Neutrino Mass: From the Terrestrial Laboratory to the Cosmos
Amherst Center for Fundamental Interactions, December 15 2015

Lepton Number Violation in Astrophysical Environments

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Los Alamos National Laboratory

Outline

- Introduction: neutrinos in hot and dense media & QKEs
- Spin-dependent effects in QKEs and ν-$\bar{\nu}$ oscillations (LNV!)
- Neutrino-antineutrino conversion in compact objects: exploration within one-flavor toy model
- Conclusions and outlook

<u>Based on collaboration with George Fuller (UCSD) and Alexey Vlasenko (UCSD → NCSU)</u>

A. Vlasenko, G. Fuller, V. Cirigliano, Phys. Rev. D 89 105004 (2014)

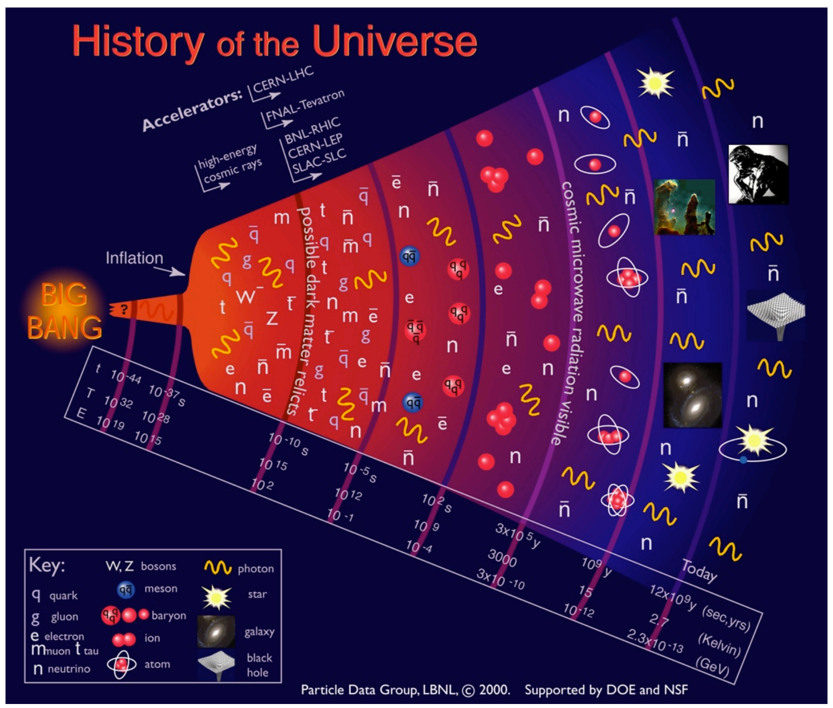
V. Cirigliano, G. Fuller, A. Vlasenko, Phys. Lett. B 747, 27 (2015)

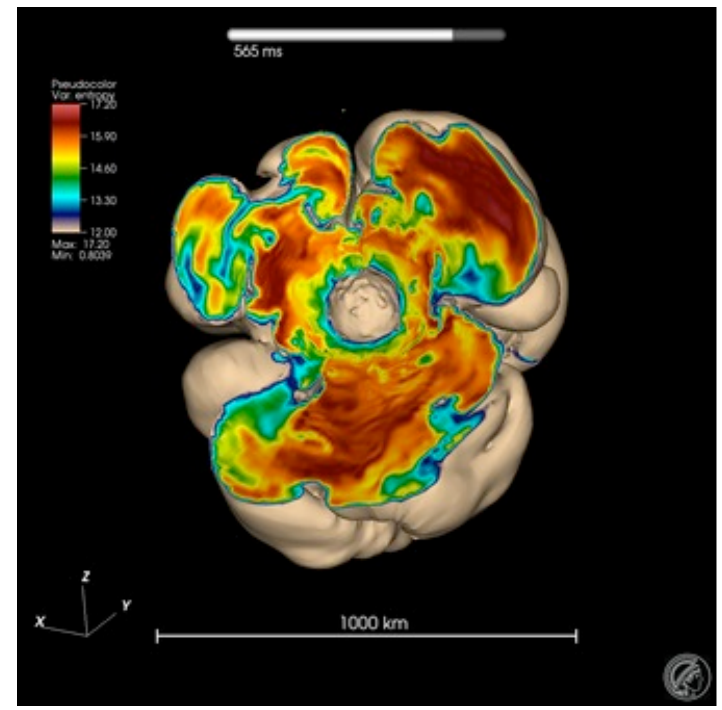
A. Vlasenko, G. Fuller, V. Cirigliano, ArXiv:1406.6724

Neutrinos in hot and dense media & QKEs

Neutrinos in hot / dense medium

- Neutrinos play a key role in early universe and astrophysical systems





- To incorporate the known physics of massive neutrinos and probe non-standard effects one must keep track of coherent flavor oscillations AND de-cohering inelastic collisions with the medium

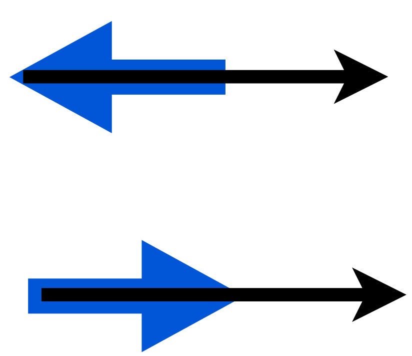
Neutrinos in hot / dense medium

- Ensemble of neutrinos described by "density matrix"
- Spin 1/2 massive neutrino: one flavor

$$F(x,p) = \begin{pmatrix} f_{LL} & f_{LR} \\ f_{RL} & f_{RR} \end{pmatrix}$$

Diagonal: "populations"
Off-diagonal: "coherences"

p, **s**



L-handed state $|\,L\,\rangle$

R-handed state $|\,R\,\rangle$

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For a pure state

$$|\psi\rangle = c_L\,|L\rangle + c_R\,|R\rangle$$

$$f_{LL} = |c_L|^2 \qquad f_{RR} = |c_R|^2 \qquad f_{LR} = c_L c_R^* \qquad f_{RL} = c_R c_L^*$$

Neutrinos in hot / dense medium

- Ensemble of neutrinos described by "density matrix"
- Spin 1/2 massive neutrino: many flavors

$$F(x,p) = \begin{pmatrix} f_{LL}^{\alpha\beta} & f_{LR}^{\alpha\beta} \\ f_{RL}^{\alpha\beta} & f_{RR}^{\alpha\beta} \end{pmatrix}$$

flavor

spin

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$$F(x,p) = \begin{pmatrix} f^{\alpha\beta}_{LL} & f^{\alpha\beta}_{LR} \\ f^{\alpha\beta}_{RL} & f^{\alpha\beta}_{RR} \end{pmatrix}$$

flavor

spin

$$\begin{pmatrix} f^{ee} & f^{e\mu} \\ f^{\mu e} & f^{\mu\mu} \end{pmatrix}$$

Usual neutrino density matrix
(for two flavors e, μ)

Neutrinos in hot / dense medium

- Ensemble of neutrinos described by "density matrix"
- Spin 1/2 massive neutrino: many flavors (Majorana)

$$F(x,p) = \begin{pmatrix} f & \phi \\ \phi^\dagger & \bar{f}^T \end{pmatrix}$$

R-handed neutrino = antineutrino ⇒ $f \equiv f_{LL}$ $\bar{f} \equiv f^T_{RR}$ $\phi \equiv f_{LR}$

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$$F(x,p) = \begin{pmatrix} f & \phi \\ \phi^\dagger & \bar{f}^T \end{pmatrix}$$

n_f x n_f blocks describing density matrix for neutrinos and antineutrinos

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- QKEs: evolution equations for F(x,p)

QKEs from Quantum Field Theory

A. Vlasenko, G. Fuller, V. Cirigliano, Phys. Rev. D 89 105004 (2014)

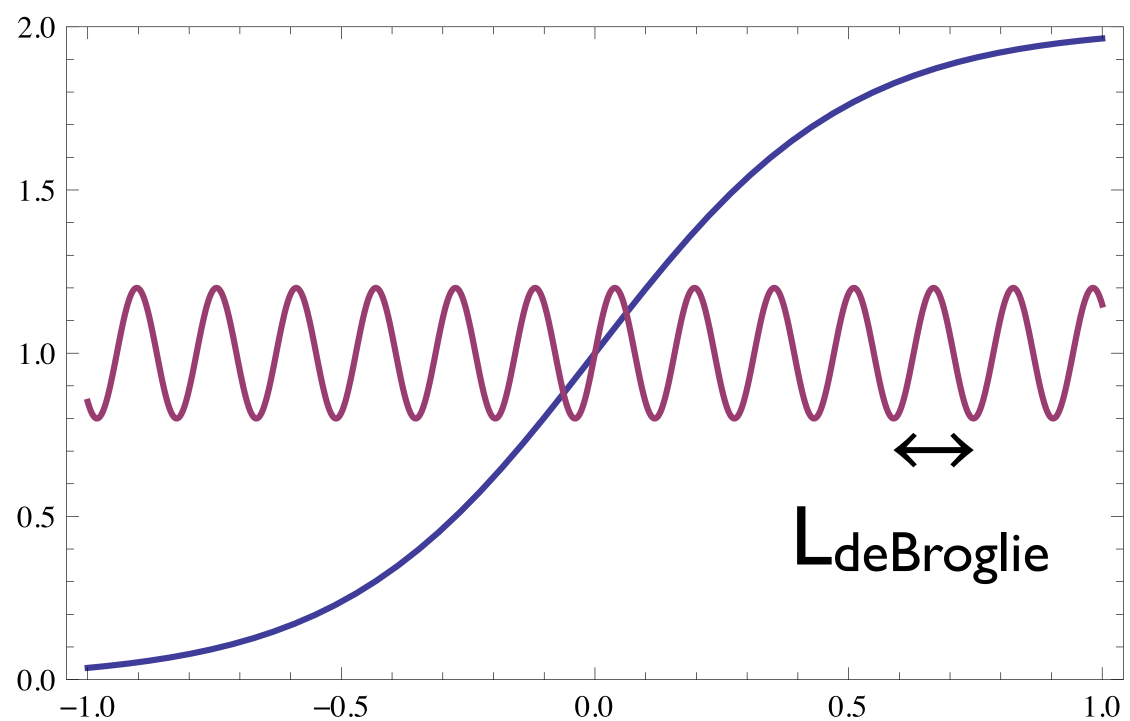
Equations of motion for Green Functions
$\langle \nu_\alpha(x) \bar{\nu}_\beta(y) \rangle$

→

Evolution equations for F(x,p)
$f^{\alpha\beta}_{hh'}(x,p) \sim \langle\, a^\dagger_\beta(p,h')\; a_\alpha(p,h)\,\rangle$

- Exploit hierarchy of scales: $L_{osc} \sim E/(\Delta m_\nu^2)$, L_{mfp}, $L_{gradients}$ >> $L_{deBroglie}$

- Work to 2nd order in small ratios of length scales



Structure of the QKEs

A. Vlasenko, G. Fuller, V. Cirigliano, Phys. Rev. D 89 105004 (2014),
Sigl-Raffelt 1993, McKellar-Thomson 1994, Enqvist-Kainulainen-Maalampi 1991, Dolgov-Barbieri 1991

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$$DF = -i[H, F] + C$$

F, H(F), C(F): $2n_f \times 2n_f$ matrices

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Derivative along ν world line

"Liouville"

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“Liouville”

Coherent evolution:
vacuum mass &
neutrino potential

“Oscillation”

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Derivative along ν world line

"Liouville"

Coherent evolution: vacuum mass & neutrino potential

"Oscillation"

Inelastic collisions

"Boltzmann"

Spin-dependent effects in QKEs

A new spin on the QKEs

$$F(x,p) = \begin{pmatrix} f & \phi \\ \phi^\dagger & \bar{f}^T \end{pmatrix}$$

$$DF = -i[H, F] + C$$

- Coherent helicity-flip is possible (even in absence of magnetic moment and magnetic field)

$$H = \begin{pmatrix} H_{LL} & H_{LR} \\ H_{RL} & H_{RR} \end{pmatrix}$$

V. Cirigliano, G. Fuller, A.Vlasenko, Phys. Lett. B 747, 27 (2015), arXiv:1406.5558

A.Vlasenko, G. Fuller, V. Cirigliano, ArXiv:1406.6724

Basic results also obtained in: Serreau-Volpe, arXiv:1409.3591 & Kartavstev-Raffelt-Vogel, arXiv:1504.03230

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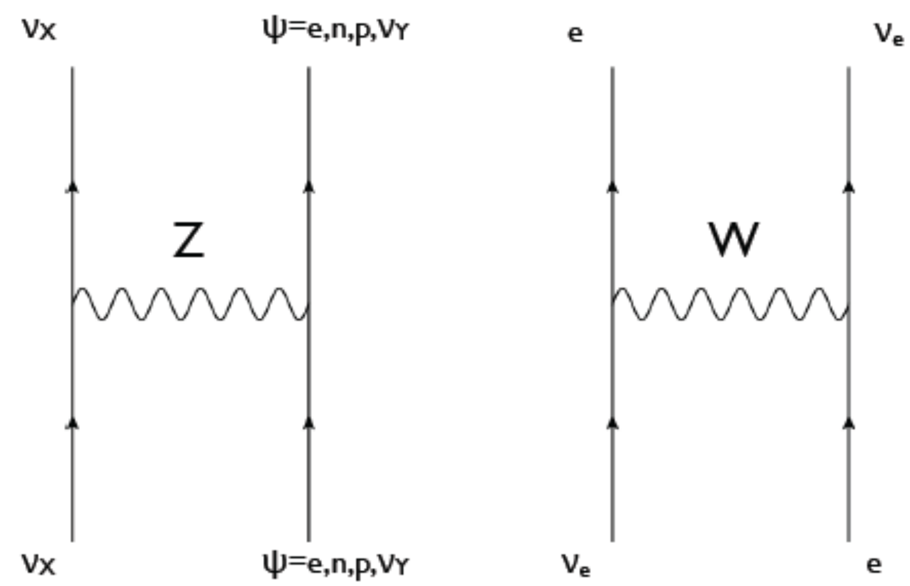
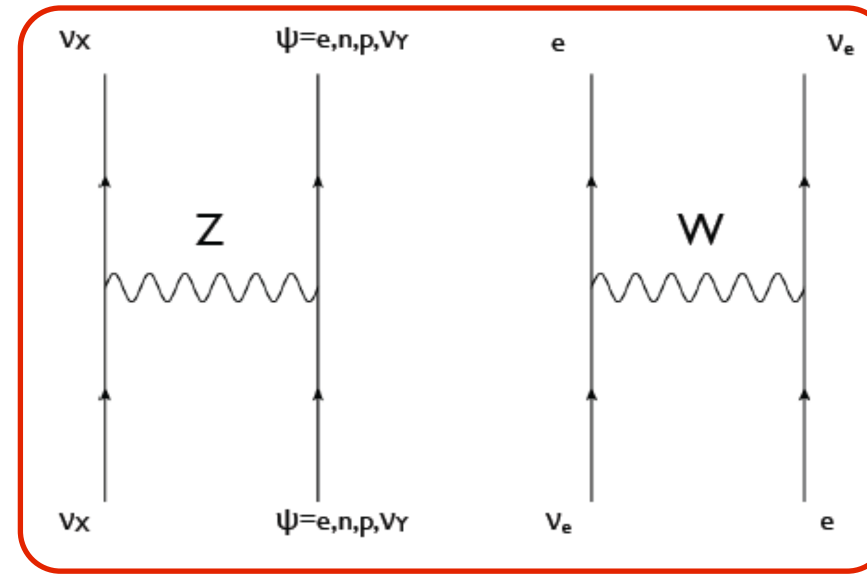
$$H = \begin{pmatrix} H_{LL} & H_{LR} \\ H_{RL} & H_{RR} \end{pmatrix}$$

- H_{LR} is determined by mass matrix and neutrino potentials, generated by forward scattering in medium



Neutrino potentials and spin effects

- Key point: weak interactions generate *4-vector* potential, with time- and space-like components in non-isotropic medium



$$\mathcal{H}_{\rm int} \ = \ -\bar{\nu}\ \Sigma_\mu \gamma^\mu \gamma_5\ \nu$$

4-potential couples to the neutrino axial current:

$$H_{\rm int} \sim \mathbf{J} \cdot \mathbf{\Sigma}$$

This terms mixes LH and RH ν states

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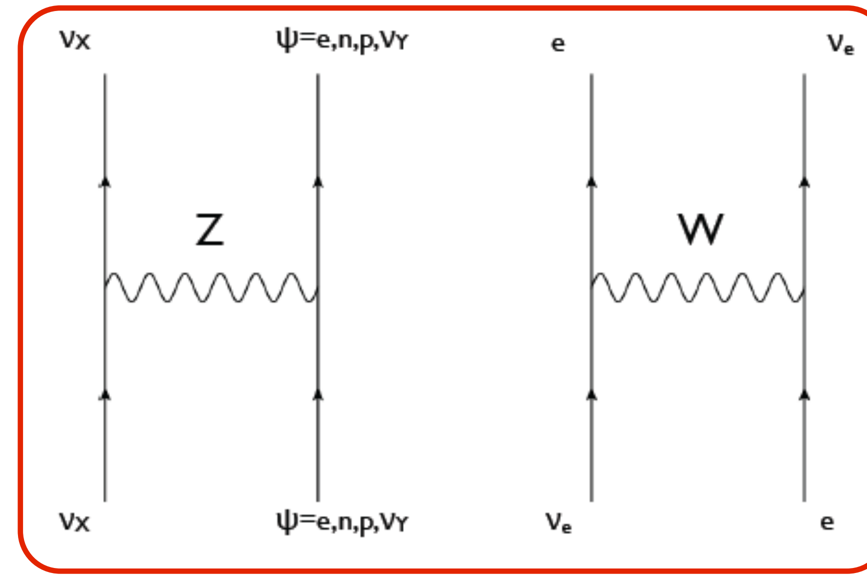
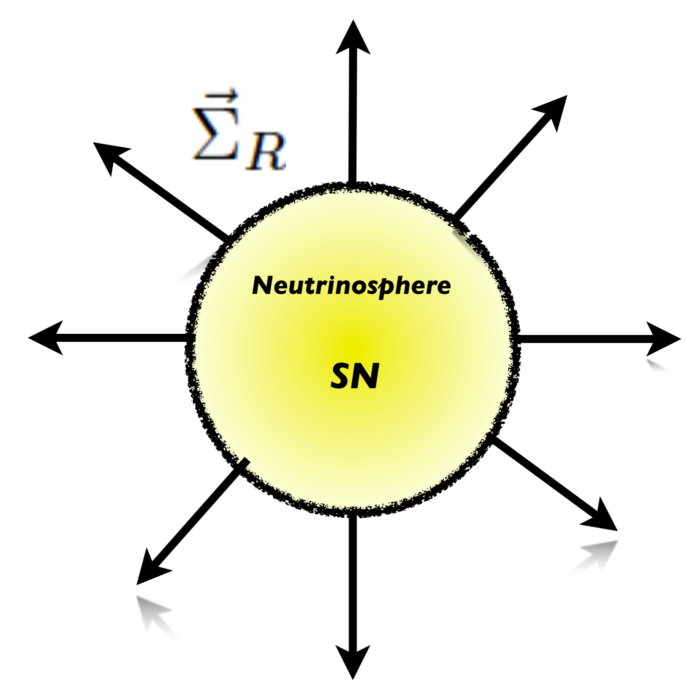


- Σ_μ receives contributions from matter and neutrinos, e.g.

$$\Sigma^\mu_{(\nu)} = \sqrt{2} G_F \left(J^\mu_{(\nu)} + \mathbf{1}\ \mathrm{tr} J^\mu_{(\nu)} \right)$$

$$J^\mu_{(\nu)}(x) = \int \frac{d^3 q}{(2\pi)^3}\, n^\mu(q) \left(f(\vec{q}, x) - \bar{f}(\vec{q}, x) \right)$$

$$n^\mu(q) = (1, \hat{q})$$

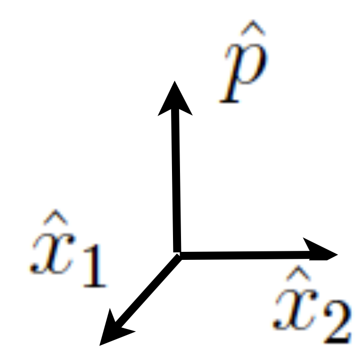


Even in simple "bulb" model for SN: $\vec{\Sigma} \neq 0$

ν-$\bar{\nu}$ mixing: one flavor

- The 2x2 effective hamiltonian H takes the form

$$H = \left(p + \frac{m^2}{2p}\right) \times I \ + \ \begin{pmatrix} \Sigma^0 - \hat{p}\cdot\vec{\Sigma} & (m/p)\,\hat{x}_+ \cdot \vec{\Sigma} \\ (m/p)\,\hat{x}_+^* \cdot \vec{\Sigma} & -(\Sigma^0 - \hat{p}\cdot\vec{\Sigma}) \end{pmatrix}$$



$$\hat{x}_+ = \hat{x}_1 + i\hat{x}_2$$

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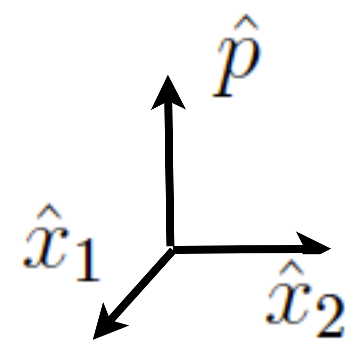
- Medium birefringence



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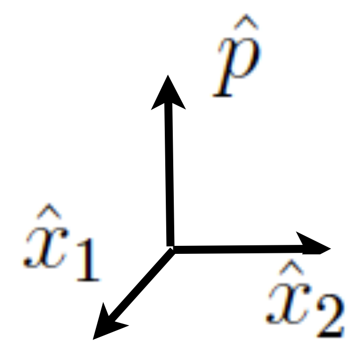

$$\hat{x}_+ = \hat{x}_1 + i\hat{x}_2$$

- Helicity mixing / oscillations:
  - *Linear* in neutrino mass: manifestation of Lepton Number Violation; sensitive to absolute mass scale
  - Proportional to component of the potential *transverse* to the neutrino momentum (anisotropy)

ν-$\bar{\nu}$ mixing: one flavor

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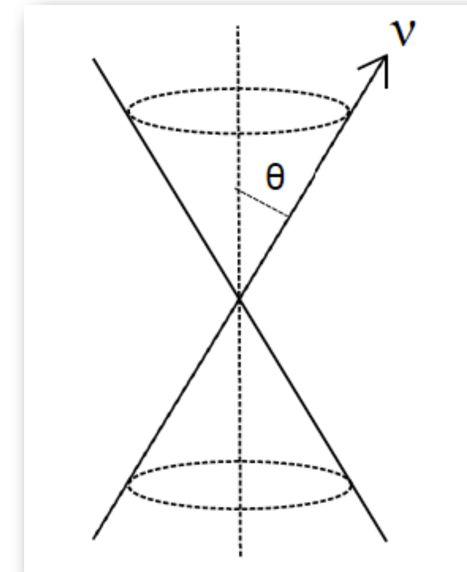
- Off-diagonal terms parametrically suppressed (m/p) compared to diagonal ones
- But resonant conversion is possible if diagonal elements vanish
- Astrophysical environments?

Neutrino-antineutrino conversion in compact objects?

ν-$\bar{\nu}$ conversion in compact objects?

A.Vlasenko, G. Fuller, V. Cirigliano, ArXiv:1406.6724



u = cosθ

- Toy problem: crossed neutrino beams of infinite width in time-varying background matter

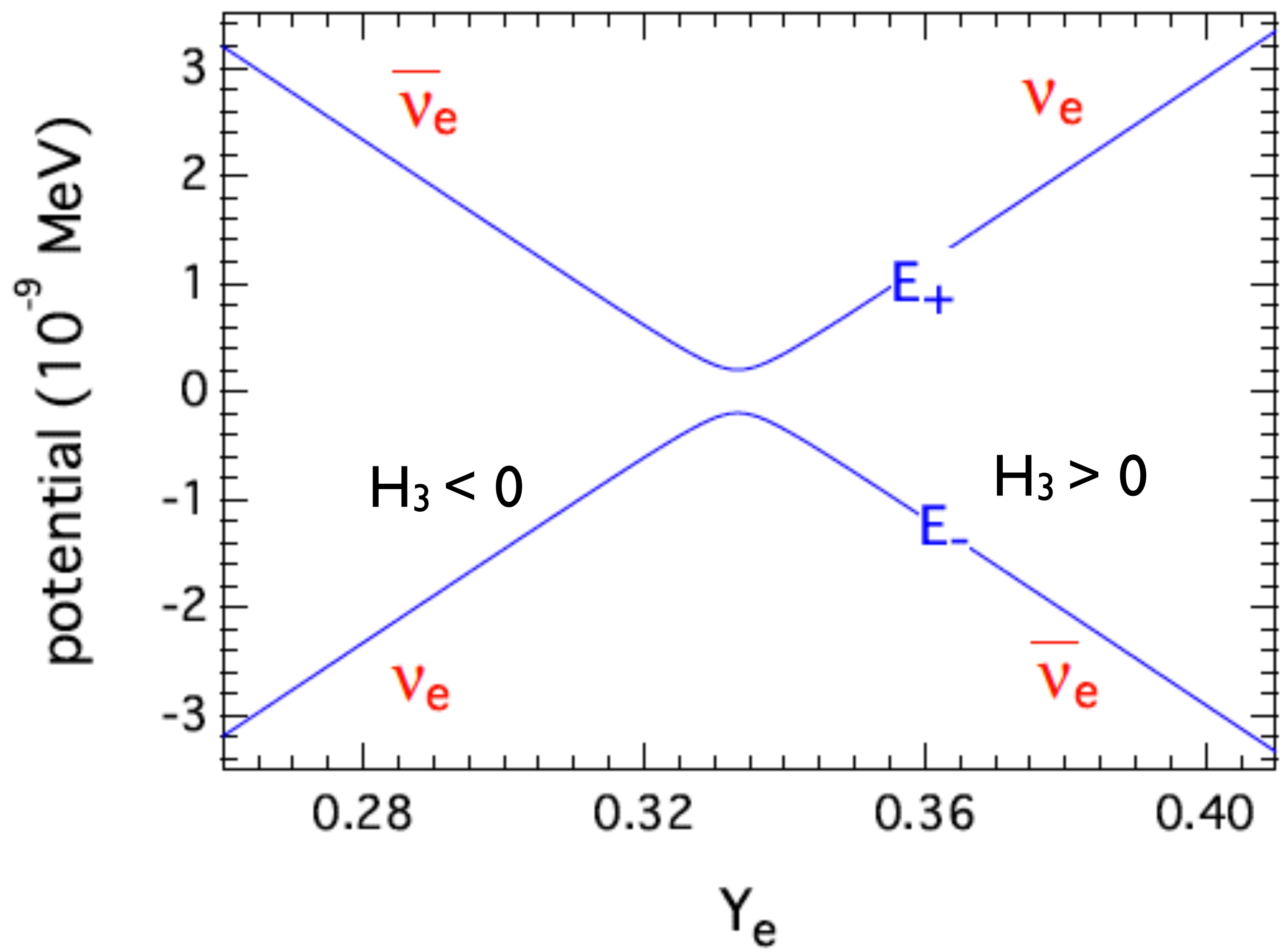
- Effective hamiltonian (one flavor)

$$H = \begin{pmatrix} H_3 & H_1 \\ H_1 & -H_3 \end{pmatrix} \qquad H_1 = 2\sqrt{2}G_F\, u\sqrt{1-u^2}\frac{m}{E}(n_\nu - n_{\bar{\nu}})$$

$$H_3 = \frac{G_F}{\sqrt{2}}\left[(3Y_e - 1)\, n_B + 4\,(1-u^2)(n_\nu - n_{\bar{\nu}})\right]$$

Resonance condition: matter and neutrino contribution to $H_3 = H_{Matter} + H_\nu$ cancel.
Can happen near protoneutron star, or in central region of compact object merger ($Y_e \sim 1/3$)

- Level crossing

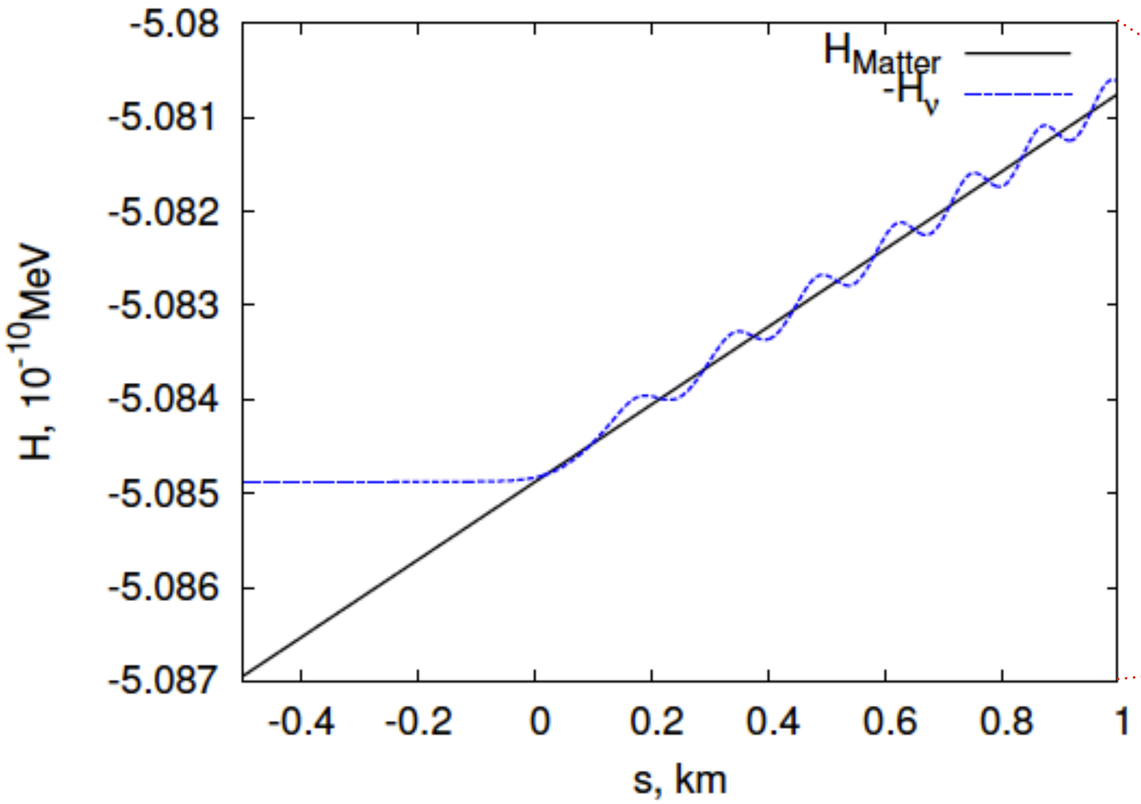


$$n_B = 300 \text{ MeV}^3$$
$$T = 4 \text{ MeV}$$
$$\mu = 8 \text{ MeV}.$$

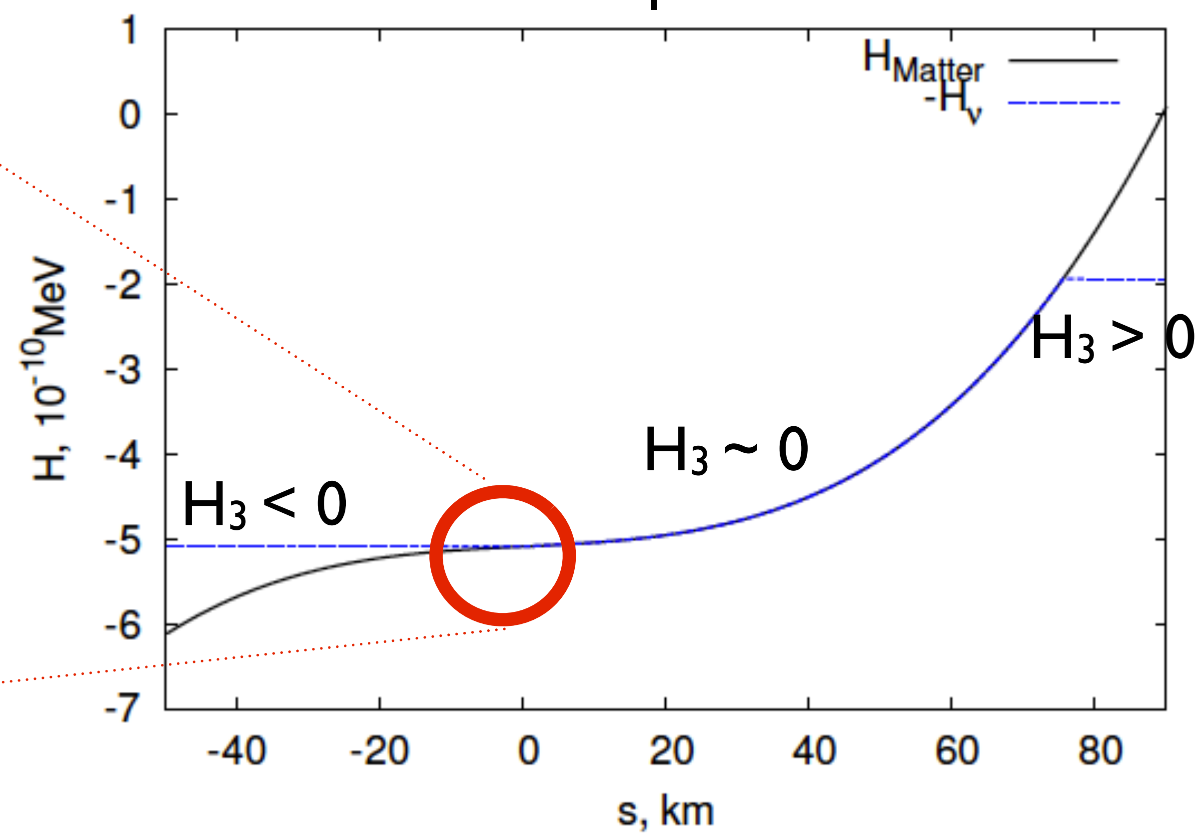
- Can we get large scale conversion?

- Neutrino feedback keeps system close to resonance: $\delta H_{Matter}>0$ while $\delta H_{\nu}<0$ as neutrinos start to convert into antineutrinos

Onset of coherent neutrino-antineutrino transformation



Cancellation between matter and neutrino potential



$$Y_e = Y_{e0} + \frac{s}{\lambda}\left(1 + \frac{s^2}{\kappa^2}\right)$$

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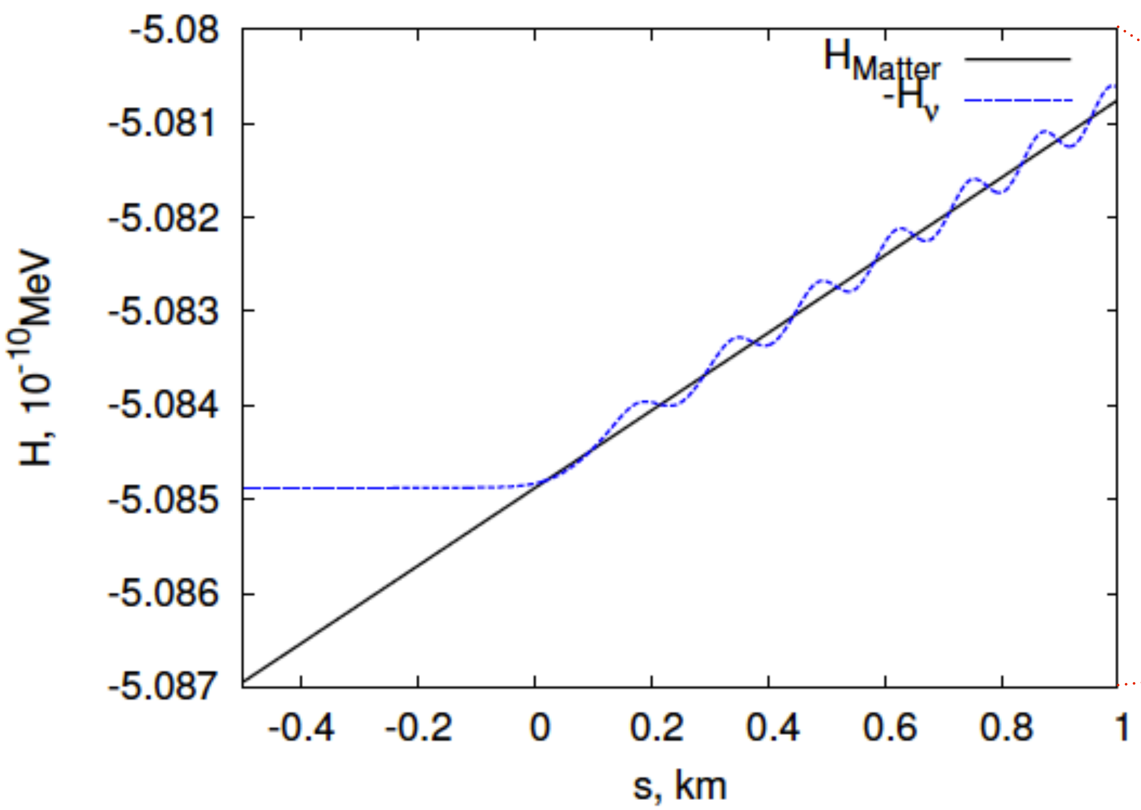

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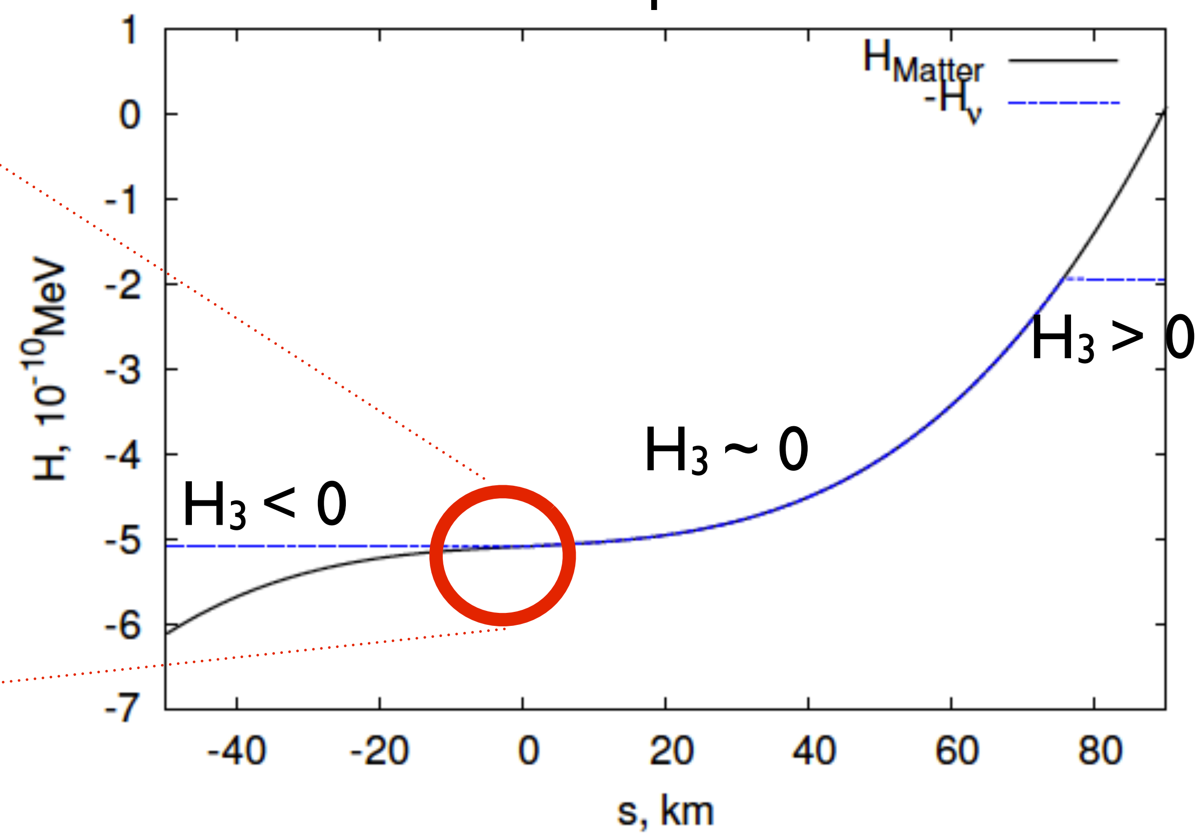


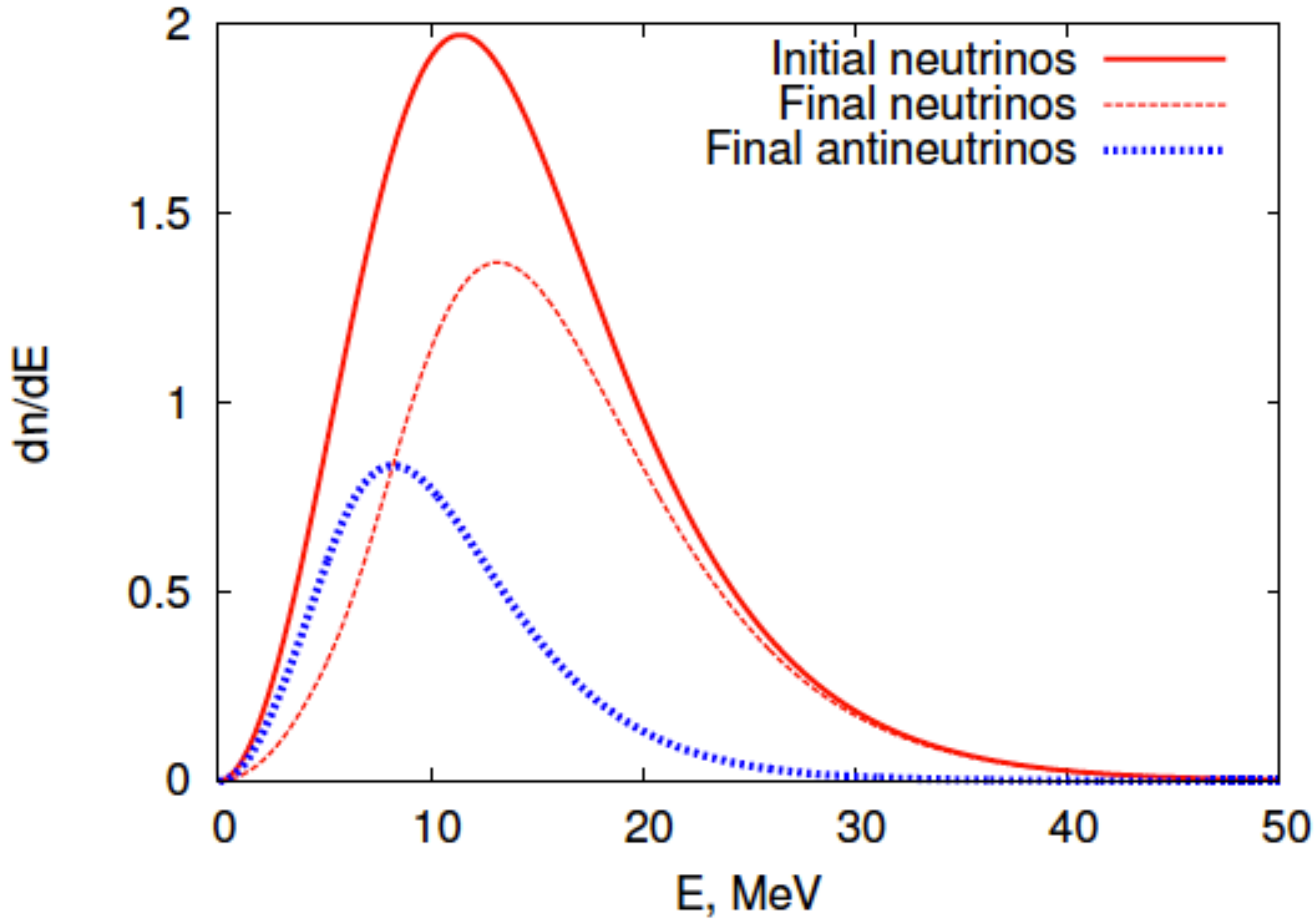
- Requires small derivative of the matter potential and largish neutrino masses (*the two effects are correlated*)

$$Y_e = Y_{e0} + \frac{s}{\lambda}\left(1 + \frac{s^2}{\kappa^2}\right)$$

$m = 1\,\mathrm{eV}$, $\lambda = 1.8 \times 10^4\,\mathrm{km}$, $\kappa = 25\,\mathrm{km}$

In supernovae typical value is $\lambda \sim 10^3$ Km

- In this case, coherent neutrino-to-antineutrino transformation occurs, mostly in the low-energy part of the spectrum



Such an effect would have significant impact on nucleosynthesis, supernova dynamics, and neutrino signal from supernovae

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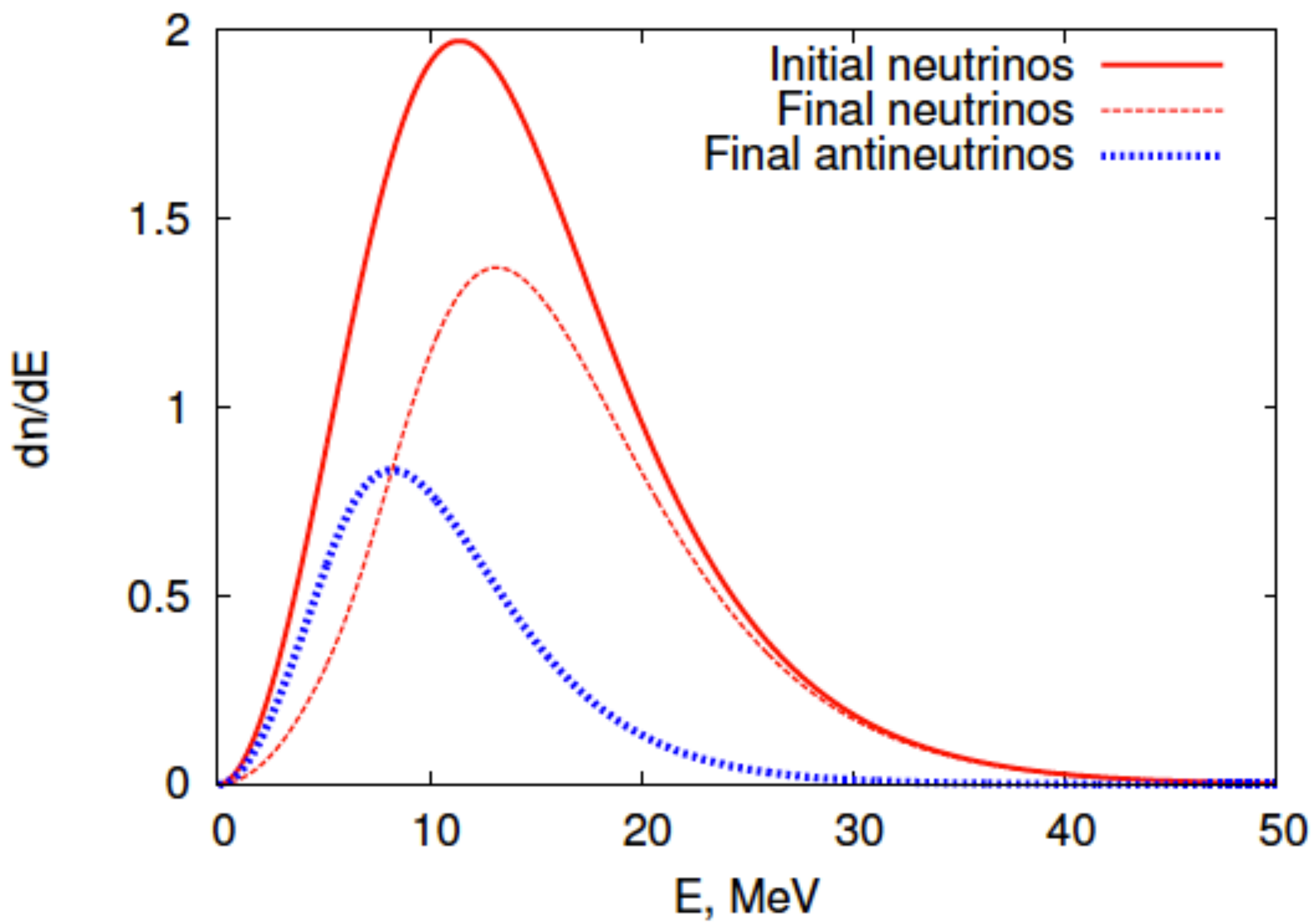

HOWEVER: this is a simplified exploratory study!
Robust conclusions require: matter (Y_e) feedback, realistic "bulb" model of Supernova (multi-angle, geometric effects), multi flavor, and eventually inelastic collisions

Generalizations, conclusion & outlook

From one to n_f flavors

- Evolution controlled by $2n_f$ x $2n_f$ Hamiltonian $H = \begin{pmatrix} H_{LL} & H_{LR} \\ H_{RL} & H_{RR} \end{pmatrix}$

$$DF = -i[H, F] + C$$

Σ_μ and m become n_f x n_f matrices

From one to n_f flavors

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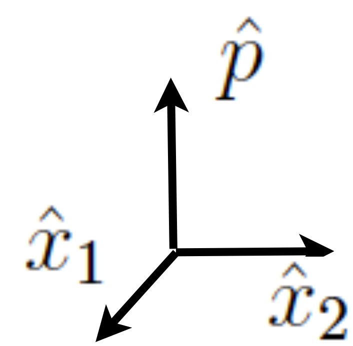
$$\begin{aligned} H_{LL} &= \Sigma^0 - \vec{\Sigma} \cdot \hat{p} + \frac{1}{2|\vec{p}|} m^\dagger m \\ H_{RR} &= -\left(\Sigma^0 - \vec{\Sigma} \cdot \hat{p}\right)^T + \frac{1}{2|\vec{p}|} m m^\dagger \\ H_{LR} &= \frac{1}{2|\vec{p}|}\left(\hat{x}_+ \cdot \vec{\Sigma}\, m^* + (\hat{x}_+ \cdot \vec{\Sigma}\, m^*)^T\right) \end{aligned}$$



$$\hat{x}_+ = \hat{x}_1 + i\hat{x}_2$$

- H_{LR} depends on absolute neutrino mass scale and Majorana phases!

From one to n_f flavors

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$$DF = -i[H, F] + C$$

- Two-flavor case (one mixing angle θ and one Majorana phase α):

$$m = m_0 \begin{pmatrix} c_\theta^2 + e^{-i\alpha} s_\theta^2 & (e^{-i\alpha} - 1) s_\theta c_\theta \\ (e^{-i\alpha} - 1) s_\theta c_\theta & s_\theta^2 + e^{-i\alpha} c_\theta^2 \end{pmatrix} + \frac{\Delta m^2}{4 m_0} \begin{pmatrix} -(c_\theta^2 - e^{-i\alpha} s_\theta^2) & (e^{-i\alpha} + 1) s_\theta c_\theta \\ (e^{-i\alpha} + 1) s_\theta c_\theta & c_\theta^2 e^{-i\alpha} - s_\theta^2 \end{pmatrix}$$

$$\Delta m^2 \equiv m_2^2 - m_1^2 \qquad m_0 \equiv (1/2)(m_1 + m_2)$$

Conclusions and outlook

- Neutrino QKEs from QFT: many expected features, some surprising ones (spin oscillations in anisotropic environment)
- Neutrino-antineutrino conversion can only happen if lepton-number is violated: astrophysical probe of Majorana nature of neutrinos
- Resonant conversion possible in compact objects
- Next challenges:
  - computational implementation in more realistic SN model
  - map out regimes in which conversion is possible, as a function of absolute mass scale and Majorana phases
  - signatures in neutrino / antineutrino spectra and their impact on nucleosynthesis and SN signal on Earth

Backup

QKEs from Quantum Field Theory

Equations of motion for Green Functions
$\langle \nu_\alpha(x) \bar{\nu}_\beta(y) \rangle$

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Evolution equations for F(x,p)
$f^{\alpha\beta}_{hh'}(x,p) \sim \langle a^\dagger_\beta(p,h')\, a_\alpha(p,h) \rangle$

- Exploit hierarchy of scales: $L_{osc} \sim E/(\Delta m_\nu^2)$, L_{mfp}, $L_{gradients}$ >> $L_{deBroglie}$

- Work to 2nd order in small ratios (E~T):

$m_\nu/E \sim \Delta m_\nu/E \sim \Sigma_{forward}/E \sim \partial_X/E \sim O(\varepsilon)$ $\qquad \Sigma_{inelastic}/E \sim O(\varepsilon^2)$

Small neutrino (Δ)masses

Small potential induced by forward scattering on matter and other neutrinos

Slowly varying background

Weak interaction rates

QKEs from Quantum Field Theory

Equations of motion for Green Functions
$\langle \nu_\alpha(x) \bar{\nu}_\beta(y) \rangle$

→

Evolution equations for F(x,p)
$f^{\alpha\beta}_{hh'}(x,p) \sim \langle \, a^\dagger_\beta(p,h') \; a_\alpha(p,h) \, \rangle$

- Advantages of this approach:
  - First principles method, forced us to think about L-R coherence
  - No guesses or fudging: diagrammatic computations in QFT (real time formalism) determine all terms of the QKEs
  - Systematic truncation based on power counting in ε's

Survival probability for different E