Horizon and extended thermodynamics of Lovelock black holes

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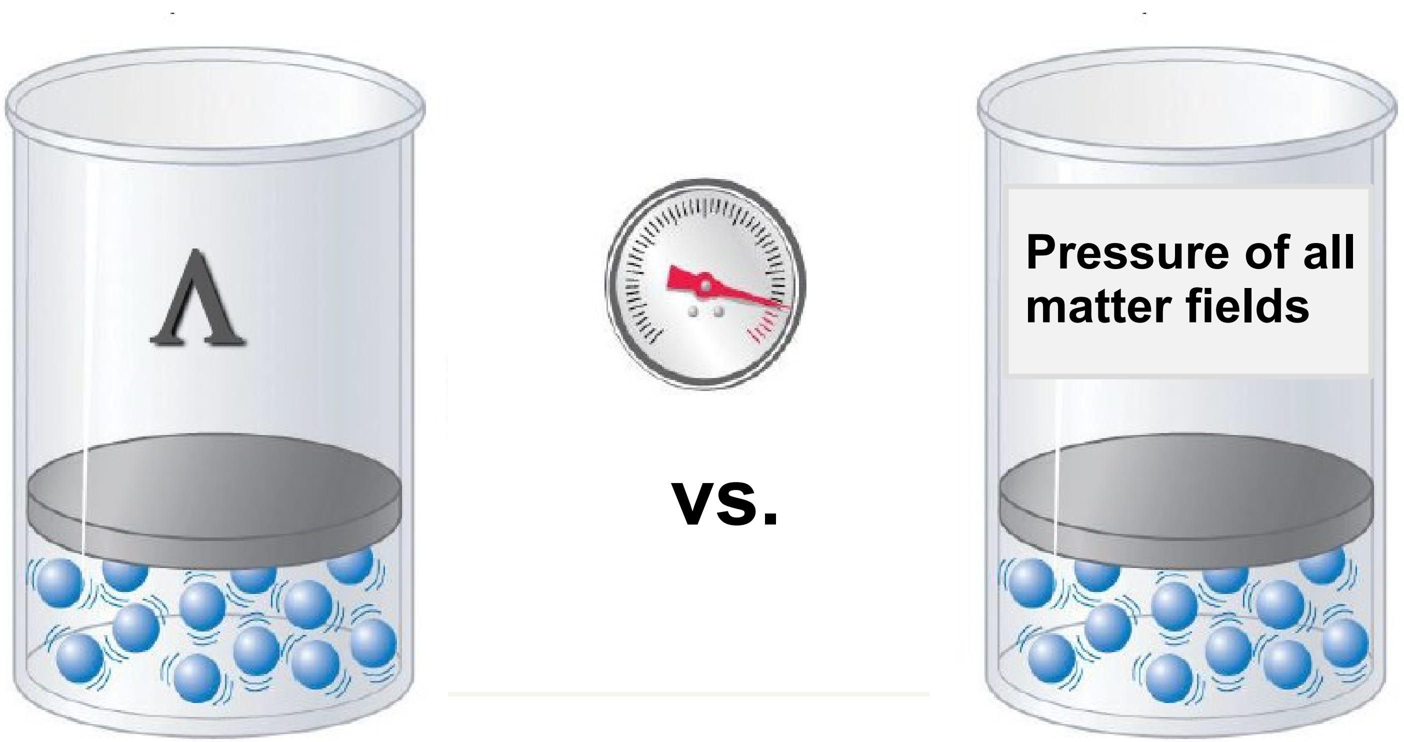


Northeast Gravity Workshop 2016, ACFI
University of Massachusetts, Amherst, USA
Friday April 22 – 24, 2016

Prelude: Extended phase space thermodynamics

(Thermodynamics with variable Lambda)

D.Kastor, S.Ray, and J.Traschen, *Enthalpy and the Mechanics of AdS Black Holes*, Class. Quant. Grav. 26 (2009) 195011, [arXiv:0904.2765].

- Consider an asymptotically AdS black hole spacetime
- Identify the cosmological constant with pressure

$$\boxed{P_\Lambda = -\frac{1}{8\pi}\Lambda = \frac{3}{8\pi}\frac{1}{l^2}}$$

- Allow it to be a thermodynamic quantity

$$\Rightarrow \quad \begin{aligned} P_\Lambda &= P_\Lambda(V_{\mathrm{TD}}, T, Q, \mathcal{J})\,, \\ \delta M &= T\delta S + V_{\mathrm{TD}}\delta P_\Lambda + \Phi\delta Q + \Omega\delta\mathcal{J} \end{aligned}$$

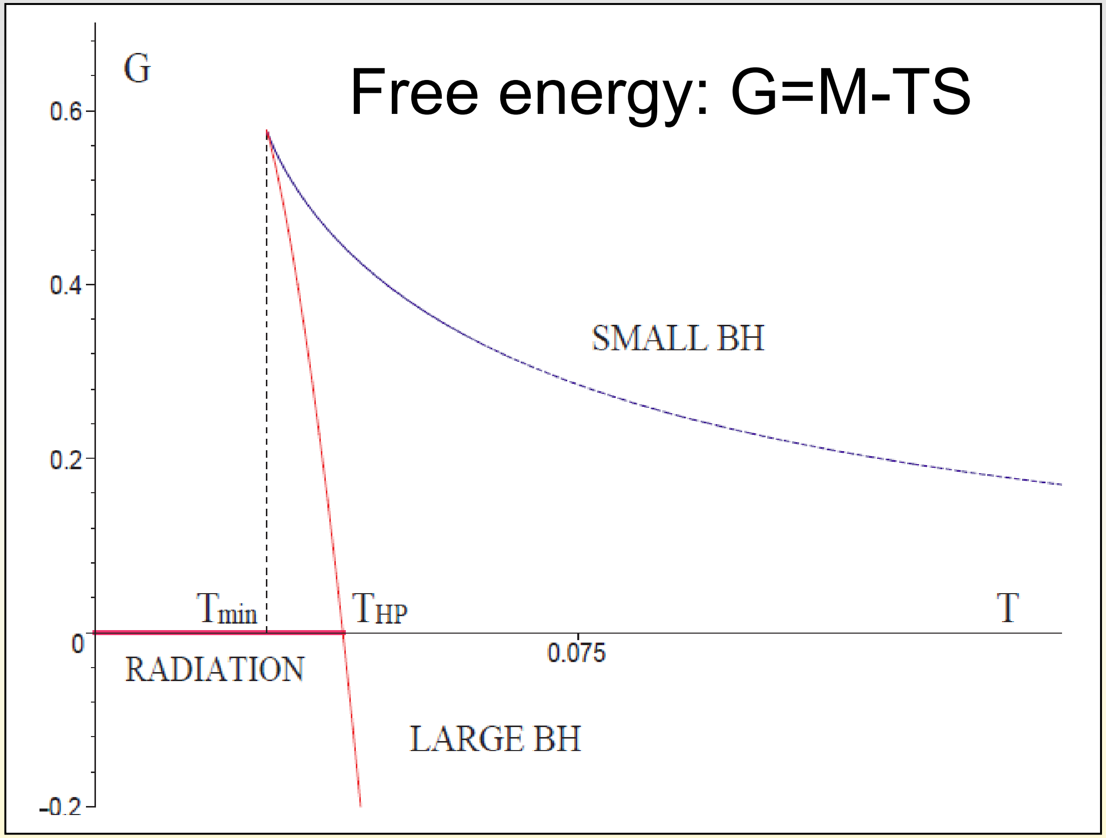
$$V_{\mathrm{TD}} = \left(\frac{\partial M}{\partial P_\Lambda}\right)_{S,Q,\mathcal{J}}$$

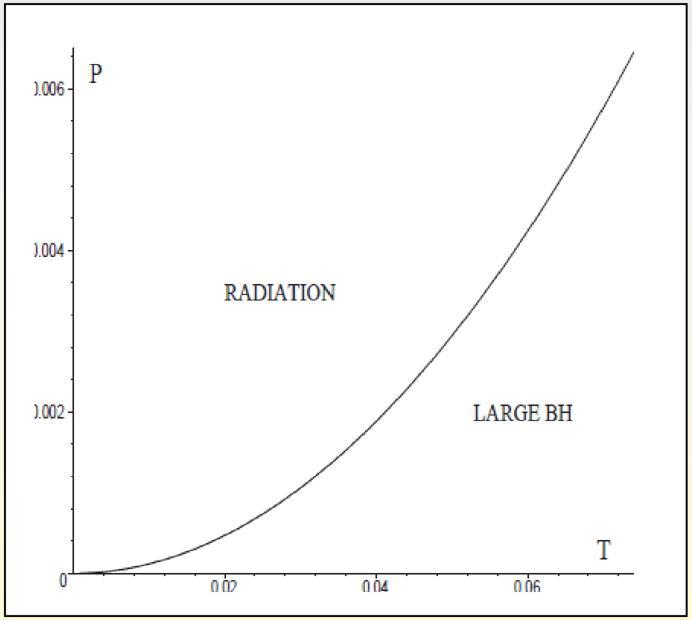
Hawking-Page transition

S.W. Hawking & D.N. Page, *Thermodynamics of black holes in anti-de-Sitter space*, Commun. Math. Phys. 87, 577 (1983).

- Schwarschild-AdS black hole $$f = k - \frac{2M}{r} + \frac{r^2}{l^2}$$

$$T = \frac{|f'(r_+)|}{4\pi} = \frac{1}{4\pi r_+}\left(1 + \frac{3r_+^2}{l^2}\right) \Leftrightarrow P_\Lambda = P_\Lambda(V,T) = \frac{T}{2r_+} - \frac{k}{8\pi r_+^2}$$

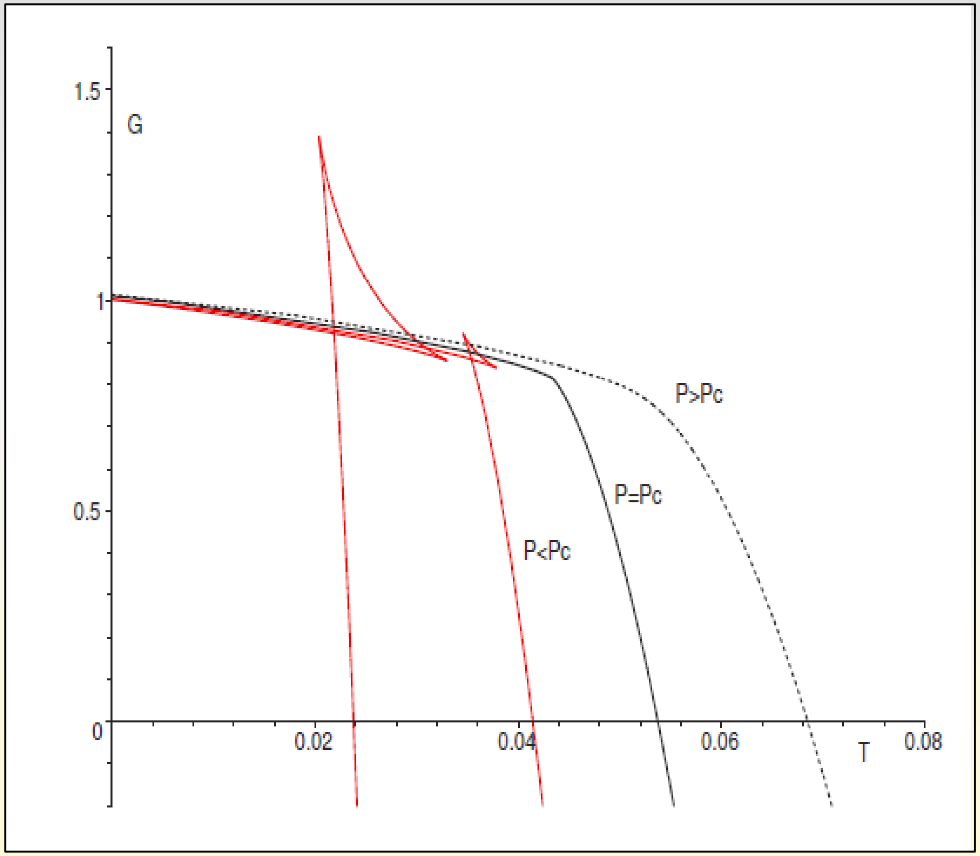
- 

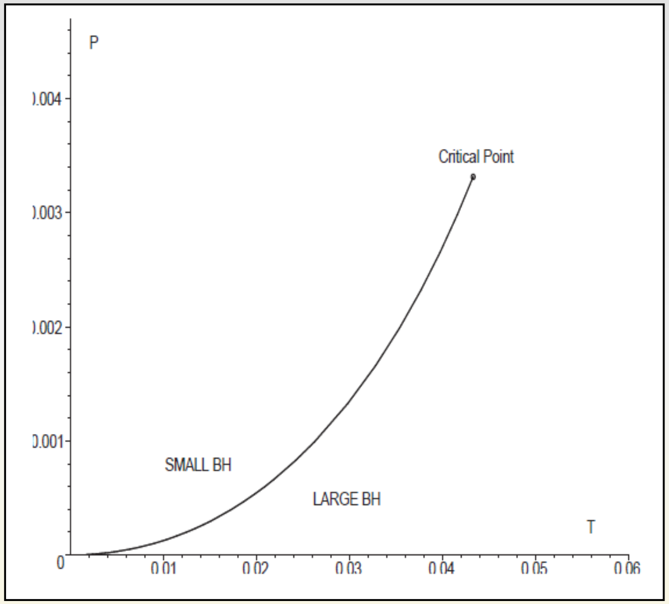


Van der Waals-like phase transition

- DK, R. Mann, *P-V criticality of charged AdS black holes*, JHEP 1207 (2012) 033.
- Charged-AdS black hole $$f = k - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2}$$

$$T = \frac{|f'(r_+)|}{4\pi} \Longleftrightarrow P_\Lambda(V,T) = \frac{T}{2r_+} - \frac{k}{8\pi r_+^2} + \frac{Q^2}{8\pi r_+^4}$$





$$\alpha = 0\,, \quad \beta = \frac{1}{2}\,, \quad \gamma = 1\,, \quad \delta = 3$$

Plan of the talk

I. Gravitational dynamics & thermodynamics

II. Horizon thermodynamics
   a) Einstein gravity
   b) Free energy & universal thermodynamic behavior
   c) Lovelock gravity: triple points, reentrant phase transitions, isolated critical point

III. Beyond spherical symmetry

IV. Summary

Based on:

1) Devin Hansen, DK, Robert Mann, *Universality of P-V criticality in Horizon Thermdoynamics*, ArXiv: 1603.05689.
2) Devin Hansen, DK, Robert Mann, *Criticality and Surface Tension in Rotating Horizon Thermodynamics*, **today**.

I) Gravitational dynamics and thermodynamics: some references

- Black hole thermodynamics: How do the completely classical Einstein equations know about QFT?

- Sakharov (1967): spacetime emerges as a MF approximation of underlying microscopics, similar to how hydrodynamics emerges from molecular physics.

- Can we understand gravity from thermodynamic viewpoint?

R. G. Cai, *Connections between gravitational dynamics and thermodynamics*, *J. Phys. Conf. Ser.* 484 (2014) 012003.

1) <u>Local Rindler horizon: Jacobson's argument</u>

- T. Jacobson, *Thermodynamics of space-time: The Einstein equation of state*, Phys. Rev. Lett. 75 (1995) 1260, gr-qc/9504004.

Are Einstein equations an equation of state?

2) <u>Black hole horizon: "horizon thermodynamics"</u>

- T. Padmanabhan, *Classical and quantum thermodynamics of horizons in spherically symmetric space-times*, Class. Quant. Grav. 19 (2002) 5387.
- D. Kothawala, S. Sarkar, and T. Padmanabhan*, Einstein's equations as a thermodynamic identity: The Cases of stationary axisymmetric horizons and evolving spherically symmetric horizons*, Phys. Lett. B652 (2007) 338.

3) <u>Apparent horizon in FRW</u>

- M. Akbar and R.-G. Cai, *Thermodynamic Behavior of Friedmann Equations at Apparent Horizon of FRW Universe*, Phys. Rev. D75 (2007) 084003, hep-th/0609128.

1) Local Rindler horizon: Jacobson's argument

- T. Jacobson, *Thermodynamics of space-time: The Einstein equation of state*, Phys. Rev. Lett. 75 (1995) 1260, gr-qc/9504004.

- Local Rindler horizon $\boxed{T = \frac{\kappa}{2\pi}}$
- Clausius relation $$\boxed{\delta Q = T\delta S \propto \kappa\delta S \propto \kappa\delta A}$$
- Heat flux $$\boxed{\delta Q = \int T_{ab}\xi^a d\Sigma^b = -\kappa \int T_{ab}k^a k^b \lambda d\lambda da}$$
- Change in area $\boxed{\delta A = \int \theta d\lambda da}$
- Raychaudhury Eq. $$\boxed{\frac{d\theta}{d\lambda} = \frac{1}{2}\theta^2 - \sigma^2 - R_{ab}k^a k^b}$$
- Recover EE: $$\boxed{R_{ab} + f g_{ab} \propto T_{ab}}$$

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II) Horizon thermodynamics

- Consider a spherically symmetric spacetime

$$ds^2 = -f\,dt^2 + \frac{dr^2}{g} + r^2 d\Sigma_2^2\,.$$

with an horizon determined from $f(r_+) = 0.$

- Identify total pressure P with $T^r{}_r$ component of energy-momentum tensor evaluated on the horizon

- Then the radial grav. field equation rewrites as **Universal**

<u>Horizon equation of state</u>: $\boxed{P = P(V,T)}$

<u>Horizon first law</u>: $\boxed{\delta E = T\delta S - P\delta V}$

a) Einstein gravity

- The radial Einstein equation

$$\boxed{8\pi T^r{}_r = \Lambda + \frac{f'g}{rf} - \frac{k-g}{r^2}}$$

$$k = \pm 1, 0$$

- Horizon is a regular null surface $g(r_+) = 0$ $f'(r_+) = g'(r_+)$

- Temperature reads

$$T = \frac{\kappa}{2\pi} = \frac{\sqrt{f'(r_+)g'(r_+)}}{4\pi} = \frac{f'(r_+)}{4\pi}$$

- Identify the total pressure as

$$\boxed{P = P_m + P_\Lambda} \quad P_m \equiv T^r{}_r\big|_{r=r_+} \quad P_\Lambda = -\frac{\Lambda}{8\pi}$$

- We recover the **Horizon Equation of State:**

$$\boxed{P = \frac{T}{2r_+} - \frac{k}{8\pi r_+^2}}$$ with volume $$\boxed{V = \frac{\Sigma_2 r_+^3}{3}}$$

Horizon first law

- Starting from the horizon equation of state

$$T = 2r_+P + \frac{k}{4\pi r_+} \qquad k = +1$$

- let's identify the entropy as quarter of the area

$$S = \frac{A}{4} = \pi r_+^2 \Rightarrow \delta S = 2\pi r_+ \delta r_+$$

- Multiply

$$\boxed{T\delta S = \underbrace{4\pi r_+^2 \delta r_+}_{\delta V} P + \underbrace{k\frac{\delta r_+}{2}}_{\delta E}}$$

- So we recovered the **horizon first law:**

$$\boxed{\delta E = T\delta S - P\delta V}$$

Horizon thermodynamics in Einstein gravity

Input:

- The radial Einstein equation where P is identified with Trr component of stress tensor
- Identification of T, S, and V by **other criteria**

Output:

- Identification of **horizon energy E**

$$\boxed{E = \frac{r_+}{2}}$$

(Misner-Sharp mass evaluated on the horizon)

- **Universal** (matter independent) horizon equations that depend only on the **gravitational theory** considered

$$\boxed{P = P(V,T) = \frac{T}{2r_+} - \frac{k}{8\pi r_+^2}}$$

$$\boxed{\delta E = T\delta S - P\delta V}$$

End of story!

b) Free energy & Universal thermodynamic behavior

- Define the Gibbs free energy (Legendre transform of E)

$$G = G(P,T) = E - TS + PV$$

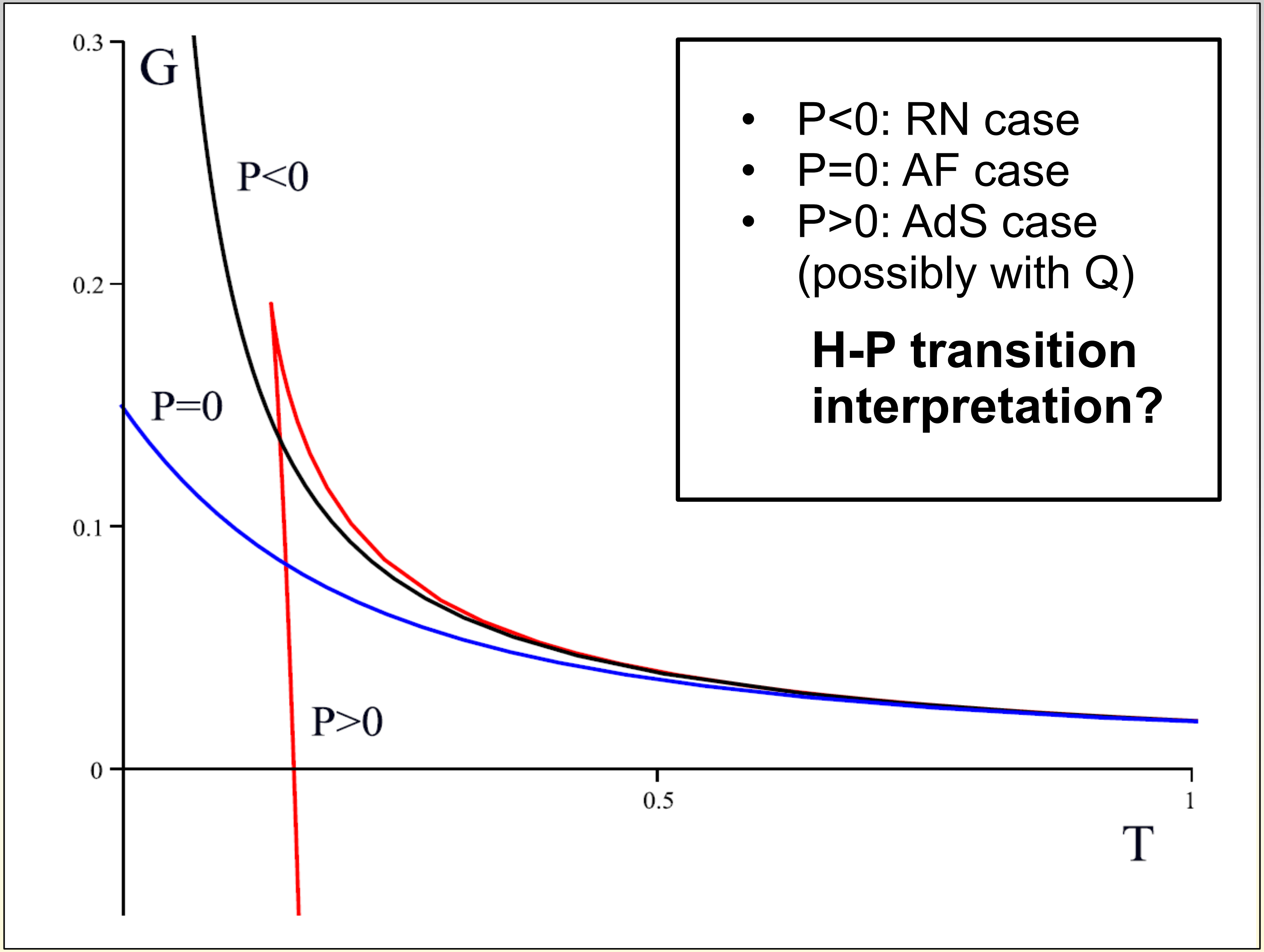
$$\Rightarrow \quad \delta G = -S\delta T + V\delta P$$

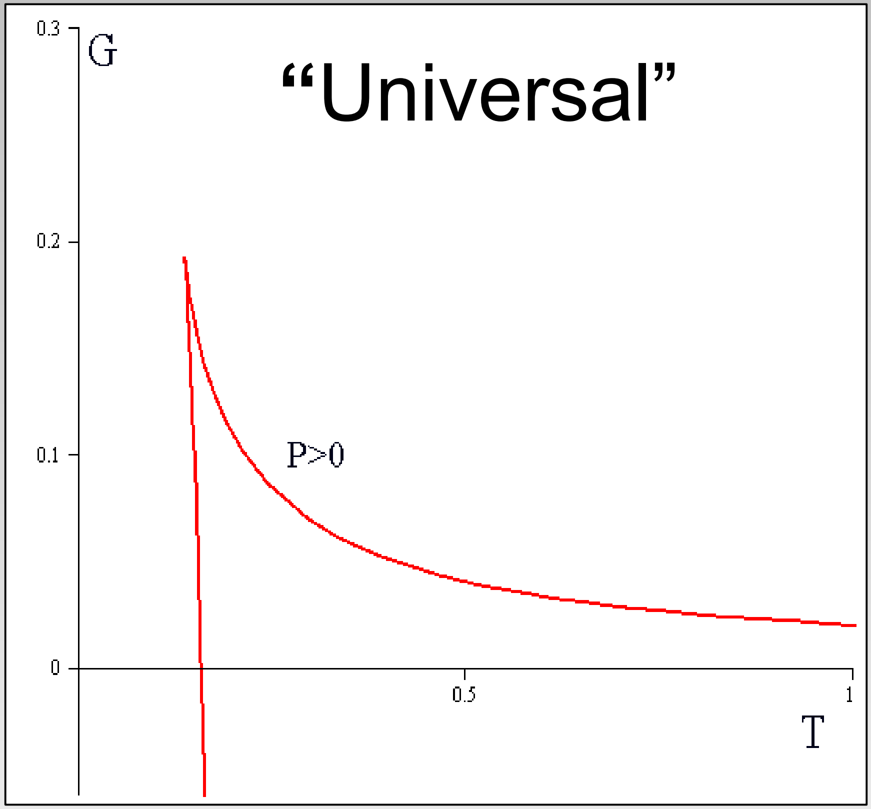
- Can plot it parametrically using the equation of state

$$G = \frac{r_+}{2} - \frac{k r_+}{4} - \frac{2}{3}\pi r_+^3 P$$

$$T = 2 r_+ P + \frac{k}{4\pi r_+}$$

- Only 3 qualitative different cases (considering all possible matter fields - values of P)





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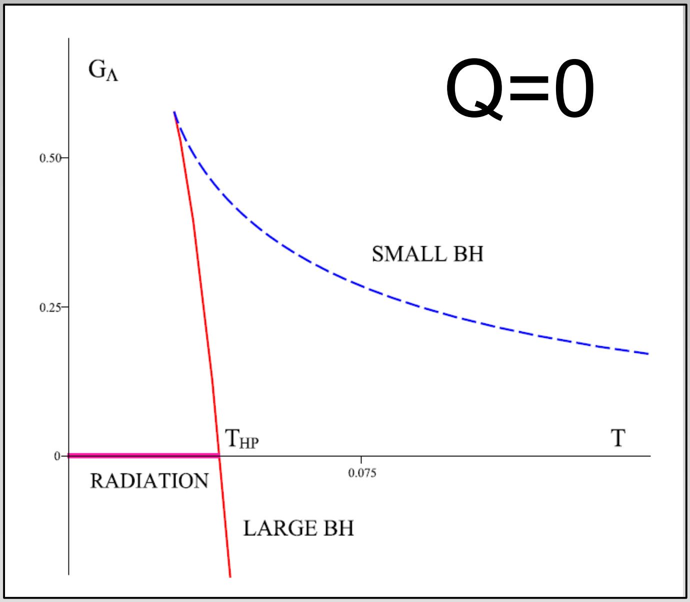
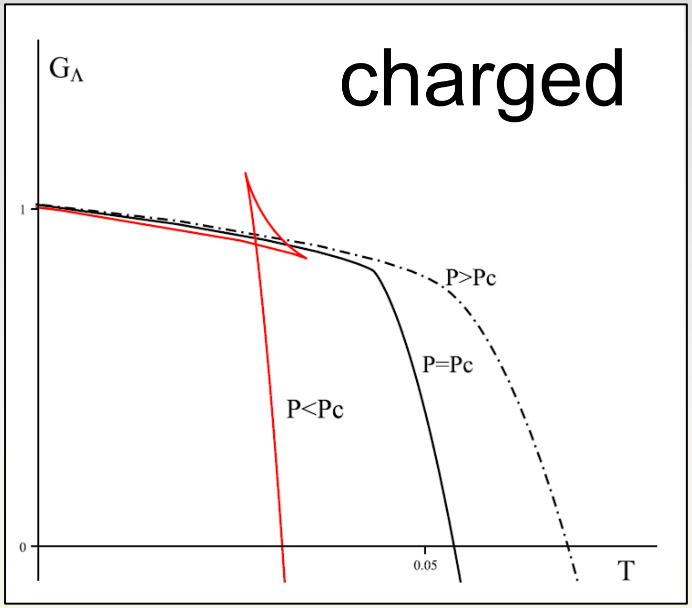




$$P_\Lambda = \frac{T}{2r_+} - \frac{1}{8\pi r_+^2} - P_m = \frac{T}{2r_+} - \frac{1}{8\pi r_+^2} + \frac{Q^2}{8\pi r_+^4}$$

- Different ensembles (can. vs. grand can.)
- Effectively reduces to "vacuum" diagrams
- Different black holes & environments

Universal but the concrete physical interpretation depends on the matter content

c) Generalization to Lovelock gravity

- Lovelock higher curvature gravity

$$\mathcal{L} = \frac{1}{16\pi G_N} \sum_{j=0}^{J} \alpha_j \mathcal{L}^{(j)}$$

- **Input:**
  - Impose spherically symmetric ansatz
  - Write the radial Lovelock equation where P is identified with Trr component of stress tensor
  - Identify the temperature T via Euclidean trick
  - Identify the true entropy S (Wald)
  - Specify the black hole volume

$$V = \frac{\Sigma_{d-2} r_+^{d-1}}{d-1}$$

- **Output:**
  - Identification of **horizon energy E**

$$E = \frac{\Sigma_{d-2}}{16\pi} \sum_{j=1}^{J} \alpha_j \frac{k^j (d-2)!}{(d-2j-1)!} r_+^{d-2j-1}$$

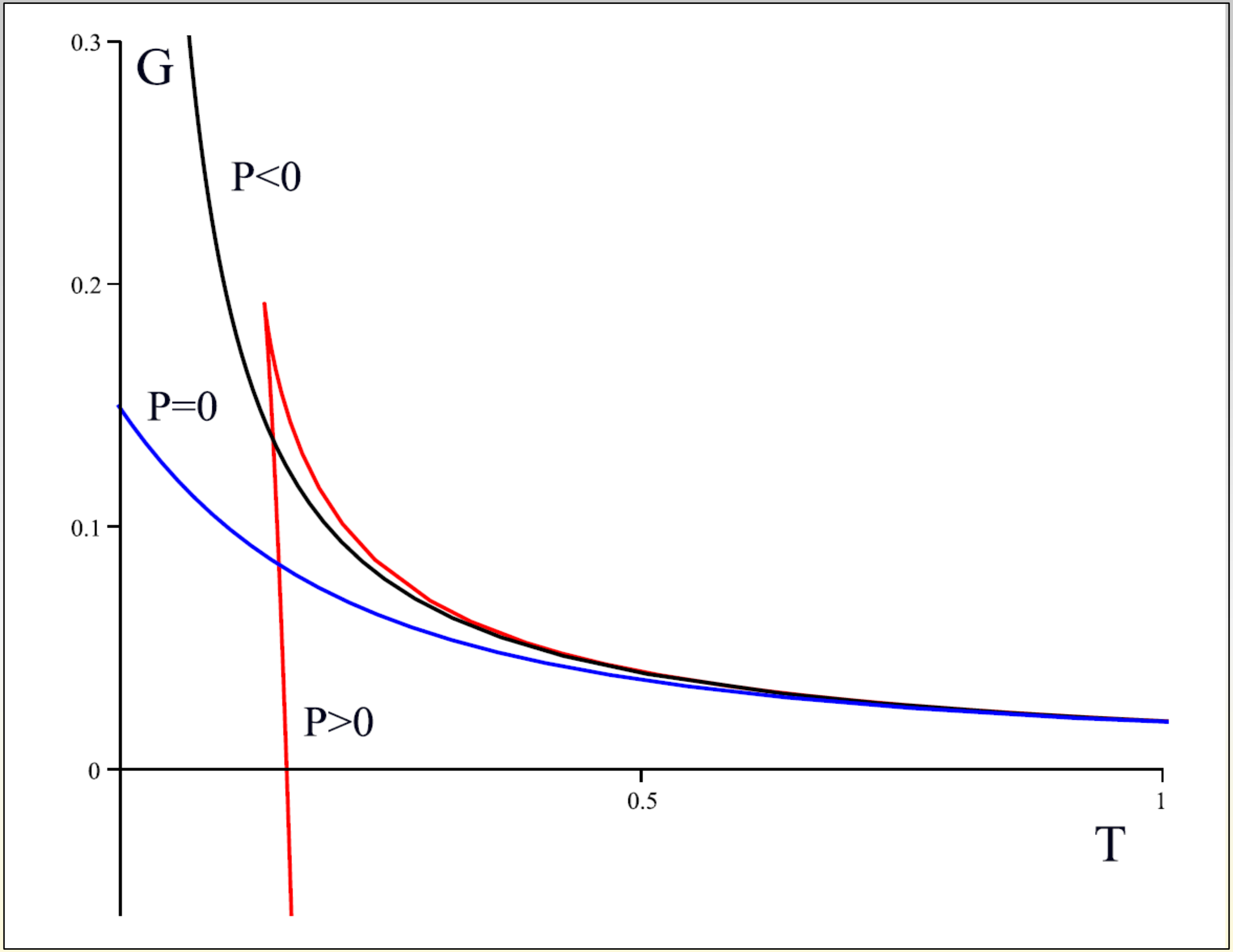
(Misner-Sharp energy evaluated on the horizon, an energy to warp the spacetime and embed the black hole horizon)

  - **Universal** (matter independent) horizon equations that depend only on the **gravitational theory** considered

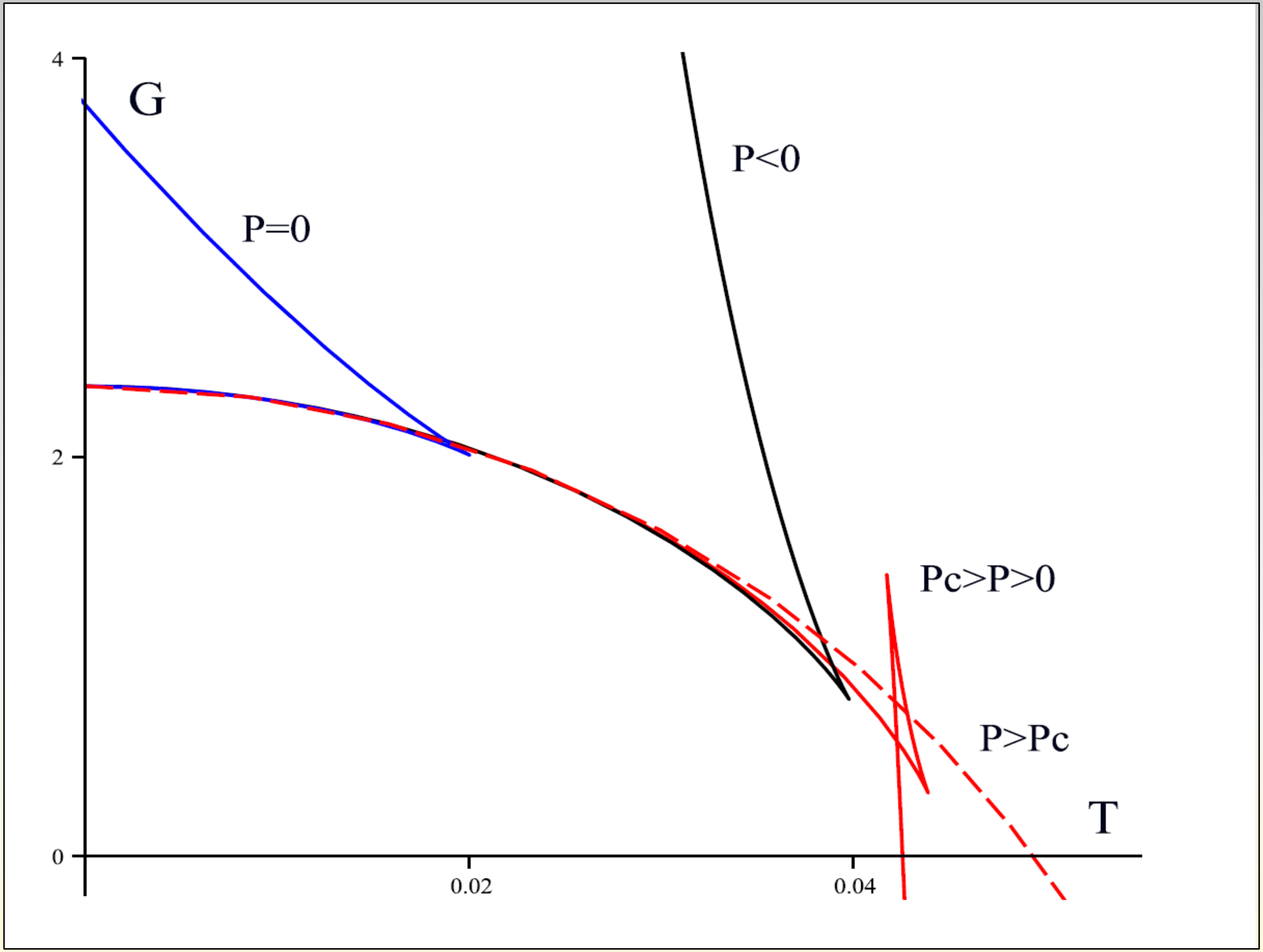
$$P = \sum_{j=1}^{J} \frac{\alpha_j}{4r_+} \frac{(d-2)!}{(d-2j-1)!} \left(\frac{k}{r_+^2}\right)^{j-1} \left[jT - \frac{k(d-2j-1)}{4\pi r_+} \right].$$

$$\delta E = T\delta S - P\delta V$$

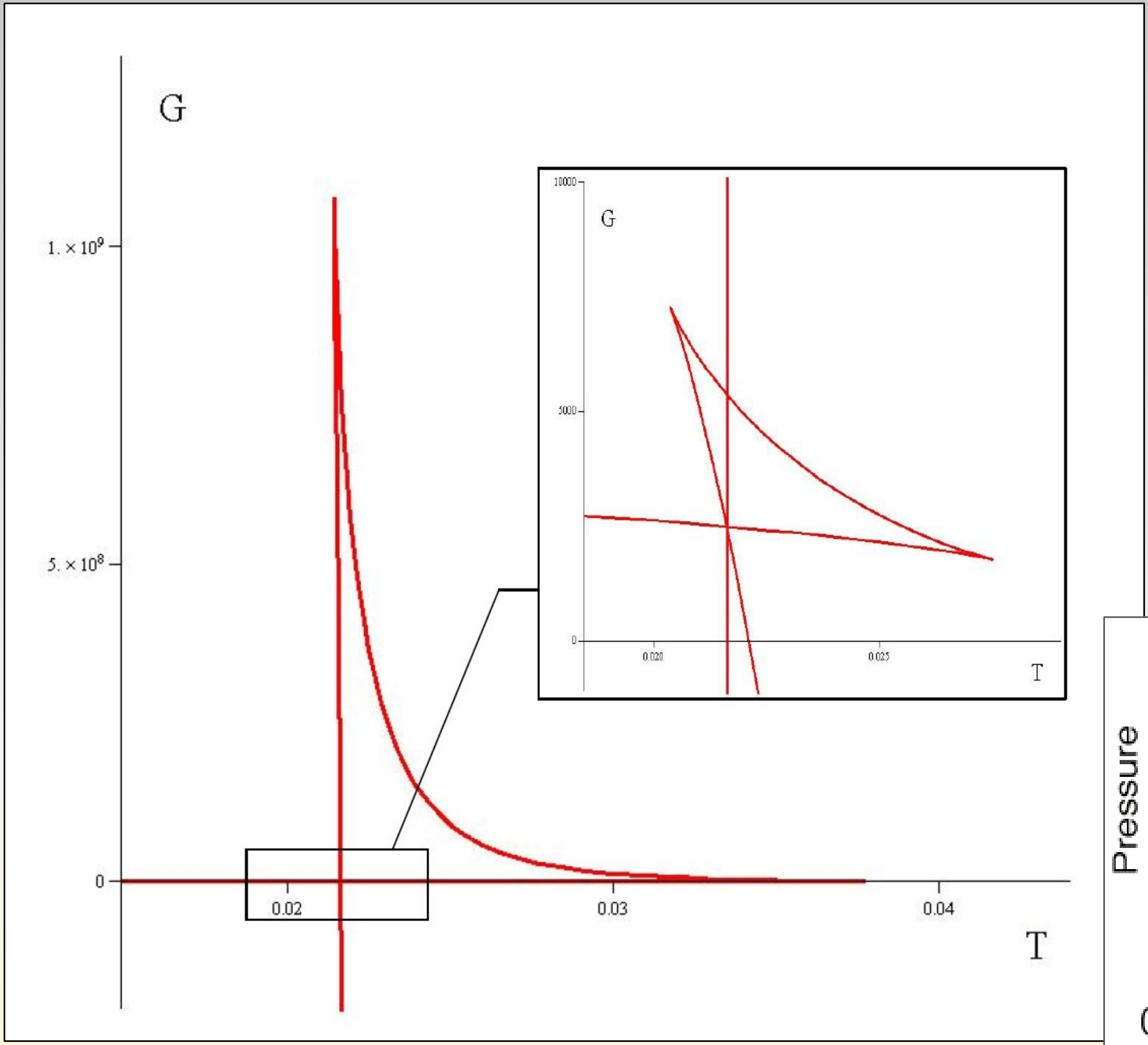
Einstein gravity (d=4)

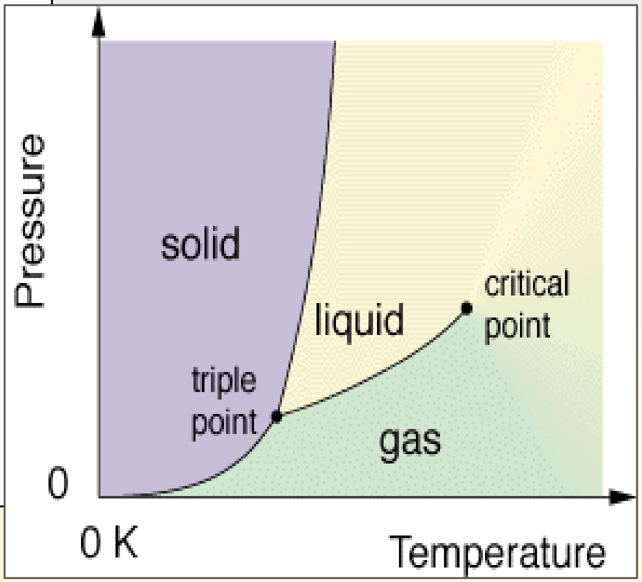


Swallow tails (Gauss-Bonnet gravity d=5)



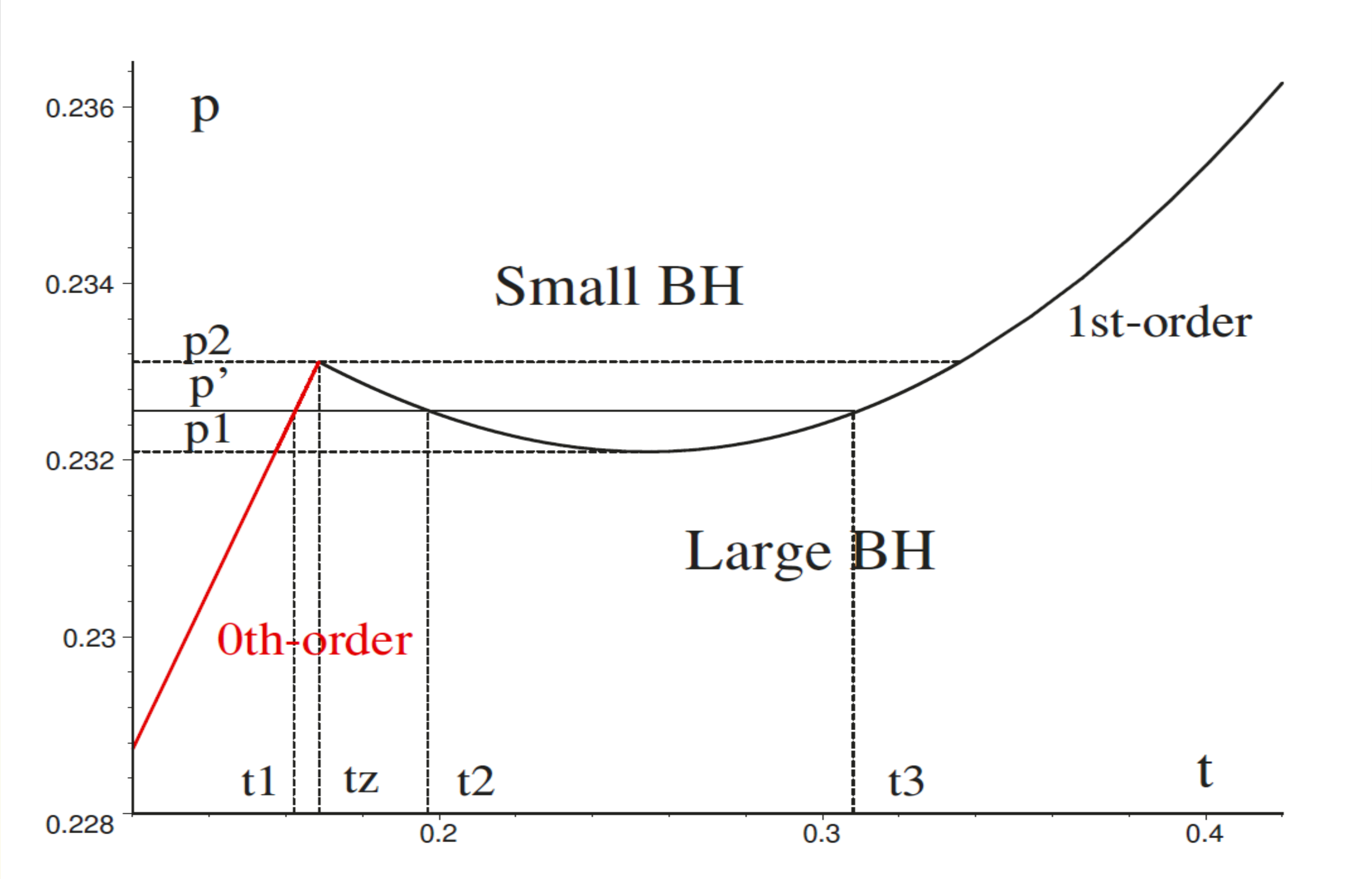
Triple point (4^{th}-order Lovelock)





Reentrant phase transitions

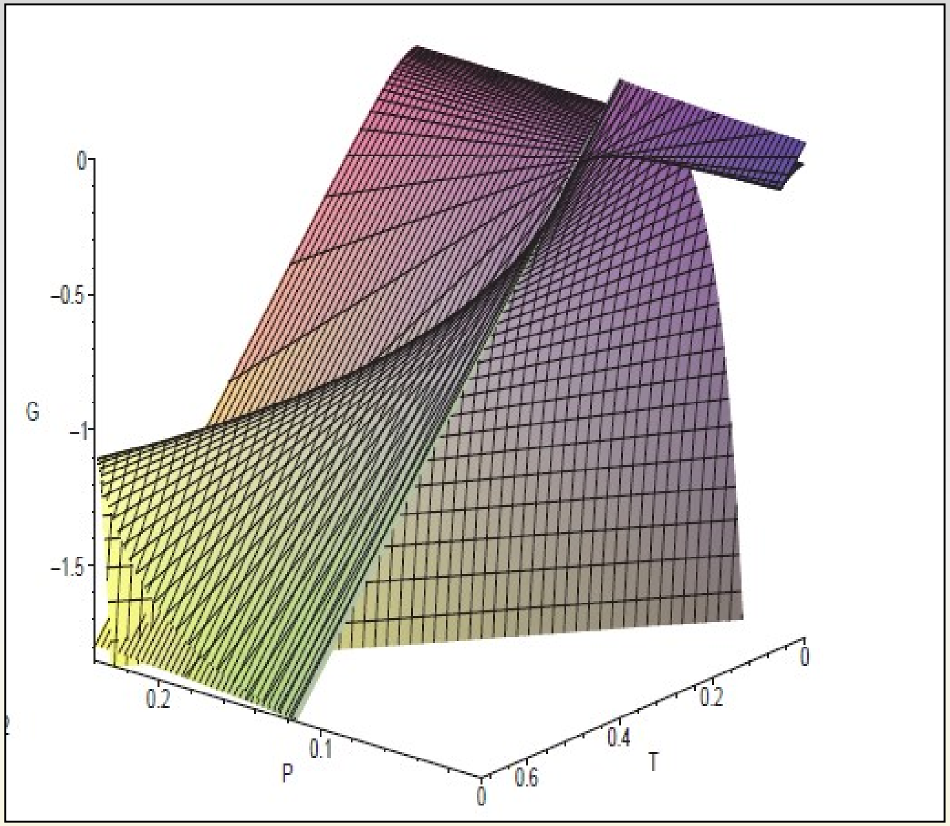
A system undergoes an RPT if a **monotonic** variation of any TD quantity results in two (or more) phase transitions such that the **final state is macroscopically similar** to the initial state.

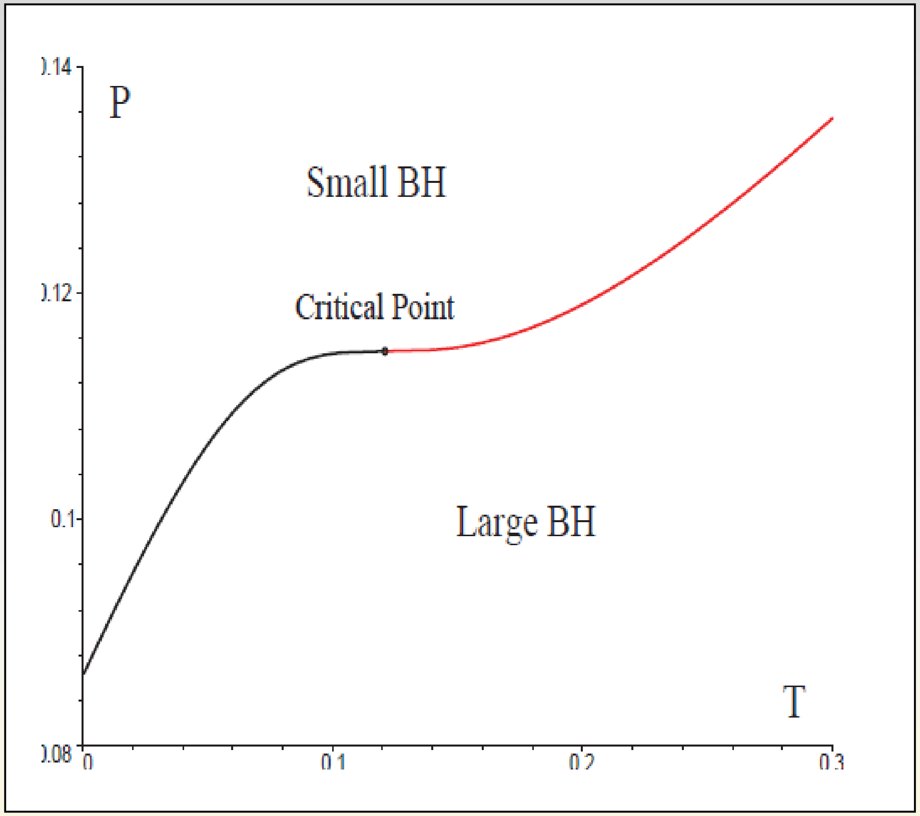


A. Frassino, DK, R. Mann, F. Simovic, arXiv:1406.7015.

Isolated critical point (3rd order and higher)

- Special tuned Lovelock couplings
- Odd-order J
- hyperbolic horizons (k=-1)





B. Dolan, A. Kostouki, DK, R. Mann, arXiv:1407.4783.

Critical exponents:

$$\boxed{\alpha = 0\,, \quad \beta = 1\,, \quad \gamma = J - 1\,, \quad \delta = J\,.}$$

Comments:

- cf. mean field theory critical exponents

$$\boxed{\alpha = 0\,, \quad \beta = \frac{1}{2}\,, \quad \gamma = 1\,, \quad \delta = 3}$$

- satisfies Widom relation and Rushbrooke inequality

$$\boxed{\gamma = \beta(\delta - 1)} \qquad \boxed{\alpha + 2\beta + \gamma \geq 2}$$

- Prigogine-Defay ratio

$$\boxed{\Pi = 1/J}$$

…indicates more than one order parameter?

III) Beyond spherical symmetry

- Ansatz

$$ds^2 = -\frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\varphi\right)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left[adt - (r^2 + a^2)d\varphi\right]^2$$

$$\rho^2 = r^2 + a^2\cos^2\theta \qquad \Delta = \Delta(r)$$

- Identify

$$T = \frac{\Delta'(r_+)}{4\pi(r_+^2 + a^2)} \qquad \Omega = -\frac{g_{t\varphi}}{g_{\varphi\varphi}}\bigg|_{r_+} = \frac{a}{r_+^2 + a^2}$$

$$\boxed{S = \frac{A}{4} = \pi(r_+^2 + a^2)}$$

$$\Rightarrow \quad \delta S = 2\pi(r_+\delta r_+ + a\delta a)$$

(cohomogeneity two)

- Rewrite the radial Einstein equation

$$8\pi T^r{}_r\big|_{r_+} = G^r{}_r\big|_{r_+} = \frac{a^2 - r_+^2 + r_+\Delta'(r_+)}{\rho_+^4}$$

as

$$T\delta S = \underbrace{\frac{2\rho_+^4 T^r{}_r\big|_{r_+}}{r_+(r_+^2+a^2)}\delta S}_{\sigma\delta A} + \underbrace{\frac{r_+^2 - a^2}{4\pi r_+(r_+^2+a^2)}\delta S}_{\delta E - \Omega\delta J}$$

- where we identified
  - horizon energy and angular momentum

$$E = \frac{r_+^2 + a^2}{2r_+}, \qquad J = Ea$$

  - "surface tension"

$$\sigma = \sigma(r_+, a) = \frac{\rho_+^4 T^r{}_r\big|_{r_+}}{2r_+(r_+^2+a^2)}$$

Rotating horizon thermodynamics

Horizon equation of state: $\boxed{\sigma = \sigma(A, J, T)}$

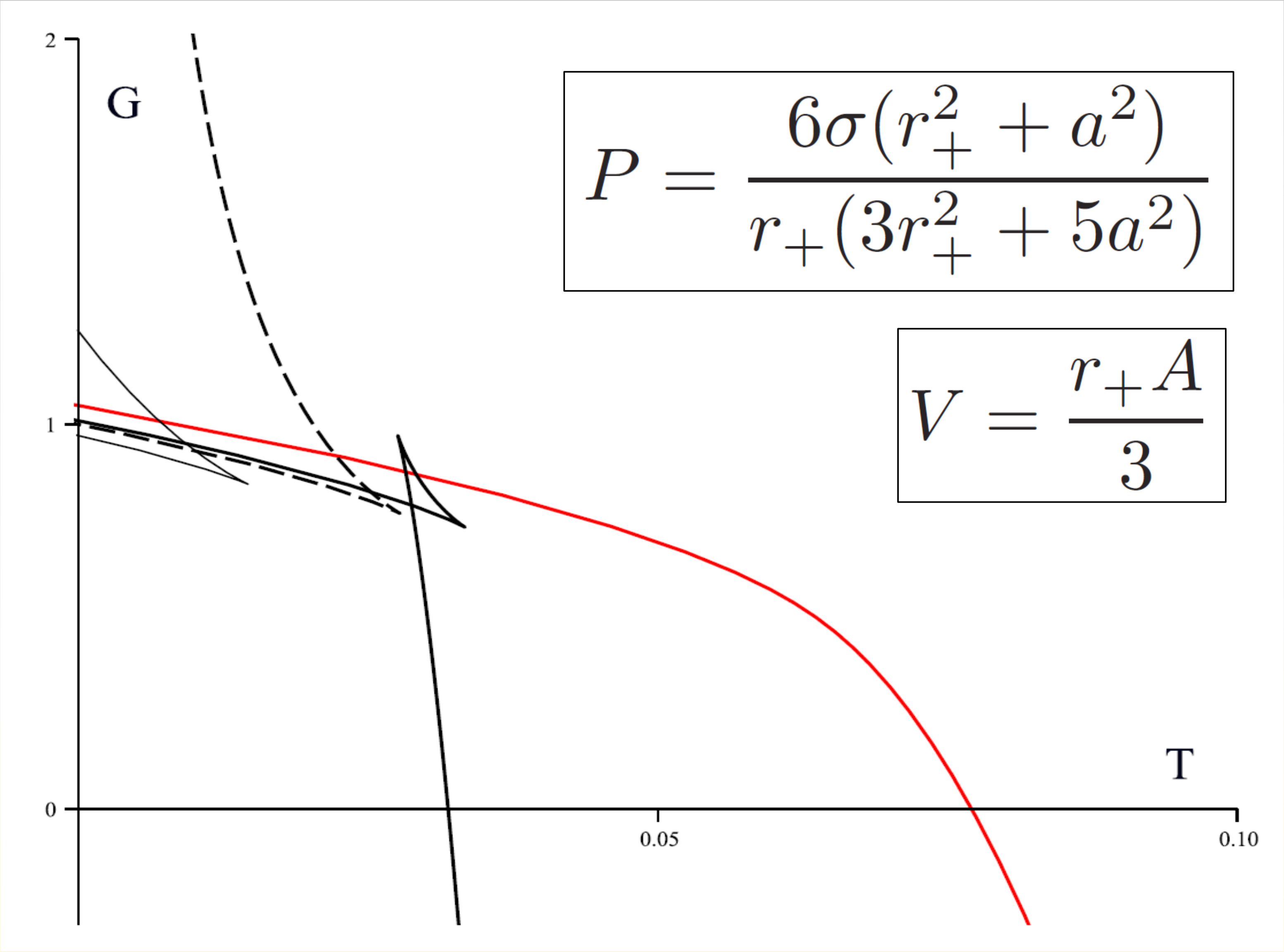
Horizon first law: $\boxed{\delta E = T\delta S + \Omega\delta J - \sigma\delta A}$

- **Cohomogeneity two** relation
- **Universal** regarding the matter content but ansatz has limited applications
- Provided a volume V of black hole is "freely" specified (not-unique) in canonical ensemble (J=const) we can rewrite these as **cohomogeneity one**

Horizon equation of state: $\boxed{P = P(V, T)}$

Horizon first law: $\boxed{\delta E = T\delta S - P\delta V}$

Swallow tail: rotating Einstein (P>0)



IV) Summary

1) We reviewed the proposal of **horizon thermodynamics** showing the "**equivalence**" of thermodynamic first law and radial Einstein equation and extended it to rotating black hole spacetimes.

2) The horizon first law is **cohomogeneity one** (two in the rotating case)

$$\boxed{\delta E = T\delta S - P\delta V}$$

$$\boxed{\delta E = T\delta S + \Omega\delta J - \sigma\delta A}$$

3) Essentially **universal** thermodynamic behaviour (independent of matter content) that depends only on the **gravitational theory** in consideration.

4) Interesting phase transitions are observed in Lovelock case (isolated critical point, re-entrance, triple points).

5) Compared to the extended phase space thermodynamics of AdS black holes adjusting P more natural?

6) What about a more general rotating ansatz? What is the physical meaning of surface tension sigma?