Ionization cross sections of neutrino electronagnetic interactions with electrons

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Neutrino-Electron Scattering at Low Energies

Thursday, April 25, 2019 - 9:00am to Saturday, April 27, 2019 - 1:00pm

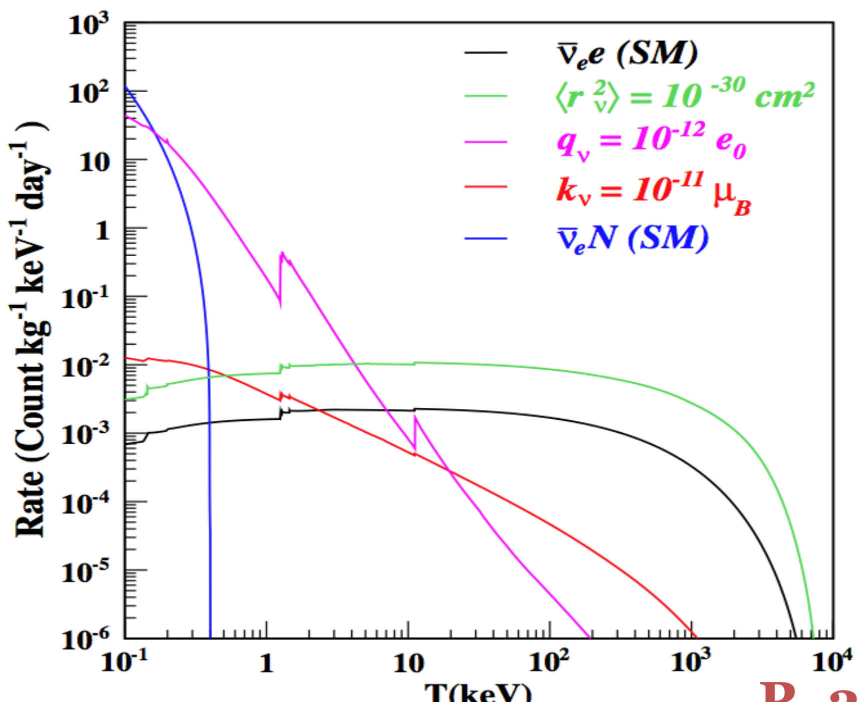
AMHERST CENTER FOR FUNDAMENTAL INTERACTIONS

Physics at the interface: Energy, Intensity, and Cosmic frontiers

University of Massachusetts Amherst

Constrain ν EM Properties by Ge

Data set	Reactor-$\bar{\nu}_e$ Flux ($\times 10^{13}$ cm^{-2} s^{-1})	Data strength Reactor on/off (kg-days)	Analysis Threshold (keV)	Bounds at 90% C.L. $\kappa_{\bar{\nu}_e}^{(\mathrm{eff})}$ ($\times 10^{-11} \mu_B$)	$q_{\bar{\nu}_e}$ ($\times 10^{-12}$)	$\langle r_{\bar{\nu}_e}^2 \rangle^{(\mathrm{eff})}$ ($\times 10^{-30}$ cm^2)
TEXONO 187 kg CsI [9]	0.64	29882.0/7369.0	3000	< 22.0	< 170	< 0.033
TEXONO 1 kg Ge [5,6]	0.64	570.7/127.8	12	< 7.4	< 8.8	< 1.40
GEMMA 1.5 kg Ge [7,8]	2.7	1133.4/280.4	2.8	< 2.9	< 1.1	< 0.80
TEXONO point-contact Ge [4,17]	0.64	124.2/70.3	0.3	< 26.0	< 2.1	< 3.20
Projected point-contact Ge	2.7	800/200	0.1	< 1.7	< 0.06	< 0.74
Sensitivity at 1% of SM	...	...	...	~ 0.023	~ 0.0004	~ 0.0014



Reference:

Phys. Lett. B **731**, 159, arXiv:1311.5294 (2014).

Phys. Rev. D **90**, 011301(R), arXiv:1405.7168 (2014).

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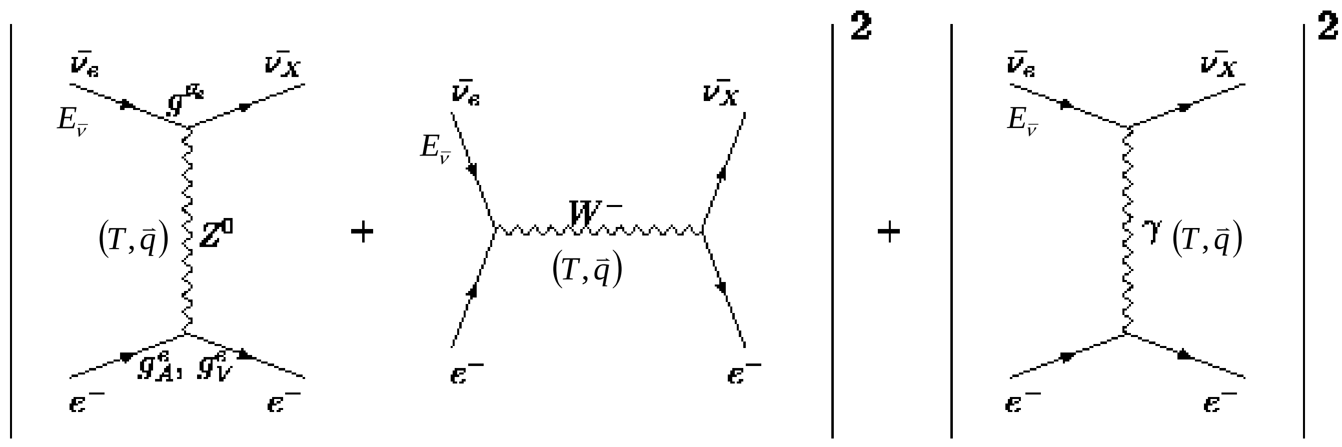
ν-e Non-standard Interactions (NSI)



$g_A = -1/2$

$g_v = 2\sin\theta_w + 1/2$

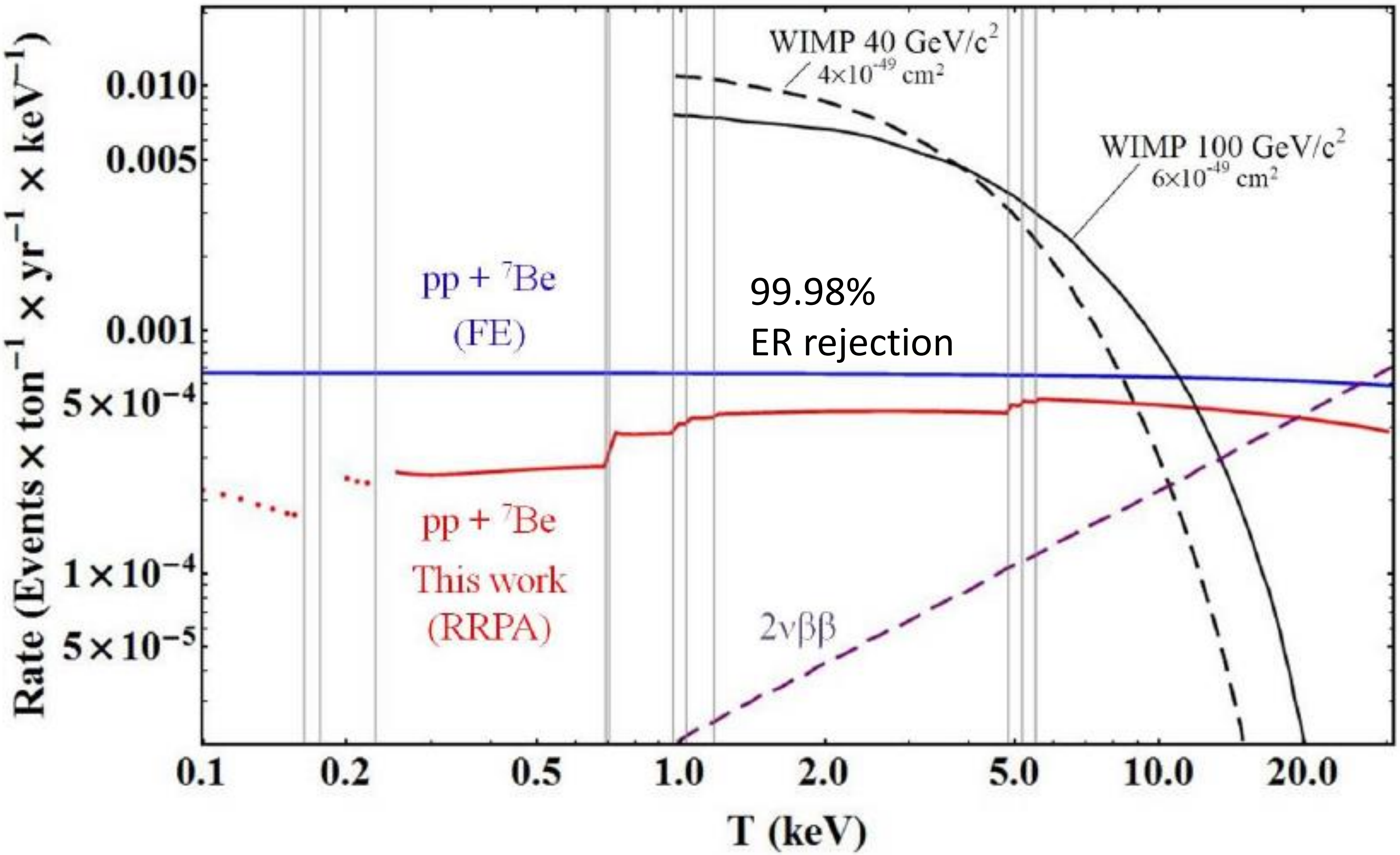
Weak Interaction

Magnetic moment

$$\frac{d\sigma}{dT} = \frac{G_F{}^2 m_e}{2\pi}[(g_V - g_A)^2 + (g_V + g_A + 2)^2(1 - \frac{T}{E_\nu})^2 + ((g_A+1)^2 - (g_V+1)^2)\frac{m_e T}{E_\nu^2}] + \frac{\pi\alpha^2\mu_\nu^2}{m_e^2}\left(\frac{1}{T} - \frac{1}{E_\nu}\right)$$

An Example for enhanced signals at low T!

Solar ν Background in LXe Detectors



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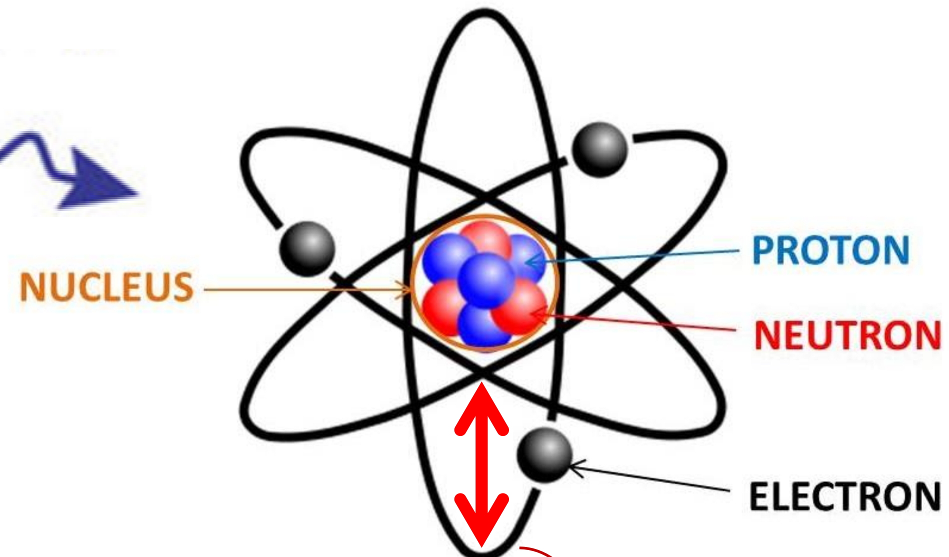
J.-W. Chen *et. al*., Phys. Lett. B **774**, 656, arXiv:1610.04177 (2017).

Outline

- Atomic ionization cross sections of neutrino-electron scattering
  - Why & when atomic effects become relevant?
  - MCRRPA: a framework of *ab initio* method
- Discovery potential of ton-scale LXe detectors in neutrino electromagnetic properties
  - arXiv: 1903.06085
  - solar nu v.s. reactor nu + Ge detector

Why Atomic Responses Become Important?



The space uncertainty is inversely proportional to its incident momentum:

$\lambda \sim 1/p$

Atomic Size is inversely proportional to its orbital momentum:

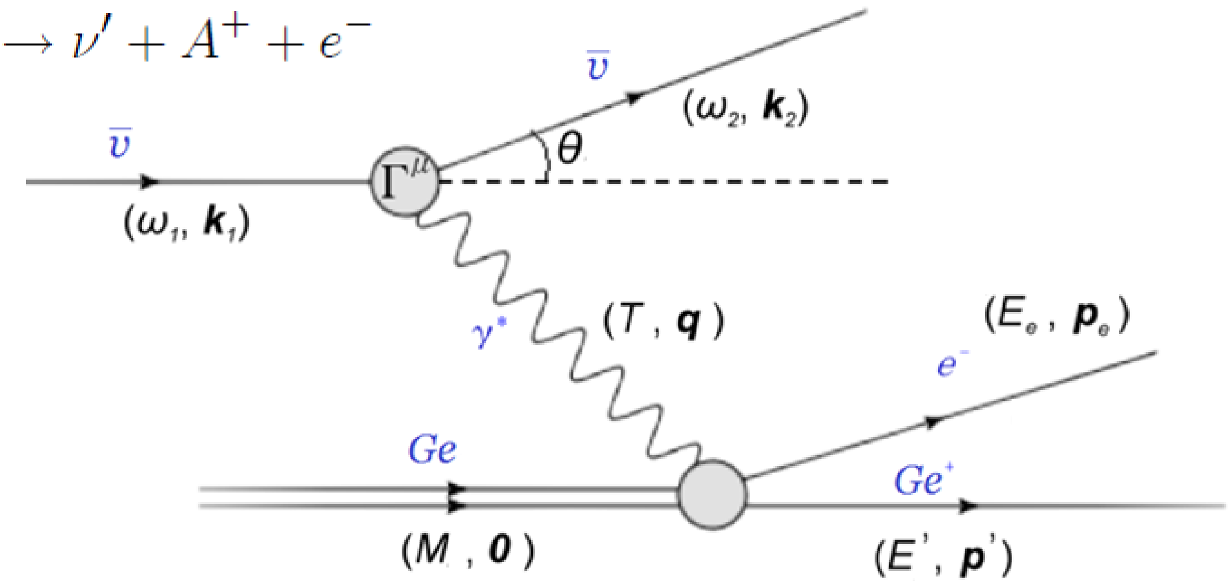
$Z m_e \alpha \sim Z$*3.7 keV

Z: effective charge

- 2 important factors:
  - neutrino momentum
  - Energy transfer T

Atomic Ionization Process for ν

$$\nu + A \rightarrow \nu' + A^+ + e^-$$



$$d\sigma = \frac{1}{|\vec{v}_1|} \frac{(4\pi\alpha)^2}{q^4} \ | M |^2 (2\pi)^4 \delta^4(k_1 + p_{\mathrm{Ge}} - k_2 - p_R - p_r) \frac{d^3\vec{k}_2}{(2\pi)^3} \frac{d^3\vec{p}_R}{(2\pi)^3} \frac{d^3\vec{p}_r}{(2\pi)^3}$$

The weak scattering amplitude:

$$\mathcal{M}^{(w)} = \frac{G_F}{\sqrt{2}} j_\mu^{(w)} (c_V \mathcal{J}^\mu - c_A \mathcal{J}_5^\mu)$$

The EM scattering amplitude:

$$\mathcal{M}^{(\gamma)} = \frac{4\pi\alpha}{q^2} j_\mu^{(\gamma)} \mathcal{J}^\mu$$

Electroweak Currents

Lepton current:

$$\langle k_2 | \hat{j}_l^{\mu} | k_1 \rangle = j_\mu^{(w)} + j_\mu^{(\gamma)}$$

w: The neutrino weak current
γ: The electromagnetic current

$$j_\mu^{(w)} = \bar{\nu}(k_2, s_2)\gamma_\mu(1-\gamma_5)\nu(k_1, s_1)$$

$$j_\mu^{(\gamma)} = \bar{\nu}(k_2, s_2)[F_1(q^2)\gamma_\mu - i(F_2(q^2) + iF_E(q^2)\gamma_5)\sigma_{\mu\nu}q^\nu + F_A(q^2)(q^2\gamma_\mu - \not{q}q_\mu)\gamma_5]\nu(k_1, s_1)$$

Atomic (axial-)vector current:

$$\langle f^{(-)} | j_A^{\mu} | i \rangle = c_V \mathcal{J}^\mu - c_A \mathcal{J}_5^\mu$$

$$c_V = -\frac{1}{2} + 2\sin^2\theta_w + \delta_{l,e} \quad , 1$$

$$c_A = -\frac{1}{2} + \delta_{l,e} \quad , 0$$

Sys. Error: $\sim\alpha \approx 1\%$

The Form Factors & Related Physical Quantities

$F_1(q^2)$: charge form factor

$F_2(q^2)$: anomalous magnetic

$F_A(q^2)$: anapole (*P*-violating)

$F_E(q^2)$: electric dipole

(*P*, *T*-violating)

neutrino millicharge :

$$\delta_Q = F_1(0),$$

charge radius squared :

$$\langle r_\nu^2 \rangle = 6 \frac{d}{dq^2} F_1(q^2) \Big|_{q^2 \to 0}$$

neutrino magnetic moment :

$$\mu_\nu = F_2(0),$$

electric dipole moment :

$$d_\nu = F_E(0),$$

anapole moment :

$$a_\nu = F_A(0)$$

“effective” Magnetic Moment

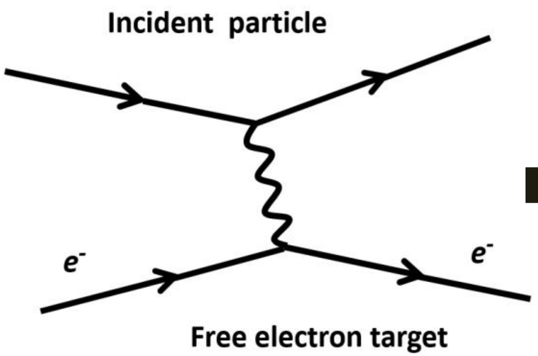
$$\bar{\nu}_R \sigma_{\mu\nu} \nu_L = -\bar{\nu}_R \sigma_{\mu\nu} \gamma_5 \nu_L \,, \quad \bar{\nu}_L \gamma_\mu \nu_L = -\bar{\nu}_L \gamma_\mu \gamma_5 \nu_L$$

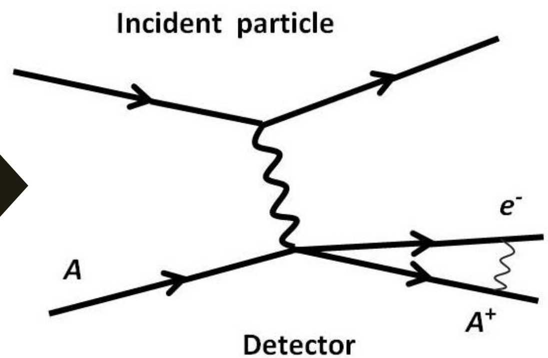
μ_ν and d_ν interactions are not distinguishable

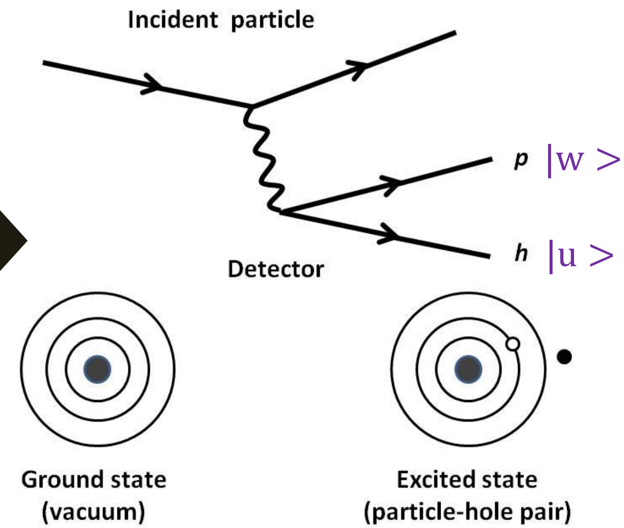
$$\left|\mu_{\nu_S}^{\text{eff}}\right|^2 = \sum_f \left| \sum_i A_{ie}(E_\nu, L) \left(\mu_{fi} - i d_{fi}\right) \right|^2$$

where f and i are the mass eigenstate indices for the outgoing and incoming neutrinos, $A_{ie}(E_\nu, L)$ describes how a solar neutrino oscillates to a mass eigenstate ν_i with distance L from the Sun to the Earth.

Bound Electron Wave Function







$\bar{e}\,\gamma^{\mu} e$

$< \Psi_f |\, \gamma^{\mu} | \Psi_i >$

$< w |\, \gamma^{\mu} | u >$

EM interaction | Weak interaction | Non-standard interactions

$$\gamma^{\mu} \;\rightarrow\; g_V \gamma^{\mu} + g_A \gamma^{\mu} \gamma^{5} \;\rightarrow\; \text{NSI Operators}$$

One-Electron Dirac Spinors

$$U_{n\kappa m}(\mathbf{r}) = \frac{1}{r}\begin{pmatrix} G_{n\kappa}(r)\,\Omega_{\kappa m}(\theta,\varphi) \\ iF_{n\kappa}(r)\,\Omega_{-\kappa m}(\theta,\varphi) \end{pmatrix}$$

$$\Omega_{\kappa m} \equiv \Omega_{jlm} = \sum_{M\mu} \langle lM\frac{1}{2}\mu|jm\rangle\, Y_{lm}(\hat{r})\,\chi_{\mu}$$

Then the radial Dirac equations can be reduced to

$$\frac{\mathrm{d}G}{\mathrm{d}r} = -\frac{\kappa}{r}G + \left[\frac{E+mc^2}{c} - \frac{V(r)}{c}\right]F$$

$$\frac{\mathrm{d}F}{\mathrm{d}r} = \frac{\kappa}{r}F - \left[\frac{E-mc^2}{c} - \frac{V(r)}{c}\right]G\,,$$

Atomic Response Functions

$$R_{\mu\nu}^{(w)} = \frac{1}{2J_i+1}\sum_{M_{J_i}}\sum_f \langle \Psi_f^{(-)} | c_V \hat{\mathcal{J}}_\mu - c_A \hat{\mathcal{J}}_{5\mu} | \Psi_i \rangle$$

$$\times \langle \Psi_f | c_V \hat{\mathcal{J}}_\nu - c_A \hat{\mathcal{J}}_{5\nu} | \Psi_i \rangle^* \delta(T + E_i - E_f)$$

Do multipole expansion with J

Final continuous wave functions could be obtained by **MCRRPA** and expanded in the (J, L) basis of orbital wave functions

Initial states could be approximated by bound electron orbital wave functions given by **MCDF**

$$R_{\mu\nu}^{(w)}\big|_{c_V=1, c_A=0} \rightarrow R_{\mu\nu}^{(\gamma)}$$

Ab initio Theory for Atomic Ionization

MCDF: multiconfiguration Dirac-Fock method

Dirac-Fock method: $\psi(t)$ is a Slater determinant of one-electron orbitals $u_a(\vec{r},t)$

and invoke variational principle $\langle \delta\bar{\psi}(t)|\, i\frac{\partial}{\partial t} - H - V_I(t)\,|\psi(t)\rangle = 0$

to obtain eigenequations for $u_a(\vec{r},t)$.

multiconfiguration: Approximate the many-body wave function $\Psi(t)$
by a superposition of configuration functions $\psi_\alpha(t)$

$$\Psi(t) = \sum_\alpha C_\alpha(t)\,\psi_\alpha(t) \qquad \text{For Ge: } \begin{cases} \psi_1 = \mathrm{Zn}(4p_{1/2})^2 \\ \psi_2 = Zn(4p_{3/2})^2 \end{cases}$$

MCRRPA: multiconfiguration relativistic random phase approximation

RPA: Expand $u_a(\vec{r},t)$ into time-indep. orbitals in power of external potential

$$u_a(\vec{r},t) = e^{i\varepsilon_a t}\left[u_a(\vec{r}) + w_{a+}(\vec{r})e^{-i\omega t} + w_{a-}(\vec{r})e^{i\omega t} + \ldots\right]$$

$$C_a(t) = C_a + [C_a]_+ e^{-i\omega t} + [C_a]_- e^{i\omega t} + \ldots$$

Here use square brackets with subscripts to designate the coefficients in powers of $e^{\pm i\omega t}$ in the expansion of various matrix elements:

$$H_{ab} \equiv \langle \psi_a(t) | H | \psi_b(t) \rangle = [H_{ab}]_0 + [H_{ab}]_+ e^{-i\omega t} + [H_{ab}]_- e^{i\omega t} + \cdots$$

MCDF Equations:

$\gamma_{\alpha\beta}$: Lagrange multipliers

$\delta_\alpha^\dagger$: functional derivatives with respect to $u_\alpha^\dagger$

$$EC_a + \sum_b C_b [H_{ab}]_0 = 0$$

$$\sum_{ab} C_a^* C_b \delta_\alpha^\dagger [H_{ab}]_0 - \sum_\beta \gamma_{\alpha\beta} u_\beta = 0$$

The zero-order equations are MCDF equations for unperturbed orbitals u_a and unperturbed weight coefficients C_a.

MCRRPA Equations:

$$(E \pm \omega) C_{a\pm} - \sum_b \left([H_{ab}]_0 C_{b\pm} + [H_{ab}]_\pm C_b \right) = \sum_b [V_{ab}]_\pm C_b$$

$$\sum_{ab} C_a^* C_b \, i\delta_\alpha^\dagger \left([\partial_{ab}]_\pm - [H_{ab}]_\pm \right) - \sum_{ab} \left(C_{am}^* C_b + C_a^* C_{b\pm} \right) \delta_\alpha^\dagger [H_{ab}]_0$$

$$- \sum_\beta \left(\gamma_{\alpha\beta} \mathrm{W}_{\beta\pm} + \gamma_{\alpha\beta\pm} u_\beta \right) = \sum_{ab} C_a^* C_b \delta_\alpha^\dagger [V_{ab}]_\pm$$

The first-order equations are the MCRRPA equations describing the linear response of atom to the external perturbation $v_\pm$.

Atomic Structure of Ge

Multiconfiguration of Ge Ground State (Coupled to total *J*=0) :

$$\Psi = C_1 \left(4p_{1/2}^2\right)_0 + C_2 \left(4p_{3/2}^2\right)_0$$

Valence Configuration	Configuration Weight	Precentage
$4p_{1/2}^2$	0.84939	72.15
$4p_{3/2}^2$	0.52776	27.85

Angular Momentum Selection Rule:

$$|j - J| \leqslant j' \leqslant |j + J|$$

Parity Selection Rule:

$$l + l' + J + \lambda - 1 = \text{even.}$$

Selection Rules for *J*=1, *λ*=1:

$$4p_{1/2} \rightarrow \epsilon s_{1/2},$$

$$4p_{1/2} \rightarrow \epsilon d_{3/2},$$

$$4p_{3/2} \rightarrow \epsilon s_{1/2},$$

$$4p_{3/2} \rightarrow \epsilon d_{3/2},$$

$$4p_{3/2} \rightarrow \epsilon d_{5/2}.$$

Multipole Expansion

Transition matrix elements of atomic ionization by nu-EM interactions:

$$\langle \Psi_f | v_+^{(\gamma)} | \Psi_i \rangle$$
$$= \frac{4\pi\alpha}{q^2} \left\{ j_0^{(\gamma)} \left\langle \Psi_f \middle| \int d^3x e^{i\vec{q}\cdot\vec{x}} \hat{\mathcal{J}}^0(\vec{x}) \middle| \Psi_i \right\rangle + \sum_{\lambda=\pm 1,0} (-1)^\lambda j_\lambda^{(\gamma)} \left\langle \Psi_f \middle| \int d^3x e^{i\vec{q}\cdot\vec{x}} \hat{\epsilon}^{-\lambda} \cdot \hat{\vec{\mathcal{J}}}(\vec{x}) \middle| \Psi_i \right\rangle \right\}$$

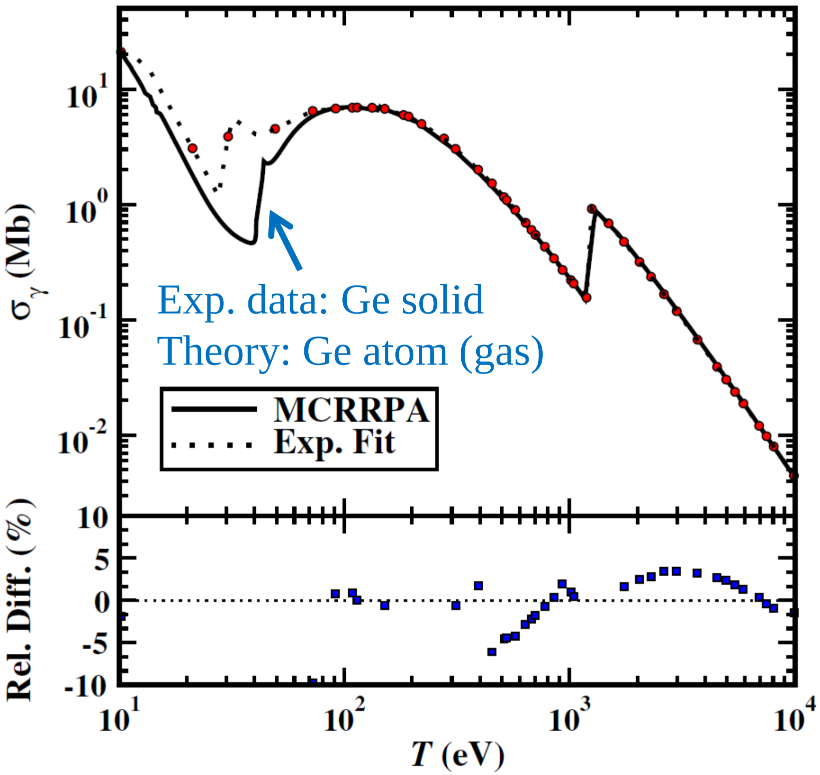
$$e^{i\vec{q}\cdot\vec{x}} = \sum_{J=0}^{\infty} \sqrt{4\pi(2J+1)}\, i^J j_J(\kappa r) Y_J^0(\Omega_x)$$

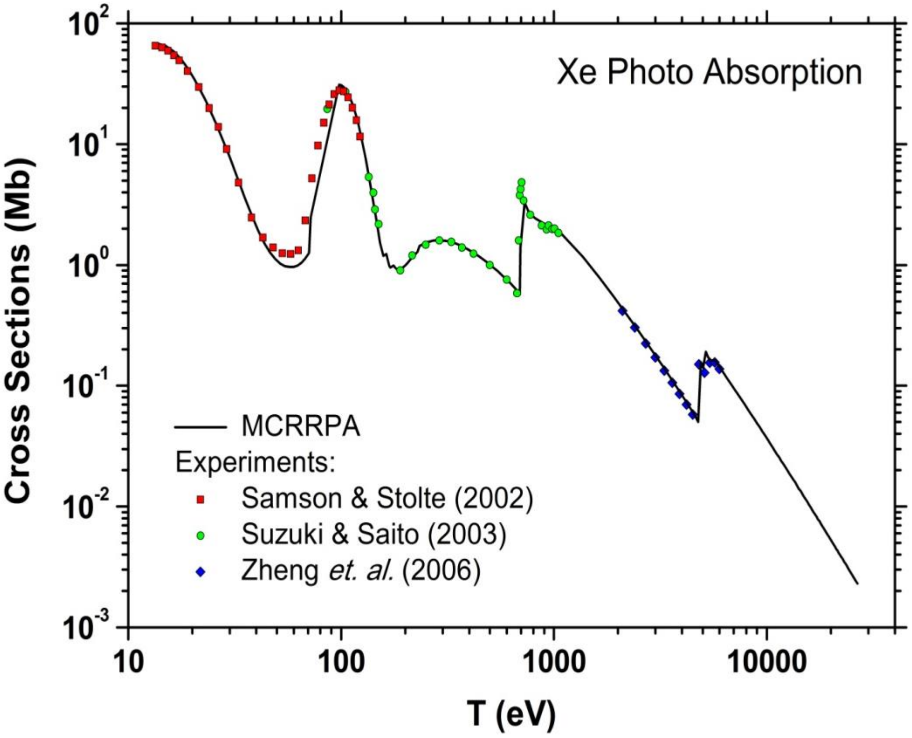
$$\hat{e}_{(\lambda=\pm1)}\, e^{i\vec{q}\cdot\vec{x}} = \sum_{J\geq 1} i^J \sqrt{2\pi(2J+1)} \left\{ \mp j_J(kr) \mathcal{Y}_{JJ1}^{\lambda} - \frac{1}{k} \nabla \times \left[j_J(kr) \mathcal{Y}_{JJ1}^{\lambda} \right] \right\}$$

$$\hat{e}_{(\lambda=0)}\, e^{i\vec{q}\cdot\vec{x}} = \frac{-i}{k} \sum_{J\geq 0} i^J \sqrt{4\pi(2J+1)}\, \nabla \left[j_J(kr)\, Y_{J0} \right]$$

$$v_+^{(\gamma)} = \frac{4\pi\alpha}{q^2} \left\{ \sum_{J=0}^{\infty} \sqrt{4\pi(2J+1)}\, i^J [j_0^{(\gamma)} \hat{C}_{J0}(k) - j_3^{(\gamma)} \hat{L}_{J0}(k)] \right.$$
$$\left. + \sum_{J\geq 1}^{\infty} \sqrt{2\pi(2J+1)}\, i^J \sum_{\lambda=\pm 1} j_\lambda^{(\gamma)} [\hat{E}_{J-\lambda}(k) - \lambda \hat{M}_{J-\lambda}(k)] \right\}$$

Benchmark: Ge & Xe Photoionization





Above 100 eV error under 5%.

B. L. Henke, E. M. Gullikson, and J. C. Davis, Atomic Data and Nuclear Data Tables **54**, 181-342 (1993).
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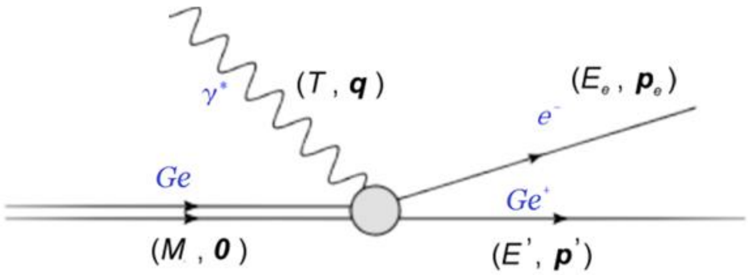
I. H. Suzuki and N. Saito, J. Electron Spectrosc. Relat. Phenom. **129**, 71 (2003).
L. Zheng *et al*., J. Electron Spectrosc. Relat. Phenom. **152**, 143 (2006).

Approximation Schemes

Longitudinal Photon Approx. (LPA) : $V_T = 0$

Equivalent Photon Approx. (EPA) : $V_L = 0, \; q^2 = 0$

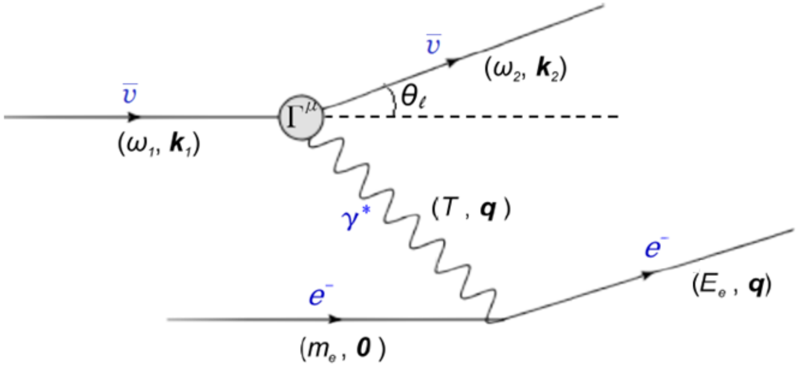


① Strong q^2-dependence in the denominator : long-range interaction

② Real photon limit $q^2 \sim 0$:
relativistic beam or soft photons $q^\mu \sim 0$

Free Electron Approx. (FEA) : $q^2 = -2\, m_e T$

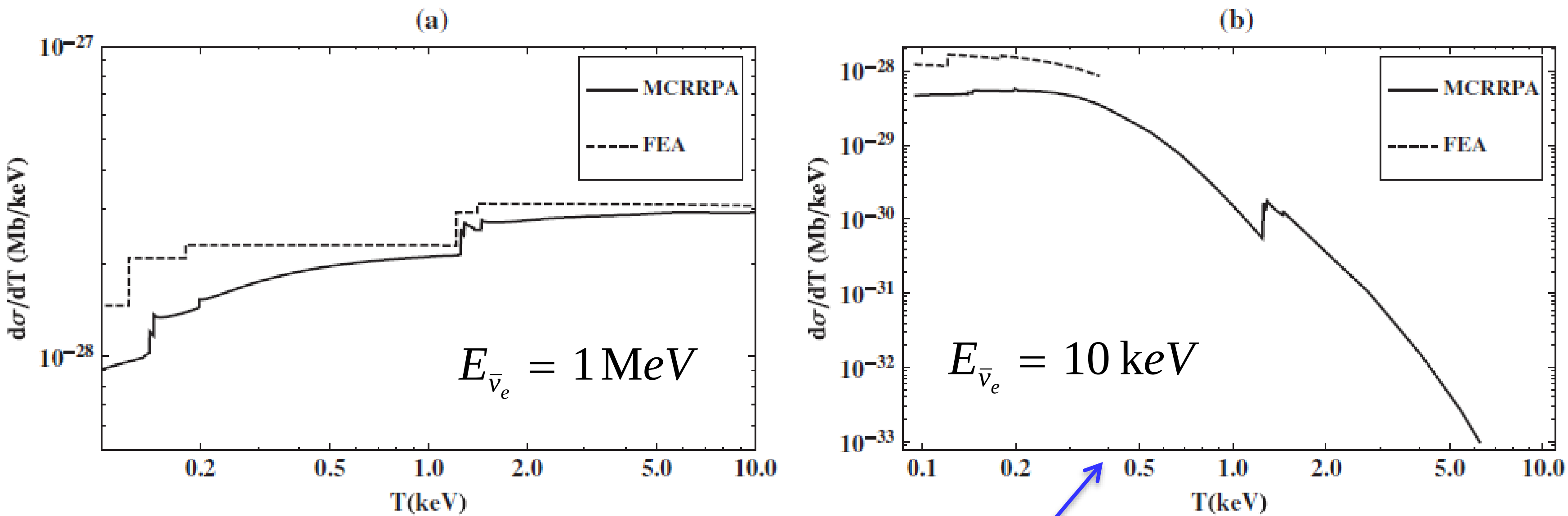
$$\left.\frac{d\sigma}{dT}\right|_{\mathrm{FEA}} = \sum_{i=1}^{Z} \theta(T - B_i) \left.\frac{d\sigma^{(0)}}{dT}\right|_{q^2=-2m_e T}$$



① Main contribution comes from the phase space region similar with 2-body scattering

② Atomic effects can be negligible :
$E_\nu >> Z\, m_e \alpha$
$T \neq B_i$ (binding energy)

Numerical Results: Weak Interaction



(1) short range interaction

(2) neutrino mass is tiny

(3) $E_\nu >> Z m_e \alpha$

FEA works well away from the ionization thresholds.

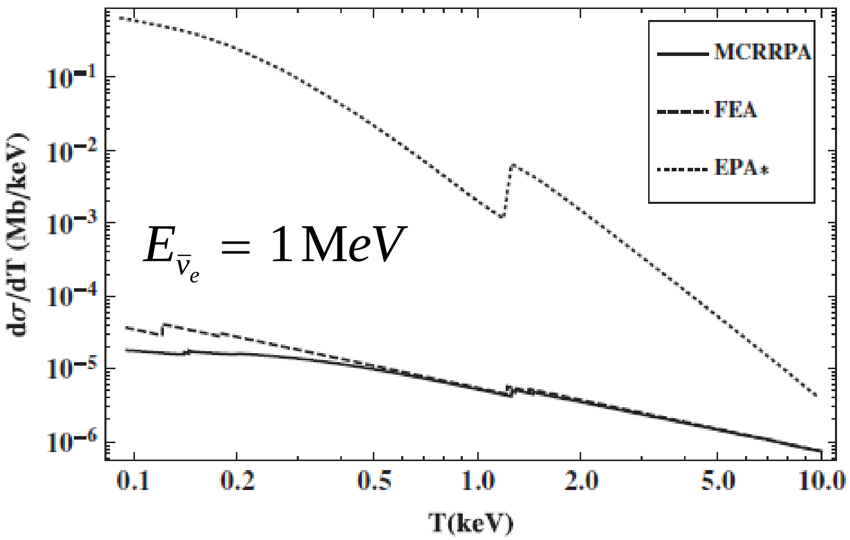
$$\text{cutoff}: T_{\text{Max}} = \frac{2E_{\bar{\nu}_e}^2}{2E_{\bar{\nu}_e} + m_e} \approx 0.38\,keV$$

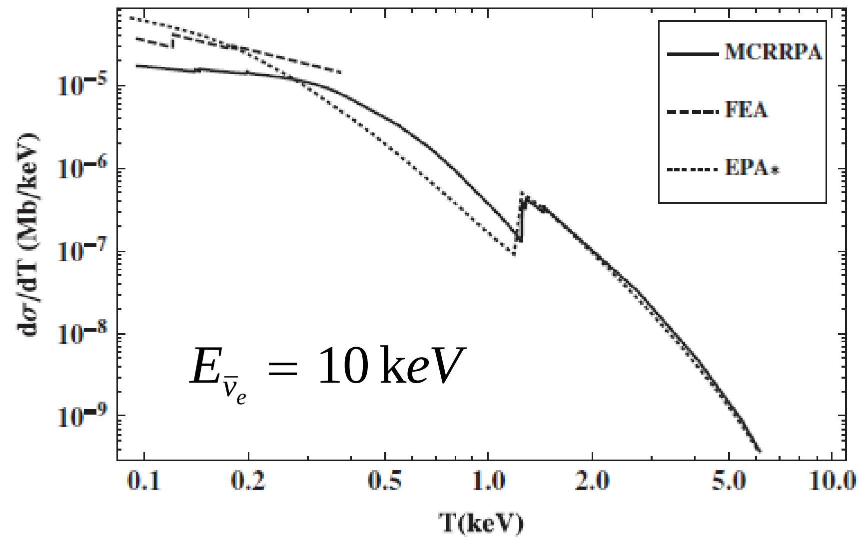
Kinematic forbidden by the inequality:

$$\vec{p}_r \approx \sqrt{2m_e T} \leq \vec{q}_{Max} \approx 2E_{\bar{\nu}_e} - T$$

(backward scattering, $m_{\bar{\nu}_e} \to 0$)

Numerical Results: NMM



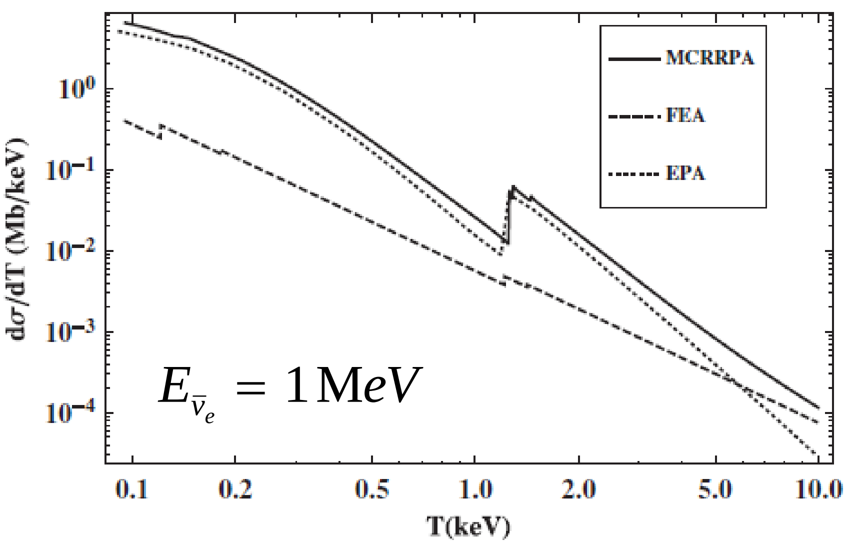


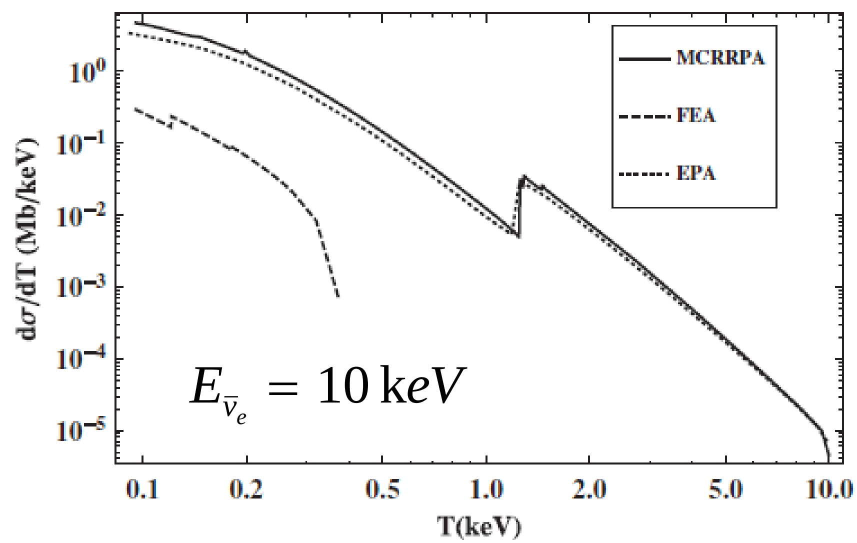
Similar with WI cases. FEA still faces a cutoff with lower E_ν.

For right plot, EPA becomes better when T approaches to E_ν (q^2 -> 0).

Consistent with analytic Hydrogen results.

Numerical Results: Millicharge



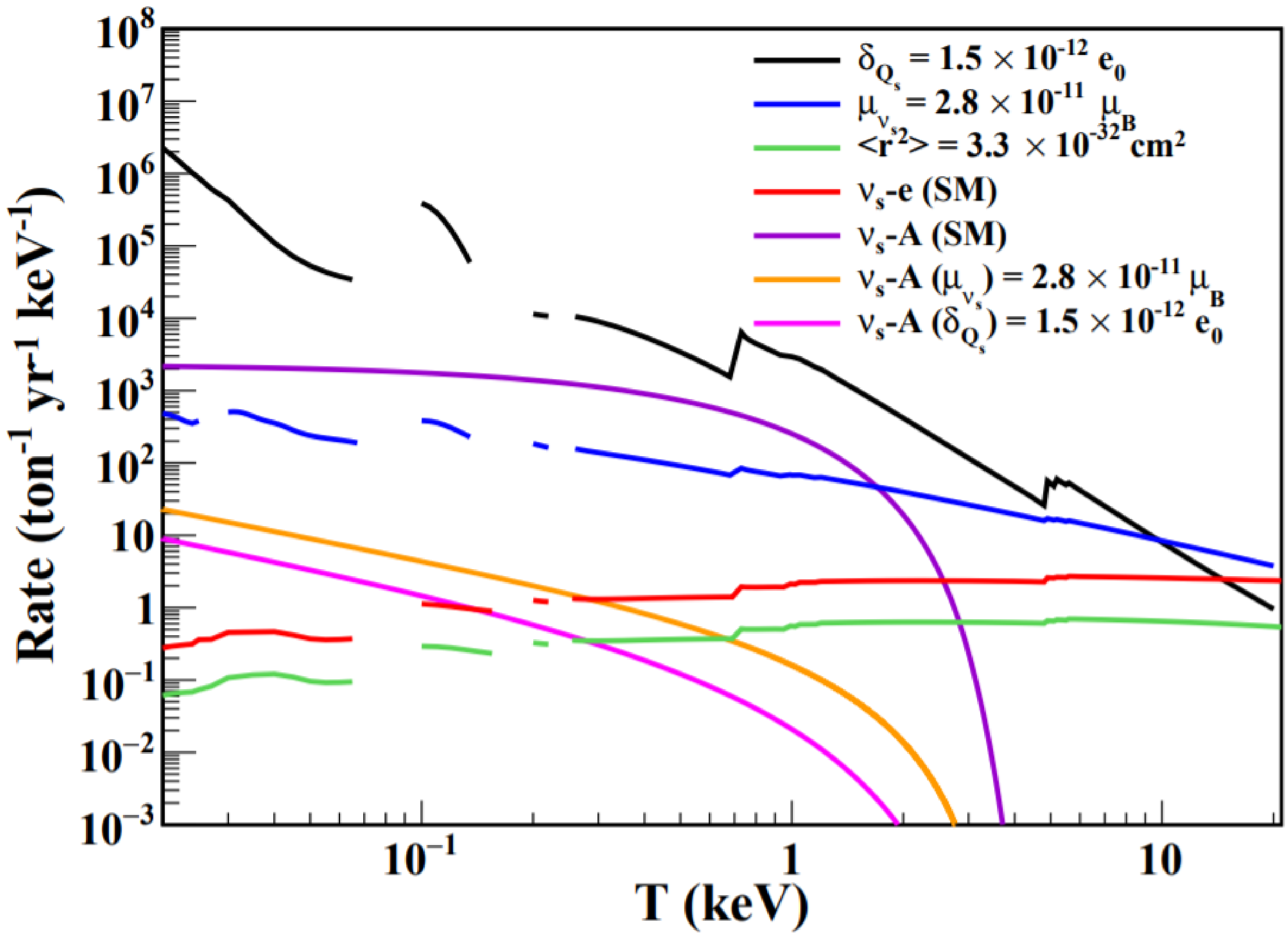


EPA worked well due to q^2 dependence in the denominator of scattering formulas of F_1 form factor (a strong weight at small scattering angles).

Double Check on Our Simulation

- We perform *ab initio* many-body calculations for atomic initial & final states WF in ionization processes, and test by
  - Comparing with photo-absorption experimental data, for typical E1 transition, the difference is <5%.
  - In general, we have confidence to report a 5~10% theoretical errors.
  - It agrees with some common approximations under the crucial condition as we know in physical picture

Solar ν As Signals in LXe Detectors



J.-W. Chen *et. al.*, arXiv:1903.06085 (2019).

Expected Experimental Limits

Experiment	Exposure (ton-year)	Threshold ($\mathrm{keV_{ER}}$) (S1 + S2)	Background Level ($\mathrm{kg^{-1}keV^{-1}day^{-1}}$)	Upper Bounds at 90% CL $\mu_{\nu_S}^{\mathrm{eff}}$ ($\times 10^{-11}\ \mu_B$)	$\delta_{Q_S}^{\mathrm{eff}}$ ($\times 10^{-12} e_0$)	$\langle r_{\nu_S}^2 \rangle^{\mathrm{eff}}$ ($\times 10^{-30}\ \mathrm{cm^2}$)
XENON-10 [40]	8.67×10^{-2}	2.0	1.1	348.74	65.45	158.86
XENON-100 [41]	2.1×10^{-2}	5.0	5.3×10^{-3}	35.13	13.03	11.20
PandaX-II [42]	7.4×10^{-2}	1.2	2.7×10^{-3}	15.46	2.06	8.13
XENON-1T [43]	1.0	1.4	2.24×10^{-4}	4.51	0.64	2.31
Projected LZ [44]	15.34	1.5/0.5	4.27×10^{-5}	$1.85/\sim 1$	$0.28/\sim 0.01$	0.93
Projected DARWIN [45]	14.0	2.0	2.0×10^{-5}	1.27	0.24	0.58

Assuming an energy resolution from the XENON100 experiment

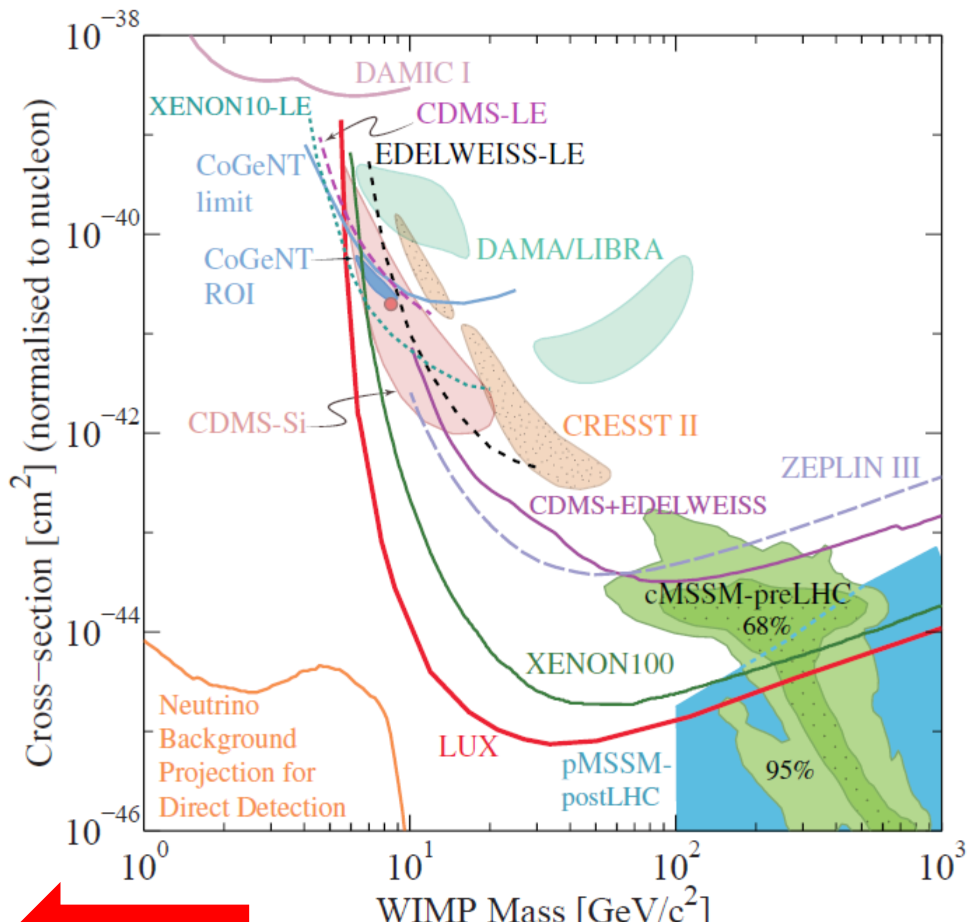
J.-W. Chen *et. al.*, arXiv:1903.06085 (2019).

What Else?

Portals to the Dark Sector:

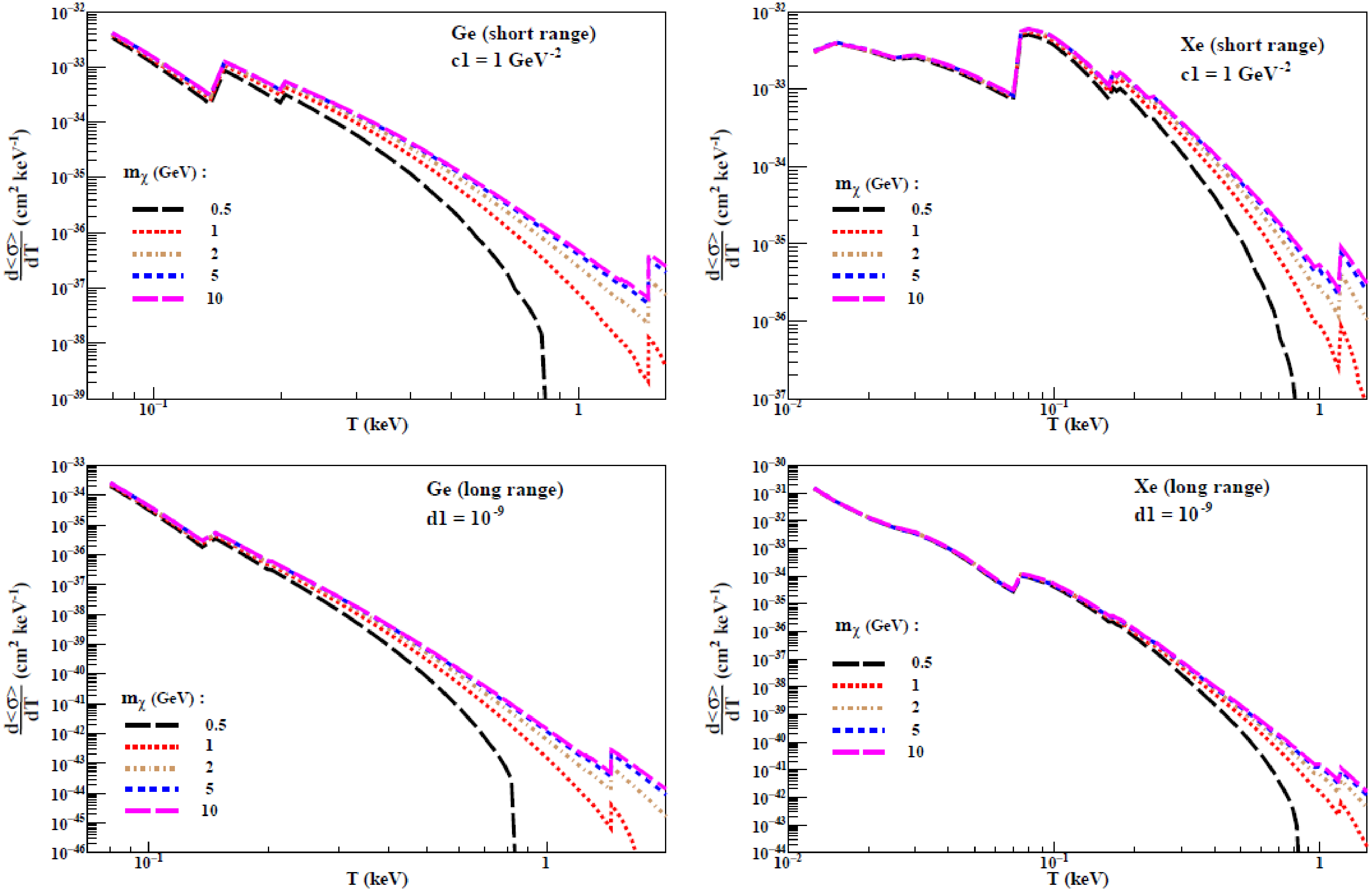
Portal	Particles	Operator(s)
"Vector"	Dark photons	$-\frac{\epsilon}{2\cos\theta_W} B_{\mu\nu} F'^{\mu\nu}$
"Axion"	Pseudoscalars	$\frac{a}{f_a} F_{\mu\nu} \widetilde{F}^{\mu\nu}$, $\frac{a}{f_a} G_{i\mu\nu} \widetilde{G}_i^{\mu\nu}$, $\frac{\partial_\mu a}{f_a} \overline{\psi}\gamma^\mu\gamma^5\psi$
"Higgs"	Dark scalars	$(\mu S + \lambda S^2) H^\dagger H$
"Neutrino"	Sterile neutrinos	$y_N LHN$

1. Remain a large region for the possibility of LDM (Ex: Dark Sectors)
2. Other interactions, or interacted with electrons



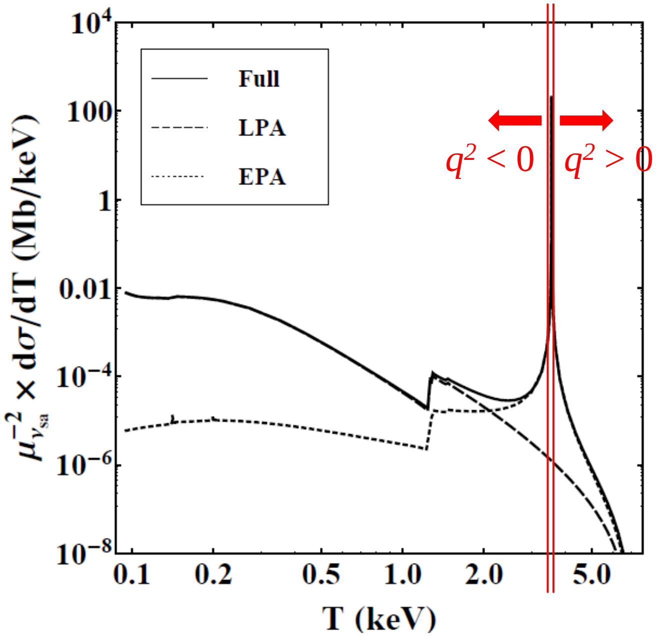
K. Olive *et al*. (Particle Data Group), Chin. Phys. C **38**, 090001 (2014).
R. Essig, J. A. Jaros, W. Wester, P. H. Adrian, S. Andreas *et al*., arXiv:1311.0029.

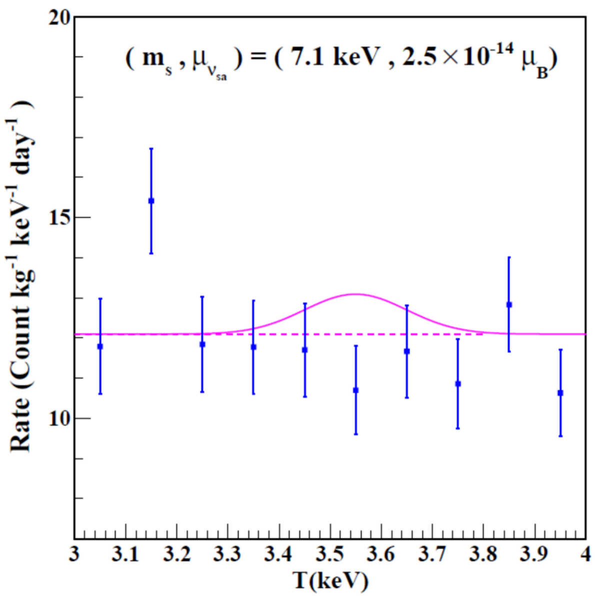
Spin-Indep. DM-e Scattering in Ge & Xe

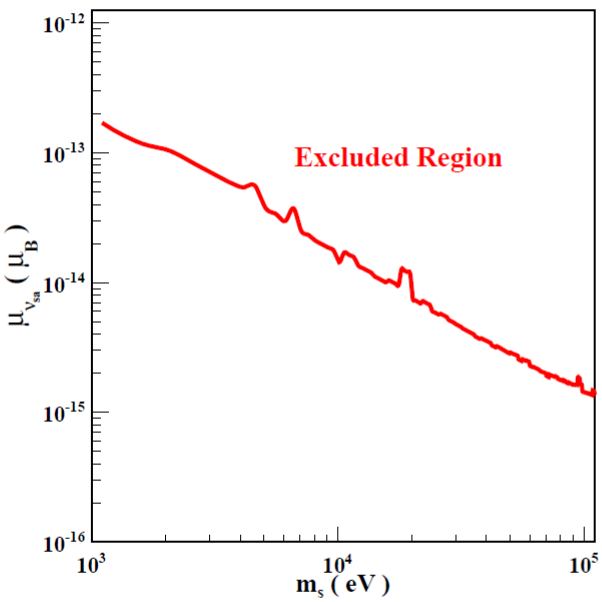


J.-W. Chen *et. al.*, arXiv:1812.11759 (2018).

Sterile Neutrino Direct Constraint



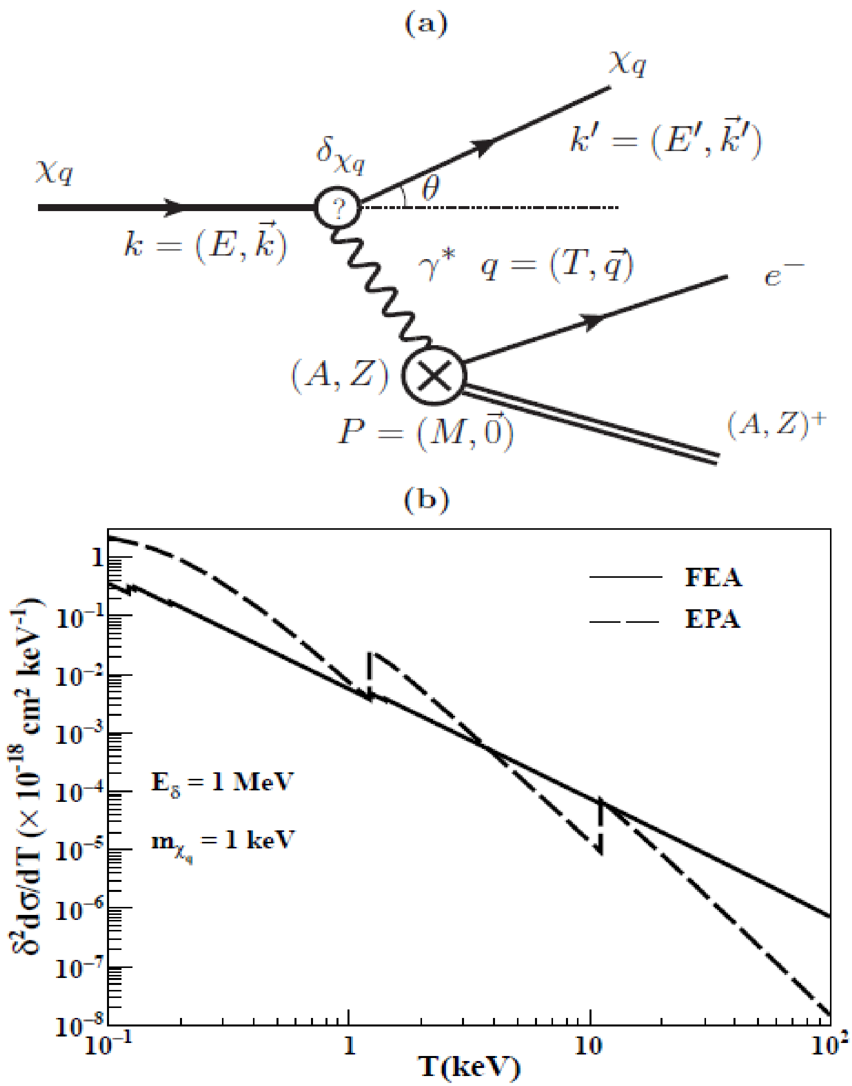


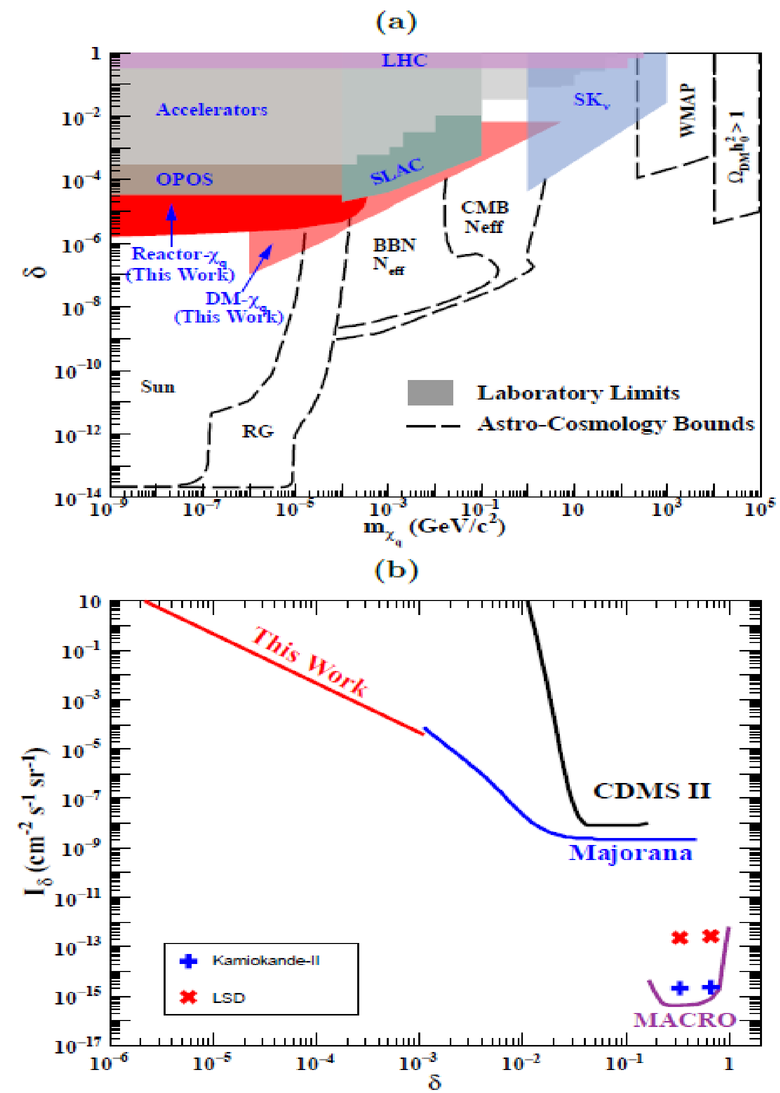


- Non-relativistic massive sterile neutrinos decay into SM neutrino.
- At m_s = 7.1 keV, the upper limit of $\mu_{\nu sa} < 2.5*10^{-14}\ \mu_B$ at 90% C.L.
- The recent X-ray observations of a 7.1 keV sterile neutrino with decay lifetime $1.74*10^{-28}\ s^{-1}$ can be converted to $\mu_{\nu sa} = 2.9*10^{-21}\ \mu_B$, much tighter because its much larger collecting volume.

J.-W. Chen *et al.*, Phys. Rev. D **93**, 093012, arXiv:1601.07257 (2016).

Constraints on millicharged DM





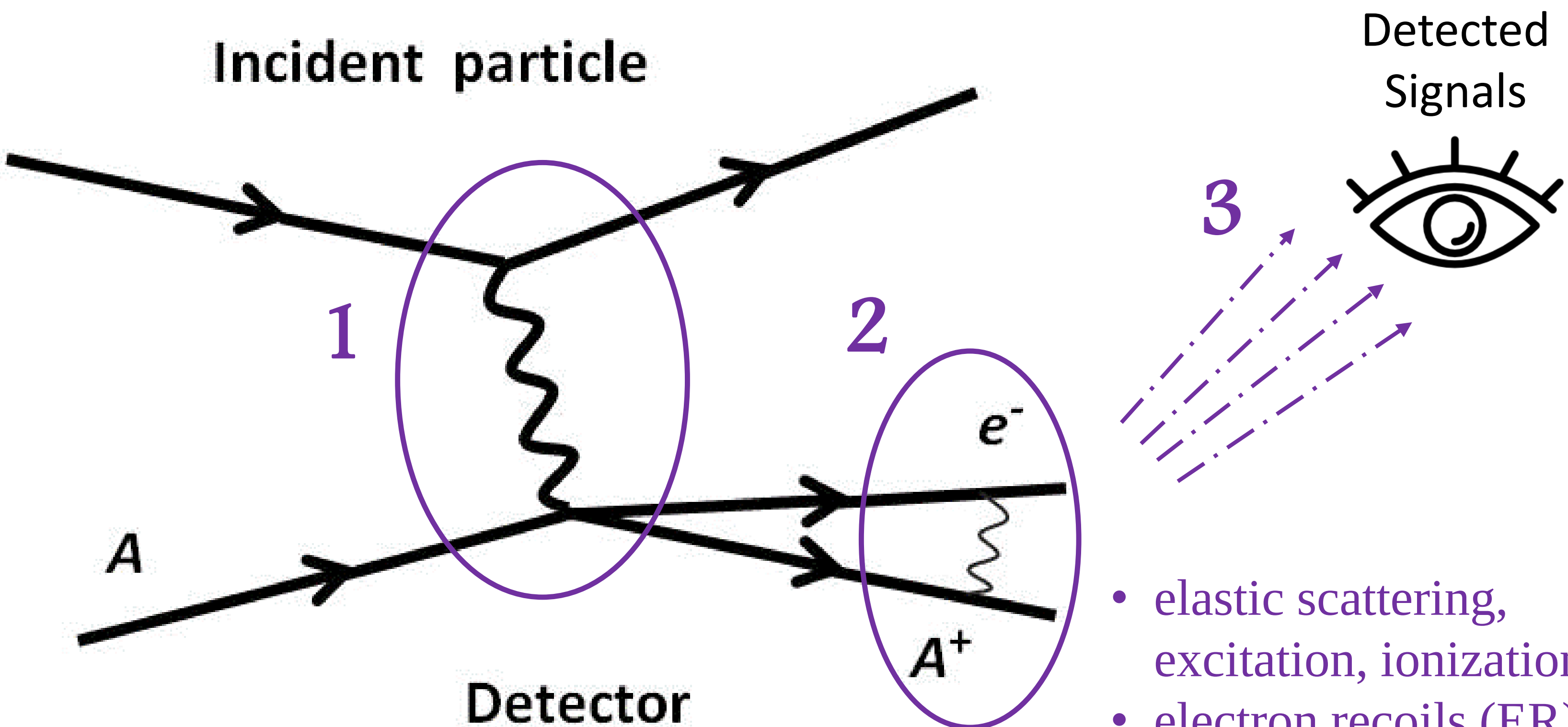
L. Singh *et. al*. (TEXONO Collaboration), arXiv:1808.02719 (2018).

Summary

- Low energies nu-e Scattering can be the signal or important background in direct detection experiments, but the atomic effects should be taken into consideration now.
- *Ab initio* atomic many-body calculations of ionization processes in Ge and Xe detectors performed with ~5% estimated error. That can be applied for
  1. Constraining neutrino EM properties,
  2. Study on solar neutrino backgrounds in DM detection,
  3. Calculating DM atomic ionization cross sections.

THANKS FOR YOUR ATTENTION!

Scattering Diagrams and Detector Response



- elastic scattering, excitation, ionization
- electron recoils (ER) or nucleus recoils (NR)

1. **The particle-detector interaction**
2. ***dσ/dT* for the primary scattering process**
3. **The following energy loss mechanism**

DM Effective Interaction with Electron or Nucleons

Leading order (LO):

short range

$$L_{\rm int}^{\rm (LO)} = \sum_{f=e,p,n} \Big\{ c_1^{(f)} (\chi^\dagger \chi)(f^\dagger f) + c_4^{(f)} (\chi^\dagger \vec{S}_\chi \chi) \cdot (f^\dagger \vec{S}_f f) + d_1^{(f)} \frac{1}{q^2} (\chi^\dagger \chi)(f^\dagger f) + d_4^{(f)} \frac{1}{q^2} (\chi^\dagger \vec{S}_\chi \chi) \cdot (f^\dagger \vec{S}_f f) \Big\}$$

spin-indep. long range spin-dep.

Differential cross section for spin-independent contact interaction with electron ($c_1^{(e)}$) :

$$d\sigma|_{c_1^{(e)}} = \frac{2\pi}{v_\chi} \sum_F \bar{\sum_I} |\langle F| c_1^{(e)} e^{i\frac{\mu}{m_e}\vec{q}\cdot\vec{r}} |I\rangle|^2 \times \delta(T - E_{\rm c.m.} - (E_F - E_I)) \frac{d^3 k_2}{(2\pi)^3}$$

Initial & final states of detector material

Toy Model: Analytic Hydrogen WFs

$$\langle 100|\vec{r}\rangle = \frac{1}{\sqrt{\pi}} Z^{\frac{3}{2}} e^{-Z\bar{r}},$$

exp.-decay with the rate ∝ orbital momentum ~ 3.7 keV

$$\langle nlm_l|\vec{r}\rangle = \frac{1}{(2l+1)!}\sqrt{\frac{(n+l)!}{2n(n-l-1)!}}\left(\frac{2Z}{n}\right)^{\frac{3}{2}} e^{-\frac{Z\bar{r}}{n}}\left(\frac{2Z\bar{r}}{n}\right)^{l}$$

$${}_1F_1\left(-(n-l-1), 2l+2, \frac{2Z\bar{r}}{n}\right) Y_l^{m_l *}(\theta,\phi),$$

$$\langle \vec{p}_r|\vec{r}\rangle = e^{\frac{\pi Z}{2\bar{p}_r}}\Gamma\left(1-\frac{iZ}{\bar{p}_r}\right) e^{-i\vec{p}_r\cdot\vec{r}}\,{}_1F_1\left(\frac{iZ}{\bar{p}_r}, 1, i(p_r r + \vec{p}_r\cdot\vec{r})\right)$$

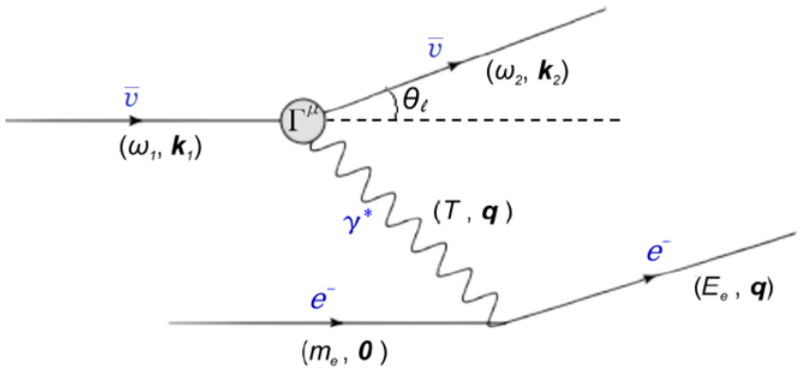
Oscillated like sin/cos function with frequency ∝ electron momentum ~ $(2m_e T)^{1/2}$

- The initial state of the hydrogen atom at the ground state, the spatial part $|I>_{\text{spat}} = |1s>$

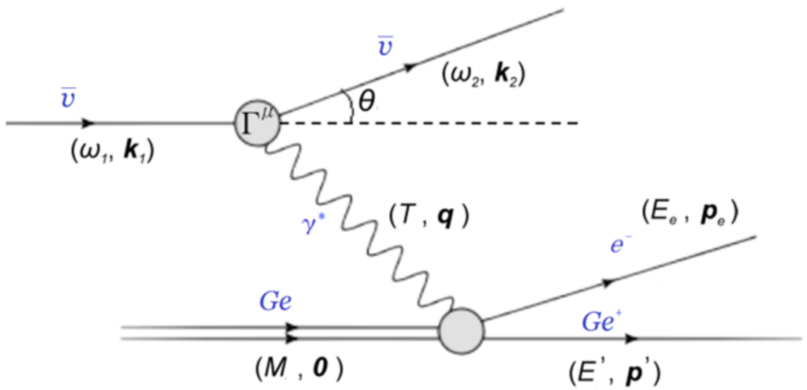
1. **elastic scattering**: $<F|_{\text{spat}} = <1s|$
2. **discrete excitation** (ex): $<F|_{\text{spat}} = <nlm_l|$
3. **ionization** (ion): $<F|_{\text{spat}} = <\vec{p}_r|$

Elastic v.s. Inelastic Scattering

$$q^2 = -2\, m_e\, T$$



$$\nu + A \rightarrow \nu' + A^+ + e^-$$



Phase space is fixed in 2-body scattering
→ 4-momentum transfer is fixed
→ scattering angle is fixed
→ Maximum energy transfer is limited

by a factor $r = \dfrac{4\, m_{inc}\, m_{tar}}{(m_{inc} + m_{tar})^2}$

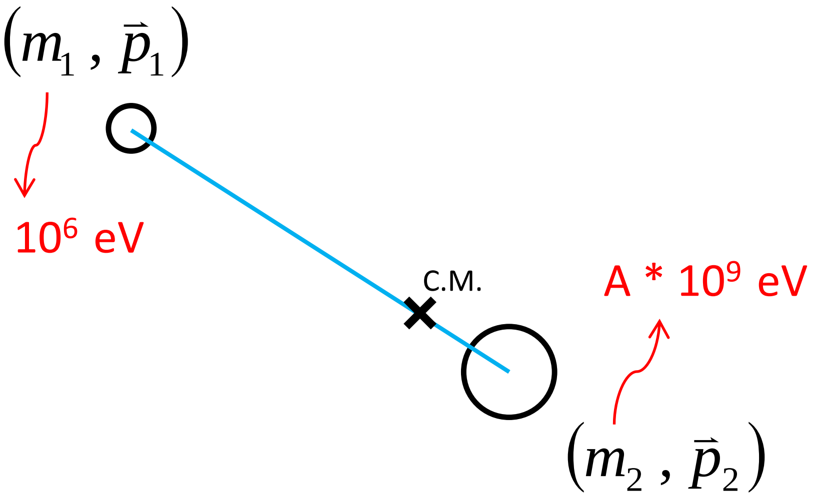
Energy and momentum transfer can be shared by nucleus and electrons
→ Inelastic scattering
(energy loss in atomic energy level)
→ Phase space suppression

Reduce Mass System for Atom

Two particles can reduce to one system at their center of mass, with internal motion:

$$\frac{\vec{p}_1^{\,2}}{2m_1} + \frac{\vec{p}_2^{\,2}}{2m_2} = \frac{\vec{p}_{tot}^{\,2}}{2M} + \frac{\vec{p}_{rel}^{\,2}}{2\mu} = T - B$$

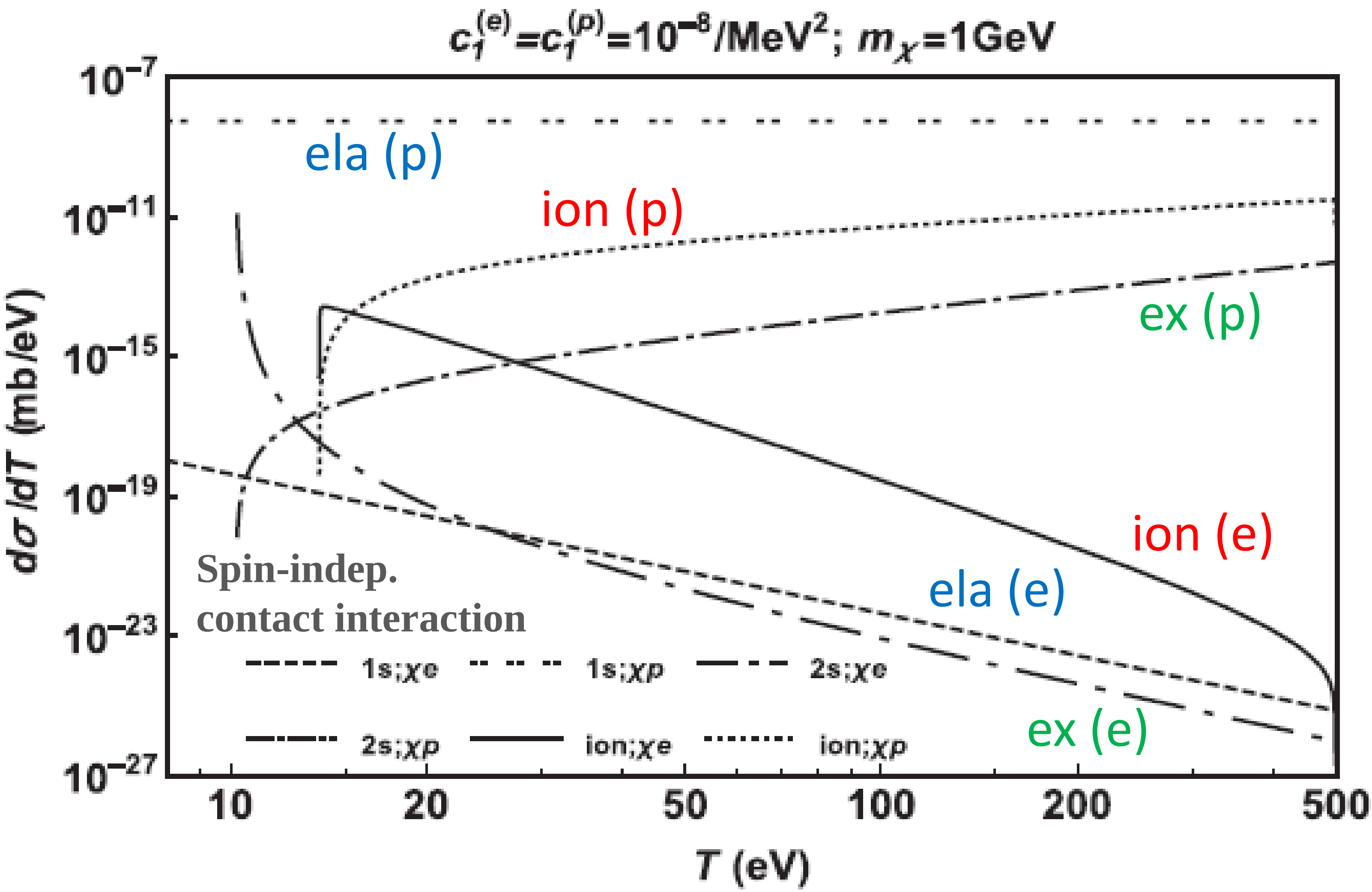
$$\begin{cases} M = m_1 + m_2 \\ \mu = \dfrac{m_1 m_2}{m_1 + m_2} \end{cases}, \quad \begin{cases} \vec{p}_{tot} = \vec{p}_1 + \vec{p}_2 \\ \vec{p}_{rel} = \mu\left(\vec{v}_1 - \vec{v}_2\right) \end{cases}$$



If the system received a 4-momentum transfer $(T , \vec{q})$, then the relative momentum would be:

$$\vec{p}_{rel} = \begin{cases} \dfrac{\mu}{m_1}\vec{q} \ \ (\text{hit } m_1) \\ \dfrac{\mu}{m_2}\vec{q} \ \ (\text{hit } m_2) \end{cases} \approx \sqrt{2\mu\,(T - B)} \ \ (\text{for } \mu << M)$$

Comparison of DM-H Cross Sections with the Electron and Proton



J.-W. Chen *et al.*, Phys. Rev. D 92, 096013 (2015).

Multipole Expansion & Operators

$$\hat{C}_{JM}(k) = \int d^3x[j_J(kr)Y_{JM}]\,\hat{J}_0(\vec{x})$$

$$\hat{L}_{JM}(k) = \frac{i}{k}\int d^3x\{\nabla[j_J(kr)Y_{JM}]\}\cdot\hat{J}(\vec{x})$$

$$\hat{E}_{JM}(k) = \frac{1}{k}\int d^3x\,[\nabla\times j_J(kr)\mathcal{Y}^M_{JJ1}]\cdot\hat{J}(\vec{x})$$

$$\hat{M}_{JM}(k) = \int d^3x[j_J(kr)\mathcal{Y}^M_{JJ1}]\cdot\hat{J}(\vec{x})$$

MCRRPA Transition Amplitude:

$$\langle\Psi_f^{(-)}|v_+|\Psi_i\rangle = \sum_\alpha \Lambda_\alpha(\langle w_{\alpha+}|v_+|u_\alpha\rangle + \langle u_\alpha|v_+|w_{\alpha-}\rangle)$$
$$+\sum_{a,b}([C_a]^\star_+ C_b + C^\star_a[C_b]_-)\langle\psi_a|v_+|\psi_b\rangle$$

Neutrino-Impact Ionization Cross Sections

neutrino weak scattering :

$$\frac{d\sigma_w}{dT} = \frac{G_F^2}{2\pi^2}(E_\nu - T)^2 \int \cos^2\frac{\theta}{2}\Big\{R_{00} - \frac{T}{|\vec{q}|}R_{03+30} + \frac{T^2}{|\vec{q}|^2}R_{33} + \left(\tan^2\frac{\theta}{2} + \frac{|\vec{q}|^2}{2\,q^2}\right)R_{11+22} + \tan\frac{\theta}{2}\sqrt{\tan^2\frac{\theta}{2} + \frac{|\vec{q}|^2}{q^2}}R_{12+21}\Big\}d\Omega_{\mathbf{k}_2}$$

neutrino magnetic moment scattering :

$$\frac{d\sigma_\mu}{dT} = \left(\frac{\alpha F_2}{2m_e}\right)^2\left(1 - \frac{T}{E_\nu}\right)\int\Big\{-\frac{(2E_\nu - T)^2 q^2}{q^4}R_{00} + \frac{q^2 + 4E_\nu(E_\nu - T)}{2|\vec{q}|^2}R_{11+22}\Big\}d\Omega_{\mathbf{k}_2}$$

neutrino millicharge scattering:

$$\frac{d\sigma_C}{dT} = F_1^2\left(\frac{E_\nu - T}{E_\nu}\right)\int\Big\{\frac{(2E_\nu - T)^2 - |\vec{q}|^2}{q^4}R_{00} - \left[\frac{q^2 + 4E_\nu(E_\nu - T)}{2q^4} + \frac{1}{q^2}\right]R_{11+22}\Big\}d\Omega_{\mathbf{k}_2}$$

ν-N Coherent Scattering through ν EM properties

$F_V(q^2)$ with normalization $F_V(0) = 1$ is the nuclear isoscalar vector form factor

NMM:

$$\frac{d\sigma_\mu(\nu A \to \nu A)}{dT_{NR}} = \frac{\pi \mu_\nu^2 \alpha^2 Z^2}{m_e^2} \left(\frac{1}{T_{NR}} - \frac{1}{E_\nu} + \frac{T_{NR}}{4E_\nu^2} \right) F_V^2(q^2)$$

Millicharge:

$$\frac{d\sigma_{\delta_Q}(\nu A \to \nu A)}{dT_{NR}} = \frac{M_A G_F^2}{4\pi} (1 - \frac{M_A T_{NR}}{2E_\nu^2})(2A \sin^2\theta_W + x)^2 F_V^2(q^2)$$

$$x = \frac{2\sqrt{2}\pi\alpha Z \delta_Q}{G_F M_A T_{NR}}$$