





Deutsche Forschungsgemeinschaft

# Nuclear uncertainties in superallowed decays and V<sub>ud</sub>

#### Misha Gorshteyn

Mainz University

Collaborators: Chien-Yeah Seng (U. Shanghai -> U. Bonn) Hiren Patel (U. Mass. -> UC Santa Cruz) Michael Ramsey-Musolf (U. Mass.)







C-Y Seng, MG, H Patel, M J Ramsey-Musolf, arXiv: 1807.10197 C-Y Seng, MG, H Patel, M J Ramsey-Musolf, arXiv: 1811.XXXX MG, arXiv: 1811.XXXX

November 1, 2018 — Workshop "Beta decay as a Probe of New Physics", ACFI UMass, Amherst, Massachusetts

# Current status of Vud and CKM unitarity



2

# Why are superallowed decays special?

Superallowed 0+-0+ nuclear decays:

- only conserved vector current (unlike the neutron decay and other mirror decays)
- many decays (unlike pion decay)
- all decay rates should be the same modulo phase space

Experiment: **f** - phase space (Q value) and **t** - partial half-life (t<sub>1/2</sub>, branching ratio)

• 8 cases with *ft*-values measured to <0.05% precision; 6 more cases with 0.05-0.3% precision.

 ~220 individual measurements with compatible precision





ft values: same within ~2% but not exactly! Reason: SU(2) slightly broken

- a. RC (e.m. interaction does not conserve isospin)
- b. Nuclear WF are not SU(2) symmetric (proton and neutron distribution not the same)

# Why are superallowed decays special?



Hardy, Towner 1973 - 2018

 $\overline{\mathcal{F}t} = 3072.1 \pm 0.7$ 

#### Corrections to superallowed decays

Parent	$\delta'_R$	$\delta_{NS}$	$\delta_{C1}$	$\delta_{C2}$	$\delta_C$
nucleus	(%)	(%)	(%)	(%)	(%)
$T_z = -1:$					
$^{10}\mathrm{C}$	1.679	-0.345(35)	0.010(10)	0.165(15)	0.175(18)
$^{14}\mathrm{O}$	1.543	-0.245(50)	0.055(20)	0.275(15)	0.330(25)
$^{18}\mathrm{Ne}$	1.506	-0.290(35)	0.155(30)	0.405(25)	0.560(39)
$^{22}Mg$	1.466	-0.225(20)	0.010(10)	0.370(20)	0.380(22)
$^{26}$ Si	1.439	-0.215(20)	0.030(10)	0.405(25)	0.435(27)
$^{30}\mathrm{S}$	1.423	-0.185(15)	0.155(20)	0.700(20)	0.855(28)
$^{34}\mathrm{Ar}$	1.412	-0.180(15)	0.030(10)	0.665(55)	0.695(56)
$^{38}$ Ca	1.414	-0.175(15)	0.020(10)	0.745(70)	0.765(71)
$^{42}$ Ti	1.427	-0.235(20)	0.105(20)	0.835(75)	0.940(78)
$T_z = 0:$			0.000(10)		0.010(10)
$^{20m}$ Al	1.478	0.005(20)	0.030(10)	0.280(15)	0.310(18)
	1.443	-0.085(15)	0.100(10)	0.550(45)	0.650(46)
<sup>38</sup> <i>m</i> K	1.440	-0.100(15)	0.105(20)	0.565(50)	0.670(54)
$^{42}\mathrm{Sc}$	1.453	0.035(20)	0.020(10)	0.645(55)	0.665(56)
$^{46}\mathrm{V}$	1.445	-0.035(10)	0.075(30)	0.545(55)	0.620(63)
$^{50}\mathrm{Mn}$	1.444	-0.040(10)	0.035(20)	0.610(50)	0.645(54)
$^{54}\mathrm{Co}$	1.443	-0.035(10)	0.050(30)	0.720(60)	0.770(67)
$^{62}$ Ga	1.459	-0.045(20)	0.275(55)	1.20(20)	1.48(21)
$^{66}\mathrm{As}$	1.468	-0.060(20)	0.195(45)	1.35(40)	1.55(40)
$^{70}\mathrm{Br}$	1.486	-0.085(25)	0.445(40)	1.25(25)	1.70(25)
$^{74}$ Rb	1.499	-0.075(30)	0.115(60)	1.50(26)	1.62(27)

TABLE X: Corrections  $\delta'_R$ ,  $\delta_{NS}$  and  $\delta_C$  that are applied to experimental ft values to obtain  $\mathcal{F}t$  values.

Hardy, Towner 2015

# General Structure of RC to Beta Decay

$$|V_{ud}|^2 = \frac{2984.432(3)}{\mathcal{F}t(1+\Delta_R^V)}$$

$$\mathcal{F}t = ft(1+\delta_R')[1-(\delta_C-\delta_{NS})]$$

Three caveats:

- 1. Calculation of the universal free-neutron RC  $\Delta_{RV}$  Talk by Chien Yeah
- 2. Splitting the full nuclear RC into free-neutron  $\Delta_{R^{V}}$  and nuclear modification  $\delta_{NS}$
- 3. Splitting the full RC into "outer" (energy-dependent but pure QED: no hadron structure) and "inner" (hadron&nuclear structure-dependent but energy-independent)
  - nucleon and nuclear case

Will address points 2. and 3.

## 2.Radiative corrections to nuclear decays: Nuclear structure modification of the free-n RC





C-Y Seng, MG, H Patel, M J Ramsey-Musolf, arXiv: 1811.xxxxx

## Caveats in the Ft values

General structure of nuclear and radiative corrections for nuclear decay

 $ft(1+RC) = Ft(1+\delta_R')(1-\delta_C+\delta_{NS})(1+\Delta_R^V)$ 

 $\delta'_R$  - coulomb distortions: QED + Z of daughter + nuclear size

δ<sub>C</sub> - Isospin breaking: correction to the tree-level matrix element of the Fermi op. implicitly a radiative correction: Coulomb interaction between the protons in a nucleus shell-model calculation w. Woods-Saxon potential (SM WS) beyond the scope of this work - but an **independent** check in nuclear models welcome

δ<sub>NS</sub> - modification of the universal RC due to nuclear environment Convention: extract the free-nucleon RC explicitly, then correct for each nucleus. Universal RC calculated by loop techniques or w. DR; Nuclear modification calculated in SM WS



If two pieces of one well-defined object are computed in two different frameworks, the subtraction might be model-dependent!

Desirable to use the same method to compute both - DR is a valid candidate!

## Universal vs. Nuclear Corrections

Define the nuclear  $\gamma$ W-box  $\Box_{\gamma W}^{VA, Nucl.} = \frac{\alpha}{N\pi M} \int_{0}^{\infty} \frac{dQ^2 M_W^2}{M_W^2 + Q^2} \int_{0}^{\infty} d\nu \frac{(\nu + 2q)}{\nu(\nu + q)^2} F_{3, \gamma W}^{(0), Nucl.}(\nu, Q^2)$ 

Need the nuclear structure function  $F_3^{(0)}$ Where is it different from the free-nucleon  $F_3^{(0)}$ ? - **Everywhere!** 

Long distances: LE nuclear structure - excited nuclear states; quasielastic knockout; ... Intermediate distances: widening of N<sup>\*</sup>, $\Delta$ -resonances (energy can be shared w. neighbors) Short distances: shadowing, EMC effect (N of active quarks may depend on kinematics)

Quite complicated... in the future all these effects **must** be addressed! But: The integral has more weight at low energies - HE modifications may be less important; N\*,Δ-resonances have no impact on the γW-box

To start: consider the long-range part





#### Universal vs. Nuclear Corrections

Long-distance content of  $\Delta_{\mathsf{R}}{}^{\mathsf{V}}$  - mostly Born contribution

$$\Delta_R^V = \frac{\alpha}{\pi} \left[ \text{Short and Intermediate Distance} + \mathbf{C_B} \right]$$

Born uniquely defined: a  $\delta$ -function in the SF

How is it modified in a nucleus? Due to binding the nucleon is slightly off-shell and has an initial momentum distribution - a broad QE peak instead of a  $\delta$ -function

Operating with nucleon d.o.f. — nuclear SF has two contributions:

Coupling to the same nucleon: Low energy - quasielastic vs. free nucleon Born

Coupling to two different nucleons: Lowest energies - nuclear excited states, QE region - 2+ nucleon knockout SM WS calculations:  $\delta_{NS} \sim -0.3\% - 0$  Hardy, Towner '15, '18









#### Modification of $C_B$ in a nucleus - QE

$$\Box_{\gamma W}^{VA, QE} = \frac{\alpha}{\pi M} \int_{0}^{2 \, GeV^2} dQ^2 \int_{\nu_{thr}}^{\nu_{\pi}} \frac{d\nu(\nu + 2q)}{\nu(\nu + q)^2} F_3^{(0) \, QE}(\nu, Q^2) = \frac{\alpha}{2\pi} \mathbf{C}_{\mathbf{QE}}$$

Exploratory calculation: disregard fine details, account for main effects Main features: Fermi momentum and break-up threshold

Problem: mismatch of the initial and final state Break-up thresholds differ by the Q-value of the decay! Solution: define an average threshold

Effective removal energies - all in a small range

 $\bar{\epsilon} = 7.5 \pm 1.5 MeV$ 

Fermi momenta also not too different for all A

 $k_F(A = 10) = 228 \text{ MeV}, \quad k_F(A = 74) = 245 \text{ MeV}$ 

Decay	$\epsilon_1 \ (MeV)$	$\epsilon_2 \ ({\rm MeV})$	$\overline{\epsilon} \ ({\rm MeV})$
$^{10}C \rightarrow^{10} B$	6.70	4.79	5.67
$^{14}O \rightarrow ^{14}N$	8.24	5.41	6.68
$^{18}Ne \rightarrow ^{18}F$	8.11	4.71	6.18
$^{22}Mg \rightarrow^{22} Na$	10.41	6.28	8.09
$^{26}Si \rightarrow^{26}Al$	11.14	6.30	8.38
$^{30}S \rightarrow^{30} P$	10.64	5.18	7.42
$^{34}Ar \rightarrow ^{34}Cl$	11.51	5.44	7.91
$^{38}Ca \rightarrow ^{38}K$	11.94	5.33	7.98
$^{42}Ti \rightarrow ^{42}Sc$	11.57	4.55	7.25
$^{26m}Al \rightarrow^{26} Mg$	11.09	6.86	8.72
$^{34}Cl \rightarrow ^{34}S$	11.42	5.92	8.22
$^{38m}K \rightarrow^{38}Ar$	11.84	5.79	8.28
$^{42}Sc \rightarrow ^{42}Ca$	11.48	5.05	7.61
${}^{46}V \rightarrow {}^{46}Ti$	13.19	6.14	9.00
$^{50}Mn \rightarrow ^{50}Cr$	13.00	5.37	8.35
$^{54}Co \rightarrow ^{54}Fe$	13.38	5.13	8.28
$^{62}Ga \rightarrow ^{62}Zn$	12.90	3.72	6.94
$^{66}As \rightarrow ^{66}Ge$	12.74	3.16	6.34
$^{70}Br \rightarrow^{70} Se$	13.17	3.20	6.49
$^{74}Rb \rightarrow^{74}Kr$	13.85	3.44	6.90

# Modification of C<sub>B</sub> in a nucleus - QE

A simple calculation of a QE cross section: nucleon momentum distribution  $\phi_{A^p} \simeq \phi_{A^n}$ 

$$\int \frac{d^3\vec{k}}{(2\pi)^3} |\phi(k)|^2 = 1$$

$$A\rangle = \sqrt{2E_A} \sum_{p \in A} \int \frac{d^3 \vec{k} \phi_A^p(k) | p(\vec{k}), A - p(-\vec{k}) \rangle}{(2\pi)^3 \sqrt{2E_{A-1} 2E_n}} \qquad A \qquad A \qquad A = A' - n \qquad A' \qquad |A'\rangle = \sqrt{2E_{A'}} \sum_{n \in A'} \int \frac{d^3 \vec{k} \phi_A^n(k) | n(\vec{k}), A' - n(-\vec{k}) \rangle}{(2\pi)^3 \sqrt{2E_{A-1} 2E_n}}$$

 $W^{-}$ 

Free Fermi gas model 
$$\frac{1}{(2\pi)^3} |\phi(k)|^2 = \frac{3}{4\pi k_F^3} \theta(k_F - |\vec{k}|)$$
  
Pauli blocking  $F_P(|\vec{q}|, k_F) = \frac{3|\vec{q}|}{4k_F} \left[1 - \frac{\vec{q}^2}{12k_F^2}\right]$  for  $|\vec{q}| \le 2k_F$ 

Result of the calculation: Born suppressed by ~ factor 2



0



QE: finite threshold; Bulk of QE shifted by k<sub>F</sub>



 $\gamma$ 

## Universal vs. Nuclear Corrections

Compare to existing estimate! Towner 1994 and ever since:



Idea: calculate Gamow-Teller and magnetic nuclear transitions in the shell model; Insert the single nucleon spin operators —> predict the strength of nuclear transitions "Quenching of spin operators in nuclei": shell model overestimates those strengths!

Each vertex is suppressed by 10-15% Hardy, Towner: just rescale the Born contribution to the  $\gamma$ W-box by that quenching, assume the integral to be the same (nucleon form factors)

Numerically: on average  $[q_S^{(0)}q_A - 1]C_B = -0.25$ 

 $\delta^{qB}_{NS} \sim$  - 0.055(5)% used in all reviews since 1998

But from dispersion relation perspective it corresponds to a contribution of an excited nuclear state, not to the modified box on a free nucleon! The correct estimate should base on quasielastic knockout with an on-shell N + spectator in the intermediate state



#### Modification of $C_B$ in a nucleus - QE

 $C_{QE} - C_B = -0.45 \pm 0.04$  compare to the H&T estimate  $[q_S^{(0)}q_A - 1]C_B = -0.25$ New  $\delta^{QE}_{NS} \sim -0.10(1)\%$  instead of the previous estimate  $\delta^{q}_{NS} \sim -0.055(5)\%$ Shifts the Ft value according to  $\overline{\mathcal{F}t} \to \overline{\mathcal{F}t}(1 + \delta_{NS}^{new} - \delta_{NS}^{old})$ Numerically:  $\mathcal{F}t = 3072.07(63)s \to [\mathcal{F}t]^{new} = 3070.65(63)(28)s$ 

Will affect the extracted V<sub>ud</sub>  $|V_{ud}|^2 = \frac{2984.432(3) \text{ s}}{\mathcal{F}t(1 + \Delta_R^V)}$ 

Compensates for a part of the shift due to a new evaluation of  $\Delta^{V_{R}}$ 

 $V_{ud}^{\text{old}} = 0.97420(21) \rightarrow V_{ud}^{\text{new}} = 0.97370(14) \rightarrow V_{ud}^{\text{new, QE}} = 0.97392(14)(04)$ 

Brings the first row a little closer to the unitarity  $(4\sigma \rightarrow 3\sigma)$ 

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9984 \pm 0.0004 \quad \rightarrow |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9988 \pm 0.0004$$

Important message:

dispersion relations as a unified tool for treating hadronic and nuclear parts of RC

# 3.Splitting of the RC into inner and outer SOON



MG, arXiv: 1811.xxxxx

## Splitting the RC into "inner" and "outer"

Radiative corrections ~  $\alpha/2\pi$  ~ 10<sup>-3</sup>

Precision goal: ~ 10-4

When does energy dependence matter? Correction ~  $E_e/\Lambda$ , with  $\Lambda$  ~ relevant mass (m<sub>e</sub>; M<sub>p</sub>; M<sub>A</sub>) Maximal  $E_e$  ranges from 1 MeV to 10.5 MeV

Electron mass regularizes the IR divergent parts - (E<sub>e</sub>/m<sub>e</sub> important) - "outer" correction

If  $\Lambda$  of hadronic origin (at least m<sub> $\pi$ </sub>) —> E<sub>e</sub>/ $\Lambda$  small, correction ~ 10<sup>-5</sup> —> negligible

- certainly true for the neutron decay
- hadronic contributions do not distort the spectrum, may only shift it as a whole

However, in nuclei binding energies ~ few MeV — similar to Q-values

A scenario is possible when RC ~ ( $\alpha/2\pi$ ) x (E<sub>e</sub>/ $\Lambda$ <sup>Nucl</sup>) ~ 10<sup>-3</sup>

Nuclear structure may distort the electron spectrum

With dispersion relations can be checked straightforwardly!

#### Nuclear structure and E-dependent RC

With DR: can include linear terms in energy Even and odd powers of energy - leading terms

E-dependent correction: estimate with nuclear polarizabilities and size

Photonuclear sum rule:

$$\alpha_E = \frac{2\alpha}{M} \int_{\epsilon}^{\infty} \frac{d\nu}{\nu^3} F_1(\nu, 0) = 2\alpha \int_{\epsilon}^{\infty} \frac{d\nu}{\nu^2} \frac{\partial}{\partial Q^2} F_2(\nu, 0)$$

Supplement with the nuclear form factor:  $\alpha_E(Q^2) \sim \alpha_E(0) \times e^{-R_{Ch}^2Q^2/6}$ 

Radius and polarizability scale with A:

 $R_{Ch} \sim 1.2 \,\mathrm{fm} \,A^{1/3}, \ \alpha_E \sim 2.25 \times 10^{-3} \,\mathrm{fm}^3 A^{5/3}$ 

Dimensional analysis estimate: 
$$\Delta_R(E) = 2 \times 10^{-5} \left(\frac{E}{MeV}\right) \frac{A}{N}$$

#### Nuclear structure and E-dependent RC

E-dependent correction: estimate in Fermi gas model (similar to E-independent)

$$\Delta_R(E) = (2.8 \pm 0.4) \times 10^{-4} \left(\frac{E}{MeV}\right)$$

Uncertainty: spread in  $\epsilon$  and k<sub>F</sub>

Use the two estimates as upper and lower bound of the effect

$$\Delta_R(E) = (1.6 \pm 1.6) \times 10^{-4} \left(\frac{E}{MeV}\right)$$

Spectrum distortion due to nuclear polarizabilities ~ 0.016 % per MeV

Roughly independent of the nucleus;

The total rate will depend on nucleus: different Q-values!

Correction to Ft values: integrate over spectrum (only total rate measured)

$$\Delta_E^{NS} = \frac{\int_{m_e}^{E_m} dEEp(Q-E)^2 \Delta_R(E)}{\int_{m_e}^{E_m} dEEp(Q-E)^2} \longrightarrow \tilde{\mathcal{F}}t = ft(1+\delta_R')(1-\delta_C+\delta_{NS}+\Delta_E^{NS})$$

#### Nuclear structure distorts the β-spectrum!

 $\tilde{\mathcal{F}}t = ft(1+\delta_R')(1-\delta_C+\delta_{NS}+\Delta_E^{NS})$ 

Absolute shift in Ft values

|--|

Decay	$Q \;({\rm MeV})$	$\Delta_E^{NS}(10^{-4})$	$\delta \mathcal{F}t(s)$	$\mathcal{F}t(s)$ [3]
$^{10}C$	1.91	1.5	0.5	3078.0(4.5)
$^{14}O$	2.83	2.3	0.7	3071.4(3.2)
$^{22}Mg$	4.12	3.3	1.0	3077.9(7.3)
$^{34}Ar$	6.06	4.8	1.5	3065.6(8.4)
$^{38}Ca$	6.61	5.3	1.6	3076.4(7.2)
$^{26m}Al$	4.23	3.4	1.0	3072.9(1.0)
$^{34}Cl$	5.49	4.4	1.4	$3070.7^{+1.7}_{-1.8}$
$^{38m}K$	6.04	4.8	1.5	3071.6(2.0)
$^{42}Sc$	6.43	5.1	1.6	3072.4(2.3)
${}^{46}V$	7.05	5.6	1.7	3074.1(2.0)
$^{50}Mn$	7.63	6.1	1.9	3071.2(2.1)
$^{54}Co$	8.24	6.6	2.0	$3069.8^{+2.4}_{-2.6}$
$^{62}Ga$	9.18	7.3	2.2	3071.5(6.7)
$^{74}Rb$	10.42	8.3	2.6	3076(11)

Shift comparable with the precision of the 7 best-known decays

$$\overline{\mathcal{F}t} = 3072.07(63)$$
s  $\rightarrow \overline{\mathcal{F}t} = 3073.6(0.6)(1.5)$ s

Decay electron polarizes the daughter nucleus

As a result the spectrum is slightly distorted towards the upper end

Changes the rate at 0.05% level

Previously found: E-independent piece lowers the Ft value by about the same amount

 $\mathcal{F}t = 3072.07(63)s \rightarrow [\mathcal{F}t]^{\text{new}} = 3070.65(63)(28)s$ 

The two effects tend to cancel each other; a good problem for hard-core nuclear theorists!

# RC to β-decay from dispersion relations: Summary

All three sources of possible model dependence addressed with DR

At each step a considerable shift beyond the previously assumed precision is observed

Universal correction: the biggest shift (2.5  $\sigma$ ) but the uncertainty reduced

Matching hadronic and nuclear corrections: shift (-  $2\sigma$ ) to the Ft value

Nuclear polarizabilities distort the  $\beta$ -spectrum, split inner-outer RC ambiguous: shift ~ (+2 $\sigma$ ) to the Ft value

Net effect: (4 $\sigma$ ) deficit for the first-row unitarity  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9984 \pm 0.0004$ 

Some increase in the nuclear uncertainty is likely

CKM first-row unitarity at a historic low. Solutions: SM or beyond?

## Discrepancy - BSM?

BSM explanation: non-standard CC interactions —> new V,A,S(PS),T(PT) terms

$$H_{S+V} = (\overline{\psi}_p \psi_n) (C_S \overline{\phi}_e \phi_{\overline{\nu}_e} + C'_S \overline{\phi}_e \gamma_5 \phi_{\overline{\nu}_e}) + (\overline{\psi}_p \gamma_\mu \psi_n) \left[ C_V \overline{\phi}_e \gamma_\mu (1+\gamma_5) \phi_{\overline{\nu}_e} \right]$$



Exp. plans: high precision measurement of <sup>6</sup>He spectrum (A. Garcia et al., U. Washington)

Complementarity to LHC searches (Gonzalez Alonso et al., arXiv: 1803.08732)

### Discrepancy - SM?

#### Hadronic correction $\Delta_{\text{R}}{}^{\text{V}}$

Neutrino data at low Q<sup>2</sup> are not precise upcoming DUNE experiment @ Fermilab may provide better data for F<sub>3</sub>

- can check the parametrization of F<sub>3</sub><sup>WW</sup> directly

Isospin rotation needs to be tested separately: axial Z-N coupling is a pure isovector —>  $4F_3^{(0)} \approx F_{3,\gamma Z}^p - F_{3,\gamma Z}^n \approx 2F_{3,\gamma Z}^p - F_{3,\gamma Z}^d$ 

Update axial  $\gamma$ Z-box —> a change in F<sub>3</sub> $\gamma$ Z —> a shift in weak charge (seems small)

Moments M<sub>3</sub><sup>(0)</sup>(N,Q<sup>2</sup>) from lattice?

#### Nuclear correction $\delta_{NS}$

DR allow to address hadronic and nuclear parts of the calculation on the same footing

But data will not guarantee the needed precision —> use nuclear model input

The trouble is with V<sub>us</sub>

Discrepancy in V<sub>us</sub> from KI3 and KI2 decays

Could be due to RC?  $\gamma$ W-box?

 $|V_{ud}|^{2} + |V_{us}^{K\ell^{2}}|^{2} + |V_{ub}|^{2} = 0.9979(5)$ PDG:  $|V_{ud}|^{2} + |\overline{V_{us}}|^{2} + |V_{ub}|^{2} = 0.9984(4)$  $|V_{ud}|^{2} + |V_{us}^{K\ell^{3}}|^{2} + |V_{ub}|^{2} = 0.9988(5)$