From Parity Violation In e-p Scattering to the Running of the Weak Mixing Angle

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PV e-p Scattering:



Weak Charge of the Proton <-> Left-Right Asymmetry:

at tree level:

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \frac{G_F Q^2}{4\sqrt{2}\pi\alpha} Q_W^{p,LO} \quad \text{very small, but finite}$$

$$Q_W^{p,LO} = 1 - 4\sin^2\theta_W = 0.0688$$

$$\Rightarrow Q_W^{p,LO} \sim A_{LR} \quad \text{link to measurement as } Q^2 \rightarrow 0$$

One-Loop Quantum Corrections to Q^p_W

$$Q_W^{p,NLO} = [\rho_{NC} + \Delta_e][1 - 4\sin^2\theta_W(0) + \Delta'_e] + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}$$





Thompson limit $(Q^2 \ll m_e^2)$: $\delta f_1 = 0$; $\delta f_2 = \frac{\alpha}{2\pi}$; $\delta g_1 = -\frac{\alpha}{2\pi}$

Scattering $(Q^2 \gg m_e^2)$: $\delta f_1 \sim \log \frac{Q^2}{m_e^2} \times \log \frac{\lambda^2}{m_e^2}$; $\delta f_2 \to 0$; $\delta g_1 \to \delta f_1$

IR divergent (cancelled by soft photon emission)

Overall Zee vertex has same (divergent) factor as γee \rightarrow Cancels in A_{PV} (or include with standard rad. corr.). Therefore take $\Delta_e = 0$ for scattering. Vertex correction: $Q_W^p = (1 + \Delta \rho + \Delta_e)(1 - 4\sin^2 \theta_W(0) + \Delta'_e) + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}(0)$

Electroweak: $\Delta'_e = -0.0014$ (1.9% of 0.0708)



Thompson limit $(Q^2 \ll m_e^2)$: $\Delta'_e(0) = -\frac{\alpha}{3\pi}(1-4\hat{s}^2)\left(\log\frac{M_z^2}{m_e^2}+\frac{1}{6}\right)$ Scattering $(Q^2 \gg m_e^2)$: $\Delta'_e(Q^2) = -\frac{\alpha}{3\pi}(1-4\hat{s}^2)\left(\log\frac{M_z^2}{Q^2}+\frac{11}{6}\right)$

$$\Delta'_{e}(Q^{2}) = \Delta'_{e}(0) + \underbrace{\frac{\alpha}{3\pi}(1 - 4\hat{s}^{2})\left(\log\frac{Q^{2}}{m_{e}^{2}} - \frac{5}{3}\right)}_{\text{OUT}}$$

+0.0006 at Q^2 =0.025 GeV²

Same size as electron contribution from running of $\sin^2 heta_W$

Corrections to $\sin^2 \theta_W$



Fermonic Loops:

Bosonic Loops:



Vertex Contributions to $\kappa^{PT}(Q^2)$



Expanding the γWW vertex gives a term which apparently cancels the neutrino propagator (aka the "Pinch Technique" [G. Degrassi, A. Sirlin, Phys. Rev. D 46, 3104 (1992)])



Pinched Diagram



This is not a physical vertex in EW theory, but some "effective" vertex given by: $C^{\delta} = \gamma^{\delta}(1 - \gamma_5)$

Solution: rewrite this effective vertex as a superposition of physical vertex factors with the Z and γ bosons:

$$C^{\delta} = V_{Zee}^{\delta} + 4\sin^2\theta_W V_{\gamma ee}^{\delta}$$



Similarly, the proton vertex correction must be pinched in this way

Revised expression

$$Q_W^p = \rho \left(1 - 4\kappa^{\rm PT}(0)\hat{s}^2 + \Delta'_e + \Delta_W \right) \qquad \Delta_W = \frac{8\alpha}{9\pi} + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}$$

• In practice, $\kappa^{\text{PT}}(Q)$ is very close to the original Czarnecki-Marciano definition, shifted upward by $2\alpha/(9\pi)=0.00052$ at Q=0, and 0.00045 at $Q=M_Z$



Leptonic & Hadronic Corrections to $\sin^2 heta_W$ at low Q^2

$$\sin^2 \theta_W(Q) = \frac{\alpha(Q)}{\alpha_2(Q)} \qquad \qquad \alpha(Q) = \frac{\alpha_0}{1 - \Delta\alpha(Q)} \qquad \Delta \alpha = \Delta \alpha_{lep} + \Delta \alpha_{had} + \Delta \alpha_{top}$$

 $\delta \sin^2 \theta_W(Q) = (\Delta \alpha(Q) - \Delta \alpha_2(Q)) \sin^2 \theta_W(0)$

This is the <u>shift</u> in the weak mixing angle from $Q^2=0$ to finite Q^2



	$\delta \sin^2 heta_W(Q)$	δQ_W^p
leptonic	-0.00015	+0.0006
hadronic	-0.00005	+0.0002
Total	-0.00020	+0.0008

- The shift in Δ_e' is also +0.0006
- It should be included in the Q²-dependent fits to get Q=0 limit