Testing CVC and CKM unitarity via superallowed nuclear $\beta$ decay

J.C. Hardy
Cyclotron Institute
Texas A&M University
SUPERALLOWED $0^+ \rightarrow 0^+$ BETA DECAY

BASIC WEAK-DECAY EQUATION

\[ ft = \frac{K}{G_v^2} <\tau>^2 \]

- $f$ = statistical rate function: $f(Z, Q_{EC})$
- $t$ = partial half-life: $f(t_{1/2}, BR)$
- $G_v$ = vector coupling constant
- $<\tau>$ = Fermi matrix element

EXPERIMENT
SUPERALLOWED $0^+ \rightarrow 0^+$ BETA DECAY

BASIC WEAK-DECAY EQUATION

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INCLUDING RADIATIVE AND ISOSPIN-SYMMETRY-BREAKING CORRECTIONS

$$ \mathcal{F} t = f_t (1 + \delta'_R) [1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_v^2 (1 + \Delta_R)} $$
SUPERALLOWED $0^+ \rightarrow 0^+$ BETA DECAY

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INCLUDING RADIATIVE AND ISOSPIN-SYMMETRY-BREAKING CORRECTIONS

$$\mathcal{F}t = ft \left(1 + \delta'_R\right) \left[1 - (\delta_C - \delta_{NS})\right] = \frac{K}{2G_v^2 (1 + \Delta_R)}$$

- $f(Z, Q_{EC}) \sim 1.5\%$
- $f(\text{nuclear structure}) \sim 0.3-1.5\%$
- $f(\text{interaction}) \sim 2.4\%$
SUPERALLOWED $0^+ \rightarrow 0^+$ BETA DECAY

**BASIC WEAK-DECAY EQUATION**

$$ft = \frac{K}{G_v^2 <\tau>^2}$$

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**INCLUDING RADIATIVE AND ISOSPIN-SYMMETRY-BREAKING CORRECTIONS**

$$\mathcal{F}t = ft (1 + \delta_R') [1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_v^2 (1 + \Delta_R)}$$

- $f(Z, Q_{EC})$ ~1.5%
- $f(\text{nuclear structure})$ 0.3-1.5%
- $f(\text{interaction})$ ~2.4%

**THEORETICAL UNCERTAINTIES** 0.05 – 0.10%
1. Radiative corrections

\[ \delta'_R = \frac{\alpha}{2\pi} [g(E_m) + \delta_2 + \delta_3 + \ldots] \]
\[ \Delta_R = \frac{\alpha}{2\pi} [4 \ln(m_z/m_p) + \ln(m_p/m_A) + 2C_{\text{Born}} + \ldots] \]

2. Isospin symmetry-breaking corrections

\[ \delta_C \quad \text{Charge-dependent mismatch between parent and daughter analog states (members of the same isospin triplet).} \]
\[ \delta_C = \delta_{C1} + \delta_{C2} \]

Difference in configuration mixing between parent and daughter.

- Shell-model calculation with well-established 2-body matrix elements.
- Charge dependence tuned to known single-particle energies and to measured IMME coefficients.
- Results also adjusted to measured non-analog \(0^+\) state energies.

0.01 – 0.3 %

Mismatch in radial wave function between parent and daughter.

- Full-parentage Saxon-Woods wave function also matched to known binding energy and charge radius from electron scattering.
- Compared with Hartree-Fock calculation matched to known binding energy.
- Core states included based on measured spectroscopic factors.

0.4 – 1.5 %
WHAT CAN WE LEARN?
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FROM A SINGLE TRANSITION

Experimentally determine $G_v^2 (1 + \Delta_R)$

$$\mathcal{F}t = ft (1 + \delta'_R)[1 - (\delta_c - \delta_{NS})] = \frac{K}{2G_v^2 (1 + \Delta_R)}$$
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FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate the correction terms

Test for presence of a Scalar current

$\mathcal{F}t$ values constant
WHAT CAN WE LEARN?

FROM A SINGLE TRANSITION

Experimentally determine $G_v^2 (1 + \Delta_R)$

\[ \mathcal{F}_t = ft (1 + \delta_R') [1 - (\delta_c - \delta_{NS})] = \frac{K}{2G_v^2 (1 + \Delta_R)} \]

FROM MANY TRANSITIONS

- Test Conservation of the Vector current (CVC)
- Validate the correction terms
- Test for presence of a Scalar current

WITH CVC VERIFIED

\[ \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{u_d} & V_{u_s} & V_{u_b} \\ V_{c_d} & V_{c_s} & V_{c_b} \\ V_{t_d} & V_{t_s} & V_{t_b} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \]

Obtain precise value of $G_v^2 (1 + \Delta_R)$

Determine $V_{u_d}^2$

$V_{u_d}^2 = G_v^2 / G_\mu^2$
WHAT CAN WE LEARN?

FROM A SINGLE TRANSITION

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\[ \mathcal{F}t = ft (1 + \delta'_R)[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_v^2 (1 + \Delta_R)} \]

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validates the correction terms

Test for presence of a Scalar current

WITH CVC VERIFIED

\[ \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \]

Obtain precise value of $G_v^2(1 + \Delta_R)$

Determine $V_{ud}^2$

Test CKM unitarity

\[ V_{ud}^2 = \frac{G_v^2}{G_\mu^2} \]

\[ V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1 \]
WHAT CAN WE LEARN?

FROM A SINGLE TRANSITION

Experimentally determine $G_v^2 (1 + \Delta_R)$

$$\mathcal{F} t = ft (1 + \delta'_R)[1 - (\delta_c - \delta_{NS})] = \frac{K}{2G_v^2 (1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate the correction terms

Test for presence of a Scalar current

WITH CVC VERIFIED

Obtain precise value of $G_v^2 (1 + \Delta_R)$

Determine $V_{ud}$

Test Conservation of the Vector current (CVC)

weak eigenstates

mass eigenstates

Cabibbo Kobayashi Maskawa (CKM) matrix

Only possible if prior conditions satisfied

$V_{ud} + V_{us} + V_{ub} = 1$
8 cases with \( ft \)-values measured to <0.05% precision; 6 more cases with 0.05-0.3% precision.

~220 individual measurements with compatible precision

\[
ft = ft (1 + \delta_R') \left[ 1 - (\delta_c - \delta_{NS}) \right] = \frac{K}{2G_v^2 (1 + \Delta_R)}
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WORLD DATA FOR $0^+ \rightarrow 0^+$ DECAY, 2014

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$ft = ft \left(1 + \delta_R'\right) \left[1 - (\delta_C - \delta_{NS})\right] = \frac{K}{2G_v^2 (1 + \Delta_R)}$

Hardy & Towner
New survey (2014) [PRC 79, 055502 (2009)]
TESTS OF $\delta_c$ CALCULATIONS

A. Agreement with CVC:

$T$ values have been calculated with different models for $\delta_c$, then tested for consistency. Normalized $\chi^2$ and confidence levels are shown.

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$\chi^2 = 1.37$
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\[ \chi^2 = 1.37 \]

\[ \chi^2 = 4.26 \] 

\[ \chi^2 = 6.38 \]
B. Measurements of mirror superallowed transitions:
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\[ t = ft (1 + \delta_R') [1 - (\delta_C - \delta_{NS})] \]

\[ \frac{ft_A}{ft_B} = \frac{(1 + \delta^B_R)[1 - (\delta^B_C - \delta^B_{NS})]}{(1 + \delta^A_R)[1 - (\delta^A_C - \delta^A_{NS})]} = 1 + (\delta^B_R - \delta^A_R) + (\delta^B_{NS} - \delta^A_{NS}) - (\delta^B_C - \delta^A_C) \]
B. Measurements of mirror superallowed transitions:

\[ t = ft \left(1 + \delta'_{R}\right)\left[1 - (\delta_{C} - \delta_{NS})\right] \]

\[ \frac{ft_{A}}{ft_{B}} = \frac{(1 + \delta'^{B}_{R})\left[1 - (\delta^{B}_{C} - \delta^{B}_{NS})\right]}{(1 + \delta'^{A}_{R})\left[1 - (\delta^{A}_{C} - \delta^{A}_{NS})\right]} \]

\[ = 1 + (\delta'^{B}_{R} - \delta'^{A}_{R}) + (\delta^{B}_{NS} - \delta^{A}_{NS}) - (\delta^{B}_{C} - \delta^{A}_{C}) \]
ACCESSIBLE MIRROR PAIRS OF SUPERALLOWED DECAYS

NUMBER OF PROTONS, Z

NUMBER OF NEUTRONS, N

\[ \frac{Q}{E_{\text{EC}}} = 6612 \]

\[ \frac{Q}{E_{\text{EC}}} = 6044 \]

\[ 0^+; 1 \]

\[ t_{1/2} \]

\[ Q_{\text{EC}} \]

\[ 1^0 \text{C} \]
ACCESSIBLE MIRROR PAIRS OF SUPERALLOWED DECAYS

\[ ^{38}\text{Ca}_{18}, Q_{\text{EC}} = 6612 \]

\[ ^{38}\text{Ar}_{20}, Q_{\text{EC}} = 6044 \]

\[ ^{74}\text{Rb} \]

\[ ^{10}\text{C} \]

\[ \text{Number of Protons, } Z \]

\[ 20 \]

\[ 30 \]

\[ 40 \]

\[ 10 \]

\[ \text{Number of Neutrons, } N \]

\[ 20 \ 30 \ 40 \ 50 \ 60 \]

\[ 38 \]

\[ 18 \]

\[ ^{38}\text{K}_{19}, Q_{\text{EC}} = 6044 \]

\[ ^{38}\text{Ar}_{20}, 99.97\% \]

\[ ^{38}\text{Ca}_{18}, 444 \text{ ms} \]

\[ ^{74}\text{Rb}, t_{1/2}, Q_{\text{EC}} \]

\[ ^{10}\text{C}, 924 \text{ ms} \]

\[ ^{38}\text{K}_{19}, 1.0\% \]

\[ ^{3978}, 0.1\% \]

\[ ^{3341}, 0.3\% \]

\[ ^{1698}, 19.5\% \]

\[ ^{458}, 2.8\% \]

\[ ^{130}, 77.3\% \]

\[ ^{0}, 924 \text{ ms} \]

\[ ^{10}\text{C} \]

\[ ^{38}\text{Ar}_{20}, 99.97\% \]

\[ ^{38}\text{Ca}_{18}, 0.1\% \]

\[ ^{444}, 444 \text{ ms} \]

\[ ^{6612}, Q_{\text{EC}} \]

\[ ^{6044}, Q_{\text{EC}} \]

\[ ^{0\uparrow1}, 100\% \]

\[ ^{0\uparrow1}, 1\% \]

\[ ^{1\uparrow0}, 1\% \]

\[ ^{0\uparrow1}, 0.1\% \]

\[ ^{0\uparrow1}, 3\% \]

\[ ^{77.3\%}, 924 \text{ ms} \]

\[ ^{2.8\%}, 444 \text{ ms} \]

\[ ^{19.5\%}, 77.3\% \]

\[ ^{0.3\%}, 2.8\% \]

\[ ^{458}, 130 \]

\[ ^{1698}, 1698 \]

\[ ^{3341}, 3341 \]

\[ ^{3978}, 3978 \]

\[ ^{10}\text{C} \]

\[ ^{18}\text{Ar}_{20} \]
PRECISION DECAY MEASUREMENTS AT TAMU

Momentum Achromat Recoil Separator (MARS)

\[ ^{1}H(^{39}K, 2n)^{38}Ca \]

\( ^{39}K \) beam (1170 MeV)

H\(_{2}\) gas target

Shielding

99.7% pure \(^{38}Ca\)

Thin scintillator and Al degraders

25,000 atoms/s
PRÉCISION DECAY MEASUREMENTS AT TAMU

Momentum Achromat Recoil Separator (MARS)

$^1\text{H}(^{39}\text{K}, 2\text{n})^{38}\text{Ca}$

$^{39}\text{K}$ beam (1170 MeV)

$\text{H}_2$ gas target

$4\pi$ proportional gas detector

Shielding

Precise decay measurements at TAMU
Momentum Achromat Recoil Separator (MARS)

$^1$H($^3$K, 2n)$^{38}$Ca

$^3$K beam (1170 MeV)

H$_2$ gas target

Shielding

HPGe detector calibrated for efficiency to ± 0.2%
BETA-DECAY BRANCHING OF $^{38}$Ca

Energy (keV)

Counts per channel

15 cm

Precisely efficiency calibrated

1-mm-thick plastic scintillator

$^{38}$Ca

$Q_{EC} = 6612$

1000 2000 3000 4000

10 100 1000 10000

3978 $1^+,0$ 0.1%
3341 $1^+,0$ 0.3%
1698 $1^+,0$ 19.5%
458 $1^+,0$ 2.8%
130 $0^+,1$ 77.3%
0 $3^+,0$

$^{38}$K

$^{38}$Ca

$^{38}$Ca

$^{38}$Ca
B. Measurements of mirror superallowed transitions:

\[ \mathcal{f}t = ft \left( 1 + \delta'_R \right) \left[ 1 - (\delta_C - \delta_{NS}) \right] \]

\[ \frac{ft_A}{ft_B} = \frac{(1 + \delta'_B) \left[ 1 - (\delta_C^B - \delta_{NS}^B) \right]}{(1 + \delta'_A) \left[ 1 - (\delta_C^A - \delta_{NS}^A) \right]} \]

\[ = 1 + (\delta_R^B - \delta_R^A) + (\delta_{NS}^B - \delta_{NS}^A) - (\delta_C^B - \delta_C^A) \]
RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

FROM A SINGLE TRANSITION

Experimentally

determine $G_V^2 (1 + \Delta_R)$

\[ \mathcal{F}t = ft (1 + \delta'_R) [1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^2 (1 + \Delta_R)} \]

FROM MANY TRANSITIONS

Test Conservation of
the Vector current (CVC)
RESULTS FROM 0⁺→0⁺ DECAY

FROM A SINGLE TRANSITION

Experimentally determine $G_V^2(1 + \Delta_R)$

$$\tau = ft \left(1 + \delta_R^2\right) \left[1 - (\delta_C - \delta_{NS})\right] = \frac{K}{2G_V^2(1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

$G_V$ constant to ± 0.013%

G_{V(1+\Delta_R)^{1/2}}/(hc)^3 = 1.14958(15) \times 10^{-5} \text{ GeV}^2$

$\chi^2/\nu = 0.6$
RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

FROM A SINGLE TRANSITION

Experimentally determine $G_v^2 (1 + \Delta_R)$

$$\mathcal{F}t = ft (1 + \delta_R^I)[1 - (\delta_c - \delta_{NS})] = \frac{K}{2G_v^2 (1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)
Validate correction terms

$G_v$ constant to $\pm 0.013\%$
RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

FROM A SINGLE TRANSITION

Experimentally determine $G_v^2 (1 + \Delta_R)$

$$\mathcal{F}t = ft (1 + \delta_R')[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_v^2 (1 + \Delta_R)}$$

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\( G_V \) constant to \( \pm 0.013\% \)

RESULTS FROM \( 0^+ \rightarrow 0^+ \) DECAY

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\[
\frac{ft^{38m\text{Ca}}}{ft^{38m\text{K}}} = 1.004 \\
\frac{ft^{38m\text{Ca}}}{ft^{38m\text{K}}} = 1.006
\]

\( A \) of mirror pairs
RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

FROM A SINGLE TRANSITION

Experimentally determine $G_v^2 (1 + \Delta_R)$

$$\mathcal{F}t = ft (1 + \delta'_R)(1 - (\delta_C - \delta_{NS})) = \frac{K}{2G_v^2 (1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)
Validate correction terms ✔
Test for Scalar current

$G_v$ constant to ± 0.013%
**RESULTS FROM $0^+ \rightarrow 0^+$ DECAY**

**FROM A SINGLE TRANSITION**

$$\mathcal{F} t = ft (1 + \delta'_R) [1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_v^2 (1 + \Delta_R)}$$

Experimentally determine $G_v^2 (1 + \Delta_R)$

**FROM MANY TRANSITIONS**

- Test Conservation of the Vector current (CVC)
- Validate correction terms
- Test for Scalar current

$G_v$ constant to $\pm 0.013\%$

Limit, $C_s/C_v = 0.0014$ (13)

---

![Graph](image-url)
FROM A SINGLE TRANSITION

Experimentally determine \( G_V^2 (1 + \Delta_R) \)

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RESULTS FROM \( 0^+ \rightarrow 0^+ \) DECAY
RESULTS FROM $0^+ \to 0^+$ DECAY

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FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)
Validate correction terms
Test for Scalar current

G$_V$ constant to ± 0.013%

limit, C$_s$/C$_V$ = 0.0014 (13)

WITH CVC VERIFIED

Obtain precise value of $G_V^2 (1 + \Delta_R)$

Determine $V_{ud}^2$

$$V_{ud}^2 = G_V^2/G_{\mu}^2 = 0.94900 \pm 0.00042$$

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The Cabibbo-Kobayashi-Maskawa matrix is a representation of the weak interactions in the Standard Model of particle physics. It describes how the mass eigenstates (u, d, s, c, b) are related to the weak eigenstates (d', s', b').
RESULTS FROM $0^+ \to 0^+$ DECAY

FROM A SINGLE TRANSITION

Experimentally determine $G_V^2 (1 + \Delta_R)$

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G_V constant to $\pm 0.013\%$

limit, $C_s/C_V = 0.0014 (13)$

WITH CVC VERIFIED

Obtain precise value of $G_V^2 (1 + \Delta_R)$
Determine $V_{ud}^2$

\[ V_{ud}^2 = \frac{G_V^2}{G_{\mu}^2} = 0.94900 \pm 0.00042 \]

RESULTS FROM $0^+ \to 0^+$ DECAY

1990 2000 2010

0.975 0.974 0.973

$V_{ud}$

weak eigenstates
mass eigenstates

Cabibbo-Kobayashi-Maskawa matrix
FROM A SINGLE TRANSITION

Experimentally determine $G_V^2 (1 + \Delta_R)$

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Determine $V_{ud}^2$

$$V_{ud}^2 = \frac{G_V^2}{G_{\mu}^2} = 0.94900 \pm 0.00042$$

Test CKM unitarity

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.99992 \pm 0.00048$$
CURRENT STATUS OF CKM UNITARITY

\[ V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.99992 \pm 0.00048 \]

- \( V_{ud}^2 \): nuclear decays, muon decay
  - 0.94900 ± 0.00042
- \( V_{us}^2 \): kaon decays
  - 0.05090 ± 0.00022
- \( V_{ub}^2 \): B decays
  - 0.00002 ± 0.00001
CURRENT STATUS OF CKM UNITARITY

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- \( V_{ud}^2 \): Nuclear decays
  - Muon decay
  - Nuclear decays
  - Neutron nuclear mirrors
  - Pion

- \( V_{us}^2 \): Kaon decays
  - B decays

<table>
<thead>
<tr>
<th>Source</th>
<th>Value</th>
</tr>
</thead>
<tbody>
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\[ V_{ud} = 0.97417 \pm 0.00021 \]
Kaon decay yields two independent determinations of $V_{us}$:

1) Semi-leptonic $K \rightarrow \pi \ell \nu_\ell$ decay ($K_{\ell3}$) yields $|V_{us}|$.

2) Pure leptonic decays $K^+ \rightarrow \mu^+\nu_\mu$ and $\pi^+ \rightarrow \mu^+\nu_\mu$ together yield $|V_{us}| / |V_{ud}|$.

Both require lattice calculations of form factors to obtain their result. Until March 2014 these gave highly consistent results for $|V_{us}|$. 
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**BUT**, Bazavov et al. [PRL 112, 112001 (2014)] produced a new lattice calculation of the form factor used for $K_{\ell 3}$ decays.

Their new result for $|V_{us}|$ is inconsistent with the $|V_{us}| / |V_{ud}|$ result and, when combined with the superallowed result for $|V_{ud}|$, leads to a unitarity sum over two standard deviations below 1.

Stay tuned ...
THE PATH FORWARD
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1. Improving $V_{ud}$ and CKM Unitarity

- Improve the calculation of the “inner” radiative correction $\Delta_R$.

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H.I. Park et al., PRL 112, 102502 (2014)

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- If theory improves, broadly improve all experimental $ff$ values
2. Search for scalar current

- Tighten uncertainties on the $ft$ values for $^{10}\text{C}$ and $^{14}\text{O}$

$C_s/C_v = 0.0014 \pm 0.002$

$\text{limit, } C_s/C_v = 0.0014 \pm 0.002$

Require (order of priority):
- $^{10}\text{C}$ branching ratio
- $^{14}\text{O}$ branching ratio
- $^{14}\text{O} Q_{EC}$ value
- $^{10}\text{C}$ half-life
3. Expand the number of transitions

- Potential FRIB measurements
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- Complete more pairs of mirror superallowed transitions: $^{46}\text{Cr}$, $^{50}\text{Fe}$, $^{54}\text{Ni}$ ...
- Add new heavy $T_z = 0$ superallowed emitters: $^{66}\text{As}$, $^{70}\text{Br}$, $^{78}\text{Y}$ ...

**Tests $\delta_c$, $\delta_{NS}$ calculations in $f_{7/2}$ shell nuclei**

**Probably most useful for studying isospin mixing**

Challenges:
- Complex decays (Pandemonium effect)
- Need spectroscopic data to constrain structure model
- Need $T_z = -1$ masses to determine IMME coefficients
1. Analysis of superallowed $0^+ \rightarrow 0^+$ nuclear $\beta$ decay is shown to confirm CVC and thus yield $V_{ud} = 0.97417(21)$.

2. The three other experimental methods for determining $V_{ud}$ yield consistent results, but are less precise by a factor of 8 or more.

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4. The largest contribution to the $V_{ud}$ uncertainty is from the inner radiative correction. Improving it is the highest priority requirement if uncertainty is to be reduced.

5. Isospin symmetry-breaking corrections in nuclei are the second largest contributor to $V_{ud}$ uncertainty. These may be improved by studying mirror superallowed transitions and by improving all experimental $ft$ values.

6. Potential FRIB experiments can expand the number of superallowed transitions and be useful for studying isospin mixing.