Neutrino Physics in the minimal gauged U(1)$_{L\mu-L\tau}$ model

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Content

- Introduction
- Neutrino mass structure in the minimal $U(1)_{L\mu-L\tau}$ model
- Implications for leptogenesis
- Conclusion

1. Introduction

Standard Model

The Standard Model (SM) is a chiral gauge theory of

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

All of the SM fields are charged under this symmetry.

- Left- and right-handed quarks
- Left-handed leptons
- Higgs

This framework describes all (low-energy) physics pretty well.

Can the SM fields have any other gauge interactions??

Anomaly cancellation

L_{Li} - L_{Lj}

It is known that

L_{Li} - L_{Lj} symmetries

are free from anomalies in the SM, thus can be gauged.

R. Foot (1991);
X. G. He, G. C. Joshi, H. Lew, and R. R. Volkas (1991).

In particular, the L_{μ} - L_{τ} gauge symmetry is often discussed since this interaction is less constrained.

Motivations

- Muon g - 2
- Flavor anomalies
- DM physics

etc…

(A part of) previous studies

S. Baek, N. G. Deshpande, X. G. He, and P. Ko (2001);
E. Ma, D. P. Roy and S. Roy (2002); J. Heeck and W. Rodejohann (2011);
T. Araki, J. Heeck, and J. Kubo (2012);
K. Harigaya, T. Igari, M. M. Nojiri, M. Takeuchi, and K. Tobe (2013);
W. Altmannshofer, S. Gori, M. Pospelov, and I. Yavin (2014);
T. Araki, F. Kaneko, T. Ota, J. Sato, T. Shimomura (2015);
K. Fuyuto, W. S. Hou, and M. Kohda (2015);
M. Ibe, W. Nakano, M. Suzuki (2016); Y. Kaneta and T. Shimomura (2017).

Neutrino mass structure in L_{Li} - L_{Lj}

To obtain a realistic model, we need to address

- (At least two) massive light neutrinos

  ➡ Introduction of right-handed neutrinos.

- Sizable mixing among all neutrinos

  ➡ The L_{Li} - L_{Lj} symmetry must be broken.

In the gauged L_{Li} - L_{Lj} models, the neutrino mass structure is tightly constrained due to the symmetry.

Today's talk

We discuss the neutrino mass structure in the minimal gauged L_μ - L_τ model.

(Only one scalar field)

- Observed pattern of neutrino mixing/masses can be obtained.
- A Dirac CP phase consistent with current observation is predicted.
- Future neutrino experiments can test the predictions.
- Leptogenesis can work in a wide range of parameter space.

  (Correct sign of baryon asymmetry)

2. Neutrino mass structure

L_μ - L_τ

We introduce a new U(1) gauge symmetry: $\mathrm{U}(1)_{L_\mu - L_\tau}$

- $\mu_{L,R}$, ν_μ: charge +1
- $\tau_{L,R}$, ν_τ: charge -1

(Others have zero charges)

We also introduce right-handed neutrinos: N_e, N_μ, N_τ

- N_e: charge 0
- N_μ: charge +1
- N_τ: charge -1

Minimal L_μ - L_τ

Charge structure of $N^c_\alpha L_\beta$:

$$Q_{L_\mu - L_\tau}(\mathrm{Dirac}) : \quad \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & -2 \\ 1 & 2 & 0 \end{pmatrix}$$

- Dirac Yukawa is always diagonal.
- Charged lepton Yukawa matrix is also diagonal.

Set to be real.

Even if a U(1)-breaking scalar is introduced, this structure is unchanged as long as renormalizable interactions are considered.

Charge structure of $N^c_\alpha N^c_\beta$:

$$Q_{L_\mu - L_\tau}(\mathrm{Majorana}) : \quad \begin{pmatrix} 0 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & -2 \end{pmatrix}$$

Block diagonal

To obtain sizable neutrino mixing angles, we introduce a U(1)-breaking scalar with charge +1 and couple it to right-handed neutrinos.

Minimal L_μ - L_τ

Lagrangian

$$
\begin{aligned}
\Delta\mathcal{L} = &- \lambda_e N_e^c (L_e \cdot H) - \lambda_\mu N_\mu^c (L_\mu \cdot H) - \lambda_\tau N_\tau^c (L_\tau \cdot H) \\
&- \frac{1}{2} M_{ee} N_e^c N_e^c - M_{\mu\tau} N_\mu^c N_\tau^c - \lambda_{e\mu} \sigma N_e^c N_\mu^c - \lambda_{e\tau} \sigma^* N_e^c N_\tau^c + \text{h.c.}
\end{aligned}
$$

Mass terms

$$
\mathcal{L}_{\text{mass}} = -(\nu_e, \nu_\mu, \nu_\tau) \mathcal{M}_D \begin{pmatrix} N_e^c \\ N_\mu^c \\ N_\tau^c \end{pmatrix} - \frac{1}{2} (N_e^c, N_\mu^c, N_\tau^c) \mathcal{M}_R \begin{pmatrix} N_e^c \\ N_\mu^c \\ N_\tau^c \end{pmatrix} + \text{h.c.}
$$

Mass matrix

$$
\mathcal{M}_D = \frac{v}{\sqrt{2}} \begin{pmatrix} \lambda_e & 0 & 0 \\ 0 & \lambda_\mu & 0 \\ 0 & 0 & \lambda_\tau \end{pmatrix} , \qquad \mathcal{M}_R = \begin{pmatrix} M_{ee} & \lambda_{e\mu} \langle\sigma\rangle & \lambda_{e\tau} \langle\sigma\rangle \\ \lambda_{e\mu} \langle\sigma\rangle & 0 & M_{\mu\tau} \\ \lambda_{e\tau} \langle\sigma\rangle & M_{\mu\tau} & 0 \end{pmatrix} .
$$

$$
\langle H \rangle = \frac{v}{\sqrt{2}}
$$

Real diagonal!

Vanish!

Neutrino mass matrix

Seesaw mechanism

$$\mathcal{M}_{\nu_L} \simeq -\mathcal{M}_D \mathcal{M}_R^{-1} \mathcal{M}_D^T$$

P. Minkowski (1977), T. Yanagida (1979)
M. Gell-Mann, P. Ramond, R. Slansky (1979)
S. L. Glashow (1980)
R. N. Mohapatra and G. Senjanovic (1980)

$$= \frac{v^2}{2(M_{ee}M_{\mu\tau} - 2\lambda_{e\mu}\lambda_{e\tau}\langle\sigma\rangle^2)} \begin{pmatrix} -\lambda_e^2 M_{\mu\tau} & \lambda_e\lambda_\mu\lambda_{e\tau}\langle\sigma\rangle & \lambda_e\lambda_\tau\lambda_{e\mu}\langle\sigma\rangle \\ \lambda_e\lambda_\mu\lambda_{e\tau}\langle\sigma\rangle & -\frac{\lambda_\mu^2\lambda_{e\tau}^2\langle\sigma\rangle^2}{M_{\mu\tau}} & \frac{\lambda_\mu\lambda_\tau(-M_{ee}M_{\mu\tau}+\lambda_{e\mu}\lambda_{e\tau}\langle\sigma\rangle^2)}{M_{\mu\tau}} \\ \lambda_e\lambda_\tau\lambda_{e\mu}\langle\sigma\rangle & \frac{\lambda_\mu\lambda_\tau(-M_{ee}M_{\mu\tau}+\lambda_{e\mu}\lambda_{e\tau}\langle\sigma\rangle^2)}{M_{\mu\tau}} & -\frac{\lambda_\tau^2\lambda_{e\mu}^2\langle\sigma\rangle^2}{M_{\mu\tau}} \end{pmatrix} .$$

Determinant

$$\det\left(\mathcal{M}_{\nu_L}\right) = \frac{\lambda_e^2\lambda_\mu^2\lambda_\tau^2 v^6}{8M_{\mu\tau}\left(M_{ee}M_{\mu\tau} - 2\lambda_{e\mu}\lambda_{e\tau}\langle\sigma\rangle^2\right)} .$$

The determinant vanishes if and only if $\lambda_\nu = 0$.

In this case, the matrix becomes block diagonal, so cannot reproduce the neutrino data.

All neutrino masses are predicted to be non-zero.

PMNS matrix

We can diagonalize the mass matrix using a unitary matrix U:

$$U^T \mathcal{M}_{\nu_L} U = \mathrm{diag}(m_1, m_2, m_3)$$

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\frac{\alpha_2}{2}} & \\ & & e^{i\frac{\alpha_3}{2}} \end{pmatrix}$$

$$c_{ij} \equiv \cos\theta_{ij} \qquad s_{ij} \equiv \sin\theta_{ij}$$

PDG convention

$$\theta_{ij} = [0, \pi/2] \qquad \delta = [0, 2\pi] \qquad m_1 < m_2 \qquad m_2^2 - m_1^2 \ll |m_3^2 - m_1^2|$$

Normal Ordering (NO)

$$m_1 < m_2 < m_3$$

Inverted Ordering (IO)

$$m_3 < m_1 < m_2$$

Two-zero minor structure

Since all the mass eigenvalues are non-zero, we have

$$\mathcal{M}_{\nu_L}^{-1} = U \mathrm{diag}(m_1^{-1}, m_2^{-1}, m_3^{-1}) U^T \simeq -(\mathcal{M}_D^{-1})^T \mathcal{M}_R \mathcal{M}_D^{-1}$$

We then notice that the (μ, μ) and (τ, τ) components of these terms vanish since M_D is diagonal and M_R has zeros in these components.

➡ **Two-zero minor**

L. Lavoura, Phys. Lett. **B609**, 317 (2005);
E. I. Lashin and N. Chamoun, Phys. Rev. **D78**, 073002 (2008).

Conditions

$$\frac{1}{m_1} V_{\mu 1}^2 + \frac{1}{m_2} V_{\mu 2}^2 \, e^{i\alpha_2} + \frac{1}{m_3} V_{\mu 3}^2 \, e^{i\alpha_3} = 0$$

$$\frac{1}{m_1} V_{\tau 1}^2 + \frac{1}{m_2} V_{\tau 2}^2 \, e^{i\alpha_2} + \frac{1}{m_3} V_{\tau 3}^2 \, e^{i\alpha_3} = 0$$

$$U = V \cdot \mathrm{diag}(1, e^{i\alpha_2/2}, e^{i\alpha_3/2})$$

These equations have little dependence on the U(1)-breaking scale and the right-handed mass scale.

Robust condition for this model.

Solving the conditions

We can solve the above conditions as follows:

$$e^{i\alpha_2} = \frac{m_2}{m_1} R_2(\delta) \ , \qquad e^{i\alpha_3} = \frac{m_3}{m_1} R_3(\delta) \ ,$$

with

$$R_2 \equiv \frac{(V_{\mu 1} V_{\tau 3} + V_{\mu 3} V_{\tau 1}) V_{e2}^*}{(V_{\mu 2} V_{\tau 3} + V_{\mu 3} V_{\tau 2}) V_{e1}^*} \ , \qquad R_3 \equiv \frac{(V_{\mu 1} V_{\tau 2} + V_{\mu 2} V_{\tau 1}) V_{e3}^*}{(V_{\mu 2} V_{\tau 3} + V_{\mu 3} V_{\tau 2}) V_{e1}^*} \ .$$

Taking the absolute values of the conditions, we find

$$\frac{m_2}{m_1} = \frac{1}{|R_2(\delta)|} \ , \qquad \frac{m_3}{m_1} = \frac{1}{|R_3(\delta)|} \ .$$

Hence, these mass ratios are given as functions of the Dirac CP δ.

Dirac CP phase

Mass differences (input)

$$\delta m^2 \equiv m_2^2 - m_1^2 \ , \qquad \Delta m^2 \equiv m_3^2 - (m_2^2 + m_1^2)/2 \ .$$

$$\delta m^2 \simeq 7.37 \times 10^{-5} \ \mathrm{eV}^2 \qquad \Delta m^2 \simeq 2.525 \times 10^{-3} \ \mathrm{eV}^2$$

Mass differences (prediction)

$$\delta m^2 = m_1^2 \left(\frac{m_2^2}{m_1^2} - 1 \right) = m_1^2 \left(\frac{1}{|R_2(\delta)|^2} - 1 \right) \ ,$$

$$\Delta m^2 + \frac{\delta m^2}{2} = m_1^2 \left(\frac{m_3^2}{m_1^2} - 1 \right) = m_1^2 \left(\frac{1}{|R_3(\delta)|^2} - 1 \right) \ .$$

Given the observed values of the neutrino mixing angles and the mass differences, Dirac CP δ and neutrino mass scale are predicted.

Input values

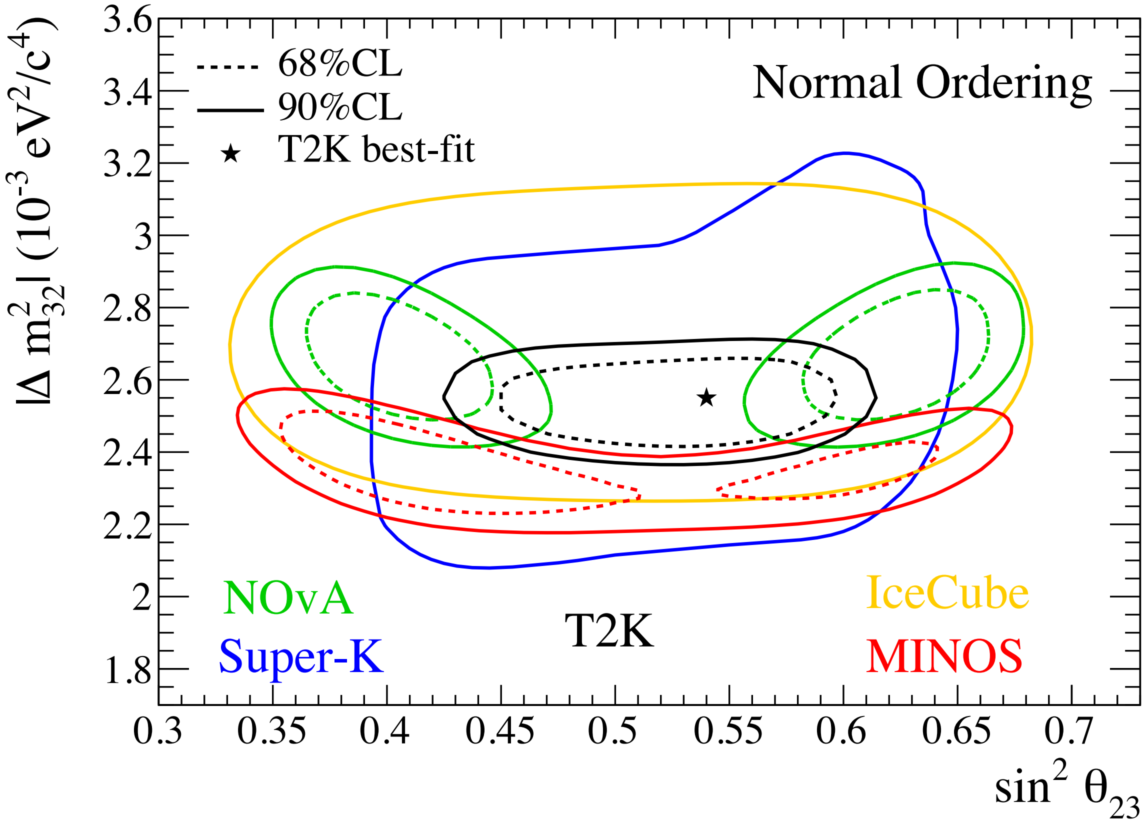
Parameter	Best fit	1σ range	2σ range
$\delta m^2/10^{-5}$ eV^2	7.37	7.21–7.54	7.07–7.73
$\Delta m^2/10^{-3}$ eV^2	2.525	2.495–2.567	2.454–2.606
$\sin^2\theta_{12}/10^{-1}$	2.97	2.81–3.14	2.65–3.34
$\sin^2\theta_{23}/10^{-1}$	4.25	4.10–4.46	3.95–4.70
$\sin^2\theta_{13}/10^{-2}$	2.15	2.08–2.22	1.99–2.31
δ/π	1.38	1.18–1.61	1.00–1.90

F. Capozzi, E. D. Valentino, E. Lisi, A. Marrone, A. Melchiorri, A. Palazzo, Phys. Rev. **D95**, 096014 (2017)

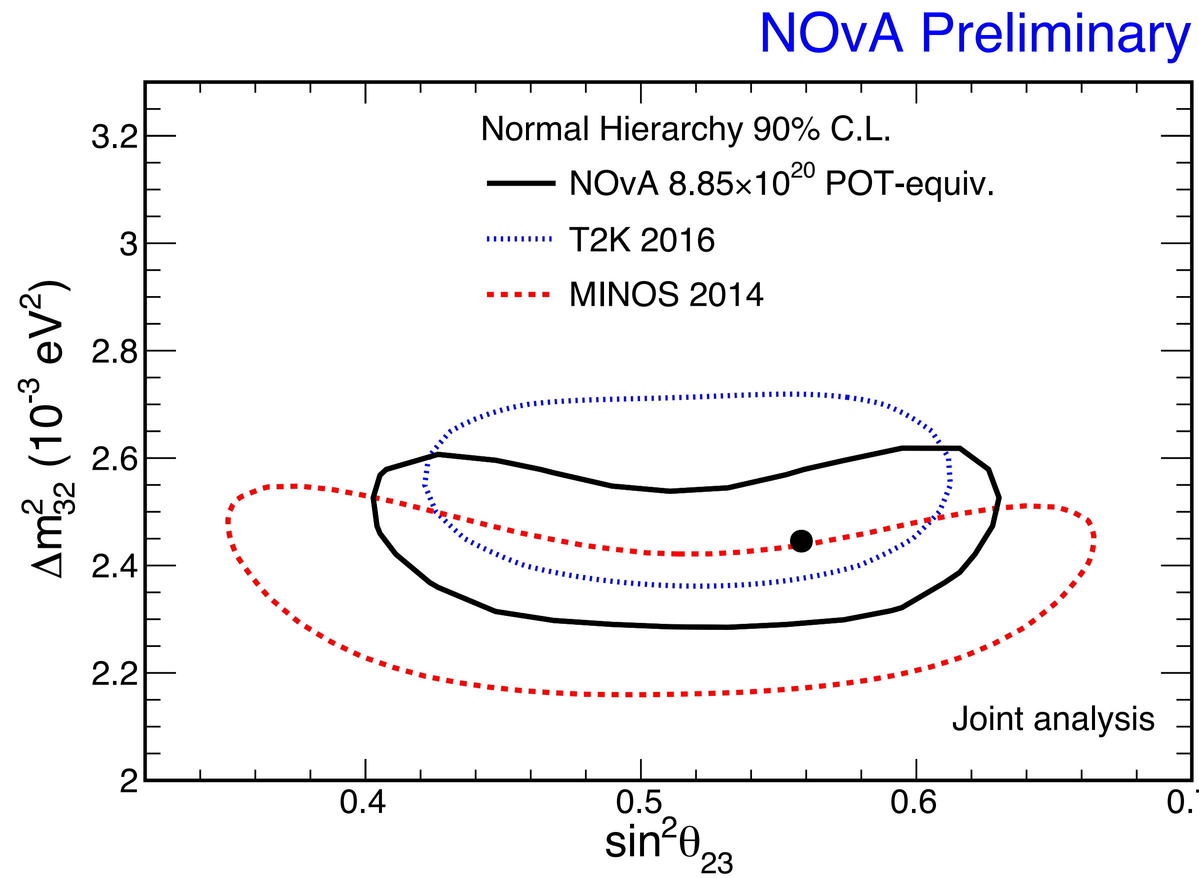
We used this result in our paper, but…

(The latest one then)

θ_{23} discrepancy resolved.



T2K [arXiv: 1707.01048]



Talk by A. Radovic, JETP, Jan., 2018.

There was a tension between T2K and NOvA results.

NOvA updated their result recently, which now agrees to the T2K result.

They changed energy response model, selection criteria, etc…

Input values

NuFIT 3.2 (2018)

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 4.14$)		Any Ordering
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	3σ range
$\sin^2\theta_{12}$	$0.307^{+0.013}_{-0.012}$	$0.272 \to 0.346$	$0.307^{+0.013}_{-0.012}$	$0.272 \to 0.346$	$0.272 \to 0.346$
$\theta_{12}/^\circ$	$33.62^{+0.78}_{-0.76}$	$31.42 \to 36.05$	$33.62^{+0.78}_{-0.76}$	$31.43 \to 36.06$	$31.42 \to 36.05$
$\sin^2\theta_{23}$	$0.538^{+0.033}_{-0.069}$	$0.418 \to 0.613$	$0.554^{+0.023}_{-0.033}$	$0.435 \to 0.616$	$0.418 \to 0.613$
$\theta_{23}/^\circ$	$47.2^{+1.9}_{-3.9}$	$40.3 \to 51.5$	$48.1^{+1.4}_{-1.9}$	$41.3 \to 51.7$	$40.3 \to 51.5$
$\sin^2\theta_{13}$	$0.02206^{+0.00075}_{-0.00075}$	$0.01981 \to 0.02436$	$0.02227^{+0.00074}_{-0.00074}$	$0.02006 \to 0.02452$	$0.01981 \to 0.02436$
$\theta_{13}/^\circ$	$8.54^{+0.15}_{-0.15}$	$8.09 \to 8.98$	$8.58^{+0.14}_{-0.14}$	$8.14 \to 9.01$	$8.09 \to 8.98$
$\delta_{\mathrm{CP}}/^\circ$	234^{+43}_{-31}	$144 \to 374$	278^{+26}_{-29}	$192 \to 354$	$144 \to 374$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.40^{+0.21}_{-0.20}$	$6.80 \to 8.02$	$7.40^{+0.21}_{-0.20}$	$6.80 \to 8.02$	$6.80 \to 8.02$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.494^{+0.033}_{-0.031}$	$+2.399 \to +2.593$	$-2.465^{+0.032}_{-0.031}$	$-2.562 \to -2.369$	$\begin{bmatrix} +2.399 \to +2.593 \\ -2.536 \to -2.395 \end{bmatrix}$

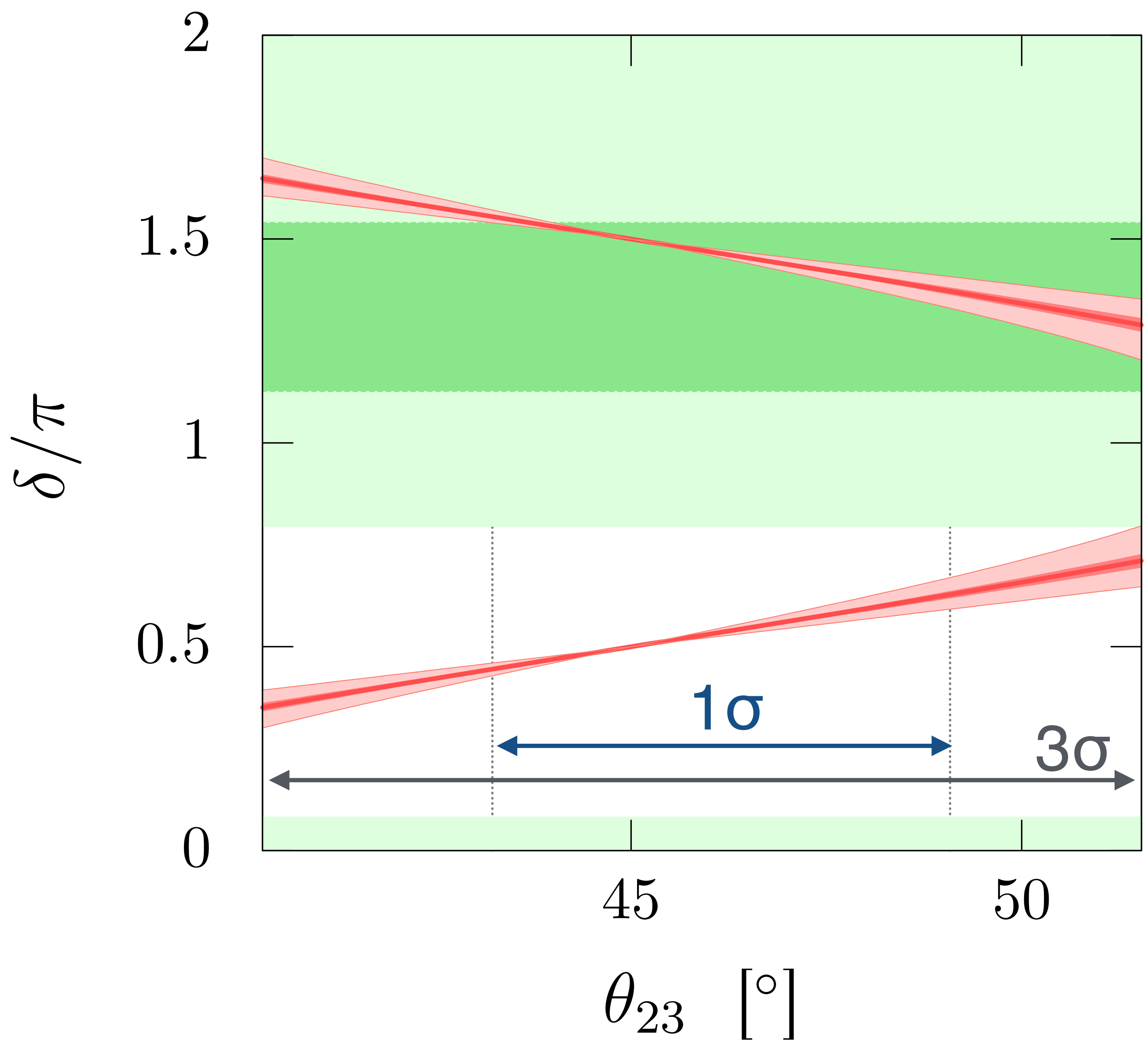
We take the three mixing angles and the two mass squared differences as input parameters:

$$\theta_{12},\ \theta_{23},\ \theta_{13},\ \delta m^2,\ \Delta m^2\ .$$

Other parameters are predicted.

Figures are different from those in our paper.

Dirac CP phase



See also

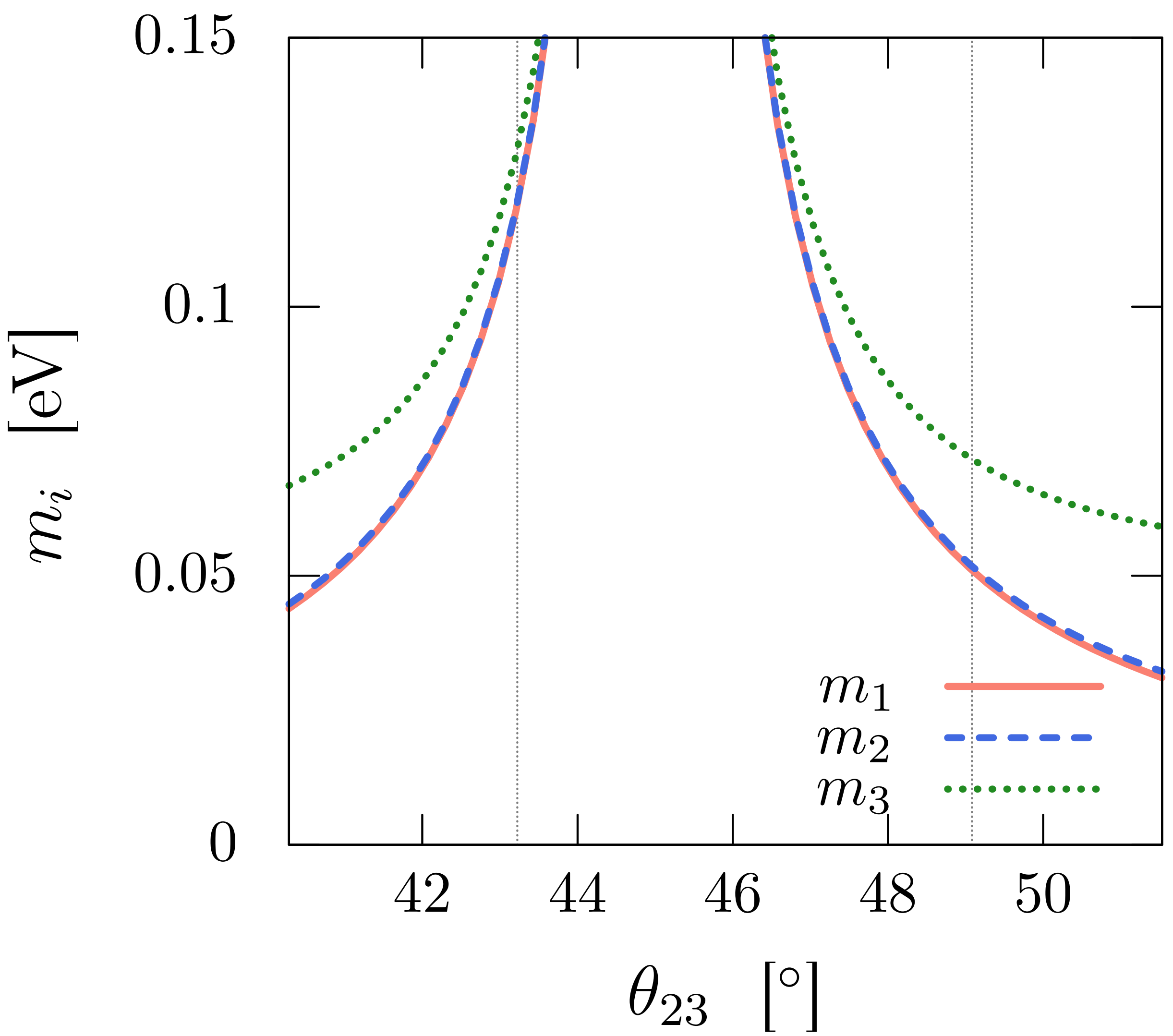
A. Crivellin, G. D'Ambrosio, J. Heeck, Phys. Rev. D91, 075006 (2015).

This model predicts a Dirac CP phase consistent with the current observation.

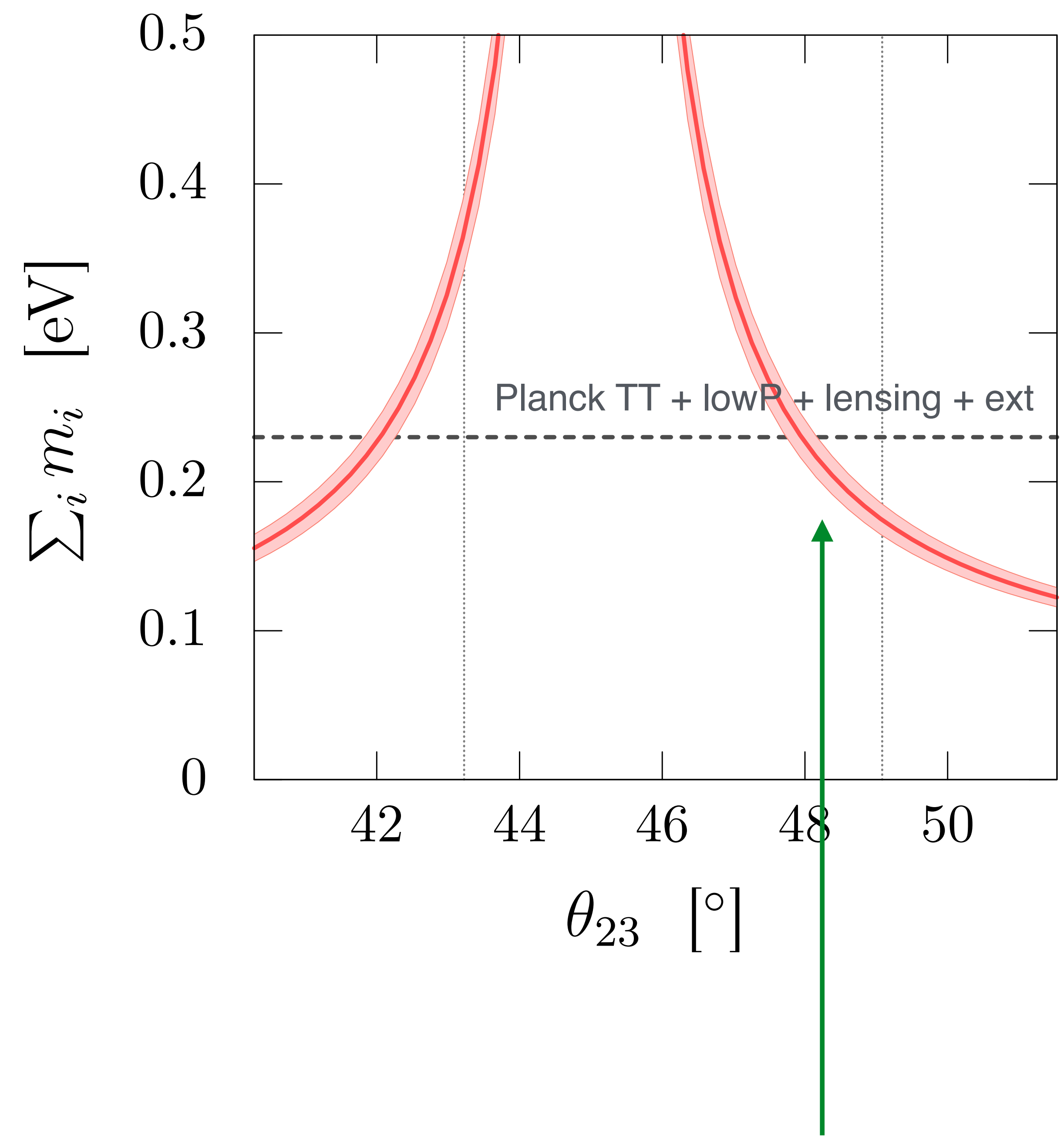
K. Asai, K. Hamaguchi, N. Nagata, Eur. Phys. J. **C77**, 763 (2017).

Neutrino masses

Mass spectrum



Sum



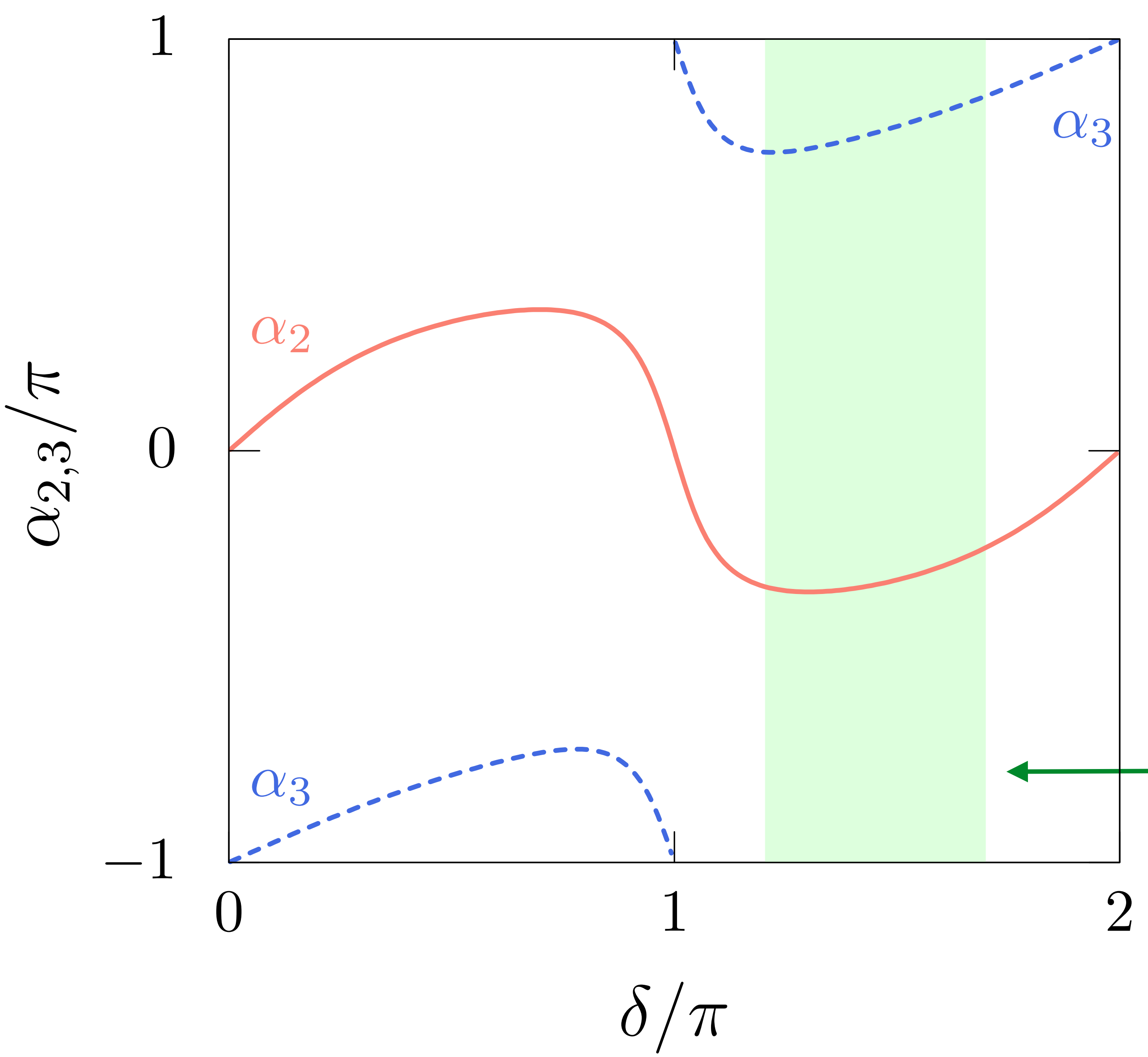
Normal ordering is predicted.

There is a parameter range which can evade the Planck limit.

K. Asai, K. Hamaguchi, N. Nagata, Eur. Phys. J. **C77**, 763 (2017).

Majorana CP phases

Majorana CP phases are also predicted.



$$e^{i\alpha_2} = \frac{m_2}{m_1} R_2(\delta) \ , \qquad e^{i\alpha_3} = \frac{m_3}{m_1} R_3(\delta)$$

Range of δ predicted in this model.

This prediction is essential when we evaluate the effective mass for neutrinoless double beta decay.

K. Asai, K. Hamaguchi, N. Nagata, Eur. Phys. J. **C77**, 763 (2017).

Neutrinoless double beta decay

Decay half life

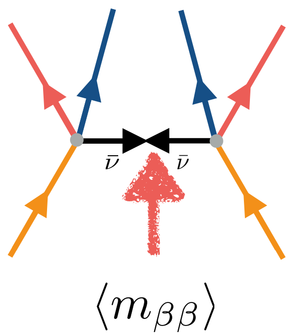
$$(T^{0\nu}_{1/2})^{-1} = G^{0\nu}|M^{0\nu}|^2\langle m_{\beta\beta}\rangle^2$$

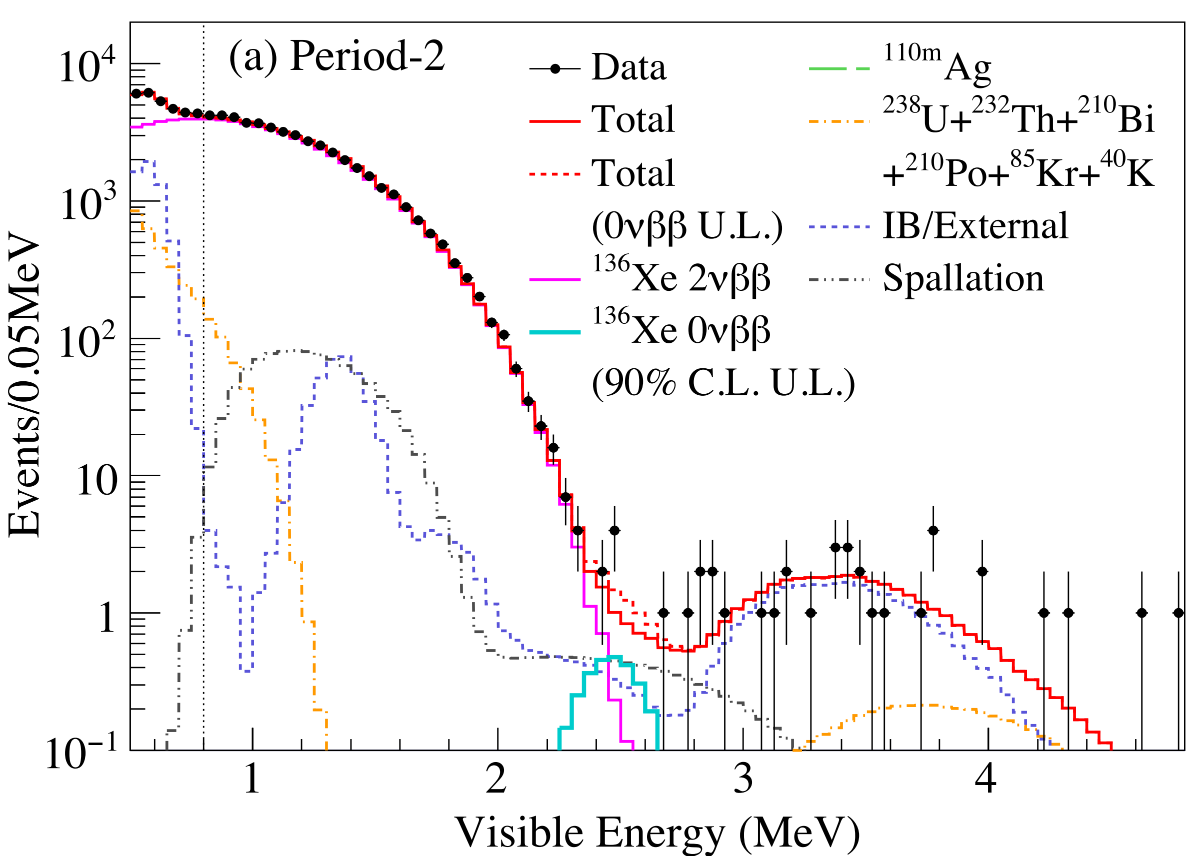
$G^{0\nu}$: Phase space factor

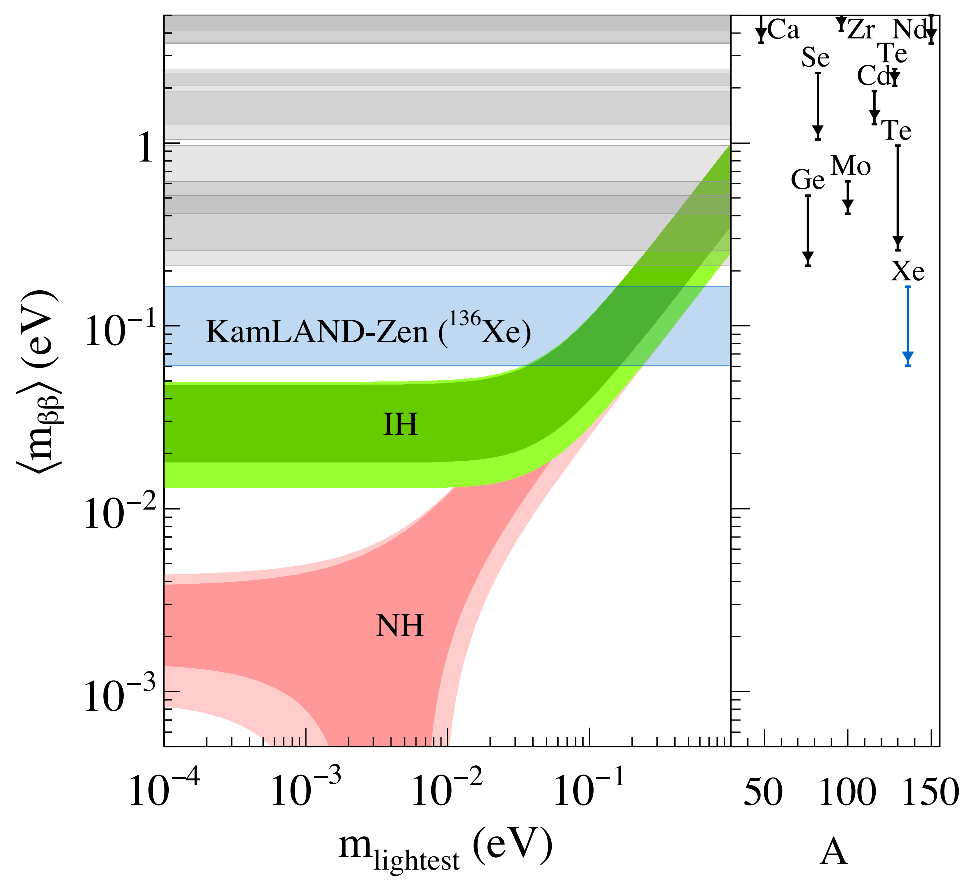
$M^{0\nu}$: Nuclear matrix element

Effective Majorana mass

$$\langle m_{\beta\beta}\rangle \equiv \left|\sum_i U_{ei}^2 m_i\right| = \left|c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{i\alpha_2} m_2 + s_{13}^2 e^{i(\alpha_3-2\delta)} m_3\right| \ .$$

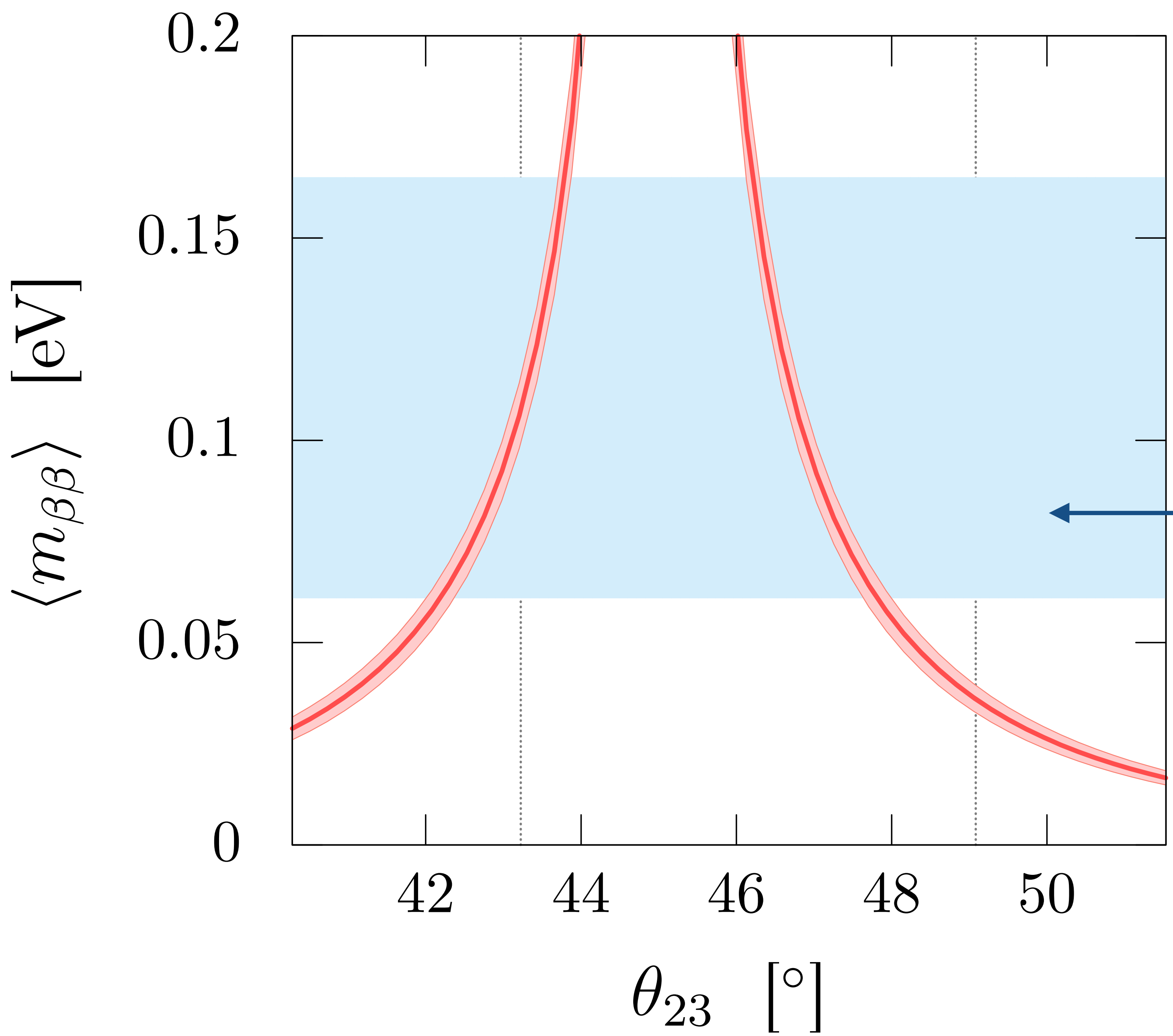






KamLAND-Zen, Phys. Rev. Lett. **117**, 109903 (2016).

Neutrinoless double beta decay



KamLAND-Zen bound.

KamLAND-Zen, Phys. Rev. Lett. **117**, 109903 (2016).

Within the reach of future experiments.

K. Asai, K. Hamaguchi, N. Nagata, Eur. Phys. J. **C77**, 763 (2017).

Future tests

The following predictions of the model can be tested in the future:

- Normal ordering
  - Currently favored at 2σ level.

    F. Capozzi, E. D. Valentino, E. Lisi, A. Marrone, A. Melchiorri, A. Palazzo, Phys. Rev. **D95**, 096014 (2017)
  - Quite a few future experiments can determine the neutrino mass hierarchy at more than 3σ level within a decade.

    Ex.) PINGU, ORCA, JUNO
- Precision measurements of θ_{23} and δ
- Sum of neutrino masses
- Neutrinoless double beta decay

3. Leptogenesis

Baryon asymmetry in the Universe

Baryon asymmetry of the Universe

$$\Omega_{\rm b} h^2 \simeq 0.022 \quad \Rightarrow \quad n_{\rm B}/s \simeq 0.87 \times 10^{-10}$$

Planck (2015)

The sign of n_B/s

The sign of the baryon asymmetry does have physical meaning.

We can define the baryon&lepton numbers unambiguously.

Ex.) K_L semileptonic decay

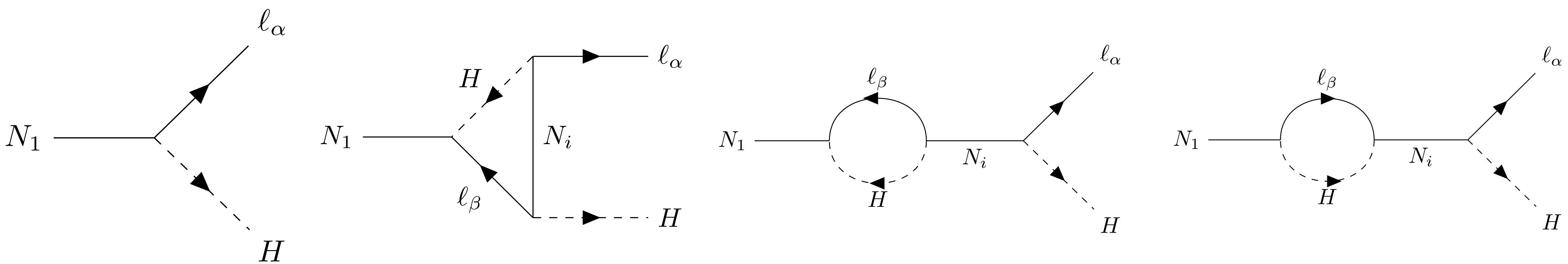
$$A_L \equiv \frac{\Gamma(K_L^0 \to \pi^- \ell^+ \nu) - \Gamma(K_L^0 \to \pi^+ \ell^- \nu)}{\Gamma(K_L^0 \to \pi^- \ell^+ \nu) + \Gamma(K_L^0 \to \pi^+ \ell^- \nu)} \simeq 3.32 \times 10^{-3}$$

Charge asymmetry

Leptogenesis M. Fukugita and T. Yanagida (1986)

Non-thermal decay of Majorana right-handed neutrinos generates lepton asymmetry.

Asymmetry parameter



$$\epsilon_1 \equiv \frac{\Gamma(N_1 \to H\ell) - \Gamma(N_1 \to H^*\bar{\ell})}{\Gamma(N_1 \to H\ell) + \Gamma(N_1 \to H^*\bar{\ell})}$$

The size of generated lepton number depends on scenarios, but the sign is determined by this asymmetry parameter.

Converted to baryon asymmetry through the sphaleron process.

V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov (1985).

Right-handed neutrino sector

Right-handed neutrino mass matrix

$$\mathcal{M}_R \simeq -\mathcal{M}_D \mathcal{M}_{\nu_L}^{-1} \mathcal{M}_D$$

There are only three additional parameters (Yukawa couplings):

λ_e, λ_μ, λ_τ in M_D

No additional phases

Lagrangian

$$\Delta\mathcal{L} = -\sum_{i=1}^{3} \sum_{\alpha=e,\mu,\tau} \widehat{\lambda}_{i\alpha} \widehat{N}_i^c (L_\alpha \cdot H) - \frac{1}{2} \sum_{i=1}^{3} M_i \widehat{N}_i^c \widehat{N}_i^c + \text{h.c.}$$

with

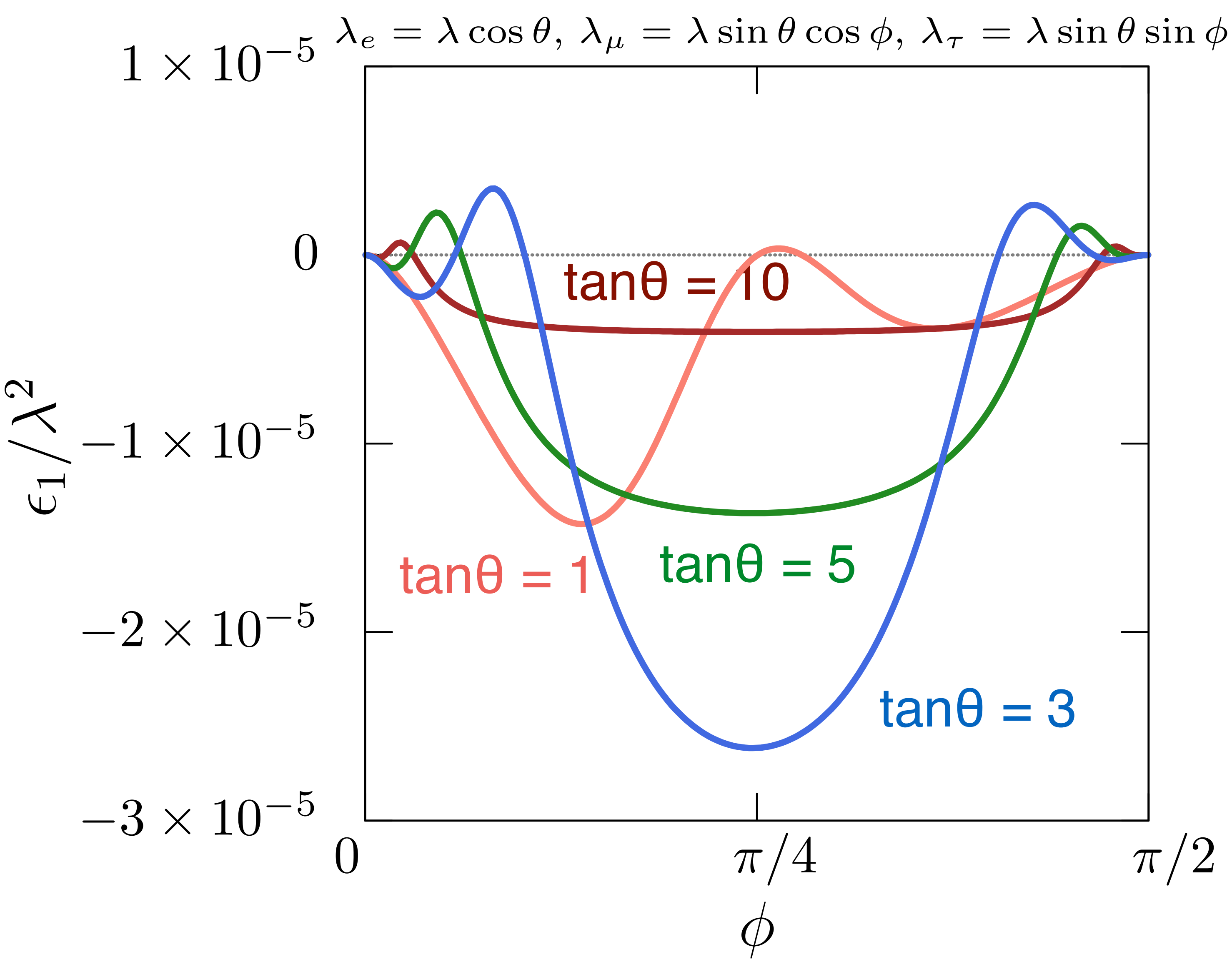
$$\mathcal{M}_R = \Omega^* \mathrm{diag}(M_1, M_2, M_3) \Omega^\dagger, \quad \Omega^\dagger \Omega = I,$$

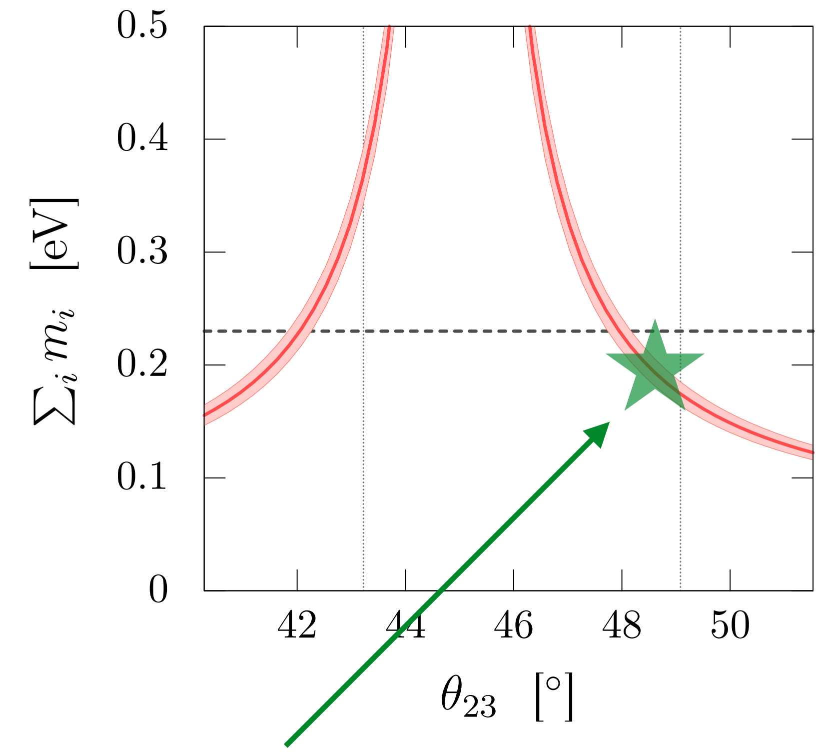
$$\widehat{N}_i^c = \sum_\alpha \Omega_{\alpha i}^* N_\alpha^c,$$

$$\widehat{\lambda}_{i\alpha} = \Omega_{\alpha i} \lambda_\alpha \quad (\text{not summed}).$$

M_i are real positive

Asymmetry parameter





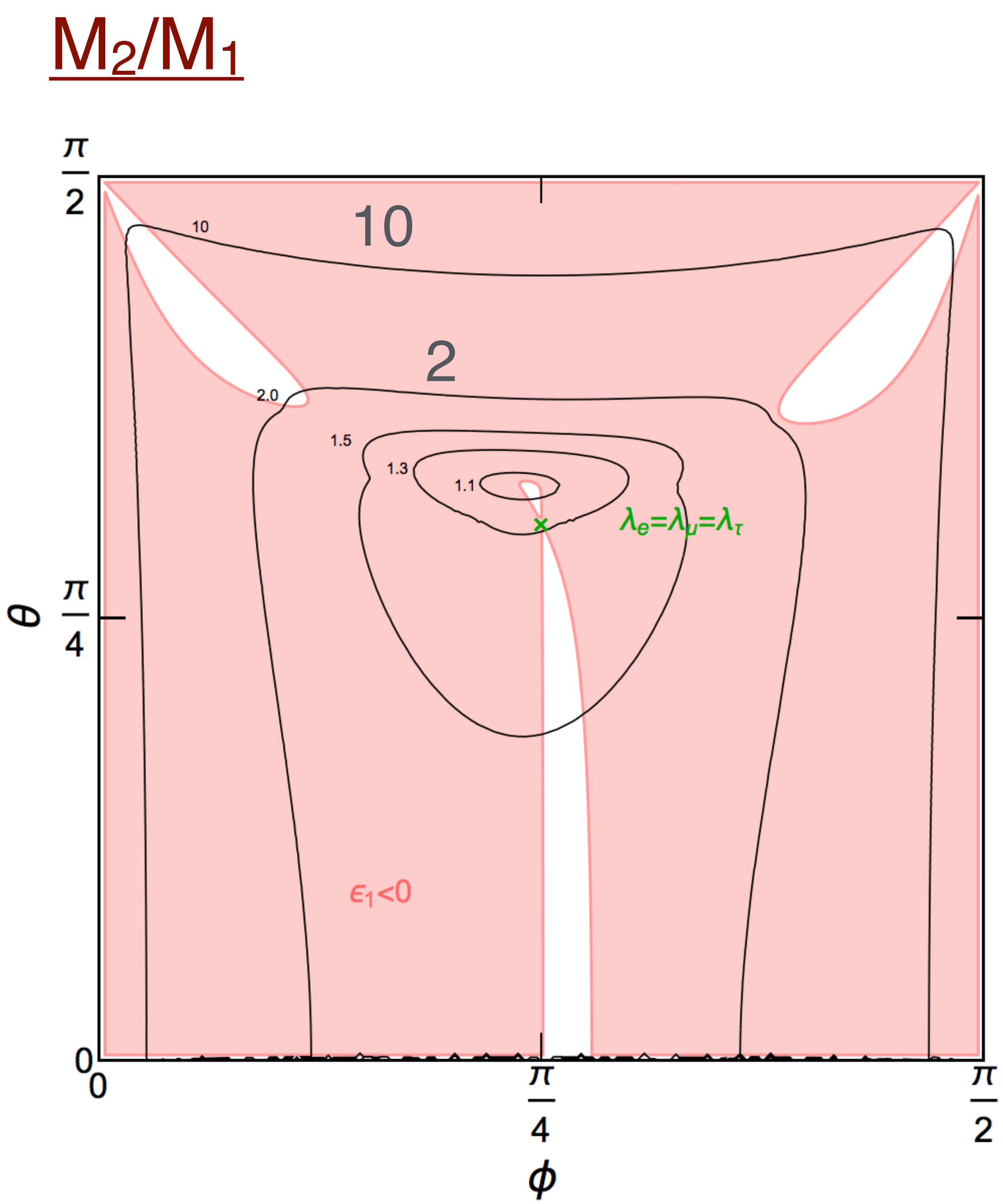
We use this parameter point.

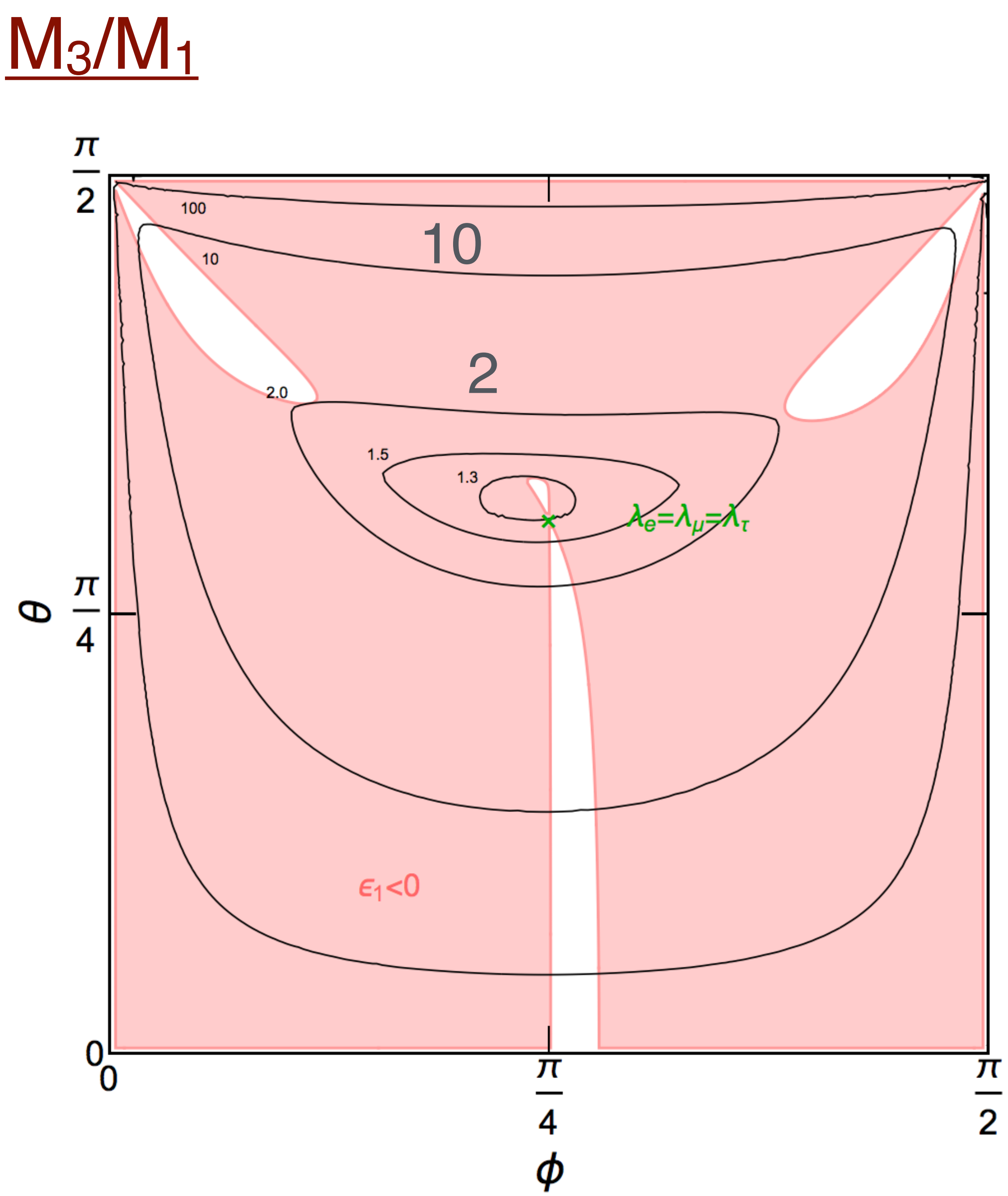
$$\frac{n_B}{n_L} = -\frac{4(2N_g + N_H)}{22N_g + 13N_H} = -\frac{28}{79}$$

$$\epsilon_1 < 0$$

Correct sign of baryon asymmetry is obtained in wide range of parameter space.

Right-handed neutrino masses





Right-handed neutrino masses tend to be close to each other.

More detailed calculation for baryon asymmetry is ongoing.

4. Conclusion

Conclusion

- Minimal gauged L_μ - L_τ model gives the two-zero minor structure for the neutrino mass matrix.
- Dirac CP phase is predicted to be in the favored region.
- This model is consistent with the current experimental limits.
- Future experiments can test this model.
- Leptogenesis in this model can give the correct sign of baryon asymmetry in a wide range of parameter space.

Backup

Anomaly cancellation

A healthy gauge theory requires anomaly cancellation.

D. J. Gross and R. Jackiw, Phys. Rev. **D6**, 477 (1972).

$$\mathrm{tr}\left[\{T_a, T_b\}T_c\right] = 0$$

$$\mathrm{tr}\left(T_a\right) = 0$$

Gravitation

Suppose that anomalies are cancelled within each generation.

SU(3)-SU(3)-U(1)

$$2Y_Q + Y_u + Y_d = 0$$

U(1)-U(1)-U(1)

$$6Y_Q^3 + 3Y_u^3 + 3Y_d^3 + 2Y_L^3 + Y_e^3 = 0$$

SU(2)-SU(2)-U(1)

$$3Y_Q + Y_L = 0$$

Graviton-graviton-U(1)

$$6Y_Q + 3Y_u + 3Y_d + 2Y_L + Y_e = 0$$

$$\Rightarrow \quad Y_Q(2Y_Q - Y_u)(4Y_Q + Y_u) = 0$$

Anomaly cancellation

$$Y_Q(2Y_Q - Y_u)(4Y_Q + Y_u) = 0$$

SM!

There are two sets of the solutions.

$$\mathrm{U}(1): \quad Y_u = -4Y_Q\ , \quad Y_d = 2Y_Q\ , \quad Y_L = -3Y_Q\ , \quad Y_e = 6Y_Q\ .$$

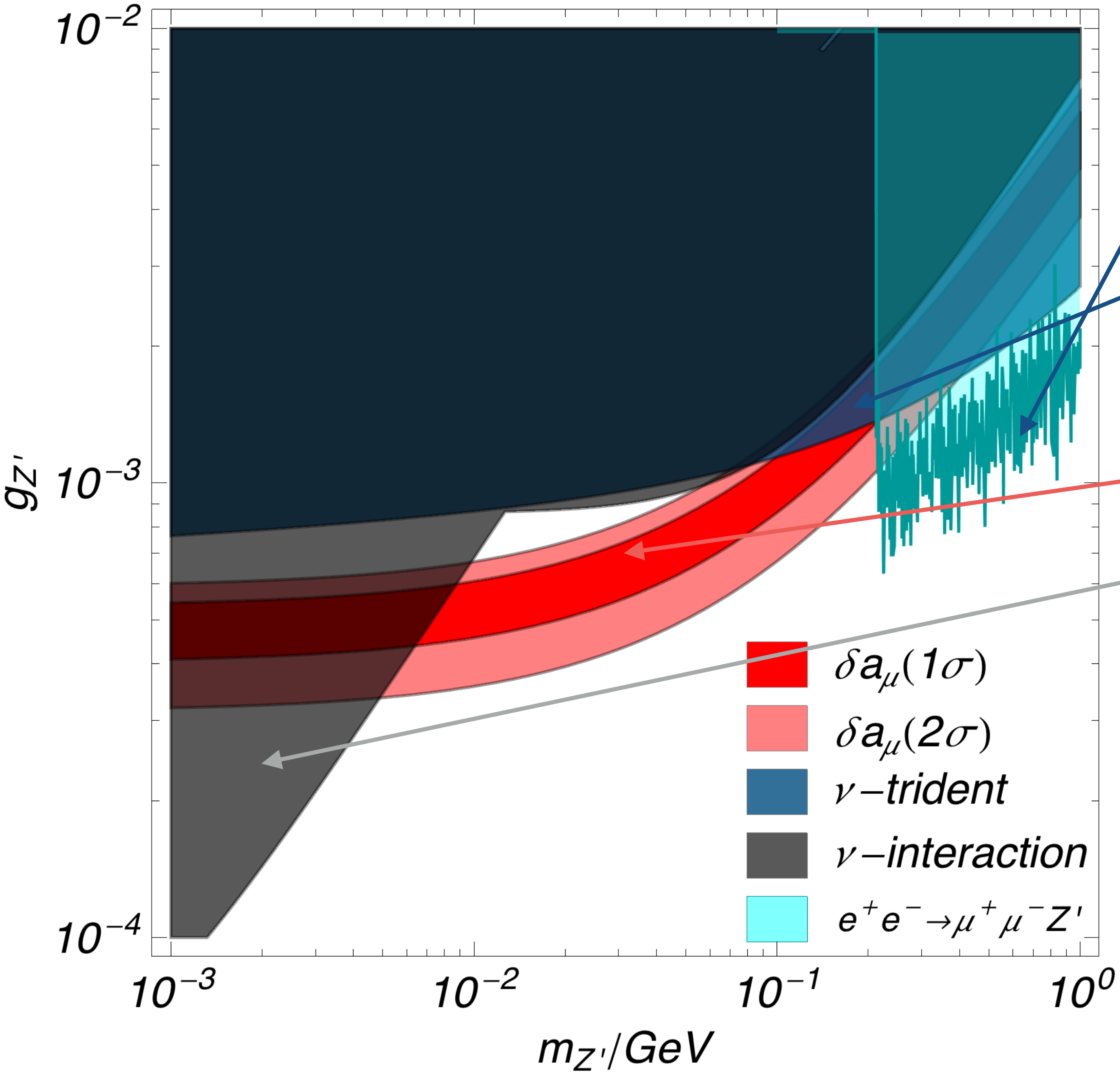
$$\mathrm{U}(1)': \quad Y_Q = Y_L = Y_e = 0\ , \quad Y_u = -Y_d\ .$$

- The $Y_Q = Y_u/2$ case results in the first case by interchanging u_R and d_R.
- The solutions are exclusive.

U(1)-U(1)-U(1)’ ∝ $(-4)^2 - (2)^2$

It is not possible to introduce an extra U(1) symmetry in addition to the hypercharge U(1).

Low-scale L_μ - L_τ model



M. Ibe, W. Nakano, M. Suzuki, Phys. Rev. **D95**, 055022 (2017).

BABAR limit

BABAR, Phys. Rev. **D94**, 011102 (2016).

Neutrino trident

W. Altmannshofer, S. Gori, M. Pospelov, and I. Yavin, Phys. Rev. Lett. **113**, 091801 (2014).

g-2

Neutron-electron scattering

S. Bilmis, I. Turan, T. M. Aliev, M. Daniz, L. Singh, H. T. Wong, Phys. Rev. **D92**, 033009 (2015).

Via kinetic mixing induced at loop level.

ΔN_{eff}

$$m_{Z'} \gtrsim 5.3 \times \left(\frac{T_{\nu,\mathrm{dec}}}{1.5\ \mathrm{MeV}}\right)\ \mathrm{MeV}$$

An O(10) MeV Z' affects neutrino star cooling, which also gives a constraint.

A. Kamada and Hai-Bo Yu, Phys. Rev. **D92**, 113004 (2015).

Kinetic mixing

In general, there is a kinetic mixing between $U(1)_Y$ and U(1)':

$$\mathcal{L}_{\text{mix}} = \frac{\epsilon}{2} B_{\mu\nu} F'^{\mu\nu}$$

B. Holdom, Phys. Lett. **B166**, 196 (1986).

We may forbid this using a discrete symmetry:

$$\mu \leftrightarrow \tau \ , \quad B_\mu \leftrightarrow B_\mu \ , \quad A'_\mu \leftrightarrow -A'_\mu$$

R. Foot, X. G. He, H. Lew, and R. R. Volkas, Phys. Rev. **D50**, 4571 (1994).

This symmetry is broken by the μ and τ masses, and thus the kinetic mixing is induced at loop level.

$$\epsilon = \frac{8}{3} \frac{e g_{Z'}}{16\pi^2} \ln\left(\frac{m_\tau}{m_\mu}\right)$$

(in the limit of low momenta)

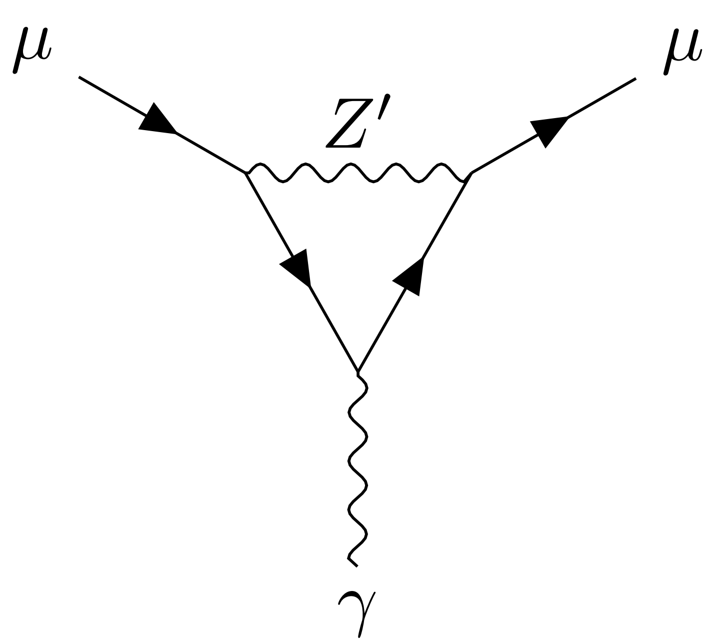
Muon g-2

Muon g-2 anomaly

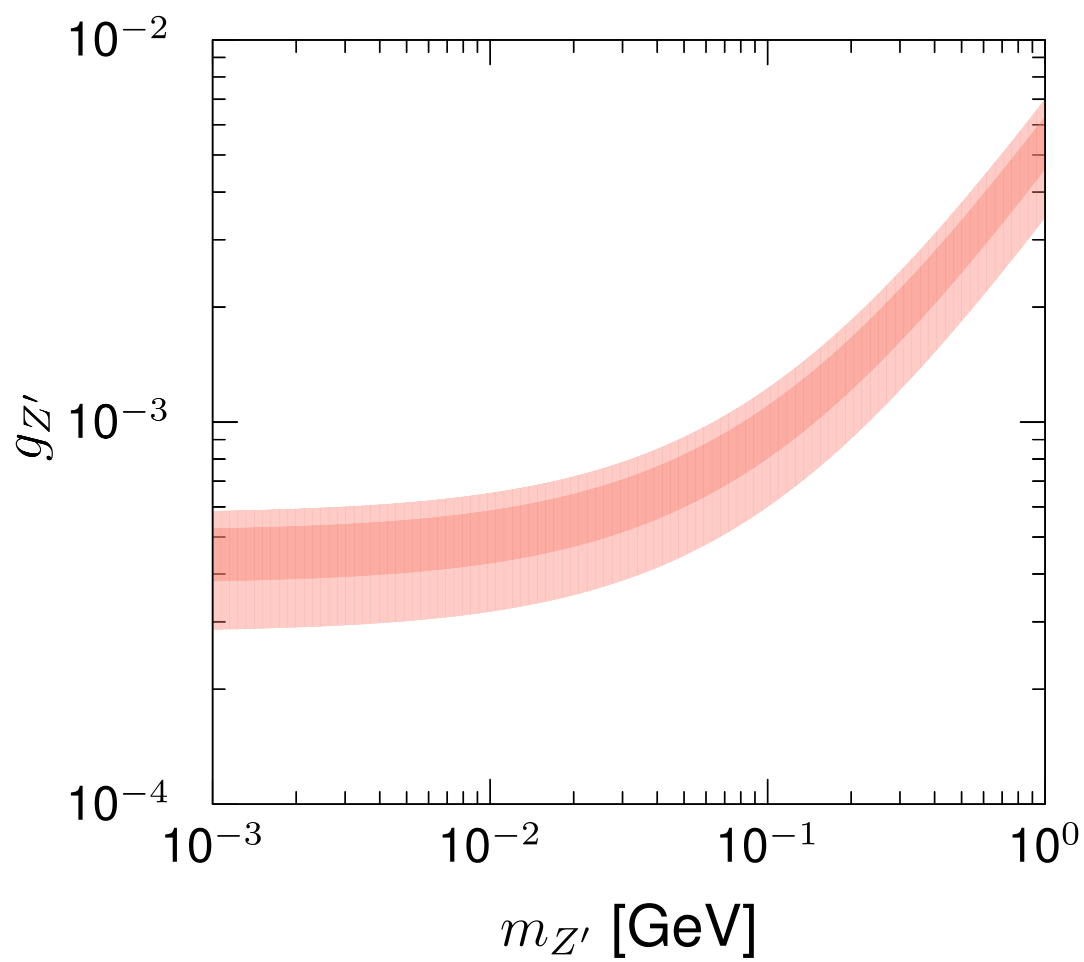
$$\delta a_\mu = a_\mu(\text{exp}) - a_\mu(\text{SM}) = (26.1 \pm 8.0) \times 10^{-10}$$

K. Hagiwara, R. Liao, A. D. Martin, D. Nomura, and T. Teubner, J. Phys. **G38**, 085003 (2011).

New contribution

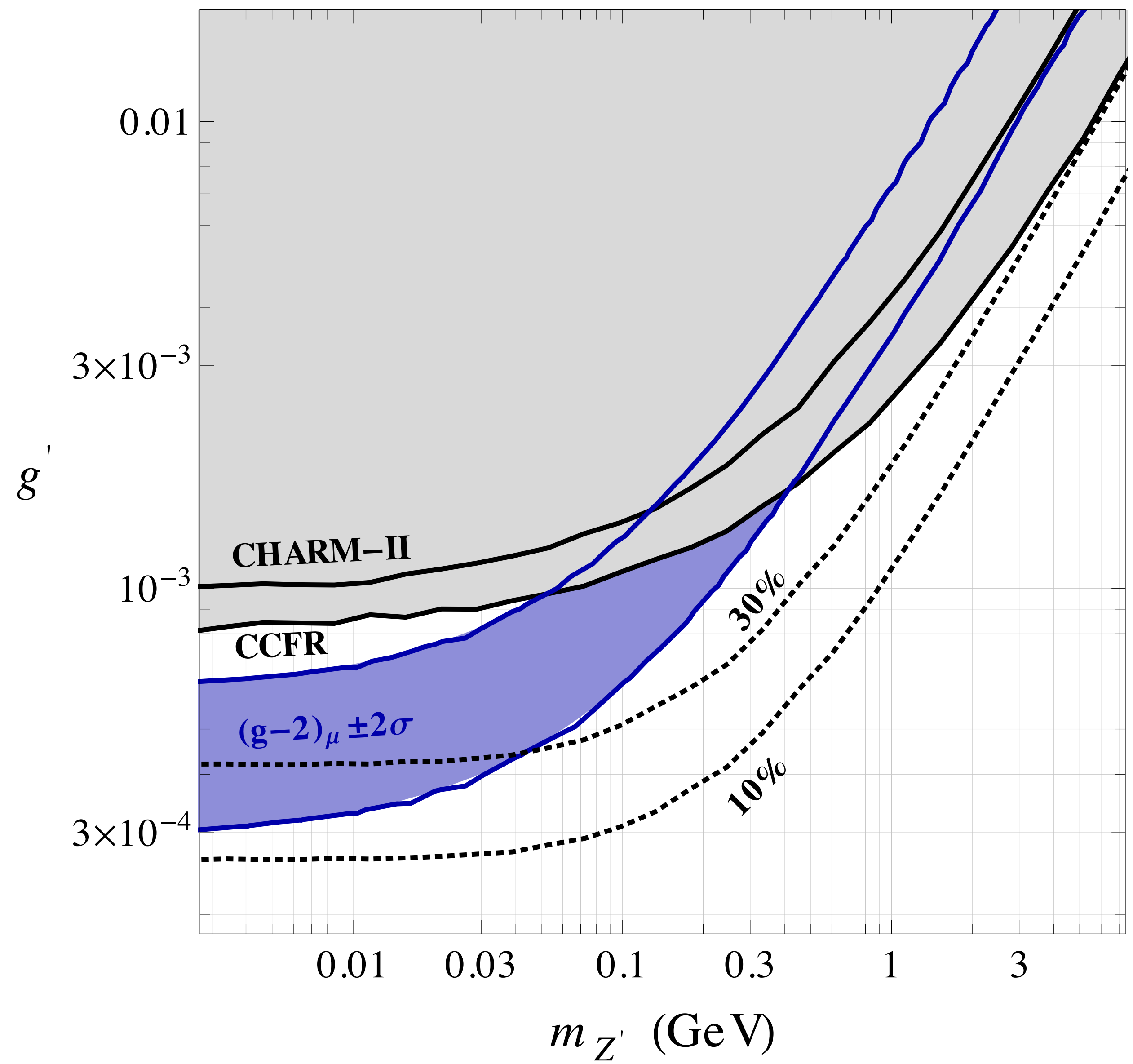


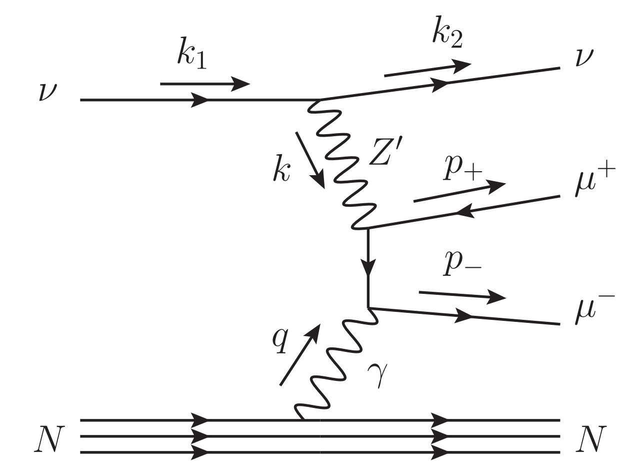
$$\delta a_\mu = \frac{g_{Z'}^2}{8\pi^2} \int_0^1 dx \frac{2m_\mu^2 x^2 (1-x)}{x^2 m_\mu^2 + (1-x) m_{A'}^2}$$



Neutrino trident production

W. Altmannshofer, S. Gori, M. Pospelov, and I. Yavin, Phys. Rev. Lett. **113**, 091801 (2014).





CCFR (1991)

$$\sigma_{\rm CCFR}/\sigma_{\rm SM} = 0.82 \pm 0.28$$

CHARM-II (1990)

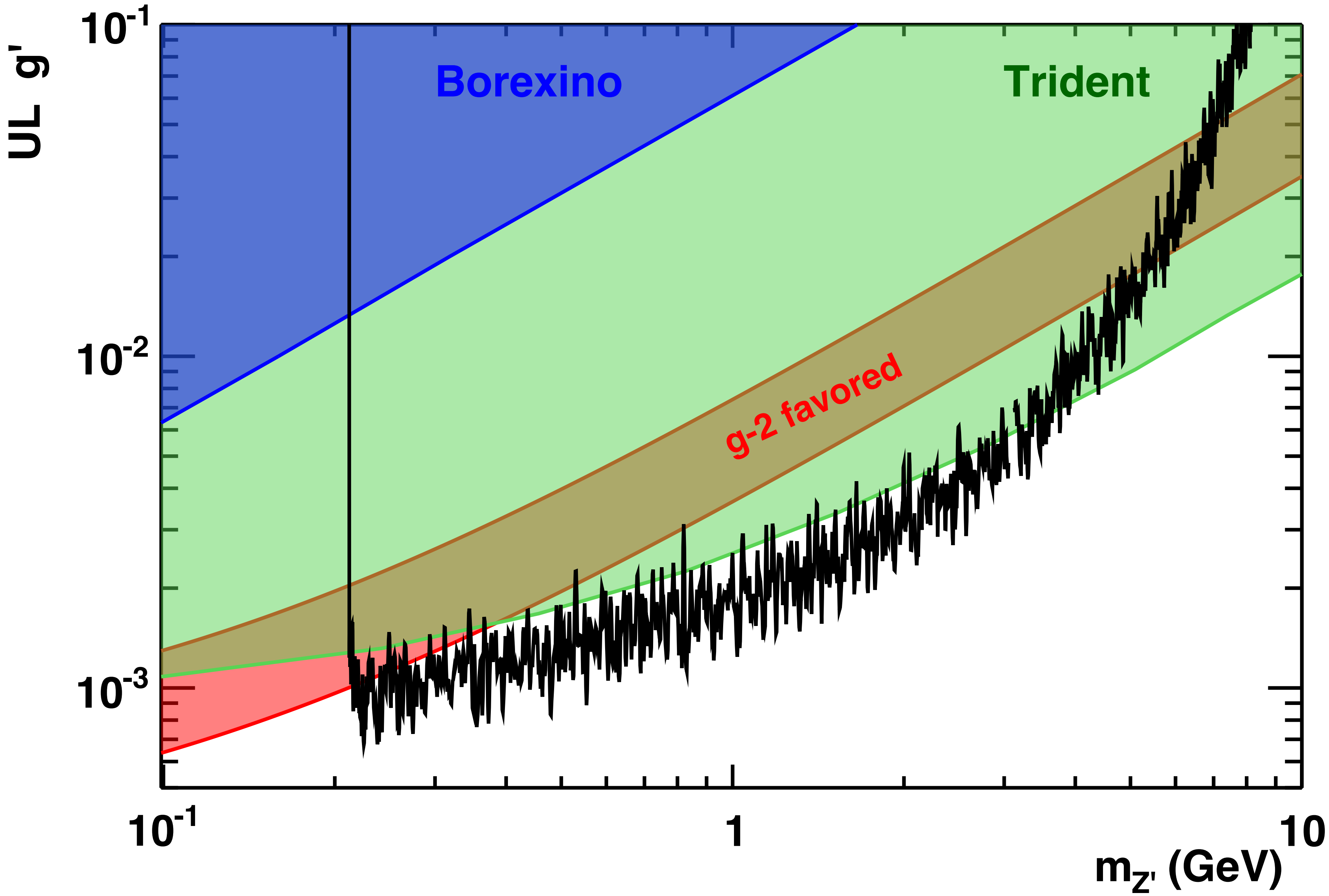
$$\sigma_{\rm CHARM-II}/\sigma_{\rm SM} = 1.58 \pm 0.57$$

····· 10 (30)% accuracy measurement with 5 GeV neutrino scattering on argon.

Z' mass larger than 400 MeV has been excluded.

BABAR constraint

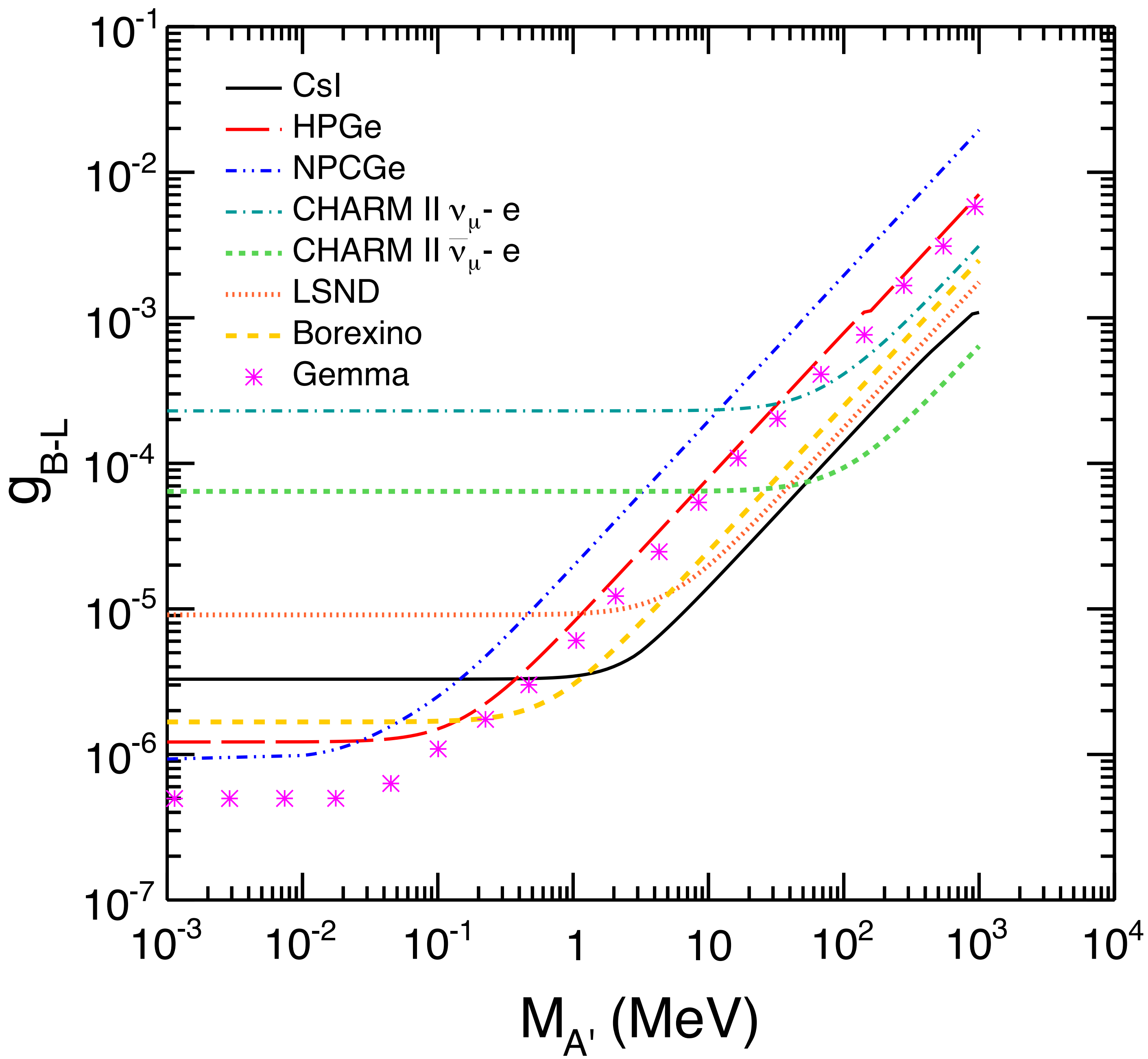
$$e^+e^- \to \mu^+\mu^- Z' \ , \quad Z' \to \mu^+\mu^-$$

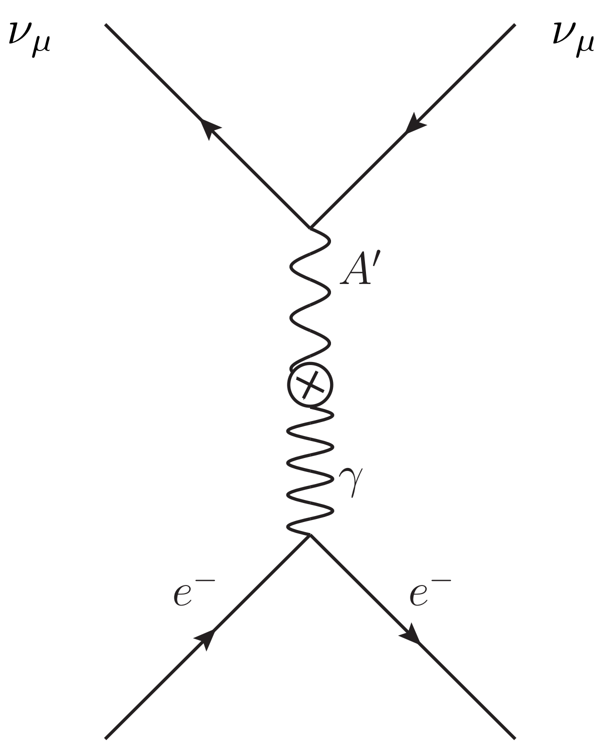


BABAR, Phys. Rev. **D94**, 011102 (2016).

Neutrino electron scattering

Neutrino-electron scattering occurs via kinetic mixing.





CHARM II and Borexino are sensitive to ν_μ-e scattering.

S. Bilmis, I. Turan, T. M. Aliev, M. Daniz, L. Singh, and H. T. Wong, Phys. Rev. **D92**, 033009 (2015).

Cosmological bound

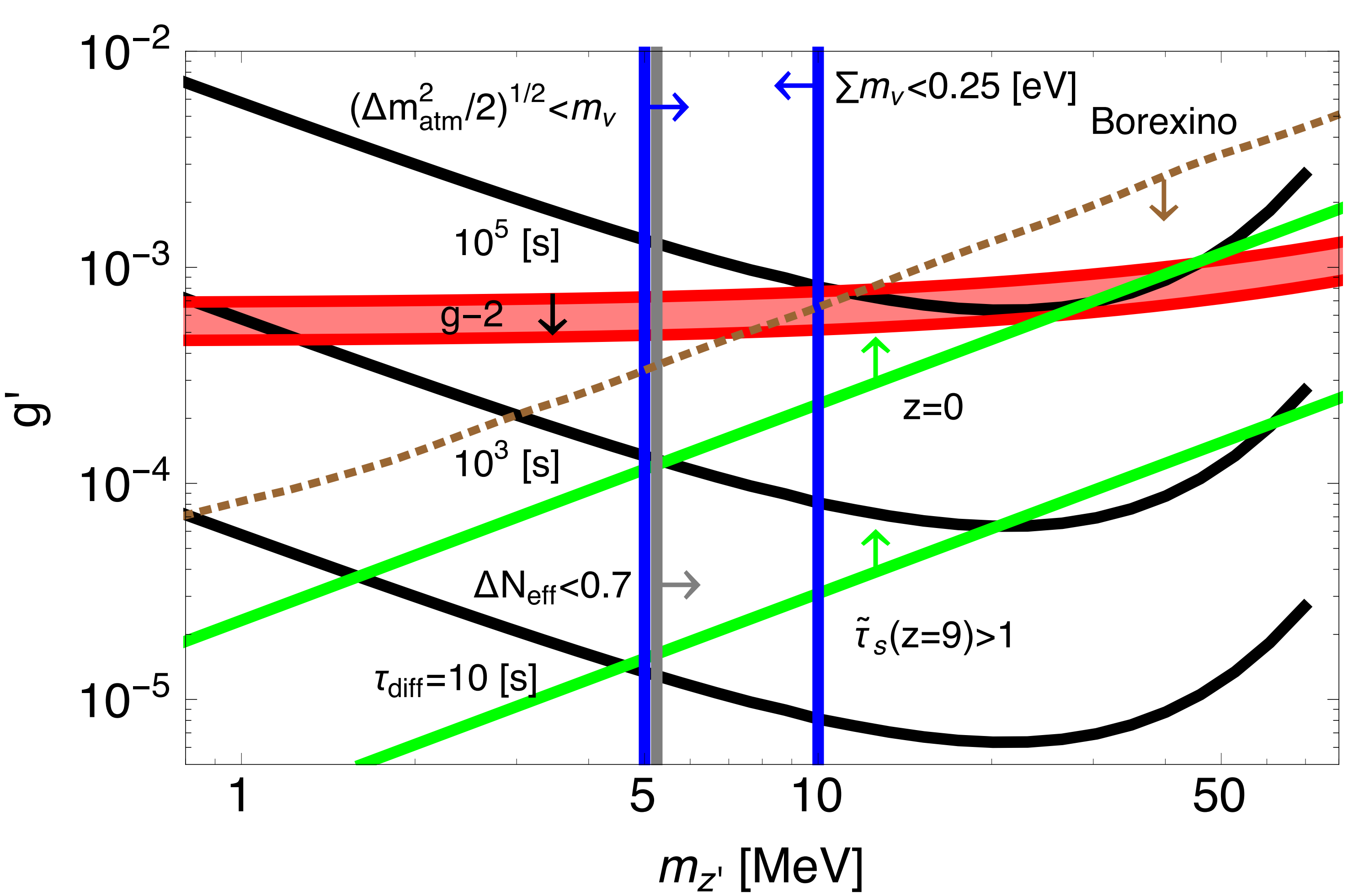
A. Kamada and Hai-Bo Yu, Phys. Rev. **D92**, 113004 (2015).

$\Delta N_{\rm eff}$

A ~ 10 MeV Z' transfers its entropy to ν_μ and ν_τ and increases their temperature after the neutrino decoupling.

$$\Delta N_{\rm eff} < 0.7 \quad \Rightarrow \quad m_{Z'} \gtrsim 5.3 \times \left(\frac{T_{\nu,\rm dec}}{1.5\ \rm MeV} \right)\ \rm MeV$$

Supernova cooling



The neutrino diffusion time exceeds 10s in a wide range of parameter space.

L_μ - L_τ

We introduce a new U(1) gauge symmetry: $\mathrm{U}(1)_{L_\mu - L_\tau}$

- $\mu_{L,R}$, ν_μ: charge +1
- $\tau_{L,R}$, ν_τ: charge -1

(Others have zero charges)

We also introduce right-handed neutrinos: N_e, N_μ, N_τ

We can assign them U(1) charges (0, a, -a) without introducing anomalies (and without loss of generality).

Charge structure of $N^c_\alpha L_\beta$:

$$Q_{L_\mu - L_\tau}(\text{Dirac}): \begin{pmatrix} 0 & 1 & -1 \\ -a & -a+1 & -a-1 \\ a & a+1 & a-1 \end{pmatrix} \qquad (a \geq 0)$$

Each component can be non-zero only if its charge is zero.

Only the a = 1 case can explain the neutrino data.

Minimal L_μ - L_τ

Charge structure of $N^c_\alpha L_\beta$ (a = 1):

$$Q_{L_\mu - L_\tau}(\text{Dirac}) : \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & -2 \\ 1 & 2 & 0 \end{pmatrix}$$

- Dirac Yukawa is always diagonal.
- Charged lepton Yukawa matrix is also diagonal.

Set to be real.

Even if a U(1)-breaking scalar is introduced, this structure is unchanged as long as renormalizable interactions are considered.

Charge structure of $N^c_\alpha N^c_\beta$:

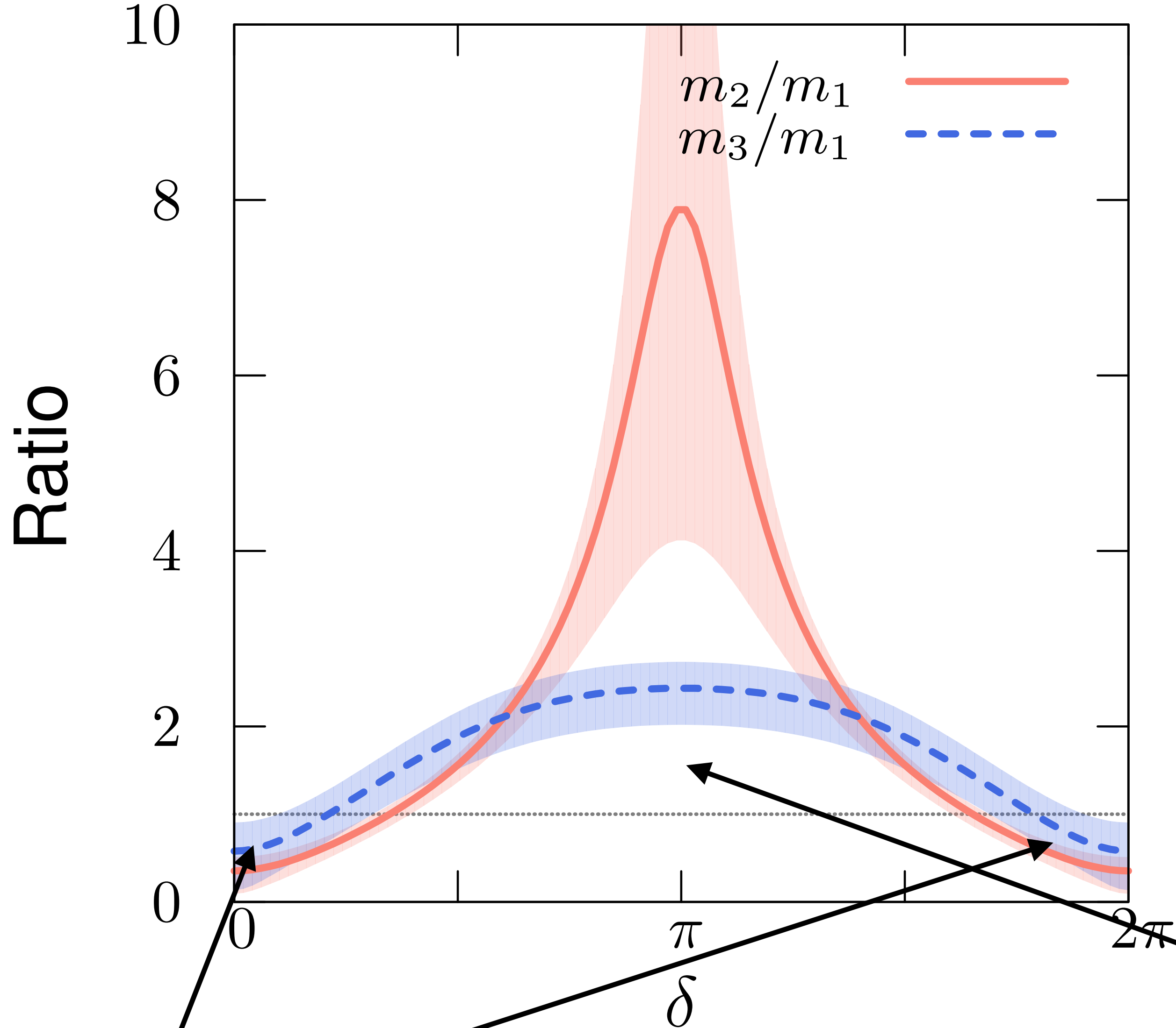
$$Q_{L_\mu - L_\tau}(\text{Majorana}) : \begin{pmatrix} 0 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & -2 \end{pmatrix}$$

Block diagonal

To obtain sizable neutrino mixing angles, we introduce a U(1)-breaking scalar with charge +1 and couple it to right-handed neutrinos.

We cannot explain the neutrino data with charge 0 or ±2 (still block diagonal).

Mass ratio



Input parameters

$$\sin^2\theta_{12} = 2.97^{+0.17}_{-0.16} \times 10^{-1}$$

$$\sin^2\theta_{13} = 2.15^{+0.07}_{-0.07} \times 10^{-2}$$

$$\sin^2\theta_{23} = 4.25^{+0.21}_{-0.15} \times 10^{-1}$$

Bands show uncertainties (mainly come from the error in θ_{23}).

Inconsistent with possible mass ordering

($m_1 < m_3 < m_2$)

Inconsistent with $m_1 < m_2$

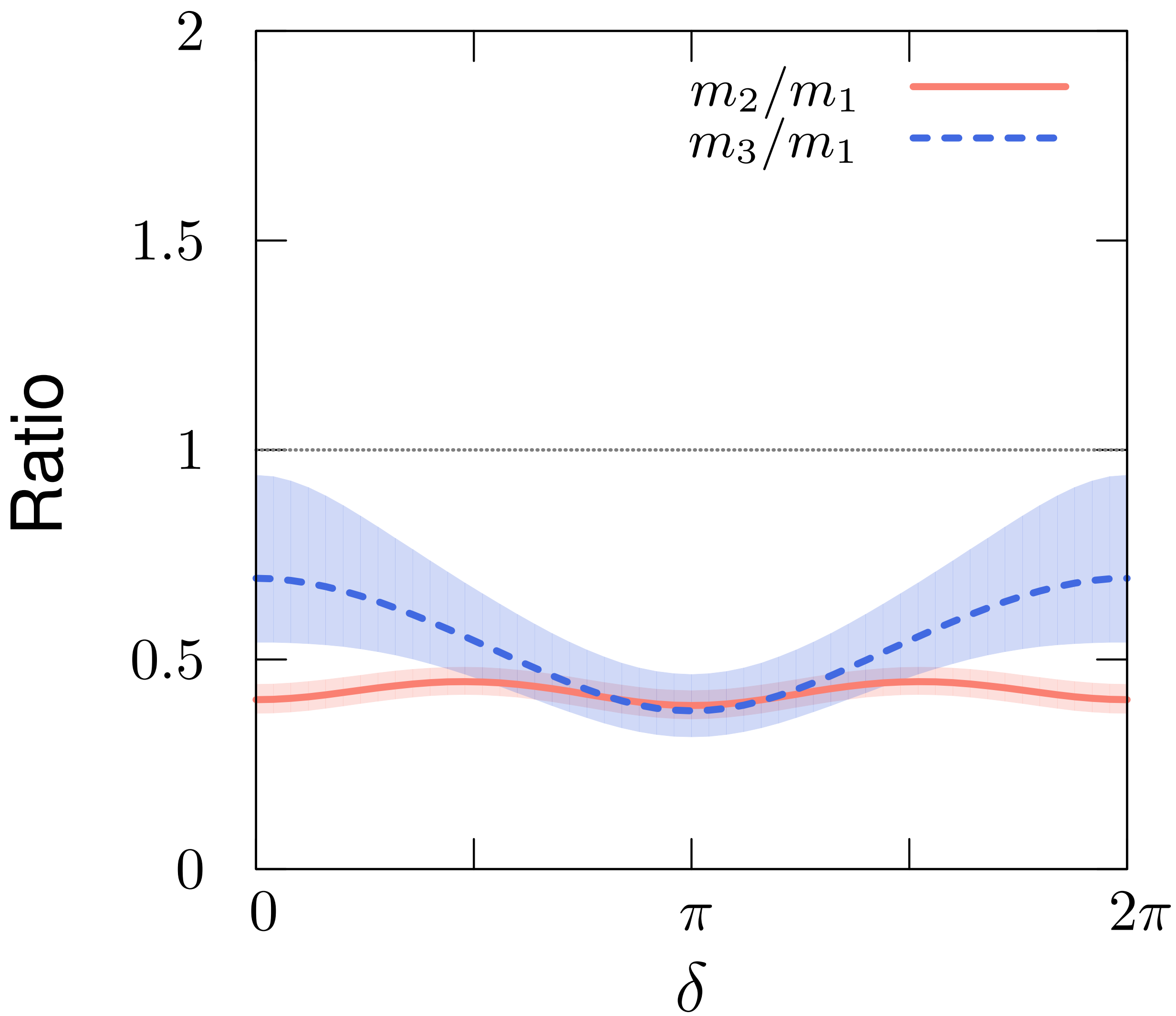
Allowed region predicts quasi-degenerate normal ordering.

($m_1 < m_2 < m_3$)

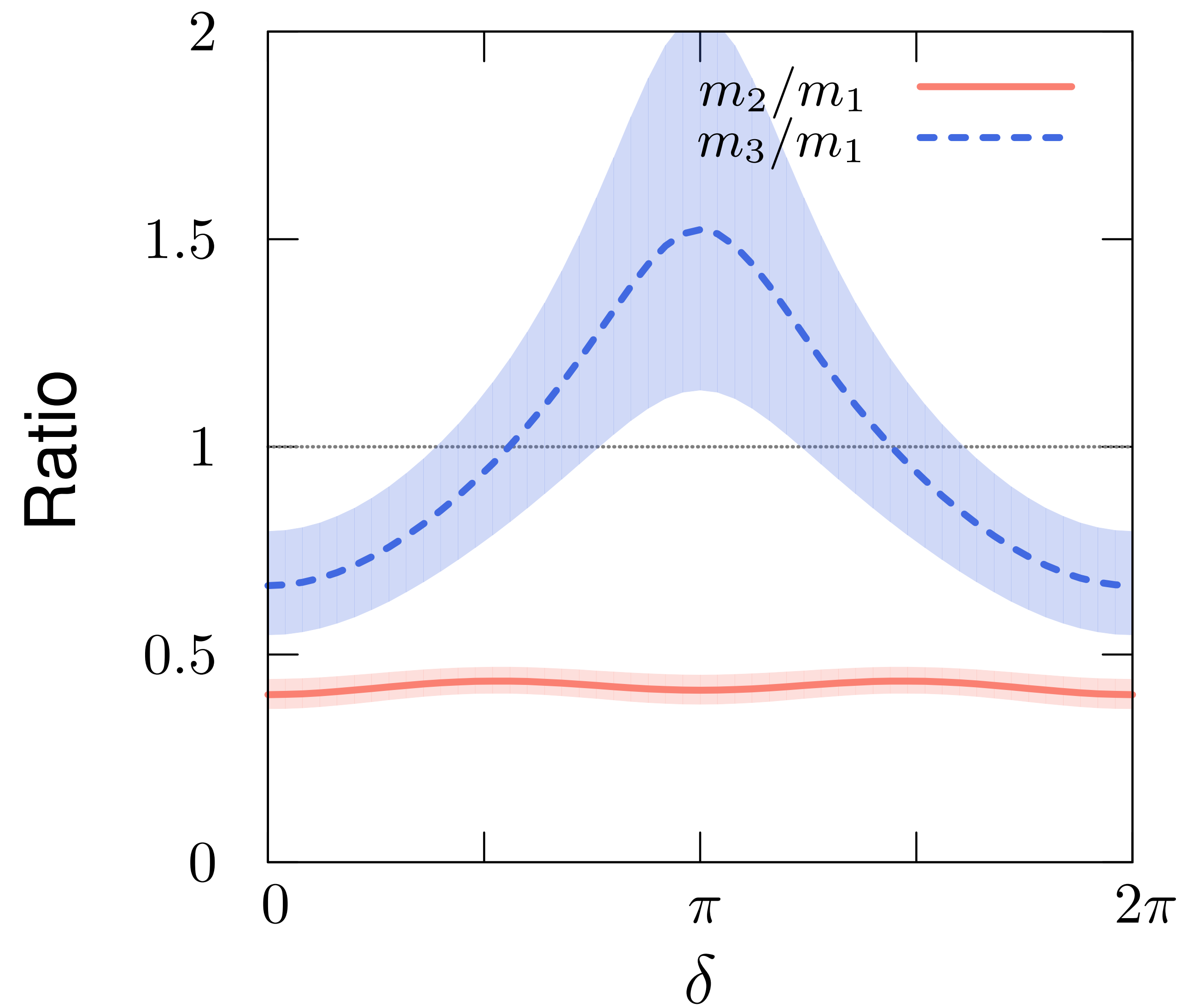
K. Asai, K. Hamaguchi, N. Nagata, Eur. Phys. J. **C77**, 763 (2017).

Other cases

L_e - L_μ



L_e - L_τ



These cases cannot reproduce possible pattern of neutrino mass spectrum.

K. Asai, K. Hamaguchi, N. Nagata, Eur. Phys. J. **C77**, 763 (2017).

Dirac CP phase

Mass differences (input)

$$\delta m^2 \equiv m_2^2 - m_1^2 \;, \qquad \Delta m^2 \equiv m_3^2 - (m_2^2 + m_1^2)/2 \;.$$

$$\delta m^2 \simeq 7.37 \times 10^{-5}\ \mathrm{eV}^2 \qquad \Delta m^2 \simeq 2.525 \times 10^{-3}\ \mathrm{eV}^2$$

Mass differences (prediction)

$$\delta m^2 = m_1^2 \left(\frac{m_2^2}{m_1^2} - 1 \right) = m_1^2 \left(\frac{1}{|R_2(\delta)|^2} - 1 \right) \;,$$

$$\Delta m^2 + \frac{\delta m^2}{2} = m_1^2 \left(\frac{m_3^2}{m_1^2} - 1 \right) = m_1^2 \left(\frac{1}{|R_3(\delta)|^2} - 1 \right) \;.$$

$\delta m^2 \ll \Delta m^2$ is realized only if $|R_2(\delta)| \sim 1$.

$$\cos\delta \simeq \frac{\cot 2\theta_{12} \cot 2\theta_{23}}{\sin\theta_{13}}$$

$$\delta \simeq 0.35\pi \ \text{or} \ 1.65\pi$$

A robust prediction for the Dirac CP phase δ.

Solving the conditions

We can solve the above conditions as follows:

$$e^{i\alpha_2} = \frac{m_2}{m_1} R_2(\delta) \ , \qquad e^{i\alpha_3} = \frac{m_3}{m_1} R_3(\delta) \ ,$$

with

$$R_2 \equiv \frac{(V_{\mu 1} V_{\tau 3} + V_{\mu 3} V_{\tau 1}) V_{e2}^*}{(V_{\mu 2} V_{\tau 3} + V_{\mu 3} V_{\tau 2}) V_{e1}^*} \ , \qquad R_3 \equiv \frac{(V_{\mu 1} V_{\tau 2} + V_{\mu 2} V_{\tau 1}) V_{e3}^*}{(V_{\mu 2} V_{\tau 3} + V_{\mu 3} V_{\tau 2}) V_{e1}^*} \ .$$

Here, we have used

$\det V = 1 \qquad \widetilde{V}^T = V^{-1} \qquad \widetilde{V}$: cofactor matrix of V

Knowledge from linear algebra

Cofactor of A ($\widetilde{A}_{ij}$): determinant of the submatrix formed by removing the i-th row and j-th column of the matrix A, multiplied by a factor of $(-1)^{i+j}$

$$\widetilde{A} \equiv (\widetilde{A}_{ij}) \qquad A^{-1} = (\det A)^{-1} \widetilde{A}^T$$

Reflection symmetry

Due to the PMNS structure, we have

$$R^*_{2,3}(-\delta) = R_{2,3}(\delta)$$

Thus, the mass ratios (and mass eigenvalues as well) are symmetric under δ → - δ. They depend only on cosδ (not sinδ).

Similarly, we have

$$\alpha_{2,3}(-\delta) = -\alpha_{2,3}(\delta)$$

Cf.)

$$e^{i\alpha_2} = \frac{m_2}{m_1} R_2(\delta) \ , \qquad e^{i\alpha_3} = \frac{m_3}{m_1} R_3(\delta) \ ;$$

$$\frac{m_2}{m_1} = \frac{1}{|R_2(\delta)|} \ , \qquad \frac{m_3}{m_1} = \frac{1}{|R_3(\delta)|} \ .$$

Stability against radiative corrections

If the U(1) breaking scale is much higher than the weak scale,
we expect sizable radiative corrections to the neutrino mass matrix.

After the right-handed neutrinos are integrated out, we obtain

$$\mathcal{L}_{\mathrm{eff}} = \frac{1}{2} C_{\alpha\beta} (L_\alpha \cdot H)(L_\beta \cdot H) + \mathrm{h.c.}$$

$C_{\alpha\beta}$ reflects the two-zero minor structure at this scale.

Renormalization group equation (RGE)

$$\mu \frac{dC}{d\mu} = -\frac{3}{32\pi^2} \left[\left(Y_e^\dagger Y_e \right)^T C + C \left(Y_e^\dagger Y_e \right) \right] + \frac{K}{16\pi^2} C \ ,$$

with

$$K = -3 g_2^2 + 2 \mathrm{Tr} \left(3 Y_u^\dagger Y_u + 3 Y_d^\dagger Y_d + Y_e^\dagger Y_e \right) + 2\lambda$$

Stability against radiative corrections

Since the charged lepton Yukawa matrix is diagonal, we can solve the RGE as follows:

$$C(t) = I_K(t)\,\mathcal{I}(t)\,C(0)\,\mathcal{I}(t) \ ,$$

where t = ln(μ/μ$_0$); μ$_0$: initial scale;

J. R. Ellis and S. Lola, Phys. Lett. **B458**, 310 (1999)

$$I_K(t) = \exp\left[\frac{1}{16\pi^2}\int_0^t K(t')\,dt'\right] \ , \qquad \mathcal{I}(t) = \exp\left[-\frac{3}{32\pi^2}\int_0^t Y_e^\dagger Y_e(t')\,dt'\right] \ .$$

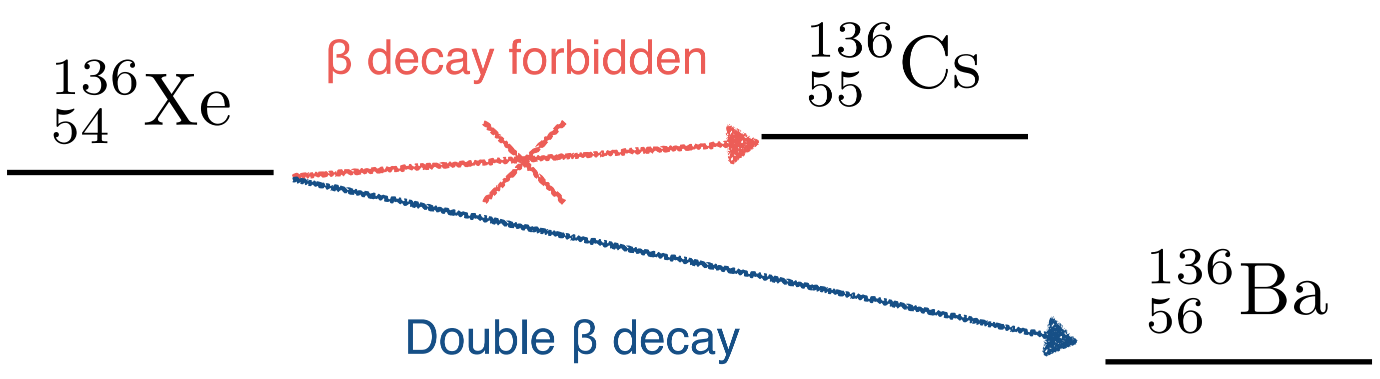
Now that I(t) is diagonal, the two-zero minor structure remains unchanged.

This structure is robust against radiative corrections.

Neutrinoless double beta decay

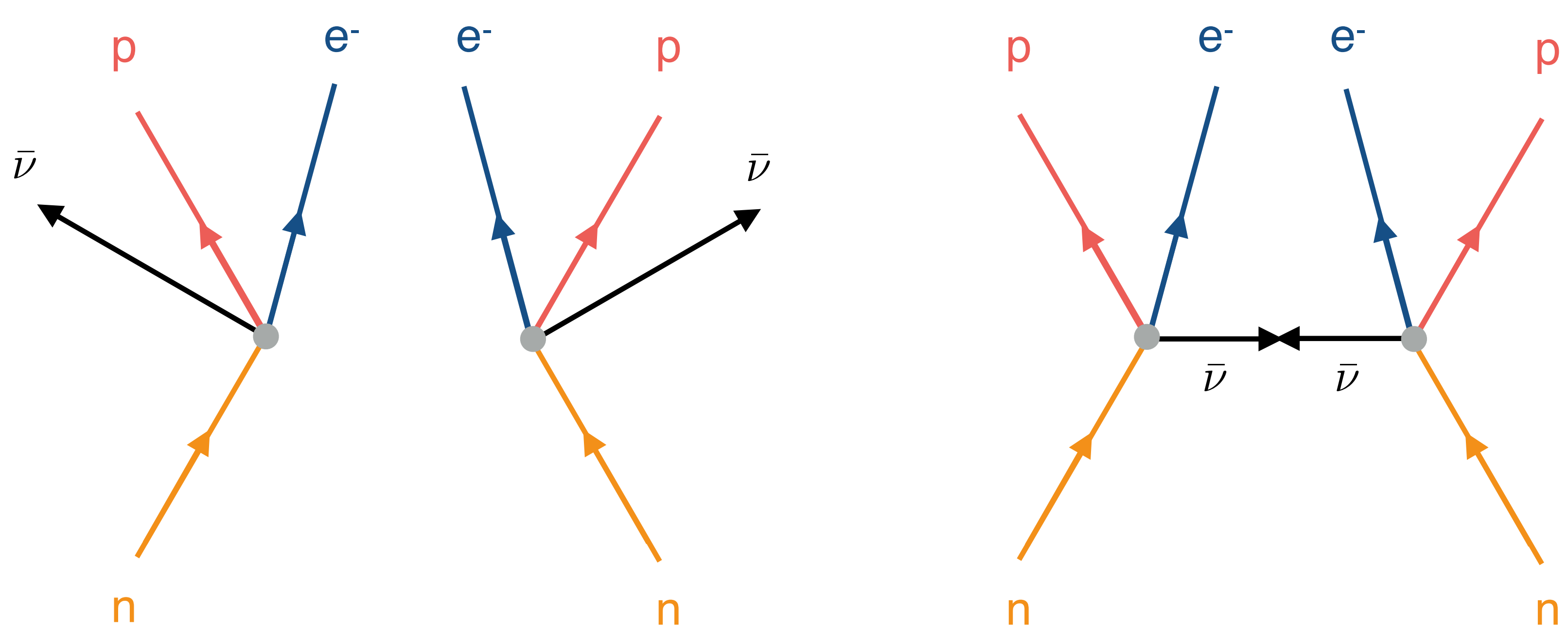
Since the model predicts Majorana neutrinos, we expect neutrinoless double beta (0νββ) decay.

Double beta decay isotopes



2νββ

0νββ



Neutrinoless double beta decay can occur for Majorana neutrinos.

Baryon asymmetry in the Universe

Sakharov's three conditions A. D. Sakharov (1967)

i) Baryon-number violation

ii) C and CP violation

iii) Out of thermal equilibrium

Violation of *CP* invariance, *C* asymmetry, and baryon asymmetry of the universe

A. D. Sakharov

(Submitted 23 September 1966)
Pis'ma Zh. Eksp. Teor. Fiz. **5**, 32–35 (1967) [JETP Lett. **5**, 24–27 (1967).
Also S7, pp. 85–88]

Usp. Fiz. Nauk **161**, 61–64 (May 1991)

The theory of the expanding universe, which presupposes a superdense initial state of matter, apparently excludes the possibility of macroscopic separation of matter from antimatter; it must therefore be assumed that there are no antimatter bodies in nature, i.e., the universe is asymmetrical with respect to the number of particles and antiparticles (*C* asymmetry). In particular, the absence of antibaryons and the proposed absence of baryonic neutrinos implies a nonzero baryon charge (baryonic asymmetry). We wish to point out a possible explanation of *C* asymmetry in the hot model of the expanding universe (see Ref. 1) by making use of effects of *CP* invariance violation (see Ref. 2). To explain baryon asymmetry, we propose in addition an approximate character for the baryon conservation law.

We assume that the baryon and muon conservation laws are not absolute and should be unified into a "combined" baryon-muon charge $n_c = 3n_B - n_\mu$. We put

for antimuons μ_+ and $\nu_\mu = \mu_0$: $n_\mu = -1,\ n_\kappa = +1.$

for muons μ_- and $\nu_\mu = \mu_0$: $n_\mu = +1,\ n_\kappa = -1.$

for baryons P and N: $n_B = +1,\ n_\kappa = +3.$

for antibaryons P and N: $n_B = -1,\ n_\kappa = -3.$

negative in the excess of μ neutrinos over μ antineutrinos).

According to our hypothesis, the occurrence of *C* asymmetry is the consequence of violation of *CP* invariance in the nonstationary expansion of the hot universe during the superdense stage, as manifest in the difference between the partial probabilities of the charge-conjugate reactions. This effect has not yet been observed experimentally, but its existence is theoretically undisputed (the first concrete example, Σ_+ and Σ_- decay, was pointed out by S. Okubo as early as 1958) and should, in our opinion, have much cosmological significance.

We assume that the asymmetry has occurred in an earlier stage of the expansion, in which the particle, energy, and entropy densities, the Hubble constant, and the temperatures were of the order of unity in gravitational units (in conventional units the particle and energy densities were $n \sim 10^{98}$ cm^{-3} and $\varepsilon \sim 10^{114}$ erg/cm^3).

M. A. Markov (see Ref. 3) proposed that during the early stages there existed particles with maximum mass of the order of one gravitational unit ($M_0 = 2\times 10^{-5}$ g in ordinary units), and called them maximons. The presence of such particles leads unavoidably to strong violation of thermodynamic equilibrium. We can visualize that neutral spinless maximons (or photons) are produced at $t < 0$ from con-

ν_μ Result- Comparison To Previous Result

Our previous result*:
2.6σ

Our rejection of maximal mixing has moved from 2.6σ to 0.8σ. This change in the character of our result comes from a few key changes which I'll break down below.

New simulation & Calibration:
~1.8σ

Driven by updates to energy response model. Drop to 2.3σ expected due to new energy resolution. Additionally we have a <70 MeV> shift in our hadronic energy response. This energy shift would be expected to move 0.5 events out of the "dip" region. However it instead pushes 3 "dip" events past a bin boundary.

New selection and analysis:
~0.5σ

For combined analysis changes 5% of pseudo-experiments in a MC study had this size shift or larger. This probability is driven by a low expected overlap in background events, and to second order the addition of resolution bins.

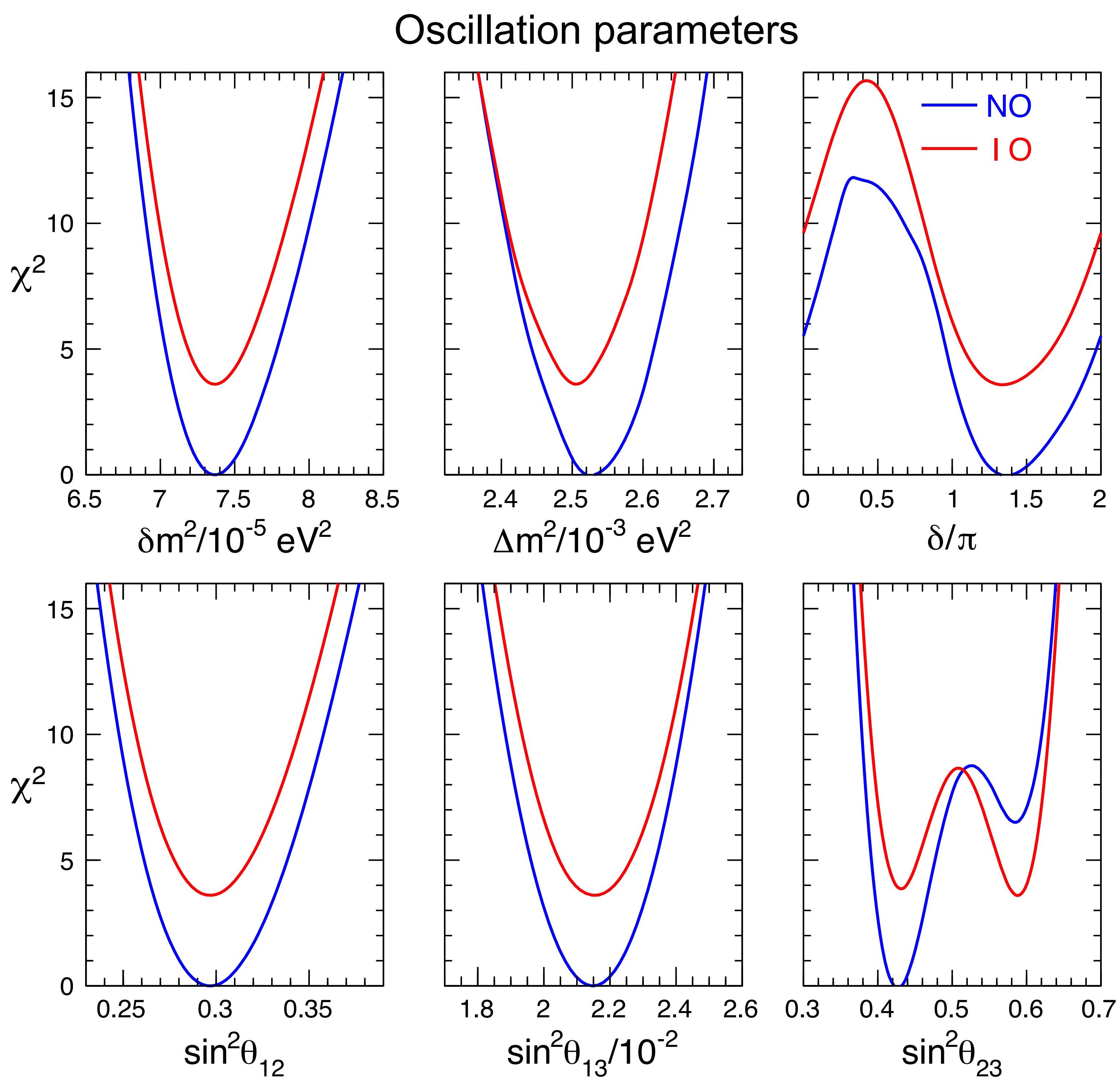
Full dataset:
~0.4σ

Full dataset*:
0.8σ

New, 2.8×10^{20} POT, data prefers maximal mixing.

*Feldman-cousins corrected significance.

Global fit



$$\Delta\chi^2_{\mathrm{IO-NO}} \equiv \chi^2_{\mathrm{min,\ IO}} - \chi^2_{\mathrm{min,\ NO}} = 3.6$$ (~2 σ)

F. Capozzi, E. D. Valentino, E. Lisi, A. Marrone, A. Melchiorri, A. Palazzo, Phys. Rev. **D95**, 096014 (2017)

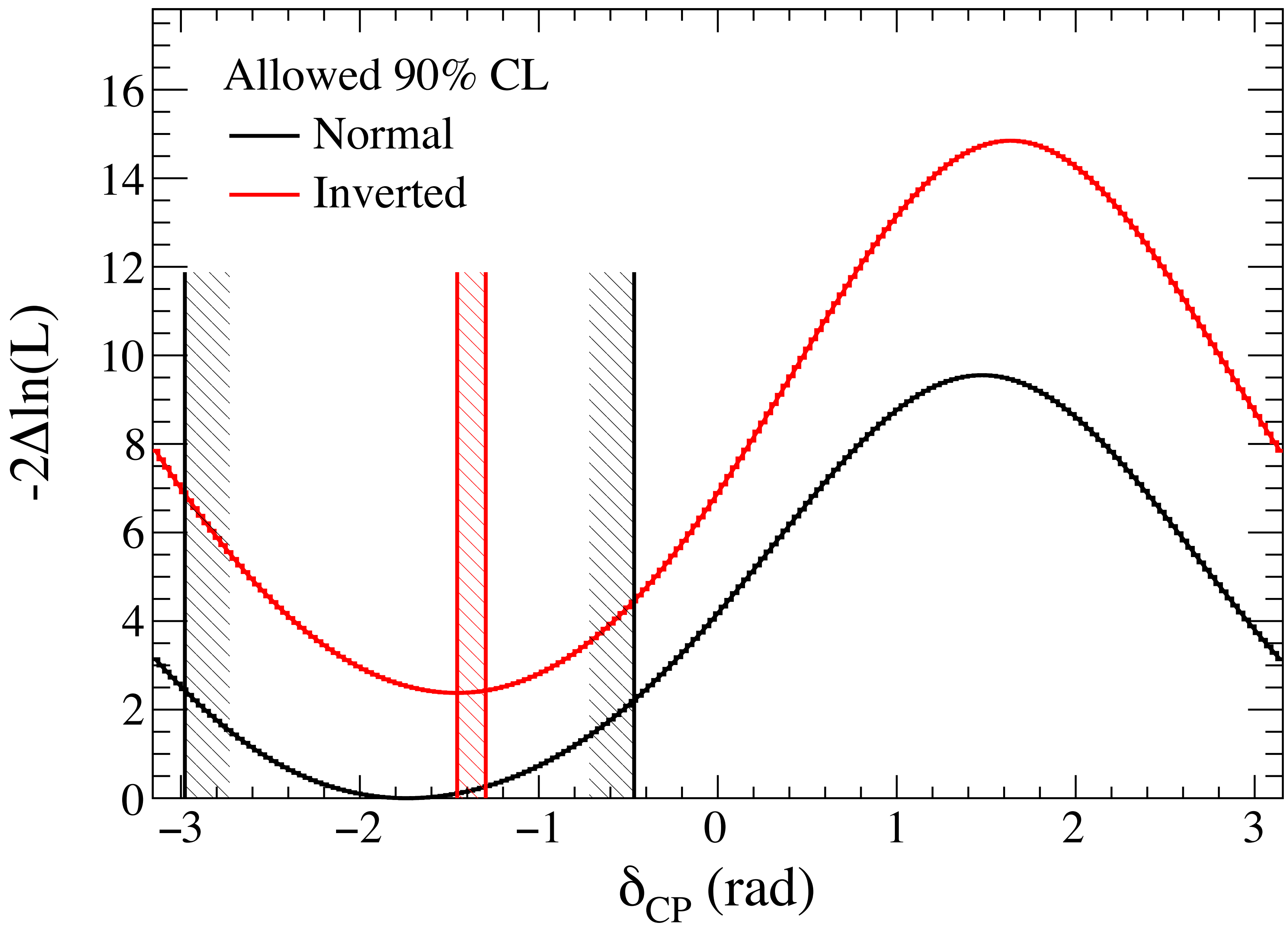
Global fit

TABLE I: Results of the global 3ν oscillation analysis, in terms of best-fit values for the mass-mixing parameters and associated $n\sigma$ ranges ($n = 1, 2, 3$), defined by $\chi^2 - \chi^2_{\rm min} = n^2$ with respect to the separate minima in each mass ordering (NO, IO) and to the absolute minimum in any ordering. (Note that the fit to the δm^2 and $\sin^2\theta_{12}$ parameters is basically insensitive to the mass ordering.) We recall that Δm^2 is defined herein as $m_3^2 - (m_1^2 + m_2^2)/2$, and that δ is taken in the (cyclic) interval $\delta/\pi \in [0, 2]$.

Parameter	Ordering	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}$ eV^2	NO, IO, Any	7.37	7.21 – 7.54	7.07 – 7.73	6.93 – 7.96
$\sin^2\theta_{12}/10^{-1}$	NO, IO, Any	2.97	2.81 – 3.14	2.65 – 3.34	2.50 – 3.54
$\lvert\Delta m^2\rvert/10^{-3}$ eV^2	NO	2.525	2.495 – 2.567	2.454 – 2.606	2.411 – 2.646
	IO	2.505	2.473 – 2.539	2.430 – 2.582	2.390 – 2.624
	Any	2.525	2.495 – 2.567	2.454 – 2.606	2.411 – 2.646
$\sin^2\theta_{13}/10^{-2}$	NO	2.15	2.08 – 2.22	1.99 – 2.31	1.90 – 2.40
	IO	2.16	2.07 – 2.24	1.98 – 2.33	1.90 – 2.42
	Any	2.15	2.08 – 2.22	1.99 – 2.31	1.90 – 2.40
$\sin^2\theta_{23}/10^{-1}$	NO	4.25	4.10 – 4.46	3.95 – 4.70	3.81 – 6.15
	IO	5.89	4.17 – 4.48 $\oplus$ 5.67 – 6.05	3.99 – 4.83 $\oplus$ 5.33 – 6.21	3.84 – 6.36
	Any	4.25	4.10 – 4.46	3.95 – 4.70 $\oplus$ 5.75 – 6.00	3.81 – 6.26
δ/π	NO	1.38	1.18 – 1.61	1.00 – 1.90	0 – 0.17 $\oplus$ 0.76 – 2
	IO	1.31	1.12 – 1.62	0.92 – 1.88	0 – 0.15 $\oplus$ 0.69 – 2
	Any	1.38	1.18 – 1.61	1.00 – 1.90	0 – 0.17 $\oplus$ 0.76 – 2

F. Capozzi, E. D. Valentino, E. Lisi, A. Marrone, A. Melchiorri, A. Palazzo, Phys. Rev. **D95**, 096014 (2017)

Neutrino mass hierarchy



T2K [arXiv: 1707.01048]

Explicit formulae

R_2 and R_3

$$R_2(\delta) = -\frac{2\sin^2\theta_{12}\cos 2\theta_{23} + \sin 2\theta_{12}\sin 2\theta_{23}\sin\theta_{13}e^{i\delta}}{2\cos^2\theta_{12}\cos 2\theta_{23} - \sin 2\theta_{12}\sin 2\theta_{23}\sin\theta_{13}e^{i\delta}} \ ,$$

$$R_3(\delta) = -\frac{\sin\theta_{13}e^{2i\delta}\left[2\cos 2\theta_{12}\cos 2\theta_{23}\sin\theta_{13} - \sin 2\theta_{12}\sin 2\theta_{23}(e^{-i\delta} + \sin^2\theta_{13}e^{i\delta})\right]}{\cos^2\theta_{13}\left[2\cos^2\theta_{12}\cos 2\theta_{23} - \sin 2\theta_{12}\sin 2\theta_{23}\sin\theta_{13}e^{i\delta}\right]} \ .$$

m_1

$$m_1 = \delta m\left[\frac{4s_{12}^4\cos^2 2\theta_{23} + 4s_{12}^3 c_{12}s_{13}\sin 4\theta_{23}\cos\delta + s_{13}^2\sin^2 2\theta_{12}\sin^2 2\theta_{23}}{2\left(2\cos 2\theta_{12}\cos^2 2\theta_{23} - s_{13}\sin 2\theta_{12}\sin 4\theta_{23}\cos\delta\right)}\right]^{\frac{1}{2}} \ .$$

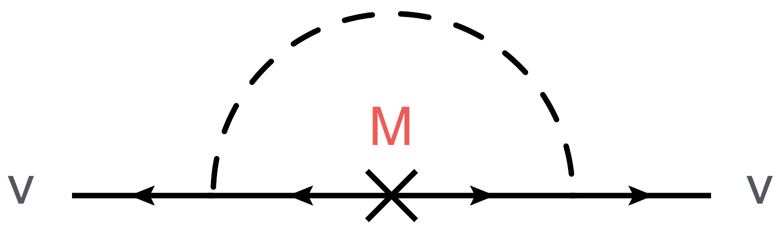
Extended possibilities

- One-loop induced neutrino masses in $U(1)_{L\mu-L\tau}$

$$\mathcal{M}_{\nu_L} = \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$$

Two-zero texture

Inverted ordering



S. Baek, H. Okada, K. Yagyu [1501.01530]; S. J. Lee, T. Nomura, H. Okada [1702.03733].

- Inverse seesaw in $U(1)_{L\mu-L\tau}$

Two-zero texture   Inverted ordering

A. Dev [1710.02878].

- Type-I seesaw in $SU(2)_{\mu\tau}$

Same neutrino structure as the minimal $U(1)_{L\mu-L\tau}$

C. W. Chiang and K. Tsumura [1712.00574].

Sphalerons

V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov (1985).

B&L violation in the SM

$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = \frac{3}{32\pi^2}\left[g^2 W^a_{\mu\nu}\widetilde{W}^a_{\mu\nu} - g'^2 B_{\mu\nu}\widetilde{B}^{\mu\nu}\right] .$$

At high temperatures (higher than the EW scale)

Thermal fluctuations → Transitions over the barrier between different EW gauge field configurations.

→ $\Delta(B+L) \neq 0 \qquad \Delta(B-L) = 0$

Sphaleron process

→ $$\frac{n_B}{n_L} = -\frac{4(2N_g + N_H)}{22N_g + 13N_H} = -\frac{28}{79}$$

N_g: # of generations

N_H: # of Higgs fields

n_L: generated lepton number

Produced lepton asymmetry converted to baryon asymmetry through the sphaleron process.

Lepton asymmetry

Asymmetry parameter

$$\epsilon_1 \equiv \frac{\Gamma(N_1 \to H\ell) - \Gamma(N_1 \to H^*\bar{\ell})}{\Gamma(N_1 \to H\ell) + \Gamma(N_1 \to H^*\bar{\ell})}$$

$$= \frac{1}{8\pi}\frac{1}{(\widehat{\lambda}\widehat{\lambda}^\dagger)_{11}} \sum_{j=2,3} \mathrm{Im}\left[\{(\widehat{\lambda}\widehat{\lambda}^\dagger)_{1j}\}^2\right] f\left(\frac{M_j^2}{M_1^2}\right)$$

with

$$f(x) = \sqrt{x}\left[1 - (x+1)\ln\left(1 + \frac{1}{x}\right) - \frac{1}{x-1}\right]$$

$$f(x) \simeq -3/(2\sqrt{x}) \quad (x \gg 1)$$

Reflection

$$\delta \to -\delta \qquad \alpha_{2,3} \to -\alpha_{2,3} \qquad U \to U^* \qquad \mathcal{M}_{\nu_L} \to \mathcal{M}_{\nu_L}^* \qquad \mathcal{M}_R \to \mathcal{M}_R^*$$

$$\Rightarrow \quad \Omega \to \Omega^* \qquad \widehat{\lambda} \to \widehat{\lambda}^* \quad \Rightarrow \quad \epsilon_1 \to -\epsilon_1$$

There is one-to-one correspondence between sign(δ) and sign(n_B).