Renormalization of Lattice Operators I

Tanmoy Bhattacharya

Los Alamos National Laboratory

Santa Fe Institute

Work done in collaboration with
Vincenzo Cirigliano, Rajan Gupta, Emanuele Mereghetti, and Boram Yoon

Jan 23, 2015

Introduction
Lattice QCD

Lattice is a nonperturbative formulation of QCD.

Lattice uses a hard regulator:

$$\bar{\psi}\not{p}\psi \to \bar{\psi}\not{W}(p)\psi\,,$$

where $W(p)$ is a periodic function:

$$W(p) = W(p + 2\pi a^{-1})\,.$$

Hard regulators introduce a scale and allow mixing with lower dimensional operators.

Hard regulators are unambiguous: no renormalon problem.

Ultraviolet divergence regulated by the periodicity:

$$\int_{-\infty}^{\infty} dp = \sum_{m=-\infty}^{\infty} \int_{\pi(m-1)/a}^{\pi(m+1)/a} dp \to \int_{-\pi/a}^{\pi/a} dp$$

Infrared controlled by calculating in a finite universe.

$$\int dp f(p) \to \sum_n (\frac{2\pi}{L}) f(\frac{2\pi n}{L} + p_0)$$

Real world reached by

$$\lim_{\substack{L\to\infty \\ a\to 0}} .$$

Current calculations $a \sim 0.05$–$0.15\,\mathrm{fm}$ and $L \sim 3$–$5\,\mathrm{fm}$.

Introduction
Euclidean Space

Lattice can calculate equal time vacuum matrix elements:
To extract $\langle n|O|n\rangle$, one starts with

$$\begin{aligned}
&\mathrm{Tr}\, e^{-\beta H}\hat{n}e^{-HT_f}Oe^{-HT_i}\hat{n}^\dagger \\
&\quad = \quad e^{-\beta E_s}\langle s|\hat{n}e^{-HT_f}Oe^{-HT_i}\hat{n}^\dagger|s\rangle \\
&\quad \xrightarrow[\beta\to\infty]{} \quad \langle\Omega|\hat{n}|n_f\rangle e^{-M_fT_f}\langle n_j|O|n_i\rangle e^{-M_iT_i}\langle n_i|\hat{n}^\dagger|\Omega\rangle \\
&\quad \xrightarrow[T_i,T_f\to\infty]{} \quad \langle n|O|n\rangle e^{-M_0(T_i+T_f)}
\end{aligned}$$

Vacuum and states chosen by the theory: can only calculate 'physical' matrix elements.

nEDM
Lattice Basics

We can extract nEDM in two ways.

- As the difference of the energies of spin-aligned and anti-aligned neutron states:

$$d_n = \frac{1}{2}\left(M_{n\downarrow} - M_{n\uparrow}\right)\Big|_{E=E\uparrow}$$

- By extracting the CP violating form factor of the electromagnetic current.

$$\begin{aligned} \langle n|J_\mu^{\mathrm{EM}}|n\rangle &\sim \frac{F_3(q^2)}{2M_n}\bar{n}\, q_\nu \sigma^{\mu\nu}\gamma_5\, n \\ d_n &= \lim_{q^2\to 0}\frac{F_3(q^2)}{2M_n} \end{aligned}$$

Difficult to perform simulations with complex ~~CP~~ action

Expand and calculate correlators of the ~~CP~~ operator:

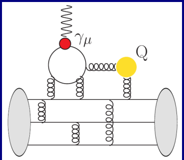
$$\begin{aligned}
\langle\, C^{\cancel{\mathrm{CP}}}(x,y,\ldots)\,\rangle_{\mathrm{CP}+\cancel{\mathrm{CP}}} &= \int [\mathcal{D}\mathcal{A}] \exp\left[-\int d^4x (\mathcal{L}^{\mathrm{CP}} + \mathcal{L}^{\cancel{\mathrm{CP}}})\right] \\
&\qquad \times C^{\cancel{\mathrm{CP}}}(x,y,\ldots) \\
&\approx \int [\mathcal{D}\mathcal{A}] \exp\left[-\int d^4x \mathcal{L}^{\mathrm{CP}}\right] \\
&\qquad \times \left(1 - \int d^4x \mathcal{L}^{\cancel{\mathrm{CP}}}\right) C^{\cancel{\mathrm{CP}}}(x,y,\ldots) \\
&= \langle\, C^{\cancel{\mathrm{CP}}}(x,y,\ldots)\, \mathcal{L}^{\cancel{\mathrm{CP}}}(p_\mu = 0)\,\rangle_{\mathrm{CP}}
\end{aligned}$$

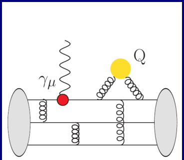
nEDM
Topological charge

To find the contribution of $\bar{\Theta}$, we note that $\int d^4x G\tilde{G} = Q$, the topological charge. So, we need the correlation between the electric current and the topological charge.

$$\left\langle n \left| (\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d) Q \right| n \right\rangle =$$

$$\frac{1}{2}\left\langle n \left| (\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) Q \right| n \right\rangle + \frac{1}{6}\left\langle n \left| (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) Q \right| n \right\rangle$$



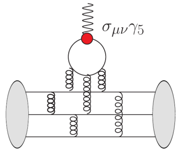


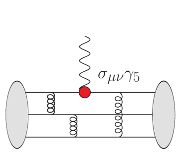
nEDM

Quark Electric Dipole Moment

Since the quark electric dipole moment directly couples to the electric field, we just need to calculate its matrix elements in the neutron state.

$$\langle n \left| d_u^\gamma \, \bar{u}\sigma^{\mu\nu} u + d_d^\gamma \, \bar{d}\sigma^{\mu\nu} d \right| \rangle \quad = \\ \frac{d_u^\gamma + d_d^\gamma}{2} \langle n \left| \bar{u}\sigma^{\mu\nu} u + \bar{d}\sigma^{\mu\nu} d \right| n \rangle + \frac{d_u^\gamma - d_d^\gamma}{2} \langle n \left| \bar{u}\sigma^{\mu\nu} u - \bar{d}\sigma^{\mu\nu} d \right| n \rangle$$

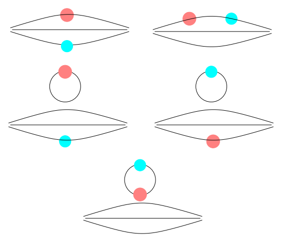




nEDM
Quark Chromoelectric Moment

nEDM from quark chromoelectric moment is a four-point function:

$$\left\langle n \left| \left(\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d\right) \int d^4x \, \left(d_u^G \, \bar{u}\sigma^{\nu\kappa}u + d_d^G \, \bar{d}\sigma^{\nu\kappa}d\right) \tilde{G}_{\nu\kappa} \right| n \right\rangle$$



Four-point functions can be calculated using noise sources. No experience yet with these.

Alternatively, we can simplify using Feynman-Hellmann Theorem:

$$\left\langle n \left| J_\mu \int d^4x (d_u^G\, \bar{u}\sigma^{\nu\kappa}u + d_d^G\, \bar{d}\sigma^{\nu\kappa}d)\, \tilde{G}_{\nu\kappa} \right| n \right\rangle = \frac{\partial}{\partial A_\mu} \left\langle n \left| \int d^4x (d_u^G\, \bar{u}\sigma^{\nu\kappa}u + d_d^G\, \bar{d}\sigma^{\nu\kappa}d)\, \tilde{G}_{\nu\kappa} \right| n \right\rangle_E$$

where the subscript E refers to the correlator calculated in the presence of a background electric field.
Needs dynamical configurations with electric fields. Can use reweighting.

nEDM
State of the Art

Neutron electric dipole moment from

- Topological charge:
  - Limits exist from lattie calculations
- Quark Electric Dipole Moment:
  - Same as the tensor charge of the nucleon
  - Preliminary results available
- Quark Chromoelectric Dipole Moment:
  - not yet calculated

Mixing
Pattern of mixing

Renormalization of the lattice operators can be performed non-perturbatively.

- Topological charge is well studied and understood.
- Electric current and Quark Elecric Dipole moment operators are quark bilinears: well understood renormalization procedure.
- Quark Chromoelectric Moment operator mixes with Quark Elecric Dipole moment: need to disentangle.
- Quark Chromoelectric Moment operator has divergent mixing with lower dimensional operators: need high precision.

Also need to calculate the influence of Chromoelectric moment of the quark on the PQ potential for Θ.

Mixing
Electric dipole moment

The operator is $\bar{\psi}\sigma^{\mu\nu}\tilde{F}_{\mu\nu}\psi$.
It is a CP-violating quark-quark-photon vertex.

- At lowest order in electroweak perturbation theory, does not mix with any lower-or-same-dimension operators.
- At one loop in electroweak,
  - chirally unsuppressed power-divergent mixing with $\bar{\psi}\gamma_5\psi$,
  - log-divergent mixing with $\bar{\psi}\sigma^{\mu\nu}\tilde{G}_{\mu\nu}\psi$, and
  - doubly chirally suppressed log-divergent mixing with $\bar{\psi}\gamma_5\psi$
- At two loops (mixed strong and electroweak), chirally suppressed, power-divergent mixing with $G\tilde{G}$.

Other mixings vanish onshell at zero four momentum, but not necessarily at zero three momentum.

If we handle electroweak perturbatively on the lattice, we can work at lowest order and avoid this mixing. In this case, all we need is the tensor charge.

Depending on accuracy needed, continuum running from high scale to hadronic scales need to account for mixing. $\overline{\mathrm{MS}}$ does not see power divergence.

Mixing
Vacuum Alignment and Phase Choice

- CP and chiral symmetry do not commute. Outer automorphism: $\mathrm{CP}_\chi \equiv \chi^{-1}\mathrm{CP}\chi$ also a CP.
  - $\psi_L^{CP} = i\gamma_4 C\bar{\psi}_L^T$ and $\psi_R^{CP} = i\gamma_4 C\bar{\psi}_R^T$.
  - $\psi_L^\chi = e^{i\chi}\psi_L$ and $\psi_R^\chi = e^{-i\chi}\psi_R$
  - $\psi_L^{CP_\chi} = e^{-2i\chi} i\gamma_4 C\bar{\psi}_L^T$ and $\psi_R^{CP_\chi} = e^{+2i\chi} i\gamma_4 C\bar{\psi}_R^T$
- In chirally symmetric theory, vacuum degenerate: spontaneously breaks all but one CP_χ.
- Addition of explicit chiral symmetry breaking breaks degeneracy of vacuum.

- Need degenerate perturbation theory unless perturbing around the right vacuum.
- Phase choice for fermions allows the preserved CP_χ to be the 'standard' CP.
- This phase choice is the one that gives

$$\langle\Omega|\mathcal{L}^{\not{C}P}|\pi\rangle = 0.$$

where $\mathcal{L}^{\not{C}P}$ is identified with the standard definition of CP, and $|\Omega\rangle$ is the true vacuum.

Consider the chiral and CP violating parts of the action

$$\mathcal{L} \supset d_i^\alpha O_i^\alpha$$

where i is flavor and α is operator index.
Consider only one chiral symmetric CP violating term: $\Theta G\tilde{G}$

Convert to polar basis

$$d_i \equiv |d_i| e^{i\phi_i} \equiv \frac{\sum_\alpha d_i^\alpha \langle \Omega | \mathcal{I}m\, O_i^\alpha | \pi \rangle}{\sum_\alpha \langle \Omega | \mathcal{I}m\, O_i^\alpha | \pi \rangle}$$

Then CP violation is proportional to:

$$\bar{d}\bar{\Theta}\, \mathcal{R}e\, \frac{d_i^\alpha}{d_i} - |d_i|\, \mathcal{I}m\, \frac{d_i^\alpha}{d_i}$$

with

$$\frac{1}{\bar{d}} \equiv \sum_i \frac{1}{d_i} \qquad \bar{\Theta} = \Theta - \sum_i \phi_i$$

CP violation depends on $\bar{\Theta}$ and on a *mismatch* of phases between d_i^α and d_i.

This is because the total chiral violation d_i is chosing the vacuum, and relative phase of d_i^α with respect to this gives the CP violation.

Only when

$$\langle\Omega|m\bar{\psi}\gamma_5\psi|\pi\rangle \gg \langle\Omega|d_i^G\bar{\psi}\gamma_5\sigma\cdot G\psi|\pi\rangle$$

we can forget about this complication and treat $\bar{\psi}\gamma_5\sigma\cdot G\psi$ as the CP violating chromoelectric dipole moment operator.

Calculation needs nonzero mass (or hold vacuum fixed).

Mixing

Lower dimensional operator

Most divergent mixing with $\frac{\alpha_s}{a^2}\bar{\psi}\gamma_5\psi$.

nEDM due to this same as due to $\frac{\alpha_s}{ma^2}G \cdot \tilde{G}$.

Current estimates of nEDM due to

- $\text{CEDM}^{\overline{\text{MS}}} \Rightarrow O(1)$
- $\frac{\alpha_s}{ma^2}\Theta G \cdot \tilde{G} \Rightarrow \frac{O(0.1)}{5\text{MeV}a^2} O(10^{-3})$e-fm $= O(1)$
  at $a \approx 0.1\text{fm}$.

Expect O(1–10) cancellation.

Renormalization
Perturbation Theory

Lattice perturbation theory can be used.

- Extra vertices: multi gauge-boson vertices.
  - Periodic functions are infinite power series.
  - Wilson lines are exponentials of gauge fields.
- Bubbles (self-loops) give large contribution.
  - Explicit scale allows bubbles to give constant non-zero contribution.
- Can choose combination of quantities that cancel ”bubble contribution”.
- Perturbation theory reasonably well behaved for these quantities.

Renormalization

Ward identity methods

Lattice QCD preserves or restores all symmetries in the continuum limit.

Ward identities lead to relations that can be tested on the lattice:

$$\delta_A O = \delta O \Rightarrow \langle \left(\int d^4x (\partial_\mu A_\mu - 2mP) O \right) O' \rangle = \langle \delta O O' \rangle \,.$$

Can be used to calculate Z_A, Z_V, Z_S/Z_P etc.

Cannot renormalize operators with anomalous dimensions.

Renormalization

Gradient flow method

Appropriately smeared operators are automatically renormalized.

Define the smearing equation

$$\frac{d}{dt}A_\mu(t) = D_\nu(t)G_{\nu\mu}(t) \qquad A_\mu(0) = A_\mu$$
$$\frac{d}{dt}\psi(t) = D_\mu(t)D_\mu(t)\psi(t) \qquad \psi(0) = \psi$$

Any $\lim_{t\to 0} O(A_\mu(t), \psi(t))$ is automatically renormalized at the scale of $\mu = 1/\sqrt{8t}$ with only fermion wavefunction renormalization.

To avoid cutoff effects, one needs $\mu a \ll 1$.

Renormalization
Position space methods

Match $\langle O(x) O(0) \rangle$ at fixed x.

Need $x \gg a$ to avoid cutoff effects.

Need $x \Lambda_{\mathrm{QCD}} \ll 1$ to be perturbative.

Needs higher loop calculations.

Renormalization

Momentum space methods

Match quark/gluon matrix elements $\langle q|O(p)|q\rangle$.

Needs gauge fixing to define external states.

- Can mix with gauge variant operators.
- BRST symmetry restricts these.
- Does not contribute to physical matrix elements.

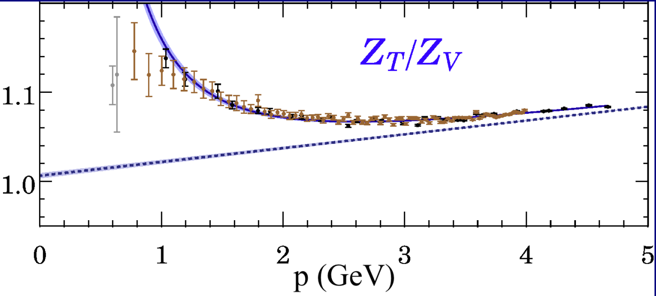
Involve contact terms

- Equation-of-motion operators do not vanish.

Needs $pa \ll 1$ to avoid cutoff effects.

Needs $p \gg \Lambda_{\mathrm{QCD}}$ to be perturbative.

Successfully carried out for dim 3 quark bilinears.


Z_T/Z_V
1.1
1.0
0
1
2
3
4
5
p (GeV)

Conclusions
Needed Calculations

Preliminary calculations needed before one can estimate errors and resource requirements.

For preliminary calculations

- Use previously generated lattices
- Study
  - Statistical signal
  - Chiral behavior
  - Dependence on lattice spacing
  - Excited state contamination

Conclusions
Outlook

Currently Quark Electric Dipole Moment ME has about 10% precision.

The calculation of chromoEDM ME need more study.

Divergent mixing leads to higher precision requirement

Remaining systematic errors not expected to be major.

- nEDM not overly sensitive to neglected EM and isospin-breaking
- Modern calculations include dynamical charm