ACFI EDM School
November 2016

Effective Field Theory and EDMs

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Lecture III outline

- EFT approach to physics beyond the Standard Model
  - Standard Model EFT up to dimension 6: guided tour
  - Simple examples of matching

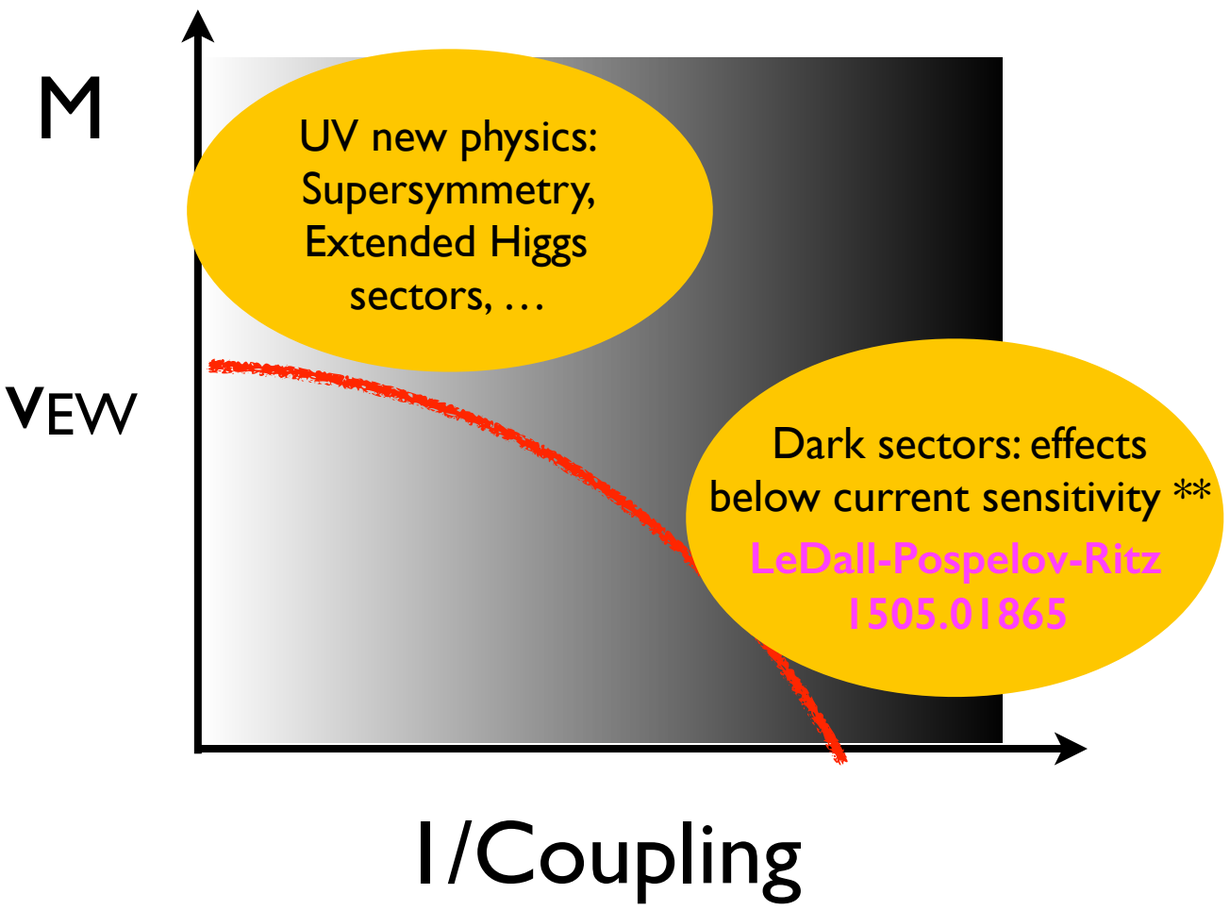
- CP violating dimension-6 operators contributing to EDMs
  - Classification
  - Evolution from the BSM scale to hadronic scale

Effective theory for new physics (and EDMs)

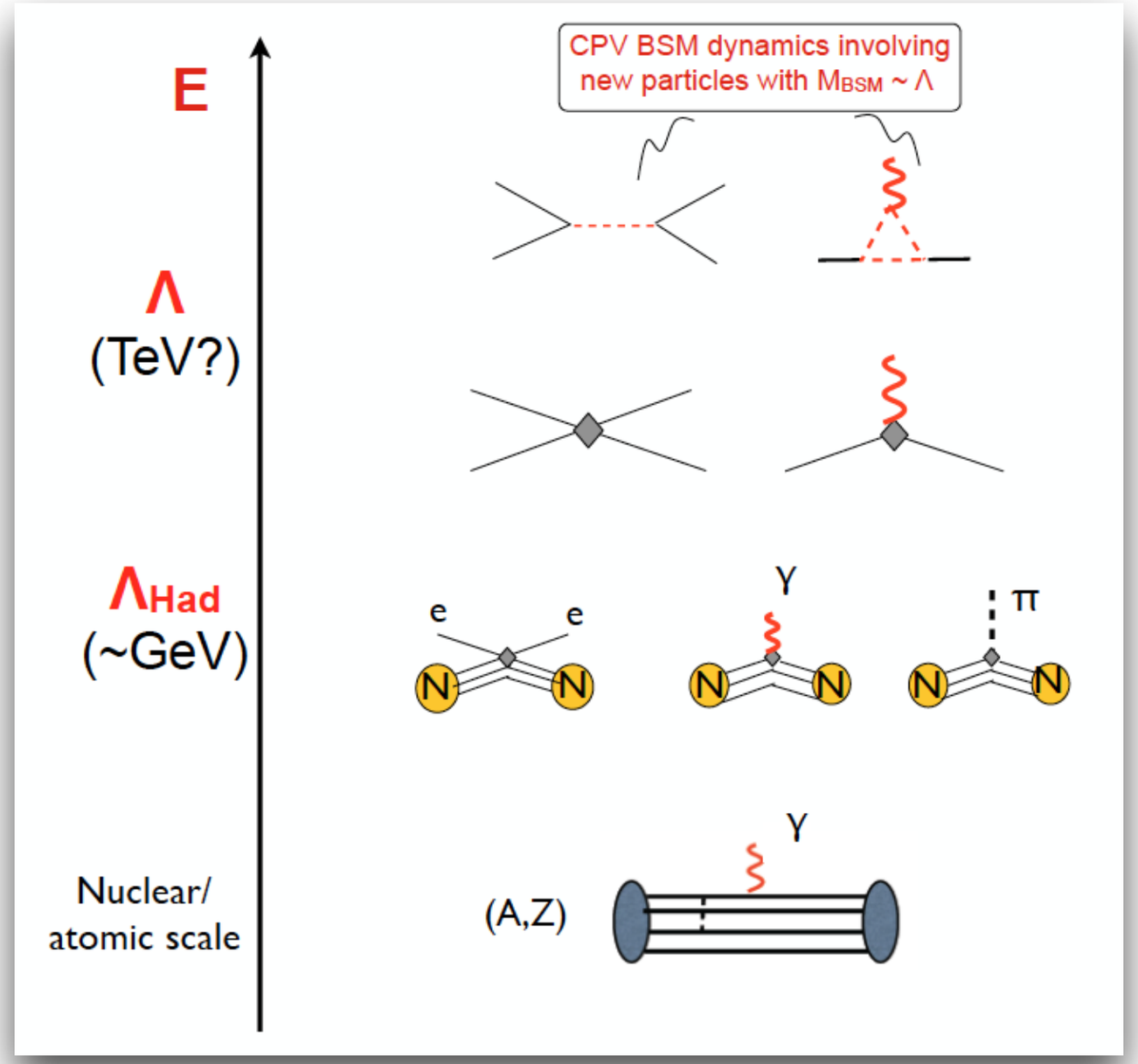
EDMs and new physics

- EDMs are a powerful probe of high-scale new physics

- Quantitative connection of EDMs with high scale models requires Effective Field Theory tools



Connecting EDMs to UV new physics

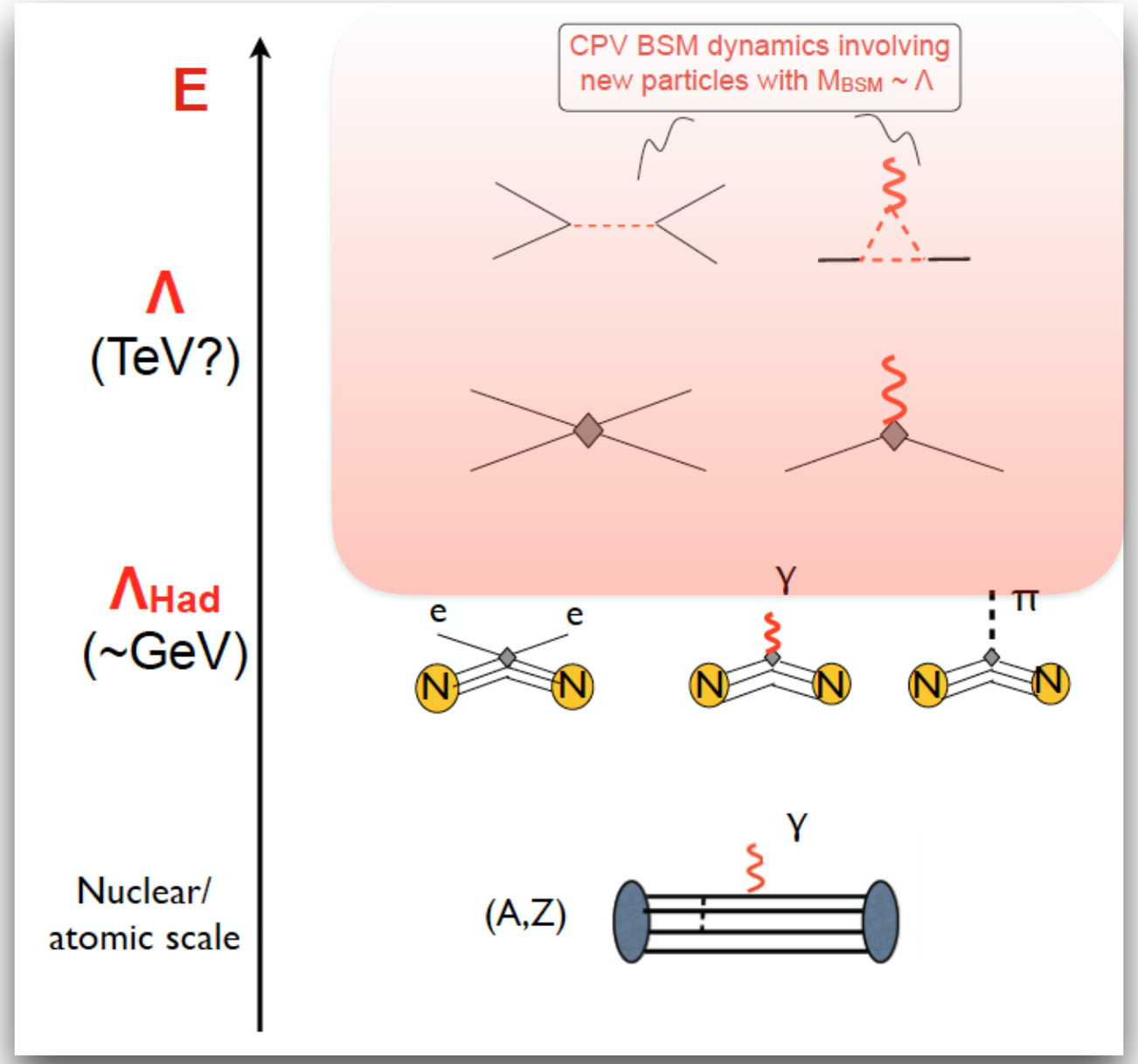


RG EVOLUTION
(perturbative)

MATRIX ELEMENTS
(non-perturbative)

Multi-scale problem: need RG evolution of effective couplings & hadronic / nuclear / molecular calculations of matrix elements

Connecting EDMs to UV new physics



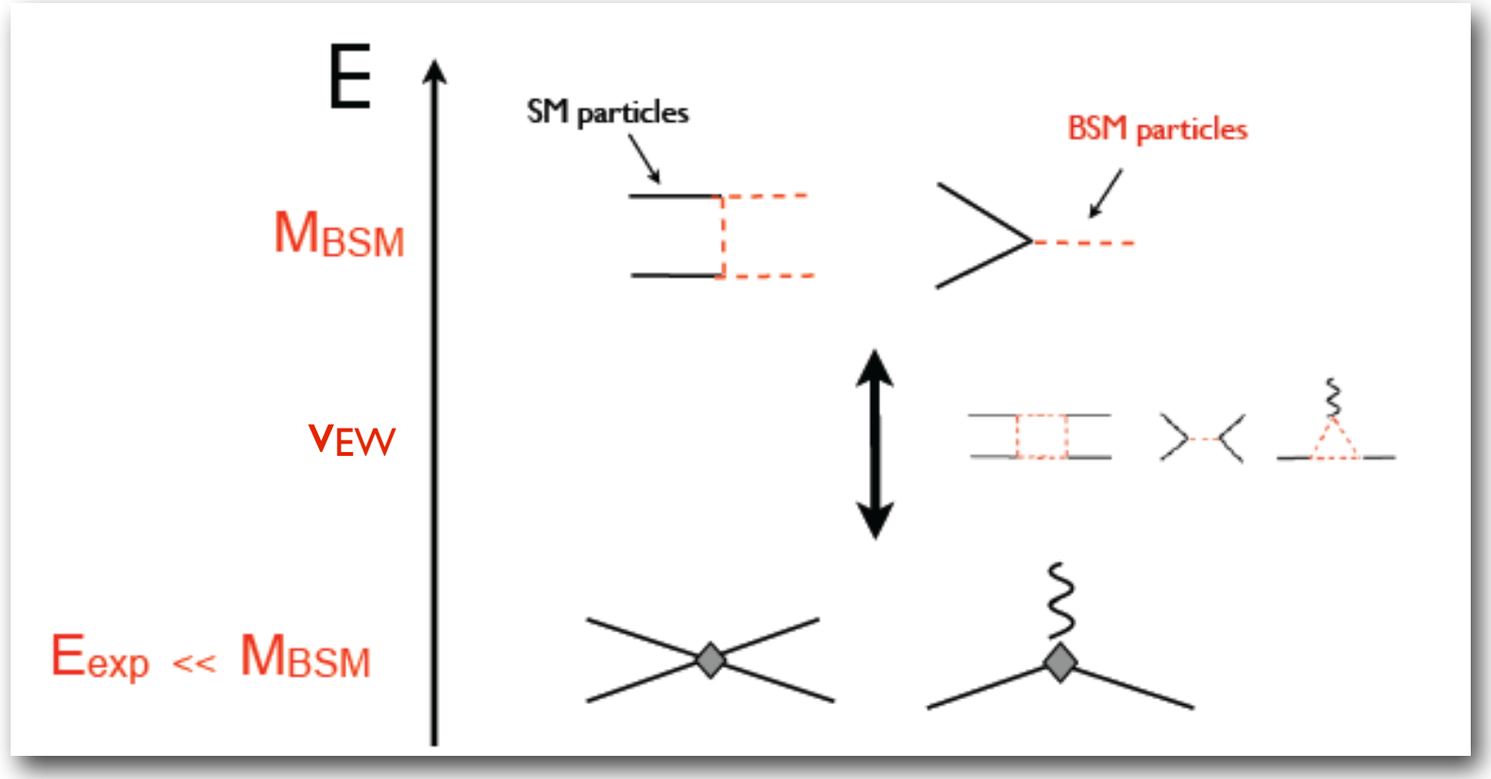
RG EVOLUTION
(perturbative)

MATRIX ELEMENTS
(non-perturbative)

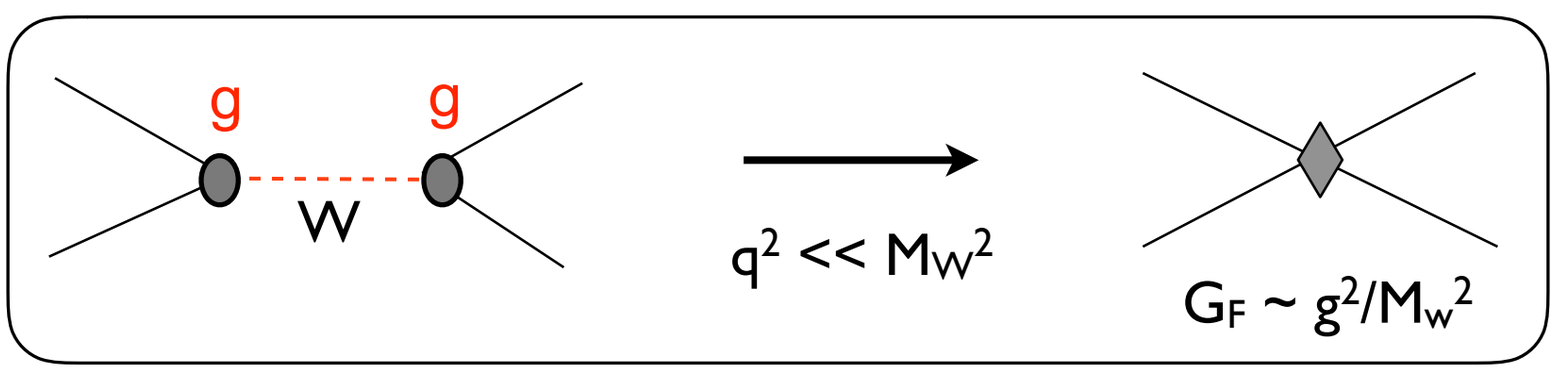
In this lecture we will cover the EFT analysis connecting physics between the new physics scale Λ and the hadronic scale $\Lambda_{had} \sim 1$ GeV

The low-energy footprints of $\mathcal{L}_{BSM}$

- At energy $E_{exp} << M_{BSM}$, new particles can be "integrated out"
- Generate new local operators with coefficients ~ $g^k/(M_{BSM})^n$



Familiar example:



Effective Field Theory emerges as a natural framework to analyze low-E implications of classes of BSM scenarios *and* inform model building

Why use EFT for new physics

- General framework encompassing classes of models
- Efficient and rigorous tool to analyze experiments at different scales (from collider to table-top)
- The steps below UV matching apply to *all* models: can be done once and for all
- Very useful diagnosing tool in this "pre-discovery" phase :)
- Inform model building (success story is SM itself**)

EFT and UV models approaches are not mutually exclusive

**EFT for β decays and the making of the Standard Model

Fermi, 1934

Current-current,
parity conserving

Lee and Yang, 1956

Parity conserving:
VV, AA, SS, TT ...
Parity violating: VA, SP, ...

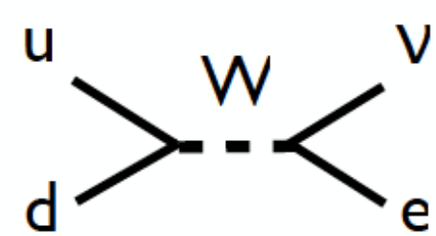
Feynman & Gell-Mann,
1958
Marshak & Sudarshan

Glashow,
Salam,
Weinberg

Sheldon Lee Glashow — Abdus Salam — Steven Weinberg

It's (V-A)*(V-A) !!

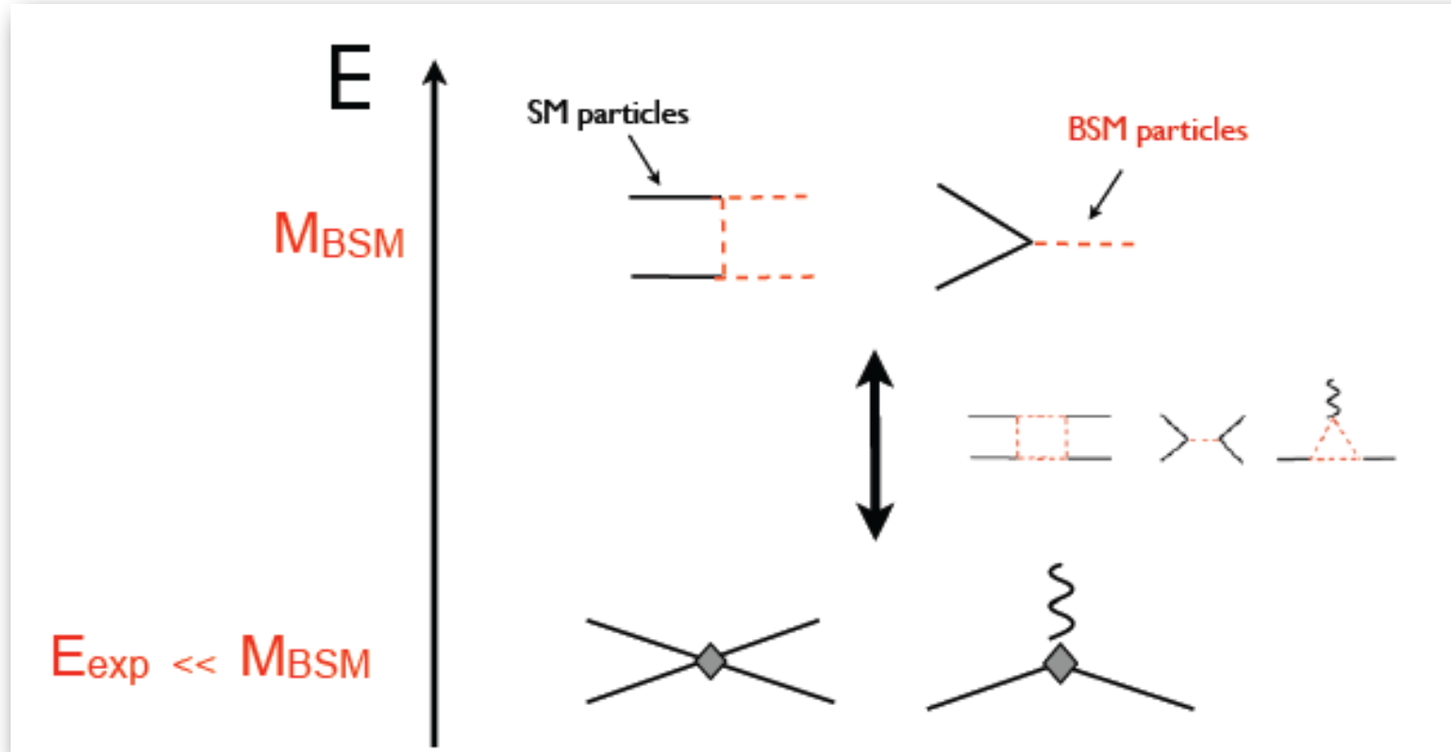
"V-A was the key"
S. Weinberg



Embed in non-abelian chiral gauge theory,
predict neutral currents

EFT framework

- Assume mass gap
  $M_{BSM} > G_F^{-1/2} \sim v_{EW}$
- Degrees of freedom:
  SM fields (+ possibly ν_R)
- Symmetries: SM gauge group; no flavor, CP, B, L



- EFT expansion in E/M_{BSM}, M_W/M_{BSM} [$O_i^{(d)}$ built out of SM fields]

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

[$\Lambda \leftrightarrow M_{BSM}$] $C_i\ [g_{\rm BSM},\ M_a/M_b]$

Guided tour of $\mathcal{L}_{\text{eff}}$

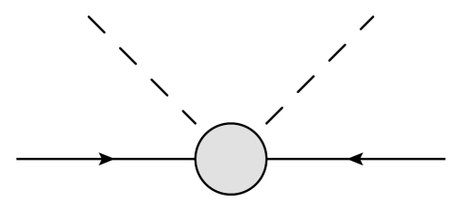
$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

Weinberg 1979

- Dim 5: only one operator

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \qquad \ell = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad \hat{O}_{\text{dim=5}} = \ell^T C \epsilon \varphi \; \varphi^T \epsilon \ell$$

$$C = i\gamma_2\gamma_0$$
$$\epsilon = i\sigma_2$$



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$$C = i\gamma_2\gamma_0$$
$$\epsilon = i\sigma_2$$

- Violates total lepton number $\ell \to e^{i\alpha}\ell \qquad e \to e^{i\alpha} e$
- Generates Majorana mass for L-handed neutrinos (after EWSB)

$$\frac{1}{\Lambda}\hat{O}_{\text{dim=5}} \xrightarrow{\langle\varphi\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}} \frac{v^2}{\Lambda}\nu_L^T C \nu_L$$

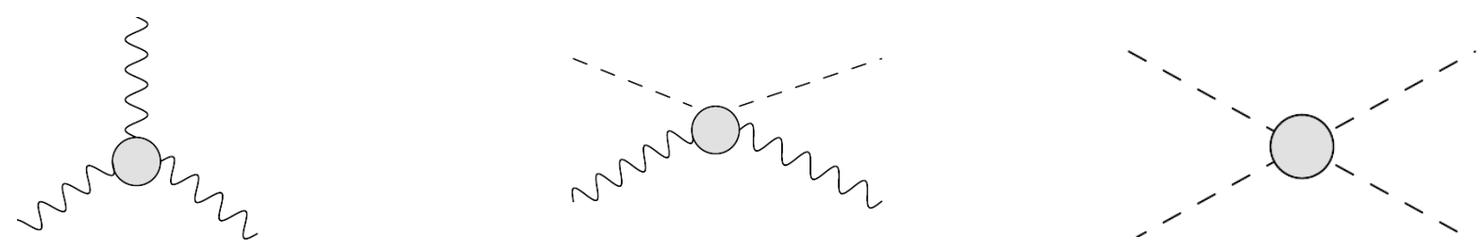
- "See-saw": $m_\nu \sim 1\,\text{eV} \;\to\; \Lambda \sim 10^{13}\,\text{GeV}$

Guided tour of $\mathcal{L}_{\text{eff}}$

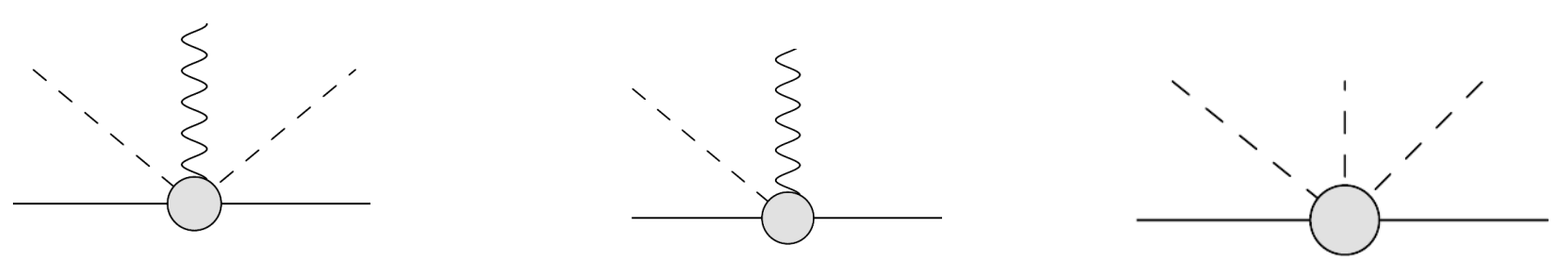
$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

- Dim 6: *many* structures (59, not including flavor)

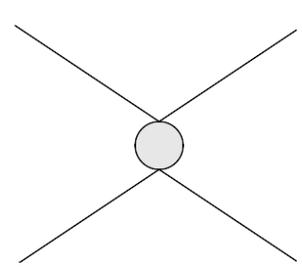
No fermions



Two fermions



Four fermions



Guided tour of $\mathcal{L}_{eff}$

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

- Dim 6: affect *many* processes
  - B violation
  - Gauge and Higgs boson couplings
  - EDMs, LFV, qFCNC, ...
  - g-2, Charged Currents, Neutral Currents, ...
- EFT used beyond tree-level: one-loop anomalous dimensions known

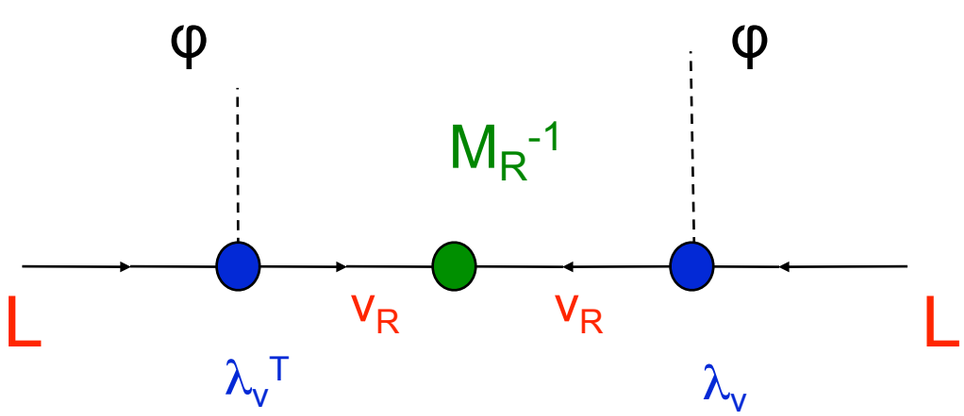
Weinberg 1979
Wilczek-Zee1979
Buchmuller-Wyler 1986, ...
Grzadkowski-Iskrzynksi-Misiak-Rosiek (2010)

Alonso, Jenkins, Manohar, Trott 2013

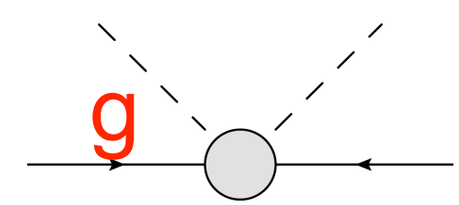
Examples of matching

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

- Explicit examples of "matching" from full model to EFT
- Dim 5: Heavy R-handed neutrino



$$\mathcal{L}_5 = g_{\alpha\beta}\, \ell_\alpha^T C \epsilon \varphi\, \varphi^T \epsilon \ell_\beta$$

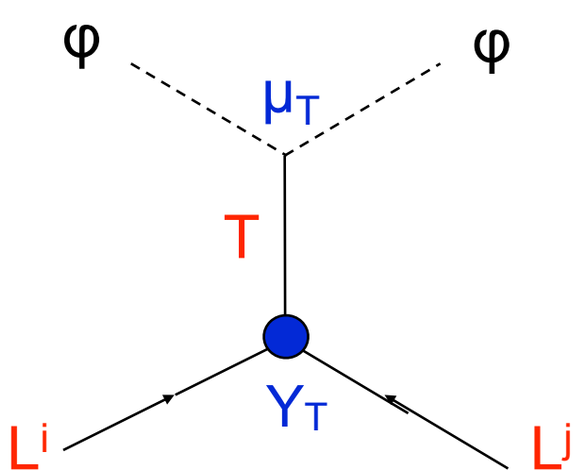


$$g \sim \lambda_\nu^T M_R^{-1} \lambda_\nu$$

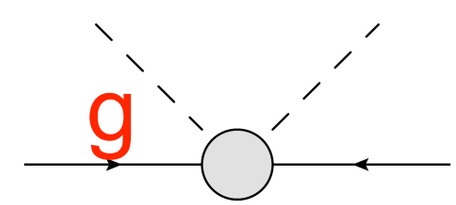
Examples of matching

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

- Explicit examples of "matching" from full model to EFT
- Dim 5: Triplet Higgs field



$$\mathcal{L}_5 = g_{\alpha\beta}\, \ell_\alpha^T C \epsilon \varphi\, \varphi^T \epsilon \ell_\beta$$



$g \sim \mu_T M_T^{-2} Y_T$

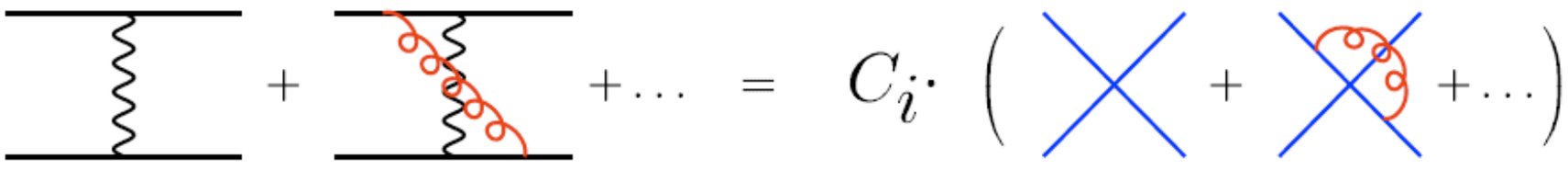
More on matching

- We just saw two simple examples of matching calculation in EFT:

$$A_{\text{full}} = \sum_i C_i \langle O_i \rangle \equiv A_{EFT}$$

★ To a given order in $E/M_{R,T}$, determine effective couplings (Wilson coefficients) from the matching condition $A_{\text{full}} = A_{\text{EFT}}$ with amplitudes involving "light" external states

★ We did matching at tree-level, but strong and electroweak higher order corrections can be included



Full theory

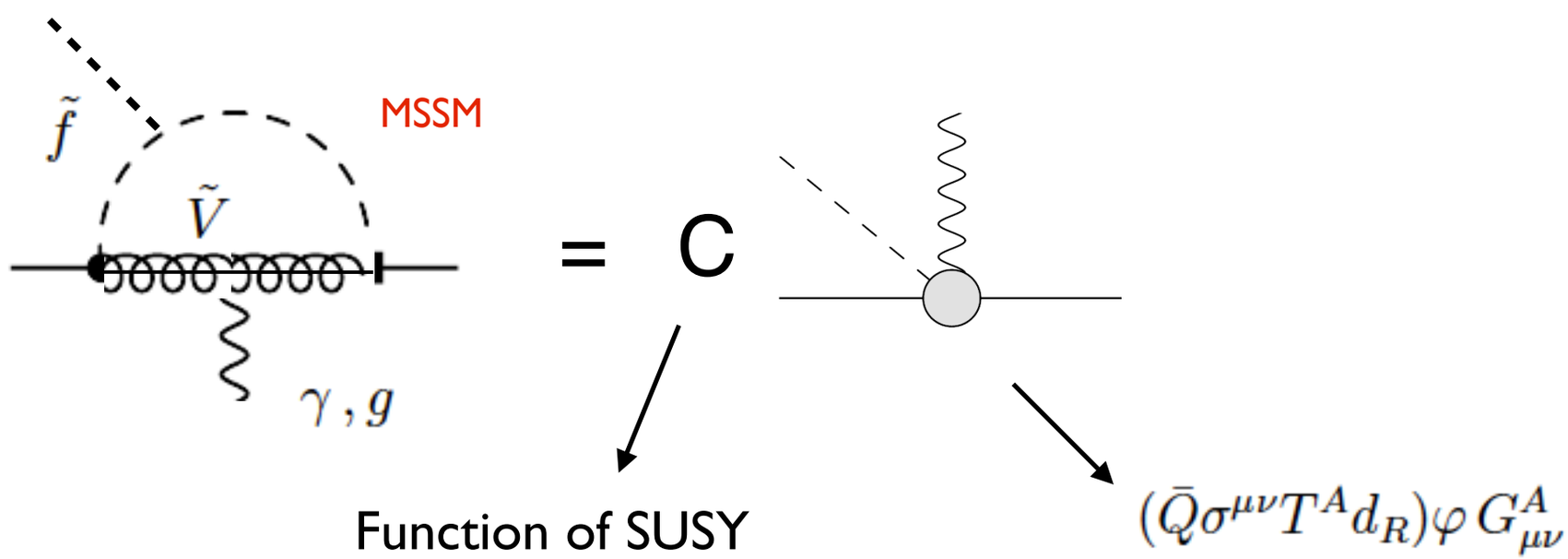
Effective theory

More on matching

- We just saw two simple examples of matching calculation in EFT:

$$A_{\text{full}} = \sum_i C_i \langle O_i \rangle \equiv A_{EFT}$$

★ In some cases A_{full} starts at loop level (highly relevant for EDMs)



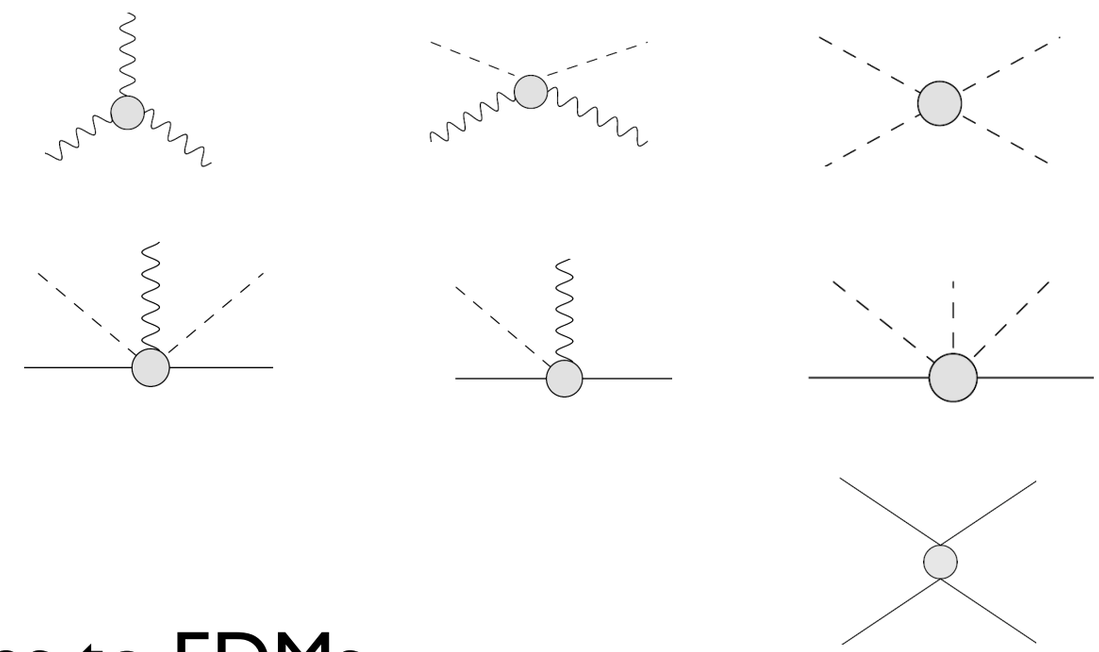
CP-violating operators contributing to EDMs:

from BSM scale to hadronic scale

Dim-6 CPV operators

- When including flavor indices, at dimension=6 there are 2499 independent couplings of which 1149 CP-violating !!

  Alonso et al.2014



- A large number of them contributes to EDMs

- Leading flavor-diagonal CP odd operators contributing to EDMs have been identified, neglecting 2nd and 3rd generation fermions**

  Dekens-DeVries 1303.3156 Engel, Ramsey-Musolf, Van Kolck 1303.2371

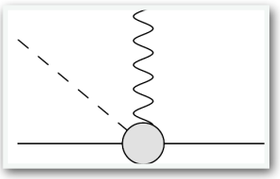
**Caveat: (i) strange quark can't really be ignored;
(ii) new physics could couple predominantly to heavy quarks;
(iii) flavor-changing operators can contribute to EDMs (multiple insertions)

High-scale effective Lagrangian

- CPV BSM dynamics dictated by:

Here follow notation of:
Engel, Ramsey-Musolf, Van Kolck
1303.2371

$$\mathcal{L}_{6,\mathrm{CPV}} = \frac{1}{\Lambda^2} \sum_i \alpha_i^{(6)} Q_i$$



Gauge-Higgs-Fermion	
Q_{uG}	$(\bar{Q}\sigma^{\mu\nu}T^A u_R)\widetilde{\varphi}\, G^A_{\mu\nu}$
Q_{dG}	$(\bar{Q}\sigma^{\mu\nu}T^A d_R)\varphi\, G^A_{\mu\nu}$
Q_{fW}	$(\bar{F}\sigma^{\mu\nu} f_R)\tau^I \Phi\, W^I_{\mu\nu}$
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$$\alpha^{(6)}_{fV_k} \equiv g_k C_{fV_k}$$

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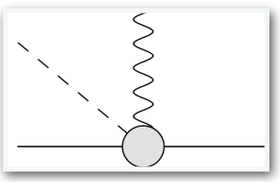

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$$\alpha^{(6)}_{fV_k} \equiv g_k C_{fV_k}$$

$\phi^T \to (0, v/\sqrt{2})$

$$\mathcal{L}^{\mathrm{CEDM}} = -i \sum_q \frac{g_3 \tilde{d}_q}{2}\, \bar{q}\sigma^{\mu\nu}T^A\gamma_5 q\; G^A_{\mu\nu}$$

$$\mathcal{L}^{\mathrm{EDM}} = -i \sum_f \frac{d_f}{2}\, \bar{f}\sigma^{\mu\nu}\gamma_5 f\; F_{\mu\nu}$$

non-relativistic limit

$$\boxed{\sum_f d_f\; \chi_f^\dagger \vec{\sigma}\chi_f \cdot \vec{E}}$$

Elementary fermion
(chromo)-electric dipole

High-scale effective Lagrangian

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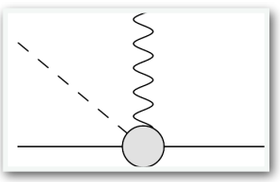

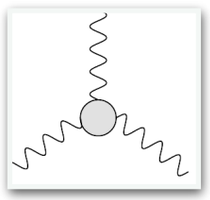

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$$\alpha^{(6)}_{fV_k} \equiv g_k C_{fV_k} \qquad \alpha^{(6)}_{\tilde{G}} \equiv g_3\, C_{\tilde{G}}$$

High-scale effective Lagrangian

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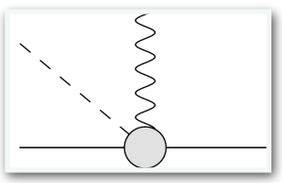
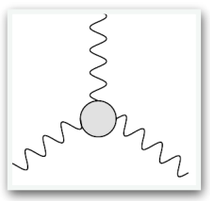
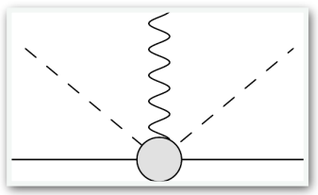

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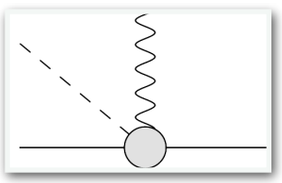

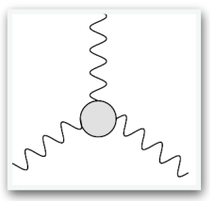

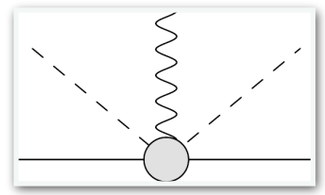

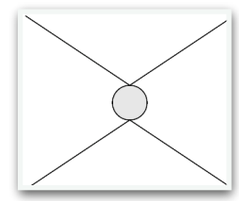

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$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$	
Q_{ledq}	$(\bar{L}^j e_R)(\bar{d}_R Q^j)$
$Q^{(1)}_{quqd}$	$(\bar{Q}^j u_R)\epsilon_{jk}(\bar{Q}^k d_R)$
$Q^{(8)}_{quqd}$	$(\bar{Q}^j T^A u_R)\epsilon_{jk}(\bar{Q}^k T^A d_R)$
$Q^{(1)}_{lequ}$	$(\bar{L}^j e_R)\epsilon_{jk}(\bar{Q}^k u_R)$
$Q^{(3)}_{lequ}$	$(\bar{L}^j \sigma_{\mu\nu} e_R)\epsilon_{jk}(\bar{Q}^k \sigma^{\mu\nu} u_R)$

$$\alpha^{(6)}_{fV_k} \equiv g_k C_{fV_k} \qquad \alpha^{(6)}_{\tilde{G}} \equiv g_3\, C_{\tilde{G}} \qquad \alpha^{(6)}_{\varphi ud} \equiv C_{\varphi ud}$$

$$\alpha^{(6)}_{\ell edq} \equiv C_{\ell edq}$$

$$\alpha^{(6)}_{\ell equ(1,3)} \equiv C^{(1,3)}_{\ell edu}$$

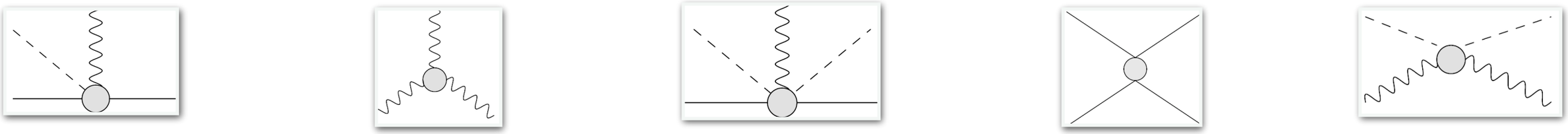
$$\alpha^{(6)}_{quqd(1,8)} \equiv g_3^2\, C^{(1,8)}_{quqd}$$

High-scale effective Lagrangian

- CPV BSM dynamics dictated by:

Here follow notation of:
Engel, Ramsey-Musolf, Van Kolck
1303.2371

$$\mathcal{L}_{6,\mathrm{CPV}} = \frac{1}{\Lambda^2} \sum_i \alpha_i^{(6)} Q_i$$



Gauge-Higgs-Fermion		Pure Gauge		
Q_{uG}	$(\bar{Q}\sigma^{\mu\nu}T^A u_R)\widetilde{\varphi}\, G^A_{\mu\nu}$	$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}^{A\nu}_{\mu} G^{B\rho}_{\nu} G^{C\mu}_{\rho}$	$Q_{\varphi ud} = i\left(\tilde{\varphi}^\dagger D_\mu \varphi\right)\bar{u}_R\gamma^\mu d_R$
Q_{dG}	$(\bar{Q}\sigma^{\mu\nu}T^A d_R)\varphi\, G^A_{\mu\nu}$	$Q_{\widetilde{W}}$	$\varepsilon^{IJK}\widetilde{W}^{I\nu}_{\mu} W^{J\rho}_{\nu} W^{K\mu}_{\rho}$	
Q_{fW}	$(\bar{F}\sigma^{\mu\nu} f_R)\tau^I \Phi\, W^I_{\mu\nu}$			
Q_{fB}	$(\bar{F}\sigma^{\mu\nu} f_R)\Phi\, B_{\mu\nu}$			

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		Gauge-Higgs	
Q_{ledq}	$(\bar{L}^j e_R)(\bar{d}_R Q^j)$	$Q_{\varphi\widetilde{G}}$	$\varphi^\dagger\varphi\, \widetilde{G}^A_{\mu\nu} G^{A\mu\nu}$
$Q^{(1)}_{quqd}$	$(\bar{Q}^j u_R)\epsilon_{jk}(\bar{Q}^k d_R)$	$Q_{\varphi\widetilde{W}}$	$\varphi^\dagger\varphi\, \widetilde{W}^I_{\mu\nu} W^{I\mu\nu}$
$Q^{(8)}_{quqd}$	$(\bar{Q}^j T^A u_R)\epsilon_{jk}(\bar{Q}^k T^A d_R)$	$Q_{\varphi\widetilde{B}}$	$\varphi^\dagger\varphi\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$
$Q^{(1)}_{lequ}$	$(\bar{L}^j e_R)\epsilon_{jk}(\bar{Q}^k u_R)$	$Q_{\varphi\widetilde{W}B}$	$\varphi^\dagger\tau^I\varphi\, \widetilde{W}^I_{\mu\nu} B^{\mu\nu}$
$Q^{(3)}_{lequ}$	$(\bar{L}^j \sigma_{\mu\nu} e_R)\epsilon_{jk}(\bar{Q}^k \sigma^{\mu\nu} u_R)$		

$$\alpha^{(6)}_{fV_k} \equiv g_k C_{fV_k} \qquad \alpha^{(6)}_{\tilde{G}} \equiv g_3\, C_{\tilde{G}} \qquad \alpha^{(6)}_{\varphi ud} \equiv C_{\varphi ud}$$

$$\alpha^{(6)}_{\ell edq} \equiv C_{\ell edq} \qquad \alpha^{(6)}_{\varphi\tilde{V}_k} \equiv g_k^2\, C_{\varphi\tilde{V}}$$

$$\alpha^{(6)}_{\ell equ(1,3)} \equiv C^{(1,3)}_{\ell edu}$$

$$\alpha^{(6)}_{quqd(1,8)} \equiv g_3^2\, C^{(1,8)}_{quqd}$$

Evolution to low-E: generalities

$$\mathcal{L}_{\text{eff}} = \sum_i C_i(\mu)\, Q_i(\mu)$$

1. Evolution of effective couplings with energy scale

- Operators in $\mathcal{L}_{\text{eff}}$ depend on the energy scale μ at which they are "renormalized" (i.e. the UV divergences are removed)
- To avoid large logs, μ should be of the order of the energy probed
- Physical results should not depend on the arbitrary scale
- The couplings C_i depend on μ in such a way to guarantee this!

$$\frac{d}{d(\ln \mu)} \left[C_i \, \langle Q_i \rangle \right] = 0$$

Evolution to low-E: generalities

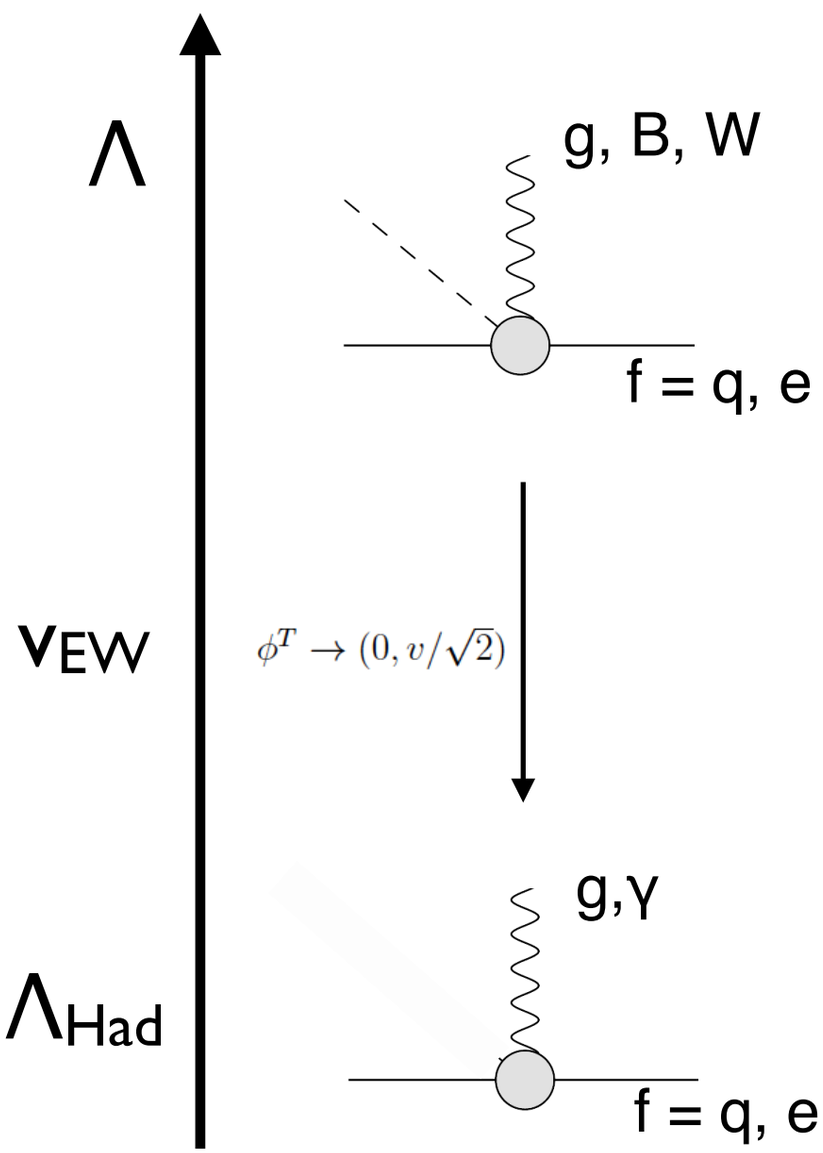
$$\mathcal{L}_{\text{eff}} = \sum_i C_i(\mu)\, Q_i(\mu)$$

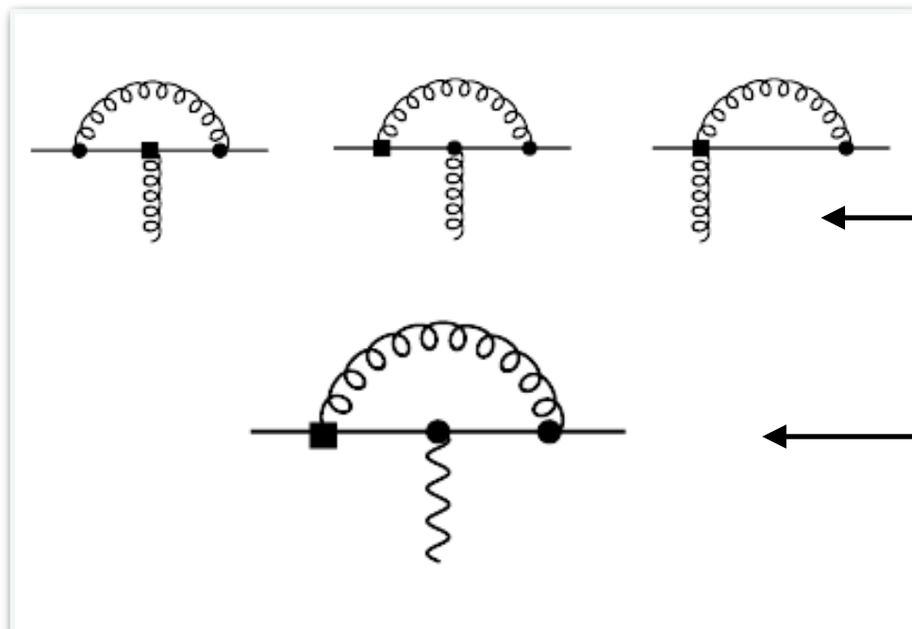
2. As one evolves the theory to low energy, need to remove ("integrate out") heavy particles

- In our case, in the evolution of $\mathcal{L}_{\text{eff}}$ we encounter the electroweak scale: remove top quark, Higgs, W, Z
- b and c quark thresholds

Dipole operators

Q_{uG}	$(\bar{Q}\sigma^{\mu\nu}T^A u_R)\widetilde{\varphi}\, G^A_{\mu\nu}$
Q_{dG}	$(\bar{Q}\sigma^{\mu\nu}T^A d_R)\varphi\, G^A_{\mu\nu}$
Q_{fW}	$(\bar{F}\sigma^{\mu\nu} f_R)\tau^I \Phi\, W^I_{\mu\nu}$
Q_{fB}	$(\bar{F}\sigma^{\mu\nu} f_R)\Phi\, B_{\mu\nu}$



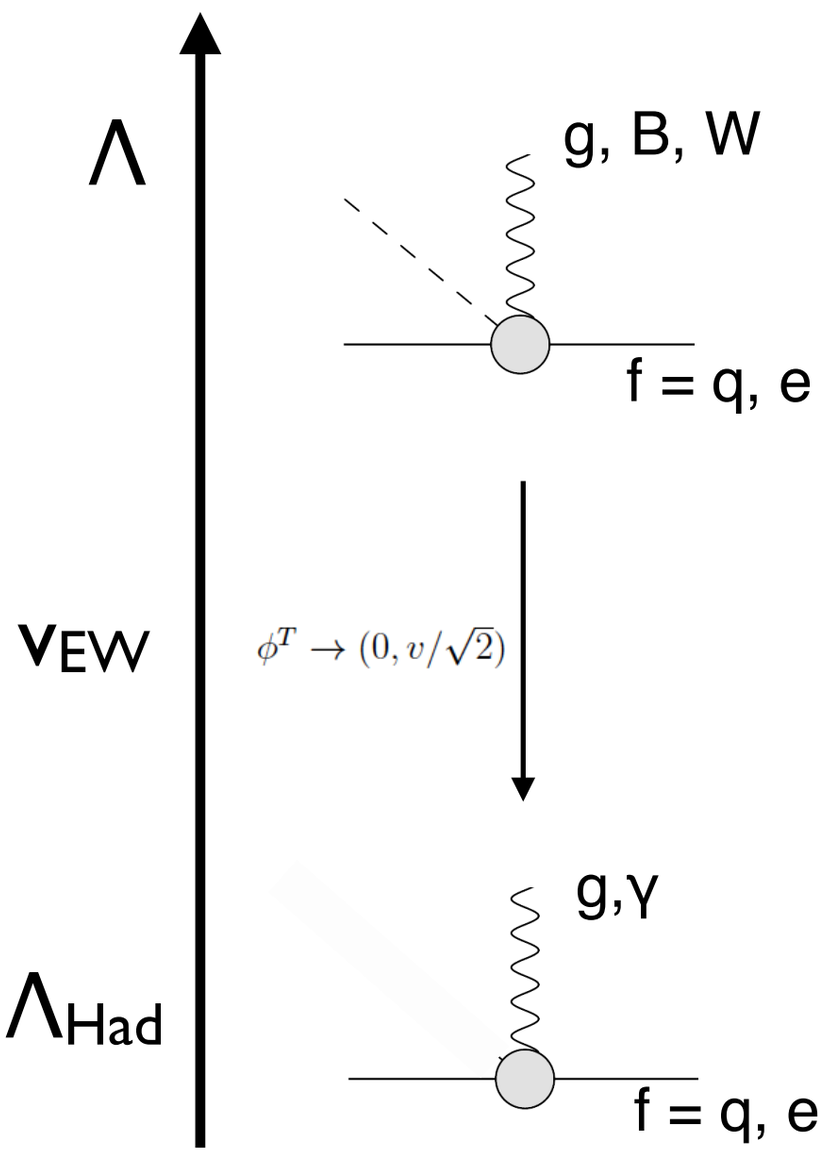


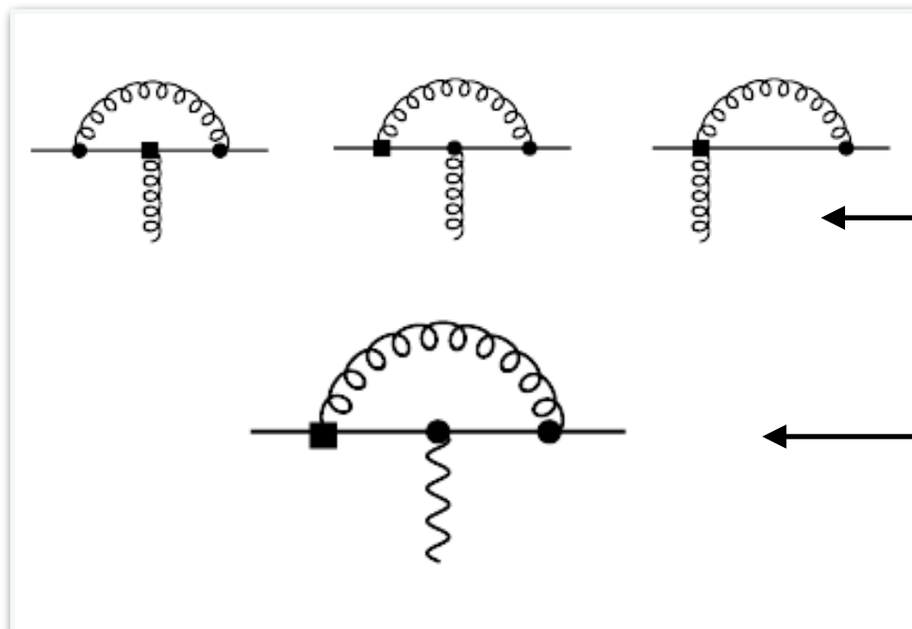
CEDM renormalization

CEDM mixing into EDM

Dipole operators

Q_{uG}	$(\bar{Q}\sigma^{\mu\nu}T^A u_R)\widetilde{\varphi}\, G^A_{\mu\nu}$
Q_{dG}	$(\bar{Q}\sigma^{\mu\nu}T^A d_R)\varphi\, G^A_{\mu\nu}$
Q_{fW}	$(\bar{F}\sigma^{\mu\nu} f_R)\tau^I \Phi\, W^I_{\mu\nu}$
Q_{fB}	$(\bar{F}\sigma^{\mu\nu} f_R)\Phi\, B_{\mu\nu}$





CEDM renormalization

CEDM mixing into EDM

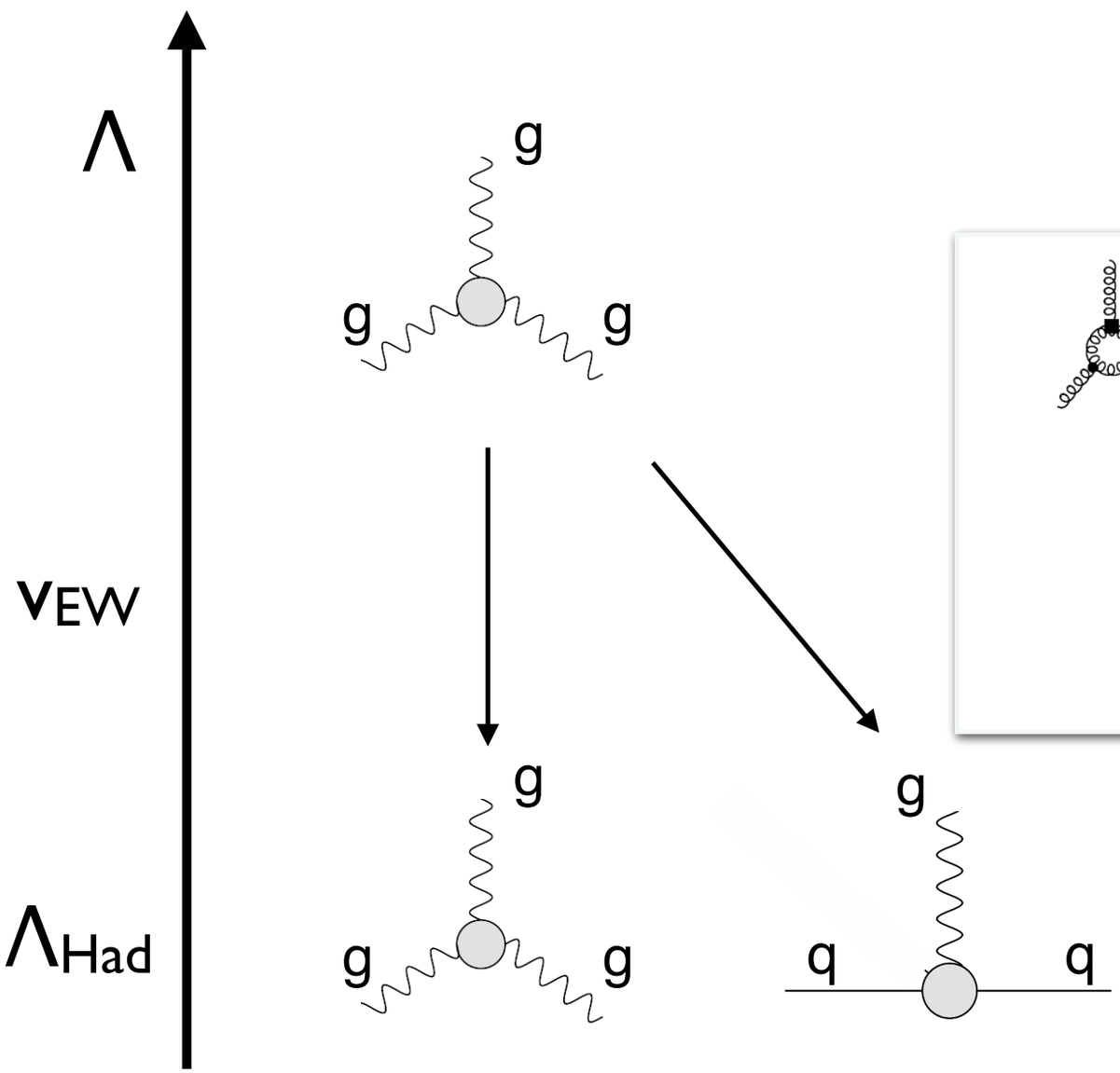
$$\tilde{d}_q = -\frac{\sqrt{2}}{v}\left(\frac{v}{\Lambda}\right)^2 \mathrm{Im}\, C_{qG}\,,$$

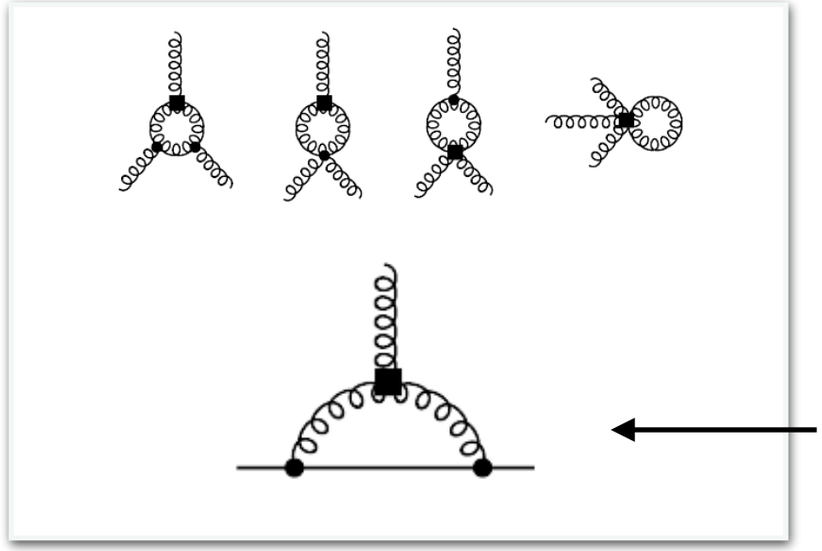
$$d_f = -\frac{\sqrt{2}e}{v}\left(\frac{v}{\Lambda}\right)^2 \mathrm{Im}\, C_{f\gamma}$$

$$\mathcal{L}^{\mathrm{CEDM}} = -i\sum_q \frac{g_3\tilde{d}_q}{2}\,\bar{q}\sigma^{\mu\nu}T^A\gamma_5 q\; G^A_{\mu\nu} \qquad \mathcal{L}^{\mathrm{EDM}} = -i\sum_f \frac{d_f}{2}\bar{f}\sigma^{\mu\nu}\gamma_5 f\; F_{\mu\nu}$$

Three gauge bosons

$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}^{A\nu}_{\mu}G^{B\rho}_{\nu}G^{C\mu}_{\rho}$



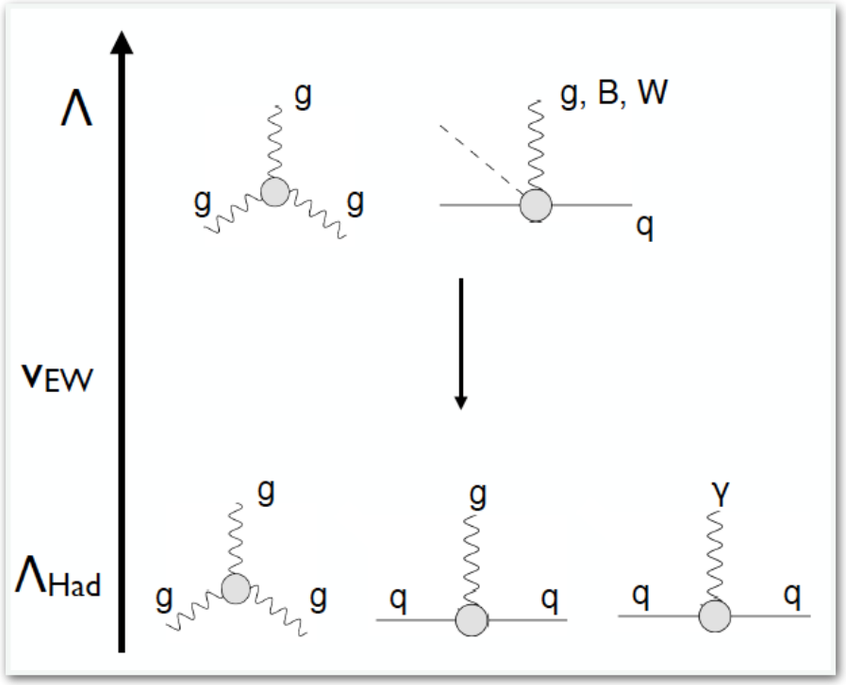


Weinberg mixing into CEDM

New structure at low-energy

$$\mathcal{L}^{\tilde{G}}_{\rm CPV} = \frac{g_3\, C_{\tilde{G}}}{\Lambda^2} f^{ABC}\widetilde{G}^{A\nu}_{\mu}G^{B\rho}_{\nu}G^{C\mu}_{\rho}$$

Dipole and three-gluon mixing



$$\mathcal{L}^{\rm EDM} = -i \sum_f \frac{d_f}{2} \bar{f} \sigma^{\mu\nu} \gamma_5 f \; F_{\mu\nu}$$

$$\mathcal{L}^{\rm CEDM} = -i \sum_q \frac{g_3 \tilde{d}_q}{2} \; \bar{q} \sigma^{\mu\nu} T^A \gamma_5 q \; G^A_{\mu\nu}$$

$$\mathcal{L}^{\tilde{\rm G}}_{\rm CPV} = \frac{g_3 \, C_{\tilde{G}}}{\Lambda^2} f^{ABC} \widetilde{G}^{A\nu}_{\mu} G^{B\rho}_{\nu} G^{C\mu}_{\rho}$$

Effect of mixing is important

Rosetta stone

$$d_q = -2 e m_q C_q$$
$$g_3 \tilde{d}_q = -2 m_q \tilde{C}_q$$
$$g_3 C_{\tilde{G}} = C_W$$

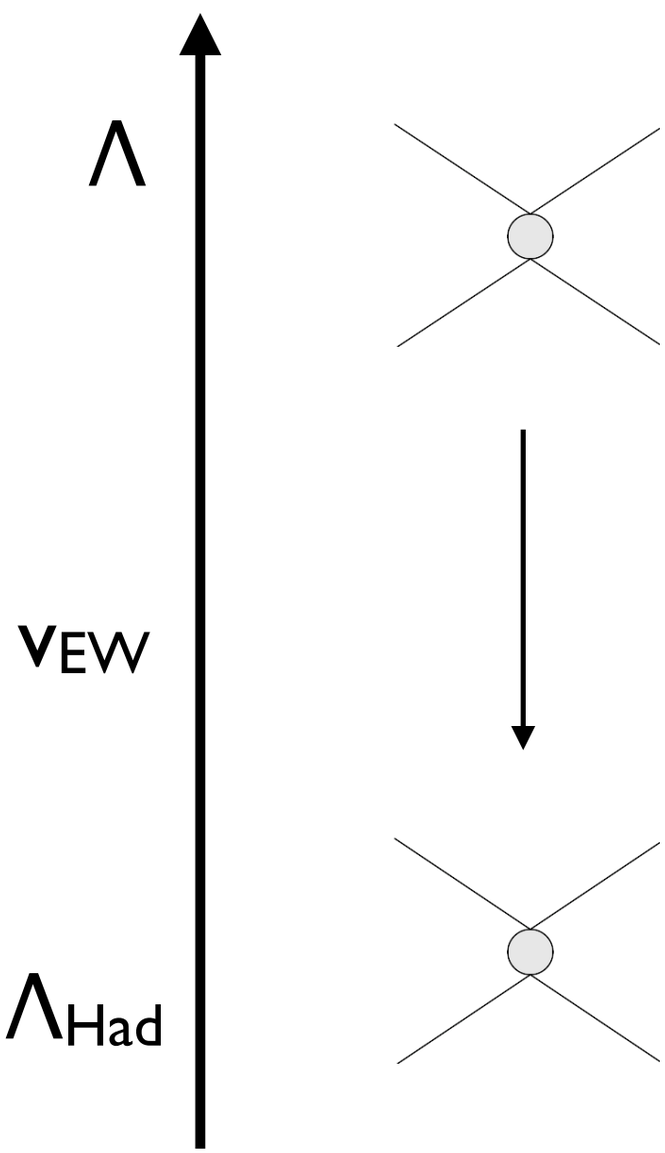
$$C_q(1\,\text{GeV}) = 0.39\, C_q(1\,\text{TeV}) + 0.37\, \tilde{C}_q(1\,\text{TeV}) - 0.072\, C_W(1\,\text{TeV})$$

$$\tilde{C}_q(1\,\text{GeV}) = + 0.88\, \tilde{C}_q(1\,\text{TeV}) - 0.29\, C_W(1\,\text{TeV})$$

$$C_W(1\,\text{GeV}) = + 0.33\, C_W(1\,\text{TeV})$$

Dekens-DeVries 1303.3156

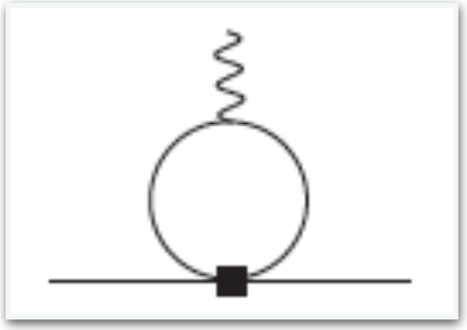
Four fermion operators (1)



Q_{ledq}	$(\bar{L}^j e_R)(\bar{d}_R Q^j)$
$Q_{quqd}^{(1)}$	$(\bar{Q}^j u_R)\epsilon_{jk}(\bar{Q}^k d_R)$
$Q_{quqd}^{(8)}$	$(\bar{Q}^j T^A u_R)\epsilon_{jk}(\bar{Q}^k T^A d_R)$
$Q_{lequ}^{(1)}$	$(\bar{L}^j e_R)\epsilon_{jk}(\bar{Q}^k u_R)$
$Q_{lequ}^{(3)}$	$(\bar{L}^j \sigma_{\mu\nu} e_R)\epsilon_{jk}(\bar{Q}^k \sigma^{\mu\nu} u_R)$

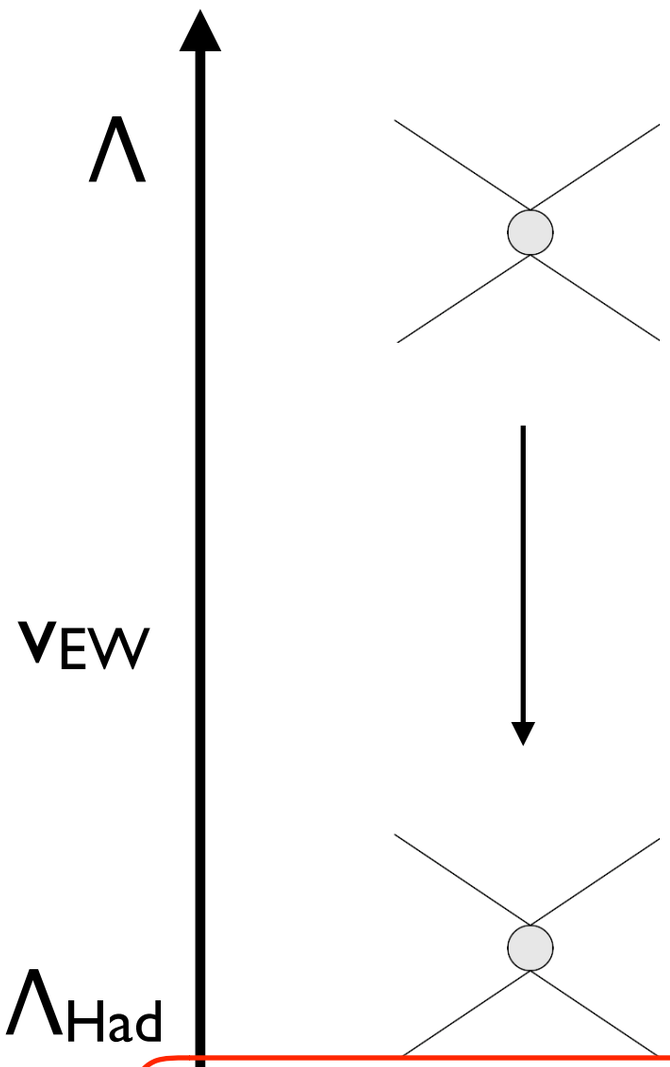
"Diagonal" QCD evolution of scalar and tensor quark bilinears

$$\mathcal{L}_{\rm CPV}^{\rm eq} = i\frac{{\rm Im}C_{\ell edq}}{2\Lambda^2}\left[\bar{e}\gamma_5 e\ \bar{d}d - \bar{e}e\ \bar{d}\gamma_5 d\right] - i\frac{{\rm Im}C_{\ell equ}^{(1)}}{2\Lambda^2}\left[\bar{e}\gamma_5 e\ \bar{u}u + \bar{e}e\ \bar{u}\gamma_5 u\right] - \frac{{\rm Im}C_{\ell equ}^{(3)}}{2\Lambda^2}\ \epsilon_{\mu\nu\alpha\beta}\ \bar{e}\sigma^{\mu\nu}e\ \bar{u}\sigma^{\alpha\beta}u$$

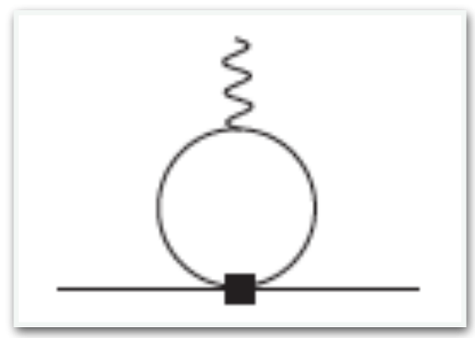


$Q_{lequ}^{(3)}$ mixes into lepton dipoles

Four fermion operators (2)



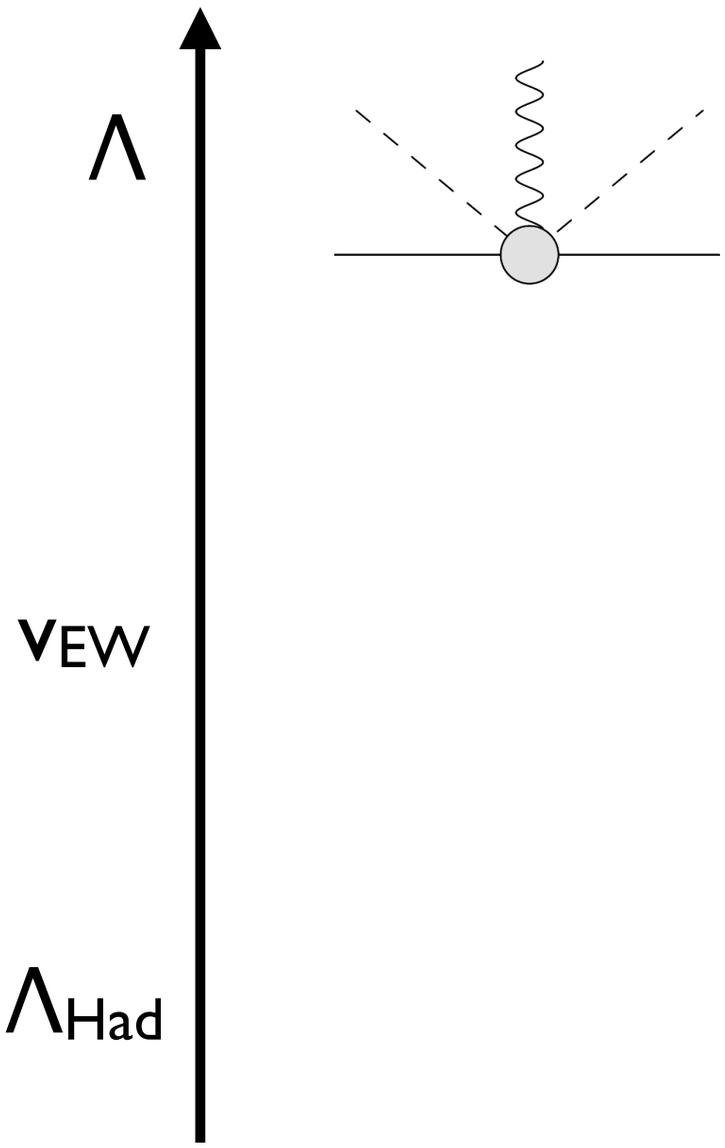
Q_{ledq}	$(\bar{L}^j e_R)(\bar{d}_R Q^j)$
$Q^{(1)}_{quqd}$	$(\bar{Q}^j u_R)\epsilon_{jk}(\bar{Q}^k d_R)$
$Q^{(8)}_{quqd}$	$(\bar{Q}^j T^A u_R)\epsilon_{jk}(\bar{Q}^k T^A d_R)$
$Q^{(1)}_{lequ}$	$(\bar{L}^j e_R)\epsilon_{jk}(\bar{Q}^k u_R)$
$Q^{(3)}_{lequ}$	$(\bar{L}^j \sigma_{\mu\nu} e_R)\epsilon_{jk}(\bar{Q}^k \sigma^{\mu\nu} u_R)$



4-quark operators mix among themselves and into quark dipoles

$$\mathcal{L}^{qq}_{\rm CPV} = i\frac{g_3^2\,{\rm Im}C^{(1)}_{quqd}}{2\Lambda^2}\left[\bar{u}\gamma_5 u\;\bar{d}d + \bar{u}u\;\bar{d}\gamma_5 d - \bar{d}\gamma_5 u\;\bar{u}d - \bar{d}u\;\bar{u}\gamma_5 d\right]$$
$$+i\frac{g_3^2\,{\rm Im}C^{(8)}_{quqd}}{2\Lambda^2}\left[\bar{u}\gamma_5 T^A u\;\bar{d}T^A d + \bar{u}T^A u\;\bar{d}\gamma_5 T^A d - \bar{d}\gamma_5 T^A u\;\bar{u}T^A d - \bar{d}T^A u\;\bar{u}\gamma_5 T^A d\right]$$

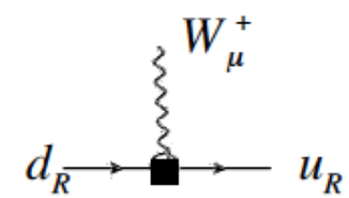
Induced 4-quark operator



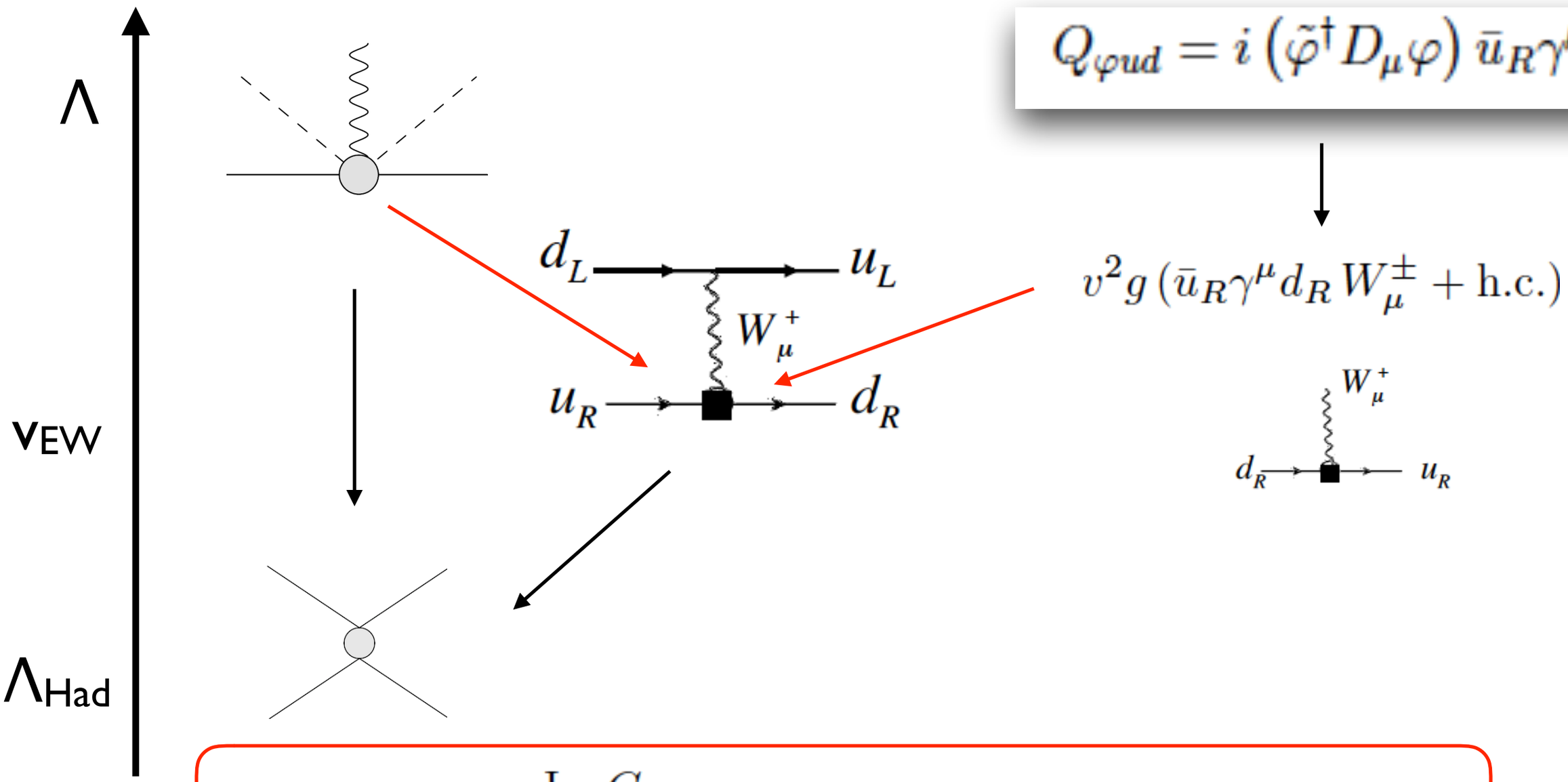
$$Q_{\varphi ud} = i\left(\tilde{\varphi}^\dagger D_\mu \varphi\right)\bar{u}_R \gamma^\mu d_R$$

$$\downarrow$$

$$v^2 g\left(\bar{u}_R \gamma^\mu d_R W_\mu^\pm + \text{h.c.}\right)$$



Induced 4-quark operator



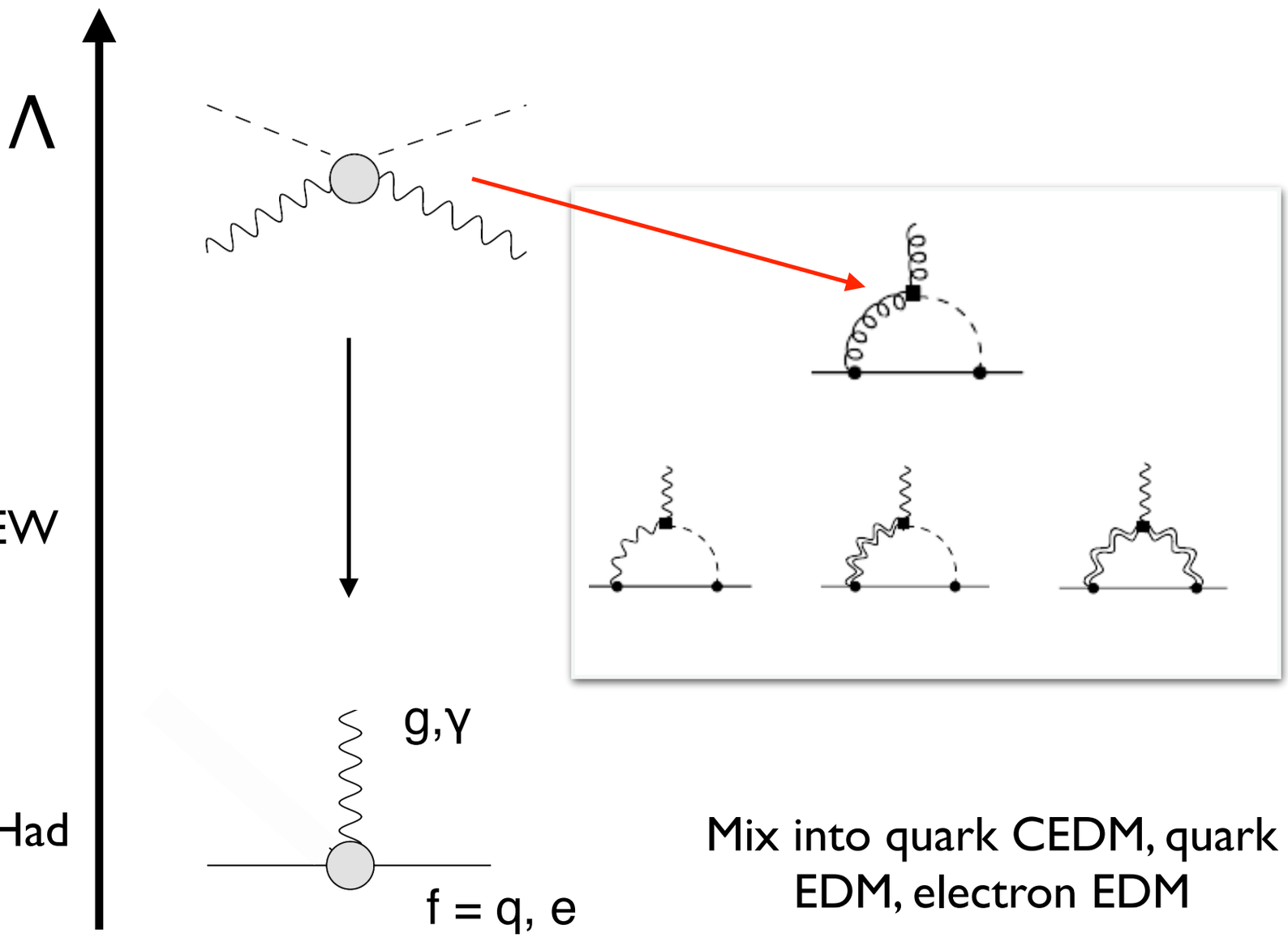
$$Q_{\varphi ud} = i\left(\tilde{\varphi}^\dagger D_\mu \varphi\right)\bar{u}_R\gamma^\mu d_R$$

$$v^2 g\left(\bar{u}_R\gamma^\mu d_R W_\mu^\pm + \text{h.c.}\right)$$

$$\mathcal{L}^{\text{eff}}_{\text{LR, CPV}} = -i\frac{\text{Im}\, C_{\varphi ud}}{\Lambda^2}\left[\bar{d}_L\gamma^\mu u_L\,\bar{u}_R\gamma_\mu d_R - \bar{u}_L\gamma^\mu d_L\,\bar{d}_R\gamma_\mu u_R\right]$$

+ color-mixed structure induced by QCD corrections

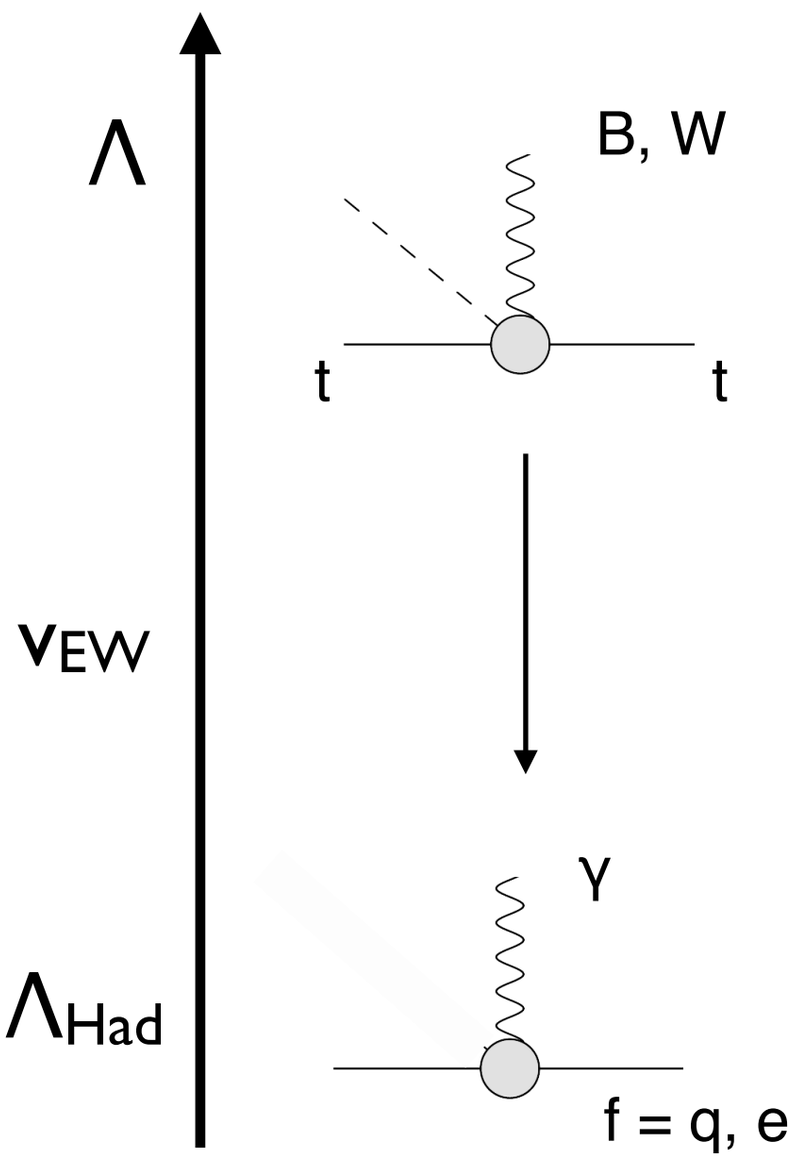
Gauge-Higgs operators



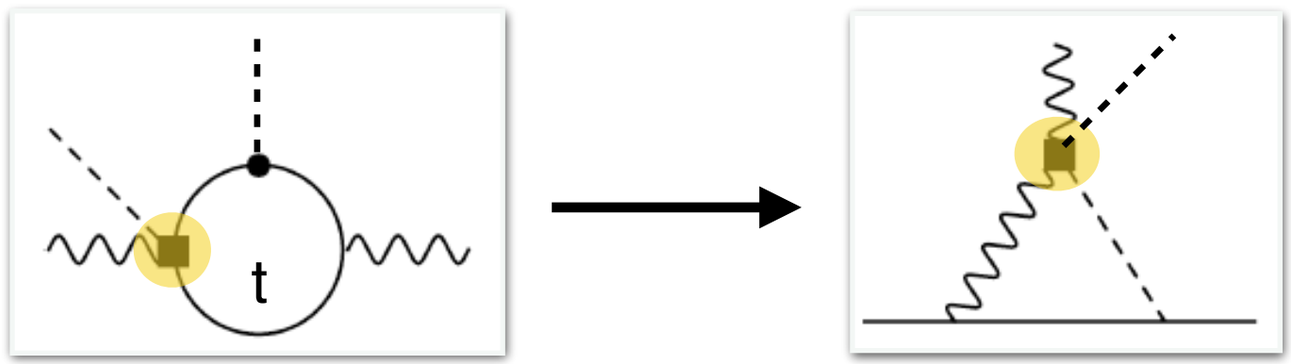
$Q_{\varphi\widetilde{G}}$	$\varphi^\dagger\varphi\,\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}$
$Q_{\varphi\widetilde{W}}$	$\varphi^\dagger\varphi\,\widetilde{W}^I_{\mu\nu}W^{I\mu\nu}$
$Q_{\varphi\widetilde{B}}$	$\varphi^\dagger\varphi\,\widetilde{B}_{\mu\nu}B^{\mu\nu}$
$Q_{\varphi\widetilde{W}B}$	$\varphi^\dagger\tau^I\varphi\,\widetilde{W}^I_{\mu\nu}B^{\mu\nu}$

Mix into quark CEDM, quark EDM, electron EDM

… and more



For example: top quark electroweak dipoles induce at two loops electron and quark EDMs — strongest constraints (by three orders of magnitude) !



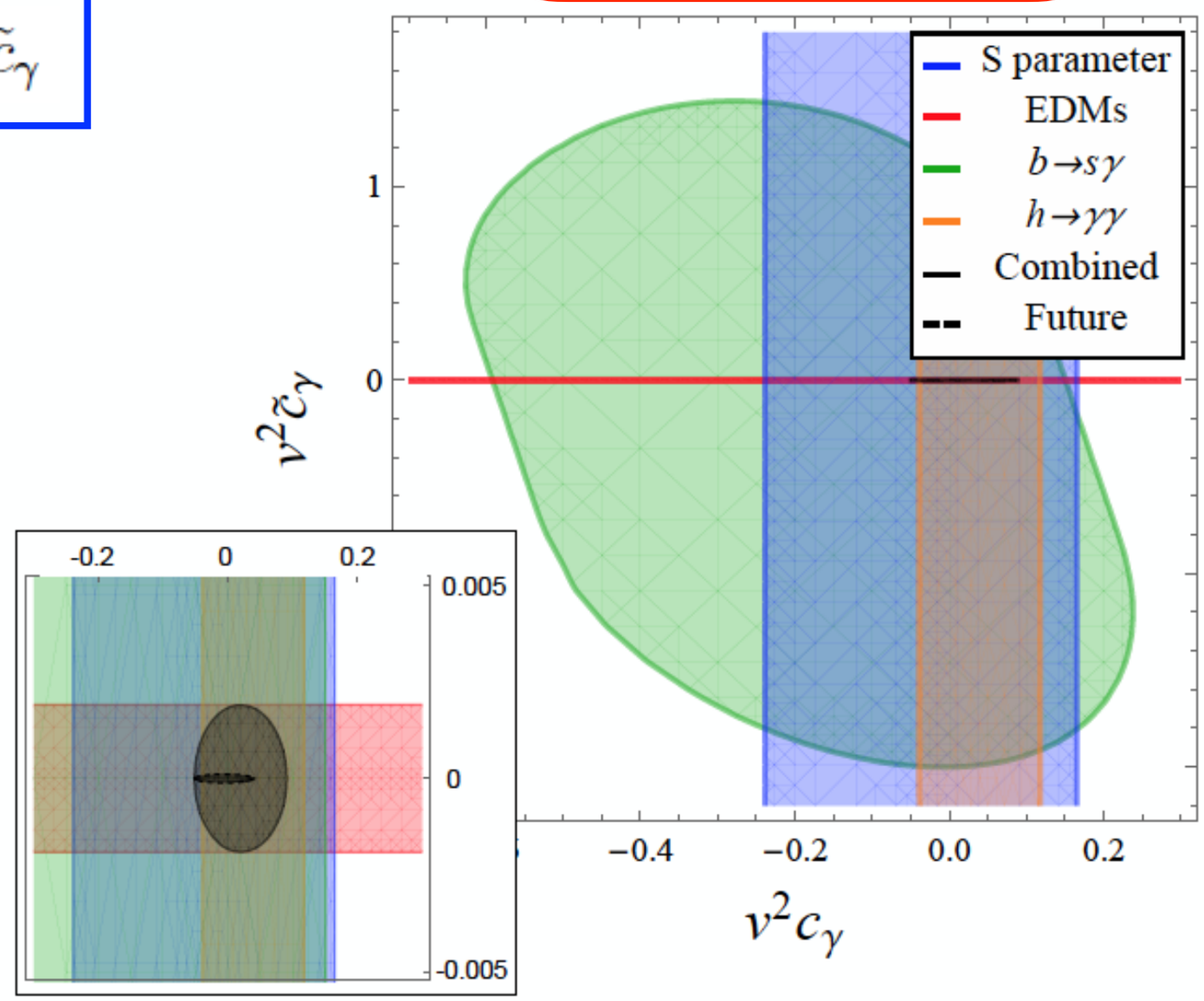
VC, W. Dekens, J. de Vries, E. Mereghetti 1603.03049 , 1605.04311

… and more

- EDM physics reach vs flavor and collider probes

$$\mu_t = eQ_t m_t c_\gamma$$
$$d_t = eQ_t m_t \tilde{c}_\gamma$$

$C_\gamma = c_\gamma + i\,\tilde{c}_\gamma$



Bound on top EDM improved by three orders of magnitude: $|d_t| < 5 \times 10^{-20}$ e cm

Dominated by eEDM

LHC sensitivity ($pp \to$ jet $t\,\gamma$) and LHeC $d_t \sim 10^{-17}$ e cm

[Fael-Gehrmann 13, Bouzas-Larios 13]

VC, W. Dekens, J. de Vries, E. Mereghetti 1603.03049

Low-energy effective Lagrangian

- When the dust settles, at the hadronic scale we have:

$$\mathcal{L}_6^{CPV} = -\frac{i}{2}\sum_{f=e,u,d,s} d_f\, \bar{f}\sigma\cdot F\gamma_5 f - \frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q\, g_s\, \bar{q}\sigma\cdot G\gamma_5 q + d_W \frac{g_s}{6} G\tilde{G}G + \sum_i C_i^{(4f)} O_i^{(4f)}$$

Low-energy effective Lagrangian

- When the dust settles, at the hadronic scale we have:

$$\mathcal{L}_6^{CPV} = -\frac{i}{2}\sum_{f=e,u,d,s} d_f\, \bar{f}\sigma\cdot F\gamma_5 f - \frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q\, g_s\, \bar{q}\sigma\cdot G\gamma_5 q + d_W \frac{g_s}{6} G\tilde{G}G + \sum_i C_i^{(4f)} O_i^{(4f)}$$

Electric and chromo-electric dipoles of fermions

$$d_f, \tilde{d}_q \sim \frac{v_{ew}}{\Lambda^2}$$

$\mathbf{J}\cdot\mathbf{E}$ $\mathbf{J}\cdot\mathbf{E_c}$

Low-energy effective Lagrangian

- When the dust settles, at the hadronic scale we have:

$$\mathcal{L}_6^{CPV} = -\frac{i}{2}\sum_{f=e,u,d,s} d_f\, \bar{f}\sigma\cdot F\gamma_5 f - \frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q\, g_s\, \bar{q}\sigma\cdot G\gamma_5 q + d_W \frac{g_s}{6} G\tilde{G}G + \sum_i C_i^{(4f)} O_i^{(4f)}$$

Electric and chromo-electric dipoles of fermions

$$d_f, \tilde{d}_q \sim \frac{v_{ew}}{\Lambda^2}$$

$\mathbf{J}\cdot\mathbf{E}$ $\mathbf{J}\cdot\mathbf{E_c}$

Gluon chromo-EDM (Weinberg operator)

$$d_W \sim \frac{1}{\Lambda^2}$$

Low-energy effective Lagrangian

- When the dust settles, at the hadronic scale we have:

$$\mathcal{L}_6^{CPV} = -\frac{i}{2}\sum_{f=e,u,d,s} d_f\, \bar{f}\sigma\cdot F\gamma_5 f - \frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q\, g_s\, \bar{q}\sigma\cdot G\gamma_5 q + d_W \frac{g_s}{6} G\tilde{G}G + \sum_i C_i^{(4f)} O_i^{(4f)}$$

Electric and chromo-electric dipoles of fermions

$$d_f, \tilde{d}_q \sim \frac{v_{ew}}{\Lambda^2}$$

$\mathbf{J}\cdot\mathbf{E}$ $\mathbf{J}\cdot\mathbf{E_c}$

Gluon chromo-EDM (Weinberg operator)

$$d_W \sim \frac{1}{\Lambda^2}$$

Semi-leptonic (3) and four-quark (2 “SP” + 2 “LR”)

Their form (and number) is strongly constrained by SU(2) gauge invariance

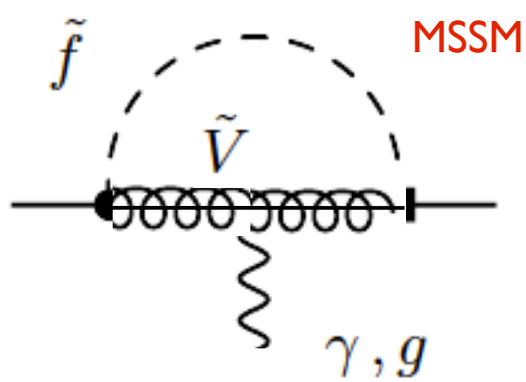
Explicit form of operators given in previous slides

Low-energy effective Lagrangian

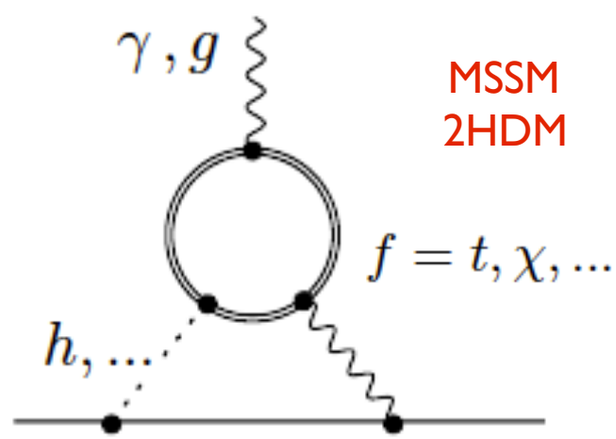
- When the dust settles, at the hadronic scale we have:

$$\mathcal{L}_6^{CPV} = -\frac{i}{2}\sum_{f=e,u,d,s} d_f\, \bar{f}\sigma\cdot F\gamma_5 f - \frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q\, g_s\, \bar{q}\sigma\cdot G\gamma_5 q + d_W \frac{g_s}{6} G\tilde{G}G + \sum_i C_i^{(4f)} O_i^{(4f)}$$

- Generated by a variety of BSM scenarios



MSSM



MSSM
2HDM

→ Quark EDM and chromo-EDM

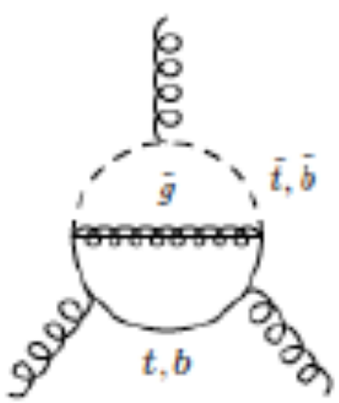
See Lecture IV for detailed discussion

Low-energy effective Lagrangian

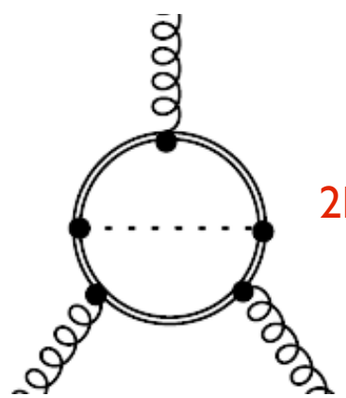
- When the dust settles, at the hadronic scale we have:

$$\mathcal{L}_6^{CPV} = -\frac{i}{2}\sum_{f=e,u,d,s} d_f\, \bar{f}\sigma\cdot F\gamma_5 f - \frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q\, g_s\, \bar{q}\sigma\cdot G\gamma_5 q + d_W \frac{g_s}{6} G\tilde{G}G + \sum_i C_i^{(4f)} O_i^{(4f)}$$

- Generated by a variety of BSM scenarios



MSSM



2HDM

Weinberg operator

See Lecture IV for detailed discussion

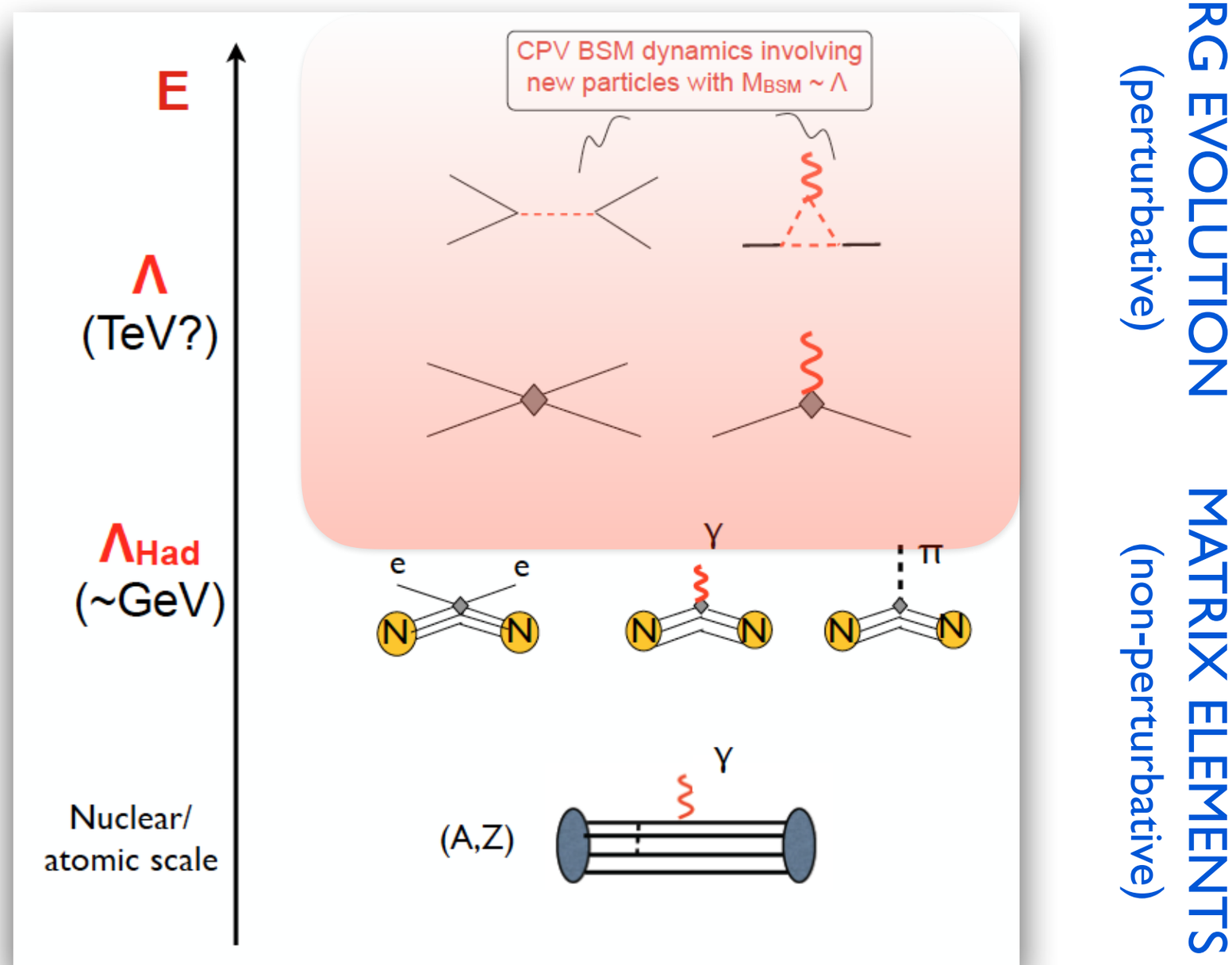
Low-energy effective Lagrangian

- When the dust settles, at the hadronic scale we have:

$$\mathcal{L}_6^{CPV} = -\frac{i}{2}\sum_{f=e,u,d,s} d_f\, \bar{f}\sigma\cdot F\gamma_5 f - \frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q\, g_s\, \bar{q}\sigma\cdot G\gamma_5 q + d_W \frac{g_s}{6} G\tilde{G}G + \sum_i C_i^{(4f)} O_i^{(4f)}$$

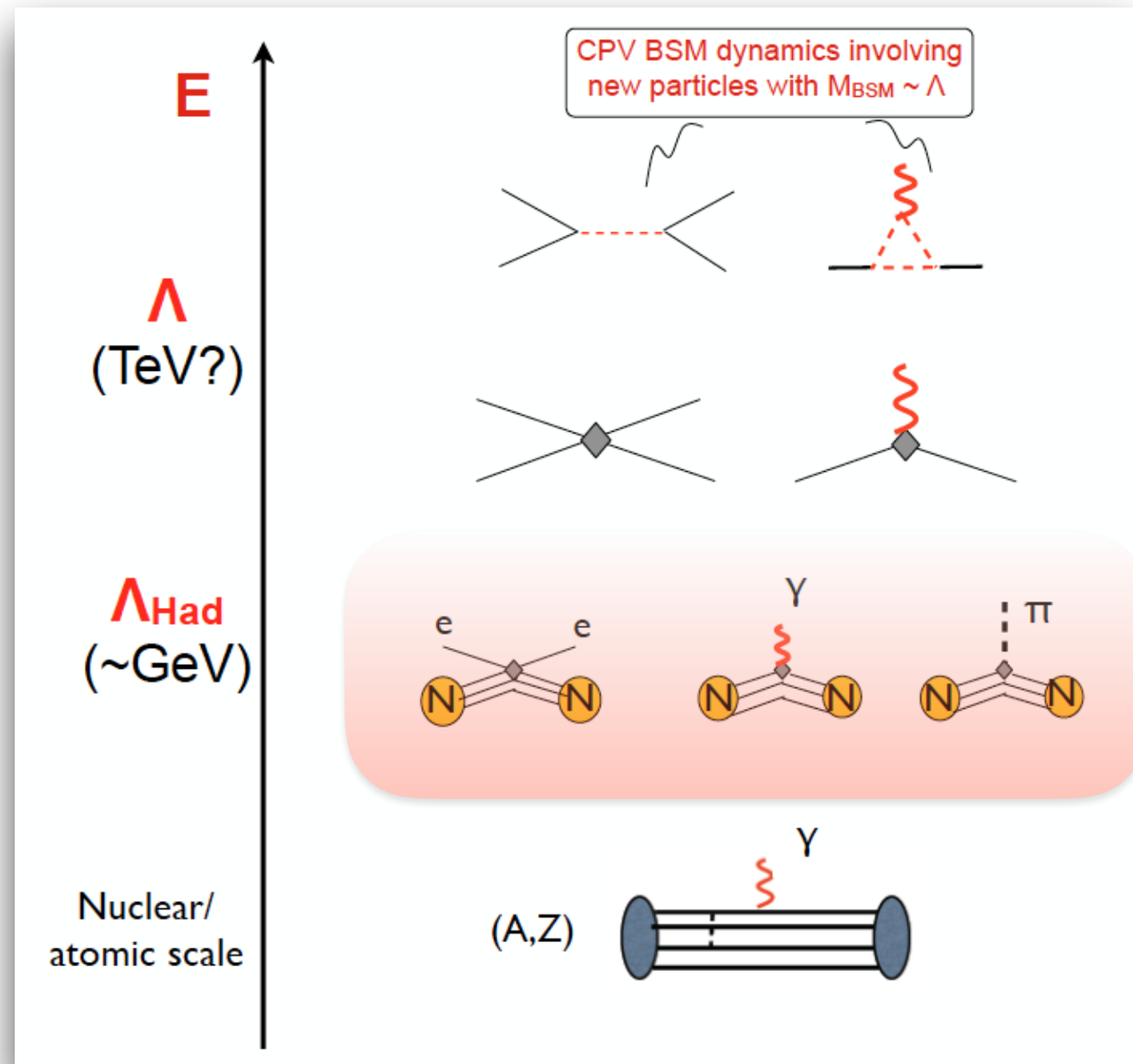
- Important points:
  - Each BSM scenario generate its own pattern of operators (and hence of EDM "signatures")
  - Within a model, relative importance of operators depends on various parameters (masses, etc)
  - So, in a post-discovery scenario, a combination of EDMs will allow us to learn about underlying sources of CP violation

But we are not done yet…



Multi-scale problem: need RG evolution of effective couplings & hadronic / nuclear / molecular calculations of matrix elements

But we are not done yet…

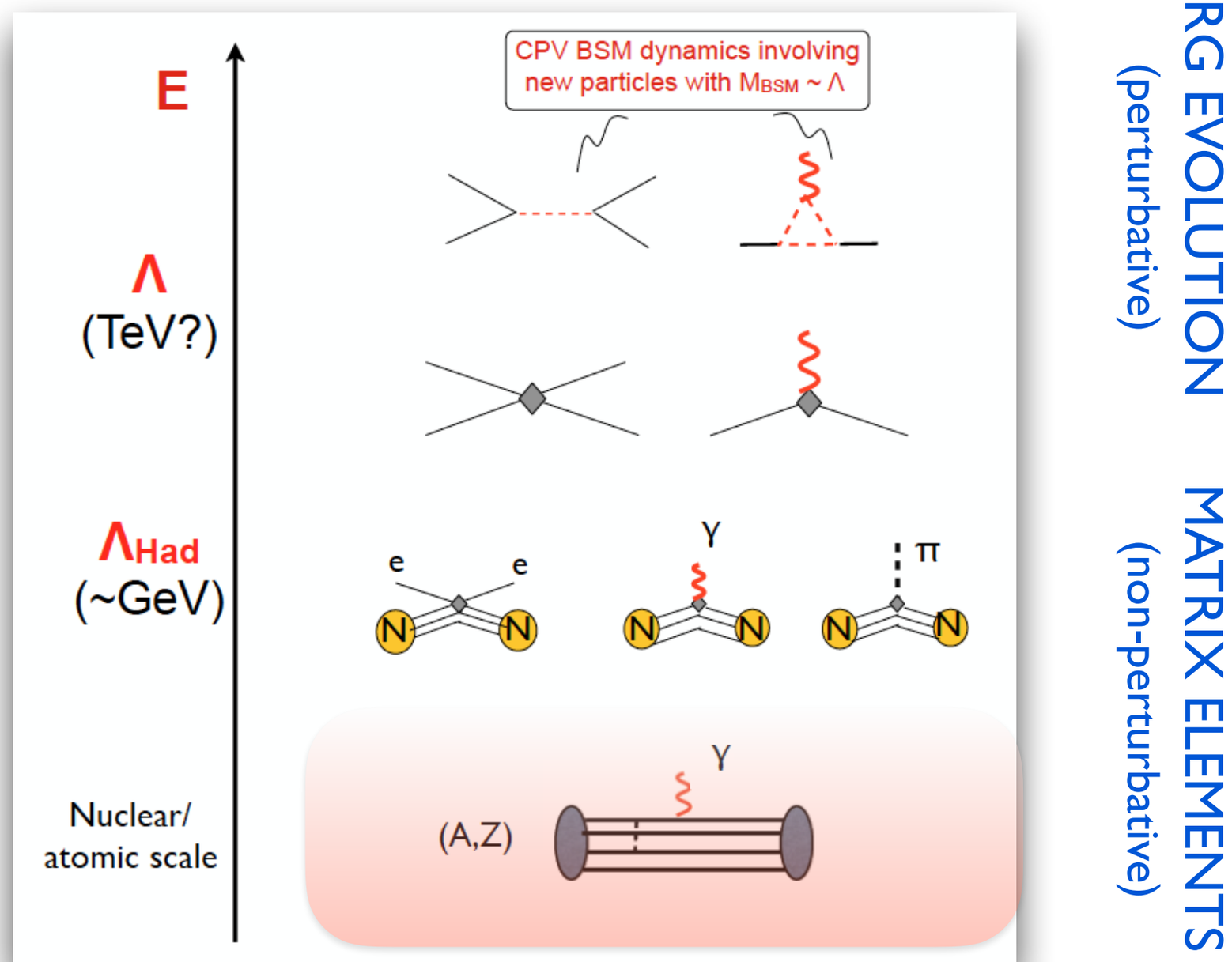


RG EVOLUTION
(perturbative)

MATRIX ELEMENTS
(non-perturbative)

Multi-scale problem: need RG evolution of effective couplings & hadronic / nuclear / molecular calculations of matrix elements

But we are not done yet…



RG EVOLUTION
(perturbative)

MATRIX ELEMENTS
(non-perturbative)

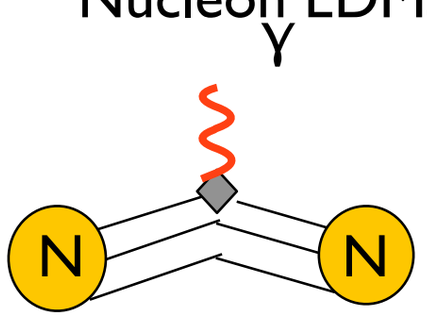
Multi-scale problem: need RG evolution of effective couplings & hadronic / nuclear / molecular calculations of matrix elements

Next step: from quarks and gluons to hadrons

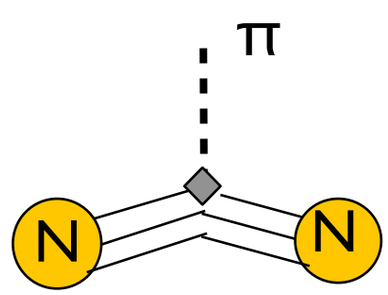
- Leading pion-nucleon CPV interactions characterized by few LECs

$$\tilde{\mathcal{L}}_{\rm CPV} = -\frac{i}{2}\sum_{i=n,p,e} d_i\,\bar{\psi}_i\,\sigma\cdot F\gamma_5\,\psi_i - \bar{N}\left[\bar{g}_0\,\vec{\tau}\cdot\vec{\pi} + \bar{g}_1\,\pi^0\right]N + \ldots$$

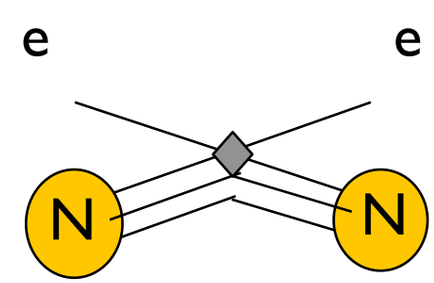
Electron and Nucleon EDMs



T-odd P-odd pion-nucleon couplings



Short-range 4N and 2N2e coupling

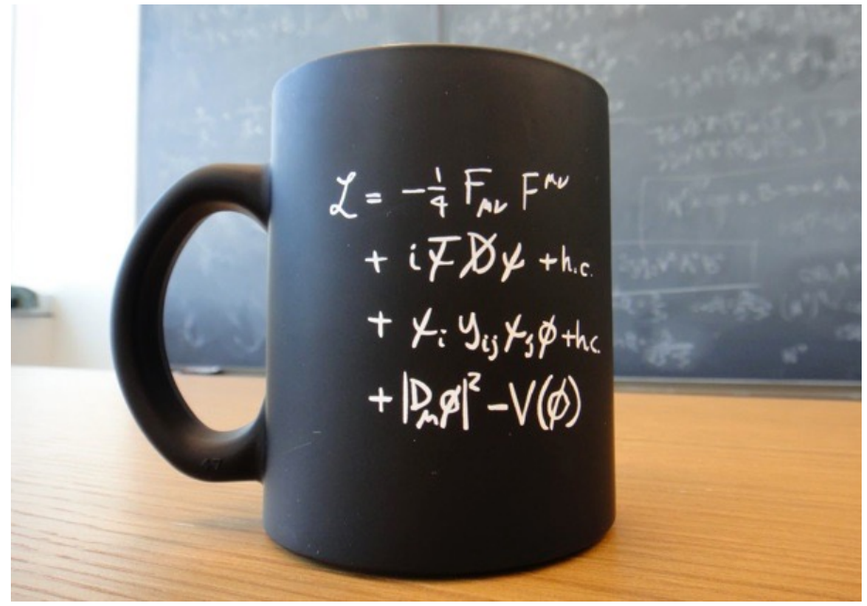


To be discussed in Lectures VI,VII,VIII

Backup slides

Standard Model building blocks

$$\psi = \begin{pmatrix} q \\ \ell \\ u \\ d \\ e \end{pmatrix}$$



	$SU(3)_c \times SU(2)_W \times U(1)_Y$ representation: (dim[$SU(3)_c$], dim[$SU(2)_W$], Y)	$SU(2)_W$ transformation
$l = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	(1,2,-1/2)	$l \to V_{SU(2)}\, l$
$e = e_R$	(1,1,-1)	
$q^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$	(3,2,1/6)	$q \to V_{SU(2)}\, q$
$u^i = u_R^i$	(3,1,2/3)	
$d^i = d_R^i$	(3,1,-1/3)	
$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$	(1,2,1/2)	$\varphi \to V_{SU(2)}\, \varphi$

		$SU(3)_c \times SU(2)_W \times U(1)_Y$ representation
gluons:	G_μ^A, $A = 1 \cdots 8$, $G_{\mu\nu}^A = \partial_\mu G_\nu^A - \partial_\nu G_\mu^A + g_s f_{ABC} G_\mu^B G_\nu^C$	(8,1,0)
W bosons:	W_μ^I, $I = 1 \cdots 3$, $W_{\mu\nu}^I = \partial_\mu W_\nu^I - \partial_\nu W_\mu^I + g \varepsilon_{IJK} W_\mu^J W_\nu^K$	(1,3,0)
B boson:	B_μ, $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$.	(1,1,0)

Gauge transformation:

$$W_{\mu\nu}^I \frac{\sigma^I}{2} \longrightarrow V(x) \left[W_{\mu\nu}^I \frac{\sigma^I}{2} \right] V^\dagger(x)$$

$$V(x) = e^{ig\beta_a(x)\frac{\sigma_a}{2}}$$

Standard Model Lagrangian

$$\mathcal{L}_{SM} = \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}$$

$$D_\mu = I\,\partial_\mu - ig_s\frac{\lambda^A}{2}G^A_\mu - ig\frac{\sigma^a}{2}W^a_\mu - ig'YB_\mu$$

$$\mathcal{L}_{\text{Gauge}} = -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$$+ \sum_{i=1,2,3}\left(i\bar{\ell}_i \not{D}\ell_i + i\bar{e}_i\not{D}e_i + i\bar{q}_i\not{D}q_i + i\bar{u}_i\not{D}u_i + i\bar{d}_i\not{D}d_i\right)$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu\varphi)^\dagger(D^\mu\varphi) - \lambda(\varphi^\dagger\varphi - v^2)^2 \xrightarrow{\text{EWSB}}$$

$$\langle\varphi\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\langle\tilde{\varphi}\rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$\tilde{\varphi} = \epsilon\,\varphi^*$$

$$\mathcal{L}_{\text{Yukawa}} = \bar{\ell}\,Y_e\,e\varphi + \bar{q}\,Y_d\,d\varphi + \bar{q}\,Y_u\,u\tilde{\varphi} + \text{h.c.}$$

Counting operators at low scale

Operator (dimension)	Number	Systems
theta term (4)	1	hadronic & diamagnetic atoms
electron EDM (6)	1	paramagenetic atoms & molecules
semi-leptonic (6)	3	
quark EDM (6)	2	hadronic & diamagnetic atoms
quark chromo EDM (6)	2	
three-gluon (6)	1	
four-quark (6)	2	
induced four-quark (6)	1	
	13	

Engel, Ramsey-Musolf, Van Kolck 1303.2371

Renormalization group

- Large logs (from widely separated scales) spoil validity of perturbation theory

	LL	NLL	N^2LL	...
NLO	$\alpha_s\,\ell$	α_s		
N^2LO	$\alpha_s^2\,\ell^2$	$\alpha_s^2\,\ell$	α_s^2	
N^3LO	$\alpha_s^3\,\ell^3$	$\alpha_s^3\,\ell^2$	$\alpha_s^3\ell$	α_s^3
	...	...	...	
	$O(1)$	$O(\alpha_s)$	$O(\alpha_s^2)$	

$$\ell = \log\frac{\Lambda}{\mu}$$

$$\alpha_s\,\ell \sim O(1)$$

- Ordinary pert. theory proceeds "by rows": NLO, N^2LO, ...
- RGE re-organize the expansion "by columns": LL, NLL, ...

- <u>RGEs</u>: exploit the fact that physics does not depend on the renormalization scale

$$\boxed{\mathcal{L}_{\rm eff} = \sum_i C_i\, Q_i}$$

- Bare operators do not depend on μ (subtraction scale)

$$\frac{d}{d(\ln\mu)} O_i^{(0)} = 0 \quad \longrightarrow \quad \frac{d}{d(\ln\mu)} O_i = -\gamma_i\, O_i$$

$$O_i^{(0)} = Z_i\, O_i$$

$$\gamma_i \equiv \frac{1}{Z_i}\frac{d}{d(\ln\mu)} Z_i = \frac{g^2}{16\pi^2}\gamma_i^{(0)} + \ldots$$

- Physical amplitudes do not depend on μ

$$\frac{d}{d(\ln\mu)}\left[C_i\, \langle Q_i\rangle\right] = 0 \quad \longrightarrow \quad \boxed{\frac{d}{d(\ln\mu)} C_i = \gamma_i\, C_i}$$

- In general, need to solve:

$$\frac{d}{d(\ln\mu)}C_i = \gamma_i\, C_i \qquad\qquad \frac{d}{d(\ln\mu)}\, g = \beta(g)$$

$$\gamma_i \;=\; \frac{g^2}{16\pi^2}\gamma_i^{(0)} + \ldots \qquad\qquad \beta(g) \;=\; -\beta_0\frac{g^3}{16\pi^2} + \ldots$$

$$\frac{C_i(\mu)}{C_i(\Lambda)} \;=\; \exp\left[\int_{g(\Lambda)}^{g(\mu)} dg'\,\frac{\gamma_i(g')}{\beta(g')}\right] \;\rightarrow\; \left[\frac{\alpha(\Lambda)}{\alpha(\mu)}\right]^{\frac{\gamma_i^{(0)}}{2\beta_0}}$$

$$\simeq\; 1 \;+\; \gamma_i^{(0)}\,\frac{\alpha}{4\pi}\,\log\frac{\mu}{\Lambda}$$

- In general, need to solve:

$$\frac{d}{d(\ln\mu)}C_i = \gamma_i\, C_i \qquad\qquad \frac{d}{d(\ln\mu)}\, g = \beta(g)$$

$$\gamma_i = \frac{g^2}{16\pi^2}\gamma_i^{(0)} + \ldots \qquad\qquad \beta(g) = -\beta_0\frac{g^3}{16\pi^2} + \ldots$$

$$\frac{C_i(\mu)}{C_i(\Lambda)} = \exp\left[\int_{g(\Lambda)}^{g(\mu)} dg'\, \frac{\gamma_i(g')}{\beta(g')}\right] \rightarrow \left[\frac{\alpha(\Lambda)}{\alpha(\mu)}\right]^{\frac{\gamma_i^{(0)}}{2\beta_0}}$$

- One-loop beta functions: $\beta_0^{\rm QCD} = \frac{11N_C - 2N_F}{3}$ $\qquad \beta_0^{\rm QED} = -\frac{4}{3}\sum_f Q_f^2$
- Needed input: $\gamma_i^{(0)}$ anomalous dimensions for relevant operators