ACFI EDM School
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EDMs from the QCD θ term

Vincenzo Cirigliano
Los Alamos National Laboratory

Los Alamos
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Lecture II outline

- The QCD θ term
- Toolbox: chiral symmetries and their breaking
- Estimate of the neutron EDMs from θ term
- The "Strong CP" problem: understanding the smallness of θ
    - Peccei-Quinn mechanism and axions
    - Induced θ term

The QCD θ term

The θ term

- The QCD Lagrangian contains in principle the following term:

$$\mathcal{L}_\theta^{CPV} = \theta \, \frac{g_s^2}{64\pi^2} \, \epsilon^{\mu\nu\alpha\beta} G^a_{\mu\nu} G^a_{\alpha\beta} \; = \theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu$$

$\varepsilon^{\mu\nu\alpha\beta}$ = 4-dim Levi-Civita symbol

g_s = strong coupling constant

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- Multiple reasons for the presence of θ term:
  - EFT perspective: at dimension=4, include all terms built out of quarks and gluons that respect $SU(3)_C$ gauge invariance
  - Diagonalization of quark mass matrix m_q induces $\Delta\theta = \arg \det m_q$ (will discuss this later)
  - Structure of QCD vacuum (won't discuss this)

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- Transformation properties under discrete symmetries: analogy with Electrodynamics

$$-\frac{1}{4} F_{\mu\nu} F_{\mu\nu} = \frac{1}{2}(\boldsymbol{E}^2 - \boldsymbol{B}^2)$$

$$F_{\mu\nu} \tilde{F}_{\mu\nu} = -4\boldsymbol{E} \cdot \boldsymbol{B}.$$

$$\xrightarrow[\textbf{B} \text{ is P-even, T-odd}]{\textbf{E} \text{ is P-odd, T-even}}$$

P-even, T-even

P-odd, T-odd

The θ term

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- θ term is P-odd and T-odd, and hence CP-odd (CPT theorem)

- How do hadronic CP-violating observables depend on θ?
(After all, no breaking of P and T observed in strong interactions)

Toolbox: chiral symmetries and their breaking

- Relevant to understand
    1. How to compute the neutron EDM from the θ term
    2. How the Peccei-Quinn mechanism works

Technical subject: I will present the main concepts and implications

Chiral symmetry

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + i\,\bar{q}_L \gamma^\mu D_\mu q_L + i\,\bar{q}_R \gamma^\mu D_\mu q_R - \bar{q}_L\, m_q\, q_R - \bar{q}_L\, m_q\, q_L$$

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \qquad m_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

Chiral symmetry

$$\boxed{\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + i\,\bar{q}_L \gamma^\mu D_\mu q_L + i\,\bar{q}_R \gamma^\mu D_\mu q_R - \bar{q}_L\, m_q\, q_R - \bar{q}_L\, m_q\, q_L}$$

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- For $m_q = 0$, action invariant under independent U(3) transformations of left- and right-handed quarks:

$$\boxed{\underbrace{SU(3)_L \times SU(3)_R}\ \times\ [U(1)_V \times U(1)_A]}$$

- Conserved vector and axial currents (T^a: SU(3) generators and identity)

$$\partial_\mu\ (\bar{q}\gamma^\mu T^a q) = 0 \qquad\qquad \partial_\mu\ (\bar{q}\gamma^\mu \gamma^5 T^a q) = 0$$

Chiral symmetry

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- Symmetry is broken by $m_q \neq 0$ and by more subtle effects

$$\partial_\mu \left(\bar{q} \gamma^\mu T^a q \right) = \bar{q}\, [T^a, m_q]\, q \qquad \partial_\mu \left(\bar{q} \gamma^\mu \gamma^5 T^a q \right) = \bar{q}\, \{T^a, m_q\}\, i\gamma_5\, q$$

Symmetry breaking

- In general, three known mechanisms for symmetry breaking
  - Explicit symmetry breaking
    - Symmetry is approximate; still very useful
  - Spontaneous symmetry breaking
    - Equations of motion invariant, but ground state is not
  - Anomalous (quantum mechanical) symmetry breaking
    - Classical invariance but no symmetry at QM level

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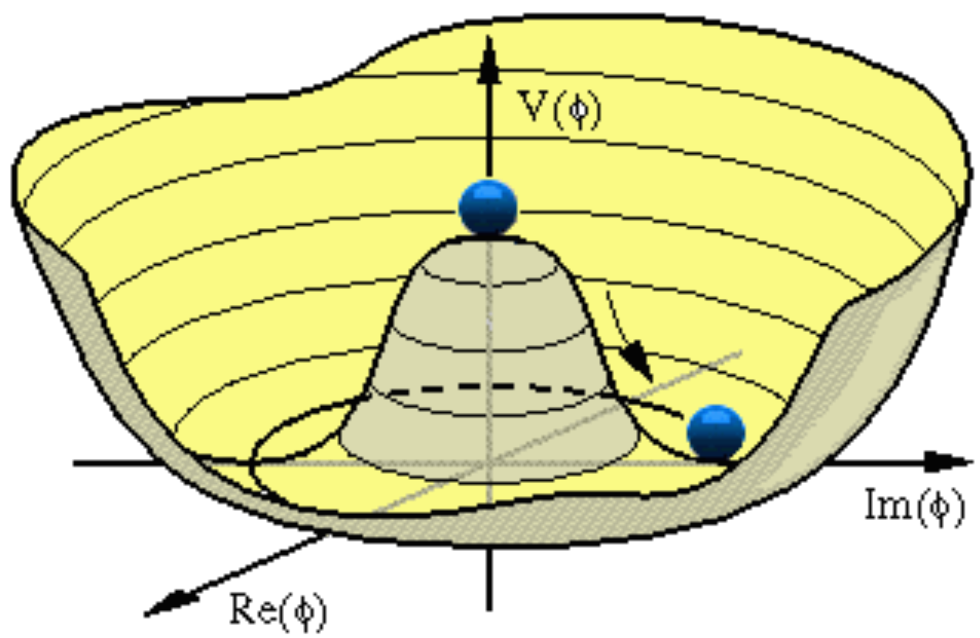
All relevant to the discussion of chiral symmetry in QCD and Peccei-Quinn symmetry

Spontaneous symmetry breaking

- Action is invariant, but ground state is not!
- Continuous symmetry: degenerate physically equivalent minima
- Excitations along the valley of minima → massless states in the spectrum (Goldstone Bosons)



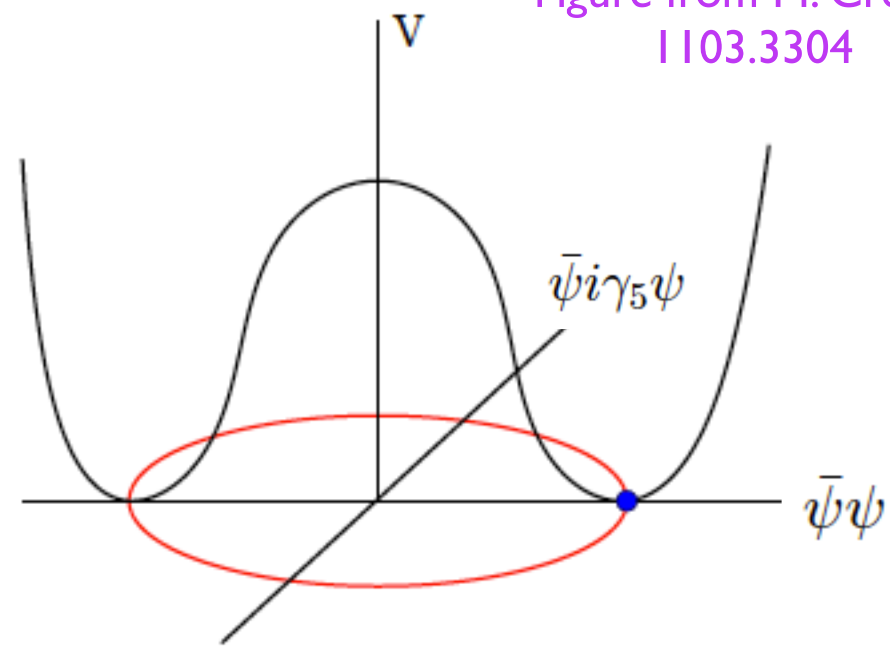
- Many examples of Goldstone bosons in physics: phonons in solids (translations); spin waves in magnets (rotations); …

Spontaneous symmetry breaking

- Pions, kaons, mesons: Goldstone bosons associated with SSB of chiral symmetry

$$\langle \bar{q}\, q \rangle \neq 0$$

Figure from M. Creutz, 1103.3304



- Axial subgroup is broken. Vector subgroup SU(3)$_V$ stays unbroken (symmetry approximately manifest in the QCD spectrum)

$$\boxed{G = SU(3)_L \times SU(3)_R \;\rightarrow\; H = SU(3)_{V=L+R}}$$

- In case of SSB currents are still conserved. Massless states appear in the spectrum. What about the U(1)$_A$ symmetry?

Anomalous symmetry breaking

- Action is invariant, but path-integral measure is not!

$$\boxed{\int [d\psi][d\bar{\psi}]\ e^{iS[\psi,\bar{\psi}]}}$$

$$S[\psi,\bar{\psi}] = S[\psi',\bar{\psi}']$$

$$\psi \to \psi' \qquad \bar{\psi} \to \bar{\psi}'$$

$$\int [d\psi][d\bar{\psi}] = \int [d\psi'][d\bar{\psi}']\,\mathcal{J} \qquad \mathcal{J} \neq 1$$

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- Chiral anomaly [$U(1)_A$]: in m_q=0 limit axial current not conserved

$$\psi \to \psi' = e^{i\frac{\alpha}{2}\gamma_5}\,\psi$$

$$\bar{\psi} \to \bar{\psi}' = \bar{\psi}\,e^{i\frac{\alpha}{2}\gamma_5}$$

Axial transformation induces a shift in the θ term

$$\log \mathcal{J} = -\alpha \int d^4x\, \frac{g_s^2}{32\pi^2}\, G^a_{\mu\nu}\tilde{G}^{\mu\nu,a}$$

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$$\boxed{\partial_\mu \left(\bar{\psi}\,\gamma^\mu \gamma^5\,\psi\right) = \frac{g_s^2}{16\pi^2} G^a_{\mu\nu}\tilde{G}^{\mu\nu,a}}$$

Implications for θ term

- Diagonalization of quark mass matrix m_q induces $\Delta\theta$ = arg det m_q
  - Diagonal m_q matrix has complex eigenvalues $(m_q)_{ii} = m_i\, e^{i\alpha_i}$
  - To make them real, additional axial rotation is needed

$$\psi \to \psi' = e^{i\frac{\alpha}{2}\gamma_5}\,\psi$$
$$\bar{\psi} \to \bar{\psi}' = \bar{\psi}\,e^{i\frac{\alpha}{2}\gamma_5}$$

  - This induces shift in θ proportional to $\arg\det m_q = \sum_i \alpha_i$

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- Physics depends only on the combination $\boxed{\bar\theta = \theta - \arg\,\det m_q}$
- Can put it in the gluonic θ term or in a complex quark mass!

Estimate of the neutron EDM from θ term

Crewther, Di Vecchia, Veneziano, Witten Phys. Lett. 88B, 123 (1979)

Rotating CPV to quark mass

- In order to analyze pion-nucleon couplings, it is more convenient to put the strong CPV in the form of pseudoscalar quark densities

$$\bar{\theta} = \theta - \mathrm{ArgDet}(\mathcal{M}_q)$$

$$\boxed{\delta\mathcal{L}_{\mathrm{CPV}} = -\bar{\theta}\frac{g_s^2}{32\pi^2}G_{\mu\nu}\tilde{G}^{\mu\nu} \;\to\; \bar{q}\, i\gamma_5 A q}$$

$$q_i \to q_i' = e^{i\frac{\alpha_i}{2}\gamma_5}\, q_i \qquad \bar{q}_i \to \bar{q}_i' = \bar{q}_i\, e^{i\frac{\alpha_i}{2}\gamma_5}$$

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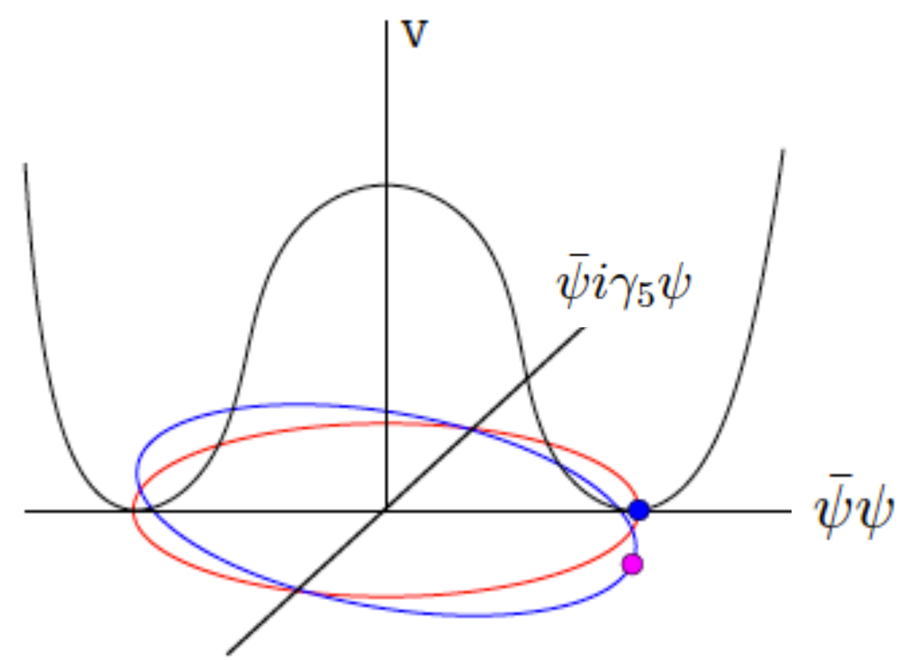
- Use freedom in $SU(3)_A$ transformation to ensure that perturbation introduces no mixing of the vacuum to Goldstone Bosons ("Vacuum alignment")

$$\boxed{\langle\pi^a|\delta\mathcal{L}_{\mathrm{CPV}}|\Omega\rangle = 0}$$

$$\pi^a \sim \bar{q}\,i\gamma^5\,T^a\,q$$



Rotating CPV to quark mass

- This requires A to be proportional to the identity, with

$$\bar{\theta} = \theta - \mathrm{ArgDet}(\mathcal{M}_q)$$

$$\delta\mathcal{L}_{\rm CPV} = -\bar{\theta}\frac{g_s^2}{32\pi^2}G_{\mu\nu}\tilde{G}^{\mu\nu} \;\rightarrow\; -m_*\,\bar{\theta}\,\bar{q}\,i\gamma_5 q$$

$$\boxed{m_* = \frac{1}{\sum_i (1/m_i)} \simeq \frac{m_u m_d}{m_u + m_d}}$$

Effect disappears if one of the quark masses vanishes

CPV pion-nucleon coupling

- Use chiral symmetry (soft pion theorem) to relate CPV pion-nucleon coupling to baryon mass splittings

$$\boxed{\langle N_f \pi^a | \bar{q} i \gamma_5 q | N_i \rangle \ \propto \ F_\pi^{-1} \ \langle N_f | \bar{q} \, \tau^a \, q | N_i \rangle}$$

Crewther-DiVecchia-Veneziano-Witten 1979

$$\lim_{q \to 0} \langle \pi^a(q) B | O | A \rangle = -\frac{i}{F_\pi} \langle B | [Q_5^a, O] | A \rangle$$

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- Equivalent way to see this: θ and mass splitting are chiral partners. Low-energy couplings controlling the two are related

$$\begin{pmatrix} \bar{q} i \gamma_5 q \\ \bar{q} \boldsymbol{\tau} q \end{pmatrix} \xrightarrow{SU_A(2)} \begin{pmatrix} -\bar{q} \boldsymbol{\alpha} \cdot \boldsymbol{\tau} q \\ \boldsymbol{\alpha}\, \bar{q} i \gamma_5 q \end{pmatrix}$$

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$$\Downarrow$$

$$\boxed{\bar{g}_0 = \frac{\delta M_{n-p}^{m_d - m_u}}{m_d - m_u} \, \frac{2 m_d m_u}{m_d + m_u} \bar{\theta}}$$

$$\frac{\bar{g}_0}{F_\pi} = (15 \pm 2) \cdot 10^{-3} \sin \bar{\theta}$$

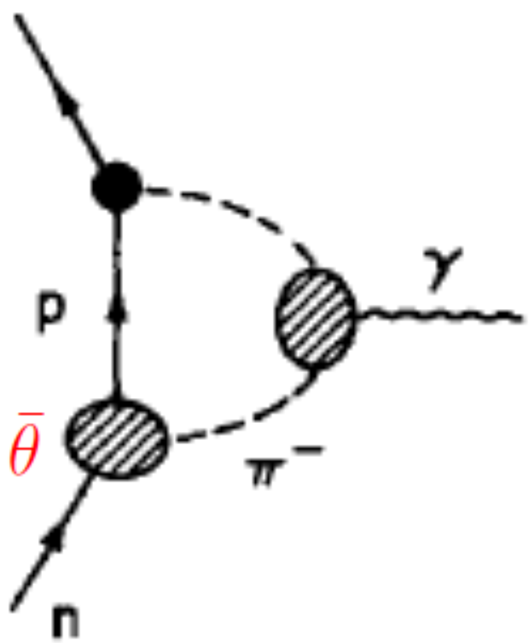
Mereghetti, van Kolck
1505.06272
and refs therein

$$\mathcal{L}_{CPV} \supset \ -\frac{1}{F_\pi} \bar{N} \left(\bar{g}_0 \, \sigma^a \pi^a \ + \ \bar{g}_1 \, \pi^3 \right) N$$

Chiral loop and estimate of d_n

- Leading contribution (for $m_q \to 0$) to neutron EDM via chiral loop

Crewther-DiVecchia-Veneziano-Witten 1979



$$d_n = \frac{e g_A \bar{g}_0}{(4\pi F_\pi)^2} \log \frac{m_n^2}{m_\pi^2} + \ldots$$

E. Mereghetti et al
Phys. Lett. B 696 (2011) 97

Counter-term (of same order) and sub-leading contributions

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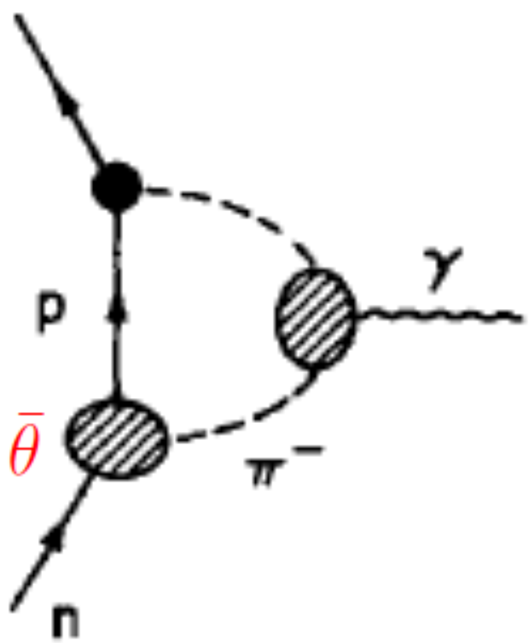

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Counter-term (of same order) and sub-leading contributions

$$d_n \simeq 2 \times 10^{-3} \, \bar{\theta} \, e\text{fm}$$

$$\downarrow$$

$$|\bar{\theta}| < 10^{-10}$$

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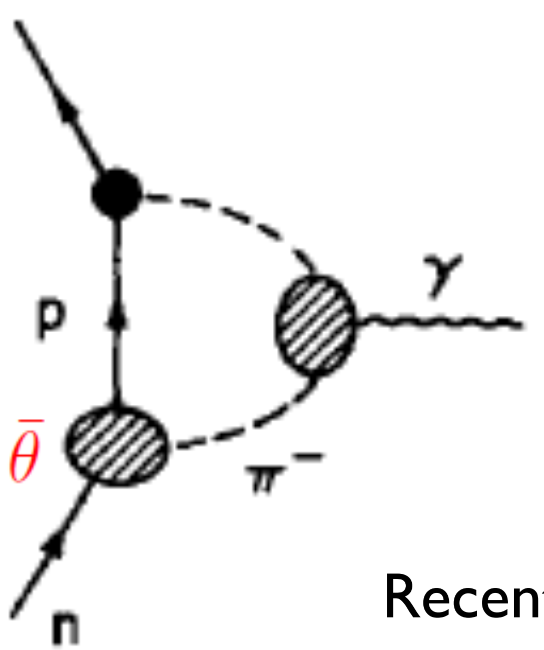

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Counter-term (of same order) and sub-leading contributions

$$d_n \simeq 2 \times 10^{-3}\, \bar{\theta}\, e\,\mathrm{fm}$$

Recent lattice QCD results** do not change qualitative picture

$$\frac{d_n}{\bar{\theta}} = -3.8(2)_{\mathrm{stat}}(9)_{\mathrm{fit}}\; 10^{-3}\, e\,\mathrm{fm}$$

Guo et al., 1502.02295

$$\frac{d_n}{\bar{\theta}} = -2.7(1.2)\; 10^{-3}\; e\,\mathrm{fm}$$

Akan et al., 1406.2882

$$\frac{d_n}{\bar{\theta}} = 45(6)\, 10^{-3}\, e\,\mathrm{fm}$$

Alexandrou et al., 151005823

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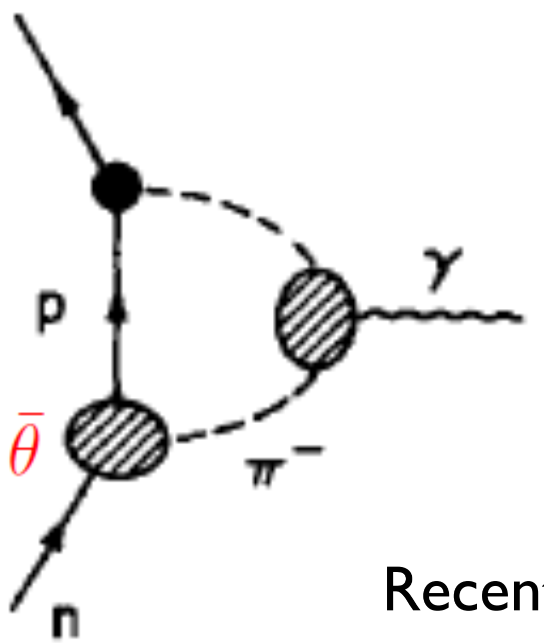

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Recent lattice QCD results** do not change qualitative picture

$$d_n \sim \frac{m_*}{\Lambda_{\mathrm{had}}^2}\, e\, \bar{\theta}$$

The "strong CP" problem: understanding the smallness of $\bar{\theta}$

Understanding the smallness of θ

- The small value of $\bar{\theta} = \theta - \mathrm{ArgDet}(\mathcal{M}_q)$ begs for an explanation

- Possible ways out:
  - One of the quark masses vanishes (so can "rotate away" θ): this is strongly disfavored by phenomenology of light quark masses**
  - Invoke some symmetry principle
    - P or CP exact at high scale, broken spontaneously at lower scale. Difficulty: keep $\theta < 10^{-10}$ while allowing large CKM phase
    - Peccei-Quinn scenarios

** See Wilczek-Moore 1[601.02937] for a reincarnation of this idea through "cryptoquarks": massless quarks confined in super-heavy bound states

Peccei-Quinn mechanism

- Basic idea: promote $\bar{\theta}$ to a field and make sure that it dynamically relaxes to zero

- How to get there: extend the SM with additional fields so that the model has an axial $U(1)_{PQ}$ global symmetry with these features:
    - $U(1)_{PQ}$ is broken spontaneously at some high scale → axion is the resulting Goldstone mode
    - $U(1)_{PQ}$ is broken by the axial anomaly → the axion acquires interactions with gluons, which generate an axion potential
    - Potential induces axion expectation value such that $\bar{\theta}=0$

- Salient features can be captured by effective theory analysis

Axion effective theory

- At energies below the $U(1)_{PQ}$ breaking scale f_a, axion effective Lagrangian is given by

$$\mathcal{L}_a = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a(x)}{f_a}\frac{\alpha_s}{8\pi} G\tilde{G}$$

We can ignore derivative terms irrelevant for strong CP problem, such as
$\partial_\mu a \bar{\psi}\gamma_\mu\gamma_5\psi$;

Goldstone nature of the axion requires the effective Lagrangian to be invariant under
a(x) → a(x) + constant **
(up to the anomaly term)

The presence of this term is required by the axial anomaly

** In simplest models, the axion is the phase of a complex scalar charge under $U(1)_{PQ}$

Hence the transformation property

Axion effective theory

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- Key point: in $\mathcal{L}_{QCD} + \mathcal{L}_a$, $a(x)$ leads to a field-dependent shift of θ

$$\bar{\theta} \to \bar{\theta} + \frac{a}{f_a}$$

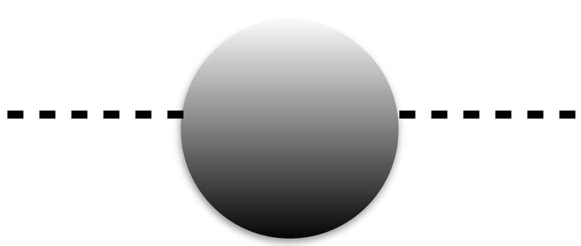
Through interactions with gluons this quantity acquires a potential

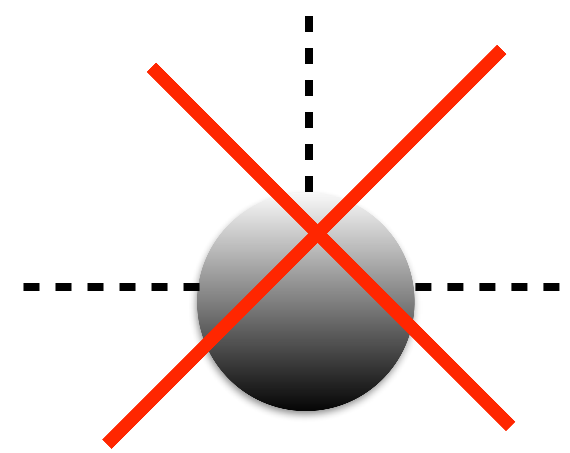
Axion effective theory

- In absence of other sources of CP violation, the potential is an *even* function of $\bar{\theta} + \dfrac{a}{f_a}$

$$\boxed{\mathcal{L}_a^{\text{eff}} = \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}\chi(0)\left(\bar{\theta} + \frac{a}{f_a}\right)^2 . + \cdots}$$

$$\chi(0) = -\frac{i}{8\pi^2}\lim_{p\to 0}\int d^4x e^{ipx}\left\langle \frac{\alpha_s}{2\pi}G\tilde{G}(x), \frac{\alpha_s}{2\pi}G\tilde{G}(0)\right\rangle$$





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- Minimum of the potential when $(\bar{\theta} + \langle a \rangle / f_a)$ vanishes. This solves the strong CP problem, independently of the initial value of θ

Axion effective theory

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- Axion mass given by $m_a \sim \frac{1}{f_a}|\chi(0)|^{1/2}$ with $\chi(0) = \frac{2F_\pi^2 m_\pi^2}{8\pi^2}\frac{4m_u m_d}{(m_u + m_d)^2}$

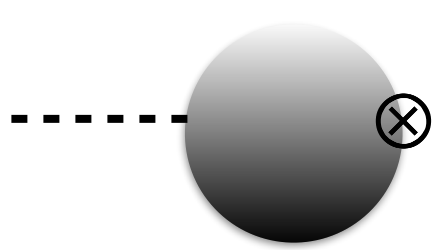
Induced θ term

- In presence of other sources of CP violation beyond the θ term, the potential is not an *even* function:

$$\mathcal{L}_a^{\rm eff} = \frac{1}{2}\partial_\mu a \partial^\mu a - \chi_{\mathcal{O}_{\rm CP}}(0)\left(\bar{\theta} + \frac{a}{f_a}\right) - \frac{1}{2}\chi(0)\left(\bar{\theta} + \frac{a}{f_a}\right)^2 + \cdots$$

$$\chi_{\mathcal{O}_{\rm CP}}(0) = -i \lim_{k\to 0} \int d^4x e^{ik\cdot x} \langle 0 | T(G\tilde{G}(x), \mathcal{O}_{\rm CP}(0)) | 0 \rangle$$

$$\boxed{\theta_{\rm ind} = -\chi_{\mathcal{O}_{\rm CP}}(0)/\chi(0)}$$



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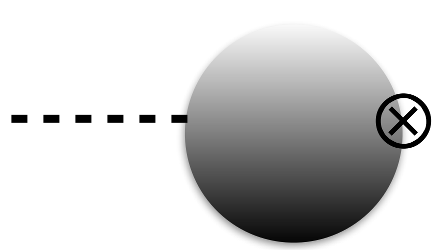


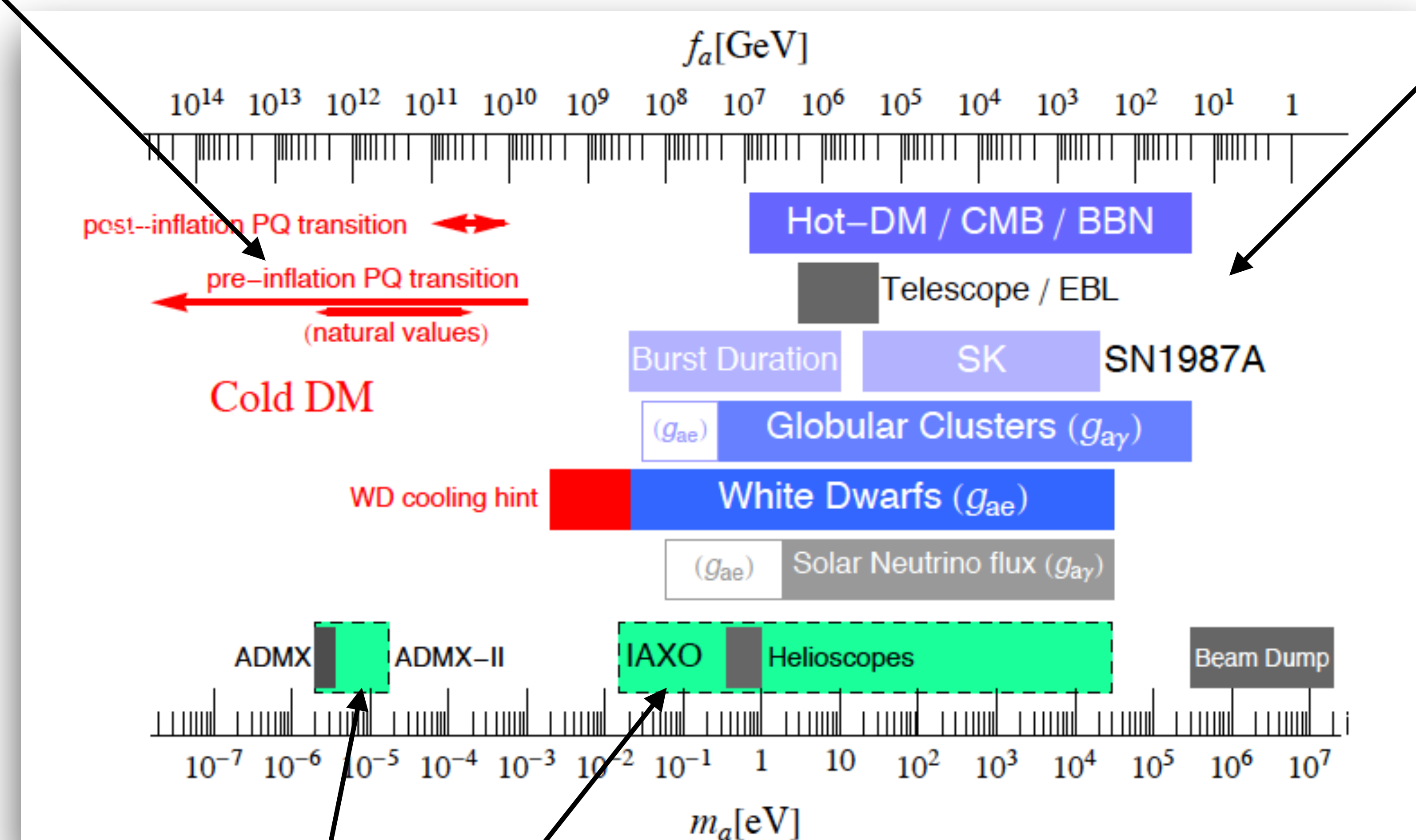
This needs to be taken into account when computing the impact of BSM operators on EDMs

Status of axion searches

Axion as cold dark matter lives here

Disfavored by astrophysics / cosmological observations (grey) or argument (blue)

$$m_a \sim 6 \text{ meV } (10^9 \text{ GeV}/f_a)$$



Sensitivity of planned experiments

ArXiv: 1311.0029

Backup slides

Abelian gauge theory

- Recall U(1) (abelian) example

$$\psi(x) \rightarrow e^{i\epsilon(x)}\,\psi(x)$$

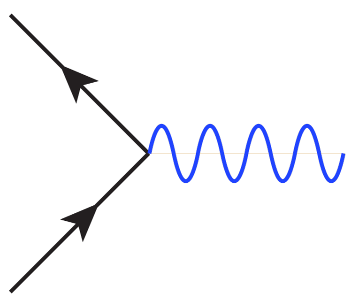
$$A^\mu \rightarrow A^\mu + \frac{1}{g}\partial^\mu \epsilon$$

$$\boxed{\mathcal{L} = \bar{\psi}\,(i\gamma_\mu \partial^\mu - m)\,\psi + g\,\bar{\psi}\gamma_\mu\,A^\mu\,\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- Form of the interaction:

$$\boxed{\mathcal{L}_{\text{int}} \; = \; g\,A_\mu\,J^\mu}$$



$$J^\mu = \bar{\psi}\gamma^\mu\psi$$

conserved current associated with global U(1)

Non-abelian gauge theory

- Generalize to non-abelian group G (e.g. SU(2), SU(3), …). $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \cdots \end{pmatrix}$

$$\psi(x) \;\rightarrow\; U(x)\,\psi(x) \qquad U(x) = e^{i\epsilon^a(x)T^a} \qquad [T^a, T^b] = i\,f^{abc}T^c$$

- Invariant dynamics if introduce new vector fields $A_\mu = A_\mu^a\,T^a$ transforming as

$$A^\mu \;\rightarrow\; U\,A^\mu\,U^\dagger - \frac{i}{g}(\partial^\mu U)U^\dagger$$

$$\boxed{\mathcal{L} = \bar{\psi}\,(i\gamma_\mu\partial^\mu - m)\,\psi \,+\, g\,\bar{\psi}\gamma^\mu\,T^a A_\mu^a\,\psi \,-\, \frac{1}{2}\mathrm{Tr}\,(F_{\mu\nu}F^{\mu\nu})}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu] \qquad F_{\mu\nu} \rightarrow U\,F_{\mu\nu}\,U^\dagger$$

Anomalous breaking of B and L

- Action is invariant, but path-integral measure is not!

$$\boxed{\int [d\psi][d\bar{\psi}]\; e^{iS[\psi,\bar{\psi}]}}$$

$$\psi \to \psi' \qquad \bar{\psi} \to \bar{\psi}'$$

$$S[\psi,\bar{\psi}] = S[\psi',\bar{\psi}']$$

$$\int [d\psi][d\bar{\psi}] = \int [d\psi'][d\bar{\psi}']\,\mathcal{J} \qquad \mathcal{J} \neq 1$$

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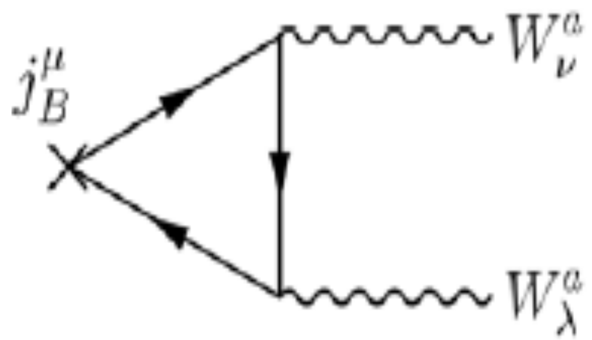
- Baryon (B) and Lepton (L) number are anomalous in the SM

$$\boxed{\partial_\mu J^\mu_B = \partial_\mu J^\mu_L = i\frac{N_F}{32\pi^2}\left(-g_2^2 F^{a\mu\nu}\widetilde{F}^a_{\mu\nu} + g_1^2 f^{\mu\nu}\widetilde{f}_{\mu\nu}\right)}$$



- Only B-L is conserved; B+L is violated; negligible at zero temperature

θ term and topology

$$\mathcal{L}_\theta^{CPV} = \theta \, \frac{g_s^2}{64\pi^2} \, \epsilon^{\mu\nu\alpha\beta} G^a_{\mu\nu} G^a_{\alpha\beta} = \theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

- θ term is total derivative (surface term) but can't ignore it due to non-trivial topological effects

$$G_a^{\mu\nu} \tilde{G}_{a\mu\nu} = \partial_\mu K^\mu \qquad K^\mu = \epsilon^{\mu\alpha\beta\gamma} A_{a\alpha} \left[G_{a\beta\gamma} - \frac{g}{3} \epsilon_{abc} A_{b\beta} A_{c\gamma} \right]$$

$$\frac{g^2}{32\pi^2} \int d^4x G_a^{\mu\nu} \tilde{G}_{a\mu\nu} = \frac{g^2}{32\pi^2} \int d\sigma_\mu K^\mu = \frac{g^3}{32\pi^2} \int d^3r K^0 |_{t=-\infty}^{t=+\infty} = n_+ - n_-$$

$$K^0 = \frac{4}{3} i g \epsilon_{ijk} \, \mathrm{Tr} \, A^i A^j A^k$$

$$n = \frac{ig^3}{24\pi^2} \int d^3r \, \mathrm{Tr} \, \epsilon_{ijk} A_n^i(\mathbf{r}) A_n^j(\mathbf{r}) A_n^k(\mathbf{r})$$

Difference in winding number of gauge fields t = ±∞

CP and chiral symmetry

- Chiral symmetry ($\Psi_{L,R} \rightarrow e^{\pm\chi}\,\Psi_{L,R}$) is spontaneously broken

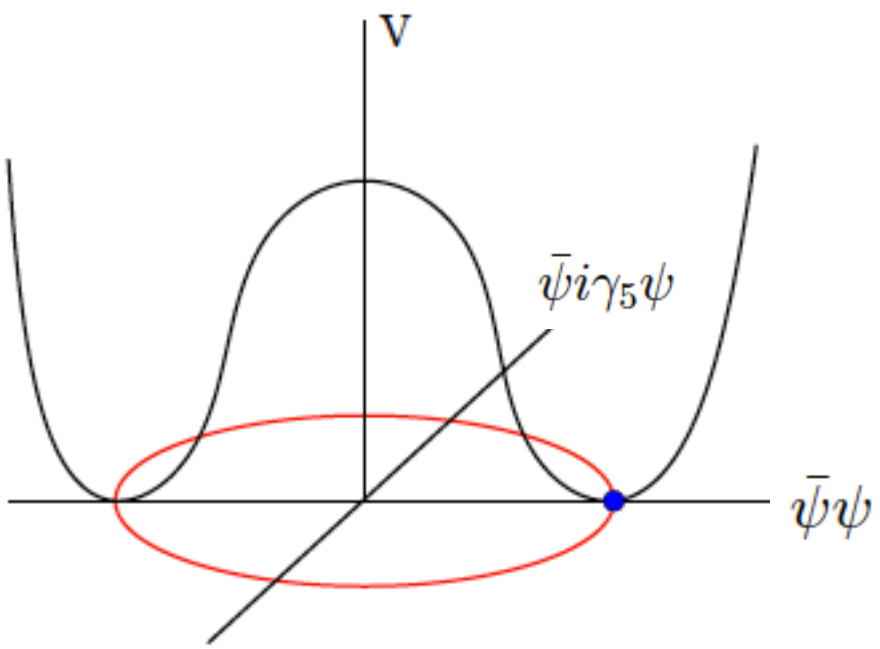


Figure from M. Creutz, 1103.3304

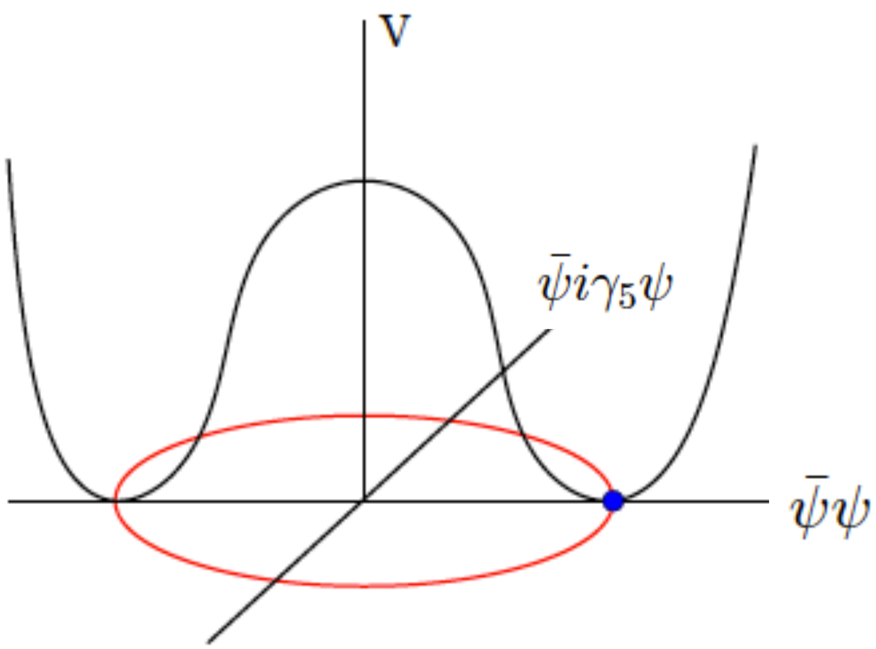
- Degenerate vacua. Each spontaneously breaks all but one $CP_\chi = \chi^{-1} CP \chi$

- Choice of fermion phases: CP_0 (standard CP) is preserved ($\langle\Omega|\, i\Psi\gamma_5\Psi |\Omega\rangle = 0$)
  This defines a "reference vacuum" $|\Omega\rangle$

CP and chiral symmetry

- Chiral symmetry ($\Psi_{L,R} \rightarrow e^{\pm\chi}\,\Psi_{L,R}$) is spontaneously broken

- Chiral symmetry is explicitly broken by quark masses <u>and</u> BSM operators



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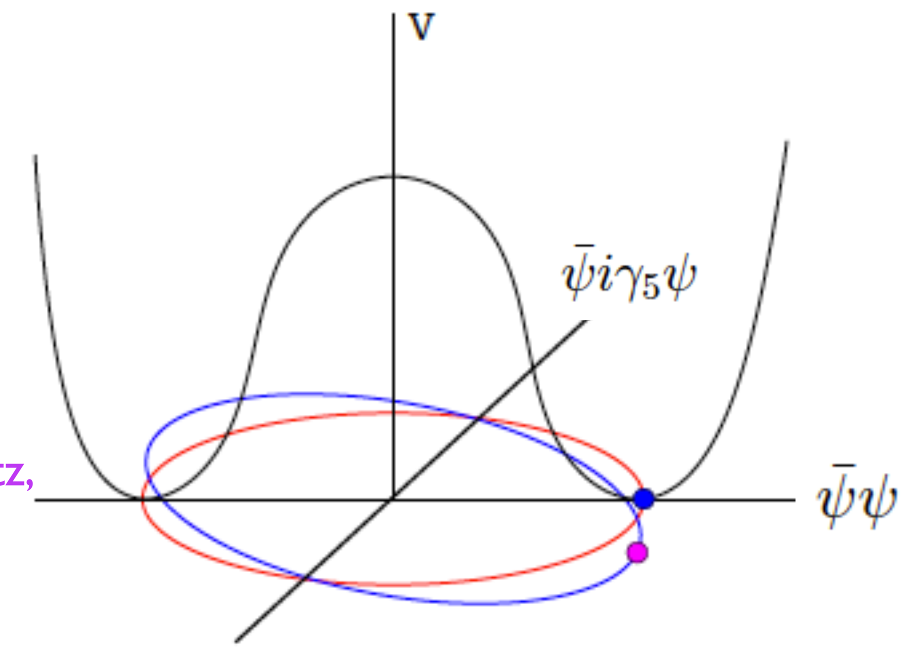

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- Choice of fermion phases: CP_0 (standard CP) is preserved ($\langle\Omega|\, i\Psi\gamma_5\Psi|\Omega\rangle = 0$) This defines a "reference vacuum" $|\Omega\rangle$

- Explicit chiral symmetry breaking δL lifts degeneracy, i.e. selects "true" vacuum and the associated unbroken CP

- If we want true vacuum to be $|\Omega\rangle$ then δL cannot be arbitrary. It satisfies

$$\langle\pi|\,\delta\mathcal{L}\,|\Omega\rangle = 0$$

"Vacuum alignment"

Chiral symmetry relations

- Prototype: theta term and mass splitting are chiral partners

$$\begin{pmatrix} \bar{q} i \gamma_5 q \\ \bar{q} \boldsymbol{\tau} q \end{pmatrix} \xrightarrow{SU_A(2)} \begin{pmatrix} -\bar{q} \boldsymbol{\alpha} \cdot \boldsymbol{\tau} q \\ \boldsymbol{\alpha}\, \bar{q} i \gamma_5 q \end{pmatrix}$$

- Nucleon matrix elements are related. At LO (soft pion theorem)

$$\langle N_f \pi^a | \bar{q} i \gamma_5 q | N_i \rangle \ \propto \ F_\pi^{-1} \ \langle N_f | \bar{q}\, \tau^a\, q | N_i \rangle$$

Crewther-DiVecchia-Veneziano-Witten 1979

$$\Downarrow$$

$$\boxed{\bar{g}_0 = \frac{\delta M_{n-p}^{m_d - m_u}}{m_d - m_u} \frac{2 m_d m_u}{m_d + m_u} \bar{\theta}}$$

$$\frac{\bar{g}_0}{F_\pi} = (15 \pm 2) \cdot 10^{-3} \sin \bar{\theta}$$

(with LQCD input)

- Corrections appear at NNLO, not log enhanced

Mereghetti, van Kolck
1505.06272
and refs therein

Toy model of invisible axion

Shifman-Vainshtein-Zakharov Nucl. Phys. B 166 (1980) 493

$$\mathcal{L}_{\psi\varphi} = \bar{\psi} i \hat{D} \psi - h(\varphi \bar{\psi}_R \psi_L + \varphi^+ \bar{\psi}_L \psi_R) + (\partial_\mu \varphi^+)(\partial_\mu \varphi) + m^2 \varphi^+ \varphi - \lambda (\varphi^+ \varphi)^2$$

$$\psi_L \to e^{i\gamma} \psi_L \qquad \psi_R \to e^{i\gamma} \psi_R \qquad \varphi \to e^{-2i\gamma} \varphi$$

Field content: new quark (only strong interactions) + New complex scalar

Yukawa interactions invariant under axial $U(1)_{PQ}$

φ acquires VEV $\quad |\langle \varphi \rangle| \equiv \varphi_0 = m/\sqrt{2\lambda} \qquad \varphi_0 \to \infty.$

Quark and "radial" scalar excitations super-heavy.
Axion is identified the phase of the scalar field:

$$\varphi(x) = \varphi_0 \exp\left(i \frac{a(x)}{\varphi_0 \sqrt{2}} \right)$$

Super-heavy quarks mediates axion-gluon interactio via triangle diagram:

$$\frac{\alpha_s}{8\pi\sqrt{2}\,\varphi_0} a(x) G^a_{\mu\nu}(x) \widetilde{G}^a_{\mu\nu}(x)$$

From this point on, the analysis proceeds as in the EFT description