## - Amherst Center for Fundamental Interactions

Physics at the interface: Energy, Intensity, and Cosmic frontiers University of Massachusetts Amherst

# Baryogenesis from a CP violating Higgs Sector II 

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## Outline

- Motivation
- The model
- Electron/Neutron EDM bounds
- Generation of the baryon asymmetry
- Conclusions


## Motivation

Sakharov's conditions:

1. B violation
2. First order EW phase transition
3. $C$ and $C P$ violation

- Standard Model is a very successful effective theory
- Plenty of motivation to go beyond (Dark Matter, hierarchy problem, ...)
- Standard Model is not sufficient to describe Baryon Asymmetry of the Universe (BAU)


## Examples of successful EW Baryogenesis

Li,Profumo, Ramesy-Musolf, 2008-2010

- MSSM

$v(y)$
$\otimes$



Bian, Liu, Shu, 2014


## Electroweak baryogenesis

Morrissey, Ramsey-Musolf, 2012

- At the boundary of two phases the particleantiparticle asymmetry is generated
- It diffuses into the symmetric phase and EW sphalerons transfer the left handed



## Extended Higgs sectors

- With the discovery of the Higgs boson, it is plausible that the new physics is hiding in the extended Higgs sector
- Adding a real singlet to Standard Model modifies the SM scalar

Profumo, Ramsey-Musolf,
Wainwright, Winslow, 2014 potential so that the $1^{\text {st }}$ order PT can be achieved without upsetting existing collider bounds

- Adding a real triplet to Standard Model provides a stable DM candidate, possibility for a two step phase transition

Perez, Patel, Ramsey-Musolf, 2008
Patel, Ramsey-Musolf, 2012
Blinov, Kozaczuk, Morrissey, Tamarit, 2015

## Phase transition in two steps?

- The first step of the phase transition is driven by the triplet acquiring VEV. Patel, Ramsey-Musolf, 2012 Easy to obtain $1^{\text {st }}$ order PT condition with heavier SM Higgs mass
- In the first step because the triplet carries $\operatorname{SU}(2)_{L}$ charge the $B+L$ violating monopole interactions inside the $\Sigma$ - bubbles are suppressed

- In the second step the net baryon asymmetry generated during the first step survives due to (sufficiently) strong $1^{\text {st }}$ order PT

Open question: explicit model realization with CPV and BAU evaluation during the first step

## Our Goal

- Our goal is to perform an explicit calculation of the BAU generated during the first step of a 2 -step phase transition
- The choice for the model is $2 \mathrm{HDM}+\mathrm{rea}$ triplet+real singlet
- Can we get BAU consistent with the EDMs?

$$
\begin{aligned}
& \Delta V\left(H_{1}, H_{2}, \Sigma\right)=-\frac{\mu_{\Sigma}^{2}}{2}(\vec{\Sigma} \cdot \vec{\Sigma})+\frac{b_{4}}{4}(\vec{\Sigma} \cdot \vec{\Sigma})^{2} \quad a_{2 \Sigma}, a_{2 S} \text { are complex } \\
& +\left[\frac{1}{2} a_{2 \Sigma} H_{1}^{\dagger} H_{2}(\vec{\Sigma} \cdot \vec{\Sigma})+\frac{1}{2} a_{2 S} H_{1}^{\dagger} H_{2} S^{2}+\text { h.c. }\right]
\end{aligned}
$$

## The Model

$$
\begin{aligned}
& V\left(H_{1}, H_{2}\right)=\frac{\lambda_{1}}{2}\left(H_{1}^{\dagger} H_{1}\right)^{2}+\frac{\lambda_{2}}{2}\left(H_{2}^{\dagger} H_{2}\right)^{2} \\
& +\lambda_{3}\left(H_{1}^{\dagger} H_{1}\right)\left(H_{2}^{\dagger} H_{2}\right)+\lambda_{4}\left(H_{1}^{\dagger} H_{2}\right)\left(H_{2}^{\dagger} H_{1}\right) \\
& +\frac{1}{2}\left[\lambda_{5}\left(H_{1}^{\dagger} H_{2}\right)^{2}+\text { h.c. }\right]-\frac{1}{2}\left\{m_{11}^{2}\left(H_{1}^{\dagger} H_{1}\right)\right. \\
& \left.+\left[m_{12}^{2}\left(H_{1}^{\dagger} H_{2}\right)+\text { h.c. }\right]+m_{22}^{2}\left(H_{2}^{\dagger} H_{2}\right)\right\} .
\end{aligned}
$$

Rephasing invariant
CPV phases:

$$
\begin{aligned}
\delta_{1} & =\arg \left[\lambda_{5}^{*}\left(m_{12}^{2}\right)^{2}\right], \\
\delta_{2} & =\arg \left[\lambda_{5}^{*}\left(m_{12}^{2}\right) v_{1} v_{2}^{*}\right], \\
\delta_{3} & =\arg \left[\lambda_{5}^{*}\left(a_{2 \Sigma}\right)^{2}\right], \\
\delta_{4} & =\arg \left[\lambda_{5}^{*}\left(a_{2 S}\right)^{2}\right] .
\end{aligned}
$$

$$
\begin{aligned}
& \Delta V\left(H_{1}, H_{2}, \Sigma\right)=-\frac{\mu_{\Sigma}^{2}}{2}(\vec{\Sigma} \cdot \vec{\Sigma})+\frac{b_{4}}{4}(\vec{\Sigma} \cdot \vec{\Sigma})^{2} \\
& +\left[\frac{1}{2} a_{2 \Sigma} H_{1}^{\dagger} H_{2}(\vec{\Sigma} \cdot \vec{\Sigma})+\frac{1}{2} a_{2 S} H_{1}^{\dagger} H_{2} S^{2}+\text { h.c. }\right]
\end{aligned}
$$

$$
\begin{aligned}
& H_{i}=\binom{H_{i}^{+}}{\frac{v_{i}+H_{i}^{0}+i A_{i}^{0}}{\sqrt{2}}}, \quad \text { where } i=1,2 . \\
& \tan \beta \equiv v_{2} / v_{1}
\end{aligned}
$$

We have two additional complex phases compared to 2HDM We assume at zero temperature the VEVs of triplet and singlet are zero Vacuum stability conditions are identical to the 2HDM case (relate first two phases)

## Mass spectrum

Charged Higgses

$$
\begin{array}{ll}
\phi_{1}^{+}=-s_{\beta} H_{1}^{+}+c_{\beta} H_{2}^{+}, & M_{\phi_{i}}^{2}=\left[\begin{array}{cc}
m_{H^{+}}^{2} & 0 \\
0 & m_{\Sigma^{+}}^{2}
\end{array}\right] \\
\phi_{2}^{+}=\Sigma^{+}, & \\
& m_{H^{+}}^{2}=\frac{1}{2}\left(2 \nu-\lambda_{4}-\operatorname{Re} \lambda_{5}\right) v^{2}, \text { where } \nu \equiv \frac{\operatorname{Re} m_{12}^{2}}{2 v^{2} c_{\beta} s_{\beta}} \\
& m_{\Sigma^{+}}^{2}=-\mu_{\Sigma}^{2}+\operatorname{Re} a_{2} v^{2} c_{\beta} s_{\beta} .
\end{array}
$$

Neutral Higgses
$H_{1}^{0}, H_{2}^{0},\left(A^{0} \equiv-\sin \beta A_{1}^{0}+\cos \beta A_{2}^{0}\right), \Sigma^{0} \quad m_{\Sigma^{0}} \equiv m_{\Sigma^{+}}$

$$
\begin{aligned}
& M_{\text {neutral }}^{2}= \\
& v^{2}\left(\begin{array}{cccc}
\lambda_{1} c_{\beta}^{2}+\nu s_{\beta}^{2} & \left(\lambda_{345}-\nu\right) s_{\beta} c_{\beta} & -\frac{1}{2} s_{\beta} \operatorname{Im} \lambda_{5} & 0 \\
\left(\lambda_{345}-\nu\right) s_{\beta} c_{\beta} & \lambda_{2} s_{\beta}^{2}+\nu c_{\beta}^{2} & -\frac{1}{2} c_{\beta} \operatorname{Im} \lambda_{5} & 0 \\
-\frac{1}{2} s_{\beta} \operatorname{Im} \lambda_{5} & -\frac{1}{2} c_{\beta} \operatorname{Im} \lambda_{5} & \nu-\operatorname{Re} \lambda_{5} & 0 \\
0 & 0 & 0 & \frac{m_{\Sigma^{0}}^{2}}{v^{2}}
\end{array}\right),
\end{aligned}
$$

No mixing among the new scalars (triplet and singlet) and the 2 Higgs doublets Mass matrices block diagonal, the top $3 \times 3$ block of the neutral matrix identical to 2HDM Inoue, Ramsey-Musolf, Zhang, 2014

## Parameters in the potential vs phenomenological parameters

| Parameters in the potential | Phenomenological parameters |
| :--- | :---: |
| $\lambda_{1}, \lambda_{2}, \lambda_{3}, \lambda_{4}, \operatorname{Re} \lambda_{5}, \operatorname{Im} \lambda_{5}$ | $v, x_{0}, \xi, \tan \beta, \nu, \operatorname{Re} a_{2 \Sigma}, \operatorname{Re} a_{2 S}$ |
| $m_{11}^{2}, m_{22}^{2}, \operatorname{Re} m_{12}^{2}, \operatorname{Im} m_{12}^{2}$ | $\alpha, \alpha_{b}, \delta_{\Sigma}, \delta_{S}$ |
| $\operatorname{Re} a_{2 \Sigma}, \operatorname{Im} a_{2 \Sigma}, \mu_{\Sigma}, b_{4}, \operatorname{Re} a_{2 S}, \operatorname{Im} a_{2 S}$ | $m_{H^{+}}, m_{h_{1}}, m_{h_{2}}, m_{h_{3}}, m_{\Sigma}, b_{4}$ |

$$
\begin{aligned}
& \delta_{\Sigma}=\arg \left[a_{2 \Sigma}^{*} v_{1} v_{2}^{*}\right]=\delta_{2}-\frac{\delta_{1}+\delta_{3}}{2} \\
& \delta_{S}=\arg \left[a_{2 S}^{*} v_{1} v_{2}^{*}\right]=\delta_{2}-\frac{\delta_{1}+\delta_{4}}{2}
\end{aligned}
$$

Compared to the 2HDM model two additional CPV phases appear

## Technical simplification for BAU analysis

Cirigliano, Lee, Ramsey-Musolf, Tulin, 2006

- When the $m_{12}^{2} \neq 0$ coupling is present it leads to flavor oscillations when the triplet and singlet undergo phase transition in the early universe
- To avoid this technical difficulty we set $m_{12}^{2}=0 \quad$ which removes CPV from the 2HDM sector ( $\alpha_{b}=\alpha_{c}=0$ ) and the only two possible phases are $\delta_{\Sigma}, \delta_{S}$

Thus, we work in the exact $Z_{2}$ symmetry limit of 2HDM
We also will assume everywhere the alignment limit $\quad \alpha=\beta-\pi / 2$

## Electron EDM bound

Bar-Zee graph

$\left|d_{e}\right|<8.7 \times 10^{-29} \mathrm{ecm} \quad$ at $90 \%$ confidence level

- In the exact $Z_{2}$ limit of our theory EDM is sensitive only to the phase $\delta_{\Sigma}$ and not $\delta_{S}$
- Amount of BAU as you will see depends on the combination $\delta_{\Sigma}-\delta_{S}$


## EDM exclusion results



Parameter dependence of electron EDM in the exact $Z_{2}$ limit

$$
\delta_{e}=\sin \delta_{\Sigma} \tan \beta F\left(m_{\Sigma}, m_{h_{3}}\right)
$$

- Electron EDM bound

$$
\left|d_{e}\right|<8.7 \times 10^{-29} \mathrm{ecm}
$$

Neutron EDM bound (100xcurrent sensitivity)

$$
\left|d_{n}\right|<2.9 \times 10^{-28} \mathrm{ecm}
$$

## Generation of the baryon asymmetry

## Theoretical framework

- We assume relying on previous studies that the $1^{\text {st }}$ order phase transition condition is satisfied Patel, Ramsey-Musolf, 2012

Blinov, Kozaczuk, Morrissey, Tamarit, 2015

- We use closed time path integral (CTP) approach to derive transport equations that describe the dynamics of the bubble nucleation during the EW phase transition
- For CPV source terms we use the VEV insertion approximation


## Coupled Boltzmann eqns

$$
\begin{aligned}
T & =n_{t_{R}} \\
Q & =n_{t_{L}}+n_{b_{L}} \\
H & =n_{H_{1}^{+}}+n_{H_{1}^{0}}-n_{H_{2}^{+}}-n_{H_{2}^{0}} \\
h & =n_{H_{1}^{+}}+n_{H_{1}^{0}}+n_{H_{2}^{+}}+n_{H_{2}^{0}} \\
\partial^{\mu} T_{\mu} & =-\Gamma_{Y}\left(\frac{T}{k_{T}}-\frac{Q}{k_{Q}}-\frac{H}{k_{H}}\right)+\Gamma_{Y} \frac{h}{k_{h}}+\Gamma_{s s}\left(\frac{2 Q}{k_{Q}}-\frac{T}{k_{T}}+\frac{9(Q+T)}{k_{B}}\right), \\
\partial^{\mu} Q_{\mu} & =-\Gamma_{Y}\left(\frac{Q}{k_{Q}}-\frac{T}{k_{T}}+\frac{H}{k_{H}}\right)-\Gamma_{Y} \frac{h}{k_{h}}-2 \Gamma_{s s}\left(\frac{2 Q}{k_{Q}}-\frac{T}{k_{T}}+\frac{9(Q+T)}{k_{B}}\right), \\
\partial^{\mu} H_{\mu} & =\Gamma_{M}^{+} \frac{h}{k_{h}}-\Gamma_{M}^{-} \frac{H}{k_{H}}-\Gamma_{H} \frac{H}{k_{H}}-\Gamma_{Y}\left(\frac{Q}{k_{Q}}-\frac{T}{k_{T}}+\frac{H}{k_{H}}\right)-\Gamma_{Y} \frac{h}{k_{h}}+S_{H}^{C P V}, \\
\partial^{\mu} h_{\mu} & =-\Gamma_{Y}\left(\frac{H}{k_{H}}+\frac{Q}{k_{Q}}-\frac{T}{k_{T}}\right)-\Gamma_{Y} \frac{h}{k_{h}} .
\end{aligned}
$$



## Coupled Boltzmann eqns

$$
\begin{aligned}
& T=n_{t_{R}} \\
& Q=n_{t_{L}}+n_{b_{L}} \\
& H=n_{H_{1}^{+}}+n_{H_{1}^{0}}-n_{H_{2}^{+}}-n_{H_{2}^{0}} \\
& h=n_{H_{1}^{+}}+n_{H_{1}^{0}}+n_{H_{2}^{+}}+n_{H_{2}^{0}} \\
& \Gamma H \sim v(x)^{2} \\
& \Delta V\left(H_{1}, H_{2}, \Sigma\right)=a_{2} v H_{1}^{\dagger} H_{2} \sigma^{0} \\
& \partial^{\mu} T_{\mu}=-\Gamma_{Y}\left(\frac{T}{k_{T}}-\frac{Q}{k_{Q}}-\frac{H}{k_{H}}\right)+\Gamma_{Y} \frac{h}{k_{h}}+\Gamma_{s s}\left(\frac{2 Q}{k_{Q}}-\frac{T}{k_{T}}+\frac{9(Q+T)}{k_{B}}\right), \\
& \left.\partial^{\mu} Q_{\mu}=-\Gamma_{Y}\left(\frac{Q}{k_{Q}}-\frac{T}{k_{T}}+\frac{H}{k_{H}}\right)-\Gamma_{Y} \frac{h}{k_{h}}-2 \Gamma_{s s} \frac{2 Q}{k_{Q}}-\frac{T}{k_{T}}+\frac{9(Q+T)}{k_{B}}\right), \\
& \partial^{\mu} H_{\mu}=\Gamma_{M}^{+} \frac{h}{k_{h}}-\Gamma_{M}^{-} \frac{H}{k_{H}}-\Gamma_{H} \frac{H}{k_{H}}-\Gamma_{Y}\left(\frac{Q}{k_{Q}}-\frac{T}{k_{T}}+\frac{H}{k_{H}}\right)-\Gamma_{Y} \frac{h}{k_{h}}+S_{H}^{C P V}, \\
& \partial^{\mu} k_{\mu}=-\Gamma_{Y}\left(\frac{H}{k_{H}}+\frac{Q}{k_{Q}}-\frac{T}{k_{T}}\right)-\Gamma_{Y} \frac{h}{k_{h}} .
\end{aligned}
$$



## Coupled Boltzmann eqns

$$
\begin{aligned}
T & =n_{t_{R}} \\
Q & =n_{t_{L}}+n_{b_{L}} \\
H & =n_{H_{1}^{+}}+n_{H_{1}^{0}}-n_{H_{2}^{+}}-n_{H_{2}^{0}} \\
h & =n_{H_{1}^{+}}+n_{H_{1}^{0}}+n_{H_{2}^{+}}+n_{H_{2}^{0}} \\
\partial^{\mu} T_{\mu} & =-\Gamma_{Y}\left(\frac{T}{k_{T}}-\frac{Q}{k_{Q}}-\frac{H}{k_{H}}\right)+\Gamma_{Y} \frac{h}{k_{h}}+\Gamma_{s s}\left(\frac{2 Q}{k_{Q}}-\frac{T}{k_{T}}+\frac{9(Q+T)}{k_{B}}\right), \\
\partial^{\mu} Q_{\mu} & =-\Gamma_{Y}\left(\frac{Q}{k_{Q}}-\frac{T}{k_{T}}+\frac{H}{k_{H}}\right)-\Gamma_{Y} \frac{h}{k_{h}}-2 \Gamma_{s s}\left(\frac{2 Q}{k_{Q}}-\frac{T}{k_{T}}+\frac{9(Q+T)}{k_{B}}\right), \\
\partial^{\mu} H_{\mu} & =\Gamma_{M}^{+} \frac{h}{k_{h}}-\Gamma_{M}^{-} \frac{H}{k_{H}}-\Gamma_{H} \frac{H}{k_{H}}-\Gamma_{Y}\left(\frac{Q}{k_{Q}}-\frac{T}{k_{T}}+\frac{H}{k_{H}}\right)-\Gamma_{Y} \frac{h}{k_{h}}+S_{H}^{C P V}, \\
\partial^{\mu} h_{\mu} & =-\Gamma_{Y}\left(\frac{H}{k_{H}}+\frac{Q}{k_{Q}}-\frac{T}{k_{T}}\right)-\Gamma_{Y} \frac{h}{k_{h}} .
\end{aligned}
$$



## Left handed quark density profile



## Resonant relaxation

CPV source

$$
\begin{aligned}
S_{H}^{\mathrm{CPV}}(x)= & \frac{\left|a_{2 \Sigma} a_{2 S}\right| \sin \left(\delta_{S}-\delta_{\Sigma}\right)}{\pi^{2}} v_{S}(x) v_{\Sigma}(x) \\
& \times\left[v_{S}(x) \dot{v}_{\Sigma}(x)-\dot{v}_{S}(x) v_{\Sigma}(x)\right] \Lambda,
\end{aligned}
$$

$\Lambda=\int \frac{k^{2} \mathrm{~d} k}{\omega_{1} \omega_{2}} \operatorname{Im}\left(\frac{n_{B}\left(\epsilon_{1}^{*}\right)-n_{B}\left(\epsilon_{2}\right)}{\left(\epsilon_{1}^{*}-\epsilon_{2}\right)^{2}}+\frac{n_{B}\left(\epsilon_{1}\right)+n_{B}\left(\epsilon_{2}\right)+1}{\left(\epsilon_{2}+\epsilon_{1}\right)^{2}}\right)$
$S_{H} \sim \sin \left(\delta_{\Sigma}-\delta_{S}\right) \Lambda\left(m_{H_{1}}(T), m_{H_{2}}(T)\right)$


$$
m_{H_{1}}^{2}(T)-m_{H_{2}}^{2}(T)=f\left(y_{b}, y_{t}, m_{2}, \tan \beta\right)
$$

Expect strong parameter dependence of the BAU on the CP even (heavy) Higgs mass $\mathrm{m}_{2}$ and $\tan \beta$

## Flavor oscillations at finite temperature


$m_{2}-\tan \beta$ parameters at and near the resonance


1. Black line corresponds to the precise resonant relaxation $m_{H_{1}}^{2}(T)=m_{H_{2}}^{2}(T)$
2. Red and Blue lines correspond to being away from the resonant relaxation so that $|\theta|=0.4$ and different CPV angle $\delta_{\Sigma}-\delta_{S}=0.1,0.9$ 3. The solid circles tell us which parameters $\left(\tan \beta, m_{2}\right)$ give maximum amount of BAU with the flavor oscillations of the order of $0.4^{*} 0.4=16 \%$


## BAU anatomy

BAU dependence on the parameters of the theory


BAU dependence on the parameters of the theory


- For a representative case $\mathrm{m}_{2}=200 \mathrm{GeV}$ we scan through $\sin \delta_{\Sigma}=0.1,0.9 \quad$ (left plot with $\delta_{S}=0$ ) and $\quad \sin \delta_{S}=-0.1,-0.9 \quad$ (right plot with $\delta_{\Sigma}=0$ ) and show the dependence of BAU on $\tan \beta$.
- In the $\delta_{S}=0$ case the electron EDM is excluding the BAU bounds (except for the red curve near resonance where the flavor oscillations are nonnegligible)


## BAU anatomy

- The result of the comprehensive analysis of the BAU anatomy plots (more in the backup) is that in order to generate the observed BAU and avoid large flavor oscillations and EDM bounds, need large phase $\delta_{S}$

$$
\delta_{S}>\sim-0.5, \delta_{\Sigma} \text { small or zero }
$$

we also assumed $m_{2}>50 \mathrm{GeV}$, otherwise exists possibility with

$$
m_{2} \sim 45 \mathrm{GeV}, \delta_{S}=0, \delta_{\Sigma} \sim 0.6, \tan \beta \sim 0.2
$$

## Parameter dependence of bounds

Electron EDM

$$
\delta_{e}=\sin \delta_{\Sigma} \tan \beta F\left(m_{\Sigma}, m_{h_{3}}\right)
$$

BAU
$Y_{B} \approx \sin \left(\delta_{\Sigma}-\delta_{S}\right) G\left(m_{h_{2}}, \tan \beta\right)$

## BAU vs EDM bounds



Blue
$Y_{B}^{\mathrm{WMAP}}=(7.3 \pm 2.5) \times 10^{-11}$
Gray: electron EDM
Pink(ish): neutron EDM (x100 sensitivity improvement)


$$
\delta_{S}=-0.65
$$

Model A: BAU vs EDM


Guided by the anatomy plots we easily find successful benchmark scenarios that satisfy both Baryogenesis and EDM bounds with negligible flavor oscillations

## Conclusions

- In an extension of the 2 HDM with a real triplet and a real singlet we have computed the BAU from CTP formalism
- We found a range for successful benchmarks consistent with both BAU and EDMs
- We neglected flavor oscillations and checked that the assumption is valid for the benchmark (additional suppression of BAU)
- This work is another step towards a consistent phenomenology of multi-step phase transitions


## Backup slides

## BAU anatomy




- Same as in the last slide, but for a value of $m_{2}$ that is a function of that $\tan \beta$ is a fixed number $|\theta|$
- Solid lines sitting on the resonance (maximum possible BAU)
- Dashed lines moving away from resonance to suppress the flavor oscillation to $16 \%$ (maximum possible BAU with fixed magnitude of the flavor oscillations)
- Conclusion from the left plot: $\delta_{S}=0$ excluded by the EDM and collider bound $\mathrm{m}_{2}>150 \mathrm{GeV}$. Right plot: need a large phase in the $\delta_{S} \sim 0.5-0.8$


## Flavor oscillations at finite temperature



Cirigliano, Lee, Ramsey-Musolf, Tulin, 2006

$$
\begin{aligned}
& \delta m_{12}^{2}(T)=\frac{a_{2 \Sigma} T^{2}}{8}+\frac{a_{2 S} T^{2}}{24} \\
& \theta \sim \frac{\delta m^{2}(T)}{m_{H_{1}}^{2}(T)-m_{H_{2}}^{2}(T)}, P_{\mathrm{osc}} \sim|\theta|^{2}
\end{aligned}
$$

- We removed the $\mathrm{m}_{12}$ coupling in order to avoid "complicated" flavor oscillations
- However at finite temperature we generate such off-diagonal mass term
- Our approach, neglect this mass term and flavor oscillations, however for selected benchmarks check that assumption is reasonable
- Note that at the point of flavor resonant relaxation $\theta=\infty$


## Thermal masses formula

$$
\begin{aligned}
& m_{H_{1}}^{2}(T)-m_{H_{2}}^{2}(T) \\
& =\left[y_{b}^{2}-y_{t}^{2}+\frac{m_{h_{2}}^{2}}{v^{2}}\left(\tan ^{2} \beta-\cot ^{2} \beta\right)\right] \frac{T^{2}}{4}
\end{aligned}
$$

