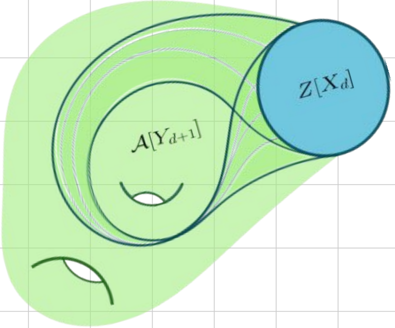
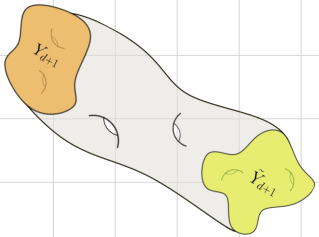


ANOMALIES & BORDISMS OF NON-SUPERSYMMETRIC STRINGS

Matilda Delgado



Based on:
[2310.06895] I. Basile, A. Debray, M.D., M. Montero

BIG PICTURE

Our world is non-supersymmetric

(**at least** at low energies)

It is crucial for phenomenology to understand String Theory (QG) in setups without supersymmetry!

On top of that: on our quest to understand the set of EFTs that come from QG,

most of our controlled top-down constructions lead to (highly) supersymmetric EFTs

Q: Is our understanding (Swampland constraints) truly representative of quantum gravity or just supersymmetry?

“Supersymmetric Lamppost” problem

BIG PICTURE

"Understand String Theory (QG) without supersymmetry"

HOW?

[1] From the **top-down?**

In principle: sure!

In reality: easier said than done...

- > as soon as we do not have (a lot of) **supersymmetry**, we lose (a lot of) computational control

BIG PICTURE

"Understand String Theory (QG) without supersymmetry"

What about from the **bottom up**?

This could work because bottom-up arguments in principle need not rely on SUSY at all

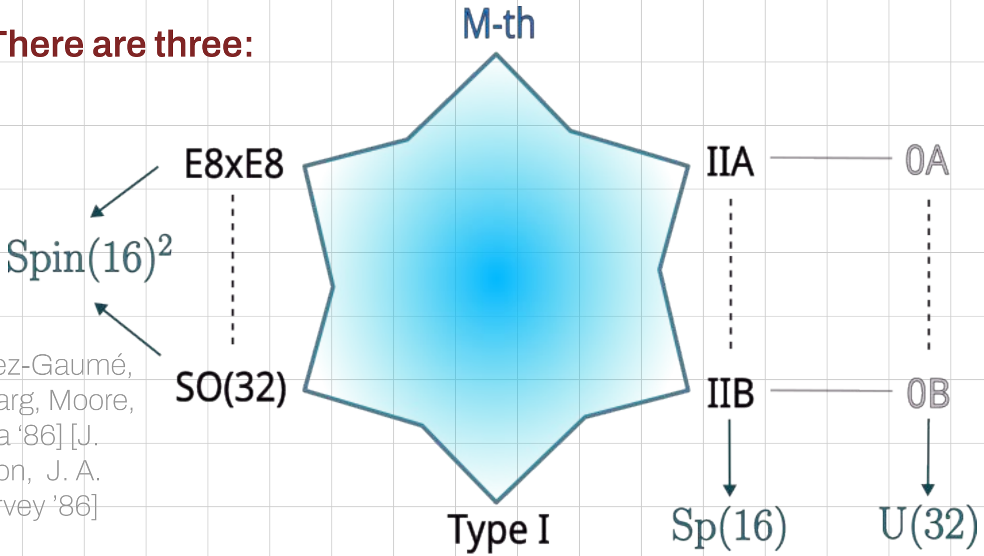
⇒ Find bottom-up black hole arguments (exploit their UV/IR mixing properties)
Ex: **no global symmetries in QG** [T. Banks and L. J. Dixon, '93]

⇒ **Gauge anomaly cancellation**
Some anomalies are specific to QG in the sense that they arise when spacetime topology change is allowed

Today: Apply this to the three 10D non-supersymmetric, non-tachyonic string theories

10D NON-SUPERSYMMETRIC STRING THEORIES

There are three:



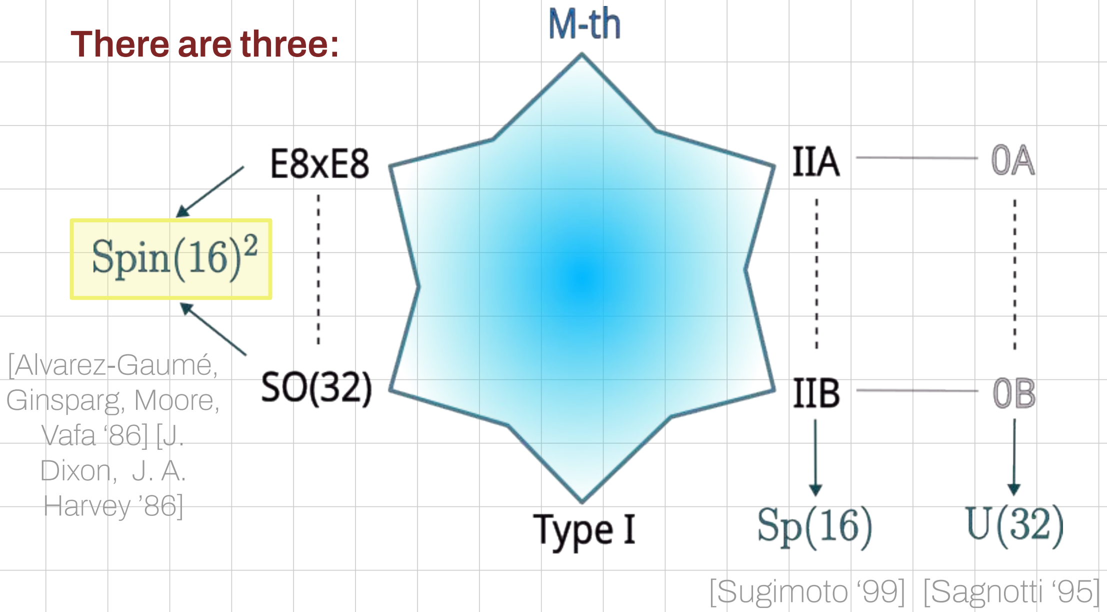
[Alvarez-Gaumé, Ginsparg, Moore, Vafa '86] [J. Dixon, J. A. Harvey '86]

[Sugimoto '99] [Sagnotti '95]

Most natural way to study Quantum Gravity away from SUSY !!

10D NON-SUPERSYMMETRIC STRING THEORIES

There are three:



Spin(16)²

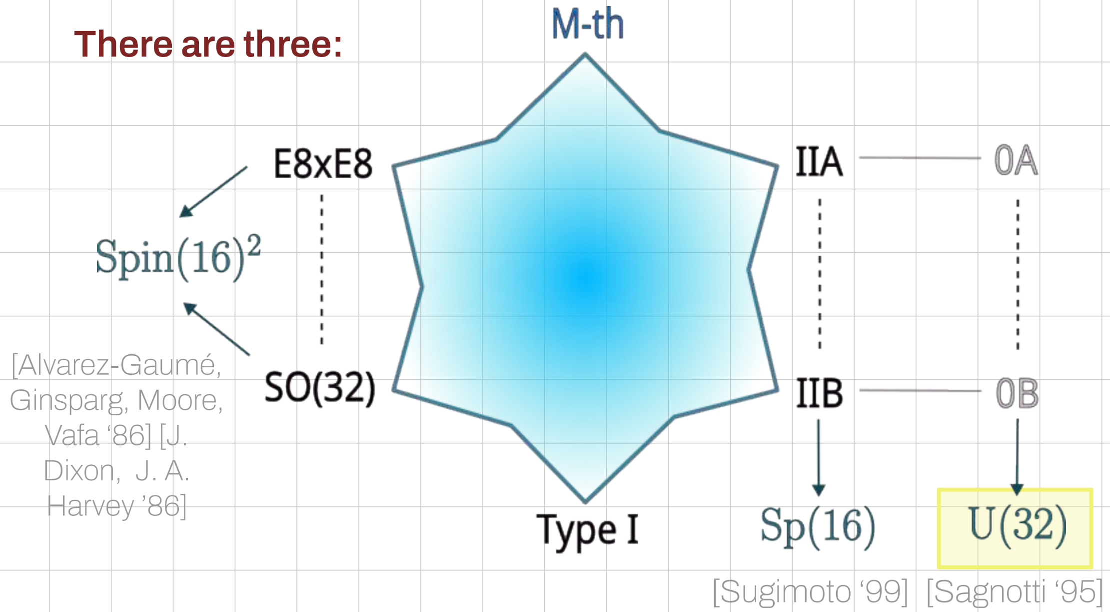
a.k.a. SO(16)xSO(16)

Can be obtained from the SO(32) and E_8 x E_8 theories by a projection on the worldsheet.

Chiral dof:
Dilatino
Gaugini
..

10D NON-SUPERSYMMETRIC STRING THEORIES

There are three:



U(32)

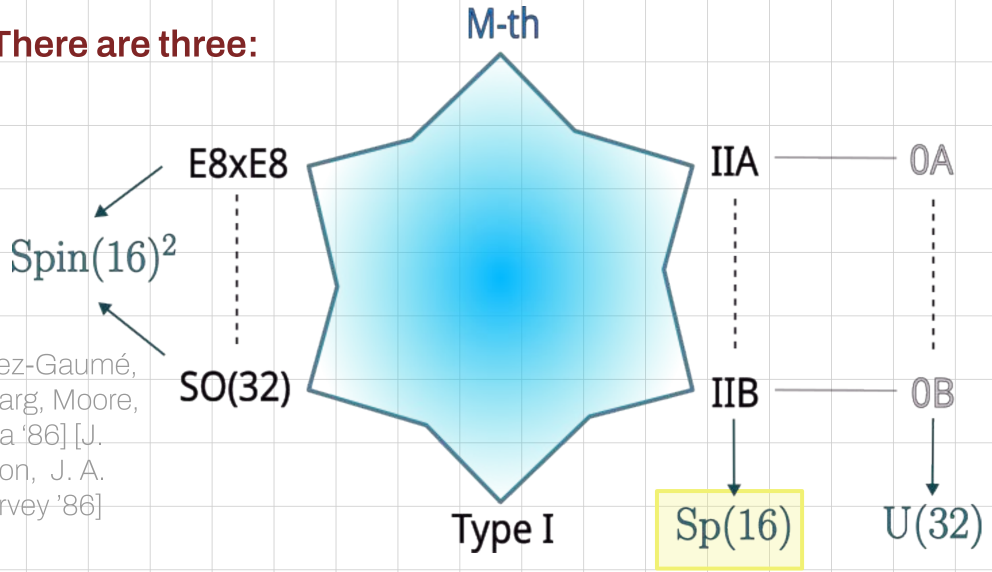
a.k.a. the Sagnotti string

Can be obtained from an orientifold of Type 0B that removes the tachyon.

<u>Chiral dof:</u>
Self-dual gauge field, Gaugini..

10D NON-SUPERSYMMETRIC STRING THEORIES

There are three:



[Alvarez-Gaumé, Ginsparg, Moore, Vafa '86] [J. Dixon, J. A. Harvey '86]

[Sugimoto '99] [Sagnotti '95]

Sp(16)

a.k.a. USp(32) or the Sugimoto string

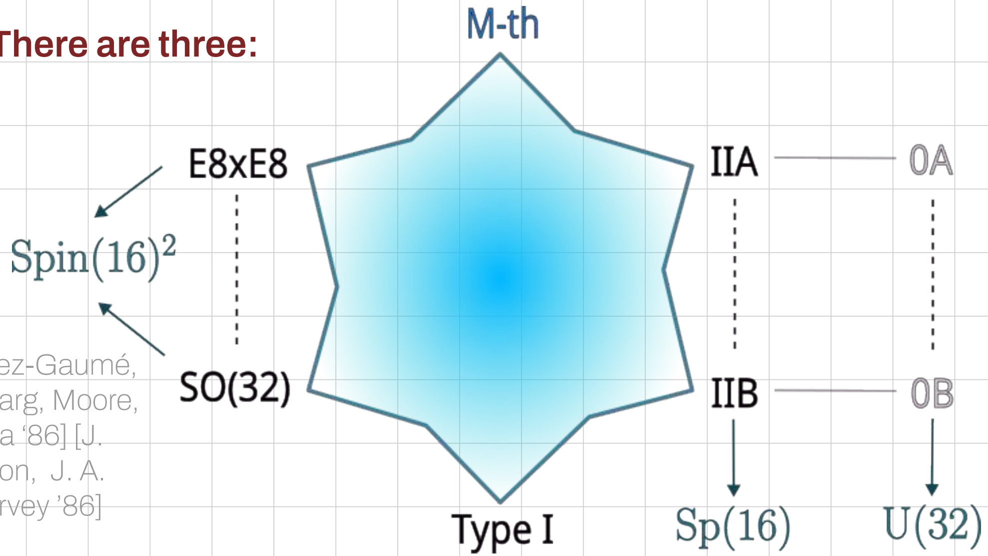
Can be obtained from a supersymmetry breaking orientifold of IIB

"non-supersymmetric sister of Type I"

Chiral dof: Gravitino, Dilatino, Gaugini..

10D NON-SUPERSYMMETRIC STRING THEORIES

There are three:



[Alvarez-Gaumé, Ginsparg, Moore, Vafa '86] [J. Dixon, J. A. Harvey '86]

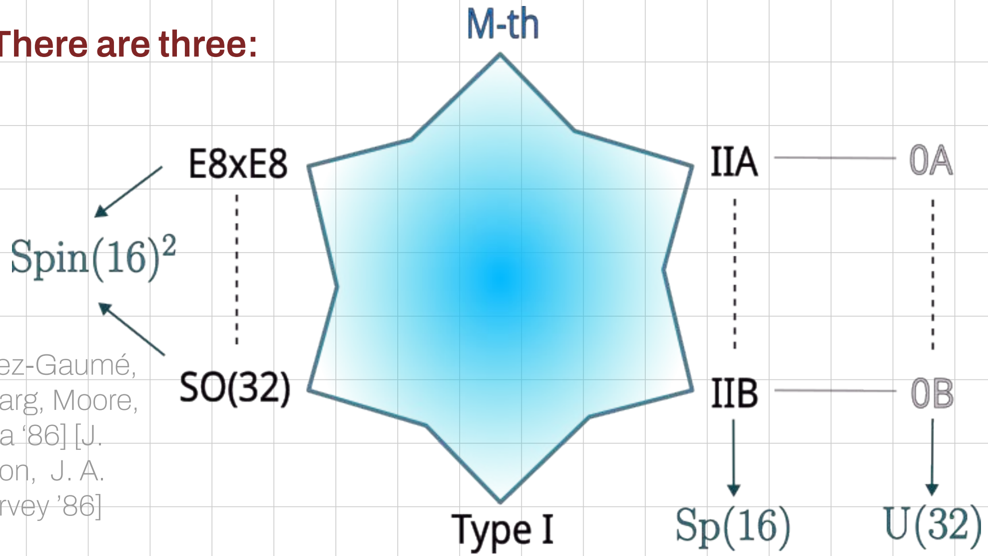
[Sugimoto '99] [Sagnotti '95]

What do we know about them?

- worldsheet theory ✅
- tachyon-free ✅
- low-energy 10D EFT (kindof ✅)
- local anomaly cancellation ✅ (through Green-Schwarz mechanism!)

10D NON-SUPERSYMMETRIC STRING THEORIES

There are three:



[Alvarez-Gaumé, Ginsparg, Moore, Vafa '86] [J. Dixon, J. A. Harvey '86]

[Sugimoto '99] [Sagnotti '95]

Still many unknowns ..

- brane content ❓
- world volume theory of NS branes ❓
- global anomaly cancellation ❓ ❗
- (...)

TODAY:

shed light on these questions

PLAN

Goal: Bottom-up methods for understanding the 10D non-supersymmetric, non-tachyonic string theories

❶

Local Anomaly Cancellation
&
5-branes worldvolumes

❷

Global Gauge Anomaly Cancelation
&
Bordisms

❸

No Global Symmetries
&
new extended objects

Questions??

PLAN

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new extended objects

ANOMALIES

In theories coupled to gauge fields and dynamical gravity, there can generally be gauge/gravitational anomalies.

Anomalies in gauge symmetries are a BIG problem (unlike for anomalies in global symmetries)

An anomaly is a lack of invariance of the path integral under a gauge transformation or diffeomorphism:

$$Z[X_d] \Longrightarrow \tilde{Z}[X_d] \neq Z[X_d]$$

- Local anomalies = "*usual ones*" , associated to gauge transformations that can be made **arbitrarily small**
  Think triangle (n-gon) diagrams

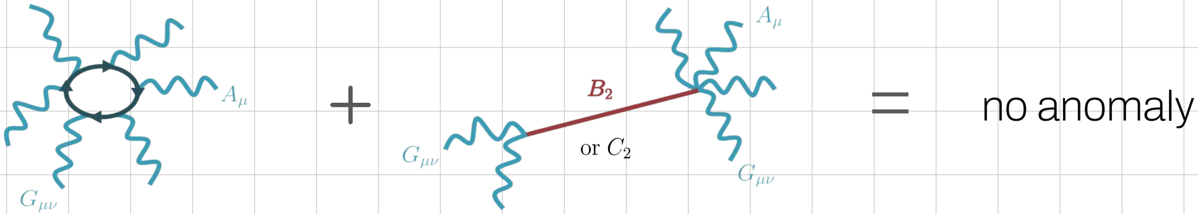
[e.g. Álvarez-Gaumé, Vázquez-Mozo '22]

[e.g. Álvarez-Gaumé, Vázquez-Mozo ‘22]

LOCAL ANOMALIES

Local anomalies = “*usual ones*” , associated to gauge transformations that can be made **arbitrarily small**

These are the anomalies that are killed by the Green-Schwarz mechanism in all three theories:



The added Green-Schwarz term cancels the anomaly by coupling the B-field to gravity and the gauge field:

$$S_{GS} = -\int_{M^{10}} B_2 \wedge X_8 \qquad X_8 = \mathrm{tr}F^4 + \mathrm{tr}R^4 + \mathrm{tr}F^2\mathrm{tr}R^2 + (\mathrm{tr}R^2)^2 + (\mathrm{tr}F^2)^2$$

or C_2

(for example)

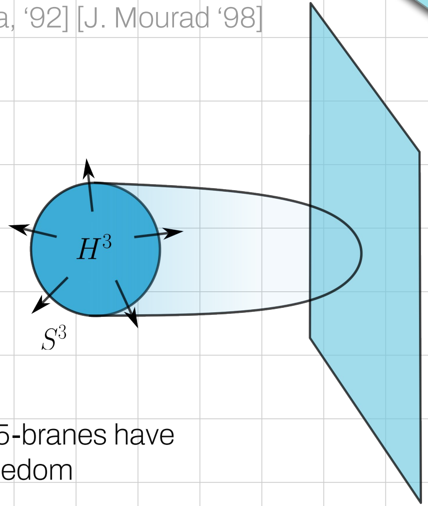
Gauge field contribution

Gravity contribution

Modified Bianchi identity → $dH \sim \mathrm{tr}F^2 - \mathrm{tr}R^2$

[J. A. Dixon, M. J. Duff, J. C. Plefka, ‘92] [J. Mourad ‘98]

ANOMALY INFLOW



Now let’s add in 5 branes that source magnetic H-flux:

$$S = S_{bulk} + S_{GS} + \underbrace{\mu \int B_6}_{} + \underbrace{S_{worldvolume}}_{}$$

Since the 5-brane sources H-flux, it participates in the modified Bianchi identity for H:

There is a new contribution to the bulk anomaly

The worldvolume theory of 5-branes have chiral degrees of freedom

They source an anomaly in the worldvolume theory

Consistency of the theory (all anomalies vanish) requires that the two contributions cancel eachother out

This is anomaly inflow

NON-SUPERSYMMETRIC 5-BRANES

The three 10D non-supersymmetric models have 5-branes that source magnetic H-flux.

For the Sugimoto and Sagnotti string: these are the D5 branes.

Because they are Dirichlet, their worldvolume theory can be computed from the worldsheet

The anomaly inflow mechanism works

[E. Dudas and J. Mourad '00] [J. A. Dixon, M. J. Duff, J. C. Plefka, '93] [J. Mourad '97]

For SO(16)xSO(16): this is the NS5 brane whose worldvolume dofs are unknown!

We can reverse the anomaly inflow argument to shed light on the chiral field content of this NS5 brane

I.e. find a chiral field content that gives the right anomalous contribution to the inflow mechanism

We get:

- Four fermion singlets
- A fermion in the (16,1) + (1,16) of SO(16)xSO(16)
- A self-dual 2-form

Non-supersymmetric interacting CFT?
(just wishful thinking)

PLAN

Goal: Bottom-up methods for understanding the 10D non-supersymmetric, non-tachyonic string theories

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- Local anomalies = "*usual ones*" , associated to gauge transformations that can be made **arbitrarily small**
  Think triangle (n-gon) diagrams ✅✅✅✅

- Global anomalies = associated to a transformation that cannot be *deformed to the identity*
  Example: Witten's SU(2) anomaly [Witten '82]

GLOBAL ANOMALIES

Review in: [García-Etxebarria, Montero '18]

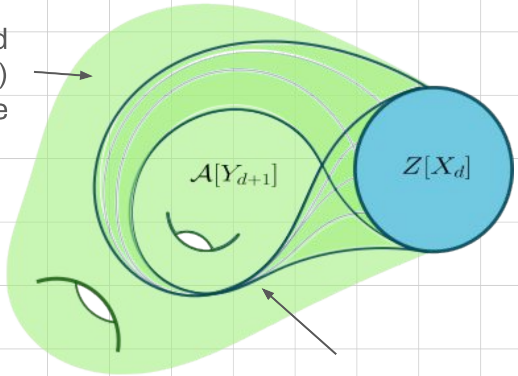
The modern way of computing **global gauge and gravitational** anomalies of a theory on X_d is through a (d+1)-dimensional *anomaly theory* on Y_{d+1} such that $\partial Y_{d+1} = X_d$

The anomaly theory is **engineered** to give the **exact (opposite) anomaly** of the one you started with.

To each anomalous dof in Z, you associate a contribution to the anomaly theory

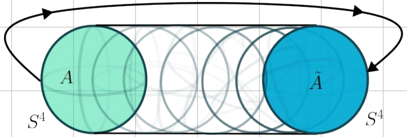
$$\boxed{\mathcal{A}(Y_{d+1})Z[X_d] \quad \text{is anomaly-free}}$$

All "structures" extend to the (d+1) dimensional space



Non-collapsible path in configuration space of gauge field / metric

~generalization of mapping torus



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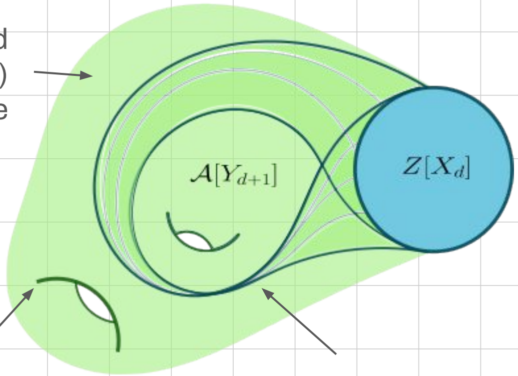


Non-collapsible path in configuration space of gauge field / metric

In QG, allow for topology-change

⇒ "Dai-Freed anomalies" Account for the possibility of a transformation that involves topology change

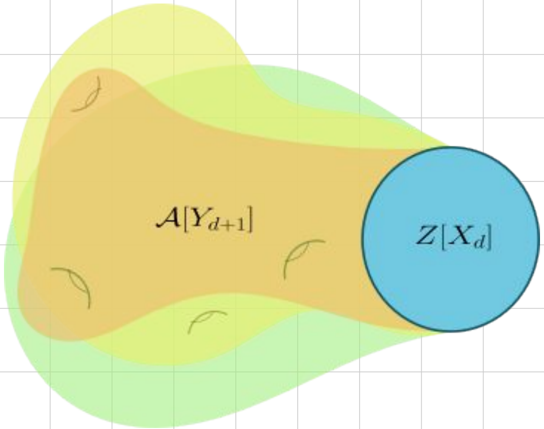
[García-Etxebarria, Montero '18]

[García-Etxebarria, Montero '18]

GLOBAL ANOMALIES

So we've constructed an anomaly theory in (d+1) dimensions that gives us the **exact (opposite) anomaly** of the one in our theory:

$$\boxed{\mathcal{A}(Y_{d+1})Z[X_d] \quad \text{is anomaly-free}}$$



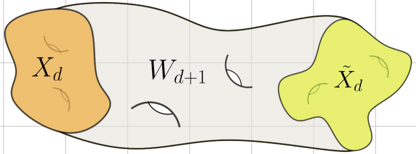
The reason is the anomaly is **much easier to detect in the anomaly theory.** Here's why:

How do we choose Y_{d+1}?

The anomaly $\mathcal{A}(Y_{d+1})$ should not depend on the choice of Y_{d+1} !

You should be able to deform any two choices of Y_{d+1} into one another!

⇒ the anomaly is a bordism invariant !

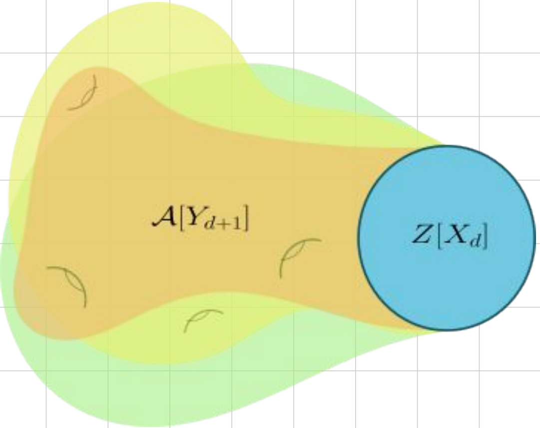


The two d-dimensional manifolds can be deformed into each other
-> They are in the same bordism class!

[García-Etxebarria, Montero '18]

GLOBAL ANOMALIES

⇒ the anomaly is a bordism invariant !



To determine the presence of a global anomaly:

Compute the (d+1)-dimensional bordism group for your theory

1. If it is non-trivial:

   a. determine its bordism invariant(s) and the corresponding generating manifold(s)

   b. Evaluate the anomaly theory on the generating manifold to get the anomaly

2. If it is trivial,
   You're done! There are no anomalies

RELEVANT BORDISM GROUPS

So then we have to start by computing the 11-d bordism groups for these theories

So what **cobordism groups** are the relevant ones for these three theories?

All three theories only make sense on backgrounds that satisfy the non-trivial **Bianchi identity** associated to H :

$$dH \sim \mathrm{tr}F^2 - \mathrm{tr}R^2 = 0$$

➡ **twisted string bordism**

Not many of them are known, we computed

$$\Omega_{11}^{string-\mathrm{Sp}(16)}, \quad \Omega_{11}^{string-\mathrm{Spin}(16)^2}, \quad \Omega_{11}^{string-U(32)}$$

using the Adams spectral sequence.

RESULTS

So what are the groups??

$$\underbrace{\Omega_{11}^{string-\mathrm{Sp}(16)} = 0 \quad \Omega_{11}^{string-\mathrm{Spin}(16)^2}} = 0 \quad \Omega_{11}^{string-U(32)} = 0 \text{ or } \mathbb{Z}_2$$

But the 3 bordism groups are trivial and so there are no cobordism invariants

i.e. GLOBAL ANOMALIES VANISH on any background for these 10d non-supersymmetric, non-tachyonic string theories

Huge consistency check!

There might (or not) be an anomaly for the Sagnotti string

We do not know the generator of the would-be non-trivial class

So we cannot evaluate the anomaly theory on it.

Maybe you are more crafty than we are?

PLAN

Goal: Bottom-up methods for understanding the 10D non-supersymmetric, non-tachyonic string theories

❶

Local Anomaly Cancellation
&
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❷

Global Gauge Anomaly Cancelation
&
Bordisms

❸

No Global Symmetries
&
new extended objects

Questions??

NO GLOBAL SYMMETRIES

The fact that there should be no global symmetries in QG is well-tested (sometimes proven) conjecture

When applied to topological charges in the compactification manifold of a string (or M) theory, it is known as:

The Cobordism Conjecture
[McNamara, Vafa '19]

All cobordism classes must be trivial in QG

$$\Omega_p = 0$$

The exact same bordism groups as for anomalies! But in smaller dimensions

The reason is that if $\Omega_p \neq 0$, then there is at least one closed, compact p-manifold that carries a sort of topological charge (the cobordism invariant):

$$Q \sim \int_{M_p} [\text{Topological OP}]_p$$

(can also be a lot more exotic than that)

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(can also be a lot more exotic than that)

Compactify on M^p and you get a (D-p-1)-form global symmetry!💀💀💀💀

You gotta **break it** or **gauge it**

Break symmetry with a new (D-p-1)-dimensional defect!

Gauge it: new consistency conditions for compactification of your theory
→ refine your notion of bordism

NO GLOBAL SYMMETRIES

So if we had non-vanishing lower-dimensional bordism groups, we impose the breaking or gauging of these global symmetries!

We'd either learn these theories have **new (D-p-1)-dimensional objects,**

Or discover **new consistency conditions** about the theories themselves!

We know these theories make sense on backgrounds that satisfy the non-trivial Bianchi identity, so we again consider twisted-string bordism groups:

A ton of classes to kill !!

k	$\Omega_k^{\text{String}-Spin(16)^2}$	$\Omega_k^{\mathbb{G}_{16,16}}$	$\Omega_k^{\text{String}-Sp(16)}$	$\Omega_k^{\text{String-}SU(32)\langle c_3\rangle}$
0	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
1	$\mathbb{Z}_2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
2	$\mathbb{Z}_2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
3	0	$\mathbb{Z}_8$	0	0
4	$\mathbb{Z}^2$	$\mathbb{Z} \oplus \mathbb{Z}_2$	$\mathbb{Z}$	$\mathbb{Z}$
5	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
6	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0 or $\mathbb{Z}_2$
7	0	$\mathbb{Z}_{16}$	$\mathbb{Z}_4$	$\mathbb{Z}_2$ or $\mathbb{Z}_4 \oplus \mathbb{Z}_2$
8	$\mathbb{Z}^6$	$\mathbb{Z}^3 \oplus \mathbb{Z}_2^i$	$\mathbb{Z}^3 \oplus \mathbb{Z}_2$	$\mathbb{Z}^3 \oplus \mathbb{Z}_2$ or $\mathbb{Z}^3 \oplus \mathbb{Z}_2^2$
9	$\mathbb{Z}_2^5$	$\mathbb{Z}_2^j$	$\mathbb{Z}_2^3$	$\mathbb{Z}_2^3$
10	$\mathbb{Z}_2^7$	$\mathbb{Z}_2^k$	$\mathbb{Z}_2^3$	$\mathbb{Z} \oplus \mathbb{Z}_2^2$ or $\mathbb{Z} \oplus \mathbb{Z}_2^3$
11	0	A	0	0 or $\mathbb{Z}_2$

NO GLOBAL SYMMETRIES

Homework Sheet

Identify generating manifold for each non trivial group

Find a reason why it is not a consistent compactification OR find new object that kills the class!

(Good luck 💅)

$$\Omega_0^{\text{String-}Sp(16)} \cong \mathbb{Z} \qquad \Omega_6^{\text{String-}Sp(16)} \cong \mathbb{Z}_2$$
$$\Omega_1^{\text{String-}Sp(16)} \cong \mathbb{Z}_2 \qquad \Omega_7^{\text{String-}Sp(16)} \cong \mathbb{Z}_4$$
$$\Omega_2^{\text{String-}Sp(16)} \cong \mathbb{Z}_2 \qquad \Omega_8^{\text{String-}Sp(16)} \cong \mathbb{Z}^{\oplus 3} \oplus \mathbb{Z}_2$$
$$\Omega_3^{\text{String-}Sp(16)} \cong 0 \qquad \Omega_9^{\text{String-}Sp(16)} \cong (\mathbb{Z}_2)^{\oplus 3}$$
$$\Omega_4^{\text{String-}Sp(16)} \cong \mathbb{Z} \qquad \Omega_{10}^{\text{String-}Sp(16)} \cong (\mathbb{Z}_2)^{\oplus 3}$$
$$\Omega_5^{\text{String-}Sp(16)} \cong \mathbb{Z}_2 \qquad \Omega_{11}^{\text{String-}Sp(16)} \cong 0.$$

Example: Sugimoto

On the quest to characterizing these new extended objects (a non-exhaustive list):
[Andriot, Angius, Blumenhagen, Buratti, Carqueville, Cribiori, Calderon-Infante, DeBiasio, Debray, Delgado, Dierigl, Friedrich, Garcia-Etxebarria, Hebecker, Heckman, Huertas, Kneissl, Makridou, Montero, McNamara, Lust, Torres, Uranga, Vafa, Valenzuela, Velazquez, Walcher, Wang...'19-'24]

CONCLUSIONS

Goal: **Bottom-up** methods for understanding the 10D non-supersymmetric, non-tachyonic string theories

❶

Local Anomaly Cancellation

told us about the worldvolume theory of NS5 branes in SO(16)xSO(16) via **anomaly inflow**

✅

❷

Global Gauge Anomaly Cancelation

was proven for these 3 theories* , through the computation the **11D bordism groups**

✅

❸

No Global Symmetries (Cobordism Conjecture)

Gave us a told of new consistency conditions and extended objects to look for, through the computation of **lower-dimensional bordism groups**

✅

LAST REMARK

These three 10D non-supersymmetric string theories are a great arena to **test QG away from the supersymmetric lamppost**

They pass every consistency check so far!

It's surprising that nobody checked global anomaly cancellation in the last 25 years,

There might be other accessible things to discover about them!

(beyond the homework sheet)

THANKS!