# Hadronic EDMs from Dyson-Schwinger: Rho-Meson & Nucleon

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# Introduction

- Introduction
- Part A: *The Theoretical Framework*
- Part B: *The $\rho$ Meson*
- Part C: *The Nucleon*

# Introduction: The Energy Scale & Effective EDM Operators for dim $\geq 4$ at scale $\sim 1$ GeV

**Calculation of hadronic EDMs naturally splits into 2 parts**

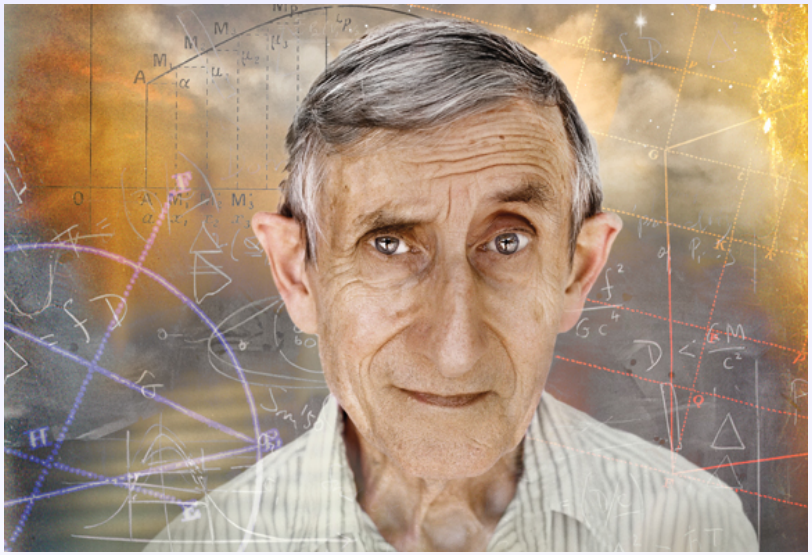
1. Calculation of Wilson coefficients
   by integrating out short distances
2. Switching from perturbative quark-gluon description to non-perturbative treatment
   – *(much harder and larger uncertainties)*

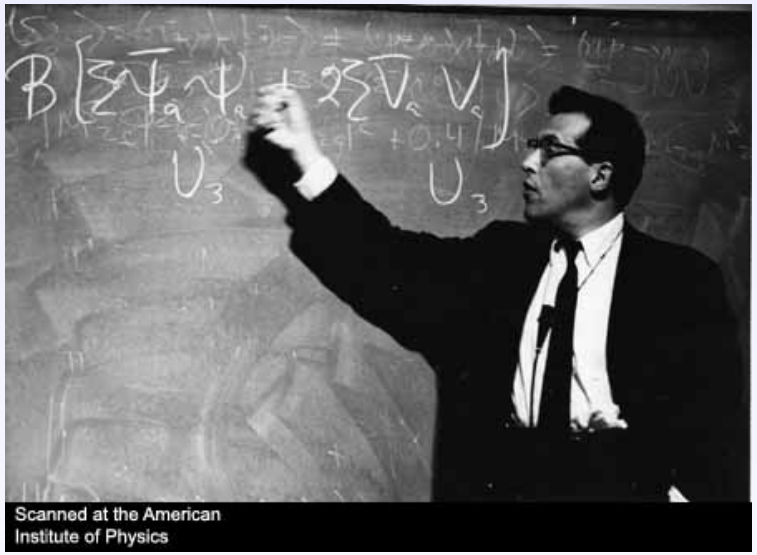
**Effective EDM Operators for dim $\geq 4$ at scale $\sim 1$ GeV**

$$\mathcal{L}_M^{1GeV} = -i\bar{\Theta}\frac{g_s^2}{32\pi^2}\, G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$$
$$-\frac{i}{2}\sum_{q=u,d} d_q\, \bar{q}\sigma_{\mu\nu}\gamma_5 q\, F_{\mu\nu} - \frac{i}{2}\, g_s \sum_{q=u,d} \tilde{d}_q\, \bar{q}\,\frac{1}{2}\lambda^a\sigma_{\mu\nu}\gamma_5 q\, G^a_{\mu\nu}$$
$$+\, i\frac{\mathcal{K}}{\Lambda^2}\,\varepsilon_{ij}\,(\bar{Q}_i d)(\bar{Q}_j\gamma_5 u) + \cdots$$

# Part A:
# The Theoretical Framework

# 1. Dyson-Schwinger Equations





# Dyson-Schwinger Equation

- *Non-perturbative continuum* approach to any QFT
- A shift in the integration variable ($\varphi(x) \to \varphi(x) + \lambda(x)$), does not change the path integral for suitable b.c., i.e.

$$\int D[\varphi] \frac{\delta}{\delta\varphi} f[\varphi] = 0$$

- Application to the *generating functional* $Z[J]$ yields

$$\int D[\varphi] \left[ -\frac{\delta S}{\delta\varphi} + J \right] e^{-S + \int d^4x\, J\varphi} = 0$$

with the action $S = \int d^4x\, \mathcal{L}$. This can be rewritten as

$$\left[ -\frac{\delta S}{\delta\varphi}\left(\frac{\delta}{\delta J}\right) + J \right] Z[J] = 0$$

# Dyson-Schwinger Equation

In QCD the *fermion propagator* is obtained by derivation of

$$\left[-\frac{\delta S}{\delta\bar{\psi}(x)}\left(\frac{\delta}{\delta\bar{\eta}},-\frac{\delta}{\delta\eta},\frac{\delta}{\delta J_\mu}\right)+\eta(x)\right]Z[\eta,\bar{\eta},J]=0$$

with respect to $\eta$ leading after several formal manipulations to the

**Gap Equation** for the *quark propagator*

$$S_F(p)^{-1}=i\not{p}\,Z_2+m_q(\mu)\,Z_4 + Z_1\int\frac{d^4q}{(2\pi)^4}\,g^2D_{\mu\nu}(k-p)\gamma_\mu\frac{\lambda^i}{2}S_F(k)\Gamma_\nu(k,p)$$

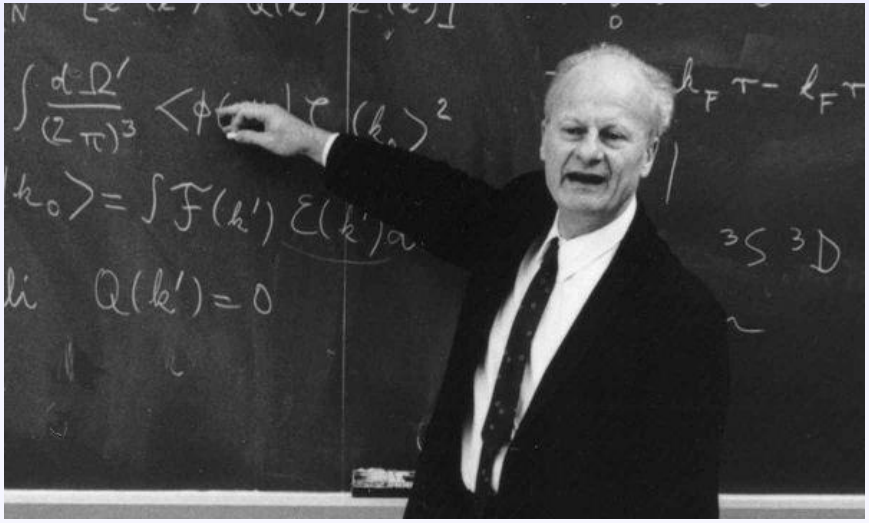
# Dyson-Schwinger Equation

1. **Gap equation** contains the *full vertex* $\Gamma_\mu$ and *full gluon propagator* $D_{\mu\nu}(k-p)$, each satisfies it's own DSE
2. DSE for the *full vertex* $\Gamma_\mu$ contains the *four-point vertex*, which has it's own DSE...

$\Longrightarrow$ DSE is an *infinite tower of equations relating all correlation functions*

- DSE are **exact relations** and are the *quantum Euler-Lagrange equations* for *any QFT*
- *Perturbative Expansion* yields *standard perturbative QFT*

# 2. Bethe-Salpeter Equations



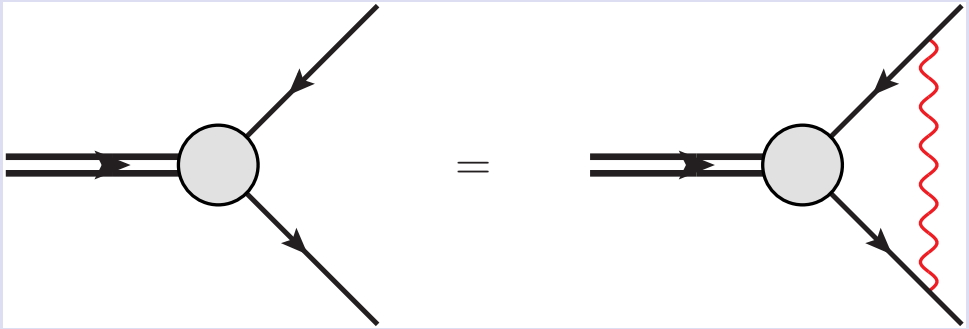
# Bethe-Salpeter Equations

*Bethe-Salpeter equation* is the DSE describing a *bound 2 body system*

Obtained by *four derivatives of the generating functional* and *several formal manipulations*

$$\Gamma(k;P) = \int \frac{d^4q}{(2\pi)^4} K(q,k;P) S_F\left(q+\frac{P}{2}\right)\Gamma(q;P) S_F\left(q-\frac{P}{2}\right)$$

Solutions for discrete set $P^2$ yield *mass spectra*



# Rainbow-Ladder Truncation

A *symmetry-preserving truncation* of the *infinite set of DSEs* which respects *relevant (global) symmetries of QCD* is the

*rainbow-ladder truncation* in combination with the *impulse approximation*

**1. In *BSE kernel***

$$K(p,p';k,k') \to -\mathcal{G}\ell(q^2)D^{\text{free}}_{\mu\nu}(q)\frac{\lambda^a}{2}\gamma_\mu \otimes \frac{\lambda^a}{2}\gamma_\nu$$

**2. In *gap equation***

$$Z_1 g^2 D_{\mu\nu}(q)\Gamma^a_\nu(k,p) \to \mathcal{G}\ell(q^2)D^{\text{free}}_{\mu\nu}(q)\frac{\lambda^a}{2}\gamma_\nu$$

R-LT is *first term* in *systematic expansion of $q\bar{q}$ scattering kernel $K(p,p';k,k')$*

# Gluon Propagator

- DSE and *unquenched QCD lattice* studies show that the

**Full gluon propagator**

$$D^{ab}_{\mu\nu}(p) = \delta^{ab}\frac{\mathcal{G}\ell(p^2)}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

is *IR finite*, i.e.

$$\lim_{p^2\to 0} D^{ab}_{\mu\nu}(p) = \textit{finite}$$

- the *gluon* has *dynamically generated mass* in the *IR*

- *EM Observables* in the static limit ($q_\mu \to 0$) probe *gluon propagator* for *small transversed momenta* $\Longrightarrow$

**Point-like vector $\otimes$ vector contact interaction**

$$g^2 D^{ab}_{\mu\nu}(p) = \delta^{ab}\delta_{\mu\nu}\frac{4\pi\alpha_{\mathrm{IR}}}{m_G^2}$$

# Contact Interaction Model

## This implies

- Non-renormalizable theory
- Introduce *proper-time regularization*

1. $\Lambda_{uv} = 1/\tau_{uv}$ *cannot be removed* but plays a dynamical role and sets the scale of all dimensioned quantities
2. $\Lambda_{ir} = 1/\tau_{ir}$ implements *confinement* by ensuring the *absence of quark production tresholds*

- Scale $m_G$, is set in agreement with *observables*
- In the *static limit* $q^2 \to 0$ results *"indistinguishable"* from any other *however sophisticated* DSE approach
- For $q^2 \gtrsim M^2_{\text{dressed}}$ *deviations* are expected from other experimental values

# Part B:

# [1] The $\rho$ Meson

[1] M. P., C. Y. Seng, M. J. Ramsey-Musolf, C. D. Roberts, S. M. Schmidt and D. J. Wilson, Phys. Rev. C **87** (2013) 015205

# The $\rho$ Meson

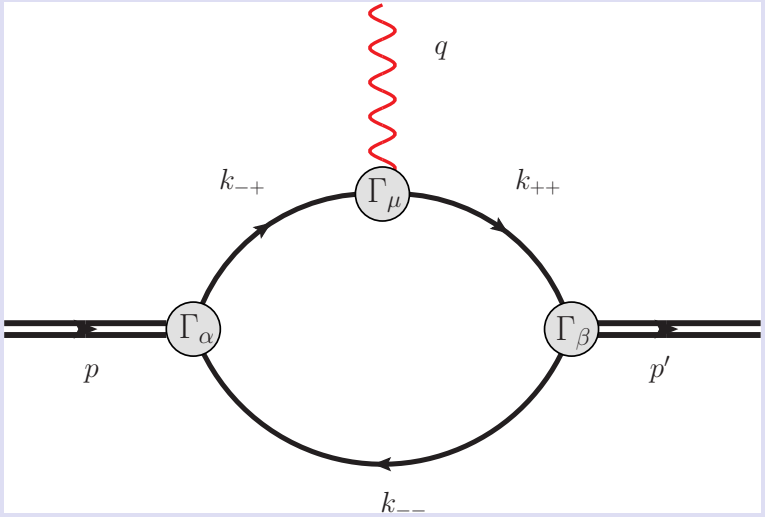
- *"Per se"* from an experimental point of view *uninteresting*
- *Short lifetime* ($\sim 10^{-24}\,s$) makes EDM measurements *hard (or rather impossible)*
- *Simplest system* possibly providing EDM and hence *perfect prototype particle*
- Results available in *QCD sum rules* and *other techniques*

## Profile

1. $I^G(J^{PC}) = 1^+(1^{--})$
2. $m = 775.49 \pm 0.34$ MeV,
   $\Gamma = 149.1 \pm 0.8$ MeV
3. Primary decay mode ($\sim 100\%$): $\rho \to \pi\pi$

# The $\rho$-Meson in Impulse Approximation

## Impulse Approximation




$$\Gamma^{(u)}_{\alpha\mu\beta} \propto \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}_{CD}\left\{\Gamma^{\rho(u)}_{\beta} S(k_{++}) \Gamma^{(u)}_{\mu} S(k_{-+}) \Gamma^{\rho(u)}_{\alpha} S(k_{--})\right\}$$

*EDM sources* induce *CP* violating corrections to the

1. $q\gamma q$ *vertex*
2. *Bethe-Salpeter amplitude*
3. *Propagator*

# The Magnetic Moment

## Results for $\mathcal{M}(0)$ in units $e/(2m_\rho)$

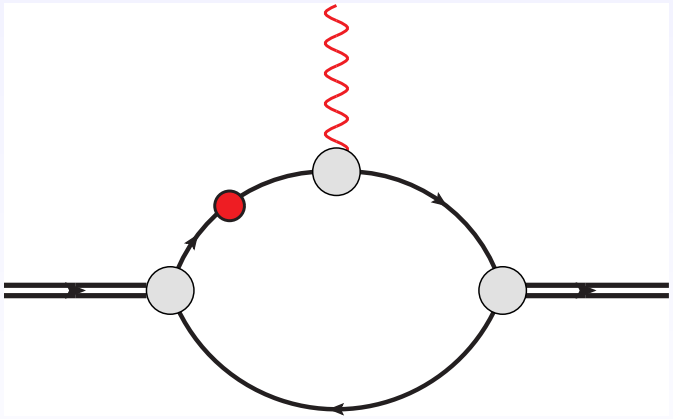
| | |
|---|---|
| DSE - *CIM* | $2.11$ |
| DSE - *RL RGI-improved* | $2.01$ |
| DSE - *EF parametrisation* | $2.69$ |
| LF - *CQM* | $2.14$ |
| LF - *CQM* | $1.92$ |
| QCD sum rules | $1.8 \pm 0.3$ |
| point particle | $2$ |

# The $\Theta$-Term

- *Only* $CP$ violating dimension 4 operator

$$\mathcal{L}_{\text{eff}} = -i\bar{\Theta}\frac{g_s^2}{32\pi^2}\, G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$$

- *No suppression by heavy scale* (strong $CP$ problem)
- $U(1)_A$ anomaly allows to rotate it into complex mass for evaluation *(effective propagator correction)*



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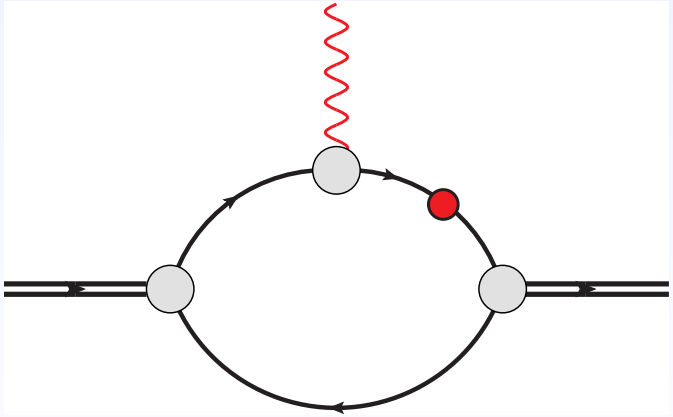

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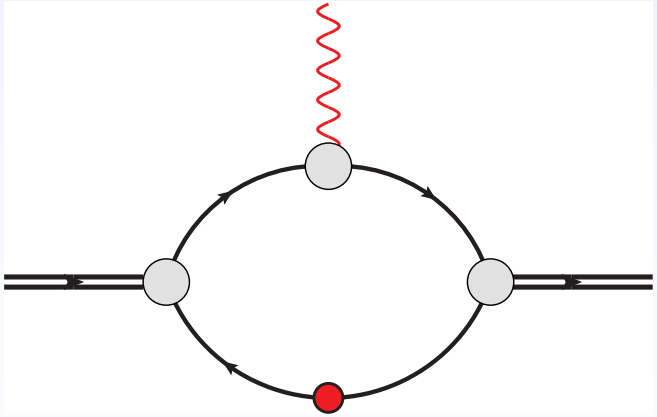

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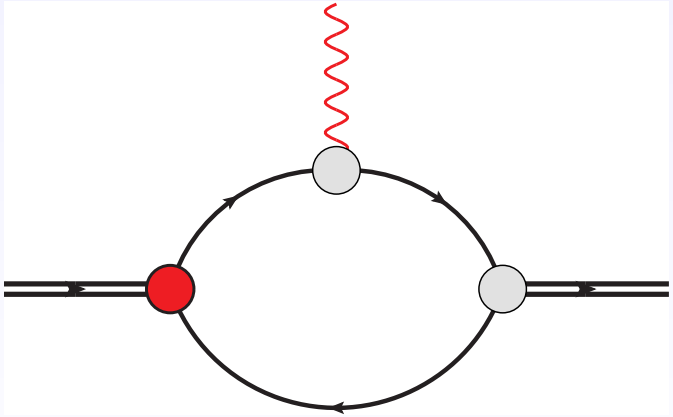

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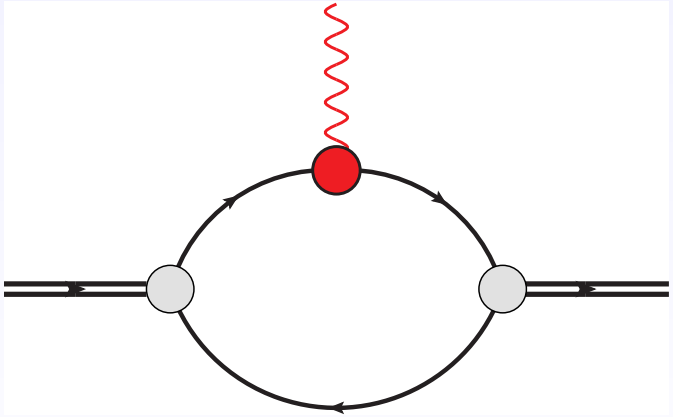

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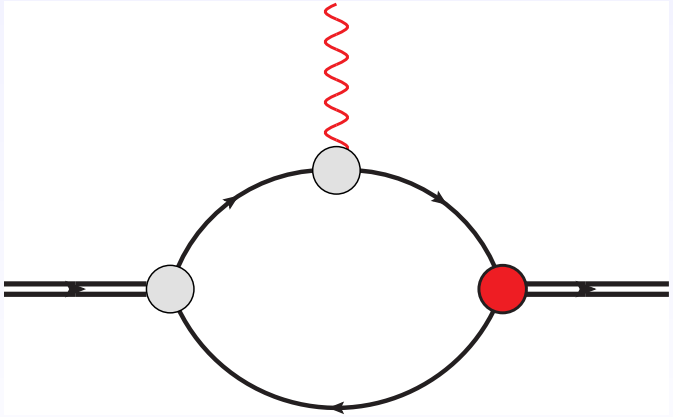

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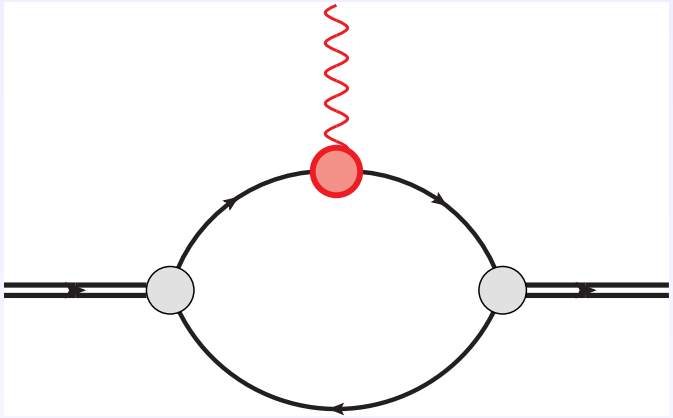
# The Quark-EDM

- The *intrinsic EDM* of a *quark* itself

$$\mathcal{L}_{\text{eff}} = -\frac{i}{2} \sum_{q=u,d} d_q \, \bar{q} \sigma_{\mu\nu} \gamma_5 q \, F_{\mu\nu}$$

- *Effective $q\gamma q$ vertex correction*



| | |
|---|---|
| DSE - *CIM* | $0.79\,(d_u - d_d)$ |
| DSE | $0.72\,(d_u - d_d)$ |
| Bag Model | $0.83\,(d_u - d_d)$ |
| QCD sum rules | $0.51\,(d_u - d_d)$ |
| Non-relativistic quark model | $1.00\,(d_u - d_d)$ |

# The Chromo-EDM

- The *Intrinsic Chromo-EDM* of a *quark* itself

$$\mathcal{L}_{\text{eff}} = -\frac{i}{2}\, g_s \sum_{q=u,d} \tilde{d}_q\, \bar{q}\, \frac{1}{2}\lambda^a \sigma_{\mu\nu}\gamma_5 q\, G^a_{\mu\nu}$$

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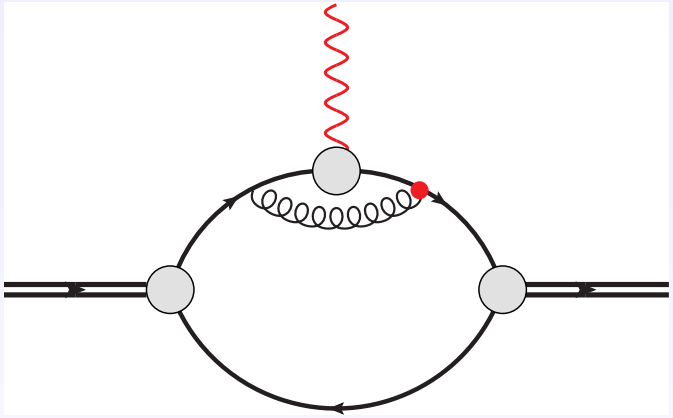

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| DSE - $q\gamma q$ | $-0.07\, e\tilde{d}_- - 0.20\, e\tilde{d}_+$ |
| DSE - *BSA* | $-0.12\, e\tilde{d}_- + 0.11\, e\tilde{d}_+$ |
| DSE - *Propagator* | $1.35\, e\tilde{d}_- - 0.60\, e\tilde{d}_+$ |
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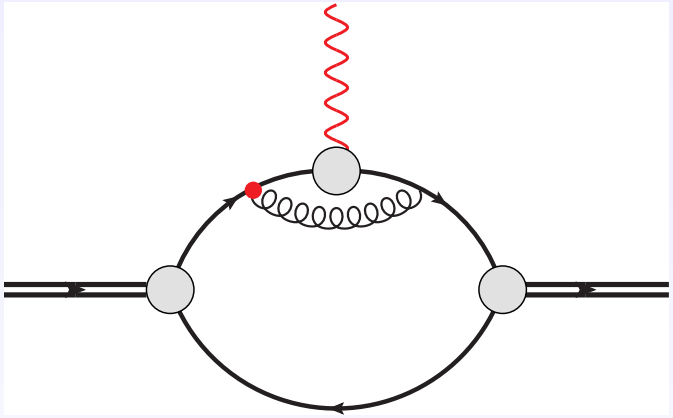

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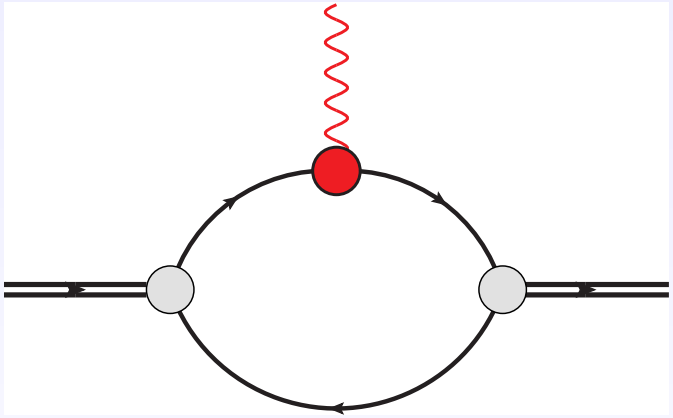

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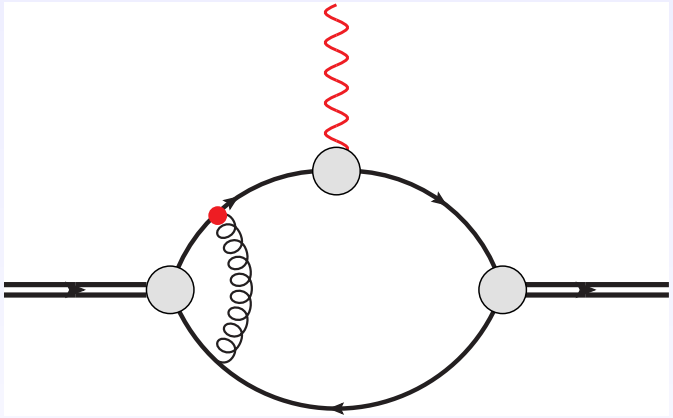

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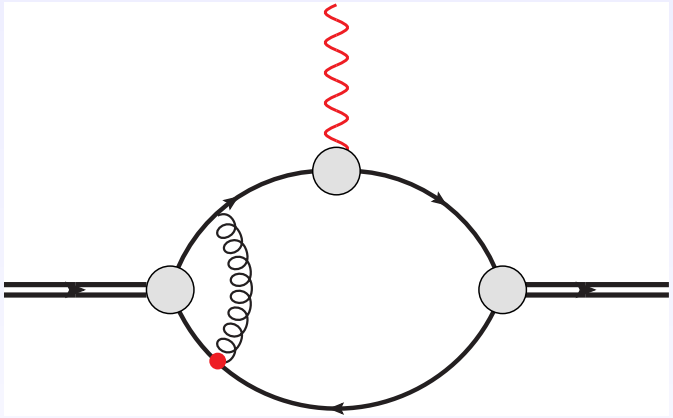

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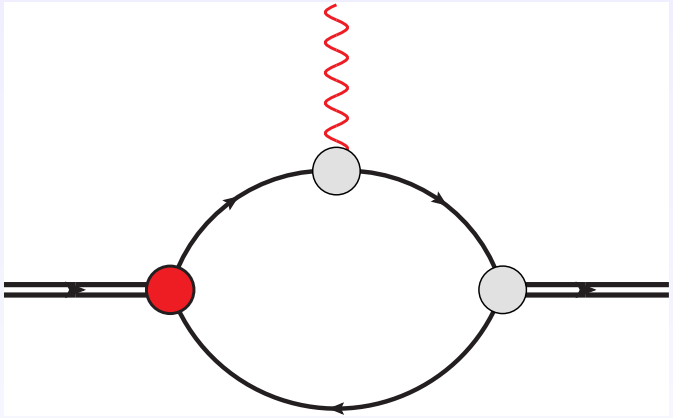

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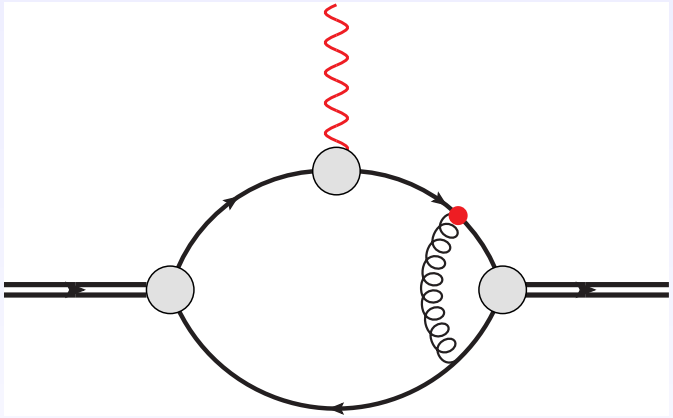

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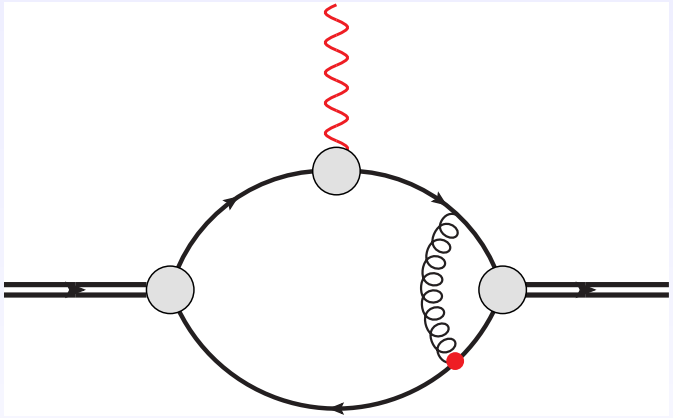

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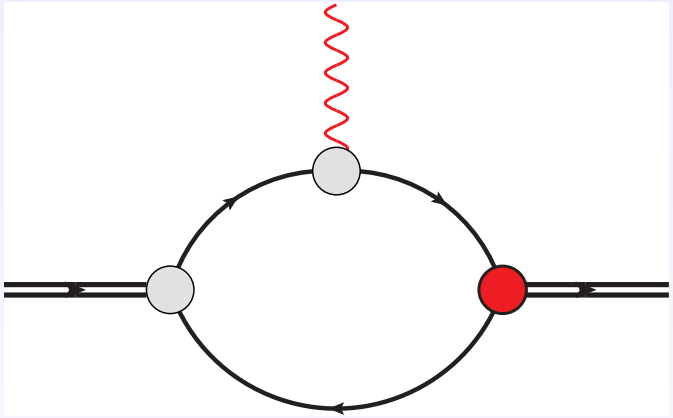

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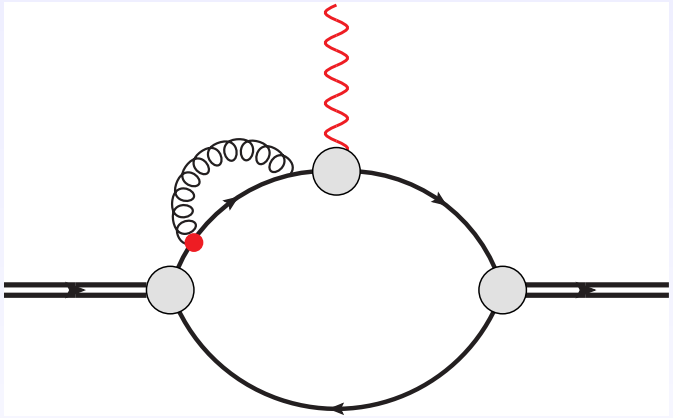

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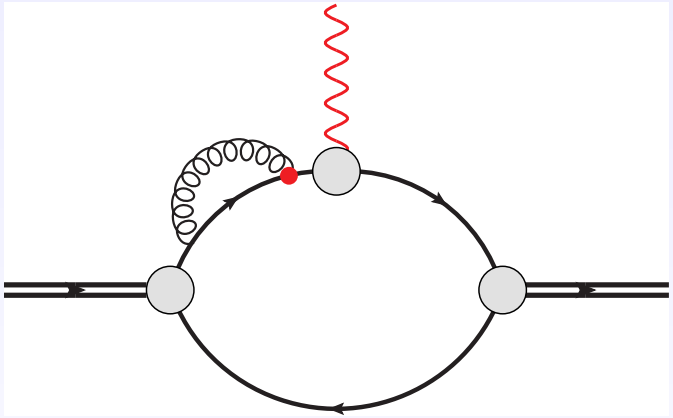

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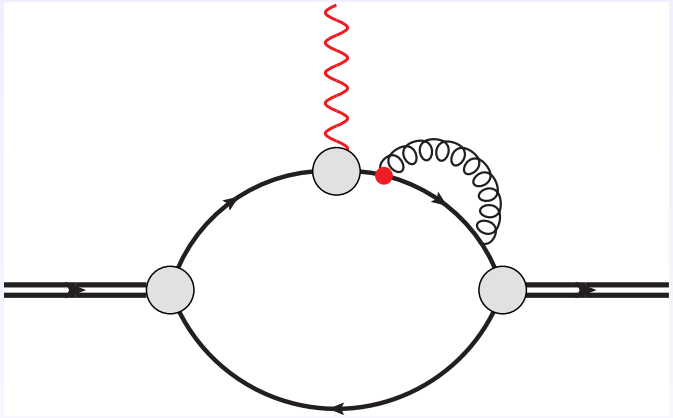

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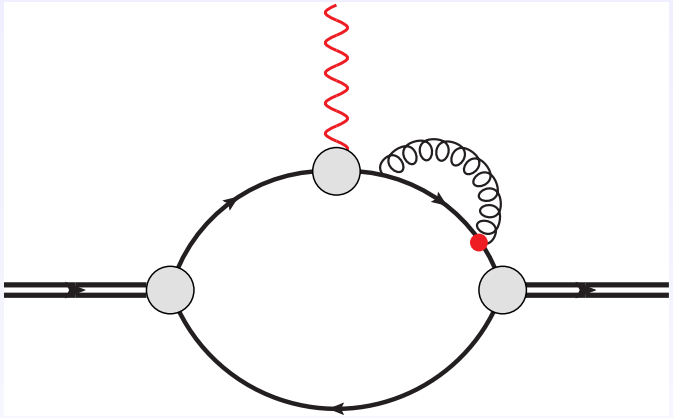

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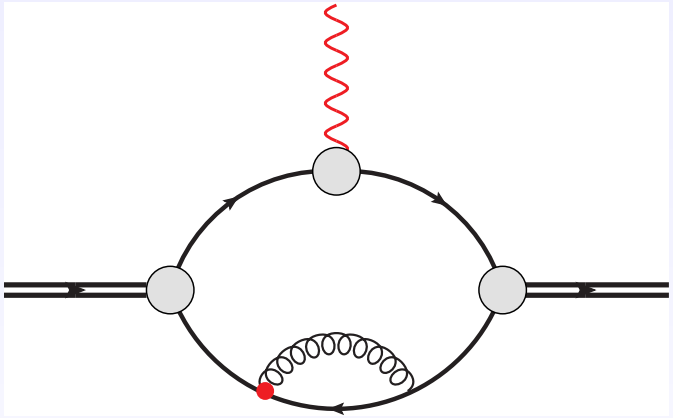

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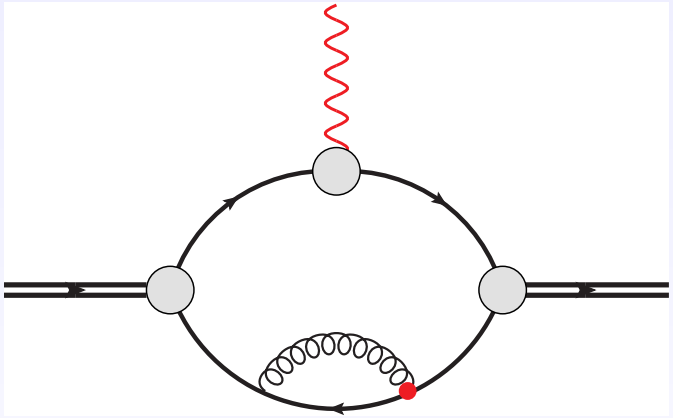

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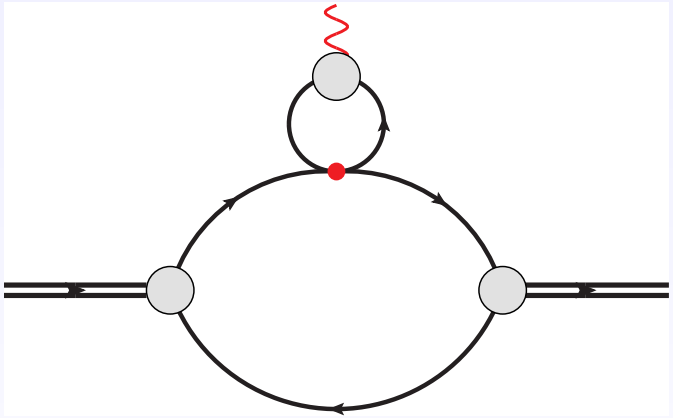

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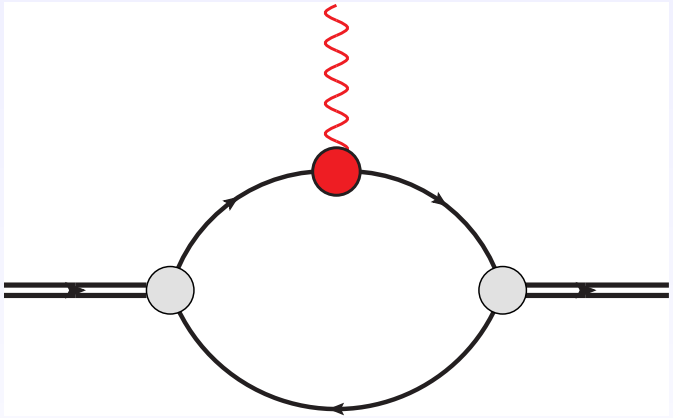

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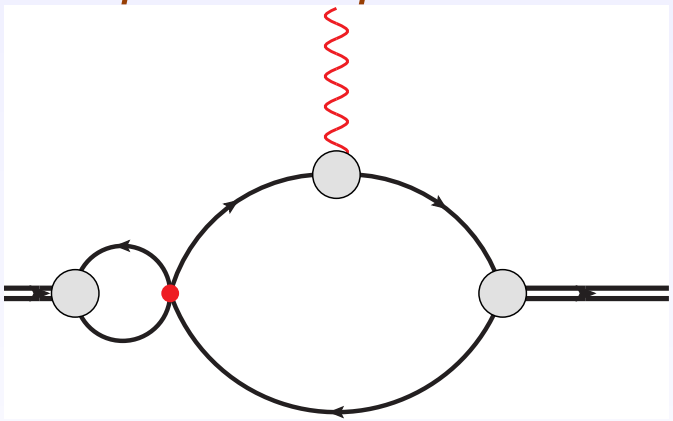

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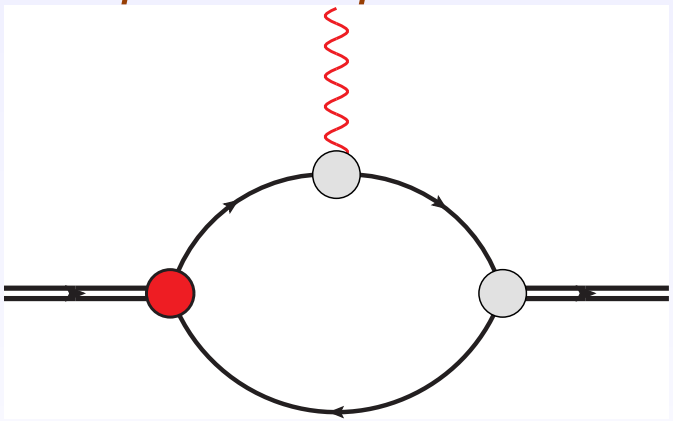

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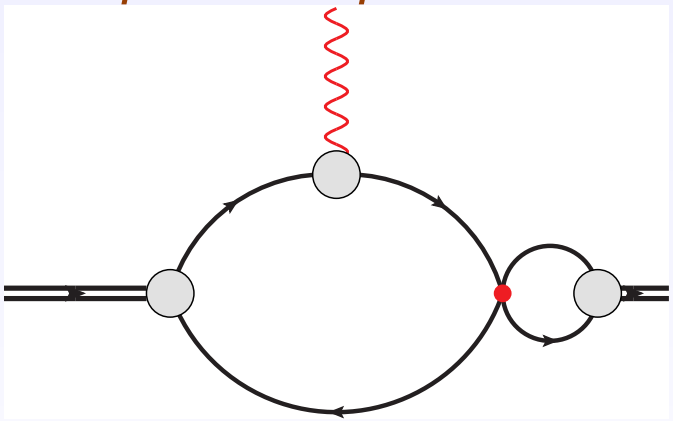

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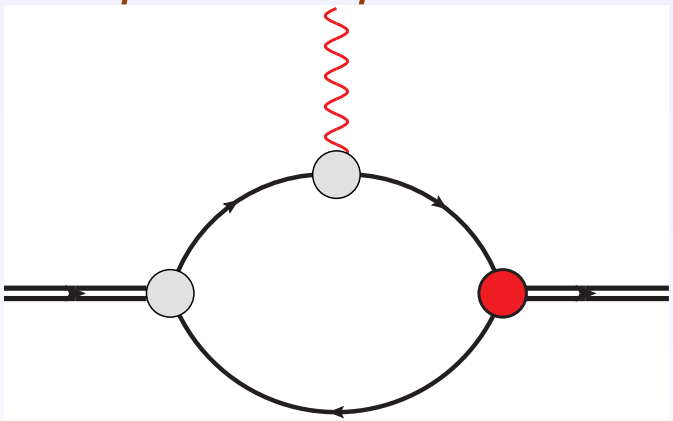

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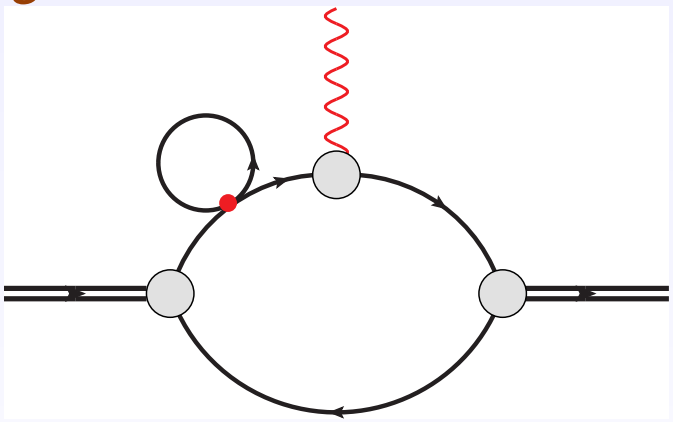

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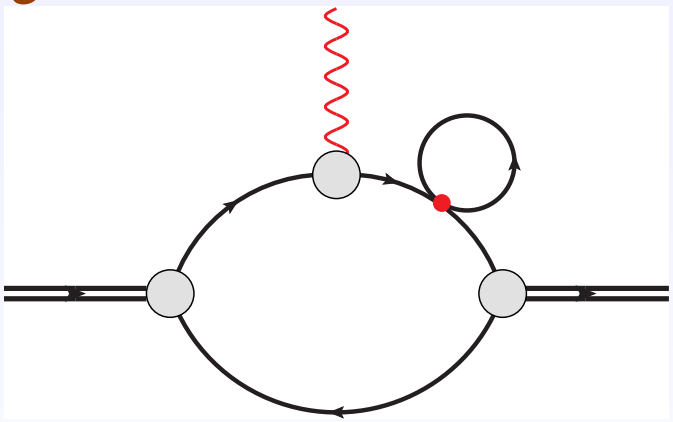

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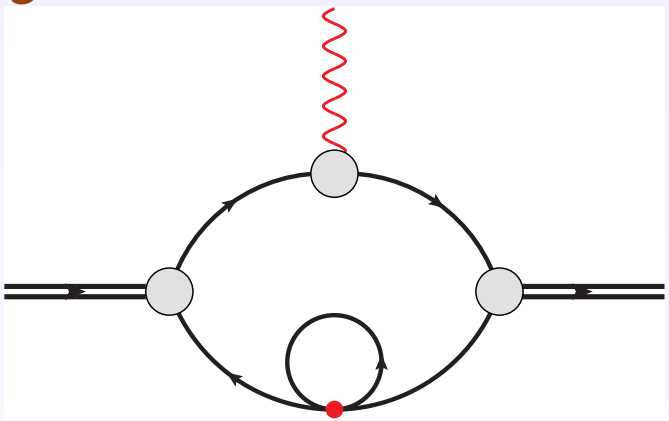

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# Part C:

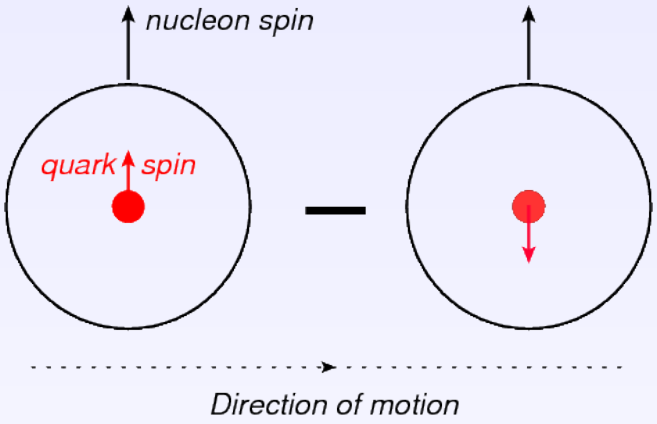
# [2]The Nucleon

[2] M. P., C. Y. Seng, C. D. Roberts and S. M. Schmidt, arXiv:1411.2052 [nucl-th].

# Introduction

## Nucleon's *tensor charge*

$$\langle P(p,\sigma)|\bar{q}\sigma_{\mu\nu}q|P(p,\sigma)\rangle = \delta_T q\, \bar{u}(p,\sigma)\sigma_{\mu\nu}u(p,\sigma) \qquad (q = u, d, \ldots)$$




$$\delta_T q = \int_{-1}^{1} dx\, h_{1T}^{q}(x) = \int_{0}^{1} dx\, [h_{1T}^{q}(x) - h_{1T}^{\bar{q}}(x)]$$

$h_{1T} \ldots$ *transversity distribution*

measures the *light-front number-density* of quarks with transverse polarisation *parallel* to that of the proton minus that of quarks with *antiparallel polarisation*

# Introduction

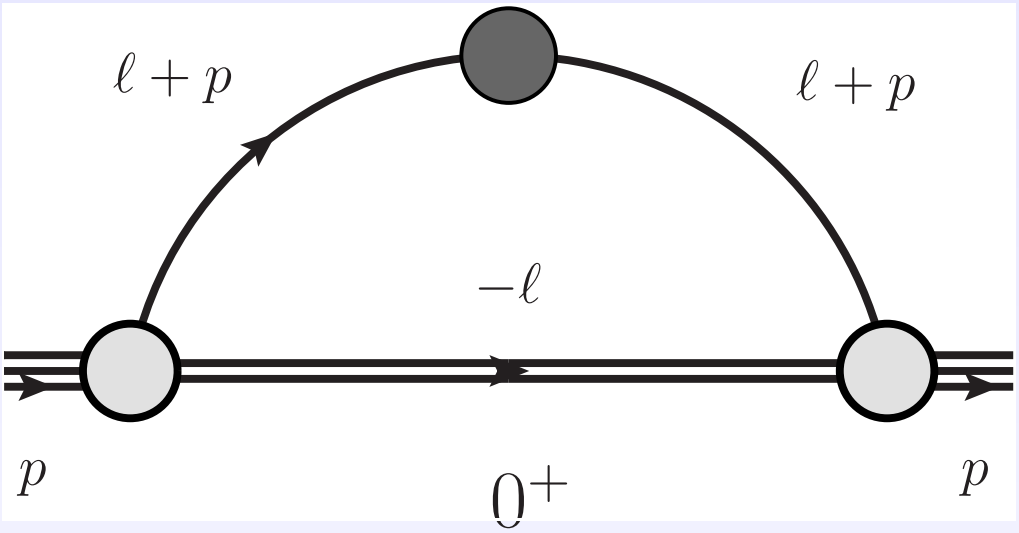
## Relation *tensor charge* to EDM

$$
\begin{aligned}
d_p\, \bar{u}(p,\sigma)\sigma_{\mu\nu}\gamma_5 u(p,\sigma) &= \sum_{q=u,d} d_q \,\langle P(p,\sigma)|\bar{q}\sigma_{\mu\nu}\gamma_5 q|P(p,\sigma)\rangle \\
&= \frac{1}{2}\,\varepsilon_{\mu\nu\alpha\beta} \sum_{q=u,d} d_q \,\langle P(p,\sigma)|\bar{q}\sigma_{\alpha\beta} q|P(p,\sigma)\rangle \\
&= \frac{1}{2}\,\varepsilon_{\mu\nu\alpha\beta}\, \bar{u}(p,\sigma)\sigma_{\alpha\beta} u(p,\sigma) \sum_{q=u,d} d_q\, \delta_T q \\
&= \bar{u}(p,\sigma)\sigma_{\mu\nu}\gamma_5 u(p,\sigma) \sum_{q=u,d} d_q\, \delta_T q
\end{aligned}
$$

Proton EDM: $d_p = d_u\, \delta_T u + d_d\, \delta_T d$

Neutron EDM: $d_n = d_u\, \delta_T d + d_d\, \delta_T u$

# The Quark-EDM



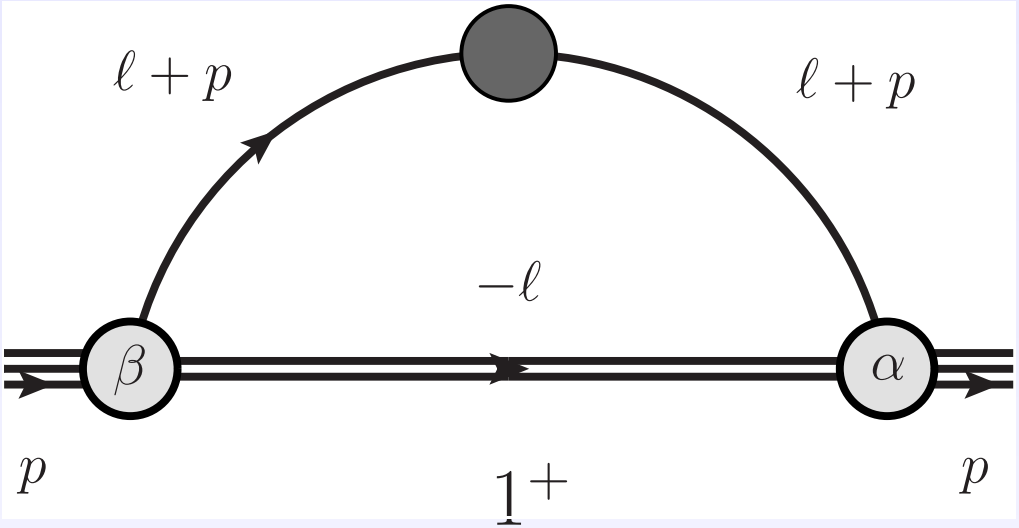

$$\Lambda^+(p)\mathcal{S}(-p)\int \frac{d^4\ell}{(2\pi)^4}\, S^{(u)}(\ell+p)\sigma_{\mu\nu}S^{(u)}(\ell+p)\Delta^{0^+}(-\ell)\mathcal{S}(p)\Lambda^+(p)$$

$$= \mathcal{N}\,\delta_T d\,\Lambda^+(p)\sigma_{\mu\nu}\Lambda^+(p)$$

$$\mathcal{S}(p) = s(p)\,\mathbf{1}_D \qquad \big(s(p) = 0.8810\big)$$

$$\Lambda^+(p) = \frac{1}{2m_N}\left(-i\gamma\cdot p + m_N\right)$$

# The Quark-EDM



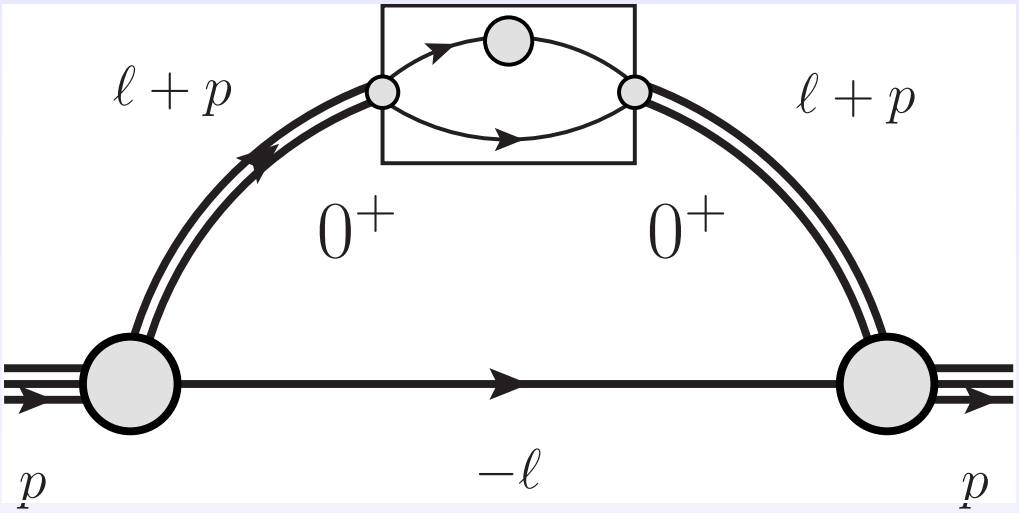

$$\Lambda^+(p)\mathcal{A}^i_\alpha(-p)\int\frac{d^4\ell}{(2\pi)^4}\,S^{(q)}(\ell+p)\sigma_{\mu\nu}S^{(q)}(\ell+p)\Delta^{1^+}_{\alpha\beta}(-\ell)\mathcal{A}^i_\beta(p)\Lambda^+(p)$$

$$= \mathcal{N}\,\delta_T q\,\Lambda^+(p)\sigma_{\mu\nu}\Lambda^+(p)$$

$$\mathcal{A}^i_\mu(p) = a^i_1(p)\,\gamma_5\gamma_\mu + a^i_2(p)\,\gamma_5\hat{p}_\mu \qquad \left(\hat{p}^2 = -1, i = +, 0\right)$$

$$a^+_1 = -0.380,\ a^+_2 = -0.065,\ a^0_1 = 0.270,\ a^0_2 = 0.046$$

# The Quark-EDM

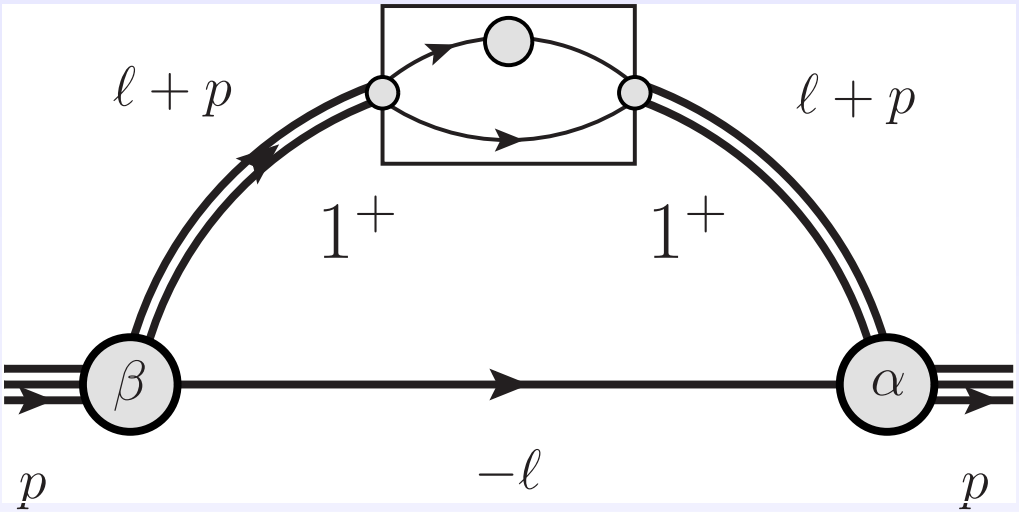



$$\Lambda^+(p)\mathcal{S}(-p)\int \frac{d^4\ell}{(2\pi)^4}\,\Delta^{0^+}_{\alpha\alpha'}(\ell+p)\Lambda_{\mu\nu}\Delta^{0^+}_{\beta'\beta}(\ell+p)S^{(q)}(-\ell)\mathcal{S}(p)\Lambda^+(p)$$

$$= 0$$

"A *spinless particle* cannot have a *vectorial/tensorial structure* of any kind!"

# The Quark-EDM



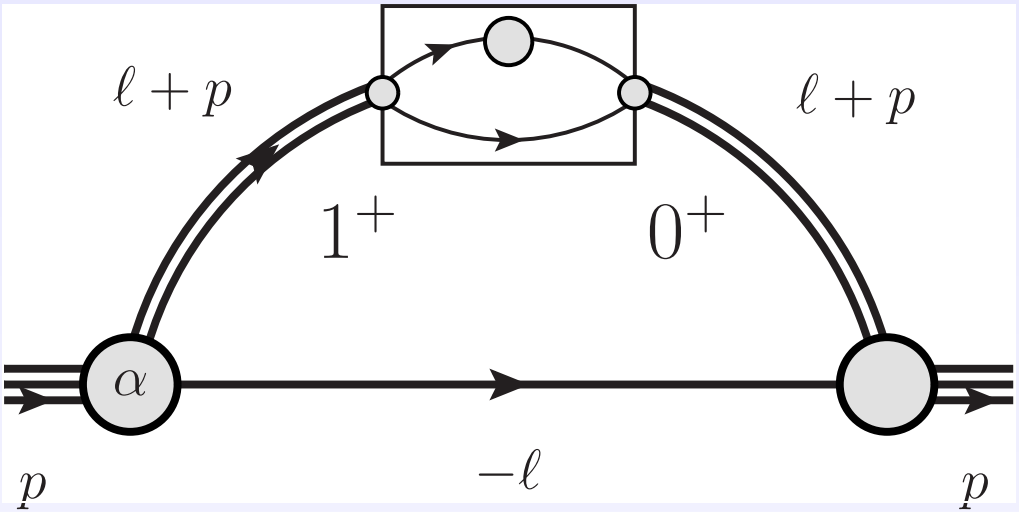

$$\Lambda^+ \mathcal{A}^i_\alpha(-p) \int \frac{d^4\ell}{(2\pi)^4} \Delta^{1^+}_{\alpha\alpha'}(\ell+p) \Lambda_{\alpha'\mu\nu\beta'} \Delta^{1^+}_{\beta'\beta}(\ell+p) S^{(q)}(-\ell) \mathcal{A}^i_\beta(p) \Lambda^+$$

$$= \mathcal{N}\, \delta_T q\, \Lambda^+(p) \sigma_{\mu\nu} \Lambda^+(p)$$

$$\mathcal{A}^i_\mu(p) = a^i_1(p)\, \gamma_5 \gamma_\mu + a^i_2(p)\, \gamma_5 \hat{p}_\mu \qquad \left(\hat{p}^2 = -1, i = +, 0\right)$$

$$a^+_1 = -0.380,\ a^+_2 = -0.065,\ a^0_1 = 0.270,\ a^0_2 = 0.046$$

# The Quark-EDM



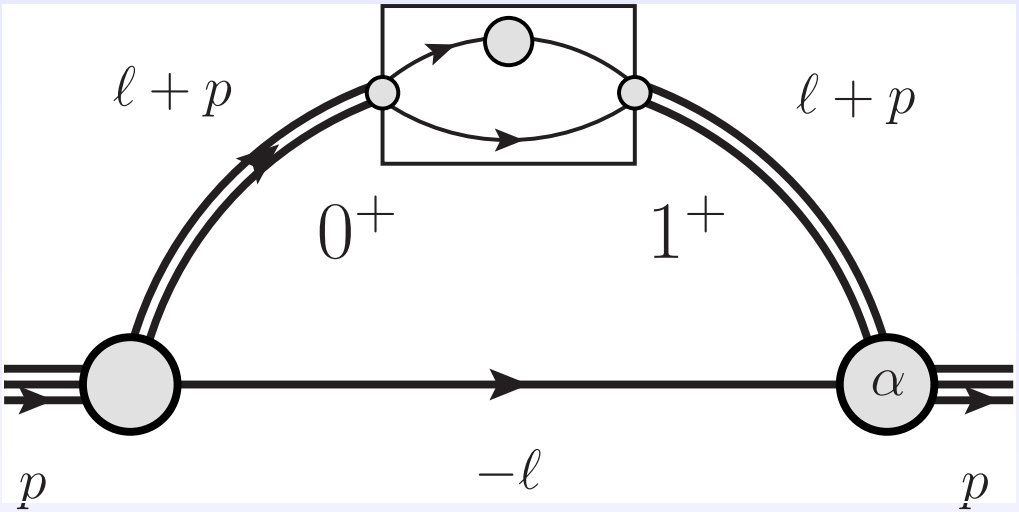

$$\Lambda^+(p)\mathcal{S}(-p)\int \frac{d^4\ell}{(2\pi)^4}\,\Delta^{0^+}(\ell+p)\Lambda_{\mu\nu\alpha}\Delta^{1^+}_{\alpha\beta}(\ell+p)S^{(u)}(-\ell)\mathcal{A}^0_\beta(p)\Lambda^+(p)$$

$$= \mathcal{N}\,\delta_T d\,\Lambda^+(p)\sigma_{\mu\nu}\Lambda^+(p)$$

$$\mathcal{S}(p) = s(p)\,\mathbf{1}_D \qquad \big(s(p) = 0.8810\big)$$

$$\mathcal{A}^0_\mu(p) = a^0_1(p)\,\gamma_5\gamma_\mu + a^0_2(p)\,\gamma_5\hat{p}_\mu \qquad \big(a^0_1 = 0.270,\ a^0_2 = 0.046\big)$$

# The Quark-EDM




$$\Lambda^+(p)\mathcal{A}^0_\alpha(-p)\int\frac{d^4\ell}{(2\pi)^4}\,\Delta^{1^+}_{\alpha\beta}(\ell+p)\Lambda_{\beta\mu\nu}\Delta^{0^+}(\ell+p)S^{(u)}(-\ell)\mathcal{S}(p)\Lambda^+(p)$$

$$= \mathcal{N}\,\delta_T d\,\Lambda^+(p)\sigma_{\mu\nu}\Lambda^+(p)$$

$$\mathcal{S}(p) = s(p)\,\mathbf{1}_D \qquad \big(s(p) = 0.8810\big)$$

$$\mathcal{A}^0_\mu(p) = a^0_1(p)\,\gamma_5\gamma_\mu + a^0_2(p)\,\gamma_5\hat{p}_\mu \qquad \big(a^0_1 = 0.270,\ a^0_2 = 0.046\big)$$

# The Quark-EDM

## The *intrinsic EDM* of a *quark* itself

$$\mathcal{L}_{\text{eff}} = -\frac{i}{2} \sum_{q=u,d} d_q \, \bar{q}\sigma_{\mu\nu}\gamma_5 q \, F_{\mu\nu}$$

| $\zeta_H \approx M$ | $\delta_T u$ | $\delta_T d$ |
|---|---|---|
| Diagram 1 | 0.581 | 0 |
| Diagram 2 | −0.018 | −0.036 |
| Diagram 3 | 0 | 0 |
| Diagram 4 | 0.292 | 0.059 |
| Diagram 5+6 | −0.164 | −0.164 |
| Total Result | 0.691 | −0.141 |

$$d_p|_{\zeta_H \approx M} = \quad 0.69(10)\, d_u - 0.14(2)\, d_d$$

$$d_n|_{\zeta_H \approx M} = -0.14(2)\, d_u + 0.69(10)\, d_d$$

# The Quark-EDM

## Significance of $\delta_T u$

$\alpha_{\text{IR}}$ *reduced* by $20\%$ leads to $\delta_T u$ *reduced* by $20\%$ with $\delta_T d$ practically *unchanged*

$\Longrightarrow$ $\delta_T u$ is a direct probe of *DCSB*

## Significance of $\delta_T d$

$\delta_T d$ is non-zero only due to *Axial-vector correlations*

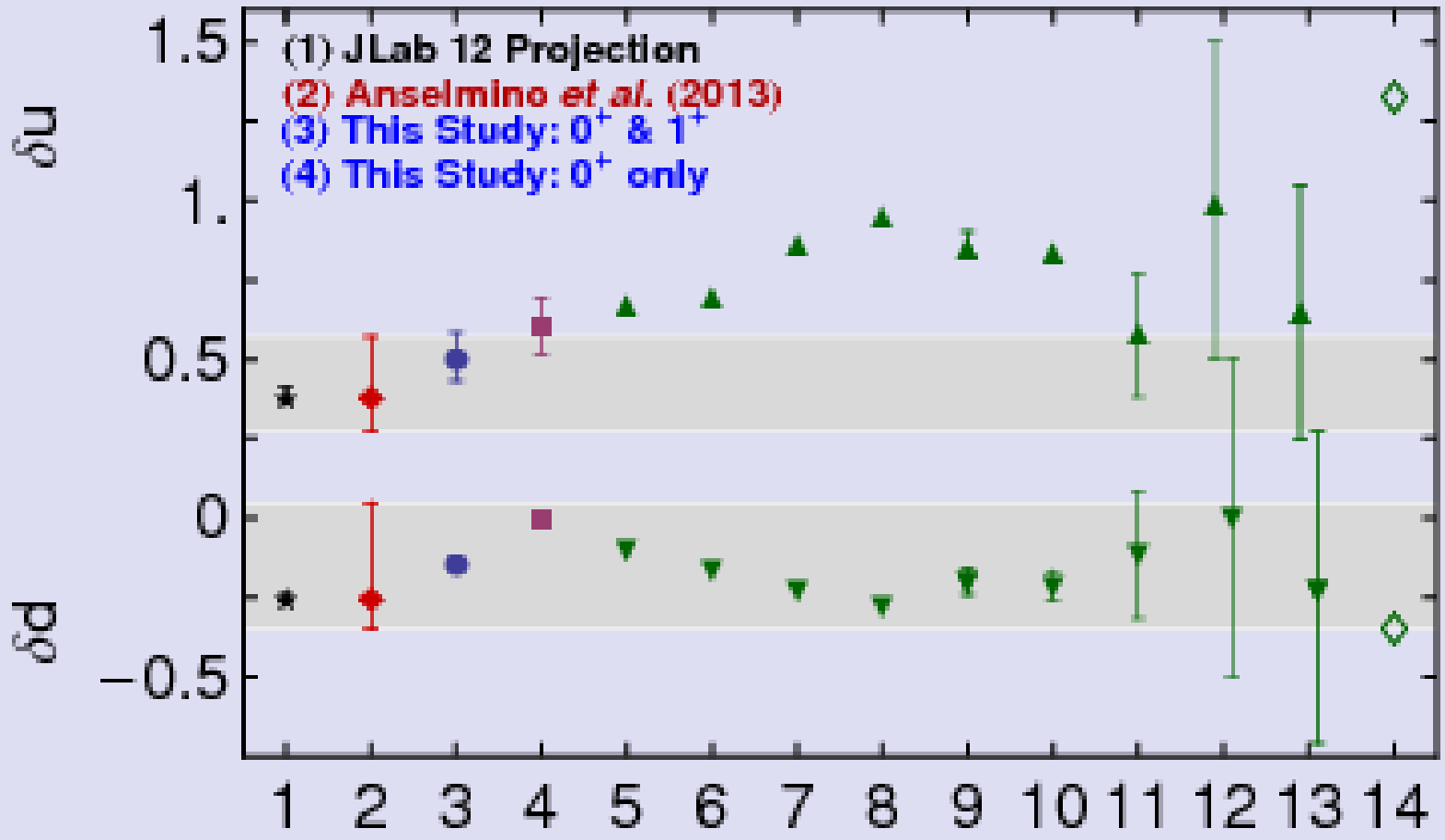
$\Longrightarrow$ $\delta_T d$ is a probe of *Axial-vector correlations*

$\delta_T u$ is increased by $11\%$ in the absence of *Axial-vector correlations*

$\Longrightarrow$ $\delta_T u$ is suppressed by *Axial-vector correlations*

# The Quark-EDM

## Flavour separation of the proton's tensor charge



# Conclusion

- *Continuum approach* to any QFT
- Originates at the *QCD current quark/gluon level*, i.e. all operators are "implemented" at that level
- "Rigid structure" – few model parameters
- Has been shown to work well in the *CP conserving sector*

# Collaboration

**The results, expounded in this talk, were obtained in Collaboration with**

- Craig D. Roberts – ANL
- Michael J. Ramsey-Musolf – UMass Amherst
- Chien-Yeah Seng – UMass Amherst

Thank You For Your Attention!