String perturbation theory in Ramond-Ramond backgrounds

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Based on Minjae Cho, MK 2311.04959

Surveying the Landscape

Summary of the talk

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- We found background solutions in SFT that corresponds to GKP type flux compactifications with small superpotential.
- With this "worldsheet" description, we can now compute stringy amplitudes in GKP backgrounds

Plan of the talk

- Introduction
- What is string field theory?
- $\bullet\,$ Review of GKP
- SFT for GKP with small flux superpotential
- Conclusions

Chapter 0: Introduction

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- We should look forward and start thinking about how to do string theory in cosmological solutions

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 - Famously, RR backgrounds are notoriously difficult to study in RNS formalism
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- This suggests that we may need to go beyond the first quantization approach to string theory
- Let's briefly review why it is difficult to study RR backgrounds in RNS formalism

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- But, RR vertex opeartors have half-integer picture numbers.
- Thus there is no way one can deform the worldsheet just by using RR fluxes.
- Furthermore, any attempts to do so will induce a non-local deformation

$$g_s^2 \int_{\Sigma} d^2 z_1 V_{RR}^{(-1/2,-1/2)}(z_1) \int_{\Sigma} d^2 z_2 V_{RR}^{(1/2,1/2)}(z_2) \,.$$

• But, this does not yet imply that we cannot compute scatterings in background field method

- Because scattering amplitudes involving RR fields are well defined, one can still attempt to compute scattering amplitudes in RR backgrounds
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- This is probably the right thing to do, if one wants to extend string perturbation theory to cosmology.
- We should use string field theory!

Chapter 1: What is string field theory?

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- The SFT action, if there is a target-space interpretation, provides target space action of fields.

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• Then, one can construct string field Ψ , by

$$\Psi = Tc\bar{c}e^{ik\cdot X} + \epsilon_{\mu\nu}c\bar{c}\partial X^{\mu}\bar{\partial}X^{\nu}e^{ik\cdot X} + \dots ,$$

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• Crucially, in SFT, on-shell condition is not imposed and k can take an arbitrary value.

• With the string field, the goal is to construct an off-shell action

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- What about interaction vertices?
- The idea is to read off Feynmann vertices from off-shell scattering amplitudes

Three-point vertex

• The three point vertex is determined by the following off-shell amplitude



• $\{\Psi^3\}$ is a complicated function of polarization/string fields.

- To compute the four-point vertex, we need to do a little more work.
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- We expect that some contributions to the four-point amplitude come from joining three-point vertices
- The goal is to isolate the contribution that comes purely from the four-point vertex

• We can put z at a generic point



• For generic z, we have a four-point vertex contribution

• We can bring z to 0



• When z is close to 0, we have t-channel

• We can bring z to ∞



• When z is close to ∞ , we have s-channel

- To find the four-point vertex contribution, we can excise local coordinate charts around 0, 1, ∞
- and integrate over z away from the blue regions



• Different choices of local coordinates correspond to field redefinitions

• Finally, we have constructed string field action

$$S(\Psi) = -\frac{1}{2g_s^2} \langle \Psi | c_0^- Q_B | \Psi \rangle + \sum_{N,g} \frac{g_s^{2-2g+N}}{N!} \{ \Psi^N \}_{\Sigma_g} \,.$$

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- The action satisfies the BV master equation, and therefore gauge invariant.
- In essence, SFT as we know is a self-consistent set of rules that allows off-shell computations in string perturbation theory
- The SFT action involves infinitely many terms for infinitely many field. So, we should carefully choose a problem

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- As we finally have SFT for all superstring theories, SFT is ripe for applications.
- As the first step, let us study flux compactification in type IIB string theory

Chapter 2: Review of GKP background.

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$$S_{bulk} \supset -\frac{1}{4\kappa_{10}^2} \int_{\mathbb{R}^{1,3} \times X/\mathcal{I}} d^{10}X \sqrt{-G} \left(\frac{|H_3|^2}{g_s^2} + |F_3|^2 \right) \,, \ S_{D3/O3} \supset \sum_i -\mu_3 Q_i \int_{\mathbb{R}^{1,3}} d^4x \sqrt{-G} \frac{1}{g_s} \int_{\mathbb{R$$

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• One can massage the above equations to obtain

$$S \supset -\frac{1}{2\kappa_{10}^2} \int_{\mathbb{R}^{1,3}} d^4 X \left[\int d^6 X \sqrt{-G} \frac{G_- \cdot \bar{G}_-}{\mathrm{Im}\tau} \right] ,$$
$$\int_{X/\mathcal{I}} H \wedge F + N_{D3} = Q_{D3} , \ G_3 := F_3 - \frac{i}{g_s} H_3 , \ G_- := G_3 + i \star_6 G_3 .$$

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- Therefore, quantized fluxes H_3 and F_3 induce potential for z and $1/g_s$.
- At the minimum of the potential, one finds

$$-\star_6 \frac{H_3}{g_s} = F_3 \,.$$

Chapter 3: SFT for GKP.

Goal

- Today we will find the background solution $\equiv B$ in string field theory for GKP backgrounds
- and show that vacua with small flux superpotential admit double scaling expansion





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, BCFT: $D^2 \to \text{Dp-branes}$, $\mathbb{RP}^2 \to \text{Op-planes}$

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• With this CFT, we can construct SFT that involves infinitely many terms for infinitely many fields

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• We want to turn on quantized fluxes F_3 , H_3 in SFT to find a nearby vacuum

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- Hence, it is absolutely crucial that the problem we want to solve is of a perturbative nature.
- But the background fluxes background fluxes H_3 , F_3 are quantized
- If deformation by H_3 , F_3 is not "small," we cannot solve eom in SFT in a reasonable manner

$$\delta \Psi = c \bar{c} H_{ijk} Y^i e^{-\phi} \psi^j e^{-\bar{\phi}} \bar{\psi}^k + g_s c \bar{c} e^{-\phi/2} \Sigma_\alpha F^{\alpha\beta} e^{-\bar{\phi}/2} \overline{\Sigma}_\beta \,.$$

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- This is a very concerning situation.
- H_{ijk} and $F^{\alpha\beta}$ are quantized fluxes. So, we cannot treat them as small numbers.
- Naively, this seems to suggest that we cannot treat quantized fluxes as a perturbation.

• Let's look at OPEs of the worldsheet fields

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• Following Demirtas, MK, McAllister, Moritz 19 (PFV), one can choose H and F such that

$$\mathcal{O}\left(H_{ijk}Y^{i}e^{-\phi}\psi^{j}e^{-\bar{\phi}}\bar{\psi}^{k}\right) = \mathcal{O}(z^{-1/2}), \ \mathcal{O}\left(g_{s}e^{-\phi/2}\Sigma_{\alpha}F^{\alpha\beta}e^{-\bar{\phi}/2}\overline{\Sigma}_{\beta}\right) = \mathcal{O}(g_{s}z^{1/2})$$

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• By taking the following double scaling expansion

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Solving EOM perturbatively

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• In this talk, we will study eom up to the second order

$$\begin{split} Q_B |\Psi_1\rangle &= 0\,,\\ Q_B |\Psi_2\rangle &= \frac{1}{2}\left[\Psi_1^2\right]_{S^2} + \left[\!\left]_{D^2 + \mathbb{RP}^2}\right., \end{split}$$

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- Let's define a projection operator \mathbb{P} that projects states to $L_0^+ := L_0 + \overline{L}_0$ nilpotent (massless) states
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$$Q_B \mathbb{P} |\Psi_2\rangle = \frac{1}{2} \mathbb{P} \left[\Psi_1^2 \right]_{S^2} + \mathbb{P} \left[\right]_{D^2 + \mathbb{R} \mathbb{P}^2}$$
$$Q_B (1 - \mathbb{P}) |\Psi_2\rangle = \frac{1}{2} (1 - \mathbb{P}) \left[\Psi_1^2 \right]_{S^2} + (1 - \mathbb{P}) \left[\right]_{D^2 + \mathbb{R} \mathbb{P}^2}$$

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• Note that b_0^+/L_0^+ corresponds to the Green's function in target space.

• Let's study the L_0^+ nilpotent part of the second-order eom

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- Because Q_B is not an invertible operator for L_0^+ nilpotent states, one needs to do an actual work here.
- The goal is to show that the right-hand side of the eom is Q_B -exact.
- After CFT gymnastics, one arrives at

$$\begin{aligned} Q_B \mathbb{P} |\Psi_2\rangle_{NSNS} &= \sum_i C_i T_3 \frac{1}{\operatorname{Vol}_{X/\mathcal{I}}} (\partial c + \bar{\partial} \bar{c}) D_{gh} + \mathcal{V} \left(\frac{\partial}{\partial g_s} V_F, \frac{\partial}{\partial G^{ij}} V_F, \int H \wedge F_3 + N_{D3} - Q_{D3} \right) \\ Q_B \mathbb{P} |\Psi_2\rangle_{RR} &= \mathcal{V}' \left(\int H \wedge F_3 + N_{D3} - Q_{D3} \right) \end{aligned}$$

 ${\mathcal V}$ and ${\mathcal V}'$ are linear combinations of derivatives of the F-term potential and the tadpole constraint

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• So, provided that the tadpole cancellation holds, and we tune the moduli such that V_F is minimized

$$Q_B \mathbb{P} |\Psi_2\rangle_{NSNS} = \sum_i C_i T_3 \frac{1}{\operatorname{Vol}_{X/\mathcal{I}}} (\partial c + \bar{\partial}\bar{c}) D_{gh} , \ Q_B \mathbb{P} |\Psi_2\rangle_{RR} = 0$$

• Hence, to show that the solution exists we need to find Ψ_2 that solves

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• And we found $\mathbb{P}|\Psi_2\rangle$

$$\frac{4\alpha'}{g_c^2} \mathbb{P}\Psi_2 = -\frac{\pi}{18\kappa_{10}^2 g_s^2 \epsilon} c\bar{c} \bigg(B_{ab} B^{ab} (\eta \bar{\partial} \bar{\xi} e^{-2\bar{\phi}} - \partial \xi \bar{\eta} e^{-2\phi}) - 2B_{ac} B^{cb} e^{-\phi} \psi^a e^{-\bar{\phi}} \bar{\psi}_b - 2i \sqrt{\frac{\alpha'}{2}} B_{ab} H^{abc} (\partial c + \bar{\partial} \bar{c}) \left(e^{-\phi} \psi_c e^{-2\bar{\phi}} \bar{\partial} \bar{\xi} + e^{-\bar{\phi}} \bar{\psi}_c e^{-2\phi} \partial \xi \right) \bigg).$$

Conclusions

- String field theory provides an *easy to use* framework to study RR backgrounds.
- Provided that sugra solutions are well controlled, finding SFT counterpart isn't very difficult.
- Using the background solution in SFT, one can now compute amplitudes in RR backgrounds.

- $\bullet\,$ We are computing tree-level/one-loop amplitudes to learn how to compute amplitudes in flux backgrounds $_{Minjae}$ Cho, MK $_{24xx,xxxxx}$
- Generalization to Calabi-Yau orientifold compactifications?
- One can also study flux compactifications in type IIA, heterotic string theories.
- Probably there are many more exciting directions! If you are interested, let's chat!