QCD(adj) on $R^3 \times S^1$: Confinement/Deconfinement Transitions

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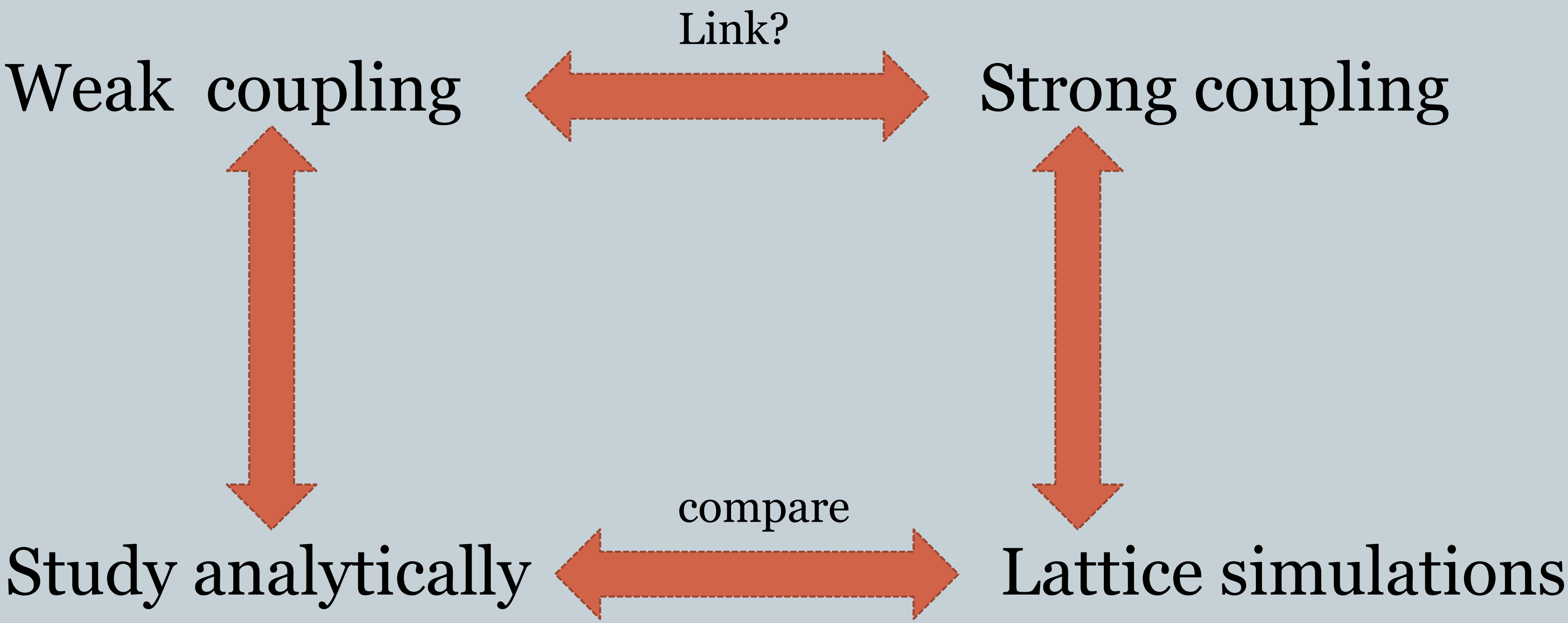
RECENT DEVELOPMENTS IN SEMICLASSICAL PROBES OF QUANTUM FIELD THEORIES

ACFI, UMASS AMHERST
MARCH 19, 2016

Preliminaries

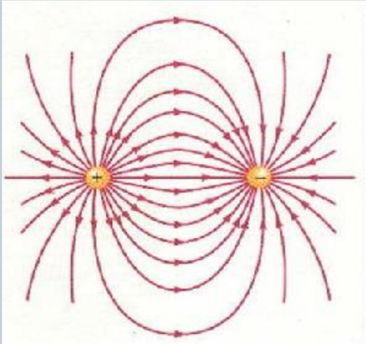
- Deconfinement transitions



Preliminaries

- Confinement is the mechanism for holding quarks inside nucleons: no isolated color charges

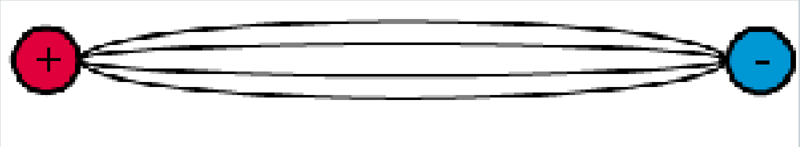
Charges in QED



Not confined

$$V = \frac{1}{R}$$

Quark-Antiquark



Confined

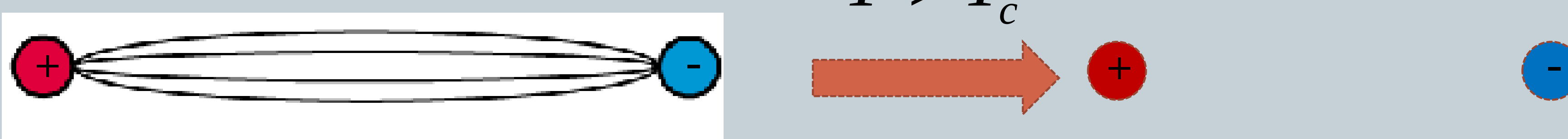
$$V = \sigma R$$

- This picture has been confirmed through extended lattice simulations

Preliminaries

- As we increase the temperature → deconfinement phase transition

$$T > T_c$$



- Quark-gluon plasma: a new state of matter
- We need an order parameter

Preliminaries

Watch out! Many circles!

Preliminaries

- Order parameter in pure YM: Polyakov loop

S

$$A_\mu(\vec{x},t) = A_\mu(\vec{x},t+\beta)$$

$$P_F = \mathrm{tr}_F[\Omega] = \mathrm{tr}_F\left[p e^{i\oint_S A_0 dx_0} \right]$$

Thermal circle: compact time

$$T = \frac{1}{\beta} = \frac{1}{\text{circumference}}$$

- P_F is gauge invariant

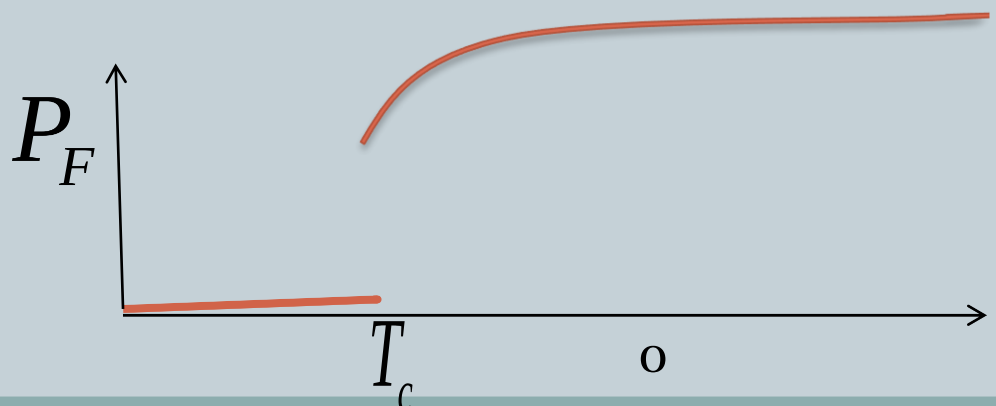
- P_F transforms under the center

$$P_F \to z P_F, \qquad Z_{SU(2)} \in \{ I_{2\times2}, -I_{2\times2} \}$$

Preliminaries

- Confined phase: the center is preserved $P_F = 0, \mathrm{T} < \mathrm{T_c}$
- Deconfined phase: the center is broken $P_F \neq 0, \mathrm{T} > \mathrm{T_c}$
- The physics is that $P_F \sim e^{-F/T}$

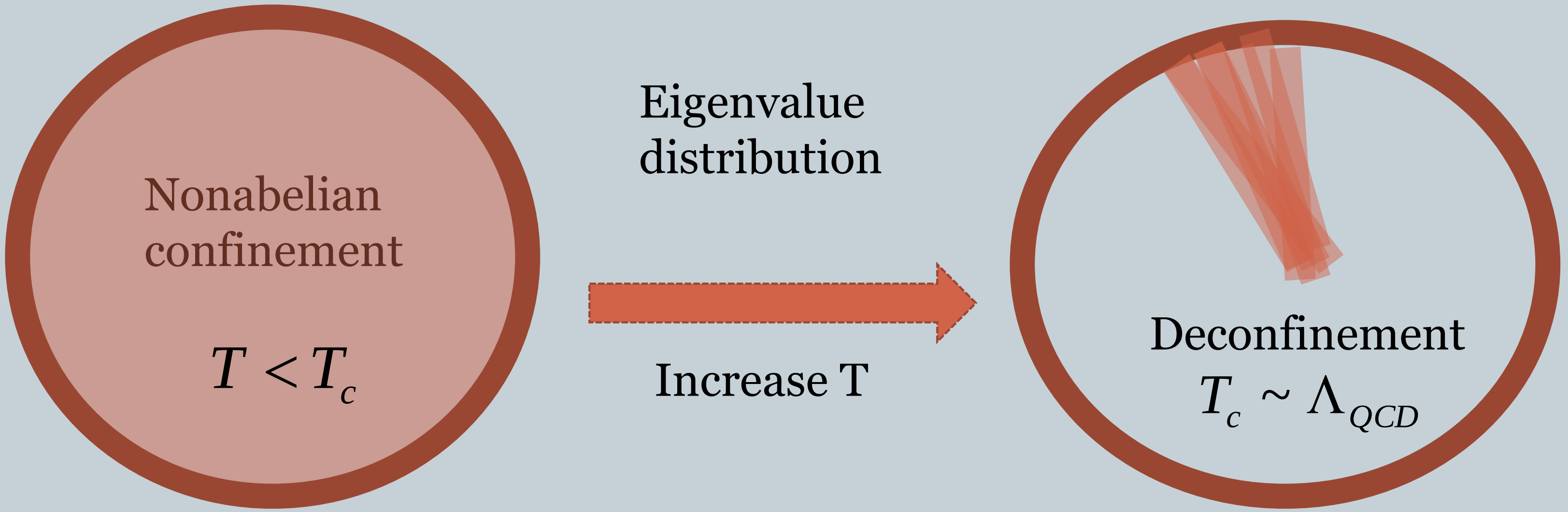
Free energy of quarks



Preliminaries

- Pure YM is very difficult since the transition happens at Λ_{QCD}

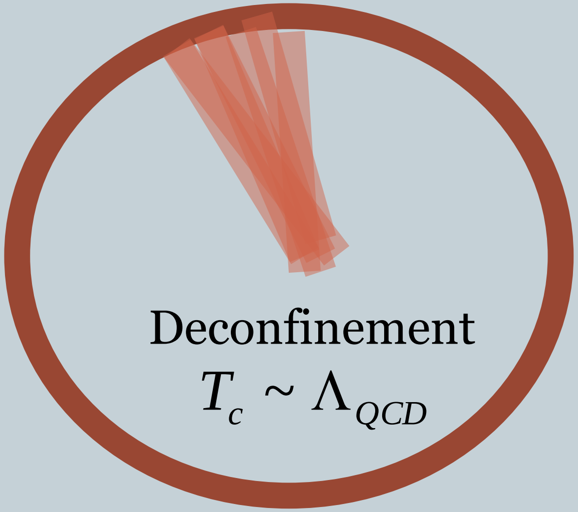
$$P_{\mathrm{F}} = \mathrm{tr}_{\mathrm{F}}[\Omega] = \mathrm{tr}_{\mathrm{F}}\left[\exp\left(i\oint_{s} A_0 dx_0\right)\right] = 0 \qquad \mathrm{tr}_{\mathrm{F}}[\Omega] \neq 0$$



Preliminaries

- Pure YM is **strongly coupled**, we **can not** do semi-classical analysis
- Brute force calculations gives

$$V_{per}(\Omega) = \frac{-1}{\pi^2 \beta^4} \sum_{n=1} \frac{2}{n^4} \left| \mathrm{tr}\Omega^n \right|^2 \left(1 + O(g^2)\right)$$



- This potential is minimized when $\mathrm{tr}\Omega = N$, so center symmetry is broken
- True for $T >> \Lambda_{QCD}$

Preliminaries

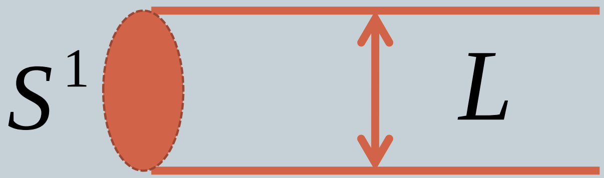
- This potential can not capture the phase transition
- The hope is that a non-perturbative part can help:

$$V(\Omega) = g^2 V_{per} + e^{-\frac{a}{g^2}} V_{non-per}$$

- But pure YM is strongly coupled at the transition and no reliable semi-classics can help

Preliminaries

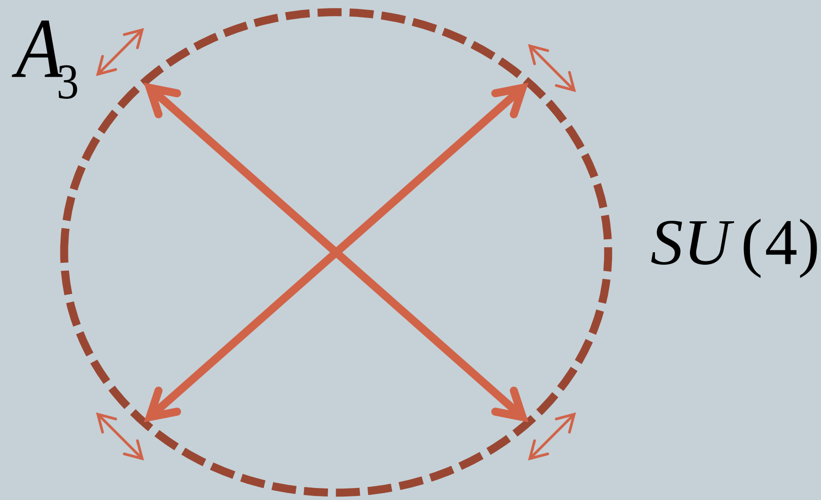
- Analytic understanding: separation of scale
- This is QCD(adj) on $R^{2,1} \times S^1 \equiv R^3 \times S^1$



$$A_\mu(x,y,t,z) = A_\mu(x,y,t,z+L)$$
$$\psi(x,y,t,z) = \psi(x,y,t,z+L)$$

Adjoint fermions

Periodic boundary conditions



$\mathrm{tr}[\Omega] = 0$

Preliminaries

Motivation:

- Hosotani Mechanism (unification)
- Egushi-Kawai reduction (Large-N volume independence, dream!)
- **Laboratory for gauge theories**

Preliminaries

Progress: (see M. Unsal and T. Sulejmanpasic talks)

- Confinement in QCD(adj) (microscopic picture)

  M. Unsal 2009; M. Unsal, E Poppitz 2010, M.A, E. Poppitz 2011; T Misumi, M. Nitta, N. Sakai 2014; T Misumi, T. Kanazawa 2014

- Deconfinement transition in hot QCD(adj)

  M. A, E. Poppitz, M. Unsal 2012; M. A, S. Collier, E. Poppitz 2013; M. A, S. Collier, S. Strimas-Mackey, E. Poppitz, B, Teeple 2013

- Deconfinement in pure YM through a continuity conjecture YM ⟷ QCD(adj)

  E. Poppitz, T. Schafer, M. Unsal 2012; E. Poppitz, T.Schafer, M. Unsal 2013; E. Poppitz, T Sulejmanpasic 2013; M.A. 2013; M.A.; M.A., B. Teeple, E. Poppitz 2014

Preliminaries

- Resurgence and the renormalon problem in QCD(adj)

  M. Unsal, P. Argyres 2012; M. A., T. Sulejmanpasic 2014

- Strings in QCD(adj)

  M.A., E. Poppitz, T. Sulejmanpasic 2015

- Global structure in QCD(adj)

  M.A., E. Poppitz 2015

- Lattice QCD(susy)

  G. Bergner, S. Piemonte 2014, G. Bergner, G. Giudice, G. Munster, S. Piemonte 2015

Preliminaries

Goals of studying the theory on a circle

- Analytic understanding of the physics
- Compare the results on the circle with lattice results on R^4
- The ultimate goal is to decompactify the theory (Clay Mathematics Institute $1000,000 question!!)

Outline

- Part I: Confinement in QCD(adj) on $R^3 \times S^1$
- Part II: Deconfinement in pure YM on R^4
- Part III: Deconfinement in hot QCD(adj) on $R^3 \times S^1$

Part I:
Formulation and confinement

QCD(adj) on $R^3 \times S^1$: Formulation

$SU(2)$:

$$S = \int_{R^3 \times S^1} \frac{1}{g^2} \mathrm{tr}\left[-\frac{1}{2} F_{mn} F^{mn} + 2i\, \overline{\lambda_I}\, \overline{\sigma}^m D_m \lambda_I \right]$$

$1 \le n_f \le 5.5$

Adjoint fermions with periodic boundary conditions along the S^1 circle

$$\underbrace{\Rightarrow}_{\text{small } L} \int_{R^3} \frac{L}{g^2} \mathrm{tr}\left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + \left(D_i A_3\right)^2 - \frac{g^2}{2} V_{per}(A_3) \right]$$

Compact scalar

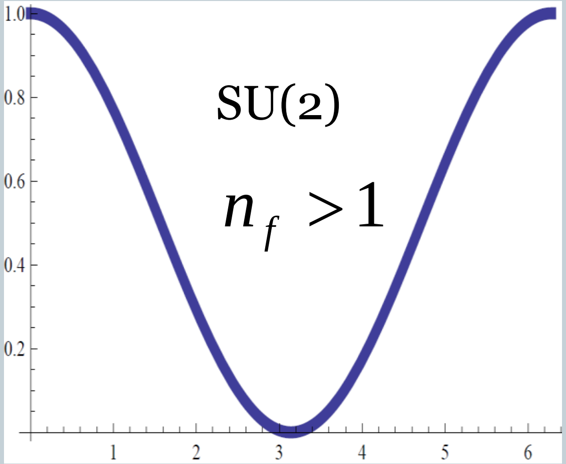
One-loop effect

Georgi-Glashow model

QCD(adj) on $R^3 \times S^1$: Formulation

- One-loop calculation

$$V_{per}(\Omega) = \frac{(-1+n_f)}{\pi^2 L^4} \sum_{n=1} \frac{2}{n^4} \left|\mathrm{tr}\,\Omega^n\right|^2$$

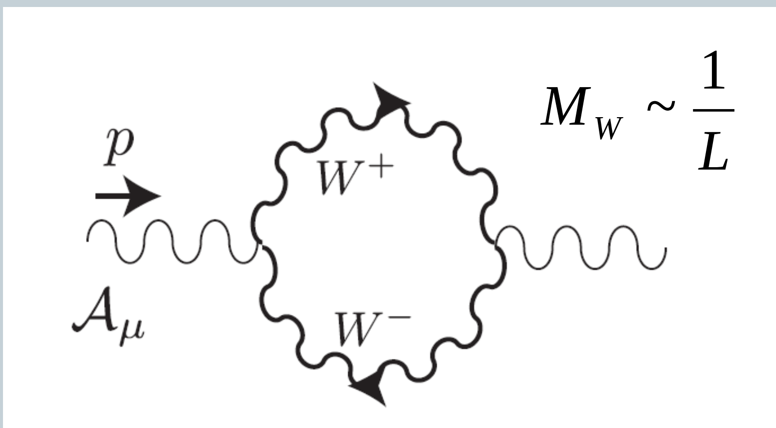


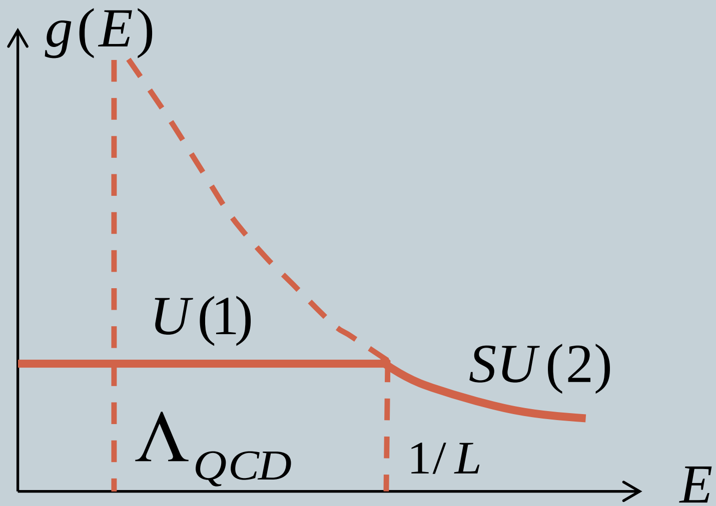
$$\Omega = e^{i\oint A_3 dx^3}$$

- For $n_f > 1$ center symmetry is preserved; $\mathrm{tr}\Omega = 0$
- At $n_f = 1$ we find $V_{per}(\Omega) = 0$. **This is** $N = 1$ **SUSY**. The center is stabilized by non-perturbative effects

QCD(adj) on $R^3 \times S^1$: Formulation

- Higsing: the theory abelianizes $SU(2) \rightarrow U(1)$
- At small $L << 1/\Lambda_{QCD}$ the gauge coupling freezes at a small value





- In 3D the photon has one degree of freedom

$$F_{\mu\nu}F^{\mu\nu} \rightarrow \left(\partial_\mu \sigma\right)^2$$

QCD(adj) on $R^3 \times S^1$: Formulation

- For $n_f > 1$

$$\ell_{eff} = \left(\partial_\mu \sigma\right)^2 + i\overline{\lambda}_I \overline{\tau_\mu} \partial^\mu \lambda_I ,$$

- For $n_f = 1$ (SUSY), the A_3 field is massless (modulus)

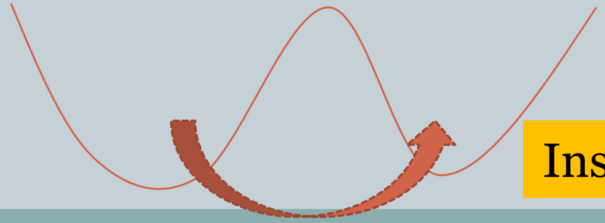
$$\ell_{eff} = \left(\partial_\mu \sigma\right)^2 + \left(\partial_\mu \Phi\right)^2 + i\overline{\lambda}_I \overline{\tau_\mu} \partial^\mu \lambda_I , \Phi \equiv A_3 L$$

QCD(adj) on $R^3 \times S^1$: Formulation

- More interesting story to tell: non-perturbative effects
- Feynman path integral

$$Z_{\text{Euclidean}} = \sum_{\text{paths}} e^{-S_E}$$

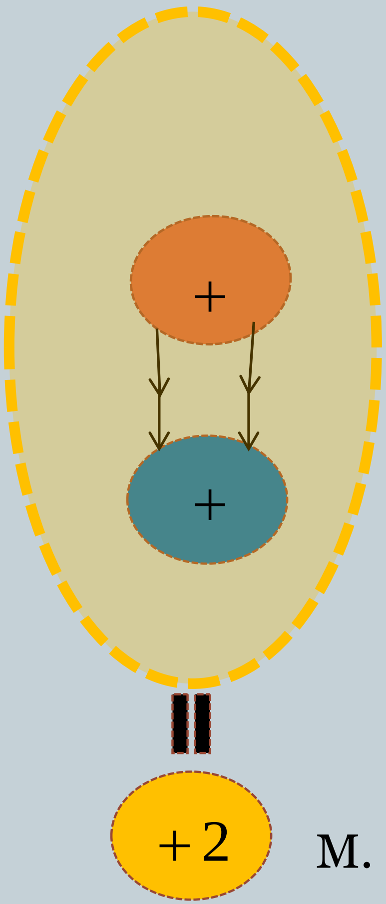
Perturbative +non-perturbative (instantons)



Instantons

QCD(adj) on $R^3 \times S^1$: Formulation

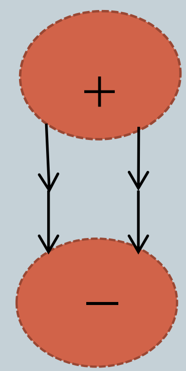
- Magnetic bion



$$e^{-\frac{8\pi^2}{g^2}} e^{\pm i 2\sigma}$$

M. Unsal 2009

Neutral bion



$$e^{-\frac{8\pi^2}{g^2}} e^{\pm 2\Phi}$$

QCD(adj) on $R^3 \times S^1$: Formulation

Summing up all contributions:

- For $n_f > 1$

$$\ell_{\text{eff}-\text{bosonic}} = (\partial\sigma)^2 - e^{-S_0} \underbrace{\cos(2\sigma)}_{\text{photon mass}}, \qquad S_0 = \frac{8\pi^2}{g^2}$$

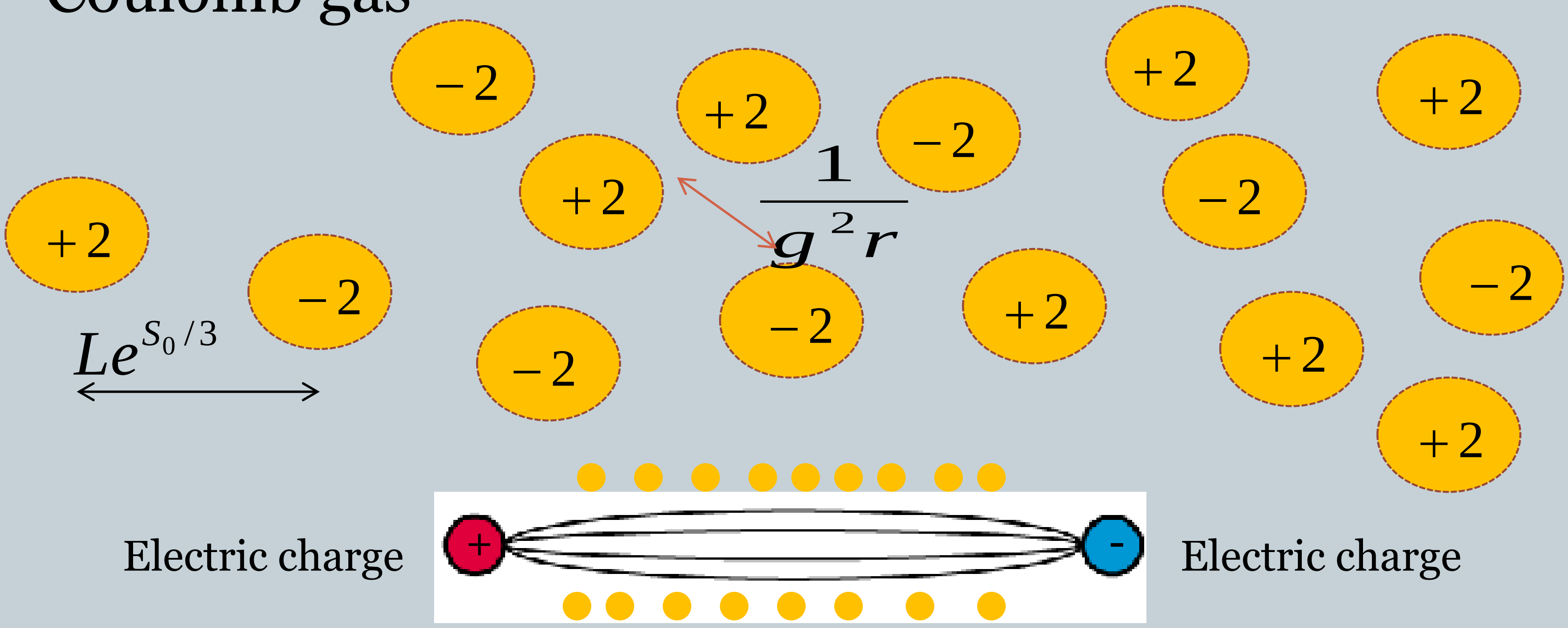
- For SUSY

$$\ell_{\text{eff}-\text{bosonic}} = (\partial\sigma)^2 + (\partial\Phi)^2 - e^{-S_0} \underbrace{\cos(2\sigma)}_{\text{photon mass}} + e^{-S_0} \cosh(2\Phi)$$

Notice the relative sign, from analytic continuation

QCD(adj) on $R^3 \times S^1$: Confinement

- Magnetic bions proliferate in the vacuum: 3D Coulomb gas



SUSY on $R^3 \times S^1$: Confinement

- One can use chiral dualities to dualize the photon
- Perturbatively: $$S = \int d^3x d^4\theta K(X, \overline{X})$$
$$X = i\sigma + \Phi$$
- Next we add the nonperturbative part:
$$S_{\text{non-pert}} = \int d^3x d^2\theta W\left(X\right) + \overline{W}(\overline{X})$$
$$W \sim e^X + e^{-X - \frac{8\pi^2}{g^2}}$$

SUSY on $R^3 \times S^1$: Confinement

- Then one obtains:

$$V = \frac{\partial W}{\partial X}\frac{\partial \overline{W}}{\partial X^+} \sim e^{-S_0}\left[-\cos(2\sigma)+\cosh(2A_4)\right]$$

$$\ell = (\partial\sigma)^2 + (\partial\Phi)^2 + e^{-S_0}\left[-\cos(2\sigma)+\cosh(2\Phi)\right]$$

Magnetic and neutral bions are the physical manifestation

QCD(adj) on $R^3 \times S^1$: Confinement

Take home messages

- QCD(adj) preserves the center ➡ separation of scales
- QCD(adj) trivial perturbatively
- Confinement is due to **magnetic bions**
- SUSY is curse and blessing!

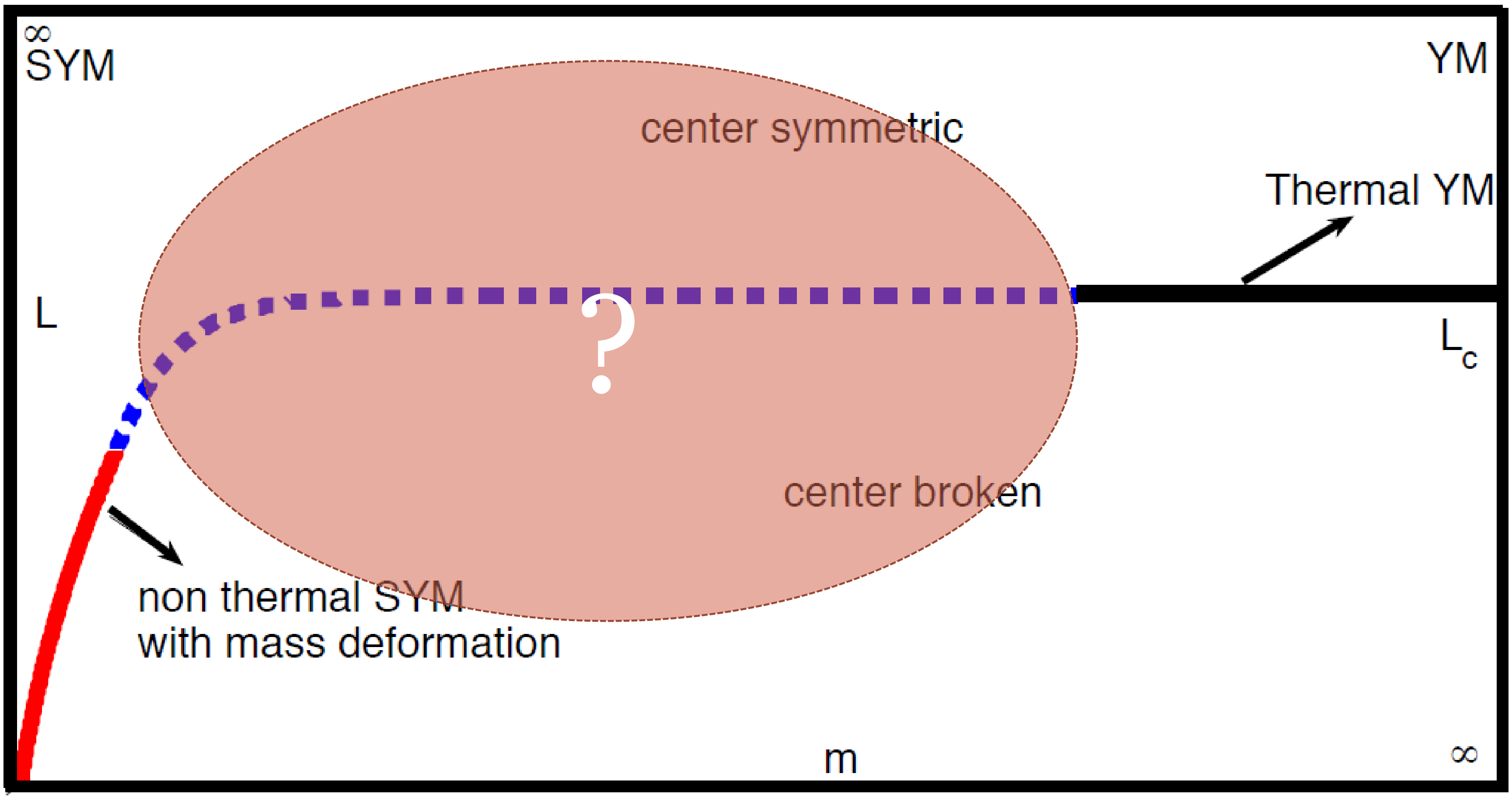
Part II: Phase Transition in pure YM on R^4 via mass deformed SUSY on $R^3 \times S^1$

Mass deformed SUSY on $R^3 \times S^1$

- Adding mass deformation (MD) to Super-Yang-Mills (QCD(adj) with $n_f = 1$), we can study phase transition in pure Yang-Mills

Mass deformed SUSY on $R^3 \times S^1$

T. Schafer, E. Poppitz, M. Unsal 2012



Mass Deformed SUSY on $R^3 \times S^1$

- Now, we add a massive fermion (gaugino)
- The mass lifts the fermions zero mode
- Monopoles contribute to the potential

$$\ell_{eff} = (\partial\sigma)^2 + (\partial\Phi)^2 + e^{-S_0/2}\left(e^{i\sigma-\Phi} \underbrace{\lambda\lambda}_{m} + e^{-i\sigma+\Phi} \underbrace{\lambda\lambda}_{m} + \text{c.c.} \right)$$

$$- e^{-S_0} \cos(2\sigma) + e^{-S_0} \cosh(2\Phi)$$

$$V_m \sim \frac{m e^{-S_0/2}}{L^2} \cos\sigma \cosh\Phi$$

Mass deformed SUSY on $R^3 \times S^1$

- Total potential

$$V_t = \underbrace{\frac{1}{L^3} e^{-\frac{8\pi^2}{g^2}} \left[-\cos(2\sigma) + \cosh(2\Phi)\right]}_{V_{bion}} + \underbrace{\frac{m}{L^2} e^{-\frac{4\pi^2}{g^2}} \cos\sigma \cosh\Phi}_{V_m}$$

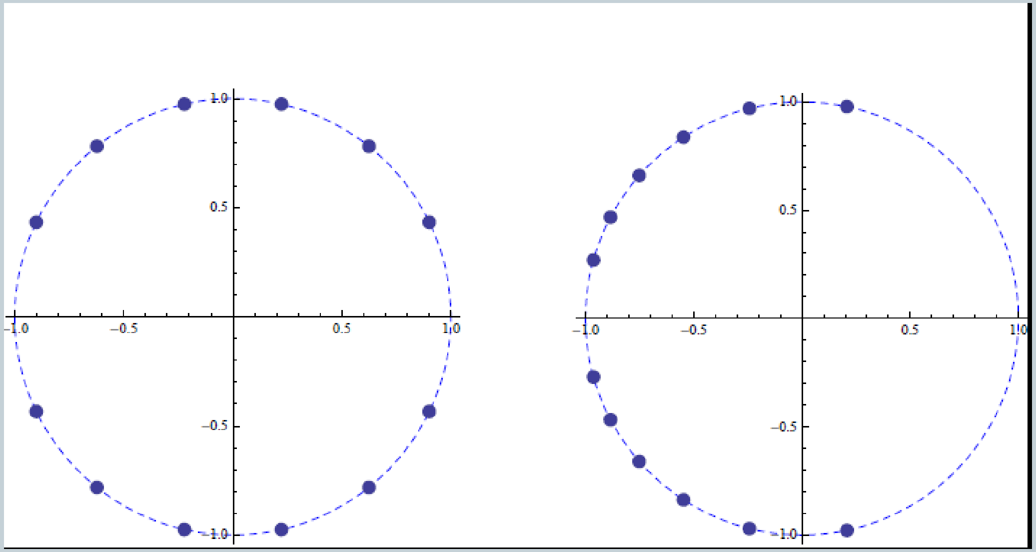
- V_{bion} stabilizes the center
- V_m destabilizes the center
- The competition between V_{bion} and V_m determines the nature of the **quantum phase transition**

Phase transitions in pure YM

- Behavior of the Polyakov loop

$$\mathrm{tr}[\Omega] = \sum_{\vec{\nu}} e^{i\vec{\nu}.\langle\vec{\Phi}\rangle}$$

$\mathrm{tr}[\Omega] = 0$ $\qquad\qquad$ $\mathrm{tr}[\Omega] \neq 0$



Phase transitions in pure YM

- **The phase transition is first order in all groups:**

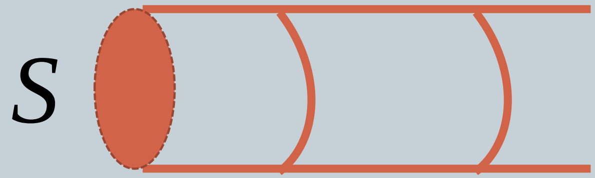
SU(N), spin(2N+1), Sp(2N), Spin(2N),

E6, E7,E8,F4,G2. M.A, B. Teeple, E. Poppitz 2014

- SU(2) and $\mathrm{Spin}(4) = SU(2) \times SU(2)$ are exception: second order transition
- **Lattice simulations for** SU(2), SU(N>2) and Sp(4) **indicated second order for** SU(2), **first order for** SU(N>2) and Sp(4) (Holland, Pepe, and Wiese 2003)

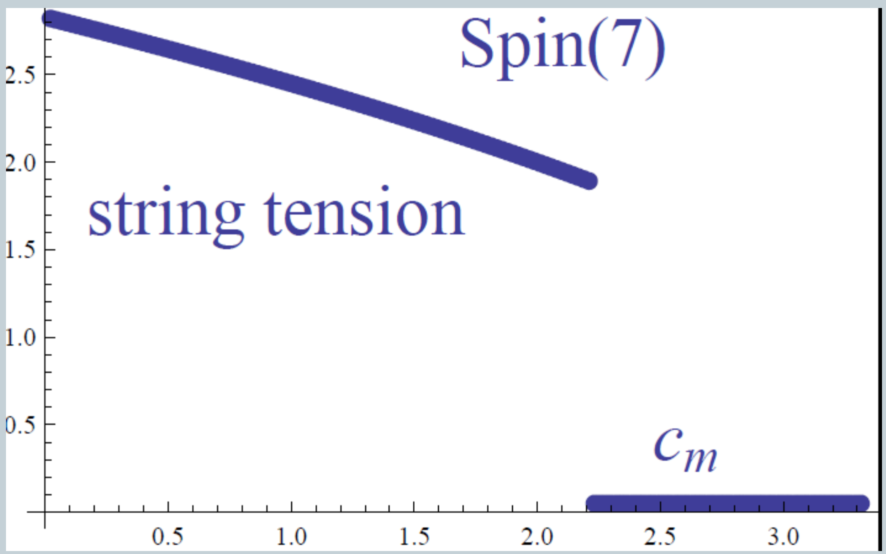
Phase transitions in pure YM

- String tension



$$\langle \mathrm{tr}\,\Omega(x)\,\mathrm{tr}\,\Omega^{+}(0)\rangle \approx e^{-\sigma r L}$$

Discontinuous transition



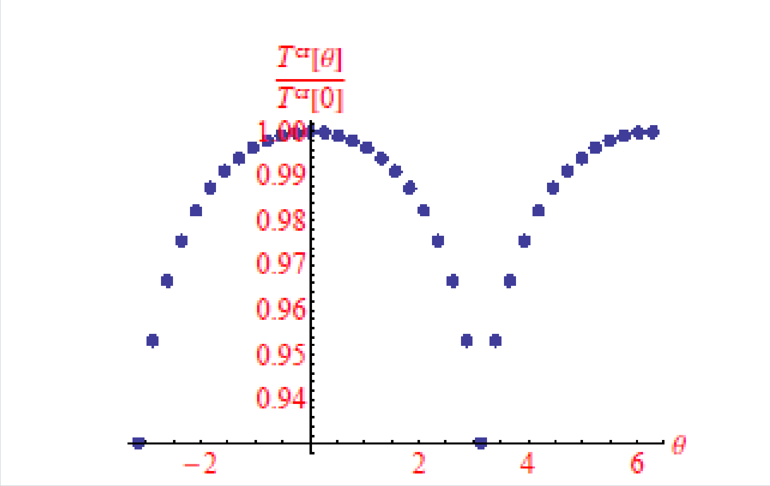
Same behavior for SU(3)

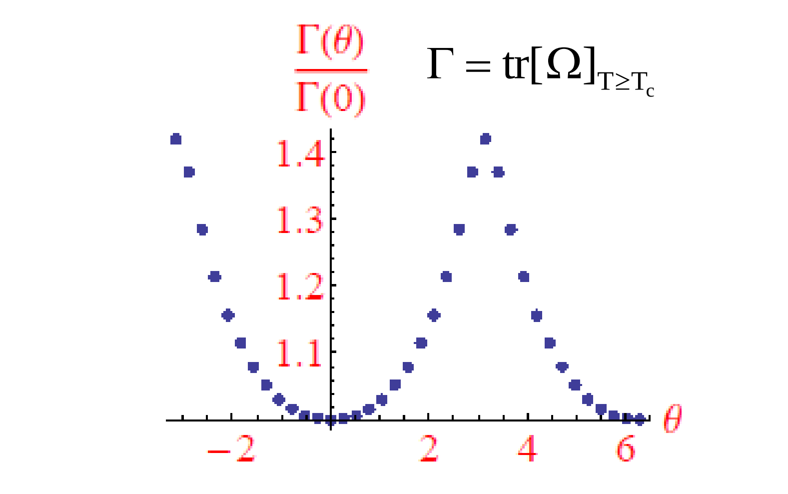
Compare lattice Simulations:
Bicudo 2010
done for SU(3)

Phase transitions in pure YM

- Topological θ angle: $\vartheta F_{mn}\tilde{F}^{mn}$

SU(3) M. A. 2013





Compare lattice studies for SU(3): Bonati, D'Elia, Panagopoulos, Vicari 2013

Phase transitions in pure YM

Take home messages

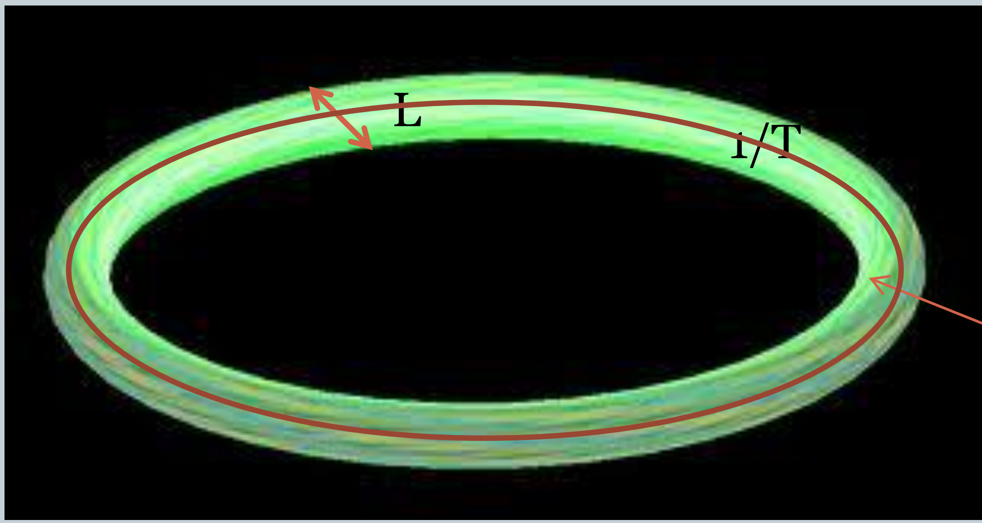
- Mass deformed SUSY is a great tool to study pure YM
- No single test has failed!
- Continuity? (see the resurgence talks)

Part III: Deconfinement transition in QCD(adj) on $R^3 \times S^1$

Thermal QCD(adj) on $R^3 \times S^1$

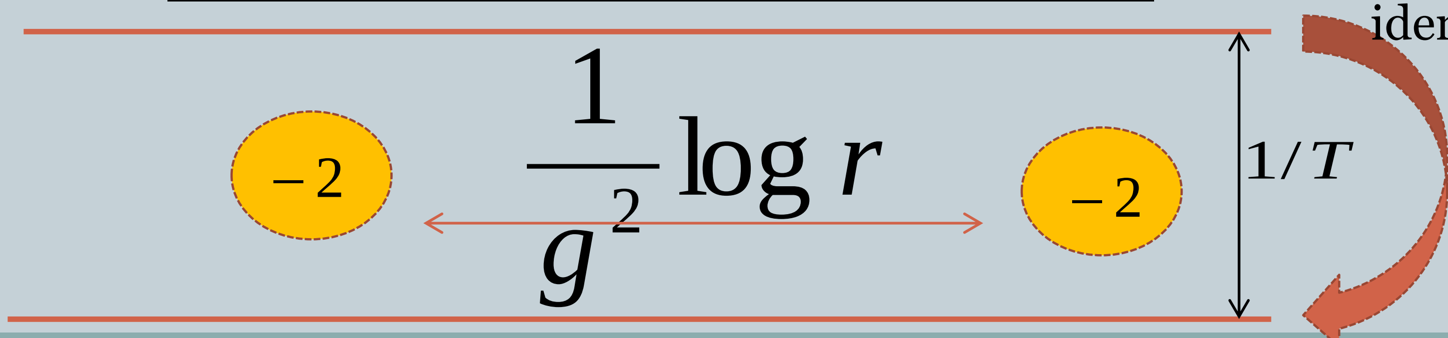
M.A., E. Poppitz, M. Unsal, 2011

- At finite temperature we compactify the time direction



$LT << 1$

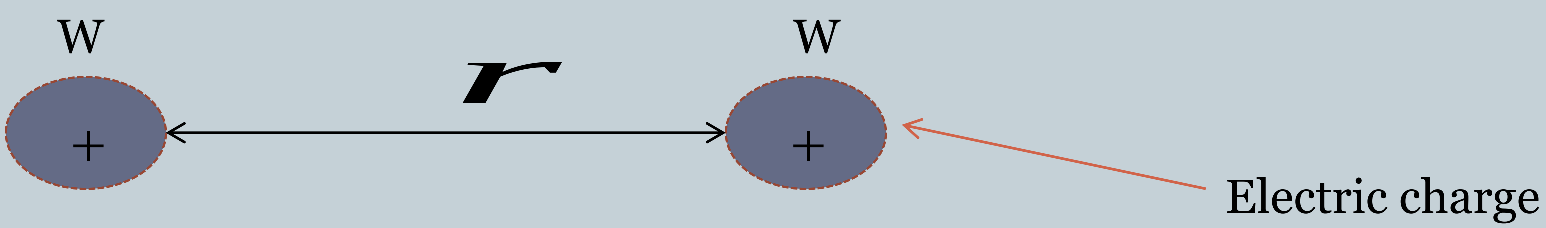
R^2



Thermal QCD(adj) on $R^3 \times S^1$

- The story has one more twist!
- At finite temperature, the W's and charged fermions are important

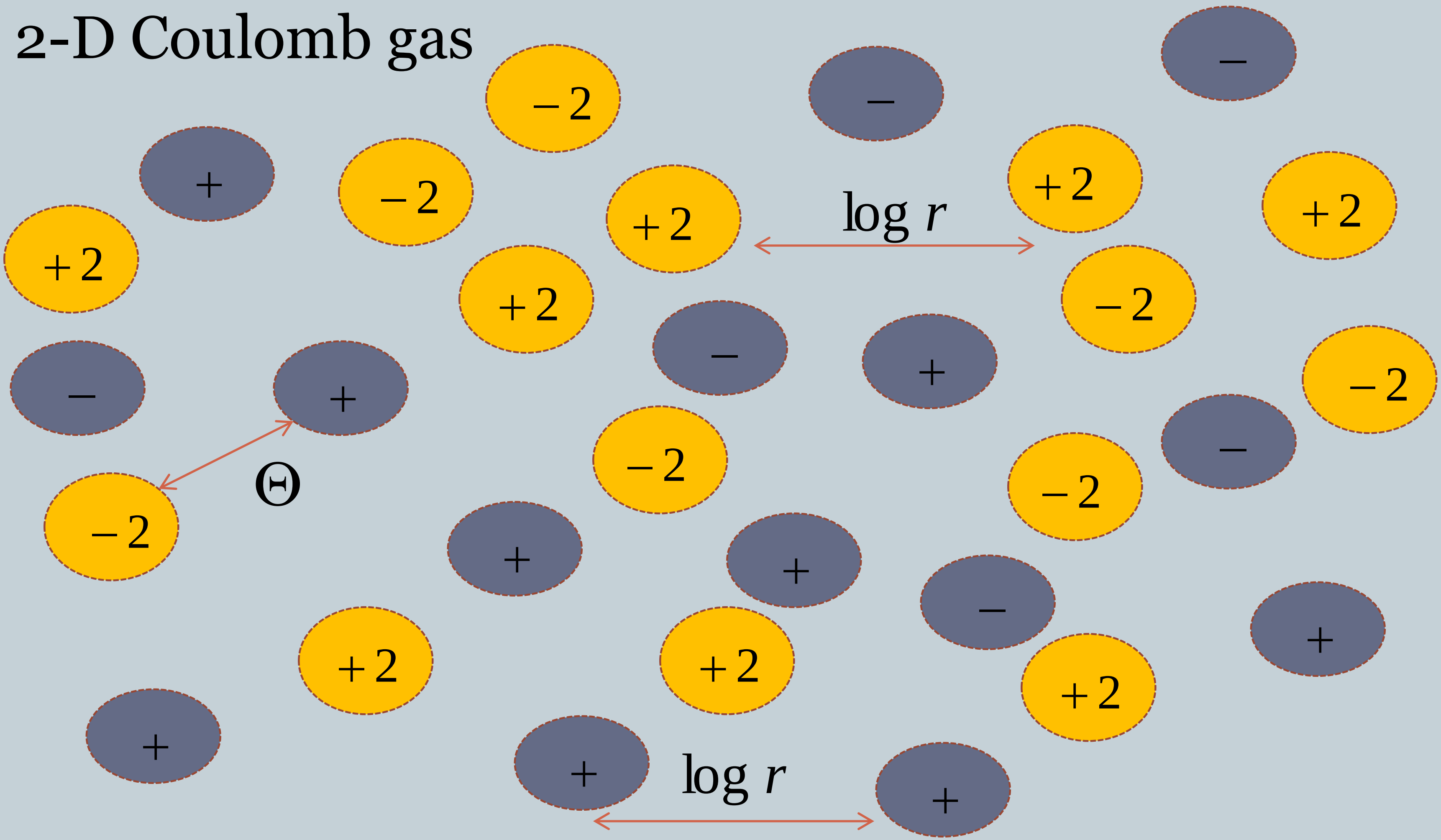
$$\text{density} \propto e^{-m_W/T}$$



$$g^2 \log r$$

Thermal QCD(adj) on $R^3 \times S^1$

- 2-D Coulomb gas



Thermal QCD(adj) on $R^3 \times S^1$

- The partition function for SU(2)

$$Z = \sum_{q_a, q_A, N_{bion-}, N_{bion+}, N_{W+}, N_{W-}} \frac{\xi_{bion}^{N^i{}_{bion+} + N^i{}_{bion-}}}{N_{bion+}! N_{bion-}!} \frac{\xi_W^{N^i{}_{W+} + N^i{}_{W-}}}{N_{W+}! N_{W-}!} \prod_{a,i} \int d^2 R^i{}_a \prod_{Ai} \int d^2 R^i{}_A$$

$$\exp\left[\sum_{i,j a,b,A,B} \frac{32\pi L T q_a q_b}{g^2} \log\left|\vec{R}^i_a - \vec{R}^j_b\right| + \frac{g^2 q_A q_B}{2\pi L T} \log\left|\vec{R}^i_A - \vec{R}^j_B\right| + 4 i q_a q_A \Theta\left(\vec{R}^i_a - \vec{R}^j_A\right) + \underbrace{\text{scalar part}}_{\text{susy}} \right]$$

Non-SUSY e-m duality

$$\xi_{bion} \Longleftrightarrow 2\xi_W, \quad \frac{32\pi L T}{g^2} \Longleftrightarrow \frac{g^2}{2\pi L T}$$

Weakly coupled self-dual point

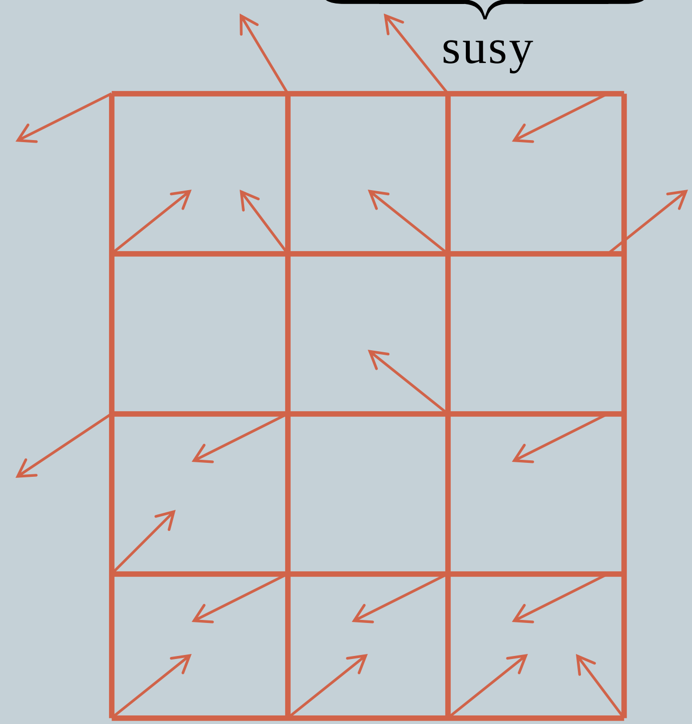
Mapping QCD(adj) to spin models

- This 2D Coulomb gas is **EXACTLY** equivalent to XY-spin models:

$$H = A\underbrace{\sum_{<ij>}\cos(\vartheta_i - \vartheta_j)}_{\vec{S}_i \cdot \vec{S}_j} + B\sum_i \cos(4\vartheta_i) + \underbrace{\text{scalar part}}_{\text{susy}}$$

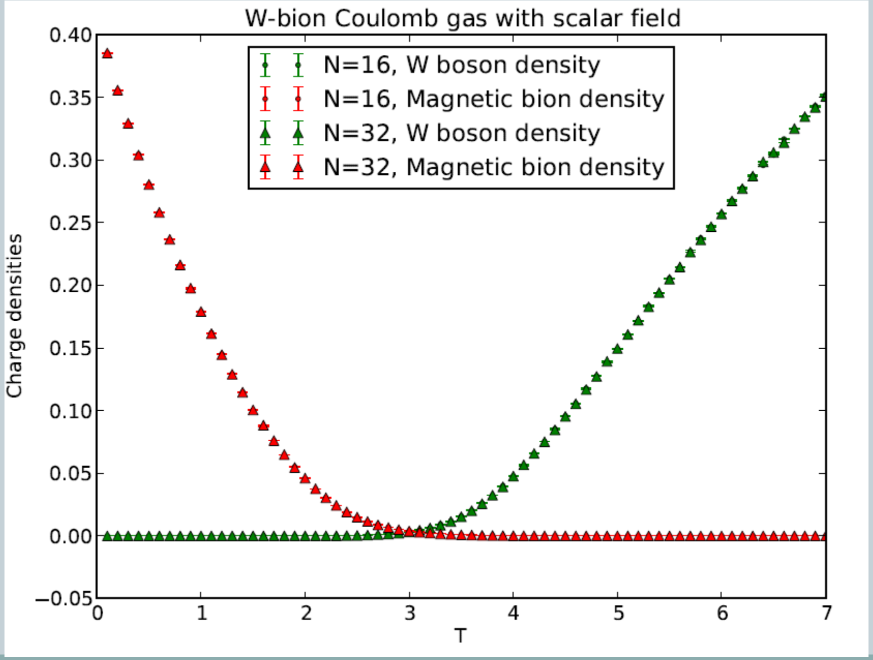
Nearest neighbor interaction

External field



Results for thermal QCD(adj) on $R^3 \times S^1$

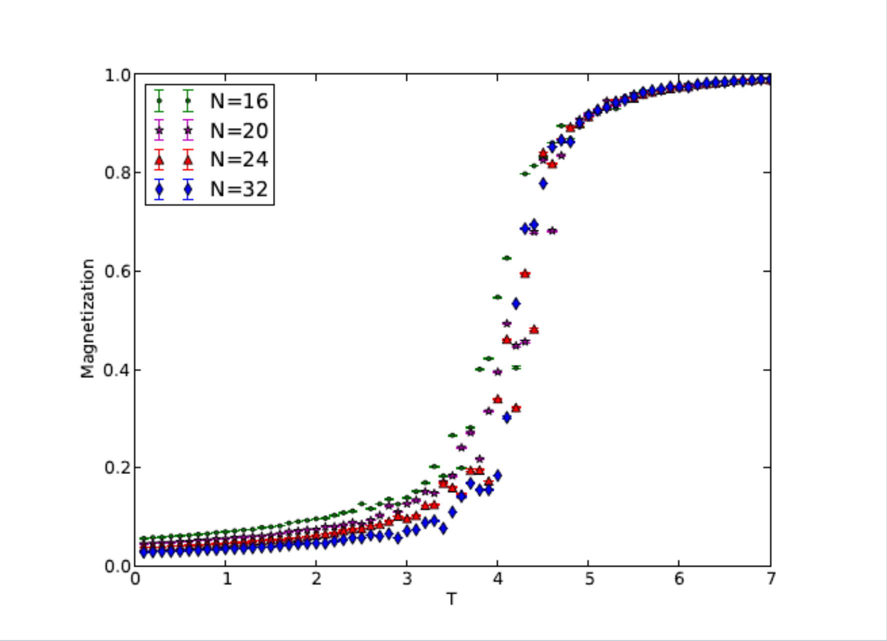
- Analytic results for non-SUSY SU(2) indicate a second transition deconfinement
- Numerical results for SUSY SU(2) indicate a second order transition

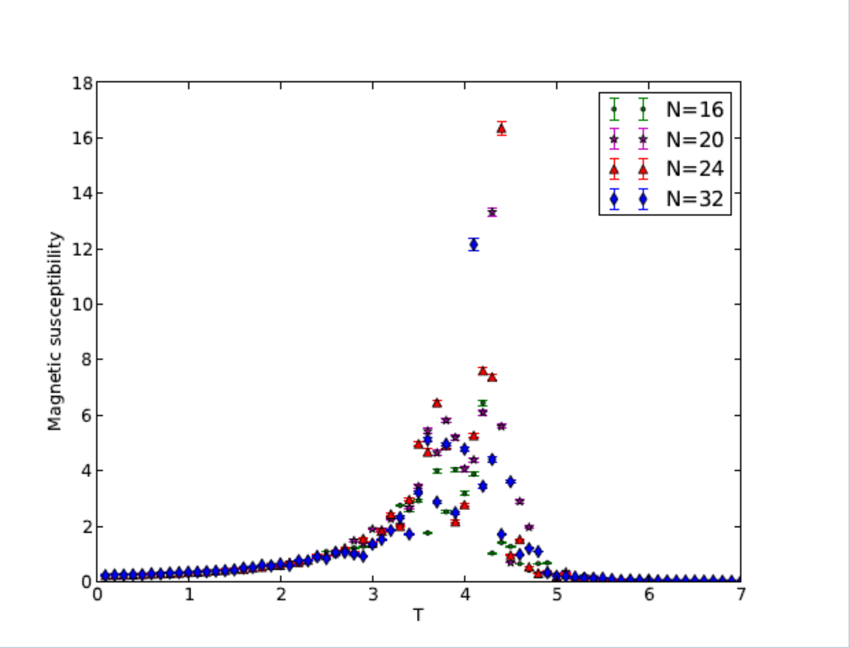


Results for thermal QCD(adj) on $R^3 \times S^1$

- XY-model simulations SUSY SU(2)

M. Anber, S. Collier, S. Strimas-Mackey, E. Poppitz, B, Teeple, 2013



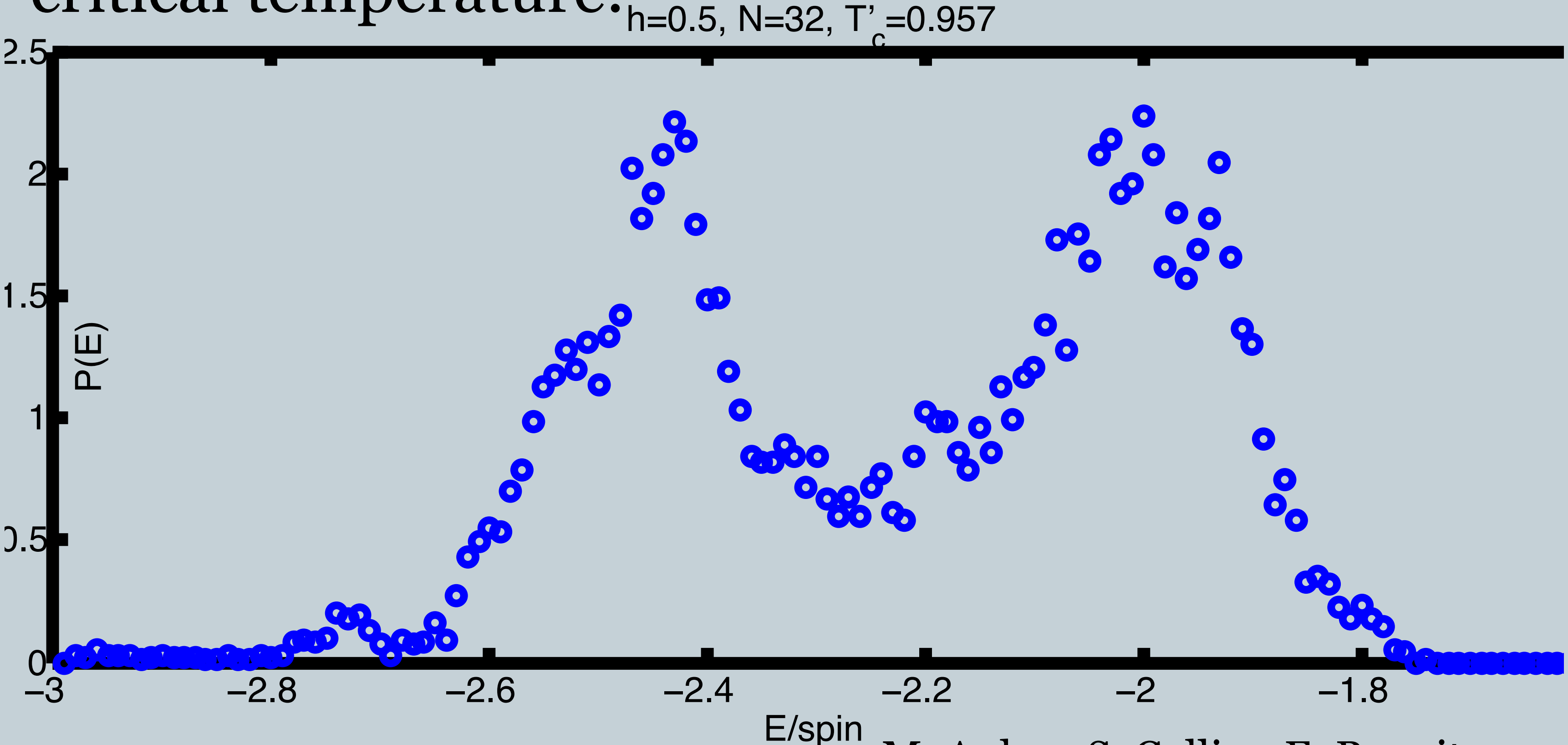


$$m = \frac{1}{N^2}\langle|\sum_x e^{i\theta_x}|\rangle = \frac{\langle|M|\rangle}{N^2}$$

$$\chi(m) = \frac{\langle|M|^2\rangle - \langle|M|\rangle^2}{N^2}$$

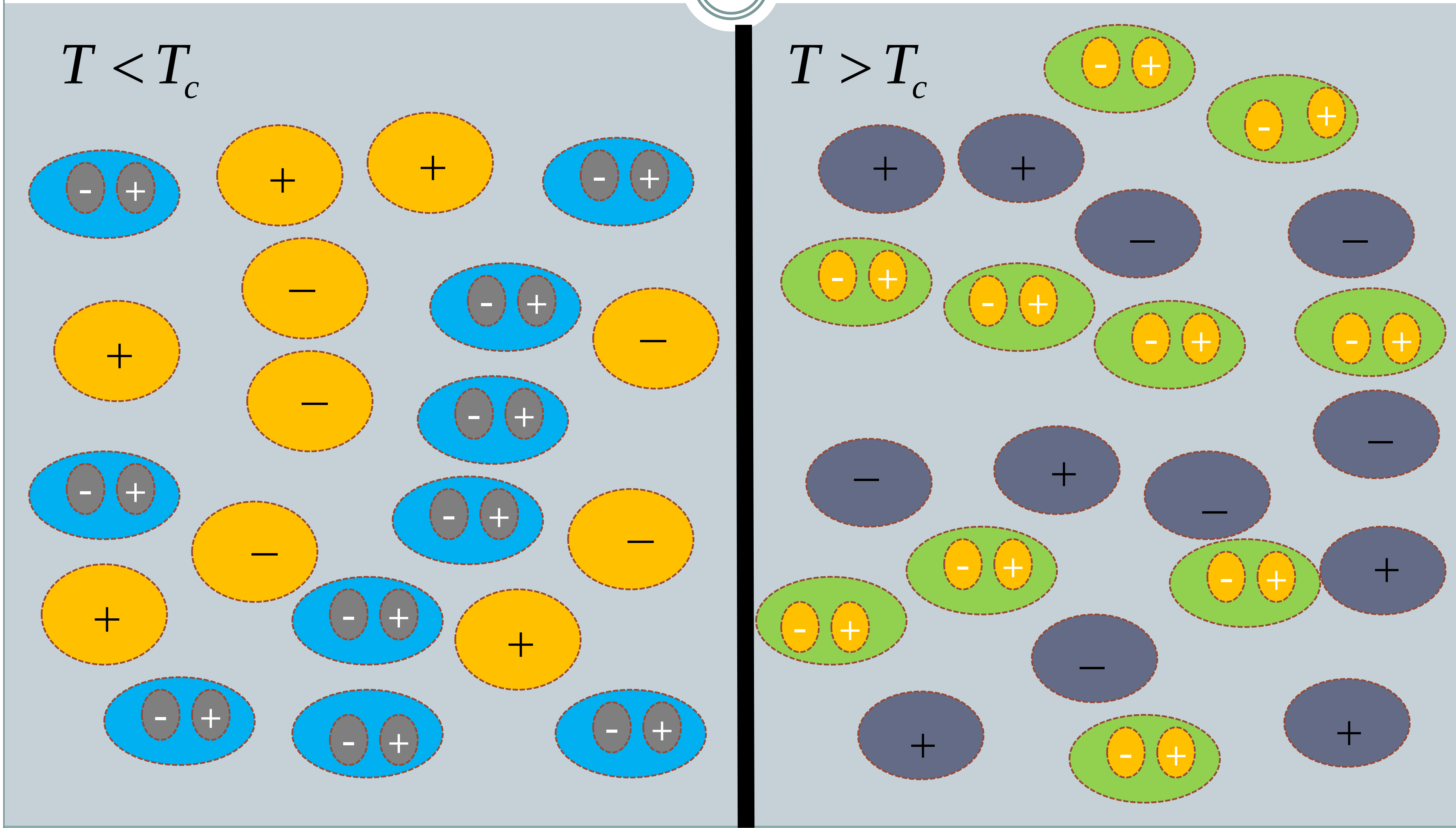
Results for thermal QCD(adj) on $R^3 \times S^1$

- Results for nonsusy SU(3): phase coexistence at the critical temperature:



M. Anber, S. Collier, E. Poppitz, 2013

Summary of the deconfinement picture



Results for thermal QCD(adj) on $R^3 \times S^1$

The transition in QCD(adj) on $R^3 \times S^1$

- Second order for SU(2)
- First order for SU(3)

Compare with lattice results of QCD(adj) on R^4

- Second order for SU(2) SUSY, G. Bergner, P. Giudice, G. Munster, S. Peimont, S. Sandbrink 2014
- First order for SU(3), Karsch and Lutgemeir 1998

Results for thermal QCD(adj) on $R^3 \times S^1$

Take home messages

- The order of the transition is the same for R^4 and $R^3 \times S^1$
- Is there a deep reason? Continuity?

Conclusion

- New insights from studying this class of theories
- Continuity? More tests are needed
- More lattice studies are needed, can we see the composite strings?
- Plenty of other works regarding compact theories on a circle (need for more future workshops)

Appendix

- To study phase transitions → order parameter

- E.g. to study magntization in 2D

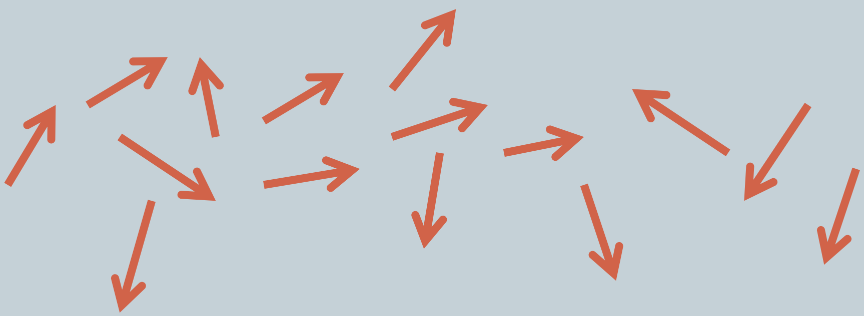
$$H = \sum_{x,\mu} \overrightarrow{S_x} . \overrightarrow{S_{x+\mu}} = \sum_{x,\mu} \cos\left(\vartheta_x - \vartheta_{x+\mu}\right)$$

- H is invariant under $SO(2): \vartheta \rightarrow \vartheta + c$

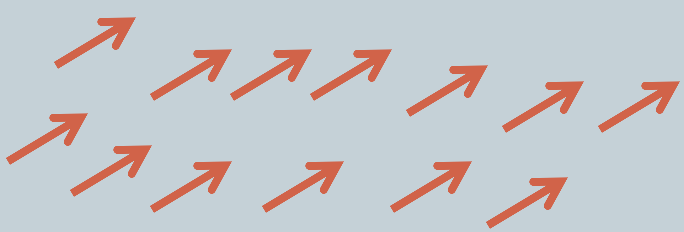
- order parameter $M = \sum_x e^{i\vartheta_x}$

Appendix

- Demagnetized phase $\langle |M| \rangle = 0$ and $SO(2)$ is unbroken



- Magnetized phase $\langle |M| \rangle \neq 0$ and $SO(2)$ is broken



Appendix

- Order parameter for pure YM: Polyakov loop

$$A_\mu(\vec{x},t) = A_\mu(\vec{x},t+\beta)$$

$$\mathrm{P}_F = \mathrm{tr}_F[\Omega] = \mathrm{tr}_F\left[p e^{i\oint_S A_0 dx_0} \right]$$

Thermal circle: compact time

$$T = \frac{1}{\beta} = \frac{1}{\text{circumference}}$$

- P_F is gauge invariant:

$$A_\mu \to U A_\mu U^+ - U\partial_\mu U^+$$

$$e^{i\int_0^\beta dx^0 A_0} \to U(\vec{x},0) e^{i\int_0^\beta dx^0 A_0} U^+(\vec{x},\beta), \quad U(\vec{x},\beta) = U(\vec{x},0)$$

Gauge transformations are periodic

Appendix

- We can also consider Center transformations:

$$U\left(\vec{x},\beta\right)=zU\left(\vec{x},0\right)$$

Element of the center

- The center of a group are the elements that commute with all the elements in the group:

$$Z=\left\{z\in G, zg=gz\right\}$$

Appendix

- For SU(N)

$$z_k = e^{i\frac{2\pi k}{N}} \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & . & & \\ & & & . & \\ & & & & 1 \end{bmatrix}_{N\times N}, k = 0,1,\ldots, N-1$$

- The Lagrangian $\ell = F^a_{\mu\nu} F^a_{\mu\nu}$ invariant under the center

- In the fundamental rep $P_F \to z P_F$