From Parity Violation In e-p Scattering to the Running of the Weak Mixing Angle

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PV e-p Scattering:

Process:

Tree Level Diagrams:

Weak Charge of the Proton <-> Left-Right Asymmetry:

at tree level:

\[ A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \frac{G_F Q^2}{4\sqrt{2}\pi\alpha} Q_{W,LO} \]

very small, but finite

\[ Q_{W,LO}^p = 1 - 4 \sin^2 \theta_W = 0.0688 \]

\[ \Rightarrow Q_{W,LO}^p \sim A_{LR} \]

link to measurement as \( Q^2 \to 0 \)
One-Loop Quantum Corrections to $Q_W^p$

$Q_{W}^{p,NLO} = [\rho_{NC} + \Delta_e][1 - 4 \sin^2 \theta_W(0) + \Delta'_e] + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}$

$\rho_{NC}$:

$\Delta'_e, \Delta_e$:

$\Box_{\gamma Z}, \Box_{ZZ}, \Box_{WW}$:
\[ Q_W^p = (1 + \Delta \rho + \Delta_e)(1 - 4 \sin^2 \theta_W(0) + \Delta_e') \]
\[ + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}(0) \]

**Electromagnetic:**

\[ \Delta_e = -\frac{\alpha}{2\pi} \sim 0.00116 \quad (0.1\% \text{ of } 0.0708) \]

\[ \gamma_\mu \rightarrow \gamma_\mu (1 + \delta f_1) + \frac{\delta f_2}{2m_e} \sigma_{\mu \nu}(-q') \]

\[ \gamma_\mu \gamma_5 \rightarrow \gamma_\mu \gamma_5 (1 + \delta g_1) + \frac{\delta g_2}{m_e} \gamma_5(-q_\mu) \]

**Thompson limit** \((Q^2 \ll m_e^2)\):

\[ \delta f_1 = 0; \quad \delta f_2 = \frac{\alpha}{2\pi}; \quad \delta g_1 = -\frac{\alpha}{2\pi} \]

**Scattering** \((Q^2 \gg m_e^2)\):

\[ \delta f_1 \sim \log \frac{Q^2}{m_e^2} \times \log \frac{\chi^2}{m_e^2}; \quad \delta f_2 \rightarrow 0; \quad \delta g_1 \rightarrow \delta f_1 \]

IR divergent
(cancelled by soft photon emission)

**Overall Zee vertex** has same (divergent) factor as \(\gamma ee\)

→ Cancels in \(A_{PV}\) (or include with standard rad. corr.).

Therefore take \(\Delta_e = 0\) for scattering.
Vertex correction: \[ Q_W^p = (1 + \Delta \rho + \Delta_e)(1 - 4 \sin^2 \theta_W(0) + \Delta'_e) + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}(0) \]

Electroweak: \[ \Delta'_e = -0.0014 \] (1.9% of 0.0708)

Thompson limit \((Q^2 \ll m_e^2)\): \[ \Delta'_e(0) = -\frac{\alpha}{3\pi} (1 - 4 \hat{s}^2) \left( \log \frac{M_Z^2}{m_e^2} + \frac{1}{6} \right) \]

Scattering \((Q^2 \gg m_e^2)\): \[ \Delta'_e(Q^2) = -\frac{\alpha}{3\pi} (1 - 4 \hat{s}^2) \left( \log \frac{M_Z^2}{Q^2} + \frac{11}{6} \right) \]

\[ \Delta'_e(Q^2) = \Delta'_e(0) + \frac{\alpha}{3\pi} (1 - 4 \hat{s}^2) \left( \log \frac{Q^2}{m_e^2} - \frac{5}{3} \right) \]

\[ +0.0006 \text{ at } Q^2=0.025 \text{ GeV}^2 \]

Same size as electron contribution from running of \(\sin^2 \theta_W\)
**Corrections to** \( \sin^2 \theta_W \)

\[
1 - 4 \sin^2 \theta_W (0) = 1 - 4\kappa(Q^2 = 0) \sin^2 \theta_W (M_Z^2)
\]

comes from all 1-Loop RC's contributing to \( \gamma Z \) mixing

good-measured

**Fermonic Loops:**

1. \( F \rightarrow Z \rightarrow Y \rightarrow F \)

**Bosonic Loops:**

1. \( G \rightarrow Z \rightarrow \gamma \rightarrow G \)
2. \( W \rightarrow Z \rightarrow \gamma \rightarrow W \)
3. \( G \rightarrow Z \rightarrow \gamma \rightarrow G \)
4. \( u_- \rightarrow Y \rightarrow \gamma \rightarrow u_- \)
5. \( u_+ \rightarrow Y \rightarrow \gamma \rightarrow u_+ \)
6. \( W \rightarrow Z \rightarrow \gamma \rightarrow \gamma \rightarrow W \)
7. \( G \rightarrow Z \rightarrow \gamma \rightarrow \gamma \rightarrow G \)
8. \( W \rightarrow Z \rightarrow \gamma \rightarrow \gamma \rightarrow W \)
1. Electron's Anapole Moment:

Expanding the $\gamma_{WW}$ vertex gives a term which apparently cancels the neutrino propagator (aka the “Pinch Technique” [G. Degrassi, A. Sirlin, Phys. Rev. D 46, 3104 (1992)])

2. Proton vertex correction:
This is not a physical vertex in EW theory, but some “effective” vertex given by: \( C^\delta = \gamma^\delta (1 - \gamma_5) \)

Solution: rewrite this effective vertex as a superposition of physical vertex factors with the \( Z \) and \( \gamma \) bosons:

\[
C^\delta = V_{Z\ell\ell}^\delta + 4 \sin^2 \theta_W V_{\gamma\ell\ell}^\delta
\]

Similarly, the proton vertex correction must be pinched in this way:

\[\text{this part contributes to } \sin^2 \theta_W !\]
Revised expression

\[ Q_{W}^{p} = \rho \left( 1 - 4k^{PT}(0)\hat{s}^2 + \Delta_e' + \Delta_W \right) \]
\[ + \Box W W + \Box Z Z + \Box \gamma Z \]

\[ \Delta_W = \frac{8\alpha}{9\pi} \]

- In practice, \( k^{PT}(Q) \) is very close to the original Czarnecki-Marciano definition, shifted upward by \( 2\alpha/(9\pi) = 0.00052 \) at \( Q=0 \), and \( 0.00045 \) at \( Q=M_Z \)

\[ k^{PT}(0)\hat{s}^2 = 0.2390 \]

Fermion contributions taken from Jegerlehner’s dispersive fits to \( e^+e^- \) data
Leptonic & Hadronic Corrections to $\sin^2 \theta_W$ at low $Q^2$

$$\sin^2 \theta_W (Q) = \frac{\alpha(Q)}{\alpha_2(Q)}$$
$$\alpha(Q) = \frac{\alpha_0}{1 - \Delta \alpha(Q)}$$

$$\Delta \alpha = \Delta \alpha_{lep} + \Delta \alpha_{had} + \Delta \alpha_{top}$$

$$\delta \sin^2 \theta_W (Q) = (\Delta \alpha(Q) - \Delta \alpha_2(Q)) \sin^2 \theta_W (0)$$

This is the shift in the weak mixing angle from $Q^2=0$ to finite $Q^2$.

<table>
<thead>
<tr>
<th></th>
<th>$\delta \sin^2 \theta_W (Q)$</th>
<th>$\delta Q_W^p$</th>
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<tbody>
<tr>
<td>leptonic</td>
<td>-0.00015</td>
<td>+0.0006</td>
</tr>
<tr>
<td>hadronic</td>
<td>-0.00005</td>
<td>+0.0002</td>
</tr>
<tr>
<td>Total</td>
<td>-0.00020</td>
<td>+0.0008</td>
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- The shift in $\Delta'_e$ is also +0.0006
- It should be included in the $Q^2$-dependent fits to get $Q=0$ limit.