# From Parity Violation In e-p Scattering to the Running of the Weak Mixing Angle 

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## PV e-p Scattering:

Process:


Tree Level Diagrams:


Weak Charge of the Proton <-> Left-Right Asymmetry:
at tree level:

$$
\begin{aligned}
& A_{L R}=\frac{\sigma_{L}-\sigma_{R}}{\sigma_{L}+\sigma_{R}}=\frac{G_{F} Q^{2}}{4 \sqrt{2} \pi \alpha} Q_{W}^{p, L O} \quad \text { very small, but finitt } \\
& \quad Q_{W}^{p, L O}=1-4 \sin ^{2} \theta_{W}=0.0688 \\
& \quad \Rightarrow Q_{W}^{p, L O} \sim A_{L R} \quad \text { link to measurement as } Q^{2} \rightarrow 0
\end{aligned}
$$

## One-Loop Quantum Corrections to $Q_{W}^{p}$

$$
Q_{W}^{p, N L O}=\left[\rho_{N C}+\Delta_{e}\right]\left[1-4 \sin ^{2} \theta_{W}(0)+\Delta_{e}^{\prime}\right]+\square_{W W}+\square_{Z Z}+\square_{\gamma Z}
$$



2
$\square \gamma Z, \quad \square Z, \quad \square W W$


Vertex correction: $Q_{W}^{p}=\left(1+\Delta \rho+\Delta_{e}\right)\left(1-4 \sin ^{2} \theta_{W}(0)+\Delta_{e}^{\prime}\right)$

$$
+\square_{W W}+\square_{Z Z}+\square_{\gamma Z}(0)
$$

Electromagnetic: $\quad \Delta_{e}=-\frac{\alpha}{2 \pi} \sim 0.00116 \quad(0.1 \%$ of 0.0708$)$


$$
\begin{array}{r}
\gamma_{\mu} \rightarrow \gamma_{\mu}\left(1+\delta f_{1}\right)+\frac{\delta f_{2}}{2 m_{e}} \sigma_{\mu \nu}\left(-q^{\nu}\right) \\
\gamma_{\mu} \gamma_{5} \rightarrow \gamma_{\mu} \gamma_{5}\left(1+\delta g_{1}\right)+\frac{\delta g_{2}}{m_{e}} \gamma_{5}\left(-q_{\mu}\right)
\end{array}
$$

Thompson limit ( $Q^{2} \ll m_{e}{ }^{2}$ ): $\delta f_{1}=0 ; \quad \delta f_{2}=\frac{\alpha}{2 \pi} ; \quad \delta g_{1}=-\frac{\alpha}{2 \pi}$ Scattering ( $Q^{2} \gg m_{e}^{2}$ ): $\delta f_{1} \sim \log \frac{Q^{2}}{m_{e}^{2}} \times \log \frac{\lambda^{2}}{m_{e}^{2}} ; \quad \delta f_{2} \rightarrow 0 ; \quad \delta g_{1} \rightarrow \delta f_{1}$
(cancelled by soft phototon emission)

Overall Zee vertex has same (divergent) factor as $\gamma e e$ $\rightarrow$ Cancels in $A_{\mathrm{PV}}$ (or include with standard rad. corr.). Therefore take $\Delta_{e}=0$ for scattering.

Vertex correction: $Q_{W}^{p}=\left(1+\Delta \rho+\Delta_{e}\right)\left(1-4 \sin ^{2} \theta_{W}(0)+\Delta_{e}^{\prime}\right)$ $+\square_{W W}+\square_{Z Z}+\square_{\gamma Z}(0)$
Electroweak: $\Delta_{e}^{\prime}=-0.0014$ (1.9\% of 0.0708)


Thompson limit $\left(Q^{2}<m_{e}^{2}\right)$ : $\quad \Delta_{e}^{\prime}(0)=-\frac{\alpha}{3 \pi}\left(1-4 \hat{s}^{2}\right)\left(\log \frac{M_{z}^{2}}{m_{e}^{2}}+\frac{1}{6}\right)$
Scattering ( $Q^{2} \gg m_{e}^{2}$ ):

$$
\Delta_{e}^{\prime}\left(Q^{2}\right)=\Delta_{e}^{\prime}(0)+\underbrace{\frac{\alpha}{3 \pi}\left(1-4 \hat{s}^{2}\right)\left(\log \frac{Q^{2}}{m_{e}^{2}}-\frac{5}{3}\right)}_{+0.0006 \text { at } Q^{2}=0.025 \mathrm{GeV}^{2}}
$$

Same size as electron contribution from running of $\sin ^{2} \theta_{W}$

## Corrections to $\sin ^{2} \theta_{W}$

$$
1-4 \sin ^{2} \theta_{W}(0)=1-4 \kappa\left(Q^{2}=0\right) \sin ^{2} \theta_{W}\left(M_{Z}^{2}\right)
$$

## Fermonic Loops:

Bosonic Loops:


5
6


3

2


4


7


## Vertex Contributions to $\kappa^{P T}\left(Q^{2}\right)$

1. Electron's Anapole Moment:

2. Proton vertex correction:


1
Expanding the $\gamma W W_{\text {vertex gives a term which apparently cancels the neutrino propagator }}$ (aka the "Pinch Technique"[G. Degrassi, A. Sirlin, Phys. Rev. D 46, 3104 (1992)])


1


## Pinched Diagram


This is not a physical vertex in EW theory, but some "effective" vertex given by: $C^{\delta}=\gamma^{\delta}\left(1-\gamma_{5}\right)$
Solution: rewrite this effective vertex as a superposition of physical vertex factors with the Z and $\gamma$ bosons:

$$
C^{\delta}=V_{Z e e}^{\delta}+4 \sin ^{2} \theta_{W} V_{\gamma e e}^{\delta}
$$



Similarly, the proton vertex correction must be pinched in this way

## Revised expression

$$
\begin{array}{rlr}
Q_{W}^{p}= & \rho\left(1-4 \kappa^{\mathrm{PT}}(0) \hat{s}^{2}+\Delta_{e}^{\prime}+\Delta_{W}\right) & \Delta_{W}=\frac{8 \alpha}{9 \pi} \\
& +\square_{W W}+\square_{Z Z}+\square_{\gamma Z} &
\end{array}
$$

- In practice, $\kappa^{\mathrm{PT}}(Q)$ is very close to the original Czarnecki-Marciano definition, shifted upward by $2 \alpha /(9 \pi)=0.00052$ at $Q=0$, and 0.00045 at $Q=M_{Z}$



## Leptonic \& Hadronic Corrections to $\sin ^{2} \theta_{W}$ at low $\mathbf{Q}^{\wedge} \mathbf{2}$

$$
\begin{aligned}
& \sin ^{2} \theta_{W}(Q)=\frac{\alpha(Q)}{\alpha_{2}(Q)} \quad \alpha(Q)=\frac{\alpha_{0}}{1-\Delta \alpha(Q)} \quad \Delta \alpha=\Delta \alpha_{\text {lep }}+\Delta \alpha_{\text {had }}+\Delta \alpha_{\text {top }} \\
& \delta \sin ^{2} \theta_{W}(Q)=\left(\Delta \alpha(Q)-\Delta \alpha_{2}(Q)\right) \sin ^{2} \theta_{W}(0)
\end{aligned}
$$

This is the shift in the weak mixing angle from $Q^{2}=0$ to finite $Q^{2}$


|  | $\delta \sin ^{2} \theta_{W}(Q)$ | $\delta Q_{W}^{p}$ |
| :--- | :---: | :---: |
| leptonic | -0.00015 | +0.0006 |
| hadronic | -0.00005 | +0.0002 |
| Total | -0.00020 | +0.0008 |

- The shift in $\Delta_{e}^{\prime}$ is also +0.0006
- It should be included in the $Q^{2}$-dependent fits to get $Q=0$ limit

