Effective Field Theory and EDMs

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Lecture III outline

• EFT approach to physics beyond the Standard Model
  • Standard Model EFT up to dimension 6: guided tour
  • Simple examples of matching

• CP violating dimension-6 operators contributing to EDMs
  • Classification
  • Evolution from the BSM scale to hadronic scale
Effective theory for new physics (and EDMs)
EDMs and new physics

- EDMs are a powerful probe of high-scale new physics

- Quantitative connection of EDMs with high scale models requires Effective Field Theory tools
Connecting EDMs to UV new physics

Multi-scale problem: need RG evolution of effective couplings & hadronic / nuclear / molecular calculations of matrix elements
In this lecture we will cover the EFT analysis connecting physics between the new physics scale $\Lambda$ and the hadronic scale $\Lambda_{\text{had}} \sim 1$ GeV.
The low-energy footprints of $\mathcal{L}_{BSM}$

- At energy $E_{\text{exp}} \ll M_{BSM}$, new particles can be “integrated out”
- Generate new local operators with coefficients $\sim g^k/(M_{BSM})^n$

Familiar example:

\[ \begin{align*}
    g & \quad g \\
    W & \quad q^2 \ll M_W^2 \\
    G_F & \sim g^2/M_W^2
\end{align*} \]

Effective Field Theory emerges as a natural framework to analyze low-E implications of classes of BSM scenarios and inform model building.
Why use EFT for new physics

• General framework encompassing classes of models
• Efficient and rigorous tool to analyze experiments at different scales (from collider to table-top)
• The steps below UV matching apply to all models: can be done once and for all
• Very useful diagnosing tool in this “pre-discovery” phase :)
• Inform model building (success story is SM itself**)

EFT and UV models approaches are not mutually exclusive
**EFT for $\beta$ decays and the making of the Standard Model**

Current-current, parity conserving

Parity conserving: VV, AA, SS, TT ...
Parity violating: VA, SP, ...

It's $(V-A)^*(V-A)$ !!

"V-A was the key" S. Weinberg

Embed in non-abelian chiral gauge theory, predict neutral currents
EFT framework

- Assume mass gap $M_{BSM} > G_F^{-1/2} \sim v_{EW}$
- Degrees of freedom:
  SM fields (+ possibly $\nu_R$)
- Symmetries: SM gauge group; no flavor, CP, B, L

- EFT expansion in $E/M_{BSM}, M_W/M_{BSM}$ $[O_i^{(d)}$ built out of SM fields$]$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \cdots$$

$[\Lambda \leftrightarrow M_{BSM}]$ $C_i [g_{BSM}, M_a/M_b]$
Guided tour of $\mathcal{L}_{\text{eff}}$

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots \]

- **Dim 5**: only one operator

\[
\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}, \quad \ell = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \hat{O}_{\text{dim}=5} = \ell^T C \epsilon \varphi \varphi^T \epsilon \ell
\]

\[
C = i\gamma_2 \gamma_0, \quad \epsilon = i\sigma_2
\]
Guided tour of $\mathcal{L}_{\text{eff}}$

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots \]

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\[ \hat{O}_{\text{dim}=5} = \ell^T C \epsilon \varphi \varphi^T \epsilon \ell \]

\[ \epsilon = i \sigma_2 \]

\[ C = i \gamma_2 \gamma_0 \]

- Violates total lepton number

\[ \ell \rightarrow e^{i\alpha} \ell \quad e \rightarrow e^{i\alpha} e \]

- Generates Majorana mass for L-handed neutrinos (after EWSB)

\[ \frac{1}{\Lambda} \hat{O}_{\text{dim}=5} \rightarrow \langle \varphi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \]

\[ \frac{v^2}{\Lambda} \nu_L^T C \nu_L \]

- “See-saw”:

\[ m_\nu \sim 1 \text{ eV} \rightarrow \Lambda \sim 10^{13} \text{ GeV} \]
Guided tour of $\mathcal{L}_{\text{eff}}$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C^{(6)}_i}{\Lambda^2} O^{(6)}_i + \ldots$$

- **Dim 6**: many structures (59, not including flavor)

- No fermions

- Two fermions

- Four fermions
Guided tour of $\mathcal{L}_{\text{eff}}$

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{C(5)}{\Lambda} O(5) + \sum_i \frac{C_i(6)}{\Lambda^2} O_i(6) + \ldots \]

- **Dim 6:** affect *many* processes
  - B violation
  - Gauge and Higgs boson couplings
  - EDMs, LFV, qFCNC, ...
  - g-2, Charged Currents, Neutral Currents, ...

- **EFT used beyond tree-level:** one-loop anomalous dimensions known

Weinberg 1979
Wilczek-Zee 1979
Buchmuller-Wyler 1986, ....

Alonso, Jenkins, Manohar, Trott 2013
Examples of matching

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} \mathcal{O}^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \ldots \]
Examples of matching

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots \]

- Explicit examples of “matching” from full model to EFT
- **Dim 5**: Triplet Higgs field

\[ \mathcal{L}_5 = g_{\alpha\beta} \ell_\alpha^T C \epsilon \varphi \varphi^T \epsilon \ell_\beta \]

\[ g \sim \mu_T M_T^{-2} Y_T \]
More on matching

• We just saw two simple examples of matching calculation in EFT:

\[ A_{\text{full}} = \sum_i C_i \langle O_i \rangle \equiv A_{\text{EFT}} \]

★ To a given order in E/M_{R,T}, determine effective couplings (Wilson coefficients) from the matching condition \( A_{\text{full}} = A_{\text{EFT}} \) with amplitudes involving “light” external states

★ We did matching at tree-level, but strong and electroweak higher order corrections can be included

\[
\begin{align*}
\text{Full theory} & \quad + \quad \text{Effective theory} \\
\end{align*}
\]

\[
\begin{align*}
\text{Full theory} & \quad + \quad \text{Effective theory} \\
\end{align*}
\]
More on matching

- We just saw two simple examples of matching calculation in EFT:

\[ A_{\text{full}} = \sum_i C_i \langle O_i \rangle \equiv A_{\text{EFT}} \]

- In some cases, \( A_{\text{full}} \) starts at loop level (highly relevant for EDMs)

\[ (\bar{Q}_{\mu\nu} T^A d_R) \varphi G^A_{\mu\nu} \]

Function of SUSY coupling and masses
CP-violating operators contributing to EDMs: from BSM scale to hadronic scale
When including flavor indices, at dimension=6 there are 2499 independent couplings of which 1149 CP-violating!!

A large number of them contributes to EDMs

Leading flavor-diagonal CP odd operators contributing to EDMs have been identified, neglecting 2nd and 3rd generation fermions**

**Caveat: (i) strange quark can’t really be ignored; (ii) new physics could couple predominantly to heavy quarks; (iii) flavor-changing operators can contribute to EDMs (multiple insertions)
High-scale effective Lagrangian

• CPV BSM dynamics dictated by:

\[ \mathcal{L}_{6,\text{CPV}} = \frac{1}{\Lambda^2} \sum_i \alpha_i^{(6)} Q_i \]

Here follow notation of:
Engel, Ramsey-Musolf, Van Kolck
1303.2371

Gauge-Higgs-Fermion

| \( Q_{uG} \) | \((\bar{Q} \sigma^{\mu\nu} T^A u_R) \tilde{\varphi} G^A_{\mu\nu} \) |
| \( Q_{dG} \) | \((\bar{Q} \sigma^{\mu\nu} T^A d_R) \varphi G^A_{\mu\nu} \) |
| \( Q_{JW} \) | \((\bar{F} \sigma^{\mu\nu} f_R) \tau^I \Phi W^I_{\mu\nu} \) |
| \( Q_{fB} \) | \((\bar{F} \sigma^{\mu\nu} f_R) \Phi B_{\mu\nu} \) |

\[ \alpha_{fV_k}^{(6)} \equiv g_k C_{fV_k} \]
High-scale effective Lagrangian

- CPV BSM dynamics dictated by:

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Elementary fermion (chromo)-electric dipole

\[ \mathcal{L}^{\text{CEDM}} = -i \sum_q \frac{g_3 d_q}{2} \bar{q} \sigma^{\mu\nu} T^A \gamma_5 q \ G^A_{\mu\nu} \]

\[ \mathcal{L}^{\text{EDM}} = -i \sum_f \frac{d_f}{2} \bar{f} \sigma^{\mu\nu} \gamma_5 f \ F_{\mu\nu} \]

\[ \sum_f d_f \chi_f \bar{\sigma} \chi_f \cdot \vec{E} \]

\[ \phi^T \rightarrow (0, v/\sqrt{2}) \]

non-relativistic limit
**High-scale effective Lagrangian**

- CPV BSM dynamics dictated by:

\[
\mathcal{L}_{6,\text{CPV}} = \frac{1}{\Lambda^2} \sum_i \alpha_i^{(6)} Q_i
\]

Here follow notation of: Engel, Ramsey-Musolf, Van Kolck [1303.2371]

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<td>(Q_{\tilde{W}})</td>
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<td>(f^{ABC} \tilde{G}^A_{\mu} C^B_{\nu} G^C_{\rho})</td>
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<td>(Q_{fB})</td>
<td>(\varepsilon^{IJK} \tilde{W}^I_{\mu} W^J_\nu W^K_\rho)</td>
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\[
\alpha_i^{(6)} \equiv g_k C_{fV_k} \quad \alpha_G^{(6)} \equiv g_3 C_{\tilde{G}}
\]
High-scale effective Lagrangian

- CPV BSM dynamics dictated by:

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<tr>
<td>( Q_{dG} )</td>
<td>( Q_{\tilde{W}} )</td>
</tr>
<tr>
<td>( (Q_{\sigma^{\mu\nu}} T^A d_R) \varphi G^A_{\mu\nu} )</td>
<td>( \varepsilon^{IJK} \tilde{W}<em>{\mu}^I W</em>{\nu}^J W_{\rho}^K )</td>
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<tr>
<td>( Q_{fW} )</td>
<td>( Q_{\varphi ud} = i (\tilde{\varphi}^\dagger D_\mu \varphi) \tilde{u}_R \gamma^\mu d_R )</td>
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<td>( (F_{\sigma^{\mu\nu}} f_R) \tau^I \Phi W_{\mu\nu}^I )</td>
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\[ \alpha_{fV_k}^{(6)} \equiv g_k C_{fV_k} \quad \alpha_{\tilde{G}}^{(6)} \equiv g_3 C_{\tilde{G}} \quad \alpha_{\varphi ud}^{(6)} \equiv C_{\varphi ud} \]
High-scale effective Lagrangian

- CPV BSM dynamics dictated by:

\[
\mathcal{L}_6,\text{CPV} = \frac{1}{\Lambda^2} \sum_i \alpha^{(6)}_i Q_i
\]

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<td>( Q_{ledq} )</td>
</tr>
<tr>
<td>( Q_{dG} )</td>
<td>( \varepsilon_{ijk} \tilde{W}^i_{\mu} W^j_{\nu} W^k_{\rho} )</td>
<td>( Q^{(1)}_{equ} )</td>
</tr>
<tr>
<td>( Q_{fW} )</td>
<td>( f_{\sigma^{\mu\nu}} f_R^{I} \tau^I \Phi W^I_{\mu} )</td>
<td>( Q^{(8)}_{quqd} )</td>
</tr>
<tr>
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<td>( f_{\sigma^{\mu\nu}} f_R^{I} \Phi B^I_{\mu} )</td>
<td>( Q^{(8)}_{equ} )</td>
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\[ \alpha^{(6)}_{fV_k} \equiv g_k C_{fV_k} \]
\[ \alpha^{(6)}_G \equiv g_3 C_G \]
\[ \alpha^{(6)}_{\varphi ud} \equiv C_{\varphi ud} \]

\[ \alpha^{(6)}_{ledq} \equiv C_{ledq} \]
\[ \alpha^{(6)}_{lequ(1,3)} \equiv C_{lequ}^{(1,3)} \]
\[ \alpha^{(6)}_{quqd(1,8)} \equiv g_3^2 C_{quqd}^{(1,8)} \]
High-scale effective Lagrangian

- CPV BSM dynamics dictated by:

\[ \mathcal{L}_{6,\text{CPV}} = \frac{1}{\Lambda^2} \sum_i \alpha_i^{(6)} Q_i \]

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Engel, Ramsey-Musolf, Van Kolck
[1303.2371]
Evolution to low-E: generalities

\[ \mathcal{L}_{\text{eff}} = \sum_i C_i(\mu) Q_i(\mu) \]

1. Evolution of effective couplings with energy scale

- Operators in \( \mathcal{L}_{\text{eff}} \) depend on the energy scale \( \mu \) at which they are “renormalized” (i.e. the UV divergences are removed)
- To avoid large logs, \( \mu \) should be of the order of the energy probed
- Physical results should not depend on the arbitrary scale
- The couplings \( C_i \) depend on \( \mu \) in such a way to guarantee this!

\[ \frac{d}{d(\ln \mu)} [C_i \langle Q_i \rangle] = 0 \]
2. As one evolves the theory to low energy, need to remove ("integrate out") heavy particles

- In our case, in the evolution of $\mathcal{L}_{\text{eff}}$ we encounter the electroweak scale: remove top quark, Higgs, $W$, $Z$
- $b$ and $c$ quark thresholds
Dipole operators

\[ Q_{uG} \quad (\bar{Q}\sigma^{\mu\nu}T^A u_R) \sigma G^A_{\mu\nu} \]
\[ Q_{dG} \quad (\bar{Q}\sigma^{\mu\nu}T^A d_R) \sigma G^A_{\mu\nu} \]
\[ Q_{fW} \quad (\bar{F}\sigma^{\mu\nu} f_R) \tau^I \Phi W^I_{\mu\nu} \]
\[ Q_{fB} \quad (\bar{F}\sigma^{\mu\nu} f_R) \Phi B_{\mu\nu} \]

CEDM mixing into EDM

CEDM renormalization

\[ \phi^T \rightarrow (0, v/\sqrt{2}) \]
Dipole operators

\[ \mathcal{L}^{\text{CEDM}} = -i \sum_q \frac{g_3 \tilde{d}_q}{2} \bar{q} \sigma^{\mu \nu} T^A \gamma_5 q \, G^A_{\mu \nu} \]

\[ \mathcal{L}^{\text{EDM}} = -i \sum_f \frac{d_f}{2} \bar{f} \sigma^{\mu \nu} \gamma_5 f \, F_{\mu \nu} \]
Three gauge bosons

\[ Q_{\tilde{G}} = f^{ABC} \tilde{G}_{\mu}^A \tilde{G}_{\nu}^B \tilde{G}_{\rho}^C \]

New structure at low-energy

\[ \mathcal{L}_{\text{CPV}}^{\tilde{G}} = \frac{g_3 C_{\tilde{G}}}{\Lambda^2} f^{ABC} \tilde{G}_{\mu}^A \tilde{G}_{\nu}^B \tilde{G}_{\rho}^C \]

Weinberg mixing into CEDM
Dipole and three-gluon mixing

\[ \mathcal{L}^{\text{EDM}} = -i \sum_f \frac{d_f}{2} \bar{f} \sigma^{\mu\nu} \gamma_5 f \ F_{\mu\nu} \]
\[ \mathcal{L}^{\text{CEDM}} = -i \sum_q \frac{g_3 \bar{d}_q}{2} \bar{q} \sigma^{\mu\nu} T^A \gamma_5 q \ G^A_{\mu\nu} \]
\[ \mathcal{L}^{\text{CPV}} = \frac{g_3 C_F}{\Lambda^2} f^{ABC} \tilde{G}^{A\mu} G^{B\nu} G^{C\rho} \]

Effect of mixing is important

Rosetta stone

\[ d_q = -2e m_q C_q \]
\[ g_3 \bar{d}_q = -2m_q \tilde{C}_q \]
\[ g_3 C_{\tilde{G}} = C_W \]
Four fermion operators (1)

\[ Q_{\text{ledq}} \]

\[ (\bar{L}^j e_R)(\bar{d}_R Q^j) \]

\[ Q_{\text{quqd}}^{(1)} \]

\[ (\bar{Q}^j u_R)\epsilon_{jk}(\bar{Q}^k d_R) \]

\[ Q_{\text{quqd}}^{(8)} \]

\[ (\bar{Q}^j T^A u_R)\epsilon_{jk}(\bar{Q}^k T^A d_R) \]

\[ Q_{\text{lequ}}^{(1)} \]

\[ (\bar{L}^j e_R)\epsilon_{jk}(\bar{Q}^k u_R) \]

\[ Q_{\text{lequ}}^{(3)} \]

\[ (\bar{L}^j \sigma_{\mu\nu} e_R)\epsilon_{jk}(\bar{Q}^k \sigma^{\mu\nu} u_R) \]

“Diagonal” QCD evolution of scalar and tensor quark bilinears

\[
\mathcal{L}_{\text{CPV}}^{\text{eq}} = \frac{i}{2\Lambda^2} \text{Im} C_{\text{ledq}} \left[ \bar{e} \gamma_5 e \bar{d} d - \bar{e} e \bar{d} \gamma_5 d \right]
\]

\[
- \frac{i}{2\Lambda^2} \text{Im} C_{\text{lequ}}^{(1)} \left[ \bar{e} \gamma_5 e \bar{u} u + \bar{e} e \bar{u} \gamma_5 u \right]
\]

\[
- \frac{\text{Im} C_{\text{lequ}}^{(3)}}{2\Lambda^2} \epsilon_{\mu\nu\alpha\beta} \bar{e} \sigma^{\mu\nu} e \bar{u} \sigma^{\alpha\beta} u
\]

mixes into lepton dipoles
Four fermion operators (2)

\[ \mathcal{L}_{\text{CPV}}^{\text{qq}} = \frac{i g_3^2 \text{Im} C^{(1)}_{\text{quqd}}}{2\Lambda^2} \left[ \bar{u} \gamma_5 u \, \bar{d} d + \bar{u} u \, \bar{d} \gamma_5 d - \bar{d} \gamma_5 u \, \bar{u} d - \bar{u} u \, \bar{d} \gamma_5 d \right] \\
+ i \frac{g_3^2 \text{Im} C^{(8)}_{\text{quqd}}}{2\Lambda^2} \left[ \bar{u} \gamma_5 T^A u \, \bar{d} T^A d + \bar{u} T^A u \, \bar{d} \gamma_5 T^A d - \bar{d} \gamma_5 T^A u \, \bar{u} T^A d - \bar{d} T^A u \, \bar{u} \gamma_5 T^A d \right] \]
Induced 4-quark operator

\[ Q_{\varphi ud} = i \left( \bar{\varphi}^\dagger D_\mu \varphi \right) \bar{u}_R \gamma^\mu d_R \]

\[ v^2 g \left( \bar{u}_R \gamma^\mu d_R W^\pm_\mu + \text{h.c.} \right) \]

\[ W^-_\mu \quad \bar{d}_R \rightarrow u_R \]
Induced 4-quark operator

\[ Q_{\varphi ud} = i \left( \bar{\varphi}^\dagger D_\mu \varphi \right) \bar{u}_R \gamma^\mu d_R \]

\[ \nu^2 g (\bar{u}_R \gamma^\mu d_R W^\pm_\mu + \text{h.c.}) \]

\[ L_{\text{LR, CPV}}^{\text{eff}} = -i \frac{\text{Im} C_{\varphi ud}}{\Lambda^2} \left[ \bar{d}_L \gamma^\mu u_L \bar{u}_R \gamma_\mu d_R - \bar{u}_L \gamma^\mu d_L \bar{d}_R \gamma_\mu u_R \right] \]

+ color-mixed structure induced by QCD corrections
Gauge-Higgs operators

\[ \Lambda \]

\[ V_{\text{EW}} \]

\[ \Lambda_{\text{Had}} \]

Mix into quark CEDM, quark EDM, electron EDM

\[ f = q, e \]

\[ g, \gamma \]

\[ Q_{\varphi G} \]

\[ Q_{\varphi W} \]

\[ Q_{\varphi B} \]

\[ Q_{\varphi \bar{W}} \]

\[ \varphi^\dagger \varphi G_{\mu \nu}^A G_{\mu \nu}^A \]

\[ \varphi^\dagger \varphi \bar{W}_{\mu \nu}^I W_{\mu \nu}^I \]

\[ \varphi^\dagger \varphi \bar{B}_{\mu \nu} B_{\mu \nu} \]

\[ \varphi^\dagger \tau^I \varphi \bar{W}_{\mu \nu}^I B_{\mu \nu} \]
For example: top quark electroweak dipoles induce at two loops electron and quark EDMs — strongest constraints (by three orders of magnitude)!

VC, W. Dekens, J. de Vries, E. Mereghetti 1603.03049, 1605.04311
... and more

- EDM physics reach vs flavor and collider probes

\[ \mu_t = e Q t m_t c_\gamma \]
\[ d_t = e Q t m_t \tilde{c}_\gamma \]

\[ C_\gamma = c_\gamma + i \tilde{c}_\gamma \]

Bound on top EDM improved by three orders of magnitude:
\[ |d_t| < 5 \times 10^{-20} \text{ e cm} \]

Dominated by eEDM

LHC sensitivity
(pp → jet t γ)
and LHeC
\[ d_t \sim 10^{-17} \text{ e cm} \]

[Fael-Gehrmann 13, Bouzas-Larios 13]
Low-energy effective Lagrangian

- When the dust settles, at the hadronic scale we have:

\[
\mathcal{L}_{6}^{CPV} = -\frac{i}{2} \sum_{f=e,u,d,s} d_f \bar{f} \sigma \cdot F \gamma_5 f - \frac{i}{2} \sum_{q=u,d,s} \bar{d}_q g_s \bar{q} \sigma \cdot G \gamma_5 q + d_W \frac{g_s}{6} G \tilde{G} \tilde{G} + \sum_i C_i^{(4f)} O_i^{(4f)}
\]
Low-energy effective Lagrangian

When the dust settles, at the hadronic scale we have:

\[ \mathcal{L}_{6}^{CPV} = -\frac{i}{2} \sum_{f=e,u,d,s} d_f \bar{f} \sigma \cdot F \gamma_5 f - \frac{i}{2} \sum_{q=u,d,s} \bar{d}_q g_s \bar{q} \sigma \cdot G \gamma_5 q + d_W \frac{g_s}{6} G \tilde{G} G + \sum_{i} C^{(A_f)}_i O^{(A_f)}_i \]

Electric and chromo-electric dipoles of fermions

\[ d_f, \tilde{d}_q \sim \frac{v_{ew}}{\Lambda^2} \]

\[ J \cdot E \quad J \cdot E_c \]
Low-energy effective Lagrangian

When the dust settles, at the hadronic scale we have:

\[ \mathcal{L}_{6}^{CPV} = -\frac{i}{2} \sum_{f=e,u,d,s} d_f \bar{f} \sigma \cdot F \gamma_5 f - \frac{i}{2} \sum_{q=u,d,s} \tilde{d}_q g_s \bar{q} \sigma \cdot G \gamma_5 q + d_W \frac{g_s}{6} G \tilde{G} G + \sum_i C_i^{(4f)} O_i^{(4f)} \]

Electric and chromo-electric dipoles of fermions
Gluon chromo-EDM (Weinberg operator)

\[ d_f, \tilde{d}_q \sim \frac{v_{ew}}{\Lambda^2} \]
\[ d_W \sim \frac{1}{\Lambda^2} \]

\[ J \cdot E \quad J \cdot E_c \]
Low-energy effective Lagrangian

- When the dust settles, at the hadronic scale we have:

\[
\mathcal{L}_6^{CPV} = -\frac{i}{2} \sum_{f=e,u,d,s} d_f \bar{f} \sigma \cdot F \gamma_5 f - \frac{i}{2} \sum_{q=u,d,s} \bar{d}_q g_s \bar{q} \sigma \cdot G \gamma_5 q + d_W \frac{g_s}{6} G \tilde{G} G + \sum_i C_i^{(4f)} O_i^{(4f)}
\]

Electric and chromo-electric dipoles of fermions

\[d_f, \bar{d}_q \sim \frac{v_{ew}}{\Lambda^2}\]

Gluon chromo-EDM (Weinberg operator)

\[d_W \sim \frac{1}{\Lambda^2}\]

Semi-leptonic (3) and four-quark (2 “SP” + 2 “LR”)

Their form (and number) is strongly constrained by SU(2) gauge invariance

Explicit form of operators given in previous slides
Low-energy effective Lagrangian

• When the dust settles, at the hadronic scale we have:

\[
\mathcal{L}^{CPV}_6 = -\frac{i}{2} \sum_{f=e,u,d,s} d_f \bar{f} \sigma \cdot F \gamma_5 f - \frac{i}{2} \sum_{q=u,d,s} \bar{d}_q g_s \bar{q} \sigma \cdot G \gamma_5 q + d_W \frac{g_s}{6} G \tilde{G} G + \sum_i C_i^{(A_f)} O_i^{(A_f)}
\]

• Generated by a variety of BSM scenarios

See Lecture IV for detailed discussion
Low-energy effective Lagrangian

• When the dust settles, at the hadronic scale we have:

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• Generated by a variety of BSM scenarios

See Lecture IV for detailed discussion
Low-energy effective Lagrangian

- When the dust settles, at the hadronic scale we have:

\[
\mathcal{L}^{\text{CPV}}_{6} = -\frac{i}{2} \sum_{f=e,u,d,s} d_{f} \bar{f} \sigma \cdot F \gamma_{5} f - \frac{i}{2} \sum_{q=u,d,s} \bar{d}_{q} g_{s} \bar{q} \sigma \cdot G \gamma_{5} q + d_{W} \frac{g_{s}}{6} G \tilde{G} G + \sum_{i} C_{i}^{(4f)} O_{i}^{(4f)}
\]

- Important points:
  - Each BSM scenario generate its own pattern of operators (and hence of EDM “signatures”)
  - Within a model, relative importance of operators depends on various parameters (masses, etc)
  - So, in a post-discovery scenario, a combination of EDMs will allow us to learn about underlying sources of CP violation
But we are not done yet…

Multi-scale problem: need **RG evolution of effective couplings & hadronic / nuclear / molecular calculations of matrix elements**
Multi-scale problem: need RG evolution of effective couplings & hadronic / nuclear / molecular calculations of matrix elements
But we are not done yet...

Multi-scale problem: need RG evolution of effective couplings & hadronic / nuclear / molecular calculations of matrix elements

RG EVOLUTION
(perturbative)

MATRIX ELEMENTS
(non-perturbative)
Next step: from quarks and gluons to hadrons

- Leading pion-nucleon CPV interactions characterized by few LECs

\[ \mathcal{L}_{\text{CPV}} = -\frac{i}{2} \sum_{i=n,p,e} d_i \bar{\psi}_i \sigma \cdot F \gamma_5 \psi_i - N \left[ \bar{g}_0 \vec{r} \cdot \vec{\pi} + \bar{g}_1 \pi^0 \right] N + \ldots \]

Electron and Nucleon EDMs

T-odd P-odd pion-nucleon couplings

Short-range 4N and 2N2e coupling

To be discussed in Lectures VI, VII, VIII
Backup slides
Standard Model building blocks

\[ \psi = \begin{pmatrix} q \\ \ell \\ u \\ d \\ e \end{pmatrix} \]

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \gamma^\mu \partial_\mu \psi + \psi \left( y_3 \gamma^5 + \psi \right) + \frac{1}{2} m^2 \psi - V(\phi) \]

| \text{SU}(3) \times \text{SU}(2) \times \text{U}(1) \text{ representation:} & \text{SU}(2)_W \text{ transformation} |
|---|---|
| \ell = \left( \begin{array}{c} \nu_L \\ e_L \end{array} \right) & (1, 2, -1/2) \quad l \rightarrow V_{\text{SU}(2)} l |
| e = e_R & (1, 1, -1) |
| \begin{array}{l} q^i = \left( \begin{array}{c} u^i_L \\ d^i_L \end{array} \right) \\ u^i = u^i_R \end{array} & (3, 2, 1/6) \quad q \rightarrow V_{\text{SU}(2)} q |
| \begin{array}{l} d^i = d^i_R \end{array} & (3, 1, 2/3) |
| \begin{array}{l} \varphi^+ = \left( \begin{array}{c} \varphi^+ \\ \varphi^0 \end{array} \right) \end{array} & (1, 2, 1/2) \quad \varphi \rightarrow V_{\text{SU}(2)} \varphi |

\text{SU}(3) \times \text{SU}(2) \times \text{U}(1) \text{ representation:}

- \text{gluons:} \quad G^A_\mu = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + g_8 f_{ABC} G^B_\mu G^C_\nu \\
  (8, 1, 0)

- W \text{ bosons:} \quad W^I_\mu = a^I_\mu W^I_\nu - a^I_\nu W^I_\mu + g_{IJK} W^J_\mu W^K_\nu \\
  (1, 3, 0)

- B \text{ boson:} \quad B_\mu = B_{\mu \nu} \delta_\nu, B_{\mu \nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \\
  (1, 1, 0)

\text{Gauge transformation:}

\[ W^I_\mu \gamma^I_2 \rightarrow V(x) \left[ W^I_\mu \gamma^I_2 \right] V(x) \]

\[ V(x) = e^{i q_\mu \beta_\mu(x) \frac{\pi}{2}} \]
Standard Model Lagrangian

\[ \mathcal{L}_{SM} = \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} \]

\[ D_\mu = i \partial_\mu - ig_s \frac{\lambda^A}{2} G^A_\mu - ig^{\sigma_a} W^{a}_\mu - ig' Y B_\mu \]

\[ \mathcal{L}_{\text{Gauge}} = -\frac{1}{4} G^A_{\mu \nu} G^{A \mu \nu} - \frac{1}{4} W^{I}_{\mu \nu} W^{\mu \nu} - \frac{1}{4} B_{\mu \nu} B^{\mu \nu} \]

\[ + \sum_{i=1,2,3} \left( i \bar{\ell}_i D \ell_i + i \bar{e}_i D e_i + i \bar{q}_i D q_i + i \bar{u}_i D u_i + i \bar{d}_i D d_i \right) \]

\[ \mathcal{L}_{\text{Higgs}} = (D_\mu \varphi)\dagger (D^\mu \varphi) - \lambda (\varphi\dagger \varphi - v^2)^2 \]

\[ \mathcal{L}_{\text{Yukawa}} = \bar{\ell} Y_e e \varphi + \bar{q} Y_d d \varphi + \bar{q} Y_u u \tilde{\varphi} + \text{h.c.} \]
## Counting operators at low scale

<table>
<thead>
<tr>
<th>Operator (dimension)</th>
<th>Number</th>
<th>Systems</th>
</tr>
</thead>
<tbody>
<tr>
<td>theta term (4)</td>
<td>1</td>
<td>hadronic &amp; diamagnetic atoms</td>
</tr>
<tr>
<td>electron EDM (6)</td>
<td>1</td>
<td>paramagnetic atoms &amp; molecules</td>
</tr>
<tr>
<td>semi-leptonic (6)</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>quark EDM (6)</td>
<td>2</td>
<td>hadronic &amp; diamagnetic atoms</td>
</tr>
<tr>
<td>quark chromo EDM (6)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>three-gluon (6)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>four-quark (6)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>induced four-quark (6)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>

Engel, Ramsey-Musolf, Van Kolck 1303.2371
Renormalization group

- Large logs (from widely separated scales) spoil validity of perturbation theory

<table>
<thead>
<tr>
<th></th>
<th>$\mathcal{LL}$</th>
<th>$\mathcal{NLL}$</th>
<th>$\mathcal{N}^2\mathcal{LL}$</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLO</td>
<td>$\alpha_s \ell$</td>
<td>$\alpha_s$</td>
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<tr>
<td>$\mathcal{N}^2\mathcal{LO}$</td>
<td>$\alpha_s^2 \ell^2$</td>
<td>$\alpha_s^2 \ell$</td>
<td>$\alpha_s^2$</td>
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<tr>
<td>$\mathcal{N}^3\mathcal{LO}$</td>
<td>$\alpha_s^3 \ell^3$</td>
<td>$\alpha_s^3 \ell^2$</td>
<td>$\alpha_s^3 \ell$</td>
<td>$\alpha_s^3$</td>
</tr>
<tr>
<td></td>
<td>$O(1)$</td>
<td>$O(\alpha_s)$</td>
<td>$O(\alpha_s^2)$</td>
<td></td>
</tr>
</tbody>
</table>

- Ordinary pert. theory proceeds “by rows”: NLO, $\mathcal{N}^2\mathcal{LO}$, ...
- RGE re-organize the expansion “by columns”: LL, NLL, ...
• **RGEs**: exploit the fact that physics does not depend on the renormalization scale

\[ \mathcal{L}_{\text{eff}} = \sum_i C_i Q_i \]

- Bare operators do not depend on \( \mu \) (subtraction scale)

\[
\frac{d}{d(\ln \mu)} O_i^{(0)} = 0 \quad \longrightarrow \quad \frac{d}{d(\ln \mu)} O_i = -\gamma_i O_i
\]

\[ O_i^{(0)} = Z_i O_i \]

\[ \gamma_i \equiv \frac{1}{Z_i} \frac{d}{d(\ln \mu)} Z_i = \frac{g^2}{16\pi^2} \gamma_i^{(0)} + \ldots \]

- Physical amplitudes do not depend on \( \mu \)

\[
\frac{d}{d(\ln \mu)} [C_i \langle Q_i \rangle] = 0 \quad \longrightarrow \quad \frac{d}{d(\ln \mu)} C_i = \gamma_i C_i
\]
In general, need to solve:

\[
\frac{d}{d(\ln \mu)} C_i = \gamma_i C_i \quad \text{and} \quad \frac{d}{d(\ln \mu)} g = \beta(g)
\]

\[
\gamma_i = \frac{g^2}{16\pi^2} \gamma_i^{(0)} + \ldots
\]

\[
\beta(g) = -\beta_0 \frac{g^3}{16\pi^2} + \ldots
\]

\[
\frac{C_i(\mu)}{C_i(\Lambda)} = \exp \left[ \int_{g(\Lambda)}^{g(\mu)} dg' \frac{\gamma_i(g')}{\beta(g')} \right] \rightarrow \left[ \frac{\alpha(\Lambda)}{\alpha(\mu)} \right]^{\gamma_i^{(0)} \frac{\gamma_i^{(0)}}{2\beta_0}}
\]

\[
\approx 1 + \gamma_i^{(0)} \frac{\alpha}{4\pi} \log \frac{\mu}{\Lambda}
\]
• In general, need to solve:

\[
\frac{d}{d(\ln \mu)} C_i = \gamma_i C_i \quad \text{and} \quad \frac{d}{d(\ln \mu)} g = \beta(g)
\]

\[
\gamma_i = \frac{g^2}{16\pi^2} \gamma_i^{(0)} + \ldots
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\[
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\]

\[
\frac{C_i(\mu)}{C_i(\Lambda)} = \exp \left[ \int_{g(\Lambda)}^{g(\mu)} dg' \frac{\gamma_i(g')}{\beta(g')} \right] \rightarrow \left[ \frac{\alpha(\Lambda)}{\alpha(\mu)} \right]^{\gamma_i^{(0)}}_{\frac{1}{2}\beta_0}
\]

• One-loop beta functions:

\[
\beta_0^{\text{QCD}} = \frac{11N_C - 2N_F}{3} \quad \beta_0^{\text{QED}} = -\frac{4}{3} \sum_f Q_f^2
\]

• Needed input: \( \gamma_i^{(0)} \) anomalous dimensions for relevant operators