Lepton Number Violation in Astrophysical Environments

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Outline

• Introduction: neutrinos in hot and dense media & QKEs
• Spin-dependent effects in QKEs and $\nu$-$\bar{\nu}$ oscillations (LNV!)
• Neutrino-antineutrino conversion in compact objects: exploration within one-flavor toy model
• Conclusions and outlook

Based on collaboration with George Fuller (UCSD) and Alexey Vlasenko (UCSD → NCSU)

A. Vlasenko, G. Fuller, V. Cirigliano, Phys. Rev. D 89 105004 (2014)
A. Vlasenko, G. Fuller, V. Cirigliano, ArXiv:1406.6724
Neutrinos in hot and dense media & QKEs
Neutrinos in hot / dense medium

- Neutrinos play a key role in early universe and astrophysical systems

- To incorporate the known physics of massive neutrinos and probe non-standard effects one must keep track of coherent flavor oscillations AND de-cohering inelastic collisions with the medium
Neutrinos in hot / dense medium

- Ensemble of neutrinos described by “density matrix”
- Spin 1/2 massive neutrino: one flavor

\[
F(x, p) = \begin{pmatrix}
    f_{LL} & f_{LR} \\
    f_{RL} & f_{RR}
\end{pmatrix}
\]

Diagonal: “populations”
Off-diagonal: “coherences”

\[ p, \quad s \]
- L-handed state \( |L\rangle \)
- R-handed state \( |R\rangle \)
Neutrinos in hot / dense medium

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$$F(x, p) = \begin{pmatrix} f_{LL} & f_{LR} \\ f_{RL} & f_{RR} \end{pmatrix}$$

Diagonal: “populations”
Off-diagonal: “coherences”

For a pure state

$$|\psi\rangle = c_L |L\rangle + c_R |R\rangle$$

$$f_{LL} = |c_L|^2 \quad f_{RR} = |c_R|^2 \quad f_{LR} = c_L c_R^* \quad f_{RL} = c_R c_L^*$$
Neutrinos in hot / dense medium

- Ensemble of neutrinos described by “density matrix”
- Spin 1/2 massive neutrino: many flavors

\[ F(x, p) = \begin{pmatrix} f_{\alpha \beta}^{LL} & f_{\alpha \beta}^{LR} \\ f_{\alpha \beta}^{RL} & f_{\alpha \beta}^{RR} \end{pmatrix} \]
Neutrinos in hot / dense medium

- Ensemble of neutrinos described by “density matrix”
- Spin 1/2 massive neutrino: many flavors

Usual neutrino density matrix (for two flavors e, \(\mu\))
Neutrinos in hot / dense medium

- Ensemble of neutrinos described by “density matrix”
- Spin 1/2 massive neutrino: many flavors (Majorana)

\[ F(x, p) = \left( \begin{array}{cc} f & \phi \\ \phi^\dagger & f^T \end{array} \right) \]

R-handed neutrino = antineutrino \[ \Rightarrow f \equiv f_{LL} \quad \bar{f} \equiv f_{RR}^T \quad \phi \equiv f_{LR} \]
Neutrinos in hot / dense medium

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$$F(x, p) = \begin{pmatrix} f & \phi \\ \phi^T & f^T \end{pmatrix}$$

$n_f \times n_f$ blocks describing density matrix for neutrinos and antineutrinos

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$n_f \times n_f$ blocks describing density matrix for neutrinos and antineutrinos

$n_f \times n_f$ block describing neutrino-antineutrino coherence

(Can be generated by LNV interactions such as a Majorana mass term or magnetic moment in external B field)

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- QKEs: evolution equations for \( F(x,p) \)
QKEs from Quantum Field Theory

A. Vlasenko, G. Fuller, V. Cirigliano, Phys. Rev. D 89 105004 (2014)

Equations of motion for Green Functions

\[ \langle \nu_\alpha(x) \bar{\nu}_\beta(y) \rangle \]

Evolution equations for \( F(x,p) \)

\[ f^{\alpha\beta}_{hh'}(x, p) \sim \langle a^\dagger_\beta(p, h') a_\alpha(p, h) \rangle \]

- Exploit hierarchy of scales: \( L_{\text{osc}} \sim E/(\Delta m^2), L_{\text{mfp}}, L_{\text{gradients}} \gg L_{\text{deBroglie}} \)

- Work to 2nd order in small ratios of length scales
Structure of the QKEs

\[ F(x, p) = \begin{pmatrix} f^\dagger & \phi \\ \phi^T & f^T \end{pmatrix} \]

\[ DF = -i[H, F] + C \]

F, H(F), C(F): 2n_f x 2n_f matrices

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Derivative along \( \nu \) world line

"Liouville"


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Structure of the QKEs

A. Vlasenko, G. Fuller, V. Cirigliano, Phys. Rev. D 89 105004 (2014),

\[
F(x, p) = \begin{pmatrix} \frac{f}{\phi^*} & \phi \end{pmatrix}
\]

\[DF = -i[H, F] + C\]

F, H(F), C(F):
2n_f x 2n_f matrices

Derivative along \( \nu \) world line

“Liouville”

Coherent evolution:
vacuum mass & neutrino potential

“Oscillation”
Structure of the QKEs

\[ F(x, p) = \begin{pmatrix} f \phi \phi^* \end{pmatrix} \]

\[ DF = -i[H, F] + C \]

Derivative along \( \nu \) world line

"Liouville"

Coherent evolution: vacuum mass & neutrino potential

"Oscillation"

Inelastic collisions

"Boltzmann"

F, H(F), C(F): 2n_f x 2n_f matrices

Spin-dependent effects in QKEs
A new spin on the QKEs

\[ F(x, p) = \begin{pmatrix} f & \phi \\ \phi^* & ft \end{pmatrix} \]

\[ DF = -i[H, F] + C \]

• Coherent helicity-flip is possible (even in absence of magnetic moment and magnetic field)

\[ H = \begin{pmatrix} H_{LL} & H_{LR} \\ H_{RL} & H_{RR} \end{pmatrix} \]


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A new spin on the QKEs

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- Coherent helicity-flip is possible (even in absence of magnetic moment and magnetic field)

- \( H_{LR} \) is determined by mass matrix and neutrino potentials, generated by forward scattering in medium
Neutrino potentials and spin effects

- Key point: weak interactions generate 4-vector potential, with time- and space-like components in non-isotropic medium

\[ \mathcal{H}_{\text{int}} = -\bar{\nu} \sum_{\mu} \gamma^\mu \gamma_5 \nu \]

4-potential couples to the neutrino axial current:

\[ \mathcal{H}_{\text{int}} \sim \mathbf{J} \cdot \mathbf{\Sigma} \]

This terms mixes LH and RH \( \nu \) states
Neutrino potentials and spin effects

- Key point: weak interactions generate 4-vector potential, with time- and space-like components in non-isotropic medium

\[ \mathcal{H}_{\text{int}} = -\bar{\nu} \sum_{\mu} \gamma^\mu \gamma_5 \nu \]

- \( \sum_{\mu} \) receives contributions from matter and neutrinos, e.g.

\[
\sum_{(\nu)}^\mu = \sqrt{2} G_F \left( J_{(\nu)}^\mu + 1 \text{ tr} J_{(\nu)}^\mu \right)
\]

\[
J_{(\nu)}^\mu(x) = \int \frac{d^3 q}{(2\pi)^3} n^\mu(q) \left( f(\vec{q}, x) - \bar{f}(\vec{q}, x) \right)
\]

\[
n^\mu(q) = (1, \hat{q})
\]

Even in simple “bulb” model for SN: \( \vec{\Sigma} \neq 0 \)
$\nu - \bar{\nu}$ mixing: one flavor

- The 2x2 effective hamiltonian $H$ takes the form

$$
H = \left( p + \frac{m^2}{2p} \right) \times I + \left( \begin{array}{cc}
\Sigma^0 - \hat{p} \cdot \vec{\Sigma} & (m/p) \hat{x}_+ \cdot \vec{\Sigma} \\
(m/p) \hat{x}_+^* \cdot \vec{\Sigma} & - (\Sigma^0 - \hat{p} \cdot \vec{\Sigma})
\end{array} \right)
$$

$$
\hat{x}_+ = \hat{x}_1 + i\hat{x}_2
$$
ν-ν̄ mixing: one flavor

- The 2x2 effective hamiltonian $H$ takes the form

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- Medium birefringence

$$\hat{x}_+ = \hat{x}_1 + i\hat{x}_2$$
\( \nu - \bar{\nu} \) mixing: one flavor

- The 2x2 effective hamiltonian \( H \) takes the form

\[
H = \left( p + \frac{m^2}{2p} \right) \times I + \left( \Sigma^0 - \hat{p} \cdot \hat{\Sigma} \right) \frac{m}{p} \hat{x}^* \cdot \hat{\Sigma} - \left( \Sigma^0 - \hat{p} \cdot \hat{\Sigma} \right)
\]

- Medium birefringence

- Helicity mixing / oscillations:
  - Linear in neutrino mass: manifestation of Lepton Number Violation; sensitive to absolute mass scale
  - Proportional to component of the potential transverse to the neutrino momentum (anisotropy)
ν-Bar mixing: one flavor

- The 2x2 effective hamiltonian $H$ takes the form

$$H = \left( p + \frac{m^2}{2p} \right) \times I + \left( \begin{array}{c}
\Sigma^0 - \hat{p} \cdot \vec{\Sigma} \\
(m/p) \hat{x}_+ \cdot \vec{\Sigma}
\end{array} \right) = \left( \begin{array}{c}
\Sigma^0 - \hat{p} \cdot \vec{\Sigma} \\
(m/p) \hat{x}_+ \cdot \vec{\Sigma}
\end{array} \right)$$

- Off-diagonal terms parametrically suppressed ($m/p$) compared to diagonal ones

- But resonant conversion is possible if diagonal elements vanish

- Astrophysical environments?
Neutrino-antineutrino conversion in compact objects?
**ν-ν̄ conversion in compact objects?**

A. Vlasenko, G. Fuller, V. Cirigliano, ArXiv:1406.6724

- **Toy problem:** crossed neutrino beams of infinite width in time-varying background matter.

- **Effective hamiltonian (one flavor)**

\[
H = \begin{pmatrix} H_3 & H_1 \\ H_1 & -H_3 \end{pmatrix}
\]

\[
H_1 = 2\sqrt{2}G_F \ u \sqrt{1-u^2} \frac{m}{E} (n_\nu - n_\bar{\nu})
\]

\[
H_3 = \frac{G_F}{\sqrt{2}} \left[ (3Y_e - 1) n_B + 4 (1 - u^2)(n_\nu - n_\bar{\nu}) \right]
\]

**Resonance condition:** matter and neutrino contribution to \( H_3 = H_{\text{Matter}} + H_\nu \) cancel. Can happen near protoneutron star, or in central region of compact object merger (\( Y_e \sim 1/3 \)).
• Level crossing

\[ H_3 < 0 \]

\[ H_3 > 0 \]

\[ n_B = 300 \text{ MeV}^3 \]
\[ T = 4 \text{ MeV} \]
\[ \mu = 8 \text{ MeV} \]

• Can we get large scale conversion?
- Neutrino feedback keeps system close to resonance: $\delta H_{\text{Matter}}>0$ while $\delta H_\nu<0$ as neutrinos start to convert into antineutrinos.

\[
Y_e = Y_{e0} + \frac{s}{\lambda} \left( 1 + \frac{s^2}{k^2} \right)
\]
- Neutrino feedback keeps system close to resonance: $\delta H_{\text{Matter}}>0$ while $\delta H_\nu<0$ as neutrinos start to convert into antineutrinos.

Onset of coherent neutrino-antineutrino transformation

Cancellation between matter and neutrino potential

- Requires small derivative of the matter potential and largish neutrino masses (*the two effects are correlated*).

\[ Y_e = Y_{e0} + \frac{s}{\lambda} \left(1 + \frac{s^2}{\kappa^2}\right) \]

\[ m = 1 \text{ eV}, \quad \lambda = 1.8 \times 10^4 \text{ km}, \quad \kappa = 25 \text{ km} \]

In supernovae typical value is $\lambda \sim 10^3 \text{ Km}$.
• In this case, coherent neutrino-to-antineutrino transformation occurs, mostly in the low-energy part of the spectrum.

Such an effect would have significant impact on nucleosynthesis, supernova dynamics, and neutrino signal from supernovae.
• In this case, coherent neutrino-to-antineutrino transformation occurs, mostly in the low-energy part of the spectrum

HOWEVER: this is a simplified exploratory study! Robust conclusions require: matter ($Y_e$) feedback, realistic “bulb” model of Supernova (multi-angle, geometric effects), multi flavor, and eventually inelastic collisions
Generalizations, conclusion & outlook
From one to $n_f$ flavors

- Evolution controlled by $2n_f \times 2n_f$ Hamiltonian

$$H = \begin{pmatrix} H_{LL} & H_{LR} \\ H_{RL} & H_{RR} \end{pmatrix}$$

$$DF = -i[H, F] + C$$

$\Sigma_\mu$ and $m$ become $n_f \times n_f$ matrices
From one to $n_f$ flavors

- Evolution controlled by $2n_f \times 2n_f$ Hamiltonian

$$H = \begin{pmatrix} H_{LL} & H_{LR} \\ H_{RL} & H_{RR} \end{pmatrix}$$

$$DF = -i[H, F] + C$$

$$H_{LL} = \Sigma^0 - \hat{\Sigma} \cdot \hat{p} + \frac{1}{2|\hat{p}|} m^\dagger m$$

$$H_{RR} = -\left(\Sigma^0 - \hat{\Sigma} \cdot \hat{p}\right)^T + \frac{1}{2|\hat{p}|} mm^\dagger$$

$$H_{LR} = \frac{1}{2|\hat{p}|} \left(\hat{x}_+ \cdot \hat{\Sigma} m^* + (\hat{x}_+ \cdot \hat{\Sigma} m^*)^T\right)$$

- $H_{LR}$ depends on absolute neutrino mass scale and Majorana phases!
From one to $n_f$ flavors

- Evolution controlled by $2n_f \times 2n_f$ Hamiltonian
  \[ H = \begin{pmatrix} H_{LL} & H_{LR} \\ H_{RL} & H_{RR} \end{pmatrix} \]

  \[ DF = -i [H, F] + C \]

- Two-flavor case (one mixing angle $\theta$ and one Majorana phase $\alpha$):

  \[ m = m_0 \begin{pmatrix} c_\theta^2 + e^{-i\alpha} s_\theta^2 & (e^{-i\alpha} - 1)s_\theta c_\theta \\ (e^{-i\alpha} - 1)s_\theta c_\theta & s_\theta^2 + e^{-i\alpha} c_\theta^2 \end{pmatrix} + \frac{\Delta m^2}{4m_0} \begin{pmatrix} -(c_\theta^2 - e^{-i\alpha} s_\theta^2) & (e^{-i\alpha} + 1)s_\theta c_\theta \\ (e^{-i\alpha} + 1)s_\theta c_\theta & c_\theta^2 e^{-i\alpha} - s_\theta^2 \end{pmatrix} \]

  \[ \Delta m^2 \equiv m_2^2 - m_1^2 \quad m_0 \equiv (1/2)(m_1 + m_2) \]
Conclusions and outlook

• Neutrino QKEs from QFT: many expected features, some surprising ones (spin oscillations in anisotropic environment)

• Neutrino-antineutrino conversion can only happen if lepton-number is violated: astrophysical probe of Majorana nature of neutrinos

• Resonant conversion possible in compact objects

• Next challenges:
  • computational implementation in more realistic SN model
  • map out regimes in which conversion is possible, as a function of absolute mass scale and Majorana phases
  • signatures in neutrino / antineutrino spectra and their impact on nucleosynthesis and SN signal on Earth
Backup
QKEs from Quantum Field Theory

- Exploit hierarchy of scales: \( L_{\text{osc}} \sim E/(\Delta m_{\nu}^2), L_{\text{mfp}}, L_{\text{gradients}} \gg L_{\text{deBroglie}} \)

- Work to 2\(^{nd}\) order in small ratios (E\(\sim\)T):

\[
\begin{align*}
m_{\nu}/E & \sim \Delta m_{\nu}/E \sim \Sigma_{\text{forward}}/E \sim \partial x/E \sim O(\epsilon) \\
\Sigma_{\text{inelastic}}/E & \sim O(\epsilon^2)
\end{align*}
\]

Small neutrino (\(\Delta\))masses \quad Small potential induced by forward scattering on matter and other neutrinos \quad Slowly varying background \quad Weak interaction rates
QKEs from Quantum Field Theory

Advantages of this approach:

- First principles method, forced us to think about L-R coherence
- No guesses or fudging: diagrammatic computations in QFT (real time formalism) determine all terms of the QKEs
- Systematic truncation based on power counting in $\varepsilon$’s

Equations of motion for Green Functions

$$\langle \nu_{\alpha}(x) \bar{\nu}_{\beta}(y) \rangle$$

Evolution equations for $F(x,p)$

$$f_{hh'}^{\alpha\beta}(x, p) \sim \langle a_{\beta}^+(p, h') a_{\alpha}(p, h) \rangle$$
Survival probability for different $E$