

## COMMENTS ON PARAMETRIC SCALE SEPARATION AND HOLOGRAPHY. (1)

Let's consider string compactifications of the form

$$\text{AdS} \times M.$$

To obtain a semi-realistic cosmology, we want

$$L(\text{AdS}) \gg L(M). \quad \equiv \text{scale separation.}$$

But this is not as simple as it seems.

### Swampland perspectives.

We can rewrite the scale separation condition as

$$\frac{m_{KK}}{|\Lambda|^{1/2}} \gg 1.$$

We expect towers of states to become light when making large distances in field space. Therefore, also when we are moving in metric field space, and have  $\Lambda \rightarrow 0$ , we expect a tower to become light.

- AdS Distance Conjecture (Lust, Palti; Vafa 2019).

$$m_{KK} \sim |\Lambda|^\alpha$$

$\alpha$  is a positive  $O(1)$  number.

- Based on susy examples, it seems most natural

that  $\frac{m_{KK}}{|\Lambda|^{1/2}} \approx O(1)$  &

excluding scale sep. in AdS vacua. This is the Strong AdS distance conjecture.

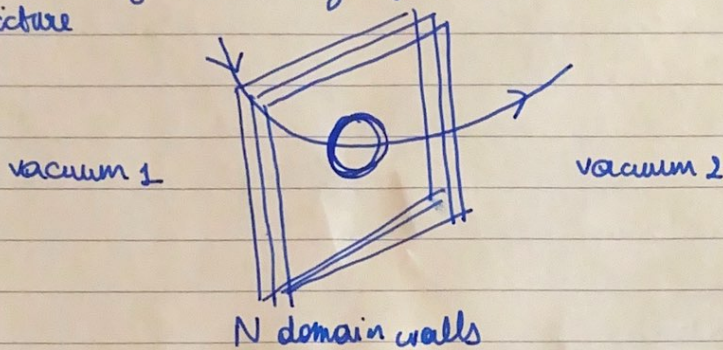
②

- There is one suggested way around it [Burrati, Calderon, Miminno, Uranga 2020].

If there is a discrete  $Z_N$  symmetry for domain walls in the theory, with  $N$  large, then this ratio gets enhanced

$$\frac{m_{KK}}{|\Lambda|^{1/2}} \sim \sqrt{N},$$

this way allowing for scale separation. (PARAMETRIC).  
Picture



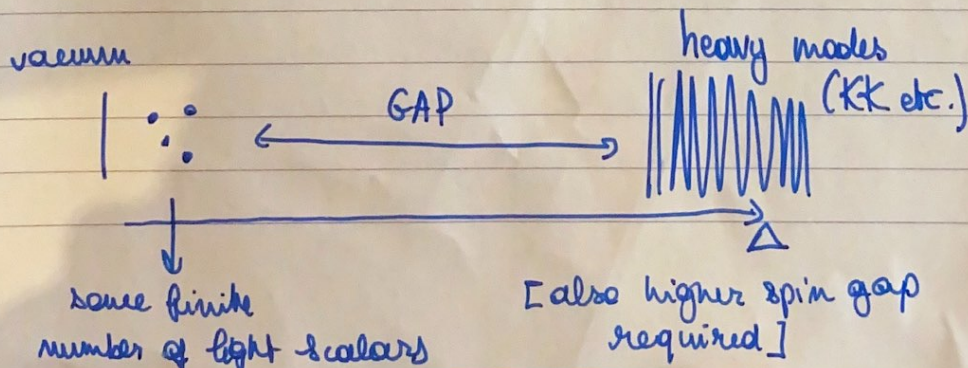
'Vacua separated by  $N$  such domain walls are equivalent!'

### Holographic perspectives

- Best understood examples like  $AdS_5 \times S^5$  do not have scale separation.
- At best partial scale separation:

$$\begin{array}{ccc} AdS_3 & \times & S^3 & \times & T^4 \\ \downarrow & & \downarrow & & \downarrow \\ \text{large} & & \text{large} & & \text{small} \end{array}$$

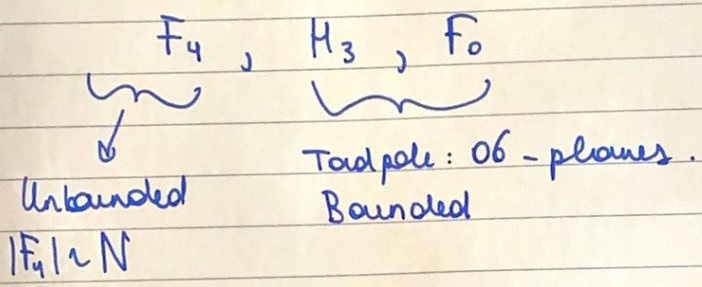
- Does holography require  $AdS \times \text{Large}$ ?
- CFT perspective: single trace spectrum



DGKT vacua (DeWolfe, Gaiotto, Kachru, Taylor 2005)  
They achieve parametric scale separation in AdS<sub>4</sub>-vacua using only very simple ingredients. (No quantum ingredients)

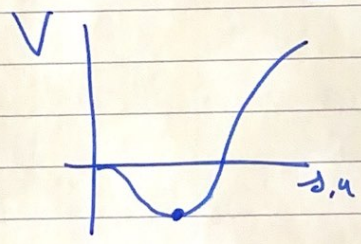
- compactification of massive IIA on a Calabi-Yau AdS<sub>4</sub> × CY<sub>6</sub>.

- Moduli Stabilization by fluxes:



$$V = \frac{1}{\delta^3} \left[ \frac{AF_4}{u \cdot \delta} + \frac{AF_0 u^3}{\delta} + \frac{AH_3 \cdot \delta}{u^3} - A_{06} \right]$$

u = volume modulus  
 $\delta$  = 4d dilaton



- minimal N=1 SUSY
- parametric control:  $V \sim u^3 \sim N^{3/2}$   
 [with string loop corrections].  $e^{\phi} \sim \delta^{-1} u^{1/2} \sim N^{-3/4}$ .

- scale separation, parametric:

$$\frac{m_{KK}}{|\Lambda|^4} \sim \sqrt{N}.$$

- only no full understanding of intersecting 06-planes.

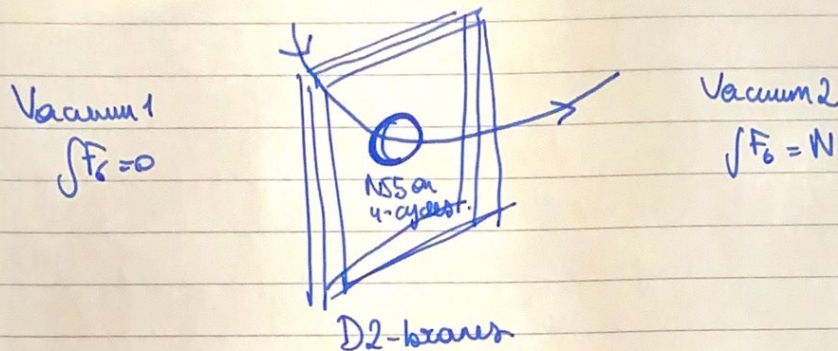
(u)

## Discrete symmetries in D&GKT-vacua

Dimensional reduction of the 10D Chern-Simons term

$$B_2 \wedge F_4 \wedge F_4 \rightarrow N \phi F_4$$

where  $\phi$  is a  $B_2$ -axion.



## Light spectrum of the D&GKT vacua

$$\Delta = \frac{3}{2} + \sqrt{\frac{9}{4} + m^2 R_{\text{AdS}}^2}$$

- Operators dual to volume moduli and dilaton have

$$\Delta = 10, 6, 6, 6, \dots$$

- The corresponding axions have ( $N=1$  multiplets)

$$\Delta = 11, 5, 5, 5, \dots$$

- No a priori reason for  $\mathbb{Z}_2$  conformal dimensions with only minimal SUSY.
- This property is independent of the flux numbers and the  $S^1$  geometry.

[See FA, Conlon, Montero, Nunez, Rovello, Weerase,  
Van Riet 2022]

# Shift Symmetries

[Danifacio, Hinterbichler, Joyce, Rosen 2018]

There are extensions of the ordinary constant shift symmetries for massless scalar fields:  
 $\phi \rightarrow \phi + c.$

A free field in  $AdS_D$  has a level- $k$  polynomial shift symmetry

$$\phi \rightarrow \phi + c_{\mu_1 \dots \mu_k} X^{\mu_1} \dots X^{\mu_k} |_{AdS}$$

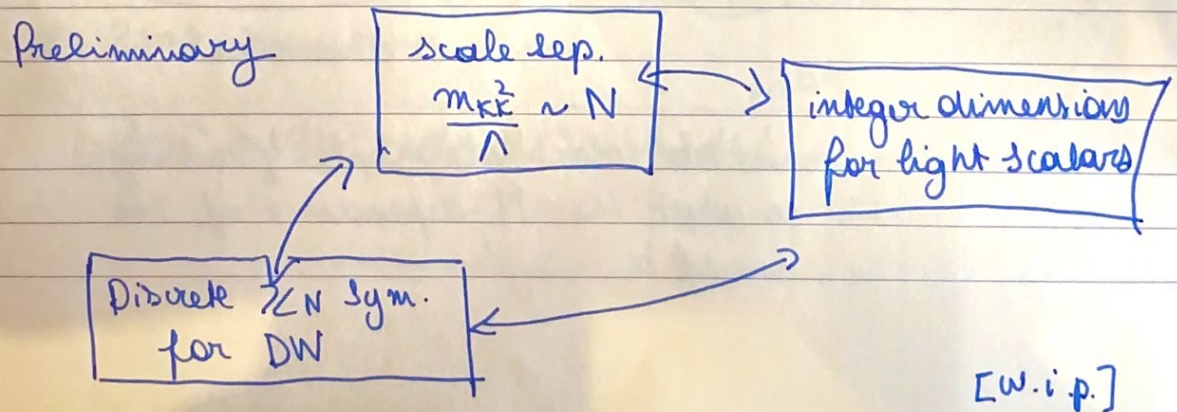
where  $X^{\mu_i}$  are embedding flat space coordinates and  $c$  is a symmetric traceless tensor iff

$$m_\phi^2 = \frac{k(k+D-1)}{R_{AdS}^2}.$$

Equivalently,  $\Delta\phi = k + D - 1.$

$\Rightarrow$  There must be polynomial shift symmetries of different levels  $k$  ( $k = 2, 3, 7, 8$ ) in the DGT vacua.

For example, the  $k=2$  symmetry is responsible for the  $\Delta=5$  axions and is related to the  $N\phi F_4$  scalar Stückelberg coupling.



[w.i.p.]

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Question: what are holographic implications of  $Z_N$  symmetries for domain walls in AdS?

The DGKT central charge

From the AdS perspective, we learn that the holographic central charge should scale like

$$c \sim N^{9/2},$$

because  $V(\phi)|_{\text{minimum}} \sim N^{-9/2}$ .

Question: How to explain from CFT perspective?

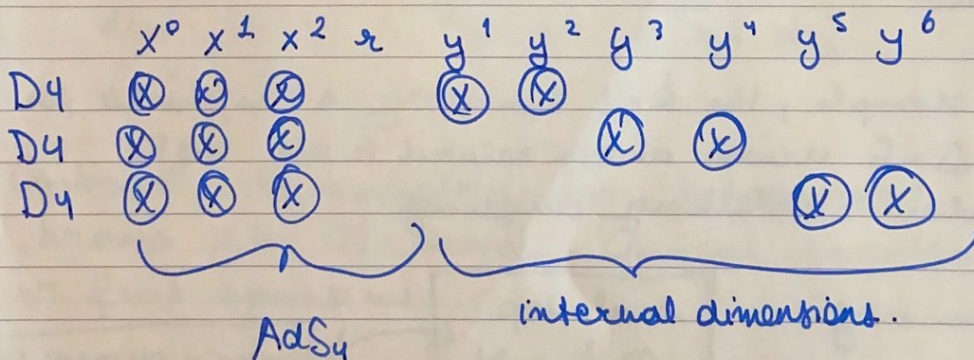
In between: BRANE DUAL.

Flux-domain wall correspondence

(see Kounnas, Lust, Petropoulos, Trnampis 2008).

$F_n$ -flux  $\leftrightarrow$  Domain wall from wrapped  $D(8-n)$ -brane.

in DGKT:  $F_4$ -flux  $\leftrightarrow$  3 sets of  $D4$ -branes wrapped on 2-cycles.



Although these branes are (non-holographic / running dilaton), let's see what the  $N$ -dependence of the near-horizon would be.

By applying the super-harmonic superposition rules, (7)  
we find that

$$\begin{aligned} d\tau_{\text{near-horizon}}^2 &= \alpha' (N^{3/2} ds_{\text{M}^4}^2 + N^{1/2} ds_6^2) \\ &= l_{\text{p},4}^2 (\underline{N^{9/2}} ds_4^2 + N^{7/2} ds_6^2) \end{aligned}$$

correctly reproducing the scalings of the string coupling  $g_s \sim N^{-3/4}$ ,  $\nu \sim N^{3/2}$  and the central charge  $C \sim N^{9/2}$ .

→ At leading order in large  $N$ , these D4-domain walls must be the brane dual somehow?

This works similarly in other large  $N$  holographic set-ups like M2-branes, or D1-D5 set-up etc.

Question microscopic counting of degrees of freedom on these intersecting D4-branes?

extra What is the background geometry?

We can summarize the presence of the other ingredients  $(F_0, H_3, G_6)$  with the following running geometry:

$$ds_{10}^2 = dt^2 + r^{-\frac{10}{9}} dx_n dx^n + r^{\frac{2}{3}} ds_6^2$$

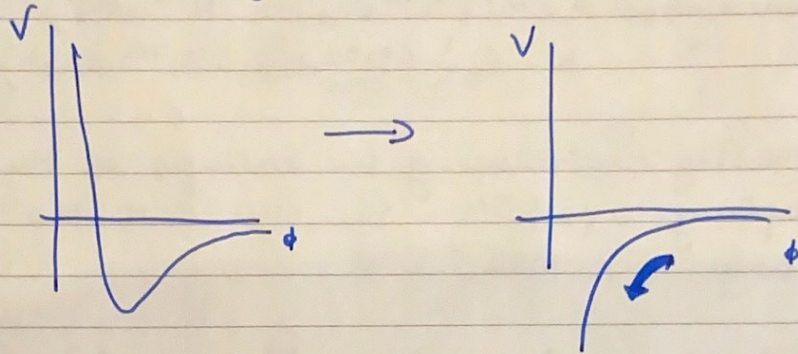
( $e^{\phi} \sim r^{-1}$ ).

Contrast with geometry for 'holographic' branes like D3-branes. Conical singularity or flat spacetime

$$ds_{10}^2 = dt^2 + dx_n dx^n + r^2 ds_6^2.$$

⑧

How to compute  $\frac{1}{2}$  from EFT?  $\rightarrow$  Consider flux potential and delete unbounded flux, compute flow (attractor) along the resulting runaway potential



[w.i.p. with Montero, Valenzuela].

Scale-separation in AdS<sub>3</sub>?

Use CFT<sub>2</sub>-technology? i.e. modular bootstrap?

[Farakos - Teringias - Van Riet 2020]:

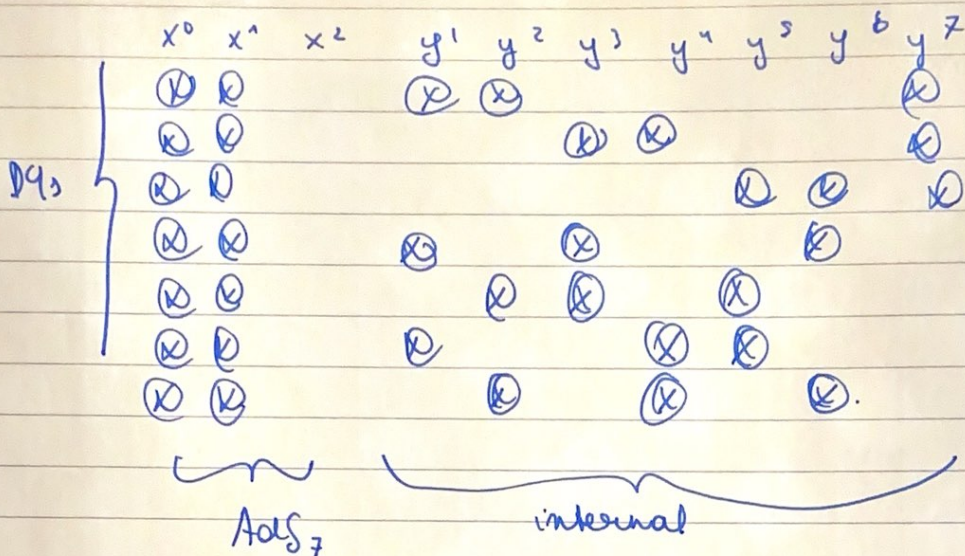
- compactification of massive IIA on a G<sub>2</sub> holonomy manifold
- $F_4 \sim N$  unbounded flux
- Bounded  $H_3, F_0$
- Ob-planes.
- $N=1$  AdS<sub>3</sub>.

Despite very similar ingredients as the DGKT vacuum, holographic properties are different.

- there are no  $\mathbb{Z}_N$  symmetries for domain walls [there are, as well as the orbifolds, projected out by the orientifold].
- the entire light spectrum consists of irrational conformal dimensions



Then the  $F_4$ -flux into domain walls:



$$\begin{aligned} \text{then } ds_{int}^2 &= \alpha' (N^{7/2} ds_{AdS_7}^2 + N^{1/2} ds_{int}^2) \\ &= l_{p,3}^2 (N^{1/4} ds_{int}^2 + N^{1/4} ds_{int}^2) \end{aligned}$$

$\Rightarrow c N^7$  which from the flux potential  $c N^4$ .  
 This is the only known large  $N$ -example where the large- $N$ -scalings from the dual domain walls don't match with the fluxes.

$$[ \text{Geometry is } ds_{10}^2 = dr^2 + r^{-4/3} dx_1 dx_1 + r^{2/3} ds_7^2 ]$$

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extra Scale separation in massive IIA

[Gubbioti, Junghans, Van Hengel, Van Riet, Wrase 21]

- T-dualize  $\times 2$  to get a new IIA solution on Iwasawa manifold

$$F_4 \rightarrow \begin{matrix} F_0 \\ F_2 \end{matrix} \quad \} \text{ unbounded}$$

$$F_0 \rightarrow F_2$$

$$H_3 \rightarrow \text{curvature}$$

- in certain anisotropic scaling limit: parametric control and scale separation

$$x^0 \ x^1 \ x^2 \ r \quad y_1 \ y_2 \ y_3 \ y_4 \ y_5 \ y_6$$

$$D2 \quad \otimes \quad \otimes \quad \otimes$$

$$D6 \quad \otimes \quad \otimes \quad \otimes \quad \otimes \quad \otimes \quad \otimes \quad \otimes$$

$$D6 \quad \otimes \quad \otimes \quad \otimes \quad \otimes \quad \otimes \quad \otimes \quad \otimes \quad \otimes$$

- Brane dual gives the correct scalings.
- Singularity probed by D2 and D6:

$$ds_{10}^2 = dr^2 + r^{-10} dx_n dx^n + r^{\frac{2}{3}} dy_{1,2}^2 + r^{\frac{2}{3}} dy_{3,4,5,6}^2$$

$$e^{\Phi} \sim r^{-5/3}$$

- uplift to M-theory is conical (pure geometry)

$$ds_{11}^2 = dr^2 + dx_n dx^n + r^2 ds_7^2$$

There is some unbounded flux absorbed into geometry however.

- there are integer conformal dimensions and the  $F_2$ -fluxes generate discrete symmetries for domain walls.