSM)

\[
M_E^{-1} = \frac{1}{N_{LP}} \sum_{i=1}^{N_{LP}} |s_i\rangle_{LP} \langle \eta_i | + \frac{1}{N_{HP}} \sum_{i=N_{LP}+1}^{N_{LP}+N_{HP}} \left( |s_i\rangle_{HP} - |s_i\rangle_{LP} \right) \langle \eta_i |
\]

- **Stochastic estimate of** \( M^{-1} \) [Bali, Collins and Schäfer, 2007]
  - Do calculate exact \( M^{-1} \), but estimate with reasonable error
  - Computational cost: \( \frac{1}{100} \sim \frac{1}{10000} \) of exact calculation
- **Same form as AMA**
  - \( C^{2pt} \longrightarrow M^{-1} \)
  - Sum over source positions
    \( \longrightarrow \) Sum over random noise sources
- \(|\eta_i\rangle\) : complex random noise vector
- \(|s_i\rangle\) : solution vector; \( M|s_i\rangle = |\eta_i\rangle \)
Removing Excited States Contamination

- Interpolating operator introduces excited state contamination
- Remove excited state by fitting to

\[
C^{2\text{pt}}(t_{\text{sep}}) = A_1 e^{-M_0 t_{\text{sep}}} + A_2 e^{-M_1 t_{\text{sep}}}
\]

\[
C^{3\text{pt}}(t_{\text{sep}}, t_{\text{ins}}) = B_1 e^{-M_0 t_{\text{sep}}} + B_2 e^{-M_1 t_{\text{sep}}}
\]

\[
+ B_{12} \left[ e^{-M_0 t_{\text{ins}}} e^{-M_1 (t_{\text{sep}}-t_{\text{ins}})} + e^{-M_1 t_{\text{ins}}} e^{-M_0 (t_{\text{sep}}-t_{\text{ins}})} \right]
\]
Removing Excited States Contamination \((a_{12m310}, s)\)

![Graph showing the effect of removing excited states contamination](image)

- \(g_{T, \text{disc}} \) values for different \(t_{\text{sep}}\) values:
  - \(t_{\text{sep}} = 8\)
  - \(t_{\text{sep}} = 9\)
  - \(t_{\text{sep}} = 10\)
  - \(t_{\text{sep}} = 11\)
  - \(t_{\text{sep}} = 12\)

- The graph illustrates the extrapolation of \(g_{T, \text{disc}}\) with respect to \(t - t_{\text{sep}}/2\).

- The shaded area represents the range of \(g_{T, \text{disc}}\) values.

- The data points are shown with error bars, indicating the uncertainty in the measurements.

- The graph includes a symbol representing the excited state contamination effect, labeled \(s\).
Proton Tensor Charge: Connected / Disconnected

- Connected Contribution

\[
\begin{align*}
g_T^u & = 0.788(64) \\
g_T^d & = -0.223(25) \\
g_T^{u-d} & = 1.020(75) \\
g_T^{u+d} & = 0.567(62)
\end{align*}
\]

- Disconnected Contribution

<table>
<thead>
<tr>
<th>Ens</th>
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<th>(g_T^s)</th>
</tr>
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<tr>
<td>a12m310</td>
<td>-0.0122(24)</td>
<td>-0.0027(24)</td>
</tr>
<tr>
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<td>-0.0030(46)</td>
<td>-0.0009(32)</td>
</tr>
<tr>
<td>a09m310</td>
<td>-0.0052(19)</td>
<td>-0.0001(25)</td>
</tr>
<tr>
<td>a09m220</td>
<td>—</td>
<td>-0.0022(69)</td>
</tr>
<tr>
<td>a06m310</td>
<td>-0.0051(94)</td>
<td>-0.0037(60)</td>
</tr>
</tbody>
</table>

- \(g_T^l,\text{disc}\) is tiny compared to the connected contributions
  \(\Rightarrow\) Take maximum value as systematic error

- No connected diagrams for \(g_T^s\) \(\Rightarrow\) Extrapolate to physical point
Simultaneous extrapolation of $g_T^s$ in $(a, M_{\pi})$

$g_T^s = 0.002(11)$
Results
Lattice Results of Nucleon Tensor Charge

Preliminary!

- **Proton Tensor Charge** ($\mu^{\overline{MS}} = 2$ GeV)

  \[
  g_T^u = 0.79(7) \\
  g_T^d = -0.22(3) \\
  g_T^{u-d} = 1.02(8) \\
  g_T^{u+d} = 0.57(6) \\
  g_T^s = -0.002(11)
  \]

- **Neutron Tensor Charge**

  In isospin limit ($m_u = m_d$), $u \leftrightarrow d$ from proton $g_T$
Proton Tensor Charge

- This study ($\mu^{\text{MS}} = 2 \text{ GeV}$)

$$
\begin{align*}
  g_T^u &= 0.79(7), & g_T^d &= -0.22(3) \\
  g_T^{u-d} &= 1.02(8), & g_T^{u+d} &= 0.57(6)
\end{align*}
$$

- Lattice QCD estimates for $g_T^{u-d}$

[LHPC, ETMC, RQCD, PNDME]
Proton Tensor Charge

- This study

\[ |g_T^{l,\text{disc}}| \leq 0.0122, \quad g_T^{s,\text{disc}} = 0.002(11) \]

- Lattice, Abdel-Rehim, et al., 2014,
  \( a = 0.082 \text{ fm}, \; M_\pi = 370 \text{ MeV}, \) Twisted mass

\[ g_T^{l,\text{disc}} = 0.0008(7) \]

- Lattice, S. Meinel, et al., 2014,
  \( a = 0.11 \text{ fm}, \; M_\pi = 317 \text{ MeV}, \) Clover

![Graph showing the relationship between 2g_T^{l,\text{disc}}, t/a, and N_{\text{Hadamard}} = 128]
Proton Tensor Charge

- This study

\[ g_u^T = 0.79(7), \quad g_d^T = -0.22(3) \quad (\mu_{\text{MS}} = 2 \text{ GeV}) \]

- Quark model

\[ g_u^T = \frac{4}{3}, \quad g_d^T = -\frac{1}{3} \]

- Dyson-Schwinger [Pitschmann, et al., 2014]

\[ g_u^T = 0.55(8), \quad g_d^T = -0.11(2) \quad (\zeta_2 = 2 \text{ GeV}) \]

- Experiments (HERMES and COMPASS)

\[ g_u^T = 0.57(21), \quad g_d^T = -0.18(33) \quad (Q^2 = 1.0 \text{ GeV}^2) \]

[Bacchetta, et al., JHEP 2013]

\[ g_u^T = 0.39_{-0.12}^{+0.18}, \quad g_d^T = -0.25_{-0.10}^{+0.30} \quad (Q^2 = 0.8 \text{ GeV}^2) \]

[Anselmino, et al., PRD 2013]
qEDM and Tensor Charge

\[ d_N = d_u \ g_{T}^{u,N} + d_d \ g_{T}^{d,N} + d_s \ g_{T}^{s,N} \]

- Known parameters

\[ |d_N| < 2.9 \times 10^{-26} e \text{ cm (90\% C.L.)} \]

\[ g_{T}^{u,N} = -0.22(3) \]

\[ g_{T}^{d,N} = 0.79(7) \]

\[ g_{T}^{s,N} = -0.002(11) \]

⇒ Place constraints on \( d_q \)
qEDM Constraints

- 90% C.L. parameter space of $d_u$ and $d_d$, assuming $g_T^s = 0$
qEDM Constraints

\[ d_N = d_u \, g_{T}^{u,N} + d_d \, g_{T}^{d,N} + d_s \, g_{T}^{s,N} \]

\[ g_{T}^{u,N} = -0.223(28), \quad g_{T}^{d,N} = 0.788(68), \quad g_{T}^{s,N} = -0.002(11) \]

- Since \( g_{T}^{s} = 0 \) within error, cannot give constraints on \( d_s \)
Conclusion

- Presented first lattice QCD calculation of nucleon tensor charge including all systematics ($a$, $M_\pi$, $M_\pi L$, disconnected diagrams)
- Constrained qEDMs by the results combined with experiment
- Need more study on $g_T^s$ to constrain $d_s$