

Quantum MC Calculations of Beta Decays in Light Nuclei

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Beta Decay as a Probe of New Physics
ACFI UMass, Amherst MA - Nov 2018



Open Questions in Fundamental Symmetries and Neutrino Physics

Majorana Neutrinos, Neutrinos Mass Hierarchy,
CP-Violation in Neutrino Sector, Dark Matter

with

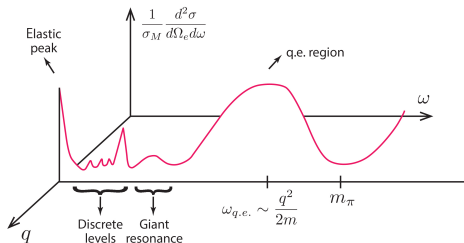
Carlson & Gandolfi (LANL) & Schiavilla (ODU+JLab)
Piarulli (WashU) & Baroni (USC) & Pieper & Wiringa (ANL)
Girlanda (Salento U.) & Marcucci & Viviani & Kievsky (Pisa U/INFN)

and with

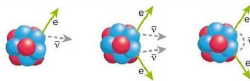
Mereghetti & Dekens & Cirigliano & Graesser (LANL)
de Vries (Nikhef) & van Kolck (AU+CNRS/IN2P3)

Towards a coherent and unified picture of neutrino-nucleus interactions

- * An **accurate** understanding of **nuclear structure and dynamics** is required to disentangle new physics from nuclear effects *



- * $\omega \sim \text{few MeV}, q \sim 0$: β -decay, $\beta\beta$ -decays
- * $\omega \sim \text{few MeV}, q \sim 10^2 \text{ MeV}$: Neutrinoless $\beta\beta$ -decays
- * $\omega \lesssim \text{tens MeV}$: Nuclear Rates for Astrophysics
- * $\omega \sim 10^2 \text{ MeV}$: Accelerator neutrinos, ν -nucleus scattering



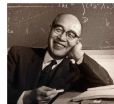
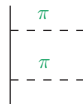
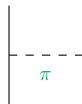
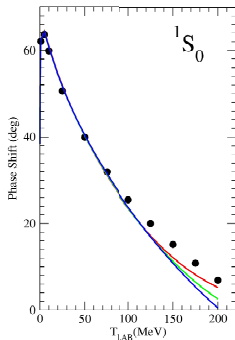
Standard β Decay Double β Decay Neutrinoless Double β Decay

Nuclear Interactions

The nucleus is made of A non-relativistic interacting nucleons and its energy is

$$H = T + V = \sum_{i=1}^A t_i + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

where v_{ij} and V_{ijk} are two- and three-nucleon operators based on EXPT data fitting and fitted parameters subsume underlying QCD



Hideki Yukawa

- * Contact terms: short-range
- * One-pion-exchange: range $\sim \frac{1}{m_\pi}$
- * Two-pion-exchange: range $\sim \frac{1}{2m_\pi}$

Quantum Monte Carlo Methods

Minimize expectation value of $H = T + V_{ij} + V_{ijk}$

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

using trial function

$$|\Psi_V\rangle = \left[\mathcal{S} \prod_{i<j} (1 + U_{ij} + \sum_{k \neq i,j} U_{ijk}) \right] \left[\prod_{i<j} f_c(r_{ij}) \right] |\Phi_A(JMTT_3)\rangle$$

Ψ_V is further improved it by “filtering” out the remaining excited state contamination

$$\Psi(\tau) = \exp[-(H - E_0)\tau] \Psi_V = \sum_n \exp[-(E_n - E_0)\tau] a_n \psi_n$$

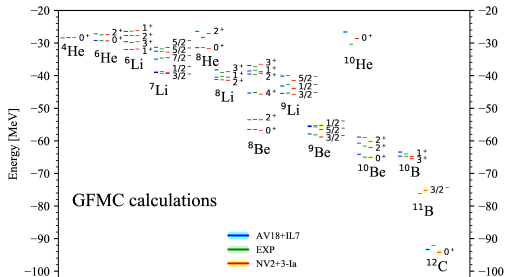
$$\Psi(\tau \rightarrow \infty) = a_0 \psi_0$$

- * QMC: AV18+UIX / AV18+IL7; Wiringa+Schiavilla+Pieper *et al.*
- * QMC: NN(N2LO)+3N(N2LO) (π &N); Gerzelis+Tews+Epelbaum+Gandolfi+Lynn *et al.*
- * QMC: NN(N3LO)+3N(N2LO) (π &N& Δ); Piarulli *et al.*

Lomnitz-Adler *et al.* NPA361(1981)399 - Wiringa PRC43(1991)1585 - Pudliner *et al.* PRC56(1997)1720 - Wiringa *et al.* PRC62(2000)014001

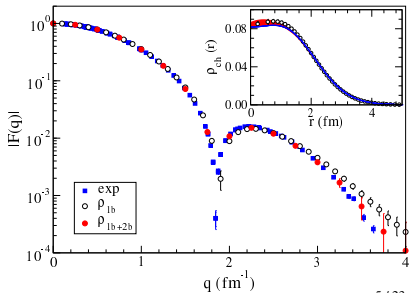
Pieper *et al.* PRC70(2004)054325 - Carlson *et al.* RevModPhys87(2014)1067

Energy Spectrum and Shape of Nuclei



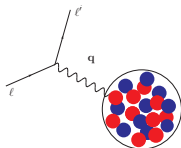
Piarulli *et al.* - PRL120(2018)052503

Lovato *et al.*
PRL111(2013)092501

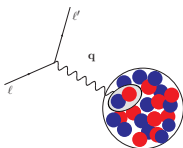


Nuclear Currents

1b



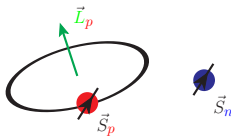
2b



$$\rho = \sum_{i=1}^A \rho_i + \sum_{i<j} \rho_{ij} + \dots,$$

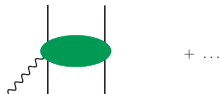
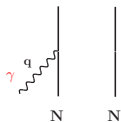
$$\mathbf{j} = \sum_{i=1}^A \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \dots$$

* Nuclear currents given by the sum of p 's and n 's currents, **one-body currents (1b)**



* **Two-body currents (2b)** essential to satisfy current conservation

* We use **Meson-Exchange Currents (MEC)** or **χ EFT Currents**



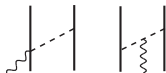
$$\mathbf{q} \cdot \mathbf{j} = [H, \rho] = [t_i + v_{ij} + V_{ijk}, \rho]$$

Electromagnetic Currents from Chiral Effective Field Theory

LO : $j^{(-2)} \sim eQ^{-2}$



NLO : $j^{(-1)} \sim eQ^{-1}$



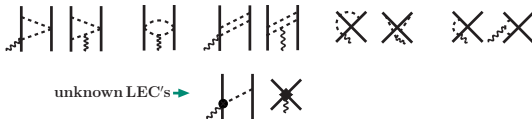
N²LO : $j^{(-0)} \sim eQ^0$



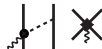
* 3 unknown Low Energy Constants:
fixed so as to reproduce d , 3H , and 3He magnetic moments

** also obtainable from LQCD calculations **

N³LO : $j^{(1)} \sim eQ$



unknown LEC's →

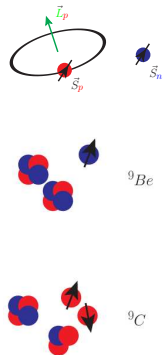
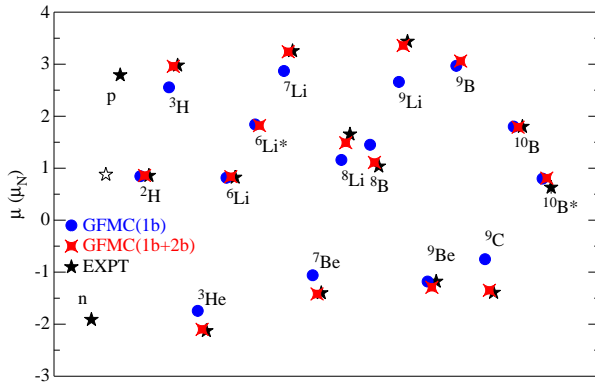


Pastore *et al.* PRC78(2008)064002 & PRC80(2009)034004 & PRC84(2011)024001

Piarulli *et al.* PRCC87(2013)014006

derived by Park+Min+Rho NPA596(1996)515 in CPT
and by Kölling+Epelbaum+Krebs+Meissner PRC80(2009)045502 & PRC84(2011)054008 with UT

Magnetic Moments of Nuclei

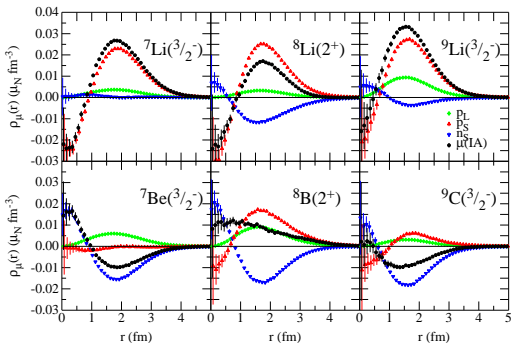


m.m.	THEO	EXP
${}^9\text{C}$	-1.35(4)(7)	-1.3914(5)
${}^9\text{Li}$	3.36(4)(8)	3.4391(6)

chiral truncation error based on [EE *et al.*](#) error algorithm, [Epelbaum, Krebs, and Meissner EPJA51\(2015\)53](#)

[Pastore *et al.* PRC87\(2013\)035503](#)

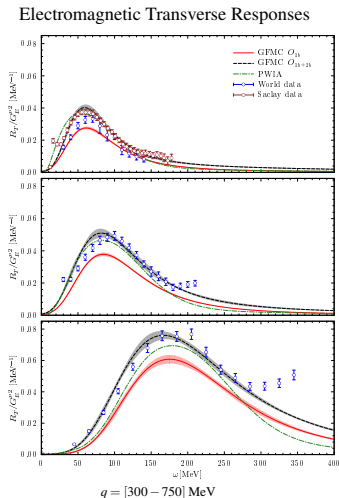
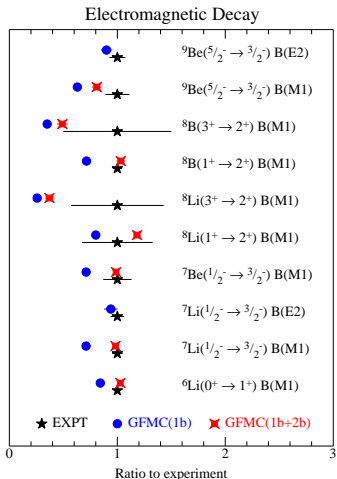
One-body magnetic densities



* one-body (IA) magnetic moment operator

$$\mu(\text{IA}) = \mu_N \sum_i [(L_i + g_p S_i)(1 + \tau_{i,z})/2 + g_n S_i(1 - \tau_{i,z})/2]$$

Electromagnetic Decays and e -scattering off nuclei



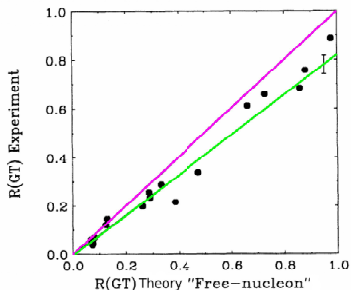
Pastore *et al.* PRC87(2013)035503 & PRC90(2014)024321

Lovato & Gandolfi *et al.* PRC91(2015)062501 &
arXiv:1605.00248

Electromagnetic data are explained when
two-body correlations and currents are accounted for!

Neutrinos and Nuclei: Challenges and Opportunities

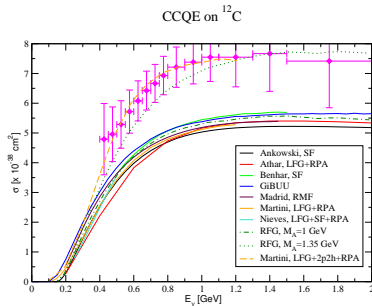
Beta Decay Rate



in $3 \leq A \leq 18 \rightarrow g_A^{\text{eff}} \simeq 0.80 g_A$

Chou *et al.* PRC47(1993)163

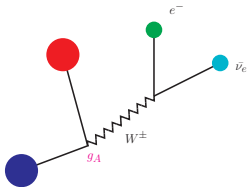
Neutrino-Nucleus Scattering



Alvarez-Ruso [arXiv:1012.3871](https://arxiv.org/abs/1012.3871)

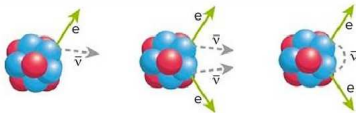
Standard Beta Decay

The “ g_A problem”
and
the role of two-body correlations and two-body currents



* Matrix Element $\langle \Psi_f | GT | \Psi_i \rangle \propto g_A$ and Decay Rates $\propto g_A^2$ *

$$(Z, N) \rightarrow (Z+1, N-1) + e + \bar{\nu}_e$$



Standard β Decay

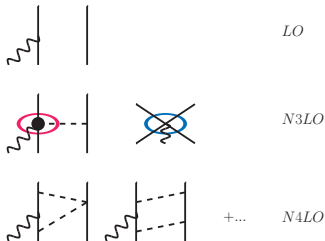
Double β Decay

Neutrinoless Double β Decay

Nuclear Interactions and Axial Currents

$$H = T + V = \sum_{i=1}^A t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

so far results are available with AV18+IL7 ($A \leq 10$)
and SNPA or chiral currents (*a.k.a.* hybrid calculations)



A. Baroni *et al.* PRC93(2016)015501

H. Krebs *et al.* Ann.Phys.378(2017)

* c_3 and c_4 are taken from Entem and Machleidt PRC68(2003)041001 & Phys.Rep.503(2011)1

* c_D fitted to GT m.e. of tritium
Baroni *et al.* PRC94(2016)024003

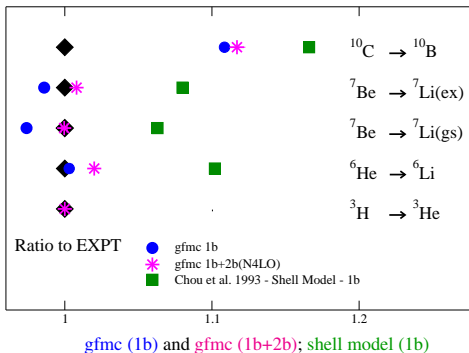
* cutoffs $\Lambda = 500$ and 600 MeV

* include also N4LO 3b currents (tiny)

* derived by Park *et al.* in the '90
used at tree-level in many calculations (Song-Ho, Kubodera, Gazit, Marcucci, Lazauskas, Navratil ...)

* pion-pole at tree-level derived
by Klos, Hoferichter *et al.* PLB(2015)B746

Single Beta Decay Matrix Elements in $A = 6-10$



Pastore *et al.* PRC97(2018)022501

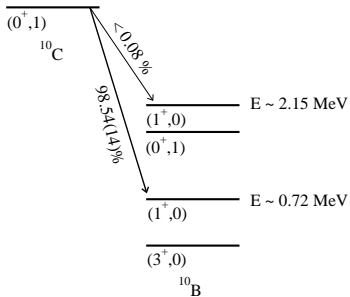
A. Baroni *et al.* PRC93(2016)015501 & PRC94(2016)024003

Based on $g_A \sim 1.27$ no quenching factor

GT in ^3H is fitted to expt - 2b give a 2% additive contribution to 1b prediction

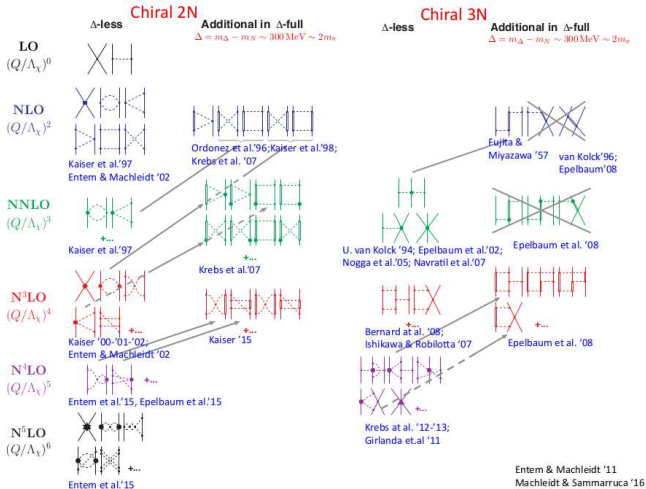
* similar results were obtained with MEC currents

* data from TUNL, Suzuki *et al.* PRC67(2003)044302, Chou *et al.* PRC47(1993)163

^{10}B 

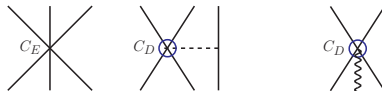
- * In ^{10}B , ΔE with same quantum numbers ~ 1.5 MeV
- * In $A = 7$, ΔE with same quantum numbers $\gtrsim 10$ MeV

Chiral calculations of beta decay m.e.'s: Nuclear Interaction



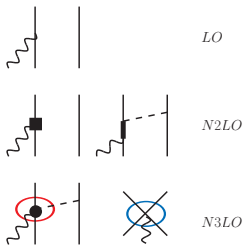
Chiral calculations of beta decay m.e.'s: Nuclear Currents

* Chiral interactions and axial currents



we now use

1. chiral 2- and 3-body interactions with πN and Δ 's developed by [Piarulli *et al.*](#) and
2. axial currents with Δ 's up to N3LO (tree-level) [A. Baroni *et al.* arXiv:1806.10245 \(2018\)](#)



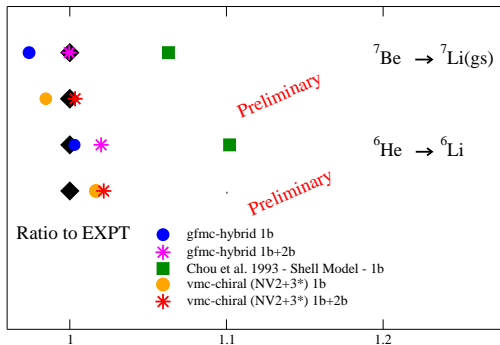
* c_3 and c_4 are taken from [Krebs *et al.* Eur.Phys.J.\(2007\)A32](#)

* c_D and c_E fitted to trinucleon B.E. and GT m.e. of tritium

[Baroni *et al.* arXiv:1806.10245 \(2018\)](#)

* based on NV2+3* chiral interactions

Single Beta Decay Matrix Elements in $A = 6-7$ in chiEFT



gfm (1b) and gfm (1b+2b); shell model (1b)

vmc-chiral-NV2+3* (1b) and vmc-chiral-NV2+3* (1b+2b)

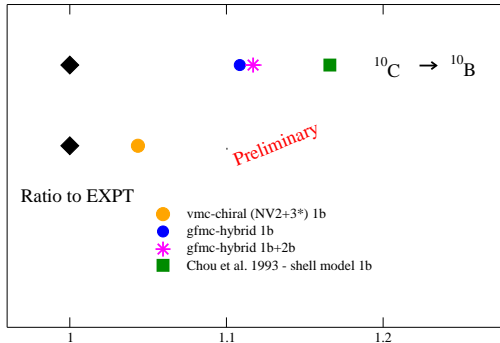
in collaboration with Piarulli *et al.*

based on chiral axial currents from A. Baroni *et al.* PRC93(2016)015501 & arXiv:1806.10245 (2018)

* qualitative agreement with hybrid calculations

* many-body currents as manifestation of many-body correlations

Single Beta Decay Matrix Elements in $A = 10$ in chiEFT



gfmc (1b) and gfmc (1b+2b); shell model (1b)

vmc-chiral-NV2+3* (1b) and vmc-chiral-NV2+3* (1b+2b)

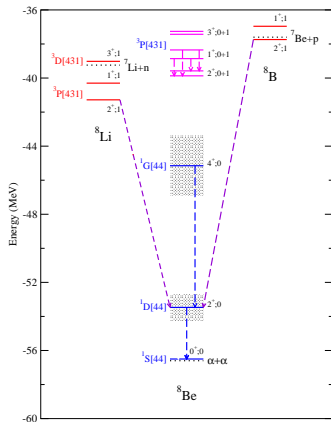
in collaboration with Piarulli *et al.*

based on chiral axial currents from A. Baroni *et al.* PRC93(2016)015501 & arXiv:1806.10245 (2018)

* improvement w.r.t. hybrid calculations

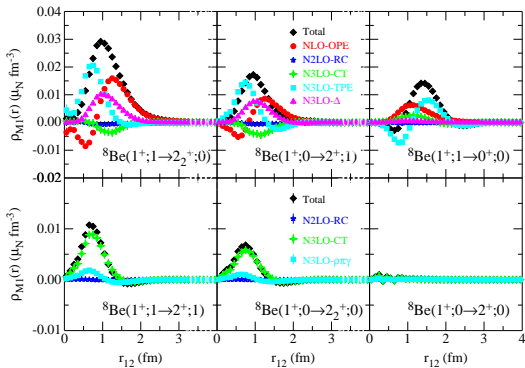
* agreement at 3%

EM and GT transitions in $A = 8$ nuclei



- * B(M1) in ^8Be are calculated at the $\sim 10\%$ level due to rich spectrum; presence of isospin-mixed states; transitions operators coupling “big” with “small components”
 - * 10% – 30% correction from two-body currents in M1 transitions
- * ^8Li and ^8B GT rme with one-body currents alone are $\sim 30\%$ smaller than expt; we expect large effect from two-body currents

Two-body M1 transitions densities



$(J_i, T_i) \rightarrow (J_f, T_f)$	IA	NLO-OPE	N2LO-RC	N3LO-TPE	N3LO-CT	N3LO- Δ	MEC
$(1^+; 1) \rightarrow (2_2^+; 0)$	2.461 (13)	0.457 (3)	-0.058 (1)	0.095 (2)	-0.035 (3)	0.161 (21)	0.620 (5)

Pastore *et al.* PRC90(2014)024321

The Present and Future of Quantum Monte Carlo Calculations

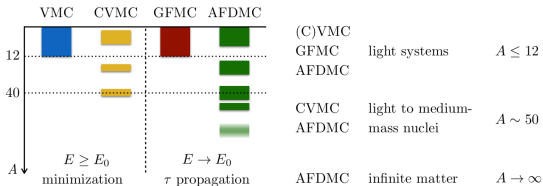
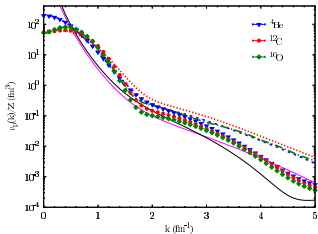


figure by Lonardon

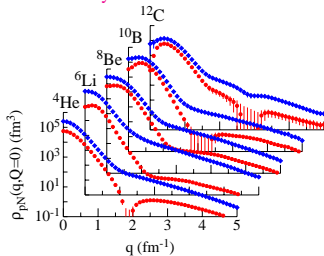
Use of Quantum Computers is being also explored - Roggero, Baroni, Carlson, Perdue *et al.*

One-body momentum distributions



Lonardon *et al.* to appear on PRC arXiv:1804.08027

Two-body momentum distributions



Wiringa *et al.* PRC89(2014)024305

One-body momentum distributions <http://www.phy.anl.gov/theory/research/momenta/>

Two-body momentum distributions <http://www.phy.anl.gov/theory/research/momenta2/>

Summary and Outlook

Two-nucleon correlations and two-body electroweak currents are crucial to explain available experimental data of both static (ground state properties) and dynamical (cross sections and rates) nuclear observables

- * We validate the computational framework vs electromagnetic data
 - * Two-body electromagnetic currents successfully tested in $A \leq 12$ nuclei
 - * $\sim 40\%$ two-body contribution found in ${}^9\text{C}$'s magnetic moments
 - * $\sim 10\text{-}30\%$ two-body contributions found in M1 transitions in low-lying states of $A \leq 8$ nuclei
 - * Calculations of β -decay matrix elements in $A \leq 10$ nuclei in agreement with the data at 2% – 3% level
 - * in $A \leq 10$ two-body currents ($q \sim 0$) are small ($\sim 2 - 3\%$) while correlations are crucial to improve agreement with expt
 - Study beta-decay within chiral framework (in progress)
 - Study beta-decay densities (in progress)
 - Extend calculations to $A \sim 40$ in AFDMC (in progress by LANL group)
 - Explore different kinematics for neutrino-nucleus interactions (including evaluation of the spectrum)
- * We are developing a coherent picture for neutrino-nucleus interactions

Bonus Material: Inclusive (e, ν) scattering

* inclusive xsecs *

$$\frac{d^2\sigma}{dE_f d\Omega_{e'}} = \sigma_M [v_L R_L(q, \omega) + v_T R_T(q, \omega)]$$

$$R_\alpha(q, \omega) = \sum_f \delta(\omega + E_0 - E_f) |\langle f | O_\alpha(\mathbf{q}) | 0 \rangle|^2$$

Longitudinal response induced by $O_L = \rho$

Transverse response induced by $O_T = \mathbf{j}$

... 5 nuclear responses in ν -scattering...

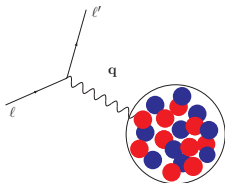
* Sum Rules *

Exploit integral properties of the response functions + closure to avoid explicit calculation of the final states

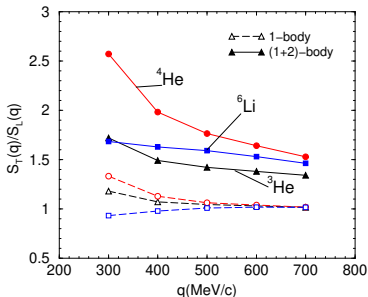
$$S(q, \tau) = \int_0^\infty d\omega K(\tau, \omega) R_\alpha(q, \omega)$$

* Coulomb Sum Rules *

$$S_\alpha(q) = \int_0^\infty d\omega R_\alpha(q, \omega) \propto \langle 0 | O_\alpha^\dagger(\mathbf{q}) O_\alpha(\mathbf{q}) | 0 \rangle$$



Sum Rules and Two-Body Physics



Carlson *et al.* [PRC65\(2002\)024002](#)

$$S_T(q) \propto \langle 0 | \mathbf{j}^\dagger \mathbf{j} | 0 \rangle$$

$$\propto \langle 0 | \mathbf{j}_{1b}^\dagger \mathbf{j}_{1b} | 0 \rangle + \langle 0 | \mathbf{j}_{1b}^\dagger \mathbf{j}_{2b} | 0 \rangle + \dots$$

- $\mathbf{j} = \mathbf{j}_{1b} + \mathbf{j}_{2b}$
- enhancement of the transverse response is due to interference between **1b** and **2b** currents **AND** presence of **two-nucleon correlations**



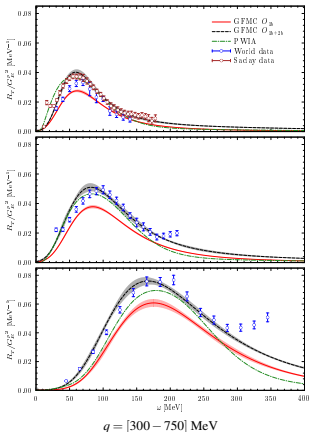
$$\langle \mathbf{j}_{1b}^\dagger \mathbf{j}_{1b} \rangle > 0$$



$$\langle \mathbf{j}_{1b}^\dagger \mathbf{j}_{2b} v_\pi \rangle \propto \langle v_\pi^2 \rangle > 0$$

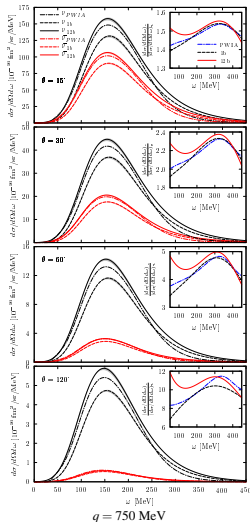
Recent Developments on Inclusive e and ν Scattering off ^{12}C

Electromagnetic Transverse Responses



Lovato & Gandolfi *et al.* PRC91(2015)062501 & arXiv:1605.00248

NC Inclusive Xsec

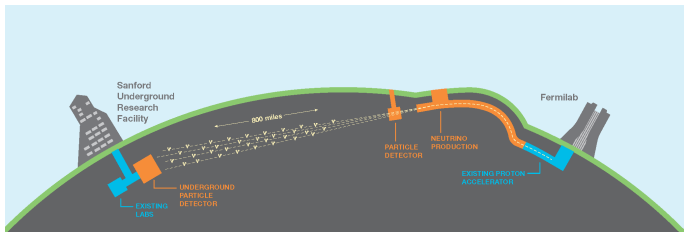


Lovato & Gandolfi *et al.* PRC97(2018)022502

~ 100 million core hours

Challenges and Opportunities

1. How to describe electroweak-scattering off $A > 12$ without losing two-body physics (*i.e.*, two-body correlations and currents)?
2. How to incorporate (more) exclusive processes?
3. How to incorporate relativistic effects?



Factorization: The Short-Time Approximation

$$R(q, \omega) = \sum_f \delta(\omega + E_0 - E_f) \langle 0 | O^\dagger(\mathbf{q}) | f \rangle \langle f | O(\mathbf{q}) | 0 \rangle$$

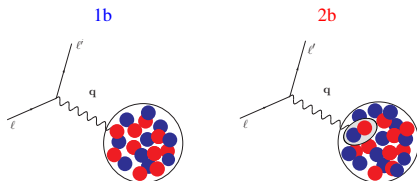
$$R(q, \omega) = \int dt \langle 0 | O^\dagger(\mathbf{q}) e^{i(H-\omega)t} O(\mathbf{q}) | 0 \rangle$$

At short time, expand $P(t) = e^{i(H-\omega)t}$ and keep up to **2b-terms**

$$H \sim \sum_i t_i + \sum_{i < j} v_{ij}$$

and

$$O_i^\dagger P(t) O_i + O_i^\dagger P(t) O_j + O_i^\dagger P(t) O_{ij} + O_{ij}^\dagger P(t) O_{ij}$$

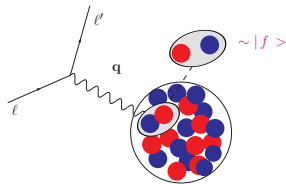


WITH

Carlson & Gandolfi (LANL) & Schiavilla (ODU+JLab) & Wiringa (ANL)

Factorization up to two-body operators: The Short-Time Approximation (STA)

Response functions are given by the scattering off pairs of fully interacting nucleons that propagate into a correlated pair of nucleons



$$R(q, \omega) = \sum_f \delta(\omega + E_0 - E_f) \langle 0 | O^\dagger(\mathbf{q}) | f \rangle \langle f | O(\mathbf{q}) | 0 \rangle$$

$$O(\mathbf{q}) = O^{(1)}(\mathbf{q}) + O^{(2)}(\mathbf{q}) = 1\mathbf{b} + 2\mathbf{b}$$

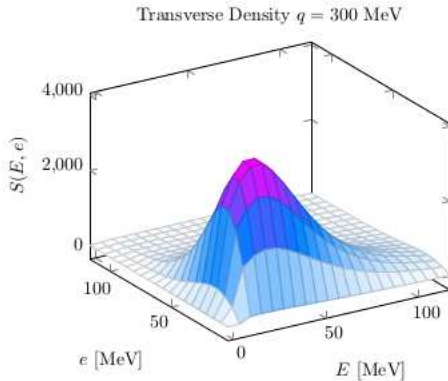
$$|f\rangle \sim |\psi_{p,P,J,M,L,S,T,M_T}(r, R)\rangle = \text{correlated two-nucleon w.f.}$$

* We retain **two-body physics** consistently **in the nuclear interactions** and **electroweak currents**

* STA can describe more two-body physics, *e.g.*, pion-production induced by e and ν

$$R(q, \omega) \sim \int \delta(\omega + E_0 - E_f) d\Omega_p d\Omega_p dP dp [p^2 P^2 \langle 0 | O^\dagger(\mathbf{q}) | \mathbf{p}, \mathbf{P} \rangle \langle \mathbf{p}, \mathbf{P} | O(\mathbf{q}) | 0 \rangle]$$

The Short-Time Approximation



Transverse “response-density” $1b + 2b$ for ${}^4\text{He}$

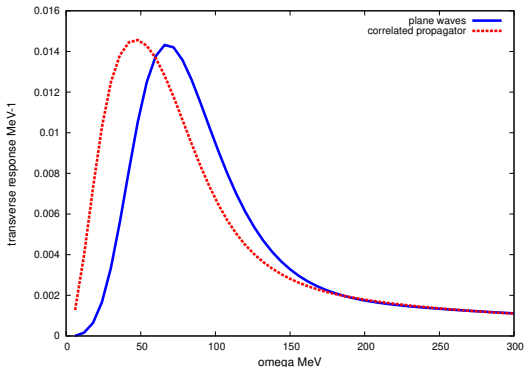
$$R(q, \omega) \sim \int \delta(\omega + E_0 - E_f) d\Omega_p d\Omega_p dP dp [p^2 P^2 \langle 0|O^\dagger(\mathbf{q})|\mathbf{p}, \mathbf{P}\rangle \langle \mathbf{p}, \mathbf{P}|O(\mathbf{q})|0\rangle]$$

* Preliminary results *

STA Transverse Response

$$q = 300 \text{ MeV}$$

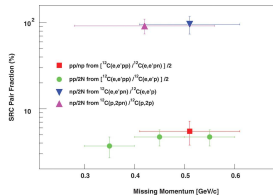
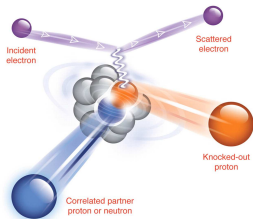
Plane Wave Propagator vs Correlated Propagator



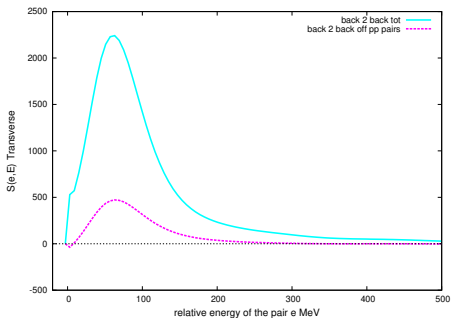
$$R_{\alpha}(q, \omega) \sim \int \delta(\omega + E_0 - E_f) d\Omega_p d\Omega_p dP dp [p^2 P^2 \langle 0 | O_{\alpha}^{\dagger}(\mathbf{q}) | \mathbf{p}, \mathbf{P} \rangle \langle \mathbf{p}, \mathbf{P} | O_{\alpha}(\mathbf{q}) | 0 \rangle]$$

* Preliminary results *

STA back to back scattering



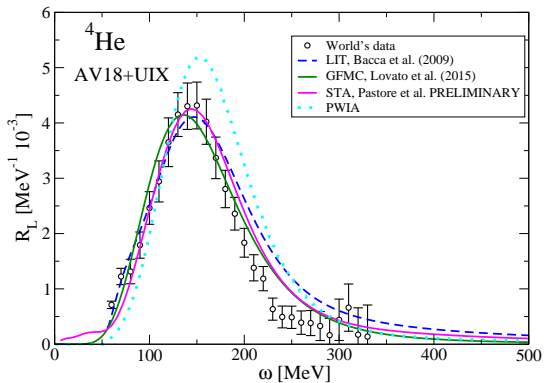
JLab, Subedi *et al.* Science320(2008)1475



$q = 500 \text{ MeV}, E = 69 \text{ MeV}$ *pp* vs *tot*

* Preliminary results *

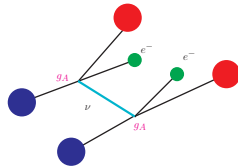
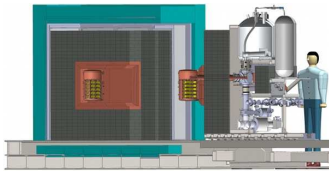
The Short-Time Approximation



Longitudinal Response function at $q = 500$ MeV

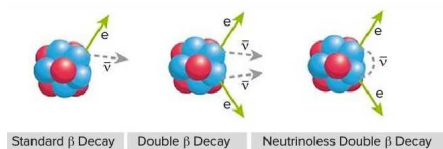
* Preliminary results *

Neutrinoless Double Beta Decay

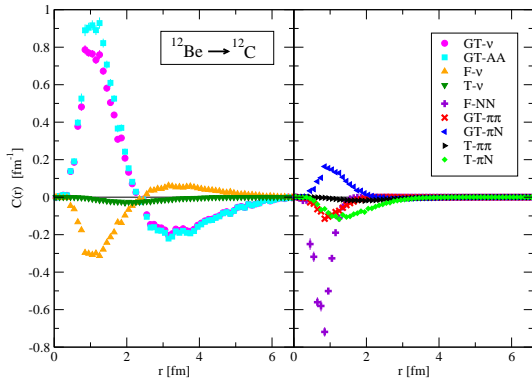


“The average momentum is about **100 MeV**, a scale set by the average distance between the two decaying neutrons” *cit. Engel&Menéndez*

* Decay rate \propto (nuclear matrix elements)² \times $\langle m_{\beta\beta} \rangle^2$ *



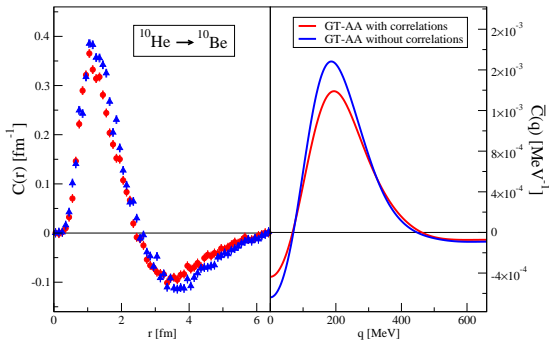
Double beta-decay Matrix Elements in $A = 12$



with Mereghetti & Dekens & Cirigliano & Carlson & Wiringa

PRC97(2018)014606

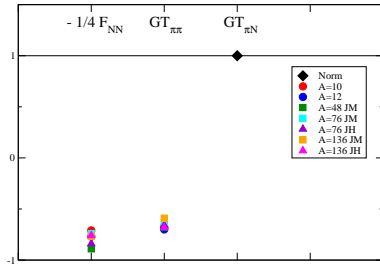
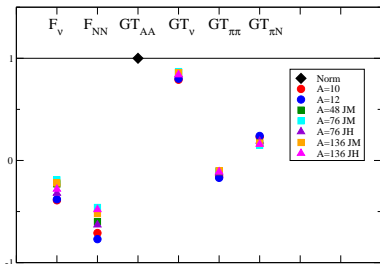
Sensitivity to 'pion-exchange-like' correlations



- * no 'pion-exchange-like' correlations
- * yes 'pion-exchange-like' correlations

Pastore, Dekens, Mereghetti *et al.* PRC97(2018)014606

Comparison with calculations of larger nuclei



JM = Javier Menendez private communication
JH = Hyvärinen *et al.* PRC91(2015)024613

- * Relative size of the matrix elements is approximately the same in all nuclei
- * Short-range terms approximately the same in all nuclei

with Mereghetti & Dekens & Cirigliano & Carlson & Wiringa

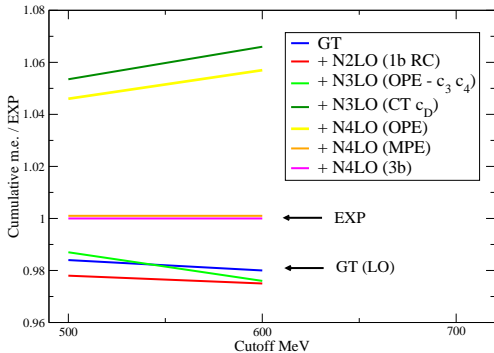
PRC97(2018)014606

Summary and Outlook

Two-nucleon correlations and two-body electroweak currents are crucial to explain available experimental data of both static (ground state properties) and dynamical (cross sections and rates) nuclear observables

- * Two-body currents can give $\sim 30 - 40\%$ contributions and improve on theory/EXPT agreement
- * Calculations of $\beta-$ and $\beta\beta-$ decay m.e.'s in $A \leq 12$ indicate two-body physics (currents and correlations) is required
- * Short-Time-Approximation to evaluate ν -A scattering in $A > 12$ nuclei is in excellent agreement with exact calculations and data
- Extend STA to study electroweak scattering in $A > 4$ nuclei
- Incorporate exclusive processes in the STA
- Study beta-decay within a chiral framework
- Explore neutrino-nucleus interaction at moderate value of momentum transfer
- * We are developing a coherent picture for neutrino-nucleus interactions *

Tritium β -decay



- * Results based on AV18+UIX and Chiral Currents are qualitatively in agreement
- * All contributions “quench” but for the N3LO OPE (tiny due to a cancellation) and CT (fitted)
- * They quench too much, and this is compensated by the fitting of c_D to EXP GT
- * Use of N4LO 2b loop currents from [H. Krebs *et al.* Ann.Phys.378\(2017\)](#) leads to a reduced value of c_D

* $\sim 2\%$ additive contribution from two-body currents *

A. Baroni *et al.* PRC93(2016)015501 & PRC94(2016)024003

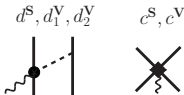
χ EFT currents in $A > 3$ systems

$A = 7$ Captures

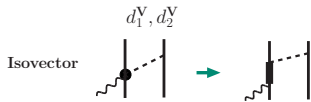
	gs	ex
LO	2.334	2.150
N2LO	-3.18×10^{-2}	-2.79×10^{-2}
N3LO(OPE)	-2.99×10^{-2}	-2.44×10^{-2}
N3LO(CT)	2.79×10^{-1}	2.36×10^{-1}
N4LO(2b)	-1.61×10^{-1}	-1.33×10^{-1}
N4LO(3b)	-6.59×10^{-3}	-4.86×10^{-3}
TOT(2b+3b)	0.050	0.046

- * Large cancellations between CT at N3LO (with c_D fitted) and other 2b currents
- * $\lesssim 3\%$ additive contribution from 2b currents in the $A \leq 10$ systems we considered
- * this is in agreement with results obtained with “conventional” axial currents
- * when using chiral axial currents $\lesssim 1\%$ error from chiral truncation (in the currents)

Electromagnetic LECs



d^S , d_1^V , and d_2^V could be determined by $\pi\gamma$ -production data on the nucleon



Isovector $d_2^V = 4\mu^* h_A / 9m_N (m_\Delta - m_N)$ and
 $d_1^V = 0.25 \times d_2^V$
 assuming Δ -resonance saturation

Left with 3 LECs: Fixed in the $A = 2 - 3$ nucleons' sector

* Isoscalar sector:

* d^S and c^S from EXPT μ_d and $\mu_S(^3\text{H}/^3\text{He})$

* Isovector sector:

* c^V from EXPT $npd\gamma$ xsec.

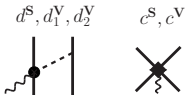
or

* c^V from EXPT $\mu_V(^3\text{H}/^3\text{He})$ m.m.

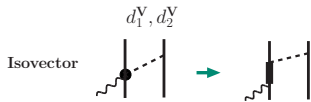
* Regulator $C(\Lambda) = \exp(-(p/\Lambda)^4)$ with $\Lambda = 500 - 600$ MeV

Λ	NN/NNN	$10 \times d^S$	c^S
500	AV18/UIX (N3LO/N2LO)	-1.731 (2.190)	2.522 (4.072)
600	AV18/UIX (N3LO/N2LO)	-2.033 (3.231)	5.238 (11.38)

Electromagnetic LECs



d^S , d_1^V , and d_2^V could be determined by $\pi\gamma$ -production data on the nucleon



Isovector $d_2^V = 4\mu^* h_A / 9m_N (m_\Delta - m_N)$ and
 $d_1^V = 0.25 \times d_2^V$
 assuming Δ -resonance saturation

Left with 3 LECs: Fixed in the $A = 2 - 3$ nucleons' sector

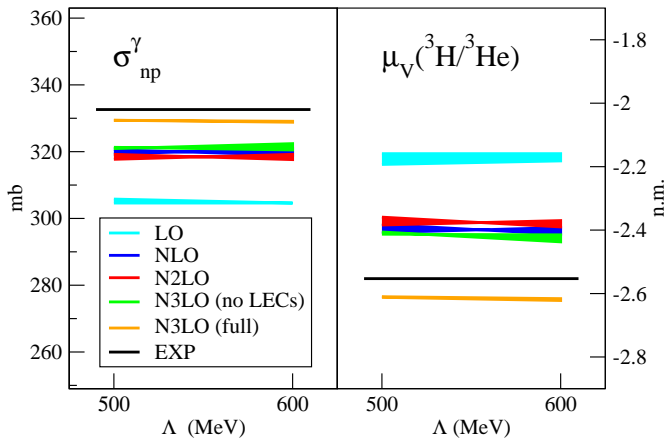
- * Isoscalar sector:
 - * d^S and c^S from EXPT μ_d and $\mu_S(^3\text{H}/^3\text{He})$
- * Isovector sector:
 - * c^V from EXPT $npd\gamma$ xsec.
 - or
 - * c^V from EXPT $\mu_V(^3\text{H}/^3\text{He})$ m.m.
 - * Regulator $C(\Lambda) = \exp(-(p/\Lambda)^4)$ with $\Lambda = 500 - 600$ MeV

Λ	NN/NNN	Current	d_1^V	c^V
600	AV18/UIX	I	4.98	-11.57
		II	4.98	-1.025

Convergence and cutoff dependence

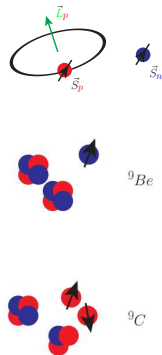
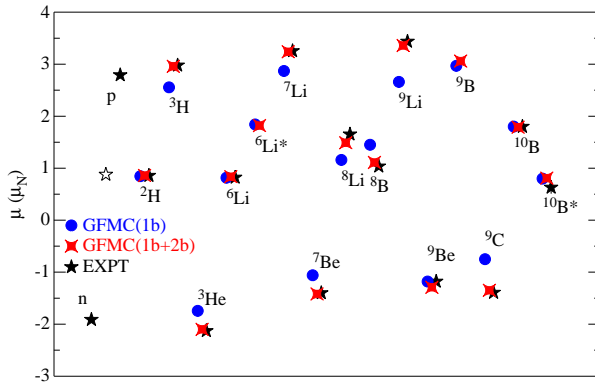
np capture x-section/ μ_V of $A = 3$ nuclei

bands represent nuclear model dependence [NN(N3LO)+3N(N2LO) – AV18+UIX]



Piarulli *et al.* PRC(2013)014006

Magnetic Moments of Nuclei



m.m.	THEO	EXP
${}^9\text{C}$	-1.35(4)(7)	-1.3914(5)
${}^9\text{Li}$	3.36(4)(8)	3.4391(6)

chiral truncation error based on [EE et al.](#) error algorithm, [Epelbaum, Krebs, and Meissner EPJA51\(2015\)53](#)

Pastore et al. PRC87(2013)035503