AMHERST CENTER FOR FUNDAMENTAL INTERACTIONS

Physics at the interface: Energy, Intensity, and Cosmic frontiers

University of Massachusetts Amherst

Current and Future Status of First-Row CKM Unitarity

UMass Amherst, 17 May 2019

The status of V_{us}

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V_{us}, CKM unitarity, gauge universality

Standard-model coupling of quarks and leptons to W:

$$\frac{g}{\sqrt{2}}W^+_\alpha \left(\overline{\mathbf{U}}_L \mathbf{V}_{\rm CKM}\gamma^\alpha \mathbf{D}_L + \bar{e}_L\gamma^\alpha\nu_{e\,L} + \bar{\mu}_L\gamma^\alpha\nu_{\mu\,L} + \bar{\tau}_L\gamma^\alpha\nu_{\tau\,L}\right) \quad + \quad \text{h.c.}$$

↑ *Single gauge coupling*

↑ *Unitary matrix*

$$|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} = 1$$

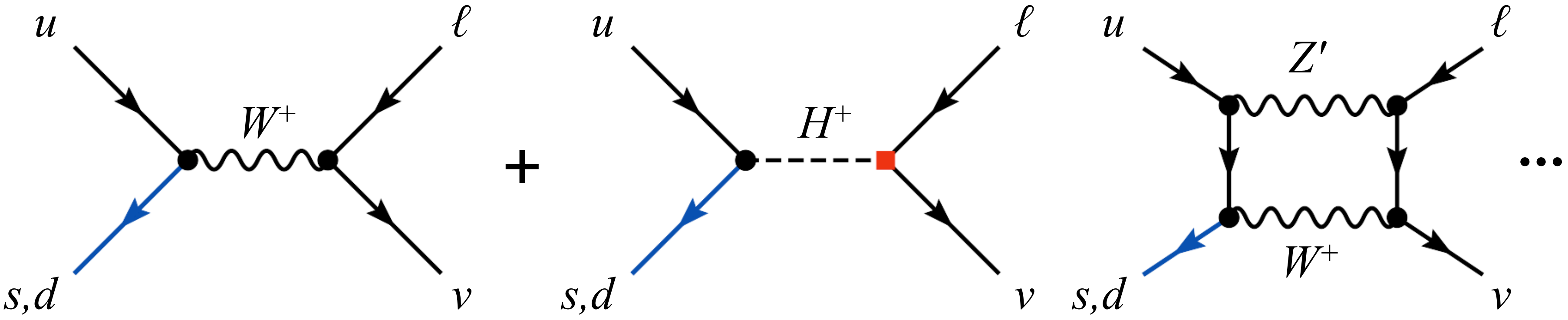
$\approx 2\times10^{-5}$

Most precise test of CKM unitarity

Universality: Is G_F from μ decay equal to G_F from π, K, nuclear β decay?

$$G_\mu^2 = (g_\mu g_e)^2/M_W^4 \quad \overset{?}{=} \quad G_{\rm CKM}^2 = (g_q g_\ell)^2\left(|V_{ud}|^2 + |V_{us}|^2\right)/M_W^4$$

Physics beyond the Standard Model can break gauge universality:



Experimental paths to V_{us}

1. **Kaon decays**

   $\Gamma(K \to \pi\ell\nu)$ $(K_{\ell3})$

   $\Gamma(K \to \mu\nu)/\Gamma(\pi \to \mu\nu)$ $(K_{\mu2}/\pi_{\mu2})$

   Most precise method: **0.3% on V_{us}**

2. **Tau decays**

   Inclusive tau decays: $\Gamma(\tau \to X_S\nu_\tau)$

   Exclusive tau decays: $\Gamma(\tau \to K\nu_\tau)/\Gamma(\tau \to \pi\nu_\tau)$ $(\tau_{K2}/\tau_{\pi2})$

   Good precision: **0.8% on V_{us}**

   Discrepancy between inclusive/exclusive results?

3. **Hyperon decays**

   $\Gamma(\Lambda \to p\ell\nu)$ etc.

   > 2% uncertainty

   Not actively pursued at the moment

V_{us} from kaon decays

Based on CKM2018 update
in collaboration with Emilie Passemar

V_{us} from kaon decays: A modern history

→ 2002 (2004 PDG)	**Old $K_{\ell 3}$ data give $1 - \vert V_{ud}\vert^2 - \vert V_{us}\vert^2 = 0.0035(15)$** A 2.3$\sigma$ hint of unitarity violation?
2003	**BNL 865 measures BR($K^+ \to \pi^0 e^+ \nu$) = 5.13(10)%** Value for V_{us} consistent with unitarity
2004-2008 (mostly)	**Many new measurements from KTeV, ISTRA+, KLOE, NA48** • **BRs, lifetimes, form-factor slopes** • **Much higher statistics** than older measurements • Importance of **radiative corrections** • Proper reporting of **correlations** between measurements
2008- beyond	**Much progress on hadronic constants from lattice QCD** **Value of V_{us} used in precision tests of the Standard Model**
2018?	**New wave of $K_{\ell 3}$ measurements imminent?** **NA62, OKA, KLOE-2, LHCb, TREK…**

Experiment, theory, and evaluation

V_{us} from $K_{\ell 3}$ & $K_{\ell 2}$ {
~100 measurements of ~10 experimental parameters
50+ (and counting!) lattice results for 2 hadronic matrix elements
Radiative and SU(2)-breaking corrections, ChPT results, etc.

FlaviAnet Kaon WG
2006-2010 (EU 6FP)

Experimental averages, fits, etc
Selection of results (experiments, corrections)
Evaluation, discussion and intepretation
Final report: EPJC 69 (2010) 399
This talk is an attempt at an update to 2019

Corresponding effort to synthesize results from **lattice QCD:**

Flavor Lattice Averaging Group (FLAG):
http://itpwiki.unibe.ch/flag

Participation by all major lattice collaborations
Biannual review of lattice results for π, K, B, D physics
2019 review: arXiv 1902.08191

Determination of V_{us} from $K_{\ell 3}$ data

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{\rm EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \times I_{K\ell}(\lambda_{K\ell}) \left(1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{\rm EM}\right)$$

with $K \in \{K^+, K^0\}$; $\ell \in \{e, \mu\}$, and:

C_K^2	1/2 for K^+, 1 for K^0
$S_{\rm EW}$	Universal SD EW correction (1.0232)

Inputs from experiment:

$\Gamma(K_{\ell 3(\gamma)})$ Rates with well-determined treatment of radiative decays:

- Branching ratios
- Kaon lifetimes

$I_{K\ell}(\{\lambda\}_{K\ell})$ Integral of form factor over phase space: λs parameterize evolution in t

- K_{e3}: Only λ_+ (or λ_+', λ_+'')
- $K_{\mu 3}$: Need λ_+ and λ_0

Inputs from theory:

$f_+^{K^0\pi^-}(0)$	Hadronic matrix element (form factor) at zero momentum transfer ($t = 0$)
$\Delta_K^{SU(2)}$	Form-factor correction for $SU(2)$ breaking
$\Delta_{K\ell}^{\rm EM}$	Form-factor correction for long-distance EM effects

Modern experimental data for V_{us} from $K_{\ell 3}$

Experiment	Measurement	Year
BNL865	BR($K^+ \to \pi^0{}_{\rm D} e^+ \nu$)/BR($K^+ \to \pi^0{}_{\rm D} X^+$)	**2003**
KTeV	$\tau(K_S)$	**2003**
	BR(K_{Le3}), BR($K_{L\mu3}$), $\lambda_+(K_{Le3})$, $\lambda_{+,0}(K_{L\mu3})$	**2004**
ISTRA+	$\lambda_+(K^-{}_{e3})$, $\lambda_{+,0}(K^-{}_{e3})$	**2004**
KLOE	$\tau(K_L)$	**2005**
	BR(K_{Le3}), BR($K_{L\mu3}$), BR(K_{Se3}), $\lambda_+(K_{Le3})$	**2006**
	$\lambda_{+,0}(K_{L\mu3})$	**2007**
	$\tau(K^\pm)$, BR(K_{Le3}), BR($K_{L\mu3}$)	**2008**
NA48	$\tau(K_S)$	**2002**
	BR(K_{Le3}/2 tracks), $\lambda_+(K_{Le3})$	**2004**
	$\Gamma(K_{Se3}/K_{Le3})$, $\lambda_{+,0}(K_{L\mu3})$	**2007**
NA48/2	BR($K^+{}_{e3}/\pi^+\pi^0$), BR($K{+}_{\mu3}/\pi^+\pi^0$)	**2007**

Above data set used for 2010 FlaviaNet review (fits, averages, etc.)

Fit to K_S rate data

6 input measurements:

- **KLOE** BR $\pi^0\pi^0/\pi^+\pi^-$
- **KLOE** BR $\pi e\nu/\pi^+\pi^-$
- **NA48** $\Gamma(K_S \to \pi e\nu)/\Gamma(K_L \to \pi e\nu)$, τ_S
- **KLOE '11** $\boldsymbol{\tau_S}$
- **KTeV '11** $\boldsymbol{\tau_S}$

2 constraints:

- **Σ BR = 1**
- **BR(K_{e3})/BR($K_{\mu3}$) = 0.66492(137)**

  From ratio of phase-space integrals from current fit to dispersive $K_{\ell3}$ form factor parameters

Parameter	Value
BR($\pi^+\pi^-(\gamma)$)	**69.20(5)%**
BR($\pi^0\pi^0$)	**30.69(5)%**
BR(K_{e3})	**$7.05(8) \times 10^{-3}$**
BR($K_{\mu3}$)	**$4.69(6) \times 10^{-3}$**
$\boldsymbol{\tau_S}$	**89.58(4) ns**

χ^2/ndf = 0.20/3 (Prob = 98%)

ρ(BR($\pi^+\pi^-$), BR($\pi^0\pi^0$)) = –0.998

Little freedom in fit

Largest effect of **2011 τ_S data:**

FlaviaNet 2010
τ_S = 89.59(6) ps

➡

Update
τ_S = 89.58(4) ps

Fit to K_L rate data

21 input measurements:

- **5 KTeV** ratios
- **NA48** BR(K_{e3}/2 track)
- **4 KLOE** BRs
  with dependence on τ_L
- **KLOE**, **NA48** BR($\pi^+\pi^-/K_{\ell 3}$)
- **KLOE**, **NA48** BR($\gamma\gamma/3\pi^0$)
- **BR($2\pi^0/\pi^+\pi^-$) from K_S fit, Re ε'/ε**
- **KLOE** τ_L from $3\pi^0$
- **Vosburgh '72** τ_L
- **KTeV** BR($\pi^+\pi^-\gamma/\pi^+\pi^-(\gamma)$)
- **E731, 2 KTeV** BR($\pi^+\pi^-\gamma_{\mathrm{DE}}/\pi^+\pi^-\gamma$)

1 constraint: Σ BR = 1

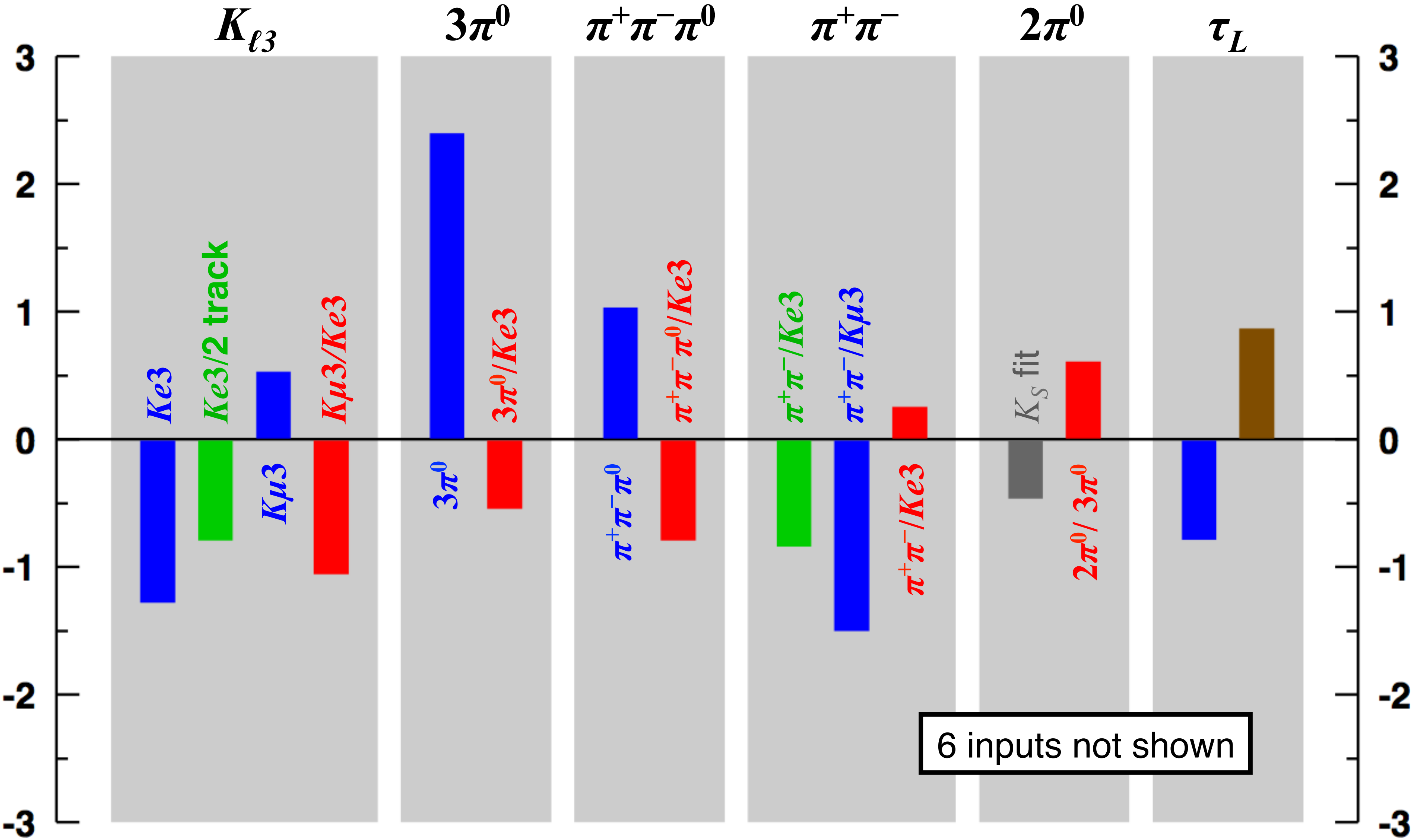
Parameter	**Value**	***S***
BR(K_{e3})	**0.4056(9)**	**1.3**
BR($K_{\mu 3}$)	**0.2704(10)**	**1.5**
BR($3\pi^0$)	**0.1952(9)**	**1.2**
BR($\pi^+\pi^-\pi^0$)	**0.1254(6)**	**1.3**
BR($\pi^+\pi^-(\gamma_{\mathrm{IB}})$)	**$1.967(7) \times 10^{-3}$**	**1.1**
BR($\pi^+\pi^-\gamma$)	**$4.15(9) \times 10^{-5}$**	**1.6**
BR($\pi^+\pi^-\gamma_{\mathrm{DE}}$)	**$2.84(8) \times 10^{-5}$**	**1.3**
BR($2\pi^0$)	**$8.65(4) \times 10^{-4}$**	**1.4**
BR($\gamma\gamma$)	**$5.47(4) \times 10^{-4}$**	**1.1**
τ_L	**51.16(21) ns**	**1.1**

χ^2/ndf = 19.8/12 (Prob = 7.0%)

Essentially same result as 2010 fit

Current PDG ('09): **37.4/17** (0.30%)

Comparison: K_L fit result vs. input data



Measurement pulls for KTeV KLOE NA48 K_S fit Vosburgh '72

Updates: $K^{\pm}$ BRs and lifetimes

KLOE-2
PLB 738 (2014)

BR($\pi^{+}\pi^{+}\pi^{-}$) = 0.05565(31)(25) (0.7%)

- **No good measurements of BR($\pi^{+}\pi^{+}\pi^{-}$) in 2010 fit**
- Reconstruct 2 tracks in small fiducial volume near interaction region; evaluate missing mass for 3rd track
- Fully inclusive of radiation, but radiative corrections handled differently from other KLOE measurements
- Significant impact on value of BR($\mu\nu$) from fit
  Correlation between BR($\mu\nu$), BR($\pi^{+}\pi^{+}\pi^{-}$) = –0.75

ISTRA+
PAN 77 (2014)

BR($K^{-}_{e3}/\pi^{-}\pi^{0}$) = 0.2423(15)(37) (1.6%)

- Claimed to be fully inclusive for $K_{e3\gamma}$
  - No mention of radiative corrections
  - Many cuts, mainly topological
  - 3 different selections, at least 1 may be largely inclusive
- Included in PDG ’15 fit
- **Treated as preliminary here (not in $K^{\pm}$ BR fit)**

Updated fit to $K^{\pm}$ rate data

17 input measurements:

- **3 old τ values in PDG**
- **KLOE τ**
- **KLOE BR $\mu\nu$, $\pi\pi^0$**
- **KLOE BR K_{e3}, $K_{\mu3}$**
  with dependence on τ
- **NA48/2 BR $K_{e3}/\pi\pi^0$, $K_{\mu3}/\pi\pi^0$**
- **E865 BR K_{e3}/KDal**
- **3 old BR $\pi\pi^0/\mu\nu$**
- **KEK-246 $K_{\mu3}/K_{e3}$**
- **KLOE BR $\pi\pi\pi$, $\pi\pi^0\pi^0$**
- (Bisi '65 BR $\pi\pi^0\pi^0/\pi\pi\pi$ removed)

1 constraint: Σ BR = 1

Much more selective than PDG fit
PDG '16: 35 inputs, 8 parameters

Parameter	**Value**	S
BR($\mu\nu$)	**63.58(11)%**	**1.1**
BR($\pi\pi^0$)	**20.64(7)%**	**1.1**
BR($\pi\pi\pi$)	**5.56(4)%**	**1.0**
BR(K_{e3})	**5.088(27)%**	**1.2**
BR($K_{\mu3}$)	**3.366(30)%**	**1.9**
BR($\pi\pi^0\pi^0$)	**1.764(25)%**	**1.0**
$\tau_{\pm}$	**12.384(15) ns**	**1.2**

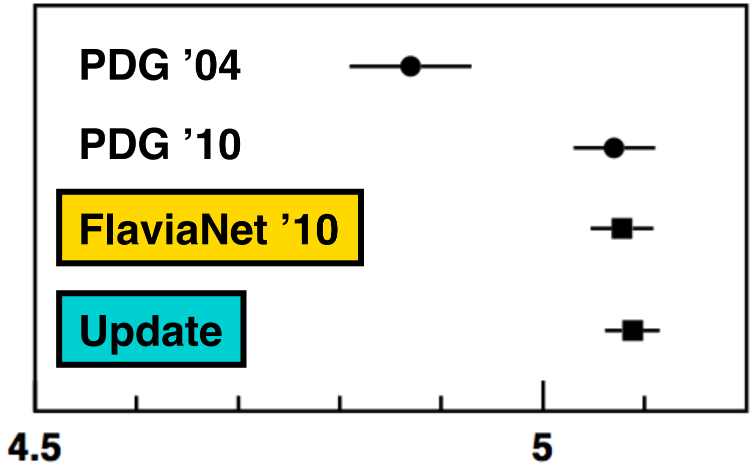
χ^2/ndf = 25.5/11 (Prob = 0.78%)
compare PDG '16: 53/28 (0.26%)

With **ISTRA+ '14 BR($K^-_{e3}/\pi^-\pi^0$)**

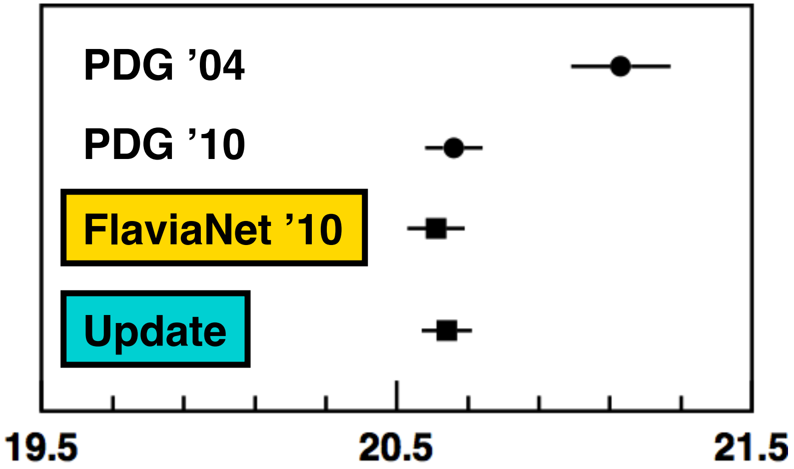
- **BR(K_{e3}) = 5.083(27)%**
- Negligible changes in other parameters, fit quality

Evolution of $K^\pm$ BRs

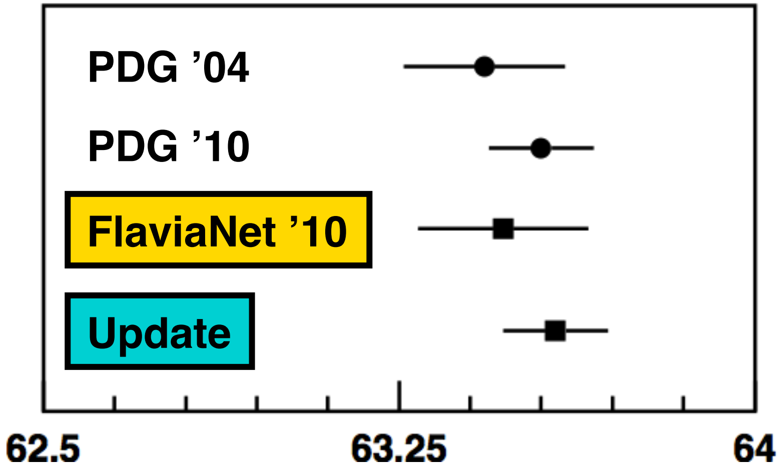
BR($K^\pm \to \pi^0 e \nu$)



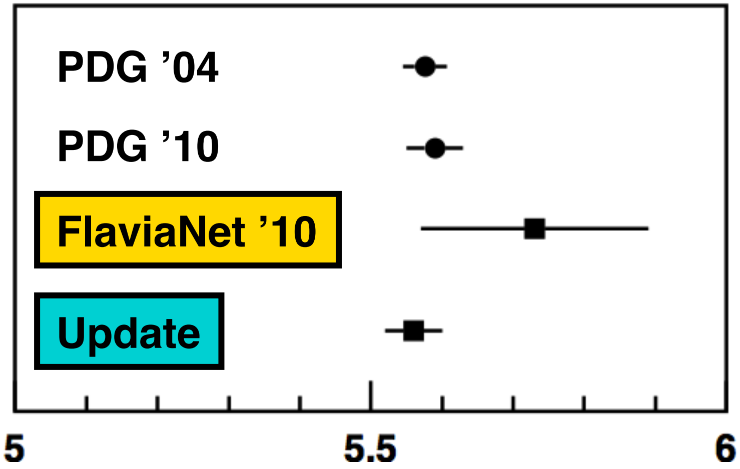
BR($K^\pm \to \pi\pi^0$)



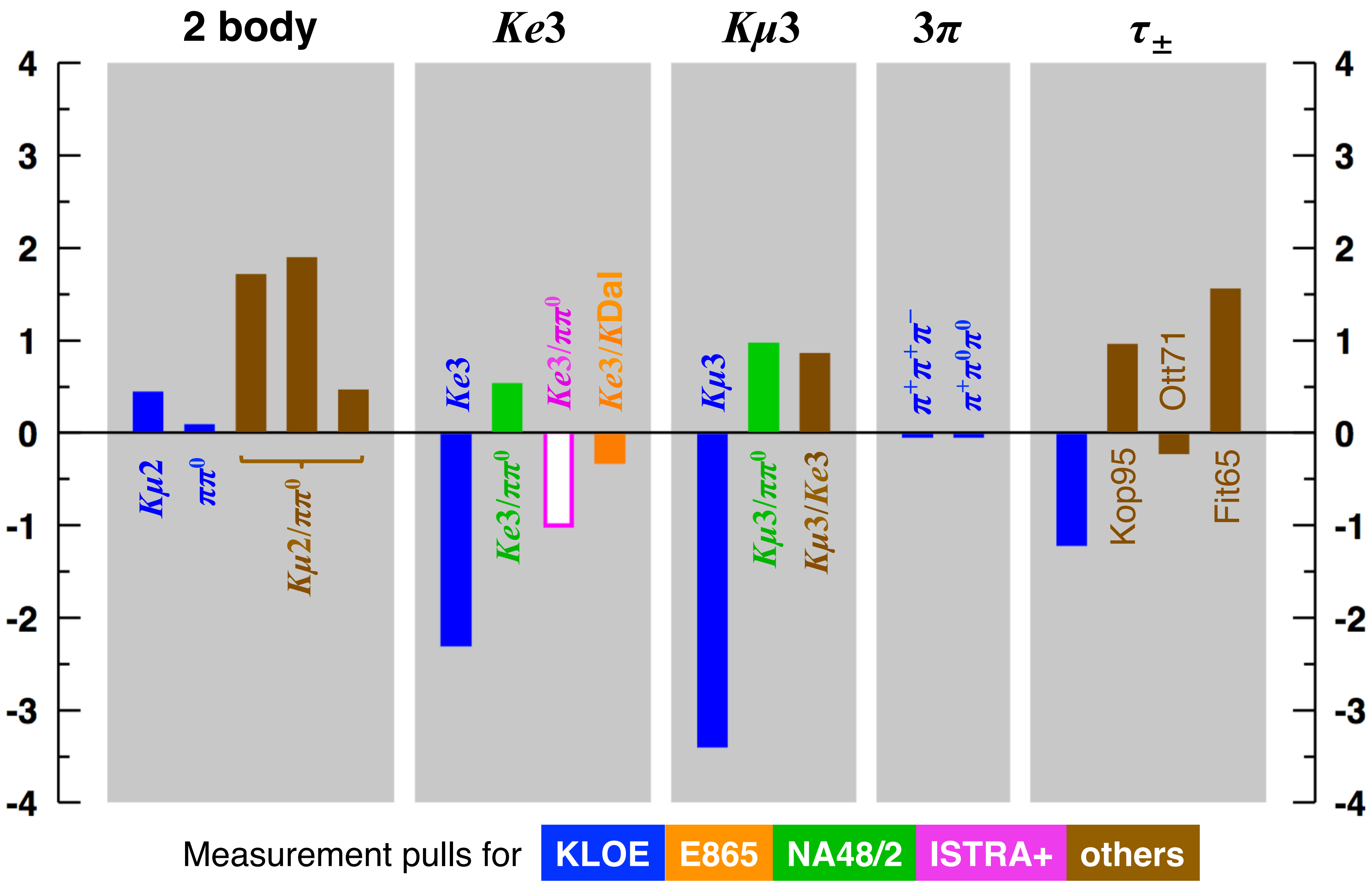
BR($K^\pm \to \mu\nu$)



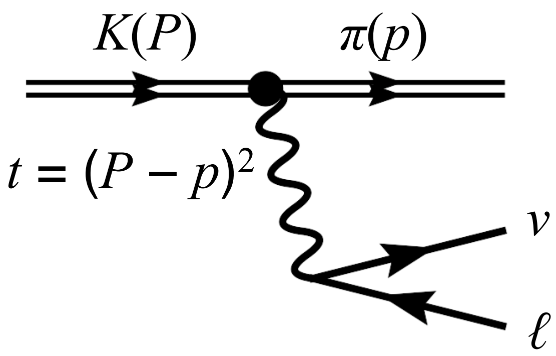
BR($K^\pm \to \pi\pi\pi$)



Comparison: $K^{\pm}$ fit result vs. input data



$K_{\ell 3}$ form factors



Hadronic matrix element:

$$\langle \pi | J_\alpha | K \rangle = f(0) \times \left[\tilde{f}_+(t)(P+p)_\alpha + \tilde{f}_-(t)(P-p)_\alpha \right]$$

K_{e3} decays: Only **vector form factor:** $\tilde{f}_+(t)$

$K_{\mu 3}$ decays: Also need **scalar form factor:** $\tilde{f}_0(t) = \tilde{f}_+ + \tilde{f}_- \dfrac{t}{m_K^2 - m_\pi^2}$

For V_{us}, need integral over phase space of squared matrix element:

Parameterize form factors and fit distributions in t (or related variables)

Parameterizations based on systematic expansions

Taylor expansion:

$$\tilde{f}_{+,0}(t) = 1 + \lambda_{+,0} \left(\frac{t}{m_{\pi^+}^2} \right)$$

$$\tilde{f}_{+,0}(t) = 1 + \lambda'_{+,0} \left(\frac{t}{m_{\pi^+}^2} \right) + \lambda''_{+,0} \left(\frac{t}{m_{\pi^+}^2} \right)^2$$

Notes:

Many parameters: λ_+', λ_+'', λ_0', λ_0''

Large correlations, unstable fits

Higher-order terms ignored

$K_{\ell 3}$ form-factor parameterizations

Parameterizations incorporating physical constraints

Pole dominance: $$\tilde{f}_{+,0}(t) = \frac{M^2_{V,S}}{M^2_{V,S} - t}$$

Notes:

What does M_S correspond to?

Dispersion relations:

$$\tilde{f}_{+}(t) = \exp\left[\frac{t}{m^2_\pi}(\Lambda_+ - H(t))\right]$$

$$\tilde{f}_{0}(t) = \exp\left[\frac{t}{m^2_K - m^2_\pi}(\ln C - G(t))\right]$$

Notes:

Allows tests of ChPT & low-energy dynamics

$H(t)$, $G(t)$ evaluated from $K\pi$ scattering data and given as polynomials

Bernard et al., PRD 80 (2009)

Uncertainties from representations $H(t)$, $G(t)$ of $K\pi$ phase-shift data contribute to fit results for Λ_+, $\ln C$

- Small compared to other uncertainties for single measurements (so far)

2010 FlaviaNet analysis used average of FF parameters from dispersive fits

- Parameterization uncertainties beginning to dominate averages for Λ_+, $\ln C$

$K_{\ell 3}$ form factor data

Form-factor parameter measurements in FlaviaNet 2010 fit:

K_L: **KTeV, KLOE, NA48** (K_{e3} only)

K^-: **ISTRA+**

Even if not in the original publications, all experiments have:

- Obtained results for Taylor, pole, and dispersive parameterizations
- Supplied parameter correlation coefficients

New measurements!

NA48/2
JHEP 1810 (2018)

$2.3 \times 10^6\ K^{\pm}_{\mu 3}$
$4.4 \times 10^6\ K^{\pm}_{e3}$

Updating 2012 preliminary

K^+ and K^- simultaneously acquired in dedicated minimum-bias run

Taylor, pole, and dispersive fits with complete investigation of systematics

OKA
JETPL 107 (2018)

$5.25 \times 10^6\ K^{+}_{e3}$

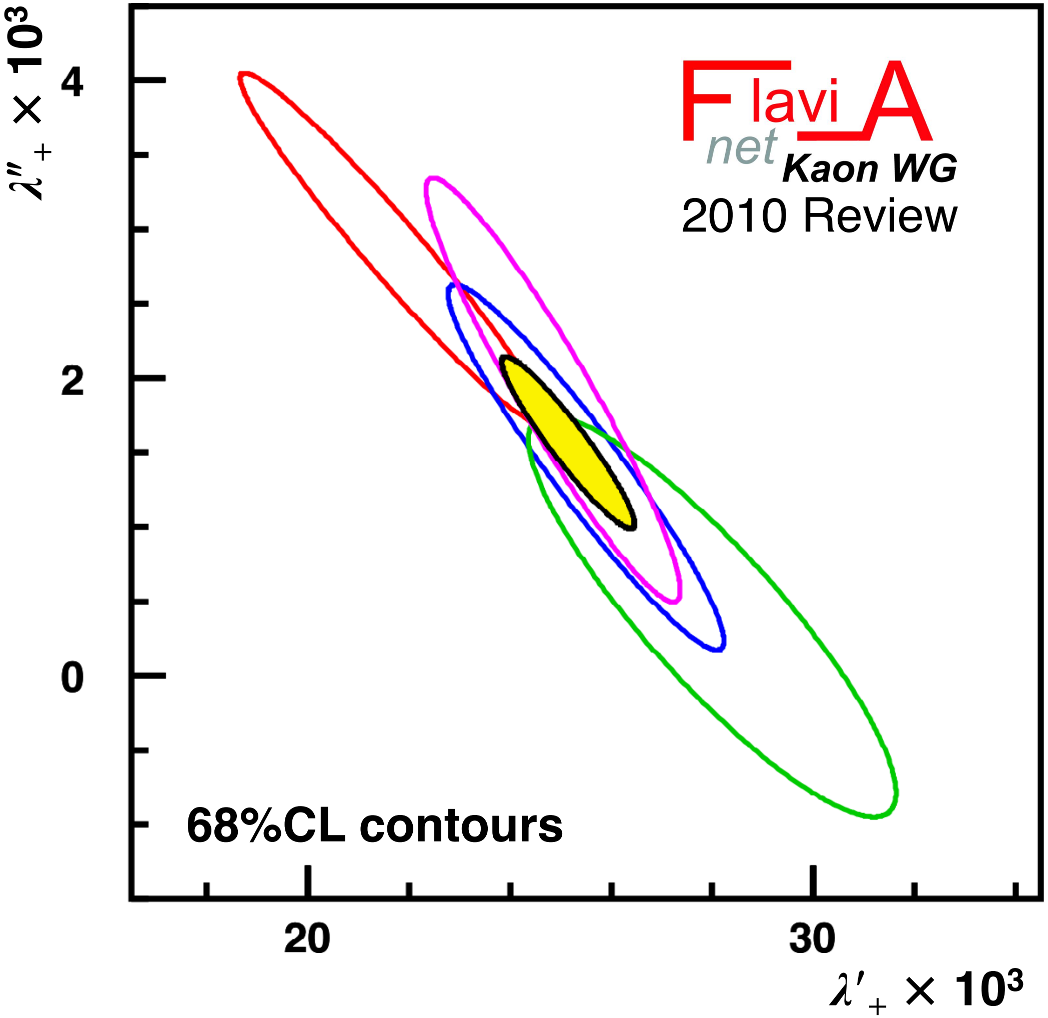
Extraordinarily high precision claimed, esp. for λ_+', λ_+''

Rudimentary discussion of systematics

Not yet included in updated K_{e3} fit

Fit to K_{e3} form-factor slopes: 2010

Slopes from **KTeV** **KLOE** **ISTRA+** **NA48** **2010 fit**



Slope parameters × 10^3

λ'_+ = **25.15 ± 0.87**

λ''_+ = **1.57 ± 0.38**

$\rho(\lambda'_+, \lambda''_+)$ = −0.941

χ^2/ndf = 5.3/6 (51%)

Excellent compatibility

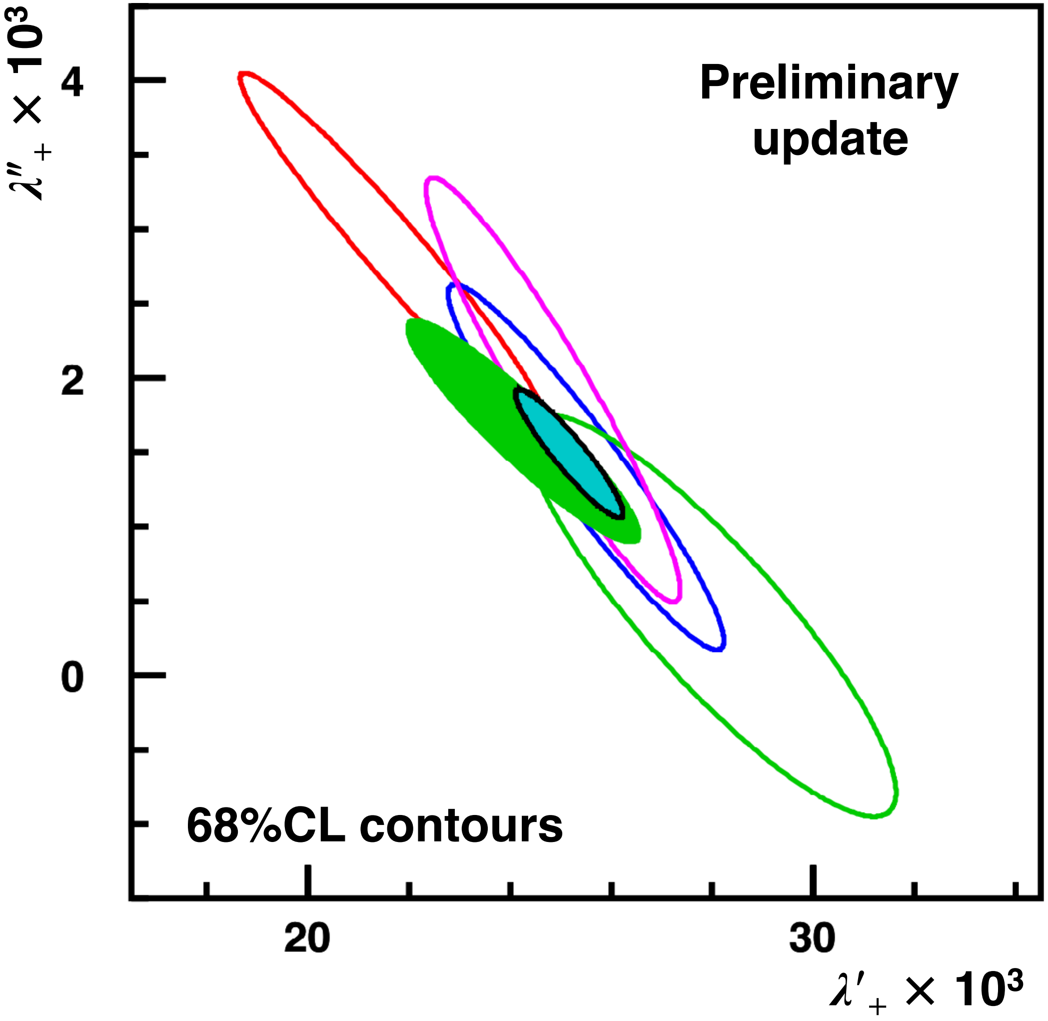
Significance of $\lambda''_+ > 4\sigma$

$I(K^0_{e3})$ = 0.15463(21)

$I(K^+_{e3})$ = 0.15900(22)

Fit to K_{e3} form-factor slopes: Update

Slopes from **KTeV** **KLOE** **ISTRA+** **NA48** **NA48/2** **Update**



Slope parameters × 10^3

λ'_+ = **25.17 ± 0.70**

λ''_+ = **1.49 ± 0.29**

$\rho(\lambda'_+, \lambda''_+)$ = −0.929

χ^2/ndf = 6.4/10 (61%)

Excellent compatibility
Very small change in λ''_+

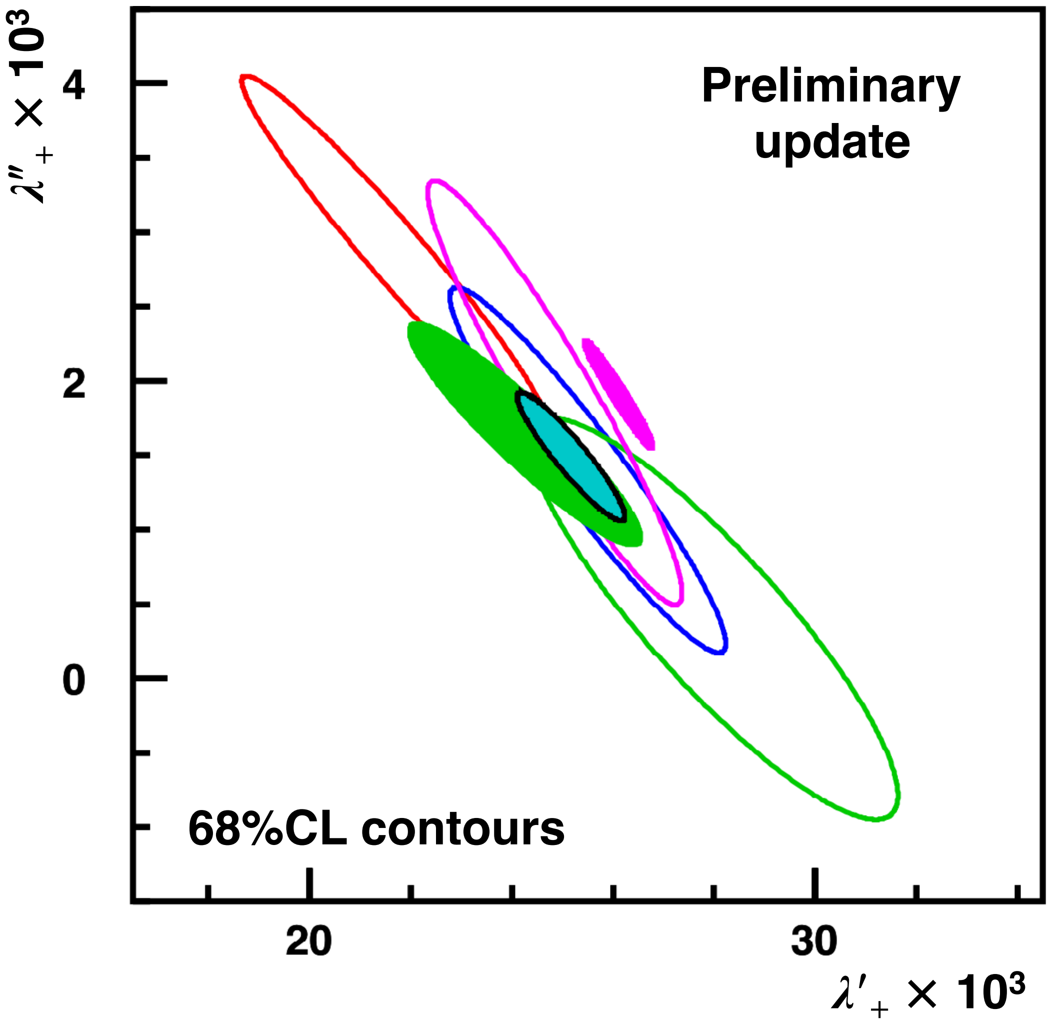
$I(K^0_{e3})$ = **0.15458(19)**

$I(K^+_{e3})$ = **0.15895(20)**

Integrals not significantly changed

Fit to K_{e3} form-factor slopes: Update

Slopes from **KTeV** **KLOE** **ISTRA+** **NA48** **NA48/2** **Update**



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χ^2/ndf = 6.4/10 (61%)

OKA
JETPL 107 (2018)

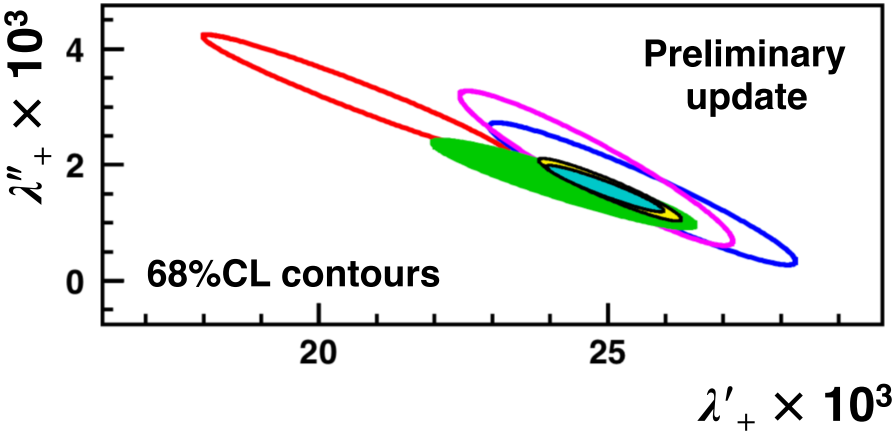
Not included in fit

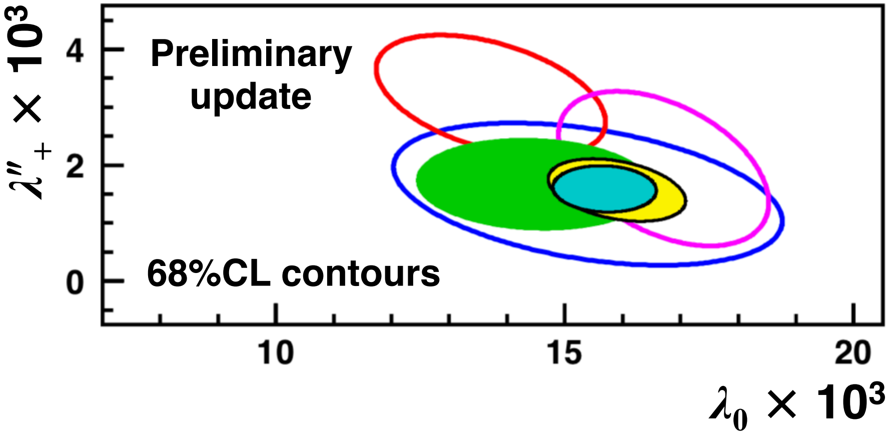
- Stated as preliminary
- If included: χ^2/ndf → 45/10 ($P \sim 10^{-6}$)

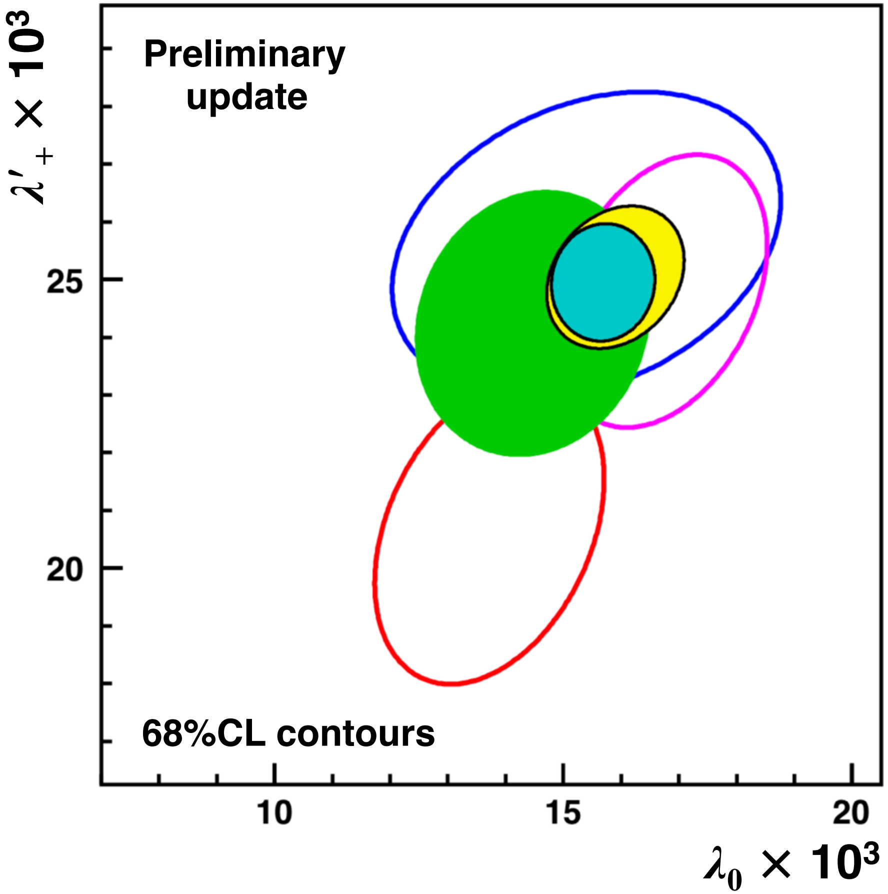
Fits to $K_{e3} + K_{\mu3}$ form-factor slopes: Update

KTeV **KLOE** **ISTRA+** **NA48/2** **2010 fit** **Update**

NA48 K_{e3} data included in fits but not shown







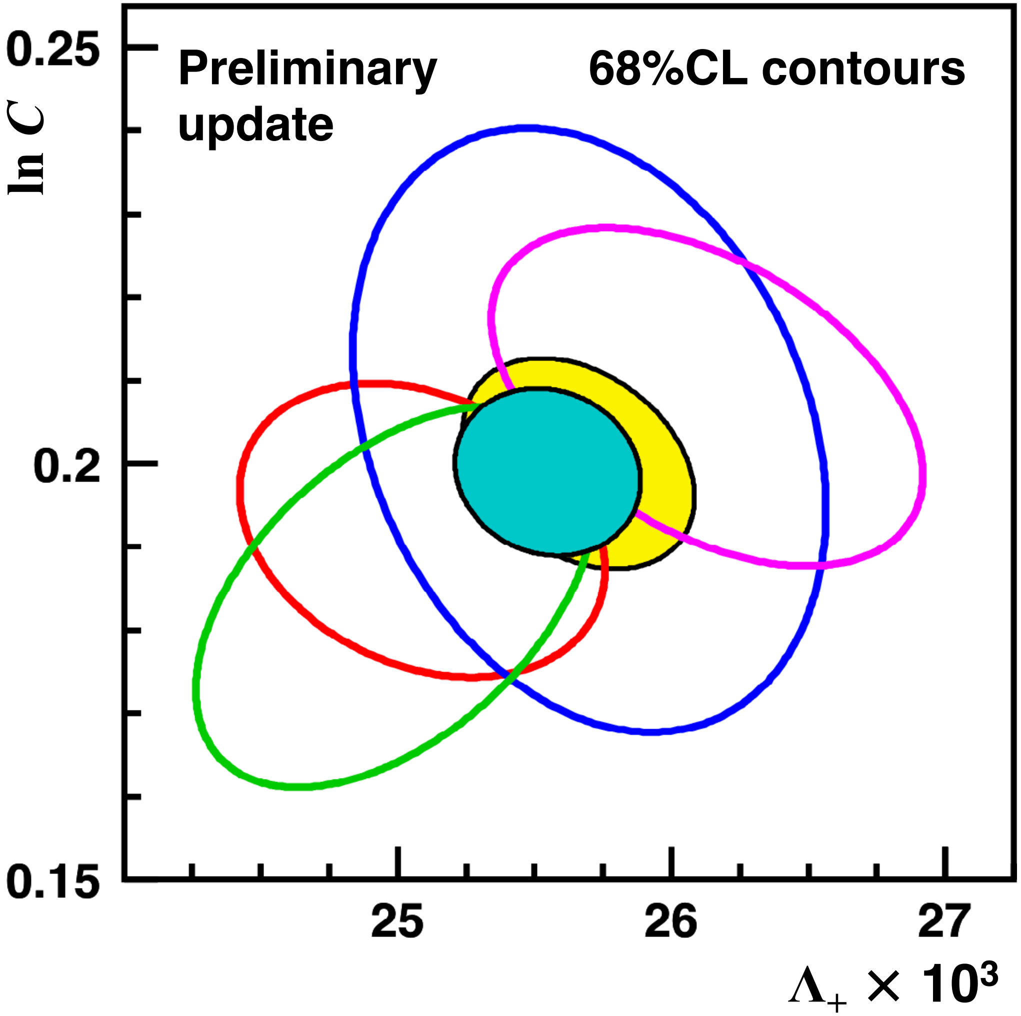
2010: $\chi^2 = 12.1/8$ ($P = 14.5\%$) Update: $\chi^2 = 13.4/11$ ($P = 26.8\%$)

Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2**

NA48 K_{e3} data included in fits but not shown

2010 fit **Update**



$\Lambda_+ \times 10^3$	=	**25.55 ± 0.38**
$\ln C$	=	**0.1992(78)**
$\rho(\Lambda_+, \ln C)$	=	−0.110
χ^2/ndf	=	7.5/7 (38%)

Integrals

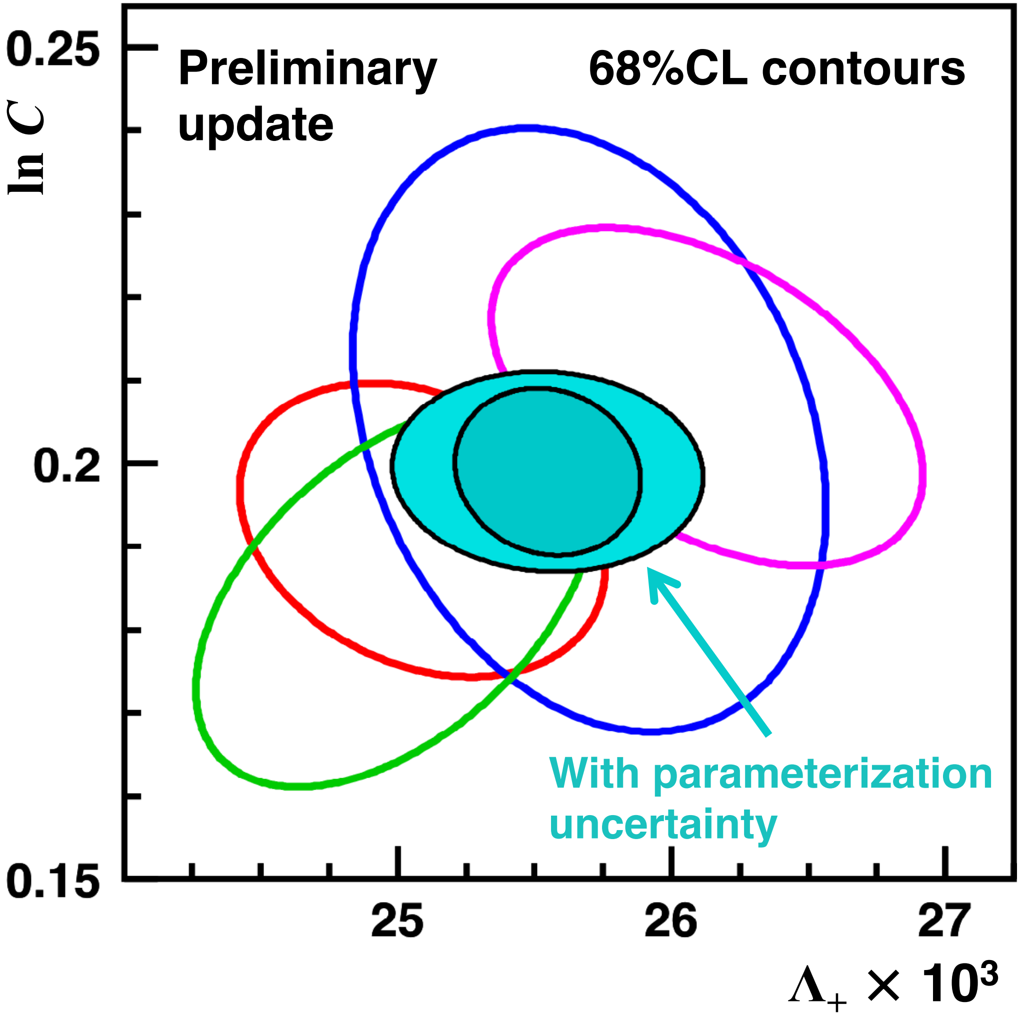
Mode	Update	2010
$K^0{}_{e3}$	**0.15470(15)**	0.15476(18)
$K^+{}_{e3}$	**0.15915(15)**	0.15922(18)
$K^0{}_{\mu 3}$	**0.10247(15)**	0.10253(16)
$K^+{}_{\mu 3}$	**0.10553(16)**	0.10559(17)

Only tiny changes in central values

Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2** **2010 fit** **Update**

NA48 K_{e3} data included in fits but not shown



$$\Lambda_+ \times 10^3 = 25.55 \pm 0.38$$
$$\ln C = 0.1992(78)$$
$$\rho(\Lambda_+, \ln C) = -0.110$$
$$\chi^2/\text{ndf} = 7.5/7\ (38\%)$$

Fit results include common uncertainty from $H(t)$, $G(t)$:

$\sigma_{\text{param}}(\Lambda_+) = 0.3 \times 10^{-3}$

$\sigma_{\text{param}}(\ln C) = 0.0040$

KTeV, Bernard et al. '09

Confidence ellipses shown **without** common uncertainty

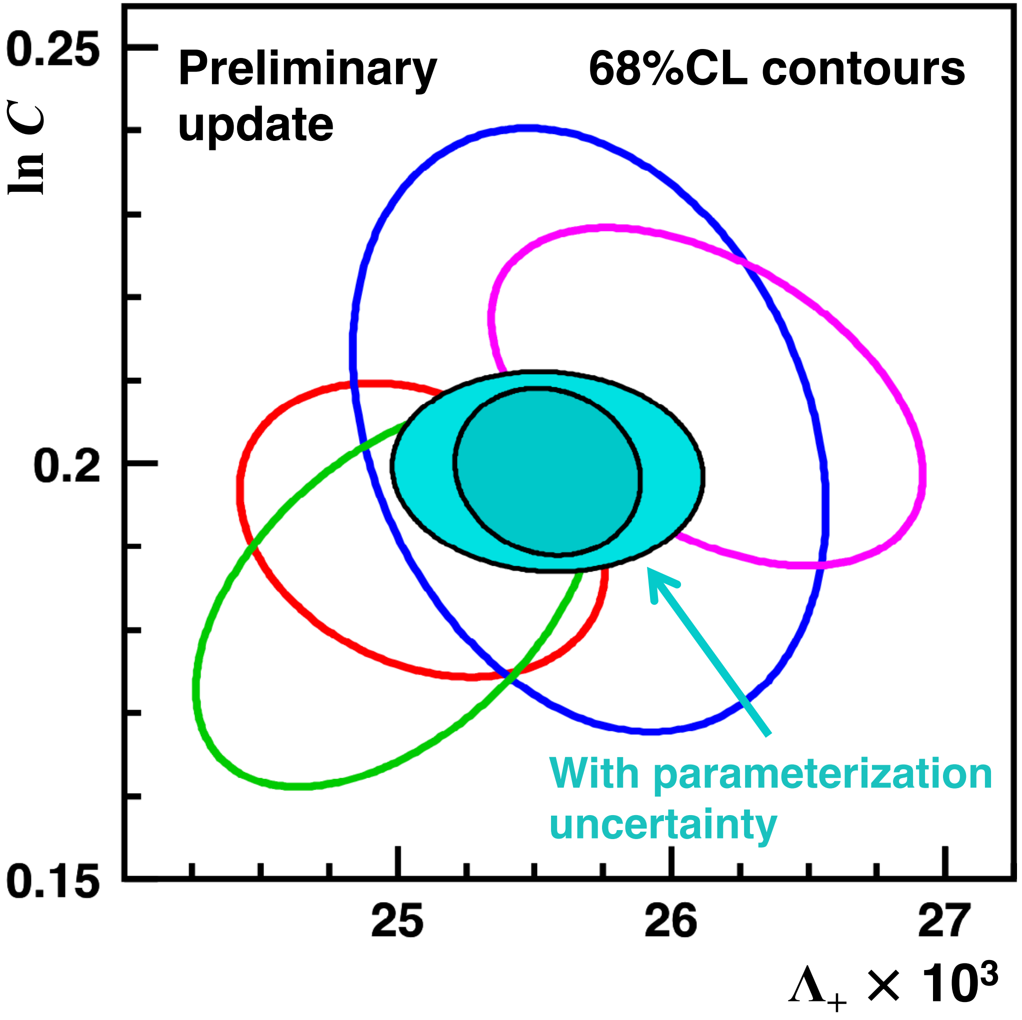
(except as indicated)

Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2**

NA48 K_{e3} data included in fits but not shown

2010 fit **Update**



$\Lambda_+ \times 10^3$ = **25.55 ± 0.38**

$\ln C$ = **0.1992(78)**

$\rho(\Lambda_+, \ln C)$ = −0.110

χ^2/ndf = 7.5/7 (38%)

Fit results include common uncertainty from $H(t)$, $G(t)$

Without common uncertainty:

$\sigma(\Lambda_+)$ (0.38 → 0.22) × 10^{-3}

$\sigma(\ln C)$ 0.0078 → 0.0067

$\sigma(K_{e3}$ int) 0.10% → 0.09%

$\sigma(K_{\mu 3}$ int) 0.15% → 0.11%

$K_{\ell 3}$ data and lepton universality

For each state of kaon charge, evaluate:

$$r_{\mu e} = \frac{(R_{\mu e})_{\rm obs}}{(R_{\mu e})_{\rm SM}} = \frac{\Gamma_{\mu 3}}{\Gamma_{e3}} \cdot \frac{I_{e3}\,(1+\delta_{e3})}{I_{\mu 3}\,(1+\delta_{\mu 3})} = \frac{[|V_{us}|\,f_+(0)]^2_{\mu 3,\,\rm obs}}{[|V_{us}|\,f_+(0)]^2_{e3,\,\rm obs}} = \frac{g_\mu^2}{g_e^2}$$

Modes	2004 BRs*,†	Current†
K_L	1.054(14)	**1.003(5)**
$K^\pm$	1.014(12)	**0.999(9)**
Avg	1.030(9)	**1.002(5)**

← Was 0.998(9) for 2010

*Assuming current values for form-factor parameters and $\Delta^{\rm EM}$ †K_S not included

As statement on lepton universality

Compare to other precise tests:

$\pi \rightarrow \ell\nu$ $(r_{\mu e})$ = **1.0020(19)**
PDG '16 with PIENU '15 result

$\tau \rightarrow \ell\nu\nu$ $(r_{\mu e})$ = **1.0002(28)**
HFLAV May '19 unofficial prelim.

As statement on calculation of $\Delta^{\rm EM}$

Cirigliano et al. '08

- Calculation at fixed order e^2p^2
- Fully inclusive for real photons
- **Confirmed at per-mil level**
- Update LECs for SD terms?

Accuracy of $SU(2)$-breaking correction

$$\Delta^{SU(2)} \equiv \frac{f_+(0)^{K^+\pi^0}}{f_+(0)^{K^0\pi^-}} - 1$$

Strong isospin breaking

Quark mass differences, η-π^0 mixing in $K^+\pi^0$ channel

$$= \frac{3}{4}\frac{1}{Q^2}\left[\frac{\overline{M}_K^2}{\overline{M}_\pi^2} + \frac{\chi_{p^4}}{2}\left(1+\frac{m_s}{\hat{m}}\right)\right] \qquad Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$$

$\chi_p{}^4 = 0.252$

NLO in strong interaction

O(e^2p^2) term $\varepsilon_{\mathrm{EM}}{}^{(4)} \sim 10^{-6}$

$=$ **+2.61(17)%** **Calculated using**

$Q = 22.1(7)$

$m_s/\hat{m} = 27.43(13)(27)$

$M_K = 494.2(3)$

$M_\pi = 134.8(3)$

Isopsin-limit meson masses

Value checked by **E. Passemar:**

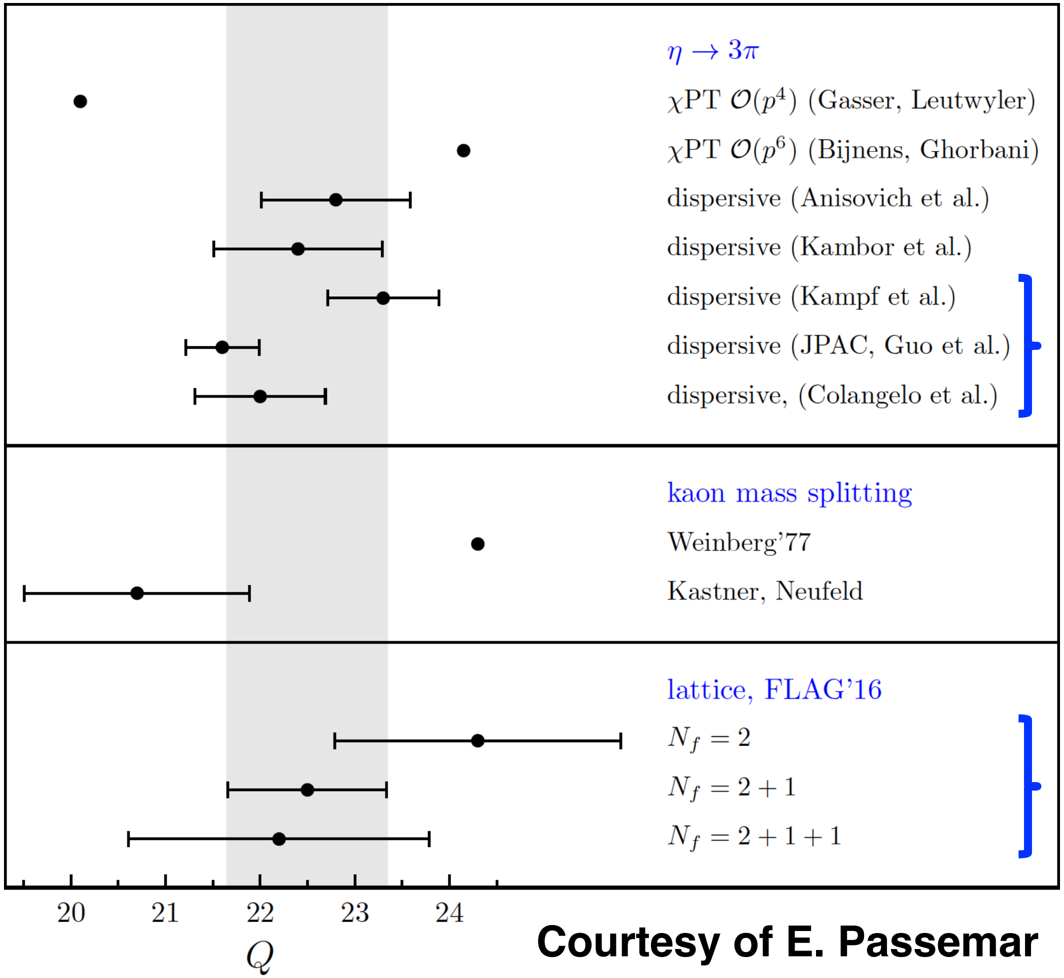
- Calculation scheme of Kastner & Neufeld ’08, Cirigliano et al. ’02
- LECs from Bijnens & Ecker ’14

Test by evaluating V_{us} from $K^\pm$ and K^0 data with **no** corrections:
Equality of V_{us} values would require **$\Delta^{SU(2)}$ = 2.82(38)%**

Accuracy of $SU(2)$-breaking correction

Previous to lattice results for Q, uncertainty on $\Delta^{SU(2)}$ was leading contributor to uncertainty on V_{us} from $K^\pm$ decays

Contribution to uncertainty still significant—can it be reduced?



Courtesy of E. Passemar

Continuing progress + systematic review of existing results for light-quark masses may help

Recent dispersion relation analyses of $\eta \to 3\pi$ Dalitz plot

E.g. **Colangelo et al.** 1610.03494

1.6 fb^{-1} KLOE '04-'05 data

Continuing progress on lattice

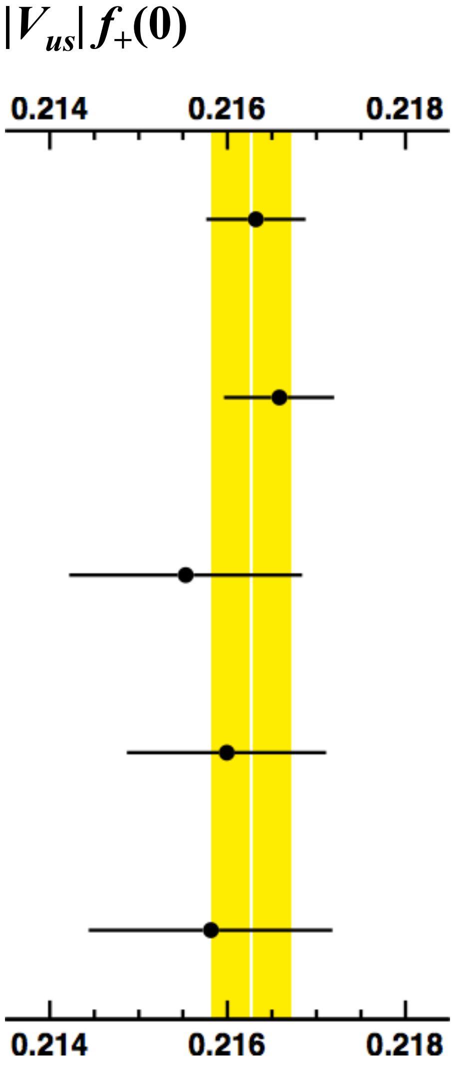
E.g. **BMW '16** PRL 117

N_f = 2+1 QCD, 5sp, m_π phys

Partially quenched QED

$Q = 23.4(4)_{\text{st}}(3)_{\text{sy}}(4)_{\text{QED}}$

$|V_{us}| f_+(0)$ from world data: 2010

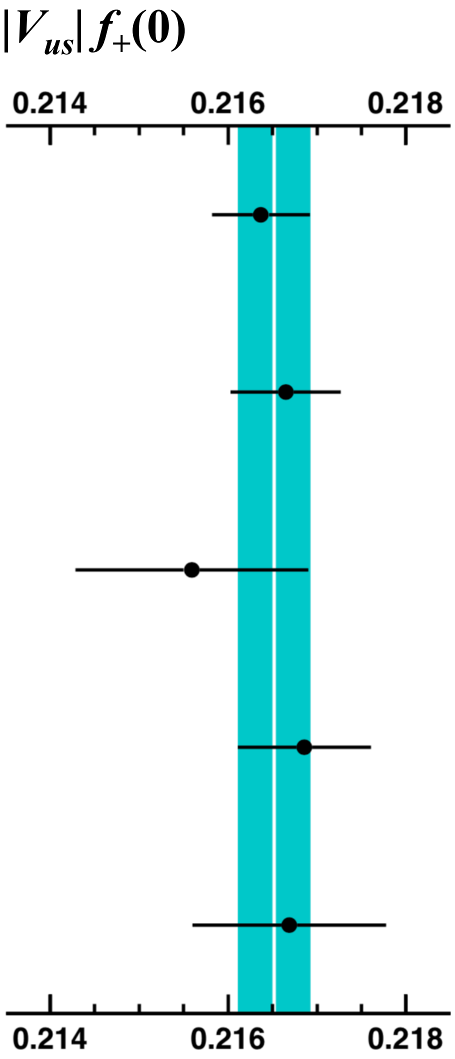


Approx. contrib. to % err from:

		% err	BR	τ	Δ	Int
K_Le3	0.2163(6)	0.26	0.09	0.20	0.11	0.06
$K_L\mu3$	0.2166(6)	0.29	0.15	0.18	0.11	0.08
K_Se3	0.2155(13)	0.61	0.60	0.03	0.11	0.06
$K^\pm e3$	0.2160(11)	0.52	0.31	0.09	0.40	0.06
$K^\pm\mu3$	0.2158(14)	0.63	0.47	0.08	0.39	0.08

Average: $|V_{us}| f_+(0)$ = 0.2163(5) χ^2/ndf = 0.77/4 (94%)

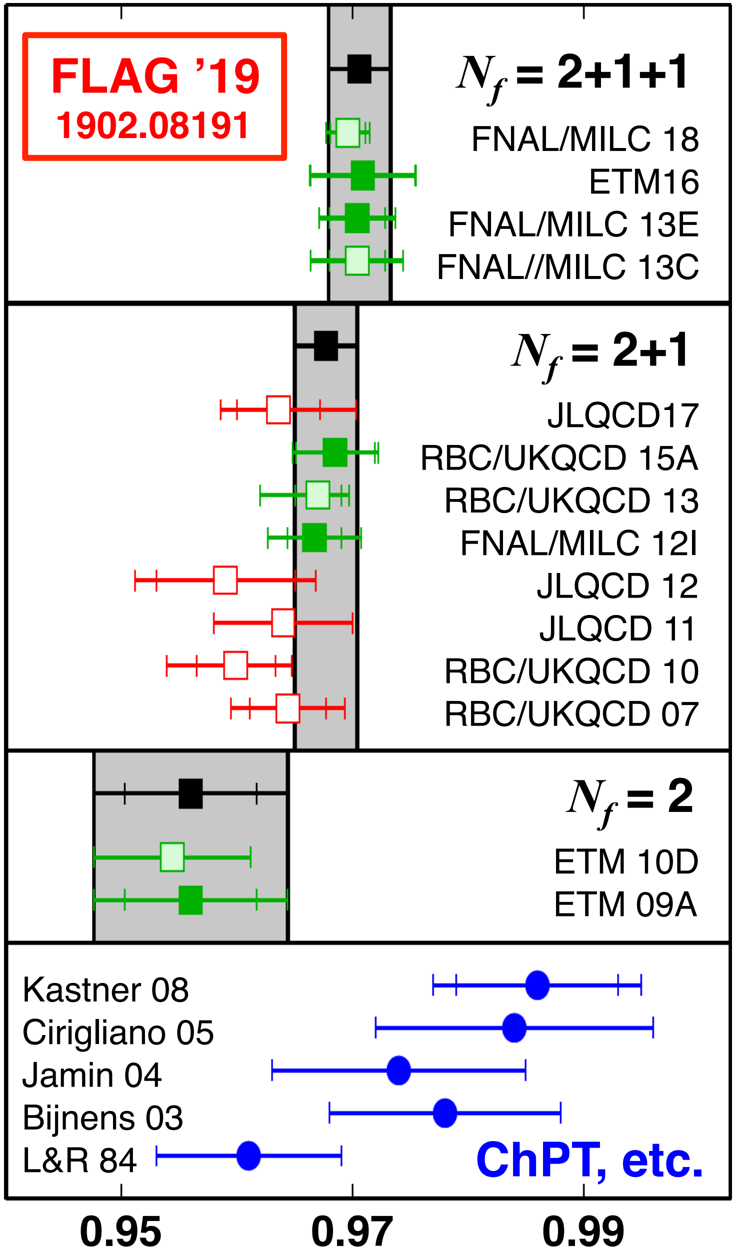
$|V_{us}|\,f_+(0)$ from world data: Update



		% err	Approx. contrib. to % err from:			
			BR	τ	Δ	Int
K_Le3	**0.2164(6)**	**0.26**	0.09	0.20	0.11	0.05
$K_L\mu3$	**0.2167(6)**	**0.29**	0.15	0.18	0.11	0.07
K_Se3	**0.2156(13)**	**0.61**	0.60	0.02	0.11	0.05
$K^{\pm}e3$	**0.2169(8)**	**0.35**	0.27	0.06	0.21	0.05
$K^{\pm}\mu3$	**0.2167(11)**	**0.50**	0.45	0.06	0.21	0.07

Average: $|V_{us}|\,f_+(0)$ = 0.21652(41) **χ^2/ndf = 0.98/4 (91%)**

Evaluations of $f_+(0)$



FLAG '19 averages:

N_f = 2+1+1 $f_+(0)$ = **0.9706(27)**

Uncorrelated average of:

FNAL/MILC 13E: HISQ, $m_\pi \to$ 135 MeV

ETM 16: TwMW, 3sp, $m_\pi \to$ 210 MeV,
Full q^2 dependence of f_+, f_0

N_f = 2+1 $f_+(0)$ = **0.9677(27)**

Uncorrelated average of:

FNAL/MILC 12I: HISQ, $m_\pi \sim$ 300 MeV

RBC/UKQCD 15A: DWF, $m_\pi \to$ 139 MeV

Recent updates:

N_f = 2+1+1 $f_+(0)$ = **0.9696(15)(11)** 1809.02827

FNAL/MILC 18: HISQ, 5sp, $m_\pi \to$ 135 MeV,
new ensembles added to FNAL/MILC 13E

ChPT:

N_f = 2+1 $f_+(0)$ = **0.970(8)** Chiral Dynamics 15

Ecker 15: According to Bijnens 03,
with new LECs from Bijnens, Ecker 14

Our 2019 averages for $f_+(0)$

N_f = 2+1+1 **$f_+(0)$ = 0.9698(17)**

FNAL/MILC18 replaces FNAL/MILC13E in FLAG average

ETM16	$0.9709(44)(9)(11)_{\text{ext}}$
FNAL/MILC18	0.9696(15)(11)

N_f = 2+1 **$f_+(0)$ = 0.9677(27)**

FLAG average, 2016 → 2019 updates

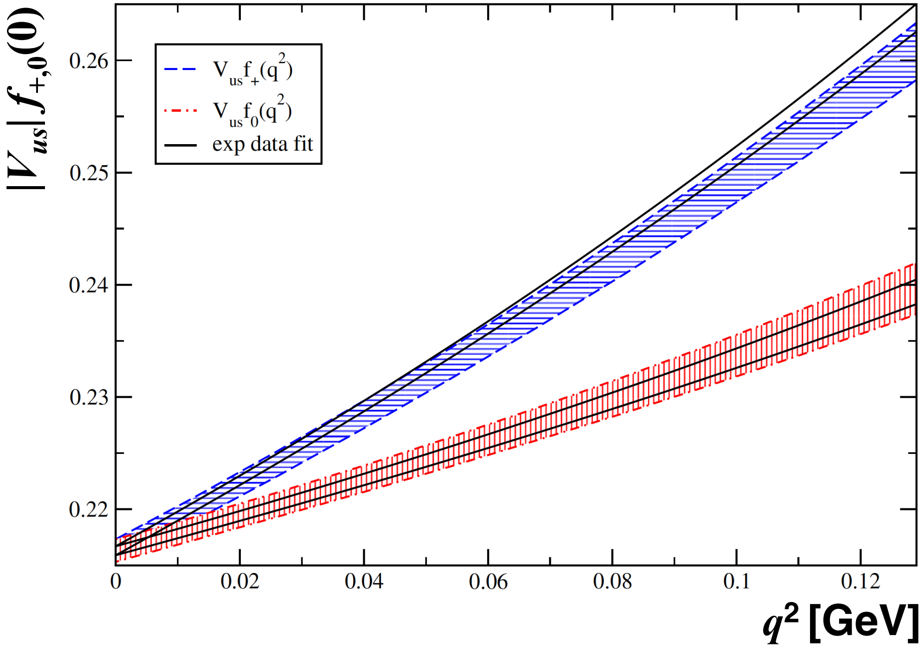
JLQCD17 not included because only 1 lattice spacing used

FNAL/MILC12I	0.9667(23)(33)
RBC/UKQCD15A	0.9685(34)(14)

Evaluations of $f_+(0)$

ETM
PRD 93 (2016)

N_f = **2+1+1** Twisted-mass Wilson fermions
3 lattice spacings, smallest $m_\pi \rightarrow$ 210 MeV
Results for full q^2 dependence of f_+, f_0



Fit synthetic data points with dispersive parameterization

Λ_+ **= 24.22(1.16) × 10^{-3}**

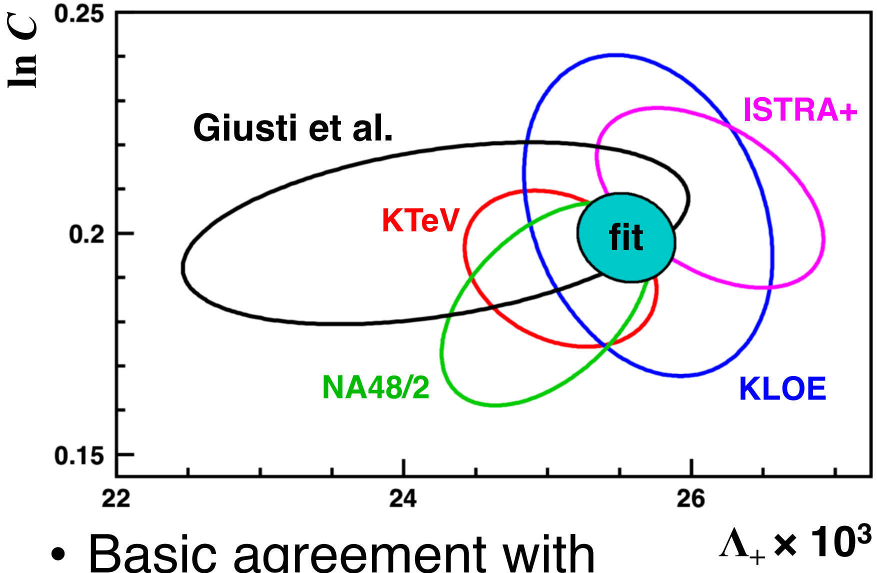
ln C = 0.1998(138)

$\rho(\Lambda_+, f_+(0)) = -0.228$

$\rho(\ln C, f_+(0)) = -0.719$

$\rho(\Lambda_+, \ln C) = +0.376$

$f_+(0) = 0.9709(44)_{st}(9)_{sy}(11)_{ext}$



- Basic agreement with experimental results
- Confirms basic correctness of lattice calculations for $f_+(0)$
- In the near future FF parameters will be obtained on lattice?

$|V_{us}|(K_{\ell 3})$ and $|V_{ud}|(0^+ \to 0^+)$: Update

Hardy & Towner, CIPANP '18
$|V_{ud}|$ = 0.97420(21)

World data set very robust

14 transitions with compatible measurements at 0.1% precision or better

From FlaviaNet 2010 $K_{\ell 3}$ analysis

$|V_{us}|\,f_+(0)$ = 0.2163(5) **$|V_{us}|$ = 0.2254(13)**

with $f_+(0)$ = 0.959(5) **with $|V_{ud}|$ = 0.97425(22)**

Δ_CKM = +0.0000(8)

Update with $|V_{us}|\,f_+(0)$ = 0.21652(41) and $|V_{ud}|$ = 0.97420(21)

N_f = 2+1
$f_+(0)$ = 0.9677(27)

V_{us} = 0.22375(43)$_\mathrm{exp}$(62)$_\mathrm{lat}$

Δ_CKM = –0.00085(19)$_\mathrm{exp}$(28)$_\mathrm{lat}$(41)$_{ud}$ = -1.6σ

N_f = 2+1+1
$f_+(0)$ = 0.9698(17)

V_{us} = 0.22326(43)$_\mathrm{exp}$(39)$_\mathrm{lat}$

Δ_CKM = –0.00107(19)$_\mathrm{exp}$(17)$_\mathrm{lat}$(41)$_{ud}$ = -2.2σ

1.5-2σ inconsistency with unitarity first seen with 2014-era lattice results

Relative to 2014 slightly better agreement between N_f = 2+1 and N_f = 2+1+1

$|V_{us}|(K_{\ell 3})$ and $|V_{ud}|(0^+ \rightarrow 0^+)$: Update

Seng. Gorchtein, Patel & Ramsey-Musolf
PRL 121 (2018)

$|V_{ud}|$ = 0.97370(14)

–1.5σ shift in V_{ud}

New calculation γW-box contribution to universal radiative correction using dispersion relations and DIS structure functions

- Revision of structure-dependent radiative corrections possibly required
- **Needs further development!**

Update with $|V_{us}|\, f_+(0)$ = 0.21652(41) and $|V_{ud}|$ = 0.97370(14)

Choice of $f_+(0)$		**V_{us}**	**$\Delta_{\mathrm{CKM}} = V_{ud}^2 + V_{us}^2 - 1$**	
N_f = 2+1	0.9677(27)	**0.2238(8)**	**–0.0019(5)**	= –4.2σ
N_f = 2+1+1	0.9698(17)	**0.2233(6)**	**–0.0021(4)**	= –5.4σ

Taken at face value, 4-5σ unitarity violation in first row!

Part of an effort to reduce systematics from radiative corrections to 10^{-4}

V_{us}/V_{ud} and $K_{\ell 2}$ decays

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1-m_\mu^2/m_{\pi^\pm}^2}{1-m_\mu^2/m_{K^\pm}^2}\left(1-\frac{1}{2}\delta_{\rm EM}-\frac{1}{2}\delta_{SU(2)}\right)$$

Inputs from theory:

Cirigliano, Neufeld ’11

$\delta_{\rm EM}$ = –0.0069(17)

Long-distance EM corrections

$\delta_{SU(2)}$ = –0.0043(5)(11)

Strong isospin breaking

$f_K/f_\pi \rightarrow f_{K\pm}/f_{\pi\pm}$

Lattice: f_K/f_π

Cancellation of lattice-scale uncertainties from ratio

NB: Most lattice results already corrected for $SU(2)$-breaking: $f_{K\pm}/f_{\pi\pm}$

Inputs from experiment:

Updated $K^\pm$ BR fit:

BR($K^\pm{}_{\mu 2(\gamma)}$) = 0.6358(11)

$\tau_{K\pm}$ = 12.384(15) ns

PDG:

BR($\pi^\pm{}_{\mu 2(\gamma)}$) = 0.9999

$\tau_{\pi\pm}$ = 26.033(5) ns

$|V_{us}/V_{ud}| \times f_{K\pm}/f_{\pi\pm}$ = 0.27599(37)

No $SU(2)$-breaking correction

V_{us}/V_{ud} and $K_{\ell 2}$ decays

Giusti et al.
PRL 120 (2018)

First lattice calculation of EM corrections to P_{l2} decays

- Ensembles from ETM
- $N_f = 2+1+1$ Twisted-mass Wilson fermions

$$\boldsymbol{\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0122(16)}$$

- Uncertainty from quenched QED included (0.0006)

Compare to ChPT result from Cirigliano, Neufeld ’11:

$$\boldsymbol{\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0112(21)}$$

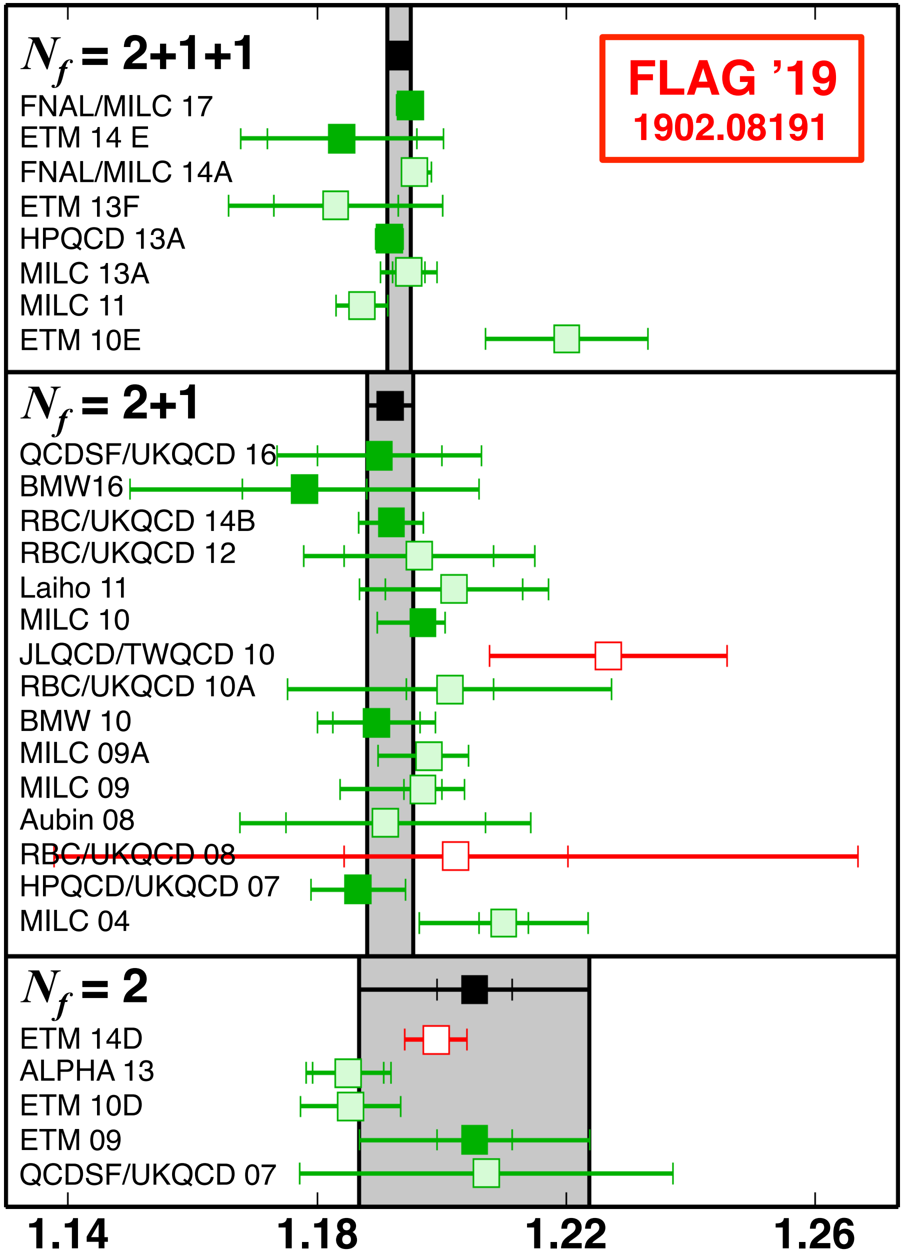
Di Carlo et al.
1904.08731

Update, extended description, and systematics of Giusti et al.

$$\boldsymbol{\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0126(14)}$$

$$\boldsymbol{|V_{us}/V_{ud}| \times f_K/f_\pi = 0.27679(28)_{\mathrm{BR}}(20)_{\mathrm{corr}}}$$

Lattice evaluations of f_K/f_π



FLAG '19 averages:

N_f = 2+1+1 **$f_{K\pm}/f_{\pi\pm}$ = 1.1932(19)**

Includes:

FNAL/MILC 17:
HISQ, 4sp, m_π phys
Updates MILC 13A, FNAL/MILC 14A

HPQCD 13A
HISQ, 3sp, m_π phys,
Same ensembles as FNAL/MILC 17

ETM 14E
TwM, 3sp, m_π = 210-450 MeV

N_f = 2+1 **$f_{K\pm}/f_{\pi\pm}$ = 1.1917(37)**

Recent measurements:

QCDSF/UKQCD 16:
Clover, 4sp, $m_\pi \to$ 220 MeV

BMW16:
Clover, 5sp, $m_\pi \to$ 139 MeV

RBC/UKQCD 14B:
DWF, m_π = 139 MeV
f_K and f_π separately (isospin limit)

Lattice results for f_K/f_π

Recalculate FLAG averages for results without $SU(2)$-breaking
Isospin-limit results as reported in original papers

$N_f = 2+1+1$ **$f_K/f_\pi = 1.1967(18)$**

FNAL/MILC17	$1.1980(^{+13}_{-19})$	Correlated uncertainties
HPQCD13A	1.1948(15)(18)	
ETM14E	1.188(15)	← Uncorrelated uncertainty

$N_f = 2+1$ **$f_K/f_\pi = 1.1946(34)$***

QCDSF/UKQCD17	1.192(10)(13)
BMW16	1.182(10)(26)
RBC/UKQCD14B	1.1945(45)
BMW10	1.192(7)(6)
HPQCD/UKQCD07	1.198(2)(7)

*MILC10 omitted from average because unpublished

$|V_{us}|(K_{\ell 2})$ and $|V_{ud}|(0^+ \rightarrow 0^+)$: Update

$|V_{us}/V_{ud}| \times f_K/f_\pi = 0.27679(34)$ and $|V_{ud}| = 0.97420(21)$

$\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0126(14)$ from Di Carlo et al. '19

$N_f = 2+1$

$f_K/f_\pi = 1.1946(34)$

$V_{us} = 0.22573(28)_{\mathrm{exp}}(64)_{\mathrm{lat}}(05)_{ud}$

$\Delta_{\mathrm{CKM}} = +0.00003(13)_{\mathrm{exp}}(29)_{\mathrm{lat}}(43)_{ud}$ $= +0.1\sigma$

$N_f = 2+1+1$

$f_K/f_\pi = 1.1967(18)$

$V_{us} = 0.22533(28)_{\mathrm{exp}}(34)_{\mathrm{lat}}(05)_{ud}$

$\Delta_{\mathrm{CKM}} = -0.00015(13)_{\mathrm{exp}}(15)_{\mathrm{lat}}(43)_{ud}$ $= -0.3\sigma$

$K_{\ell 2}$ results give better agreement with unitarity via V_{ud} than $K_{\ell 3}$ results (-2σ)

Exercise:

- Assume $|V_{ud}|$, $|V_{us}/V_{ud}| \times f_K/f_\pi$, and f_K/f_π all correct
- In $K_{\ell 3}$ does the discrepancy arise from data or from lattice results for $f_+(0)$?

Form factors & the Callan-Treiman relation

Callan-Treiman relation:

$$\tilde{f}_0(t_{\mathrm{CT}}) = \frac{f_K}{f_\pi}\frac{1}{f_+(0)} + \Delta_{\mathrm{CT}}$$

$t_{\mathrm{CT}} = m_K^2 - m_\pi^2$

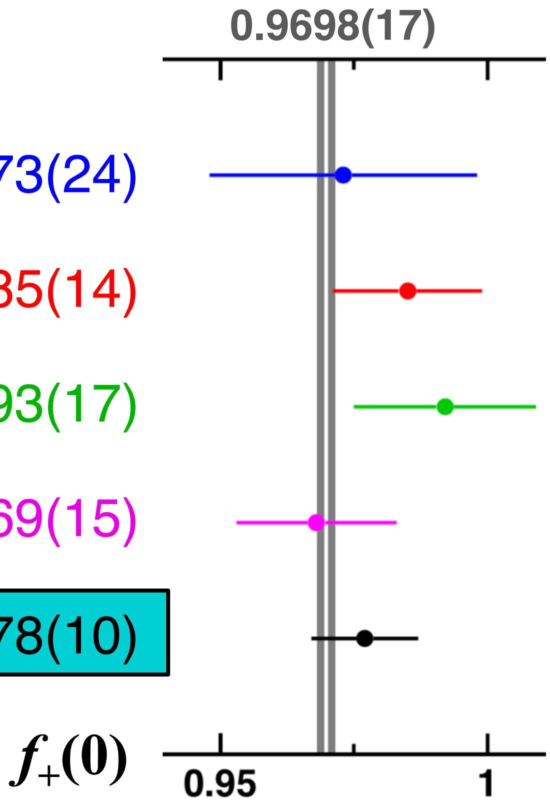
$\Delta_{\mathrm{CT}} = (-3.5 \pm 0.8) \times 10^{-3}$ ~O(m_u, m_d)
Gasser, Leutwyler ’85

Dispersive representation: $f_0(t_{\mathrm{CT}}) \equiv C$

Use ChPT & form-factor data to test N_f = 2+1+1 lattice consistency:

- Use lattice reference value f_K/f_π = **1.1967(18)**
- Obtain $f_+(0)$ corresponding to each result for $\ln C$
- Compare to lattice reference value $f_+(0)$ **= 0.9698(17)**
- Basic consistency **(0.7σ)** between lattice values for f_K/f_π and $f_+(0)$ and measurements of $\ln C$
- **Uses no experimental information on decay widths**

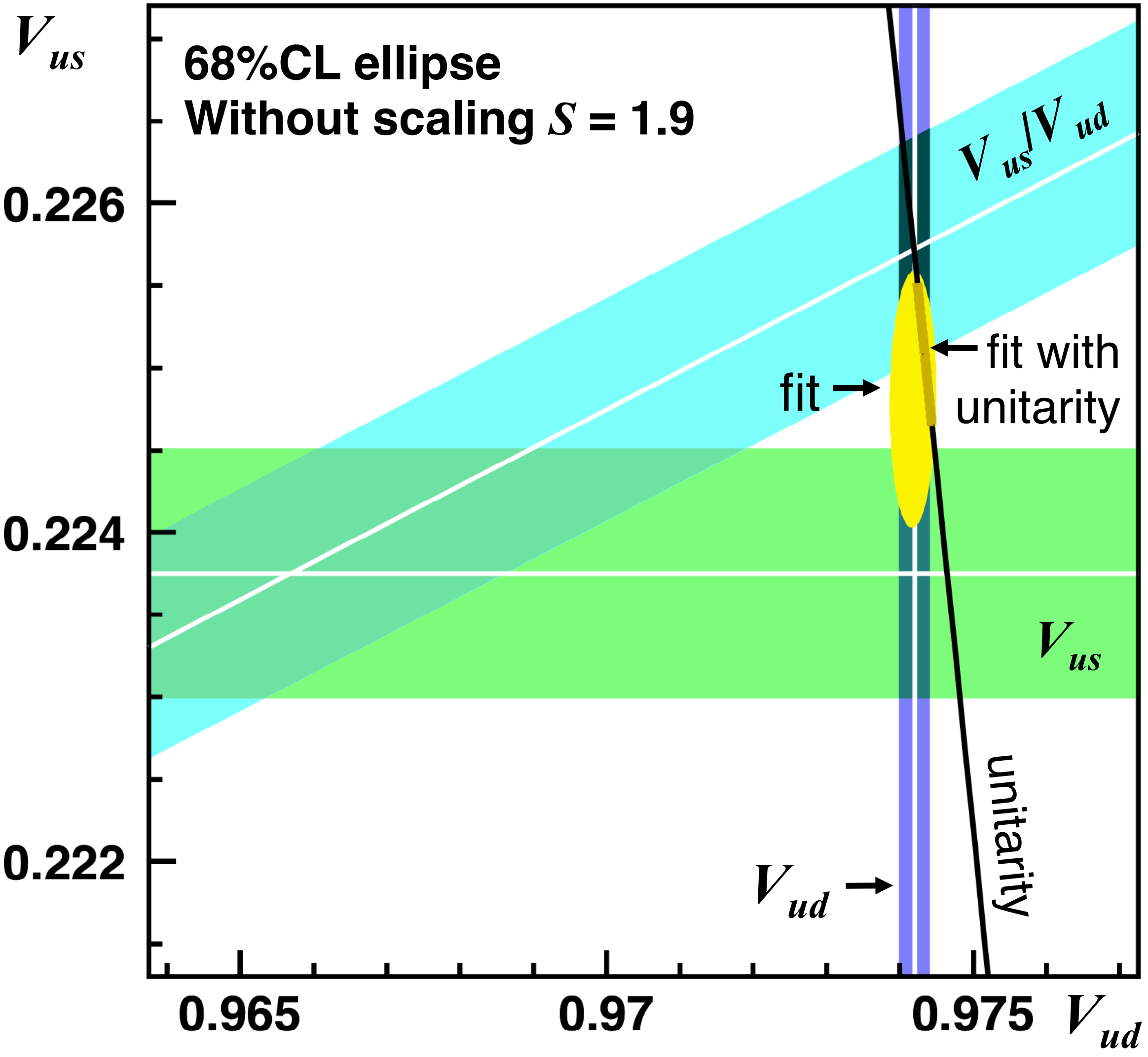
KLOE	0.973(24)
KTeV	0.985(14)
NA48/2	0.993(17)
ISTRA+	0.969(15)
Update	0.978(10)



V_{us} and CKM unitarity: All data

N_f = 2+1: Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$
$f_+(0)$ = 0.9677(27), f_K/f_π = 1.1946(34)

$|V_{ud}|$ = 0.97420(21)
$|V_{us}|$ = 0.2238(8)
$|V_{us}|/|V_{ud}|$ = 0.2317(7)



Fit results, no constraint

V_{ud} = 0.97418(21)
V_{us} = 0.2248(5)
χ^2/ndf = 3.7/1 (6%)
Δ_{CKM} = –0.0004(5)
–0.9σ

With scale factor S = 1.9

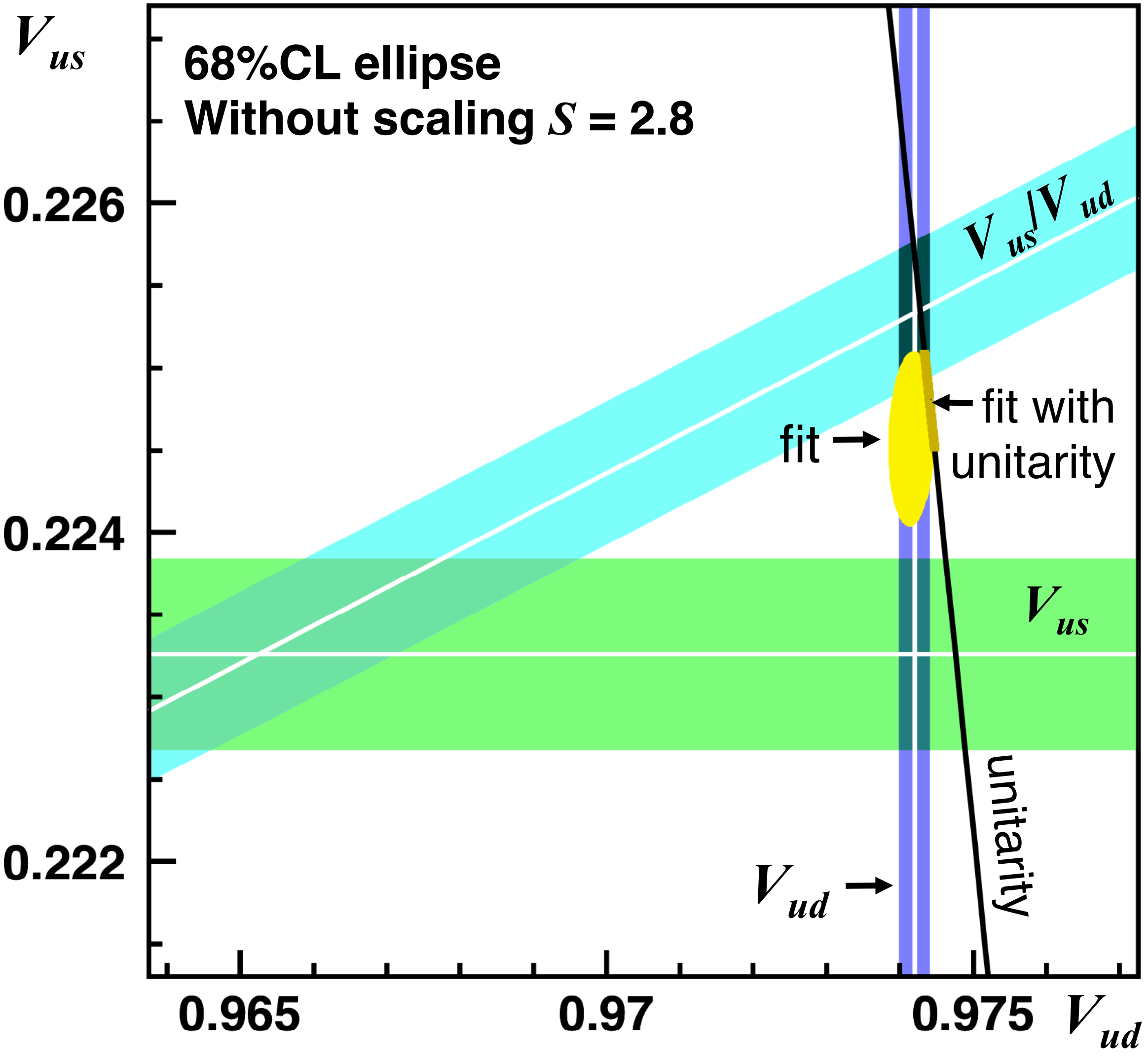
V_{ud} = 0.97418(40)
V_{us} = 0.2248(10)

V_{us} and CKM unitarity: All data

$N_f = 2+1+1$: Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$
$f_+(0) = 0.9698(17)$, $f_K/f_\pi = 1.1967(18)$

→

$|V_{ud}| = 0.97420(21)$
$|V_{us}| = 0.2233(6)$
$|V_{us}|/|V_{ud}| = 0.2313(5)$



Fit results, no constraint

$V_{ud} = \mathbf{0.97416(21)}$

$V_{us} = \mathbf{0.22457(35)}$

$\chi^2/\text{ndf} = 8.0/1$ (0.5%)

$\Delta_{\text{CKM}} = \mathbf{-0.0005(4)}$

-1.3σ

With scale factor $S = 2.8$

$V_{ud} = \mathbf{0.97416(60)}$

$V_{us} = \mathbf{0.2240(10)}$

Preliminary conclusions: V_{us} from kaons

Experimental results from kaons	**With $\lvert V_{ud}\rvert(0^+ \rightarrow 0^+)$ and $N_f = 2+1+1$ lattice**
$\lvert V_{us}\rvert f_+(0) = 0.21652(41)$	$\Delta_{\mathrm{CKM}} = -0.00107(48) = -2.2\sigma$
$\lvert V_{us}/V_{ud}\rvert \times f_K/f_\pi = 0.27679(34)$	$\Delta_{\mathrm{CKM}} = -0.00015(47) = -0.3\sigma$

2σ inconsistency between $K_{\ell 3}$ and $K_{\ell 2}$ results for V_{us}

$K_{\ell 2}$ result shows good agreement with unitarity and V_{ud}

$K_{\ell 3}$ result 2σ smaller than expected from unitarity and V_{ud}

- Change occurred after 2014-era more precise evaluations of $f_+(0)$
- No significant impact from choice of $N_f = 2+1$ or $N_f = 2+1+1$ for $f_+(0)$
  (Combined fit for $K_{\ell 3}$ and $K_{\ell 2}$ agrees better with unitarity with $N_f = 2+1$)
- Experimental results for $K_{\ell 3}$ have changed little since 2010

Are residual systematics in the data and/or calculations becoming important as stated uncertainties shrink?

Prospects for new measurements

NA48/2

NA62

Can measure BRs and form-factor parameters for K^+

NA48/2 (2003-2004) recently measured $K_{\ell 3}$ form factors

NA62-RK (2007) has O(10M) $K_{\ell 3}$ decays

NA62 has O(few M) K_{e3} from minimum bias runs (2015-16)

Relative to NA48/2, NA62 has

- Better particle identification π/μ
- Better systematics for t reconstruction:
  - full beam tracking, better σ_p in spectrometer

ISTRA+

OKA

Fixed target experiment at U-70 (Protvino), like ISTRA+

- New beamline with RF-separated K^+ beam

Can measure BRs and form-factor parameters

- Need more analysis of systematics for K_{e3} form factors

Runs from 2010-2013: ~17M K^+_{e3} events

- Additional runs in 2016-2018; more planned in future

Prospects for new measurements

KLOE

KLOE-2

Can measure all observables: BRs, τs, FFs: $K^{\pm}$, K_L, K_S

5.5 fb^{-1} of data from KLOE-2 running (2015-2018)

- +2 fb^{-1} of original KLOE data not yet analyzed for V_{us}

Measurements that can be improved with KLOE-2 statistics:

- K_S BRs ($K_S \to \pi e\nu$, but also $K_S \to \pi\mu\nu$)
  See e.g. KLOE-2 measurement of A_S 1806.08654
  70k $K_S \to \pi e\nu$ decays
- $K^{\pm}$, K_L form factors (particularly $K_{\ell 3}$), K_L mean life?

LHCb

Proven capability to measure K_S decays to muons

- 10^{13} K_S/fb^{-1} produced
- EPJC 77 (2017): BR($K_S \to \mu\mu$) $< 1.0 \times 10^{-9}$ 95%CL
  Limited by hardware trigger efficiency ($\varepsilon_{\text{trig}} \sim 1\%$)

Can LHCb measure BR($K_S \to \pi\mu\nu$) to < 1% in Run III?

- Would require dedicated software HLT line

$K_S \to \pi\mu\nu$ never yet measured – a new channel for V_{us}

- τ_S known to 0.04% (vs 0.41% for τ_L, 0.12% for $\tau_{\pm}$)

Prospects for new measurements

KEK-246

TREK E36

Primary focus is BR($K_{e2}/K_{\mu2}$) to 0.25%

+ Invisible heavy neutrino searches

+ T violation in $K_{\mu3}$ (as E06)

Upgraded KEK-246 setup, moved to J-PARC

- Stopped K^+ in active target
- Toroidal spectrometer surrounding target
- e/μ particle ID by time of flight, Cerenkov counters, lead-glass calorimetry

KEK-246 measured BR($K_{\mu3}/K_{e3}$) and K_{e3} FF, so TREK could potentially use calibration data to measure at least some BRs and FFs of interest for V_{us}

Progress on V_{us} from kaons: Final notes

- **$K_{\ell 3}$ FFs do not directly contribute significantly to uncertainty on V_{us}**
  - However, uncertainties on high-statistics BR ratio measurements may be so low that FFs become a major systematic
    - e.g. BR($K_{\mu 3}/\pi\pi^0$), BR($K_{\mu 3}/K_{e3}$)
- **Uncertainties from parameterization of $K\pi$ phase shift data now limit precision for $K_{\ell 3}$ FFs and phase space integrals**
  - Better parameterization will require old data to be re-fit!
  - Imperative for future averages that experiments publish full FF data so that it can be re-fit as parameterizations improve
  - Direct lattice calculation of $K_{\ell 3}$ FFs may help
- **For $K^{\pm}$, normalization BRs have significant uncertainties**
  - Effect of any precise new BR($K_{e3}/\pi\pi^0$) results will limited by uncertainty on BR($\pi\pi^0$)
  - Very important to measure absolute BRs or ratios involving BRs of other modes, e.g. $\pi\pi^0/\mu\nu$, $\pi\pi\pi/\pi\pi^0$, $\pi\pi\pi/\mu\nu$

Summary: V_{us} from kaons

Experimental results from kaons

With $|V_{ud}|(0^+ \to 0^+)$ and $N_f = 2+1+1$ lattice

$|V_{us}|\,f_+(0) = 0.21652(41)$ → $\Delta_{\mathrm{CKM}} = -0.00107(48) = -2.2\sigma$

$|V_{us}/V_{ud}| \times f_K/f_\pi = 0.27679(34)$ → $\Delta_{\mathrm{CKM}} = -0.00015(47) = -0.3\sigma$

2σ inconsistency between $K_{\ell 3}$ and $K_{\ell 2}$ results for V_{us}

- **$K_{\ell 2}$ result shows good agreement with unitarity and V_{ud}**
- **$K_{\ell 3}$ result 2σ smaller than expected from unitarity and V_{ud}**
  - Change occurred after 2014-era more precise evaluations of $f_+(0)$
  - Experimental result for $|V_{us}|\,f_+(0)$ has changed little since 2010

Continuing to see impressive progress on the lattice

- Not only $f_+(0)$ and $f_{K\pm}/f_{\pi\pm}$, but also full t-dependence of FFs, EM corrections, etc.

Good prospects for new round of measurements to reduce uncertainty on $|V_{us}|\,f_+(0)$ from current 0.18% to ~0.12% within next few years:

NA62, OKA, KLOE-2, LHCb, TREK…

V_{us} from τ decays

Based mainly on work by HFLAV and talks by Alberto Lusiani

V_{us} from $\Gamma(\tau \to K\nu_\tau)/\Gamma(\tau \to \pi\nu_\tau)$

$$\frac{\Gamma(\tau \to K\nu_\tau)}{\Gamma(\tau \to \pi\nu_\tau)} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{f_K^2}{f_\pi^2} \frac{(m_\tau^2 - m_K^2)^2}{(m_\tau^2 - m_\pi^2)^2} \frac{\delta\tau_{K2}}{\delta\tau_{\pi2}}$$

Inputs from theory:

f_K/f_π

1.1967(18) N_f = 2+1+1

1.1946(34) N_f = 2+1

$$\frac{\delta\tau_{K2}}{\delta\tau_{\pi2}} = \frac{\delta\tau_{K2}}{\delta K_{\mu2}} \times \frac{\delta\pi_{\mu2}}{\delta\tau_{\pi2}} \times \frac{\delta K_{\mu2}}{\delta\pi_{\mu2}}$$

$\delta K_{\mu2}/\delta\pi_{\mu2}$ = –0.0126(14)

Di Carlo et al. ’19

$\delta\tau_{K2}/\delta K_{\mu2}$ = 0.0090(22)

$\delta\tau_{\pi2}/\delta\pi_{\mu2}$ = 0.0016(14)

Decker & Finkemeier ’94

→ $\delta\tau_{K2}/\delta\tau_{\pi2}$ = 1.0053(30)

Inputs from experiment:

BR($K^-\nu_\tau/\pi^-\nu_\tau$) = 0.06438(94)

HFLAV ’17 fit

$|V_{us}/V_{ud}| \times f_K/f_\pi$ = 0.2740(20)

V_{ud}/V_{us} = 0.2290(17)

with V_{ud} = 0.97420(21) [$0^+ \to 0^+$]

f_K/f_π = 1.1967(18) [N_f = 2+1+1]

V_{us} = 0.2228(20)

Δ_{CKM} = –0.0013(10) = –1.3σ

V_{us} from inclusive hadronic τ decays

$$R_\tau = \frac{\Gamma(\tau^- \to [\text{hadrons}]^- \nu_\tau(\gamma))}{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau(\gamma))}$$

$$= R_{\tau\,\text{non-strange}} + R_{\tau\,\text{strange}}$$

vector + axial
current

$SU(3)$ breaking:

$$\delta R_\tau^{\text{th}} = \frac{R_{\tau\,\text{non-strange}}}{|V_{ud}|^2} - \frac{R_{\tau\,\text{strange}}}{|V_{us}|^2}$$

Experimental inputs:

$R_{\tau\ \text{non-strange}}/|V_{ud}|^2 \approx 3.7$

$R_{\tau\ \text{strange}} \approx 0.17$

Theoretical inputs:

$\delta R_\tau^{\text{th}} = 0.242(32)$ for $m_s(m_\tau) = 95 \pm 5$ MeV

- OPE with fixed-order or contour-improved perturbation theory for contributions up to $D = 2$

  E. Gamiz et al., hep-ph/0612154v1

$\delta R_\tau^{\text{th}}$ from finite-energy sum rules (FESR):

$$R_\tau^w(s_0) \equiv \int_0^{s_0} ds\, \frac{w(s)}{w_\tau(s)} \frac{dR_\tau(s)}{ds}$$

$$\delta R_\tau^w(s_0) = \frac{R^w_{\tau\,\text{non-strange}}(s_0)}{|V_{ud}|^2} - \frac{R^w_{\tau\,\text{strange}}(s_0)}{|V_{us}|^2}$$

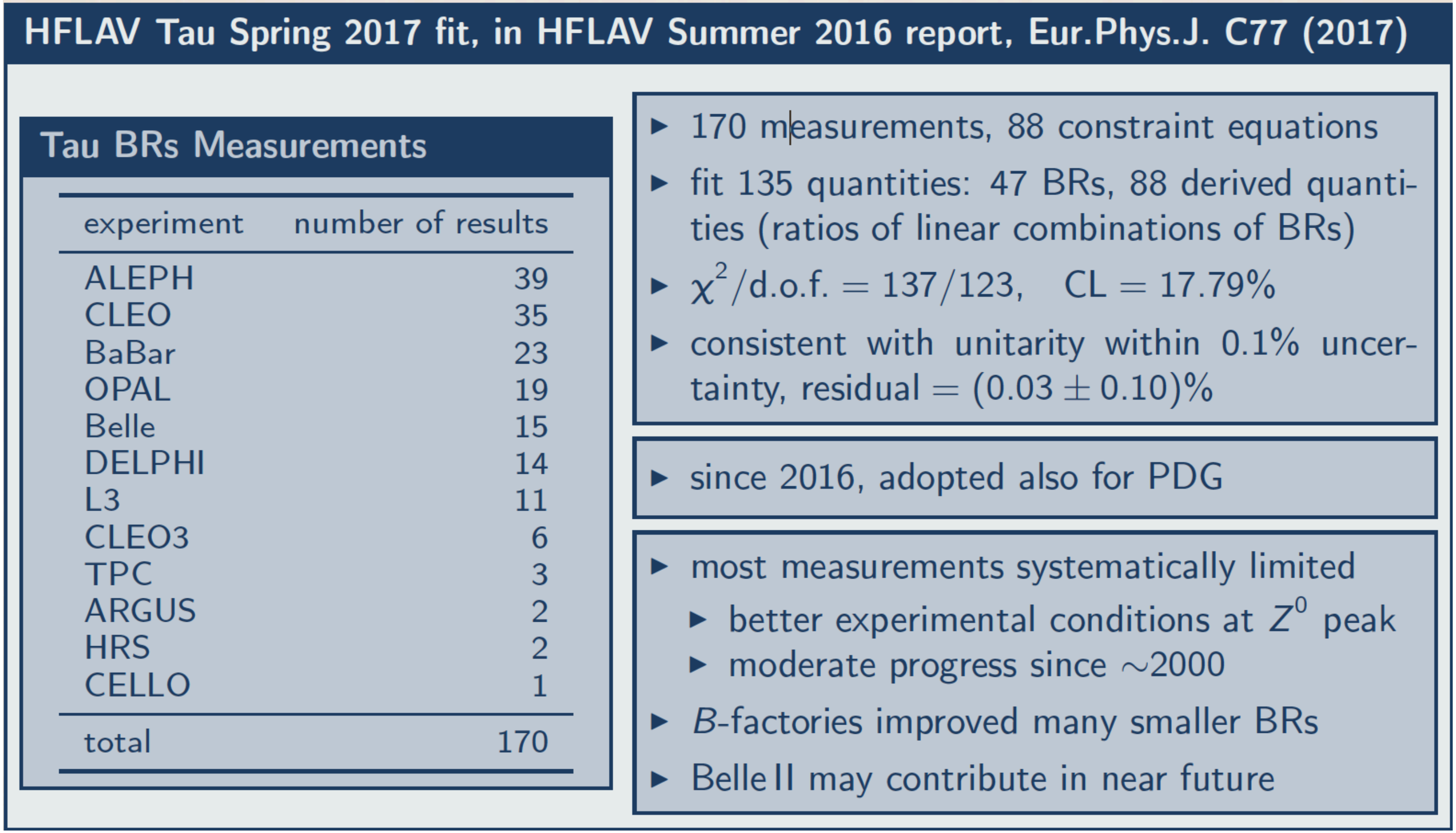
- $\delta R_\tau^w(s_0)$ has contributions up to $D = 8$
- $\delta R_\tau^w(s_0)$ has substantial dependence on s_0, w if contributions with $D > 4$ not negligible

  Hudspith et al., 2017
- Can use lattice QCD inputs for $D = 6, 8$ contributions

  Boyle et al. (RBC/UKQCD), 2018

Input data for HFLAV 2017 τ BR fit

A. Lusiani, CKM 2018



HFLAV Tau Spring 2017 fit, in HFLAV Summer 2016 report, Eur.Phys.J. C77 (2017)

Tau BRs Measurements

experiment	number of results
ALEPH	39
CLEO	35
BaBar	23
OPAL	19
Belle	15
DELPHI	14
L3	11
CLEO3	6
TPC	3
ARGUS	2
HRS	2
CELLO	1
total	170

- 170 measurements, 88 constraint equations
- fit 135 quantities: 47 BRs, 88 derived quantities (ratios of linear combinations of BRs)
- $\chi^2/\text{d.o.f.} = 137/123, \quad \text{CL} = 17.79\%$
- consistent with unitarity within 0.1% uncertainty, residual $= (0.03 \pm 0.10)\%$

- since 2016, adopted also for PDG

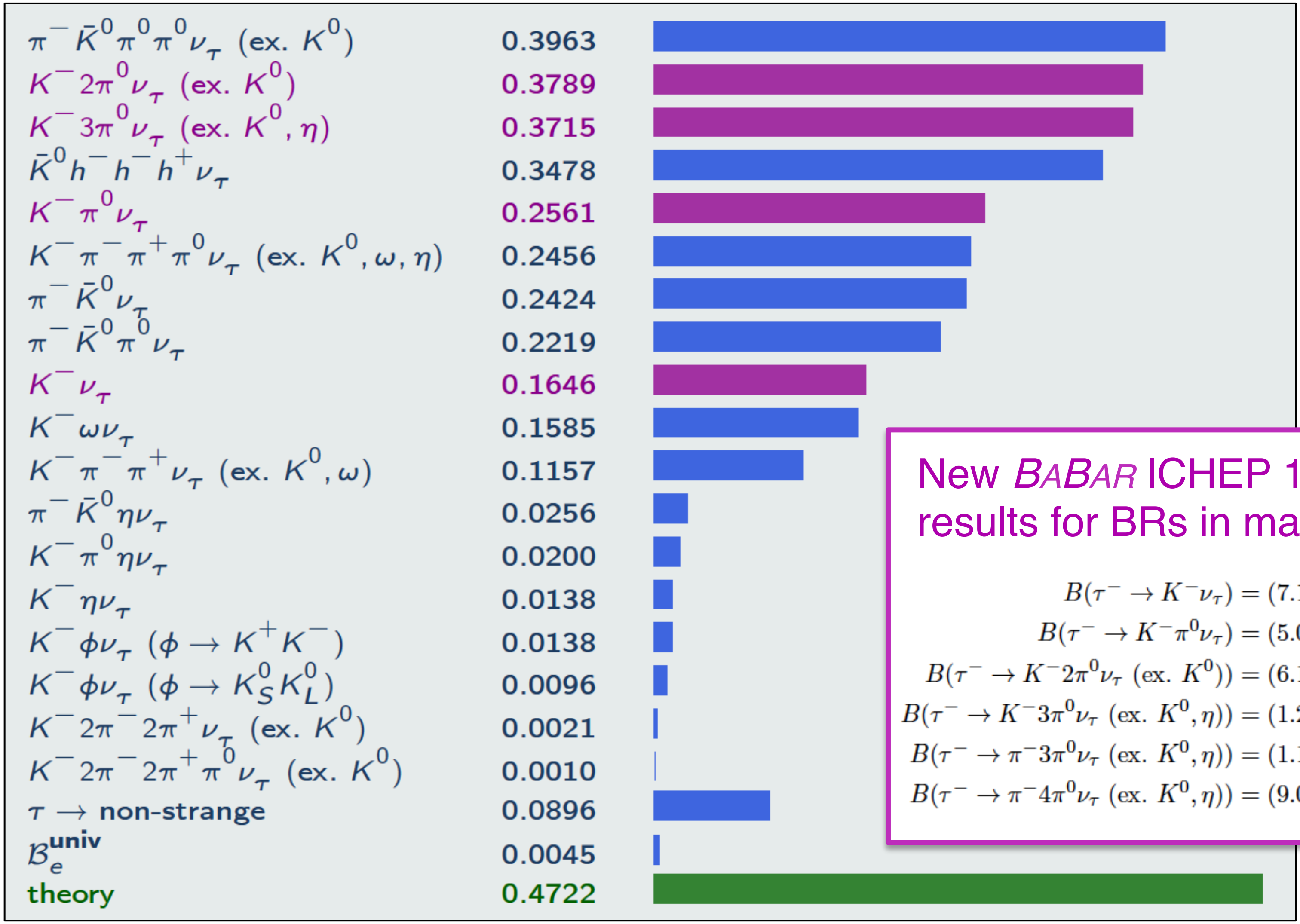
- most measurements systematically limited
  - better experimental conditions at Z^0 peak
  - moderate progress since ~2000
- *B*-factories improved many smaller BRs
- Belle II may contribute in near future

NB: HFLAV fit is not unitarity constrained

ICHEP 2018 data from *BABAR*

Uncertainty budget (%) for the HFLAV 2017 determination of V_{us} from inclusive τ decays

A. Lusiani, CKM 2018



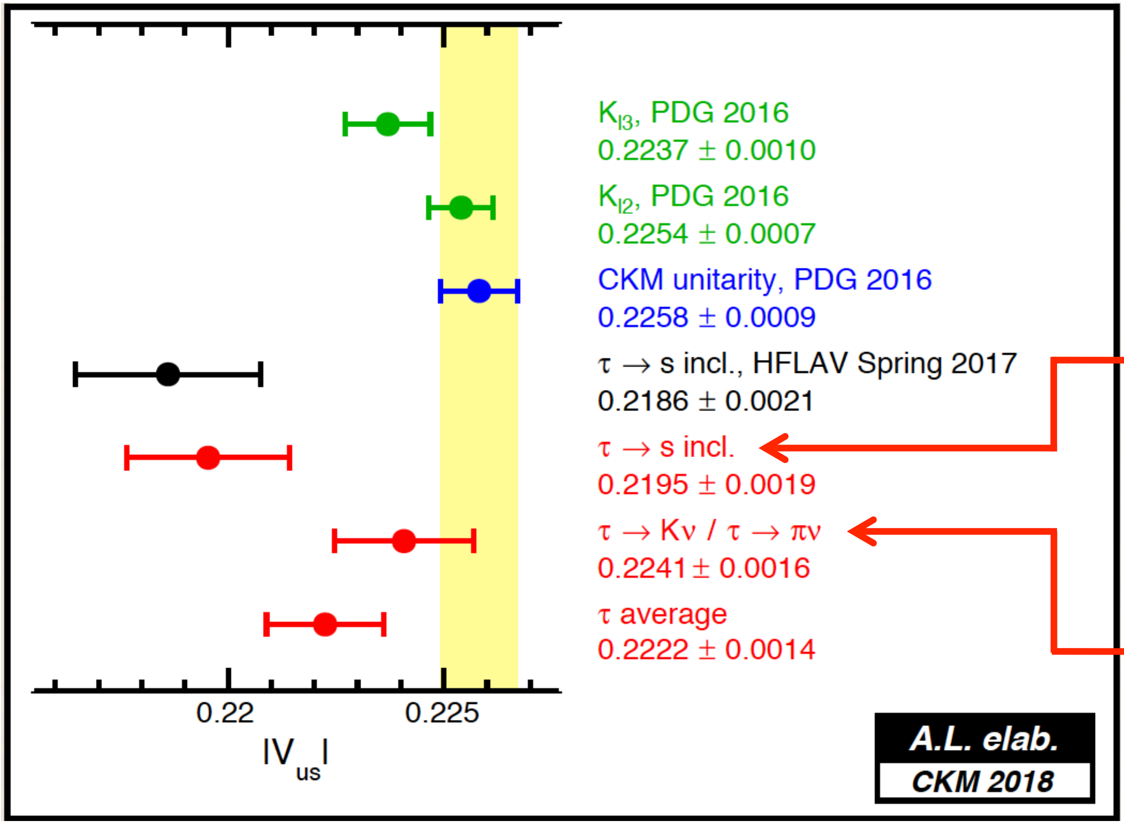
$\pi^- \bar{K}^0 \pi^0 \pi^0 \nu_\tau$ (ex. K^0)	0.3963	
$K^- 2\pi^0 \nu_\tau$ (ex. K^0)	0.3789	
$K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	0.3715	
$\bar{K}^0 h^- h^- h^+ \nu_\tau$	0.3478	
$K^- \pi^0 \nu_\tau$	0.2561	
$K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	0.2456	
$\pi^- \bar{K}^0 \nu_\tau$	0.2424	
$\pi^- \bar{K}^0 \pi^0 \nu_\tau$	0.2219	
$K^- \nu_\tau$	0.1646	
$K^- \omega \nu_\tau$	0.1585	
$K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	0.1157	
$\pi^- \bar{K}^0 \eta \nu_\tau$	0.0256	
$K^- \pi^0 \eta \nu_\tau$	0.0200	
$K^- \eta \nu_\tau$	0.0138	
$K^- \phi \nu_\tau$ ($\phi \to K^+ K^-$)	0.0138	
$K^- \phi \nu_\tau$ ($\phi \to K^0_S K^0_L$)	0.0096	
$K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	0.0021	
$K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	0.0010	
$\tau \to$ non-strange	0.0896	
$\mathcal{B}_e^{\text{univ}}$	0.0045	
theory	0.4722	

New *BABAR* ICHEP 18 preliminary results for BRs in magenta

$$B(\tau^- \to K^- \nu_\tau) = (7.174 \pm 0.033 \pm 0.213) \cdot 10^{-3}$$
$$B(\tau^- \to K^- \pi^0 \nu_\tau) = (5.054 \pm 0.021 \pm 0.148) \cdot 10^{-3}$$
$$B(\tau^- \to K^- 2\pi^0 \nu_\tau \text{ (ex. } K^0)) = (6.151 \pm 0.117 \pm 0.338) \cdot 10^{-4}$$
$$B(\tau^- \to K^- 3\pi^0 \nu_\tau \text{ (ex. } K^0, \eta)) = (1.246 \pm 0.164 \pm 0.238) \cdot 10^{-4}$$
$$B(\tau^- \to \pi^- 3\pi^0 \nu_\tau \text{ (ex. } K^0, \eta)) = (1.168 \pm 0.006 \pm 0.038) \cdot 10^{-2}$$
$$B(\tau^- \to \pi^- 4\pi^0 \nu_\tau \text{ (ex. } K^0, \eta)) = (9.020 \pm 0.400 \pm 0.652) \cdot 10^{-4}$$

HFLAV update: V_{us} from τ decays

A. Lusiani, CKM 2018



HFLAV inclusive τ w/ *BaBar*

$\delta R_\tau^{\mathrm{th}} = 0.242(32)$

for $m_s(m_\tau) = 95 \pm 5$ MeV

as per Gamiz et al. 2006

HFLAV from $\tau_{K2}/\tau_{\pi2}$ w/ *BaBar*

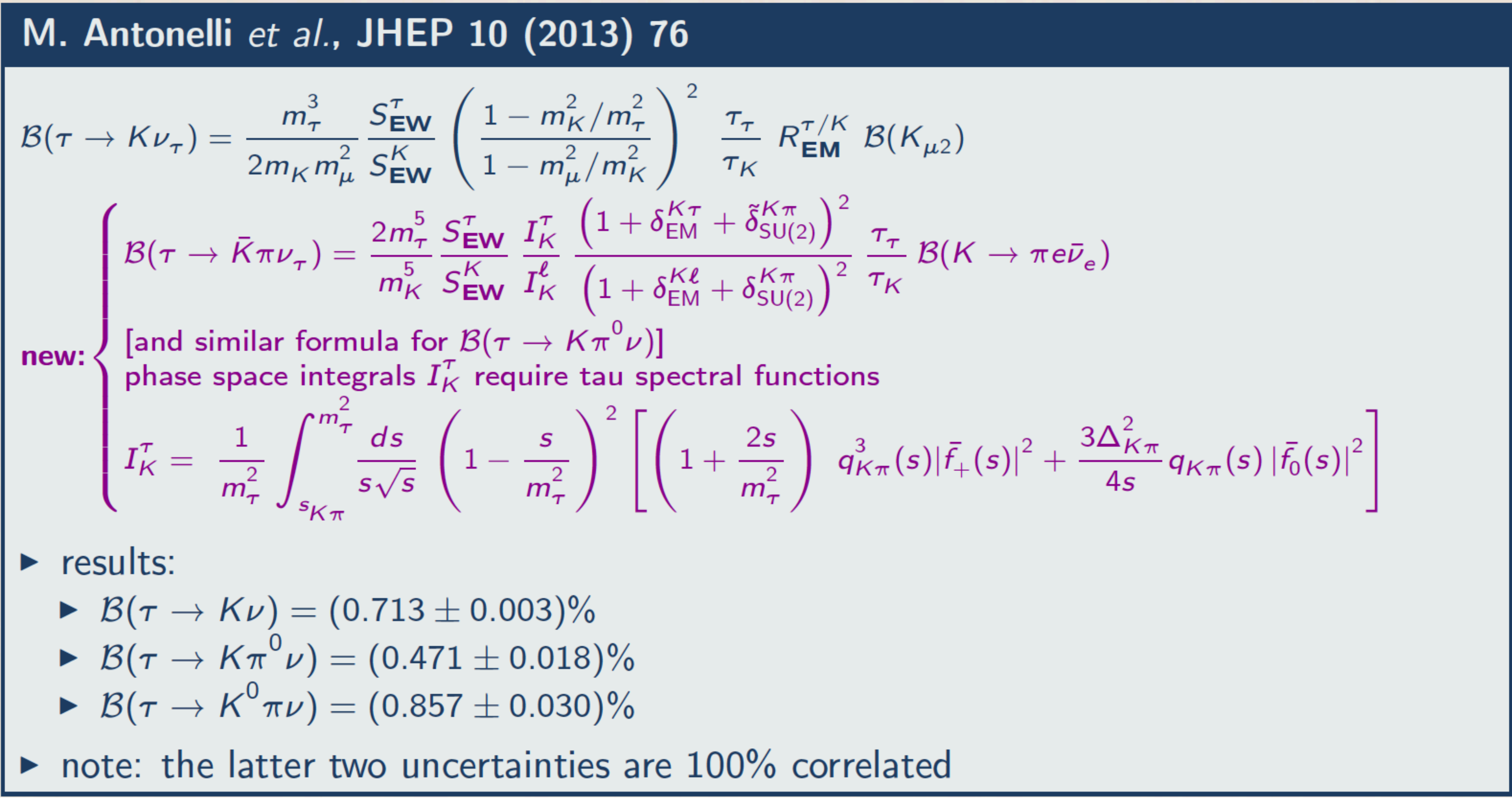
$f_K/f_\pi = 1.1930(30)$

FLAG 2016 $N_f = 2+1+1$

$\Delta_{\mathrm{CKM}} = -3\sigma$ from inclusive τ decays

No significant changes since the first HFLAV fit in 2010

Progress on V_{us} from inclusive τ decays



M. Antonelli *et al.*, JHEP 10 (2013) 76

$$\mathcal{B}(\tau\to K\nu_\tau) = \frac{m_\tau^3}{2m_K m_\mu^2}\,\frac{S_{\mathrm{EW}}^\tau}{S_{\mathrm{EW}}^K}\left(\frac{1-m_K^2/m_\tau^2}{1-m_\mu^2/m_K^2}\right)^2\frac{\tau_\tau}{\tau_K}\,R_{\mathrm{EM}}^{\tau/K}\,\mathcal{B}(K_{\mu2})$$

new:

$$\mathcal{B}(\tau\to\bar K\pi\nu_\tau) = \frac{2m_\tau^5}{m_K^5}\,\frac{S_{\mathrm{EW}}^\tau}{S_{\mathrm{EW}}^K}\,\frac{I_K^\tau}{I_K^\ell}\,\frac{\left(1+\delta_{\mathrm{EM}}^{K\tau}+\tilde\delta_{\mathrm{SU(2)}}^{K\pi}\right)^2}{\left(1+\delta_{\mathrm{EM}}^{K\ell}+\delta_{\mathrm{SU(2)}}^{K\pi}\right)^2}\,\frac{\tau_\tau}{\tau_K}\,\mathcal{B}(K\to\pi e\bar\nu_e)$$

[and similar formula for $\mathcal{B}(\tau\to K\pi^0\nu)$]

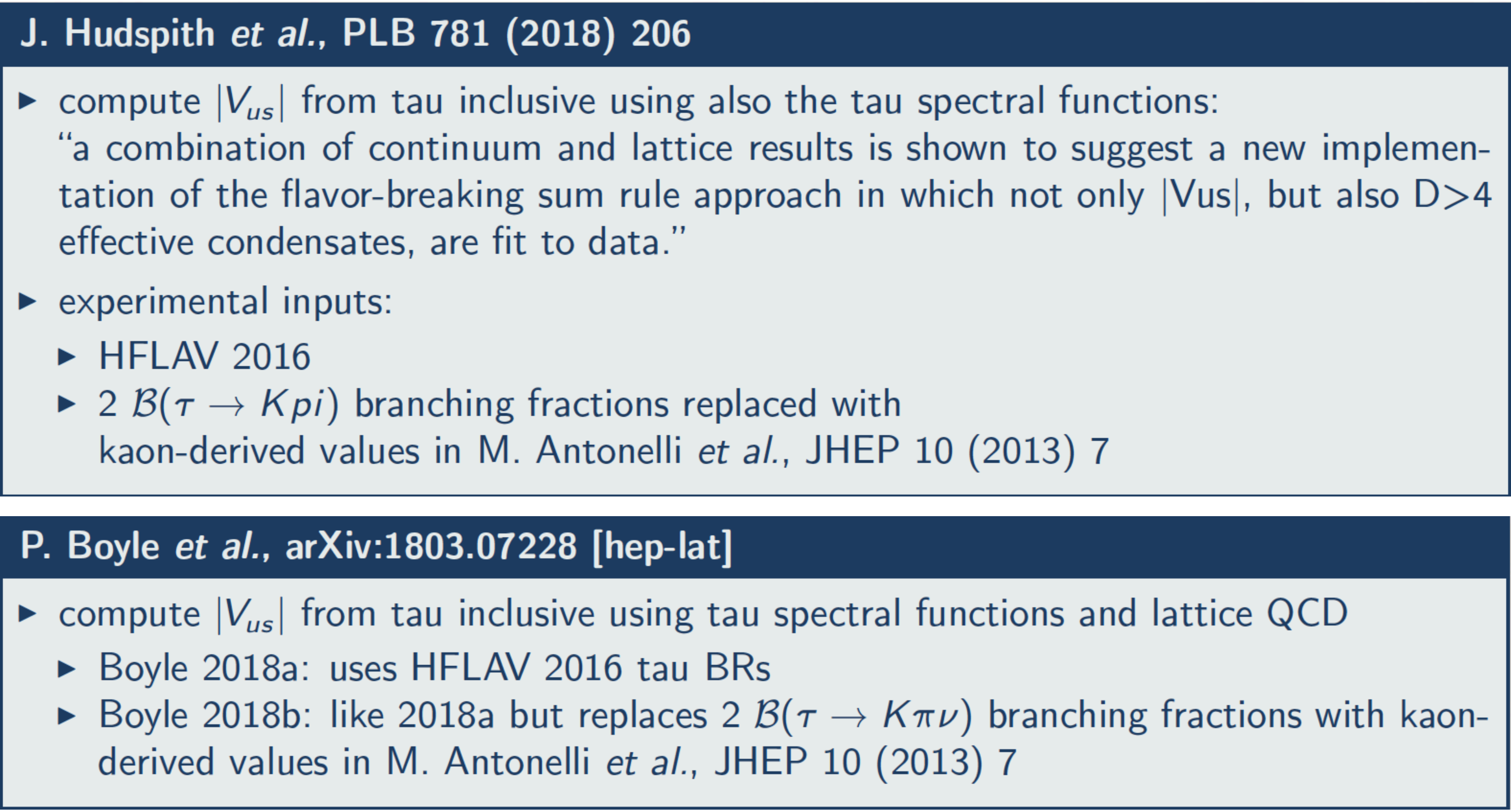
phase space integrals I_K^τ require tau spectral functions

$$I_K^\tau = \frac{1}{m_\tau^2}\int_{s_{K\pi}}^{m_\tau^2}\frac{ds}{s\sqrt{s}}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+\frac{2s}{m_\tau^2}\right)q_{K\pi}^3(s)|\bar f_+(s)|^2+\frac{3\Delta_{K\pi}^2}{4s}q_{K\pi}(s)\,|\bar f_0(s)|^2\right]$$

- results:
  - $\mathcal{B}(\tau\to K\nu) = (0.713\pm0.003)\%$
  - $\mathcal{B}(\tau\to K\pi^0\nu) = (0.471\pm0.018)\%$
  - $\mathcal{B}(\tau\to K^0\pi\nu) = (0.857\pm0.030)\%$
- note: the latter two uncertainties are 100% correlated

A. Lusiani, CKM 2018

Progress on V_{us} from inclusive τ decays



J. Hudspith *et al.*, PLB 781 (2018) 206

- compute $|V_{us}|$ from tau inclusive using also the tau spectral functions:
  "a combination of continuum and lattice results is shown to suggest a new implementation of the flavor-breaking sum rule approach in which not only |Vus|, but also D>4 effective condensates, are fit to data."
- experimental inputs:
  - HFLAV 2016
  - 2 $\mathcal{B}(\tau \to Kpi)$ branching fractions replaced with kaon-derived values in M. Antonelli *et al.*, JHEP 10 (2013) 7

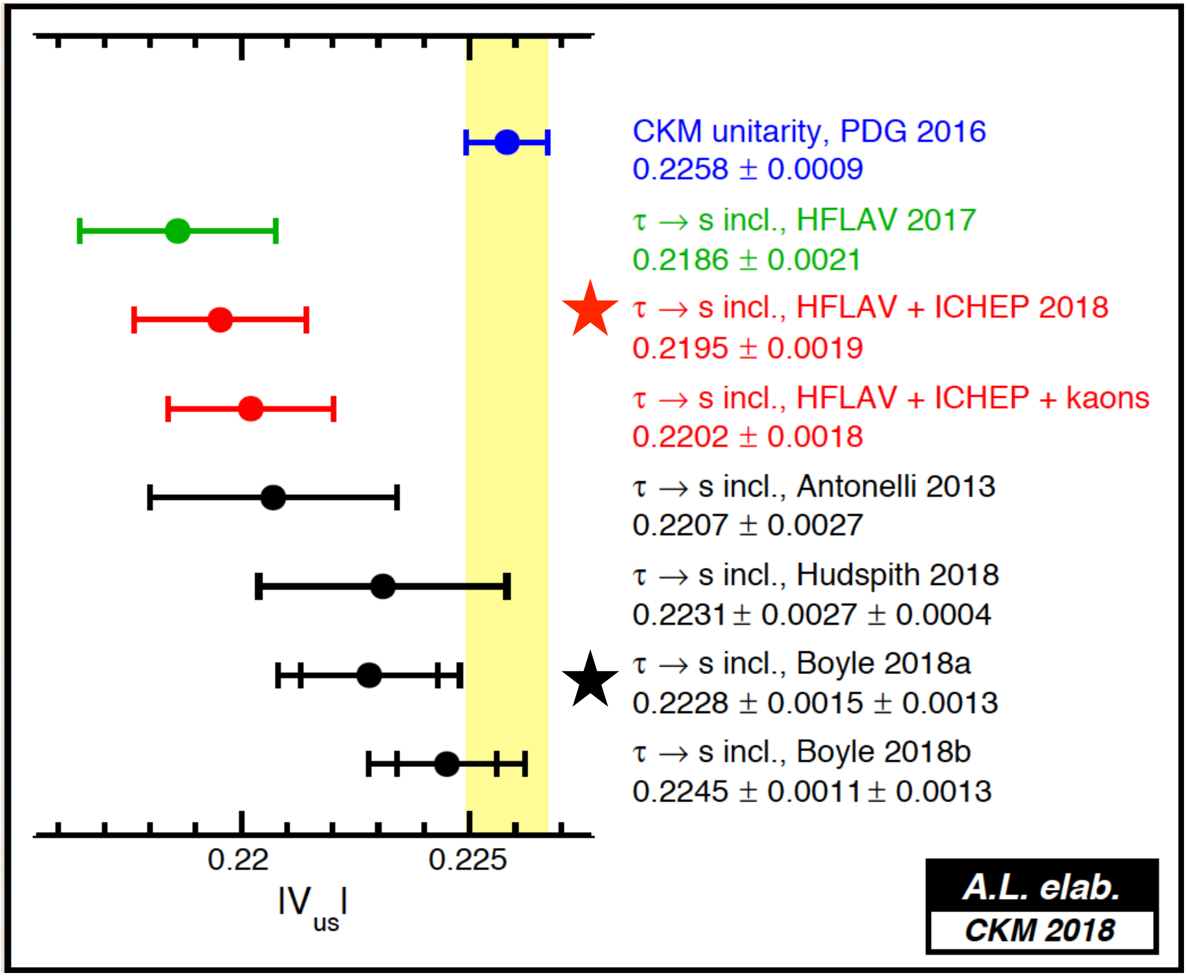
P. Boyle *et al.*, arXiv:1803.07228 [hep-lat]

- compute $|V_{us}|$ from tau inclusive using tau spectral functions and lattice QCD
  - Boyle 2018a: uses HFLAV 2016 tau BRs
  - Boyle 2018b: like 2018a but replaces 2 $\mathcal{B}(\tau \to K\pi\nu)$ branching fractions with kaon-derived values in M. Antonelli *et al.*, JHEP 10 (2013) 7

A. Lusiani, CKM 2018

Recent progress on τ decays for V_{us}

A. Lusiani, CKM 2018



Includes *BABAR*, no input from K

Shows effect of input from K

Constraints from τ spectral data,
with K input

Constraints from LQCD, no K input

Constraints from LQCD, with K input

Continuing series of improvements in analysis systematics resulting in better agreement between inclusive and exclusive determinations

Final conclusions: V_{us} from K and τ decays

Experimental results from K decays

With $|V_{ud}|(0^+ \to 0^+)$ and $N_f = 2+1+1$ lattice

$|V_{us}|\, f_+(0) = 0.21652(41)$ → **$\Delta_{\mathrm{CKM}} = -0.00107(48) = -2.2\sigma$**

$|V_{us}/V_{ud}| \times f_K/f_\pi = 0.27679(34)$ → **$\Delta_{\mathrm{CKM}} = -0.00015(47) = -0.3\sigma$**

- Continuing to see impressive progress on the lattice
- Good prospects for new round of measurements to reduce uncertainty on $|V_{us}|\, f_+(0)$ from current 0.18% to ~0.12% within next few years:

Results from τ decays

With $|V_{ud}|(0^+ \to 0^+)$

$|V_{us}| = 0.2228(20)$
RBC/UKQCD ’18 $N_f = 2+1$
→ **$\Delta_{\mathrm{CKM}} = -0.0013(10) = -1.3\sigma$**

$|V_{us}| = 0.2228(20)$
My eval. from HFLAV ’17, $N_f = 2+1+1$
→ **$\Delta_{\mathrm{CKM}} = -0.0013(10) = -1.3\sigma$**

- Continuing improvements in analysis systematics resulting in better agreement between inclusive and exclusive determinations