Fierz Interference in Neutron Decay

Leah Broussard

Oak Ridge National Laboratory

Beta Decay as a Probe of New Physics November 1-3, 2018 University of Massachusetts Amherst

A probe for new physics

- CKM unitarity
 - Competitive with LHC limits¹
 - LQCD calc g_A now $1\%^2$
 - New Δ_R^V calc shifts from unitarity³

$$\begin{pmatrix} d'\\ s'\\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d\\ s\\ b \end{pmatrix}$$
$$U^{2} + |V||^{2} + |V||^{2} = 0.9994(5) \quad (PDC.1)$$

 $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994(5)$ (PDG 18) $\longrightarrow 0.9984(4)$

- Beyond Standard Model
 - Scalar, Tensor, Right-handed currents
 - Improved LQCD calcs of g_A , g_S , g_T^4

¹Gonzalez-Alonso, Naviliat-Cuncic, and Severijns, arXiv:1803.08732 ²Chang *et al*, *Nature* **558** (2018) 91-94 ³Seng, Gorchtein, Patel, Ramsey-Musolf, arXiv:1807.10197 ⁴Gupta *et al*, PRD **98** (2018) 034503





Leah Broussard

Beta Decay as a Probe of New Physics, November 1-3, 2018

Neutron β -decay observables



UCNA collaboration



M. A.-P. Brown,¹ E. B. Dees,^{2,3} E. Adamek,⁴ B. Allgeier,¹ M. Blatnik,⁵ T. J. Bowles,⁶ L. J. Broussard,⁶ R. Carr,⁵ S. Clayton,⁶ C. Cude-Woods,² S. Currie,⁶ X. Ding,⁷ B. W. Filippone,⁵ A. García,⁸ P. Geltenbort,⁹ S. Hasan,¹ K. P. Hickerson,⁵ J. Hoagland,² R. Hong,⁸ G. E. Hogan,⁶ A. T. Holley,¹⁰ T. M. Ito,⁶ A. Knecht,⁸ C.-Y. Liu,⁴ J. Liu,¹¹ M. Makela,⁶ J. W. Martin,^{5, 12} D. Melconian,¹³ M. P. Mendenhall,⁵ S. D. Moore,² C. L. Morris,⁶ S. Nepal,¹ N. Nouri,¹ R. W. Pattie, Jr.,^{2,3} A. Pérez-Galván,⁵ D. G. Phillips II,² R. Picker,⁵ M. L. Pitt,⁷ B. Plaster,¹ J. C. Ramsey,⁶ R. Rios,^{6, 14} D. Salvat,⁸ A. Saunders,⁶ W. Sondheim,⁶ S. J. Seestrom,⁶ S. Sjue,⁶ S. Slutsky,⁵ X. Sun,⁵ C. Swank,⁵ E. Tatar,¹⁴ R. B. Vogelaar,⁷ B. VornDick,² Z. Wang,⁶ J. Wexler,² T. Womack,⁶ C. Wrede,^{8,15} A. R. Young,^{2,3} and B. A. Zeck² (UCNA Collaboration) ¹Department of Physics and Astronomy, University of Kentucky, Lexington, Kentucky 40506, USA ²Department of Physics, North Carolina State University, Raleigh, North Carolina 27695, USA ³Triangle Universities Nuclear Laboratory, Durham, North Carolina 27708, USA ⁴Department of Physics, Indiana University, Bloomington, Indiana 47408, USA ⁵ W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125, USA ⁶Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA ⁷Department of Physics, Virginia Tech, Blacksburg, Virginia 24061, USA ⁸Department of Physics, University of Washington, Seattle, Washington 98195, USA ⁹Institut Laue-Langevin, 38042 Grenoble Cedex 9, France ¹⁰Department of Physics, Tennessee Technological University, Cookeville, Tennessee 38505, USA ¹¹Department of Physics, Shanghai Jiao Tong University, Shanghai, 200240, China ¹²Department of Physics, University of Winnipeg, Winnipeg, MB R3B 2E9, Canada ¹³Cyclotron Institute, Texas A&M University, College Station, Texas 77843, USA ¹⁴Department of Physics, Idaho State University, Pocatello, Idaho 83209, USA ¹⁵Department of Physics and Astronomy and National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824, USA



¹Brown et al, PRC **97** 035505 (2018)

Fierz term from UCNA

- UCNA: $4\pi \beta$ acceptance, low neutron/ambient backgrounds, energy reconstruction \rightarrow direct spectral extraction of \mathbf{b}_n
- "Super-sum" removes distortion from **A** $\Sigma = \frac{1}{2} \sqrt{N(E)_1^+ N(E)_2^-} + \frac{1}{2} \sqrt{N(E)_1^- N(E)_2^+}$
- 2010 data set dominant error: energy calibration



¹Hickerson et al, PRC 96 (2017) 042501

Complementary approaches for b

- 2 new datasets: 2011-12, 2012-13
 - 2012 improved E reconstruction: b vs octet \rightarrow
- 2 techniques: spectrum Σ vs asymmetry $\frac{A}{1+b\frac{m}{E}}$
 - Σ limited by E calibration, A_m by statistics
- <u>Preliminary</u>: $\boldsymbol{b}_n = x \cdot xx \pm 0.03$ (blinded)





-0.09 W -0.1 -0.10 -0.12 -0.12 -0.12 -0.13 -0.14 -0.15 -0.15 -0.14 -0.15 -0.14 -0.15 -0.14 -0.15 -0.14 -0.15 -0.14 -0.15 -0.14 -0.15 -0.14 -0.15 -0.14 -0.15 -0.14 -0.15 -0.14 -0.15 -0.15 -0.14 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -0.15 -



Thanks X. Sun (Caltech) for slide content

-0.08

Fundamental Neutron Physics at the Spallation Neutron Source



Nab Collaboration

Active and recent collaborators:

R. Alarcon^a, S. Baessler^{b,c} (Project Manager), S. Balascuta^a, L. Barrón Palosⁿ, T. Bailey^m, K. Bassⁱ, N. Birgeⁱ, A. Blose^f, D. Borissenko^b, J.D. Bowman^c (Co-Spokesperson), L. J. Broussard^c, A.T. Bryant^b, J. Byrne^d, J.R. Calarco^{c,i}, J. Caylorⁱ, K. Chang^b, T. Chupp^o, T.V. Cianciolo^c, C. Crawford^f, X. Ding^b, M. Doyle^b, W. Fan^b, W. Farrar^b, N. Fominⁱ, E. Frlež^b, J. Fry^b, M.T. Gericke^g, M. Gervais^f, F. Glück^h, G.L. Greene^{c,i}, R.K. Grzywaczⁱ, V. Gudkovⁱ, J. Hamblen^e, C. Hayes^m, C. Hendrus^o, T. Ito^k, A. Jezghani^f, H. Li^b, M. Makela^k, N. Macsey^g, J. Mammei^g, R. Mammei^l, M. Martinez^a, D.G. Matthews^f, M. McCrea^f, P. McGaughey^k, C.D. McLaughlin^b, P. Mueller^c, D. van Petten^b, S.I. Penttilä^c (On-site Manager), D. E. Perrymanⁱ, R. Picker^p, J. Pierce^c, D. Počanić^c (Co-Spokesperson), Y. Qian^b, J. Ramsey, G. Randall^a, G. Rileyⁱ, K.P. Rykaczewski^c, A. Salas-Bacci^b, S. Samiei^b, E.M. Scottⁱ, T. Shelton^f, S.K. Sjue^k, A. Smith^b, E. Smith^k, E. Stevens^b, J. Wexler^m, R. Whiteheadⁱ, W.S. Wilburn^k, A.Young^m, B.Zeck^m

^a Department of Physics, Arizona State University, Tempe, AZ 85287-1504

- ^b Department of Physics, University of Virginia, Charlottesville, VA 22904-4714
- ^c Physics Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831
- ^d Department of Physics and Astronomy, University of Sussex, Brighton BN19RH, UK
- ^e Department of Chemistry and Physics, University of Tennessee at Chattanooga, Chattanooga, TN 37403
- ^f Department of Physics and Astronomy, University of Kentucky, Lexington, KY 40506
- ^g Department of Physics, University of Manitoba, Winnipeg, Manitoba, R3T 2N2, Canada

^h KIT, Universität Karlsruhe (TH), Kaiserstraße 12, 76131 Karlsruhe, Germany

Main project funding:





Office of Science



ⁱ Department of Physics and Astronomy, University of Tennessee, Knoxville,

^j Department of Physics and Astronomy, University of South Carolina,

¹Department of Physics, University of Winnipeg, Winnipeg, Manitoba

^m Department of Physics, North Carolina State University, Raleigh, NC

ⁿ Universidad Nacional Autónoma de México, México, D.F. 04510, México

^kLos Alamos National Laboratory, Los Alamos, NM 87545

^o University of Michigan, Ann Arbor, MI 48109 ^p TRIUMF, Vancouver, Canada, V6T 2A3



Leah Broussard

Beta Decay as a Probe of New Physics, November 1-3, 2018

TN 37996

Columbia, SC 29208

R3B2E9, Canada

27695-8202

Nab measurement principles

- Goal: $\Delta a/a \sim 10^{-3}$ and $\Delta b \sim 3 \times 10^{-3}$
- $4\pi\beta$ acceptance : "tear-drop"
- $p_p^2 = p_e^2 + 2p_e p_v \cos \theta_{ev} + p_v^2$

Yield:
$$\propto 1 + a \frac{p_e}{E_e} \cos \theta_{ev}$$



Leah Broussard

Nab measurement principles

 Asymmetric spectrometer with long TOF arm: proton TOF ⇒ momentum

$$t_p = L \frac{m_p}{p_p} = \frac{f(\cos\theta)}{p_p}$$

• Adiabatic field expansion

$$t_{p} = \frac{m_{p}}{p_{p}} \int_{z_{0}}^{L} \frac{dz}{\sqrt{1 - \frac{B(z)}{B_{0}}\sin^{2}\theta + \frac{q(V(z) - V_{0}}{E_{0}}}}$$

8000

6000

4000

2000

0.001

0.002

- For each E_e fit central 75% to obtain **a**
- Edges verify spectrometer response



Nab spectrometer installation



Installation crew nominated for ORNL "Significant Event Award"

Expected statistical uncertainty

- 1.4 MW routine at SNS
 - Expect 1600 decays/s = 200 p/s in top detector
- Up to $\Delta a/a \sim 2 \times 10^{-3}$ each run-cycle
- Require 2 years SNS running for statistics goal
 - Including 50% duty factor, 10% background, several systematic runs

 $\Delta a/a \sim 7 \times 10^{-4}$

Statistical uncertainties σ_a for a SM fit $[b \equiv 0]$

varied	E _{e,min} :	0	100 keV	100 keV	100 keV	300 keV
\Downarrow	t _{p,max} :	-	-	40 μ s	30 μs	40 μ s
N _u , a		$2.4/\sqrt{N_u}$	$2.4/\sqrt{N_u}$	$2.6/\sqrt{N_u}$	$2.8/\sqrt{N_u}$	$3.1/\sqrt{N_u}$
$+ E_{calib}, L_{TOF}$		$2.6/\sqrt{N_u}$	$2.6/\sqrt{N_u}$	$2.8/\sqrt{N_u}$	$3.1/\sqrt{N_u}$	$3.5/\sqrt{N_u}$
+ use 75% [§]		$3.3/\sqrt{N_u}$	$3.4/\sqrt{N_{u}}$	$3.6/\sqrt{N_u}$	$4.0/\sqrt{N_{u}}$	$4.6/\sqrt{N_u}$
+ 10% bgd.		$4.3/\sqrt{N_u}$	$4.4/\sqrt{N_u}$	$4.5/\sqrt{N_u}$	$4.9/\sqrt{N_u}$	$5.5/\sqrt{N_u}$

[§] fits for **a** use only the inner 75% of p_p^2 data. [N_u ... number of protons detected in upper detector.]

Statistical uncertainties σ_a for a BSM fit [variable b]

varied	E _{e,min} :	0	100 keV	100 keV	100 keV	300 keV
\Downarrow	t _{p,max} :	-	-	40 μ s	30 μ s	40 μ s
N _u , a , b	•	$2.4/\sqrt{N_u}$	$2.5/\sqrt{N_u}$	$2.7/\sqrt{N_u}$	$3.0/\sqrt{N_u}$	$3.6/\sqrt{N_u}$
$+ E_{ca}$	_{lib} , L _{TOF}	$2.6/\sqrt{N_u}$	$2.7/\sqrt{N_u}$	$2.9/\sqrt{N_u}$	$3.2/\sqrt{N_u}$	$3.9/\sqrt{N_u}$
+ use	e 75% [§]	$3.4/\sqrt{N_{u}}$	$3.5/\sqrt{N_u}$	$3.8/\sqrt{N_u}$	$4.4/\sqrt{N_{u}}$	$5.1/\sqrt{N_u}$
+ 109	% bgd.	$4.4/\sqrt{N_u}$	$4.6/\sqrt{N_u}$	$4.7/\sqrt{N_u}$	$5.2/\sqrt{N_u}$	$6.3/\sqrt{N_u}$

Projected statistical uncertainties σ_b (for completeness)

$E_{\rm e,min}$	0	100 keV	200 keV	300 keV
σ_{b}	$7.5/\sqrt{N}$	$10.1/\sqrt{N}$	15.6/ \sqrt{N}	$26.3/\sqrt{N}$
$\sigma_{\it b}{}^{\dagger}$	$7.7/\sqrt{N}$	$10.3/\sqrt{N}$	$16.3/\sqrt{N}$	$27.7/\sqrt{N}$

[†] with **E**_{calib} variable.

[N ... number of n-decay electrons detected in either detector.]

D. Pocanic

Nab expected systematics for a

Experimental parameter	Principal specification (comment)	$(\Delta a/a)_{ m SYST}$
Magnetic field:		
curvature at pinch	$\Delta\gamma/\gamma=2\%$ with $\gamma=({ m d}^2B_z(z)/{ m d}z^2)/B_z(0)$	$5.3 imes10^{-4}$
ratio $r_{\scriptscriptstyle B}=B_{\scriptscriptstyle TOF}/B_{\scriptscriptstyle O}$	$(\Delta r_B)/r_B=1\%$	$2.2 imes 10^{-4}$
ratio $r_{ m B,DV}=B_{ m DV}/B_{ m 0}$	$(\Delta \textit{r}_{ extsf{B}, extsf{DV}})/\textit{r}_{ extsf{B}, extsf{DV}}=1\%$	$1.8 imes10^{-4}$
L_{TOF} , length of TOF region		(*)
U inhomogeneity:		
in decay / filter region	$ \mathit{U}_{ extsf{F}} - \mathit{U}_{ extsf{DV}} < 10 extsf{mV}$	$5 imes 10^{-4}$
in TOF region	$ U_{ m F}-U_{ m TOF} <200{ m mV}$	$2.2 imes10^{-4}$
Neutron beam:		
position	$\Delta \langle z_{{ m DV}} angle < 2{ m mm}$	$1.7 imes10^{-4}$
profile (incl. edge effect)	slope at edges $< 10\%/{ m cm}$	$2.5 imes10^{-4}$
Doppler effect	(analytical correction)	small
unwanted beam polarization	$\Delta \langle P_{ m n} angle < 2 \cdot 10^{-3}$ (with spin flipper)	$1 imes 10^{-4}$
Adiabaticity of proton motion		$1 imes 10^{-4}$
Detector effects:		
E_{e} calibration	$\Delta E_{\scriptscriptstyle ext{e}} < 200 ext{eV}$	$2 \cdot 10^{-4}$
shape of $E_{ m e}$ response	$\Delta N_{ ext{tail}}/N_{ ext{tail}} \leq 1\%$	$4.4 imes 10^{-4}$
proton trigger efficiency	$\epsilon_{ t p} < 100 { m ppm/keV}$	$3.4 imes10^{-4}$
TOF shift (det./electronics)	$\Delta t_{ extsf{p}} < 0.3 extsf{ns}$	$3 imes 10^{-4}$
TOF in accel. region	$\Delta r_{ extsf{ground EL.}} < 0.5 extsf{mm} extsf{(preliminary)}$	$3.4 imes10^{-4}$
BGD/accid. coinc's	(will subtract out of time coinc)	small
Residual gas	$P < 2 \cdot 10^{-9}$ torr	3.8×10^{-4}
Overall sum		$1.2 imes 10^{-3}$

(*) Free fit parameter

Leah Broussard

Systematics for Fierz term b

- Full β energy collected, except:
 - Bounce history (deadlayer), bremsstrahlung, detector response...
 - Also backgrounds, edge effects, timing cutoff, proton efficiency...
- Statistical uncertainty $\sim 3 \times 10^{-4}$
 - if gain free parameter $\rightarrow 5 \times 10^{-4}$
- Initial (partial) parametric study of systematics:

Systematic	Requirement
Gain	Free parameter
Offset	±0.06 keV
Max. nonlinearity	± 0.05 keV
Resolution	$\pm 2 \text{ keV}$
Energy tail	$\pm 10\%$

Magnetic field

- Precise (relative) field mapping
- Locate electron/proton flux tubes







Detector effects

- Calibration (dE ~ 0.2 keV for **a**)
 - Linearity to ~10⁻⁴ → radioactive sources; need high precision calibrated pulser
 - Temperature stability to 0.5 K → sensors, leakage current, pulser gain?
 - Detector response vs. event energy/hit location; uniformity → collimated radioactive sources; electron-gun studies
 - Cross-talk → radioactive sources, proton beam (physical); pulsers (electronic)



- Si detector: 2 mm thick, 11 cm diameter active area, 100 nm deadlayer, 127 hex pixels
- 40-50 ns rise times
- 3 keV @ 30 keV FWHM

Detector effects

- Energy loss $(dN_{tail}/N_{tail} < 1\%$ for **a**)
 - Backscattering = sum both detectors but...
 - Bounce history of electrons → radioactive source studies to benchmark simulations
 - Detector deadlayer uniformity → measure with proton and electron gun
 - Bremsstrahlung needs ×10 improvement → characterize in situ with radioactive sources; electron-gun and gamma detector
- Rate-dependent effects: backgrounds, accidentals, deadtime



E. Frlez, UVA

Detector effects

- Proton trigger efficiency < 100 ppm/keV (efficiency slope 50%) → proton gun
- TOF bias <0.3 ns on average between electrons and protons (from detector response) → collimated fast timing source, electron-gun



S. Baessler



⁴⁵Ca Fierz term

- Learn high precision calibration
- Also nice BSM target
 - Source on thin 6F6F foil
 - UCNA spectrometer with UCNB/Nab detectors

UCNB Si

detectors

²⁰⁷Bi, ¹³⁹Ce, ¹¹³Sn

source insert

• 10⁸ events collected



¹Gonzalez-Alonso and Naviliat-Cuncic, PRC **94** (2016) 035503 ²Hayen *et al*, RMP **90** (2018) 015008

⁴⁵Ca source

Leah Broussard

Polarized Nab (abBA/PANDA)

Only major modification: Addition of a neutron beam polarizer

Main uncertainties in previous best experiments: statistics, detector, background, polarization

- Statistics @ SNS or NIST is sufficient for a competitive measurement of *A*, but could be better
- Superior detector energy resolution, good enough time resolution
- Keep coincidence detection (electrons and protons) to improve background
- Polarization measurement seems manageable (Crossed supermirrors or He-3)



Goal: $\Delta A/A \leq 10^{-3}$, $\Delta B/B \leq 10^{-3}$

Thanks S. Baessler for slide

Summary

- UCNA has produced first spectral determination of $-0.041 < b_n < 0.225 (90\% \text{ CL}), \Delta b \sim 0.03$ in analysis
- Nab is now commissioning, aiming for $\Delta a/a \sim 10^{-3}$ and $\Delta b \sim 3 \times 10^{-3}$
- Key systematics for **a** include relative magnetic field determination and electron energy reconstruction
- Detection systematics both challenging and an opportunity for other physics targets

This research was sponsored by the LDRD program [project 8215] of ORNL, managed by UT-Battelle, LLC, and the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, contract DE-AC05-00OR2272.

Nab uncertainty analysis

- Simple model for parametric studies $t_p = \frac{m_p}{p_p} \int_{z_0}^{L} \frac{dz}{\sqrt{1 - \frac{B(z)}{B_0} \sin^2 \theta + \frac{q(V(z) - V_0)}{E_0}}}$
- Piecewise quadratic approximation
- Compute analytically
- Neglect *V(z)* term for speed, then apply correction factor



UCNA measurement principles

- $W \propto 1 + \frac{v}{c} \langle P \rangle A(E) \cos \theta$
- Magnetic spectrometer: $\langle \cos \theta \rangle = \pm \frac{1}{2}$
- Measure asymmetry: 2 detectors, 2 spin directions
 - Spin-dependent and detector-dependent efficiencies?
- Cancel systematics with Super-Ratio

$$S(E) = \frac{N(E)_1^+ N(E)_2^-}{N(E)_1^- N(E)_2^+}$$
$$A_{SR} = \frac{1 - \sqrt{R}}{1 + \sqrt{R}} = \frac{\nu}{c} \langle P \rangle A(E) \cos \theta$$



p

θ.

n