Recoil Frontier in Beta Decay

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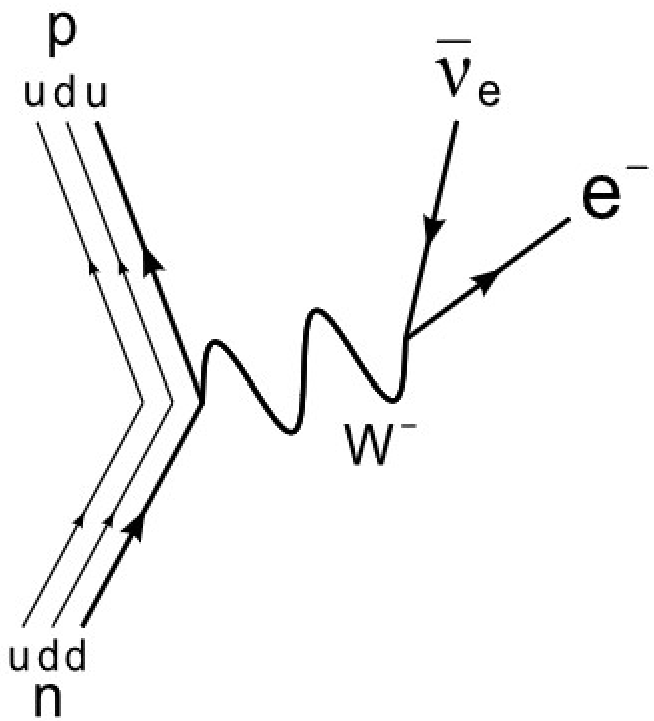
What are we talking about? Consider measurement of neutron beta decay–



Typically write

$$\frac{\mathrm{d}T^3}{\mathrm{d}E_\mathrm{e}\mathrm{d}\Omega_e\mathrm{d}\Omega_\nu} = \Phi(E_\mathrm{e})G_\mathrm{F}^2|V_\mathrm{ud}|^2(1+3\lambda^2)$$

$$\times\left(1+b\frac{m_\mathrm{e}}{E_\mathrm{e}}+a\frac{\boldsymbol{p}_\mathrm{e}\cdot\boldsymbol{p}_\mathrm{i}}{E_\mathrm{e}E_\mathrm{i}}+\boldsymbol{\sigma}\left[A\frac{\boldsymbol{p}_\mathrm{e}}{E_\mathrm{e}}+B\frac{\boldsymbol{p}_\mathrm{i}}{E_\mathrm{i}}+D\frac{\boldsymbol{p}_\mathrm{e}\times\boldsymbol{p}_\mathrm{i}}{E_\mathrm{e}E_\mathrm{i}}\right]\right),$$

with

$$a=\frac{1-|\lambda|^2}{1+3|\lambda|^2}, \qquad A=-2\frac{|\lambda|^2+R\mathrm{e}(\lambda)}{1+3|\lambda|^2},$$

$$B=2\frac{|\lambda|^2-R\mathrm{e}(\lambda)}{1+3|\lambda|^2}, \qquad D=2\frac{\mathrm{Im}(\lambda)}{1+3|\lambda|^2}.$$

Here

$$\lambda = \frac{g_A(0)}{g_V(0)} \simeq 1.275$$

and "Fierz interference term"

$$b = 0$$

if only V, A interactions.

Precision analysis requires inclusion of E_e, E_ν dependence in order to find possible BSM effects. But interesting physics also in these "recoil" effects.

Historical Evolution of Analysis:

1957: Jackson, Treiman, and Wyld use general weak interaction Lorentz-invariant form

$$
\begin{aligned}
H_{\rm int} = & (\bar{\psi}_p \psi_n)(C_S \bar{\psi}_e \psi_\nu + C_S' \bar{\psi}_e \gamma_5 \psi_\nu) \\
& + (\bar{\psi}_p \gamma_\mu \psi_n)(C_V \bar{\psi}_e \gamma_\mu \psi_\nu + C_V' \bar{\psi}_e \gamma_\mu \gamma_5 \psi_\nu) \\
& + \tfrac{1}{2} (\bar{\psi}_p \sigma_{\lambda\mu} \psi_n)(C_T \bar{\psi}_e \sigma_{\lambda\mu} \psi_\nu + C_T' \bar{\psi}_e \sigma_{\lambda\mu} \gamma_5 \psi_\nu) \\
& - (\bar{\psi}_p \gamma_\mu \gamma_5 \psi_n)(C_A \bar{\psi}_e \gamma_\mu \gamma_5 \psi_\nu + C_A' \bar{\psi}_e \gamma_\mu \psi_\nu) \\
& + (\bar{\psi}_p \gamma_5 \psi_n)(C_P \bar{\psi}_e \gamma_5 \psi_\nu + C_P' \bar{\psi}_e \psi_\nu)
\end{aligned}
$$

JTW introduced notation

$$\omega(\langle J\rangle | E_e,\Omega_e,\Omega_\nu)dE_e d\Omega_e d\Omega_\nu$$

$$= \frac{1}{(2\pi)^5} p_e E_e (E^0 - E_e)^2 dE_e d\Omega_e d\Omega_\nu \xi \left\{ 1 + a\frac{\mathbf{p}_e\cdot\mathbf{p}_\nu}{E_e E_\nu} + b\frac{m}{E_e} \right.$$

$$+ c\left[\frac{1}{3}\frac{\mathbf{p}_e\cdot\mathbf{p}_\nu}{E_e E_\nu} - \frac{(\mathbf{p}_e\cdot\mathbf{j})(\mathbf{p}_\nu\cdot\mathbf{j})}{E_e E_\nu}\right]\left[\frac{J(J+1)-3\langle(\mathbf{J}\cdot\mathbf{j})^2\rangle}{J(2J-1)}\right]$$

$$\left. + \frac{\langle\mathbf{J}\rangle}{J}\cdot\left[A\frac{\mathbf{p}_e}{E_e} + B\frac{\mathbf{p}_\nu}{E_\nu} + D\frac{\mathbf{p}_e\times\mathbf{p}_\nu}{E_e E_\nu}\right]\right\}.$$

with

$$\xi = |M_F|^2(|C_S|^2 + |C_V|^2 + |C_S'|^2 + |C_V'|^2) + |M_{GT}|^2(|C_T|^2 + |C_A|^2 + |C_T'|^2 + |C_A'|^2),$$

$$a\xi = |M_F|^2(-|C_S|^2 + |C_V|^2 - |C_S'|^2 + |C_V'|^2) + \frac{|M_{GT}|^2}{3}(|C_T|^2 - |C_A|^2 + |C_T'|^2 - |C_A'|^2),$$

$$b\xi = \pm 2\,\mathrm{Re}[|M_F|^2(C_S C_V^* + C_S' C_V'^*) + |M_{GT}|^2(C_T C_A^* + C_T' C_A'^*)],$$

and

$$A\xi = 2\,\mathrm{Re}\Big[\pm|M_{GT}|^2\lambda_{J'J}(C_T C_T'^* - C_A C_A'^*)$$
$$+\delta_{J'J}|M_F||M_{GT}|\left(\frac{J}{J+1}\right)^{\frac{1}{2}}(C_S C_T'^* + C_S' C_T^* - C_V C_A'^* - C_V' C_A^*)\Big],$$

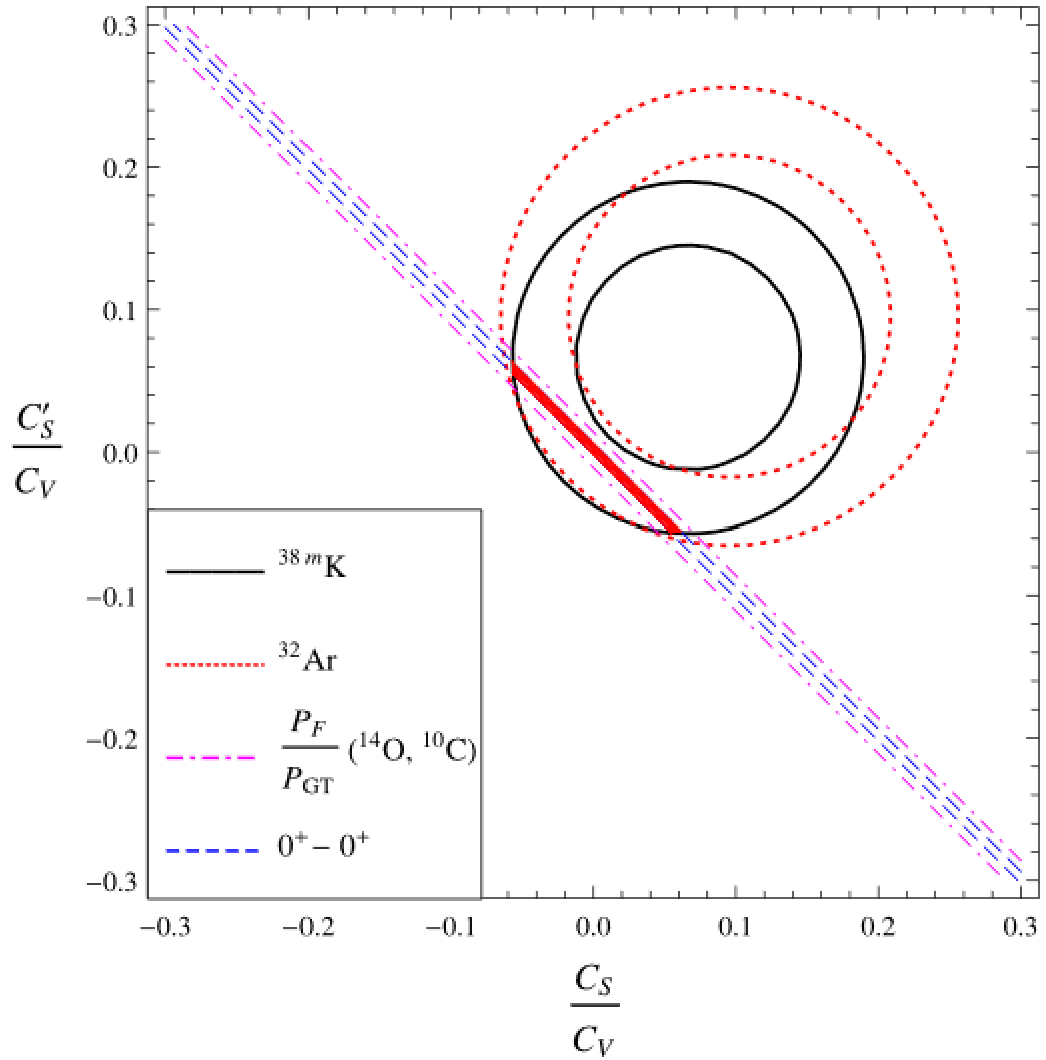
$$B\xi = 2\,\mathrm{Re}\Big\{|M_{GT}|^2\lambda_{J'J}\left[\frac{m}{E_e}(C_T C_A'^* + C_T' C_A^*) \pm (C_T C_T'^* + C_A C_A'^*)\right] - \delta_{J'J}|M_F||M_{GT}|\left(\frac{J}{J+1}\right)^{\frac{1}{2}}$$
$$\times\left[(C_S C_T'^* + C_S' C_T^* + C_V C_A'^* + C_V' C_A^*) \pm \frac{m}{E_e}(C_S C_A'^* + C_S' C_A^* + C_V C_T'^* + C_V' C_T^*)\right]\Big\},$$

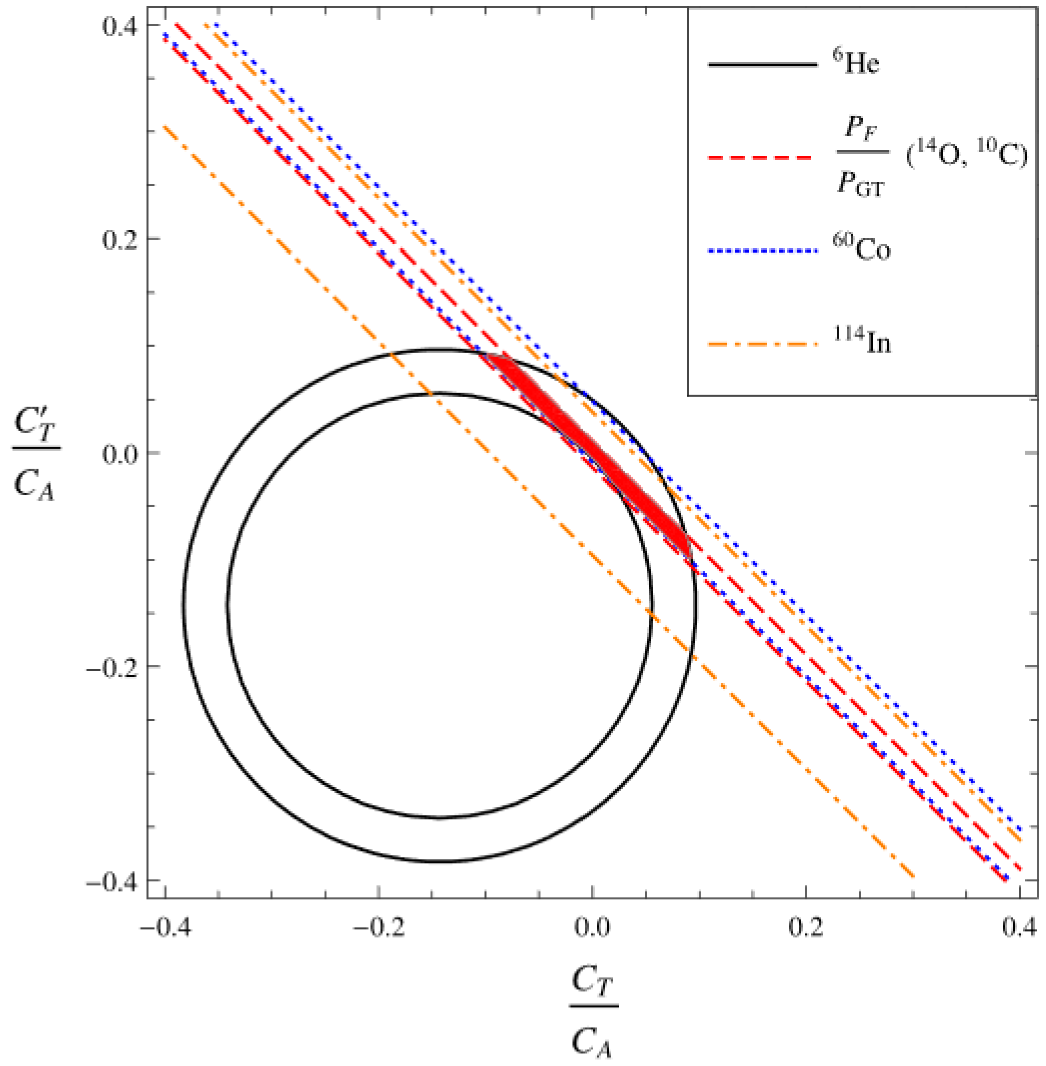
where

$$M_F = <\beta||\sum_i \tau_i^{\pm}||\alpha>, \quad M_{GT} = <\beta||\sum_i \tau_i^{\pm}\boldsymbol{\sigma}_i||\alpha>$$

are the Fermi, Gamow-Teller matrix elements.

JTW forms used in many BSM searches—





New challenges presented by use of recoil—

1965: Kim amd Primakoff—"elementary particle" method. In BRH, RMP **46**, 789 (1974) notation

$$\mathrm{Amp}_{EP}(s_f, s_i) = \frac{G_F}{\sqrt{2}} V_{ud} \ell^\mu \left\{ a\left(1+\frac{\alpha}{4\pi}e_V\right)\frac{P_\mu}{2m_N} C^{M'0;M}_{J'0;J} \right.$$

$$- \left(1+\frac{\alpha}{4\pi}e_A\right)\frac{1}{4m_N}\left(cP^\gamma - dq^\lambda\right)\epsilon_{ijk}\epsilon_{ij\gamma\mu} C^{M'k;M}_{J'1;J}$$

$$\left. +i\frac{b}{4m_N^2}\epsilon_{k\delta\gamma\mu} q^\delta P^\gamma C^{M'k;M}_{J'1;J} \right\}$$

where

$$\ell^\mu = \bar{u}_e(p_e)\gamma^\mu(1+\gamma_5)v_\nu(k)$$

which contains recoil effects to $\mathcal{O}(q)$. Now spectrum takes the form, for arbitrary allowed transitions

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = F_\mp(Z,E_e)\frac{\tilde{G}_F^2 V_{ud}^2}{(2\pi)^5} p_e E_e (E_0 - E_e)^2 \Bigg\{ f_1(E_e) + f_2(E_e)\frac{\boldsymbol{p}_e}{E_e}\cdot\hat{\boldsymbol{k}}$$

$$+ f_3(E_e)\left(\left(\frac{\boldsymbol{p}_e}{E_e}\cdot\hat{\boldsymbol{k}}\right)^2 - \frac{1}{3}\frac{p_e^2}{E_e^2}\right) + \left(f_4(E_e) + f_5(E_e)\frac{\boldsymbol{p}_e}{E_e}\cdot\hat{\boldsymbol{k}}\right)\Lambda^{(1)}\hat{\boldsymbol{n}}\cdot\frac{\boldsymbol{p}_e}{E_e}$$

$$+ \left(f_6(E_e) + f_7(E_e)\frac{\boldsymbol{p}_e}{E_e}\cdot\hat{\boldsymbol{k}}\right)\Lambda^{(1)}\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{k}}$$

$$+ \Lambda^{(2)} T^{(2)}(\hat{\boldsymbol{n}}) : \left(f_{10}(E_e)[\frac{\boldsymbol{p}_e}{E_e},\frac{\boldsymbol{p}_e}{E_e}] + f_{12}(E_e)[\frac{\boldsymbol{p}_e}{E_e},\hat{\boldsymbol{k}}] + f_{14}(E_e)[\hat{\boldsymbol{k}},\hat{\boldsymbol{k}}]\right)\Bigg\}$$

with

$$\Lambda^{(1)} = \frac{< M >}{J} \qquad \text{and} \qquad \Lambda^{(2)} = 1 - \frac{3 < M^2 >}{J(J+1)}$$

and

$$T^{(2)}(\hat{\boldsymbol{n}}) : [\boldsymbol{a},\boldsymbol{b}] = \hat{\boldsymbol{n}}\cdot\boldsymbol{a}\hat{\boldsymbol{n}}\cdot\boldsymbol{b} - \frac{1}{3}\boldsymbol{a}\cdot\boldsymbol{b}$$

Spectral functions $f_i(E_e)$ now contain recoil corrections of first order in E_e, E_0, m_e^2/ME_e.

with

$$f_1(E_e) = (a^2 + \tilde{c}^2)\left[1 + \frac{\alpha}{2\pi}\delta_\alpha^{(1)}(E_e)\right]$$

$$-\frac{1}{3M}\left[2\tilde{c}(\tilde{c} + d \pm b)E_0 + (2\tilde{c}^2 + \tilde{c}d \pm 2\tilde{c}b)\frac{m_e^2}{E_e} - 2(3a^2 + 5\tilde{c}^2 \pm 2\tilde{c}b)E_e\right]$$

$$f_2(E_e) = (a^2 - \frac{1}{3}\tilde{c}^2)\left[1 + \frac{\alpha}{2\pi}(\delta_\alpha^{(1)}(E_e) + \frac{\alpha}{2\pi}\delta_\alpha^{(2)}(E_e))\right]$$

$$+\frac{2E_0}{3M}\tilde{c}(\tilde{c} + d \pm b) - \frac{4E_e}{3M}\tilde{c}(3\tilde{c} \pm b)$$

$$f_3(E_e) = -\frac{E_e}{M}(3a^2 - \tilde{c}^2)$$

$$f_4(E_e) = \frac{2}{3}\tilde{c}(\delta_{J,J'}\sqrt{\frac{3J}{J+1}}a \mp \frac{\gamma_{J,J'}}{J+1}\tilde{c})\left[1 + \frac{\alpha}{2\pi}(\delta_\alpha^{(1)}(E_e) + \delta_\alpha^{(2)}(E_e))\right]$$

$$+\frac{E_0}{3M}(\frac{\gamma_{J,J'}}{J+1}\tilde{c} - \delta_{J,J'}\sqrt{\frac{3J}{J+1}}a)(\tilde{c}+d\pm b) + \frac{E_e}{3M}(\delta_{J,J'}\sqrt{\frac{3J}{J+1}}a(7\tilde{c}+d+b) - \frac{\gamma_{J,J'}}{J+1}\tilde{c}(5\tilde{c}+d\pm 3b))$$

$$f_5(E_e) = -\frac{E_e}{3M}(\delta_{J,J'}\sqrt{\frac{3J}{J+1}}a - \frac{\gamma_{J,J'}}{J+1}\tilde{c})(5\tilde{c} - d \pm b)$$

$$f_6(E_e) = \frac{2}{3}\tilde{c}(\sqrt{3}a + \tilde{c})\left[1 + \frac{\alpha}{2\pi}\delta_\alpha^{(1)}(E_e)\right]$$

$$+\frac{E_e}{3M}\left(\delta_{J,J'}\sqrt{\frac{3J}{J+1}}a(5\tilde{c} - d \pm b) + \frac{\gamma_{J,J'}}{J+1}\tilde{c}(\tilde{c} + d \pm 3b)\right) - \frac{2E_0}{3M}\frac{\gamma_{J,J'}}{J+1}\tilde{c}(\tilde{c}+d+b)$$

$$-\frac{m_e^2}{3ME_e}((\delta_{J,J'}\sqrt{\frac{3J}{J+1}}a + \frac{\gamma_{J,J'}}{J+1}\tilde{c})(\tilde{c} + b) + \tilde{c}d)$$

$$f_7(E_e) = \frac{\delta_{J,J'}\sqrt{3}a + \frac{\gamma_{J,J'}}{J+1}\tilde{c}}{3M}\left[E_0(\tilde{c} + d \pm b) - E_e(7\tilde{c} + d \pm b)\right]$$

$$f_{10}(E) = -\theta_{J,J'}\frac{E_e}{2M}c(c + d \mp b)$$

$$f_{12}(E) = -\theta_{J,J'}\left[\frac{E_e}{2M}c(3c \pm b) - \frac{E_0}{2M}c(c + d \pm b)\right]$$

$$f_{14}(E) = -\theta_{J,J'}\frac{E_0 - E_e}{2M}c(c + d \pm b)$$

Here

$$\gamma_{J,J'} = \begin{cases} J+1 & J = J'+1 \\ 1 & J = J' \\ -J & J = J'-1 \end{cases}$$

$$\theta_{J,J'} = \begin{cases} -\frac{J+1}{2J-1} & J = J'+1 \\ 1 & J = J' \\ -\frac{J}{2J+3} & J = J'-1 \end{cases}$$

$$\tilde{G}_F = G_F\left(1 + \frac{\alpha}{4\pi} e_V\right) \quad \text{and} \quad \tilde{c} = c\left(1 + \frac{\alpha}{4\pi}(e_A - e_V)\right)$$

$$\delta^{(1)}(E_e) = 3\ln\left(\frac{m_N}{m_e}\right) + \frac{1}{2} + \frac{4}{\beta}L\left(\frac{2\beta}{1+\beta}\right) + \frac{2}{\beta}\tanh^{-1}\beta\left[1+\beta^2 - 2\tanh^{-1}\beta\right.$$

$$\left.+\frac{1}{12}\left(\frac{(E_0-E_e)^2}{E_e^2}\right)\right] + 4\left[\frac{1}{\beta}\tanh^{-1}\beta - 1\right]\left[\ln\left(\frac{2(E_0-E_e)}{m_e}\right) + \frac{1}{3}\left(\frac{E_0-E_e}{E_e}\right) - \frac{3}{2}\right]$$

$$\delta^{(2)}(E_e) = 2\frac{1-\beta^2}{\beta}\tanh^{-1}\beta + \left(\frac{E_0-E_e}{E_e}\right)\frac{4(1-\beta)^2}{3\beta^2}\left[\frac{1}{\beta}\tanh^{-1}\beta - 1\right]$$

$$+\left(\frac{E_0-E_e}{E_e}\right)^2\frac{1}{6\beta^2}\left[\frac{1-\beta^2}{\beta}\tanh^{-1}\beta - 1\right]$$

are model-independent bremsstrahlung corrections and $F_{\mp}(Z, E_e)$ is the Fermi function.

In standard model and impulse approximation

$$a = g_V(0)M_F + \ldots \qquad \text{and} \qquad c = g_A(0)M_{GT} + \ldots$$

$$b = A\left[(\kappa_p - \kappa_n)M_{GT} + G_V(0)M_L\right] + \ldots$$

$$d = Ag_A(0)M_{\sigma L} + \ldots$$

with

$$M_L =< \beta|| \sum_i \tau_i^{\pm}(\boldsymbol{r}_i \times \boldsymbol{p}_i)||\alpha >$$

$$M_{\sigma L} =< \beta|| \sum_i \tau_i^{\pm}(\boldsymbol{\sigma}_i \times (\boldsymbol{r}_i \times \boldsymbol{p}_i))||\alpha >$$

After 2000: EFT methods. For neutron decay Ando et al., Phys. Lett. **B595**, 250-259 (2004), define

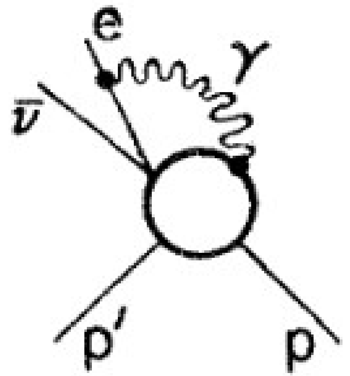
$$K_{\mu\nu} = \eta_{\mu\nu} + i v_\mu \frac{(\overleftarrow{\partial} - \overrightarrow{\partial})_\nu}{2m_N}$$

$$\mathrm{Amp}_{EFT}(s_f, s_i) = \frac{G_F}{\sqrt{2}} V_{ud} \ell^\mu \bar{N}_f \tau_+ \left\{ g_V \left(1 + \frac{\alpha}{4\pi} e_V\right) \left[K_{\mu\nu} v^\nu - i \frac{(\overleftarrow{\partial} - \overrightarrow{\partial})_\mu}{2m_N} \right] \right.$$
$$-2g_A \left(1 + \frac{\alpha}{4\pi} e_A\right) K_{\mu\nu} S^\nu - 2i \frac{g_M}{2m_N} \left[S_\mu, S \cdot (\overleftarrow{\partial} + \overrightarrow{\partial}) \right]$$
$$\left. -2i \frac{d_{II}}{2m_N} v_\mu S \cdot (\overleftarrow{\partial} + \overrightarrow{\partial}) \right\} N_i$$

which is *identical* to EP expression to $\mathcal{O}(q)$ provided we identify

$$a = g_V,\, c = \sqrt{3}g_A,\, b = \sqrt{3}g_M,\, d = \sqrt{3}g_{II}$$

Note that leading—$\mathcal{O}(Z\alpha)$—effects must also be included via one photon exchange diagrams



$$F_{\mp}(Z,E_e) = 1 \pm Z\alpha\frac{E_e}{p_e}$$

$$\delta f_1(E_e) = \mp\frac{8Z\alpha}{3\pi}\left\{a^2\left[4E_e(X+Y) + E_0X + \frac{m_e^e}{E_e}(X+2Y)\right]\right.$$

$$\left.+\tilde{c}^2\left[E_e(\frac{16}{3}X+4Y) - \frac{1}{3}E_0X + \frac{m_e^2}{E_e}(X+2Y)\right]\right\}$$

$$\delta f_2(E_e) = \mp\frac{8Z\alpha}{3\pi}\left\{a^2\left[4E_e(X+Y)+E_0X\right] - \tilde{c}^2\left[\frac{4}{3}E_e(2X+Y) - E_0X\right]\right\}$$

$$\delta f_4(E_e) = \mp\frac{8Z\alpha}{3\pi}\left[\delta_{J,J'}\sqrt{\frac{J}{J+1}}2ac \mp \frac{\gamma_{J,J'}}{J+1}\tilde{c}^2\right]E_e(5X+4Y)$$

$$\delta f_6(E_e) = \mp\frac{8Z\alpha}{3\pi}\left\{\delta_{J,J'}\sqrt{\frac{J}{J+1}}2ac\left[4E_e(X+Y) + E_0X + \frac{m_e^2}{E_e(X+2Y)}\right]\right.$$

$$\left.\pm\frac{\gamma_{J,J'}}{J+1}\tilde{c}^2\left[E_e(6X+4Y) - E_0X + \frac{m_e^2}{E_e}(X+2Y)\right]\right\}$$

$$\delta f_7(E_e) = \mp \frac{8Z\alpha}{3\pi}\left[\delta_{J,J'}\sqrt{\frac{J}{J+1}}2ac \pm \frac{\gamma_{J,J'}}{J+1}\tilde{c}^2\right](E_0 - E_e)X$$

$$\delta f_{12}(E_e) = \mp \frac{8Z\alpha}{3\pi}\theta_{J,J'}\tilde{c}^2 E_e(5X + 4Y)$$

$$\delta f_{14}(E_e) = \mp \frac{8Z\alpha}{3\pi}\theta_{J,J'}\tilde{c}^2(E_0 - E_e)X$$

where

$$X = \int_0^\infty dk F_{wk}(k^2)F'_{ch}(k^2) \qquad \text{and} \qquad Y = \int_0^\infty F'_{wk}(k^2)F_{ch}(k^2)$$

Note that if we write

$$F_{wk}(k^2) = \frac{1}{1 - a^2k^2} \qquad \text{and} \qquad F_{ch}(k^2) = \frac{1}{1 - b^2k^2}$$

then

$$X = \frac{\pi}{4}\frac{b^2(2a+b)}{(a+b)^2} \qquad \text{and} \qquad Y = \frac{\pi}{4}\frac{a^2(a+2b)}{(a+b)^2}$$

Hence three sources of recoil terms at $\mathcal{O}(q)$—

i) kinematic—$\mathcal{O}(1) \times \frac{q}{M}$

ii) dynamic—$\mathcal{O}(A) \times \frac{q}{M}$

iii) Coulomb—$\mathcal{O}(Z\alpha MR) \times \frac{q}{M}$

All the physics is in the terms of type ii)—dynamic terms. Strictures are:

a) For “weak magnetism” term b CVC requires value given in terms of difference of magnetic moments if between isotopic analog states

$$b = AM_F(\mu_\beta - \mu_\alpha)$$

while for mirror decays $b^+ = b^-$ is given in terms of corresponding M1 width of analogous electromagnetic decay.

b) For "weak electricity" term d G-parity considerations require vanishing if between isotopic analog states while for mirror decays it is required that $d^+ = d^-$. Violation of either condition indicates the presence of (BSM) "second class currents."

Recoil Experiments

First was Gell-Mann suggestion—slope of shape factor in ${}^{12}B \to {}^{12}C + e^- + \bar{\nu}_e$ and ${}^{12}N \to {}^{12}C + e^+ + \nu_e$

$$S(E) \approx 1 + \frac{2}{3M}\left(5 \pm 2\frac{b}{c}\right)E_e \quad \Rightarrow \quad \frac{dN}{dE} = \frac{4}{3M_n}\frac{b}{Ac} \quad \propto \; 0.5\% \; \text{MeV}^{-1}$$

(for an allowed GT transition and neglecting terms $\propto 1/M^2$ and $\propto m_e^2/E$)

For $A = 12$ predictions are

$$\frac{dS^{CVC}}{dE} = \begin{cases} 0.42\%/\mathrm{MeV} & {}^{12}\mathrm{B} \\ -0.52\%/\mathrm{Mev} & {}^{12}\mathrm{N} \end{cases}$$

Lee, Mo, and Wu, Phys. Rev. Lett. **10**, 253 (1963) quote measured values

$$\frac{dS^{exp}}{dE} = \begin{cases} 0.55 \pm 0.10\%/\mathrm{MeV} & {}^{12}\mathrm{B} \\ -0.52 \pm 0.06\%/\mathrm{Mev} & {}^{12}\mathrm{N} \end{cases}$$

in good agreement with CVC, but see Calaprice and Holstein, Nucl. Phys. A273, 301 (1976).

~1970, Sir Denys Wilkinson looked at ft values of mirror decays. Note that weak magnetism b cannot contribute since no parity violating signal in total decay rate. However, weak electricity term d *does*, so writing

$$d = d_I + d_{II}$$

where d_I arises from first class currents while d_{II} arises from second class currents, we find

$$\frac{ft^+}{ft^-} \simeq \left(\frac{c^{-2} - \frac{2}{3}\frac{E_0^-}{M}c^- d^-}{c^{+2} - \frac{2}{3}\frac{E_0^+}{M}c^+ d^+} \right)$$

$$\simeq 1 - \frac{2}{3M}\frac{d_I}{c}(E_0^- - E_0^+) + \frac{2}{3M}\frac{d_{II}}{c}(E_0^- + E_0^+)$$

Possible $d_{II} \neq 0$ suggested, but not clear because of difficult to calculate electromagnetic effects, which lead to $c^+ \neq c^-$, $d_I^+ \neq d_I^-$, $E_0^+ \neq E_0^-$. Hence use correlation experiments.

Calaprice et al. measured slope of $\boldsymbol{J} \cdot \boldsymbol{p}_e$ correlation A in analog $J^P = \frac{1}{2}^+ - \frac{1}{2}^+$ decay ${}^{19}\mathrm{Ne} \rightarrow {}^{19}F + e^+ + \nu_e$ for which

$$\frac{dA}{dE_e} = -\frac{1}{3M}\left[\frac{\sqrt{3}a(b + d_{II}) - c(5b - d_{II})}{\sqrt{3}ac - c^2} - \frac{4bc}{a^2 + c^2}\right]$$

Using

$$a(0) = 1, \quad b(0) = -148.60(3), \quad c(0) = -1.604(3)$$

found

$$\frac{d_{II}}{Ac} = -8.6 \pm 3.3 \qquad \text{vs.} \qquad \frac{b}{Ac} = 4.88$$

but plagued by rescattering effects.

Similar result found in A=12 system by what Telegdi called the "Axis Collaboration" (Sugimoto, Tanihata, and Göring) who measured slope of the $\boldsymbol{J} \cdot \boldsymbol{p}_e$ correlation. Using value of b from M1 decay width of analog ^{12}C state, experiment found

$$\left(\frac{dA}{dE_e}\right)^{-} - \left(\frac{dA}{dE_e}\right)^{+} = \begin{cases} 0.52 \pm 0.09\%/\mathrm{MeV} & \mathrm{exp} \\ 0.27\%/\mathrm{MeV} & \mathrm{CVC} \end{cases}$$

suggesting $\frac{d_{II}}{Ac} \sim -20$ but again experimental problems.

Solution to look at correlations, such as those involving alignment, which are purely recoil. Achieved by Minamisono et al. at Osaka, who looked at delayed α emission in $A = 8$ decays of ^{8}Li and ^{8}B and delayed γ emission in $A = 20$ decays of ^{20}Na and ^{20}F. (Disadvantage is that these are spin-2 nuclei so additional form factors required in analysis.)

Results for $A = 20$ are

$$\frac{b}{Ac} = 8.58 \pm 0.28$$

from M1 width of ^{20}Ne analog state and

$$\left(\frac{b - d_{II}}{Ac}\right)^{exp} = 8.41 \pm 0.31 \pm 0.24$$

or

$$\frac{d_{II}}{Ac} = 0.18 \pm 0.42 \pm 0.24$$

Note also that

$$\frac{d_I}{Ac} = 8.00 \pm 0.64 \pm 0.35 \pm 0.05$$

while for $A = 8$

$$\frac{b}{Ac} = 7.5 \pm 0.2 \pm 0.03$$

from M1 width of ^{8}Be analog state found

$$\frac{d_{II}}{Ac} = -0.28 \pm 0.46 \pm 0.19$$

Note that

$$\frac{d_I}{Ac} = 5.5 \pm 1.7$$

Osaka group gave up on use of polarization in $A = 12$ and used only alignment. Using

$$\frac{b}{Ac} = 4.04 \pm 0.2 \pm 0.03$$

from M1 width of ^{12}C analog state found

$$\frac{d_{II}}{Ac} = -0.15 \pm 0.12 \pm 0.05$$

Note that

$$\frac{d_I}{Ac} = 4.96 \pm 0.12 \pm 0.05$$

Conclude that new challenges but also opportunities are opened by use of recoil.